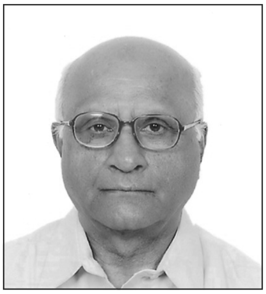
Hydraulic Machines

About The Author



K Subramanya is a retired Professor of Civil Engineering at the Indian Institute of Technology, Kanpur. He obtained his bachelor's degree in Civil Engineering from Mysore University and a master's degree from the University of Madras. Further, he obtained another master's degree and PhD degree from the University of Alberta, Edmonton, Canada. He has taught at IIT Kanpur for over 30 years and has extensive teaching and research experience in the area of Fluid Mechanics, Hydrology and Hydraulic Engineering. During his tenure at IIT Kanpur, Prof. Subramanya worked as Visiting Faculty at the Asian Institute of Technology, Bangkok, and at Concordia University, Montreal, Canada, for a short while.

He has authored several successful books for McGraw Hill Education (India). Besides the current book, his other books include *Flow in Open Channels* (3rd Ed., TMH, 2009); *Engineering Hydrology* (3rd Ed., TMH, 2008); and *Fluid Mechanics and Hydraulic Machines–Problems and Solutions* (TMH, 2011).

Dr Subramanya has published over eighty technical papers in national and international journals and conferences. He is a Fellow of the Institution of Engineers (India), Fellow of the Indian Society for Hydraulics, Member of Indian Society for Technical Education and Member, Indian Water Resources Association.

The Indian Society for Hydraulics has conferred the *Prof. R.J. Garde Lifetime Achievement Award* to Prof. Subramanya in 2012 for his contribution in education and research related to Fluid Mechanics, Hydraulic Engineering and Water Resources Engineering.

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Dedicated
to
My Wife, Prabha

Contents

Preface

Overview

Hydroelectric power plants produce roughly 17% of all the electrical energy produced in the world and the hydraulic turbines form the heart of the systems that produce such energy. Pumps consume about 25% of all the electrical energy produced in the world. Further, pumps being an important component of equipment in the manufacturing industry, the pump industry itself is commercially very important and vast in all parts of the world. In view of such great importance, the principles of hydraulic machines form one of the compulsory subjects prescribed in the syllabus for degree programmes in mechanical and civil engineering disciplines. Many other engineering disciplines also include this subject in their curriculum. While some degree programmes in the country have a full course on hydraulic machines, in many technical universities and autonomous engineering colleges, a first course in hydraulic machines is taught as part of a course typically called *Fluid Mechanics and Hydraulic Machines.*

Objective of this Book

The subject matter of hydraulic machines covered in the above-mentioned engineering courses is quite extensive and due to many constraints such as shortage of time, lack of adequate resource material, etc., students face considerable difficulty in studying and understanding the subject of hydraulic machines. The need for a good, comprehensive book that not only covers the requisite course content adequately but also gives good guidance through illustrative examples to solve various types of problems in the subject has been expressed by many teachers and students in my various interactions with them.

This book is my response to such a need. Primarily, the book is meant as a textbook for a first course in Hydraulic Machines at the undergraduate level. An attempt is made in the book to indicate present trends in analysis and practice in various classes of hydraulic machines. Further, the book contains sufficient material beyond the basic need of a standard prescribed syllabus to make it useful as a source of reference to senior students as well as practicing engineers. Working engineers and organizations functioning in the field of hydropower would find this book a useful source of current information. The contents of the book are designed to be of use in self-study also. Further, candidates taking competitive examinations like Engineering Services Examinations of UPSC will find this book very useful in their preparations.

Salient Features

- Complete and up-to-date coverage on Hydraulic Machines
- Designed to be of use in self-study
- Dedicated chapter on Elements of Hydroelectric Power Engineering: direct application of hydraulic machines

- Dimensional Analysis and Similitude added as an appendix
- Dedicated discussion on Reaction Turbines, Impulse Turbines, Centrifugal Pumps and Reciprocating Pumps
- Miscellaneous Hydraulic Devices like hydraulic press, hydraulic accumulators, hydraulic intensifiers, torque converters, etc., are also covered
- Pedagogy including Solved Examples, Practice Problems, Review Questions and MCQs are graded as Easy(*), Medium (**) and Difficult (***)
  - Figures: 293
  - Solved Examples: 190
  - Numerical Problems: 188
  - Review Questions: 142
  - Objective-type Questions: 223

Chapter Organisation

The subject matter of hydraulic machines is covered in eight chapters: *Chapters 1 through 4* cover the topic of hydro turbines; *Chapters 5 and 6* cover centrifugal and reciprocating pumps respectively; and *Chapters 7 and 8* deal with miscellaneous hydraulic devices and elements of hydroelectric engineering, respectively. The theme has been developed in a logical and coherent manner and covers substantially the prescribed syllabi of various Indian universities. Present-day (21st century) Indian students will find the book easy and simple to use. I have made deliberate attempts to make the book student friendly and SI units are used throughout the book. Internet-based references are provided at the end of each chapter to enable students to pursue further study of interested topics easily at their own pace and place. Basic knowledge of Fluid Mechanics is assumed and mathematics is kept to the level of essentials. If the book is used for a combined course in Fluid Mechanics and Hydraulic Machines, obviously the basic topics of fluid mechanics are to be taught first before taking up the topics of hydraulic machines covered in this book.

In sequencing the contents of the chapters, the study of reaction turbines is taken up first in Chapters 2 and 3, and is followed by the study of impulse turbines in Chapter 4. This is done in recognition of the fact that theories and basic equations are developed in a generalised manner while dealing with reaction turbines and the equations used in connection with impulse turbines happen to be a particular, simpler, case of these general equations.

Appendices A through D contain (i) a brief introduction to Dimensional Analysis and Similitude, (ii) a list of abbreviations used in the book, (iii) Some important Standards associated with turbines and pumps, and (iv) Answers to Objective Questions.

In each of the eight chapters, the theoretical derivations and technical descriptions are followed by a set of carefully chosen and graded *Illustrative Examples* covering all the important sub-areas of the chapter theme. A set of *Review Questions* and a set of *Problems* with answers are provided in each chapter to help students tone up their skills through practice. Further, a set of *Objective-type Questions* is included at the end of each chapter and answers to these are provided at the end of the book. This will help students in quick review of the chapter and also will be an aid in thorough understanding of the subject matter. Each of the illustrative examples, practice problems and objective questions, are graded in three levels (*Simple, Medium* and *Difficult*) and designated by the markings of *, ** and *** respectively. These markings are provided at the beginning of each item. It is believed that this feature will be of use to teachers in selecting problems for class work, assignments, quizzes and examinations.

Online Learning Centre

The online Learning Centre of this book can be accessed at http://www.mhhe.com/subramanya/hm1 and contains the following material:

For Instructors: Solution Manual and PowerPoint slides

For Students: Sample Question Papers and Web links for further reading

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The following reviewers of the typescript have provided valuable inputs to the contents of this edition. I would like to express my thanks to these reviewers and to all those who have directly or indirectly helped me in bringing out this book.

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Feedback

Comments and suggestions for further improvement of the book would be greatly appreciated. I can be contacted at the following e-mail address: ***subramanyak1@gmail.com***.

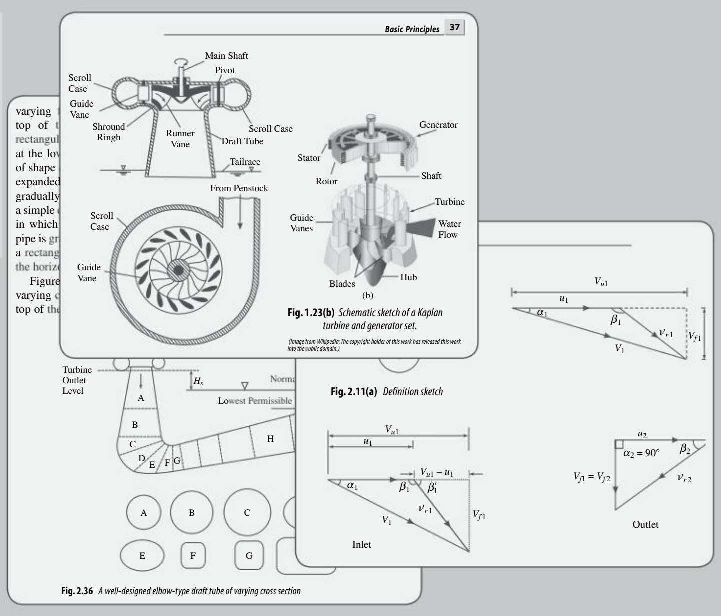
K Subramanya

Publisher's Note

Remember to write to us! We look forward to receiving your feedback, comments and ideas to enhance the quality of this book. You can reach us at tmh.mechfeedback@gmail.com. *Kindly mention the title and author's name as the subject. In case you spot piracy of this book, please do let us know.*

Guided Tour

More than 290 **illustrations and diagrams** are present to enhance the concepts.



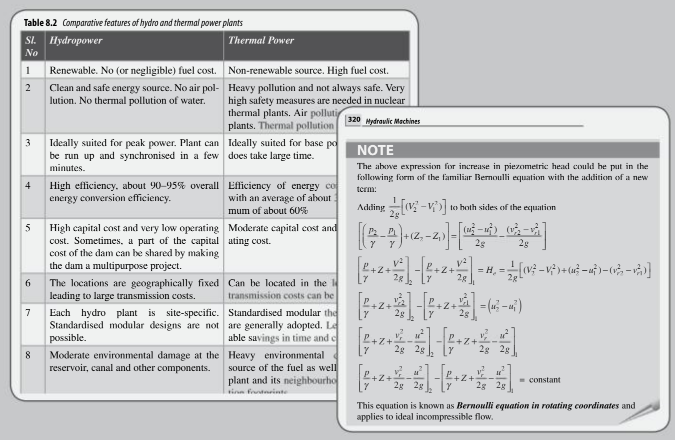


Table 8.2 *Comparative features of hydro and thermal power plants*

Sl. No	Hydropower	Thermal Power
1	Renewable. No (or negligible) fuel cost.	Non-renewable source. High fuel cost.
2	Clean and safe energy source. No air pollution. No thermal pollution of water.	Heavy pollution and not always safe. Very high safety measures are needed in nuclear thermal plants. Air polluti… plants. Thermal pollution…
3	Ideally suited for peak power. Plant can be run up and synchronised in a few minutes.	Ideally suited for base po… does take large time.
4	High efficiency, about 90–95% overall energy conversion efficiency.	Efficiency of energy co… with an average of about … mum of about 60%
5	High capital cost and very low operating cost. Sometimes, a part of the capital cost of the dam can be shared by making the dam a multipurpose project.	Moderate capital cost and… ating cost.
6	The locations are geographically fixed leading to large transmission costs.	Can be located in the l… transmission costs can be…
7	Each hydro plant is site-specific. Standardised modular designs are not possible.	Standardised modular the… are generally adopted. Le… able savings in time and c…
8	Moderate environmental damage at the reservoir, canal and other components.	Heavy environmental … source of the fuel as well… plant and its neighbourho…

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NOTE

The above expression for increase in piezometric head could be put in the following form of the familiar Bernoulli equation with the addition of a new term:

Adding $\frac{1}{2g}\left[(V_2^2 - V_1^2)\right]$ to both sides of the equation

$$\left[\left(\frac{p_2}{\gamma}-\frac{p_1}{\gamma}\right)+(Z_2-Z_1)\right]=\left[\frac{(u_2^2-u_1^2)}{2g}-\frac{(v_{r2}^2-v_{r1}^2)}{2g}\right]$$

$$\left[\frac{p}{\gamma}+Z+\frac{V^2}{2g}\right]_2-\left[\frac{p}{\gamma}+Z+\frac{V^2}{2g}\right]_1=H_e=\frac{1}{2g}\left[(V_2^2-V_1^2)+(u_2^2-u_1^2)-(v_{r2}^2-v_{r1}^2)\right]$$

$$\left[\frac{p}{\gamma}+Z+\frac{v_{r2}^2}{2g}\right]_2-\left[\frac{p}{\gamma}+Z+\frac{v_{r1}^2}{2g}\right]_1=\left(u_2^2-u_1^2\right)$$

$$\left[\frac{p}{\gamma}+Z+\frac{v_r^2}{2g}-\frac{u^2}{2g}\right]_2-\left[\frac{p}{\gamma}+Z+\frac{v_r^2}{2g}-\frac{u^2}{2g}\right]_1$$

$$\left[\frac{p}{\gamma}+Z+\frac{v_r^2}{2g}-\frac{u^2}{2g}\right]_2-\left[\frac{p}{\gamma}+Z+\frac{v_r^2}{2g}-\frac{u^2}{2g}\right]_1 = \text{constant}$$

This equation is known as ***Bernoulli equation in rotating coordinates*** and applies to ideal incompressible flow.

Useful **Notes and Tables** given at appropriate places to provide additional information

Most chapters have a set of **References** for additional reading.

References

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- ***http //bit.ly/rmZA10***

(ii) *Moving Plate:*

(a) When $u = 5$ m/s equivalent relati
to be considered. Relative velocit
Relative discharge $Q_r = Av_r = 0.0$
Normal force on the plate, $F_n =$
= 61.09 N
Since there is no friction, there is
plate.
Also, since $u = 5$ m/s, work done
$P = 61.09 \times 5 \times \cos 30° = 264.5$ V

(b) When the plate is moving in opp
Relative velocity $v_r = (V + u) = ($
Relative discharge $Q_r = Av_r = 0.0$
Normal force on the plate $F_n = \rho$
= 549.8 N
Since there is no friction, there is no tangential compo
plate.

*EXAMPLE 1.3

A two-dimensional free jet of water of thickness B and discha
the jet strikes a stationary, frictionless, plate at angle β to t
Calculate (a) the force on the plate, (b) the ratio of dischar
that move on the plate on either side of the impact zone, and (
when $\beta = 15°$, $B = 10$ cm and $q = 1.5$ m^3/s.

Solution

β is the angle made by the axis of the jet with the normal to t

(a) Consider a unit width of the jet. In the direction normal t
reaction of the plate on the control volume enclosing t
1.15).

**EXAMPLE 4.8

A double-jet Pelton wheel has specific speed of 26 and is required to deliver 10 MW of power. The turbine is supplied through a pipeline from a reservoir whose level is 400 m above the nozzles. Allowing 5% for friction loss in the pipe, calculate (a) speed in rpm, (b) diameter of the jets and (c) pitch diameter of the wheel.
[Assume K_u = 0.

Solution

Net available

Specific spee

***EXAMPLE 8.4

A run-of-river plant supplies power to a variable load as indicated below:

Time	0–4 hours	4–8 hours	8–12 hours	12–16 hours	16–20 hours	20–24 hours
Load in MW	2.1	3.2	7.4	8.2	6.2	4.1

(a) Draw the load curve and determine (i) average load, and (ii) daily load factor.
(b) If the net head is 8.0 m and overall efficiency is 82%, determine the average flow required and the pondage required to meet daily load fluctuation.

Solution

$$\text{Average load} = (2.1 + 3.2 + 7.4 + 8.2 + 6.2 + 4.1) = \frac{31.2}{6} = 5.2 \text{ MW}$$

$$\text{Daily load factor} = \frac{\text{Average load in a day}}{\text{Peak load in a day}} = \frac{5.2}{8.2} = 0.634 = 63.4\%$$

The load curve is plotted from given data and is shown in Fig. 8.12.

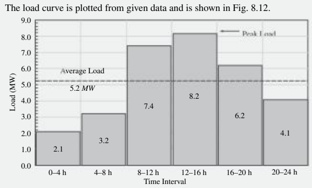


Fig. 8.12 *Load curve*

In each chapter, the theoretical derivations and technical descriptions are followed by a set of carefully chosen and **graded Illustrative Examples**

Review Questions to test the student's subjective grasp on the topics

Review Questions

5.1 What are manometric head and manometric efficiency of a centrifugal pump?
5.2 With the aid of neat sketches, describe three kinds of casings adopted in centrifugal pumps.
5.3 Explain different types of energy losses encountered in centrifugal pumps.
5.4 Explain different types of efficiencies used in connection with centrifugal pumps.
5.5 Derive Euler's equation for a centrifugal pump and explain the physical significance of each of the terms in the equation.
5.6 How are pumps classified? List the further subclassifications of centrifugal pumps.
5.7 Explain with appropriate sketches the components of a typical centrifugal pump.
5.8 Draw the inlet and outlet velocity diagram of a centrifugal pump with radial inlet velocity. Label the various velocities and angles involved.
5.9 Describe the need for priming of centrifu
5.10 Distinguish between Euler head and ma
5.11 Describe a multistage centrifugal pump
5.12 Explain why the head-discharge curve
that of an actual pump is non-linear.
5.13 Explain the head-discharge characteris
factor.
5.14 Discuss the main and operating charac
importance of constant efficiency curv
5.15 Describe the salient features of a mixe

2.23 What are the main features in classifying the runners of a Francis turbine as slow, medium and fast?

Problems

1. [All **Problems** and **Objective Questions** have been graded in three levels: – Simple, Medium and Difficult. The markings for these are: Simple = *, Medium = **, Difficult = ***]

Velocity Triangles

P2.1 *At what angle should the wicket gates of a Francis turbine be set to extract 8000 kW of power from a flow of 30 m^3/s when running at a speed of 200 rpm? The diameter of the runner at inlet is 3.0 m and the breadth of the openings at inlet is 0.9 m. The flow can be assumed to leave the runner radially and the blade angle is obtuse. [*Ans:* α_1 = 22.58°]

P2.2 *An inward-flow reaction turbine has inlet and outlet diameters of 1.2 m and 0.6 m respectively. The breadth at the inlet is 0.25 m and at the outlet, it is 0.35 m. At a speed of rotation of 250 rpm, the relative velocity at the entrance is 3.5 m/s and it is
entrance and its inclination
(c) the velocity of flow at

P2.3 *A reaction turbine works
and the inlet-blade angle
discharge at the outlet an
efficiency of the turbine.

Numerical problems for practice

Objective-type Questions

O6.1 *In positive-displacement pumps, the slip can sometimes be negative when the actual discharge is greater than the theoretical discharge. This happens in
(a) small suction pipes coupled with low delivery head
(b) small suction pipes coupled with medium delivery head
(c) long suction pipes coupled with low delivery head
(d) long suction pipes coupled with medium delivery head

O6.2 *In a reciprocating pump without air vessels, the friction head in the delivery pipe is maximum at crank angle θ =
(a) 0° (b) 90° (c) 60° (d) 180°

O6.3 *In a reciprocating pump without air vessels, the acceleration head in the suction pipe is maximum at the crank angle value of θ =
(a) 0° (b) 90° (c) 60° (d) 180°

O6.4 **In a single-acting reciprocating pump, without air vessels, the average velocity in delivery pipe is given by V_d =
(a) $\left(\frac{A}{A_d}\right)\omega r$ (b) $\left(\frac{A}{A_d}\right)\frac{\omega r}{\pi}$ (c) $\left(\frac{A}{A_d}\right)\frac{\pi\omega r}{2}$ (d) $\left(\frac{A}{A_d}\right)\frac{\omega r}{2\pi}$
where ω = Angular velocity of the crank, r = Radius of crank, A = Area of cylinder, A_d = Area of delivery pipe.

O6.5 **In a single reciprocating pump without air vessels, the ratio of the time-averaged friction head to the instantaneous maximum friction head in delivery pipe is given by $\frac{h_{fdav}}{h_{fdm}}$ =
(a) 1/2 (b) 1/3 (c) 2/3 (d) 4/3

Objective-type questions for quick revision and recapitulation of concepts

ONE

Basic Principles

1.1 INTRODUCTION

1.1.1 Definitions

Fluid-flow machines are very broadly classified as turbo-machines and positive displacement machines. A turbo-machine is a device that adds energy to a fluid or extracts energy from the fluid by virtue of a rotating system of blades. These machines require a rotating element called *rotor* and relative motion between the fluid and the rotor. If the machine adds energy it is called a *pump*; if it extracts energy it is called a *turbine*. The fluid to be pumped can be in incompressible mode throughout its passage in the machine or compressibility effects may come in to picture at different phases of its interaction in the machine. If the fluid is water, the pump device is labeled as *water pump*. For compressible fluids, the pump devices have nomenclatures such as fans, blowers and compressors. In positive displacement pumps, the fluid mass is physically displaced from one position to another owing to mechanical action of a boundary, for example a piston. There is a flow of fluid to fill the void created by this displacement and by repeated action, a continuous flow system is developed. A reciprocating pump is a typical example of a positive displacement pump.

This book deals with only incompressible fluid-flow machinery and the title *Hydraulic Machines* is intended to convey this attribute. The aspects dealing with compressibility are beyond the scope of this book.

1.1.2 Outline of Chapters

In this book, attention is focused only on the incompressible fluid flow. Water being the most widely used fluid for turbines, the chapters on turbines deal only with water turbines. Water pumps can reflect the behaviour of most incompressible flow pumps and hence the attention of this book is confined essentially to water as the working fluid. The role of viscosity is included or highlighted wherever necessary, to make the treatment applicable to liquids other than water. The book is aimed at an introductory treatment of hydraulic machines and thus is essentially an elementary treatment of a wide variety of common hydraulic machines at the first-course level. A basic knowledge of elementary fluid mechanics is a pre-requisite to follow this book. The mathematics is kept to a level of essential basics.

The water turbines that form the core of the waterpower development are dealt in detail in chapters 1, 2, 3 and 4. All the commonly used turbine types, viz., Francis, Kaplan and Pelton, are covered in sufficient detail. The coverage includes the basic theory, working ratios, operational characteristics, similarity relations, cavitation aspects and selection criteria for turbines. Separate basic equations are derived for turbines in chapters 1, 2, 3 and 4; and for pumps in chapters 5 and 6. The reaction turbines which form nearly 80% of all hydro turbines used worldwide are taken up first and described in sufficient detail in chapters 2 and 3. Chapter 4 is devoted to a full description of important aspects of impulse turbines, with major emphasis on Pelton turbine. Other types of turbines such as, Deriaz, Banki (Mitchell, Ossberger), Tube and Bulb turbines are dealt at an introductory level at appropriate locations.

Pumps form the theme of chapters 5 and 6. Chapter 5 is the largest chapter in the book and deals with a variety of aspects related to centrifugal pumps. In this chapter, besides basic flow equations of rotodynamic pumps, similarity laws, working ratios, operational characteristics of radial flow pumps are covered in detail. Mixed-flow and axial-flow pumps are dealt at introductory level. Chapter 6 describes positive displacement pumps with emphasis on reciprocating pumps. Chapter 7 covers the basic aspects of several miscellaneous hydraulic machines and devices that form important components of a hydraulic system.

The last chapter, chapter 8, is a brief introduction to the important subject of hydroelectric engineering. Having discussed the turbines in detail in chapters 1 through 4, it is necessary that the reader should know the basics about the end use of these turbines, viz., the hydropower plant. However, hydroelectric engineering is a vast interdisciplinary subject and excellent treatises are available which do full justice to the subject. Keeping this in mind, chapter 8 presents a brief note on the salient features of hydroelectric plant to compliment the earlier study of turbines discussed in chapters 2 through 4.

It is expected the reader is familiar with the basic aspects of *Dimensional Analysis* in an earlier exposure to basic course in Fluid Mechanics. However, for the benefit of those who are not exposed to this topic earlier, a brief introduction to dimensional analysis, along with a few illustrative examples, is given Appendix-A1 titled *Dimensional Analysis and Similitude*.

1.2 PRINCIPLES OF IMPINGEMENT OF JETS

Towards understanding the mechanism of transfer of power from water to the rotor in an impulse turbine, basic concepts of linear momentum equation are reviewed briefly in this section.

1.2.1 Control Volume

A control volume (*CV*) is a fixed volume in space of the fluid flow field. While the boundaries of control volume do not change, the mass and energy can cross its boundaries. In the application of linear momentum equation, it is necessary to have a control volume encompassing the fluid and suitably accounting for the presence of the boundaries. It is usual practice to draw the control volume for a given flow situation in such a way that:

1. Its boundaries are normal to the direction of flow at inlets and outlets
2. It is inside the flow boundary and has same alignment as the flow boundary
3. Wherever the magnitude of the boundary forces (due to pressure and shear stresses) are not known, their resultant is taken as a reaction force R (with components R_x, R_y and R_z along the coordinate directions X, Y and Z respectively) on the control volume. This reaction R is the force acting on the fluid in the control volume arising out of the reaction from the boundary. The force F of the fluid on the boundary will be equal to and opposite to the reaction force R.
4. The direction of R can be taken arbitrarily and the correct direction will emerge after the analysis through linear momentum equation.

As an example, Fig.1.1 (a) shows the control volume in the case of impingement of a free jet on a flat plate. In this, a jet of liquid of density amounting to a discharge Q and having a velocity V_1 impinges normally on a fixed plate. The control volume normally taken for the analysis is shown in the figure.

As another example, Fig. 1.1(b) shows a flow through a bend. The control volume that would normally be adopted, the pressures and velocities at the two ends and the reaction force R on the control volume and its components in X and Y directions, are shown in the figure.

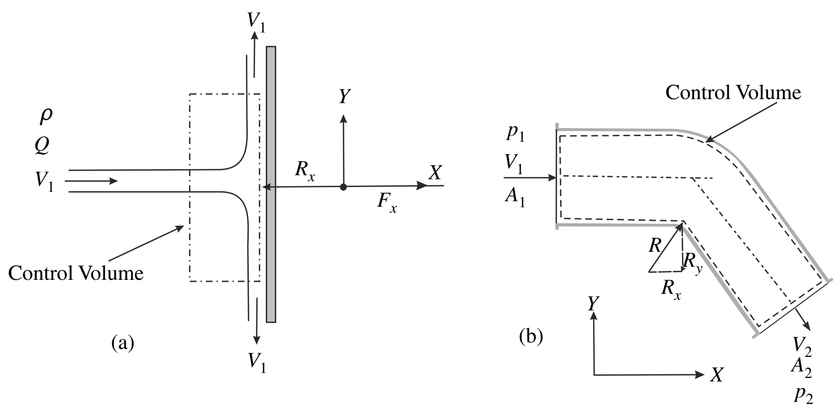


Fig. 1.1 *Examples of use of control volume*

1.2.2 Linear Momentum Equation

The linear momentum equation states:

The vector sum of all external forces acting on a control volume in a fluid flow equals the time rate of change of linear momentum vector of the fluid mass in the control volume.

The external forces are of two kinds, viz. (i) boundary forces, and (ii) body forces.

(i) Boundary forces consist of

- Force (F_p) due pressure intensities acting normal to a boundary, and
- Force (F_s) due to shear stresses acting tangential to a boundary.

(ii) Body forces (F_b) are those that depend upon the mass of the fluid in the control volume, for example the weight of fluid.

The linear momentum equation for a *general flow* can be written for any coordinate direction x as

$$\sum F_x = F_{px} + F_{sx} + F_{bx} = \frac{\partial}{\partial t}(M_x)_{cv} + M_{x\,out} - M_{x\,in} \tag{1.1}$$

where(a) M_x = Momentum flux in x-direction = $\rho Q V_x$.

In this, ρ = density of the fluid, Q = discharge and V = velocity of flow. Suffixes *out* represents momentum flux going out of the control volume and *in* represents the momentum flux coming into the control volume. Similarly, suffix x represents the x-component.

(b) F_{px}, F_{sx} and F_{bx} represent respectively the x-component of pressure force, shear force and body force acting on the control volume.

$\frac{\partial}{\partial t}(M_x)_{cv}$ = rate of change of x-momentum within the control volume. (This component is zero for a steady flow.)

Thus for steady fluid flow, the linear momentum equation in x-direction is

$$\begin{aligned}\sum F_x = F_{px} + F_{sx} + F_{bx} &= M_{x\,out} - M_{x\,in} \\ &= (\rho Q V_x)_{out} - (\rho Q V_x)_{in}\end{aligned} \tag{1.2}$$

Similar momentum equations are applicable to other coordinate directions y and z also.

Use of steady-state linear momentum equations is illustrated in the following jet impingement situations.

1.2.3 Impingement of Free Jets

In the following parts of this section, seven cases of impingement of a free jet of water on stationary and moving plates (or vanes) are discussed. This study of impingement of jets provides the basic background on the application of linear momentum equation, concepts of energy transfer, relative velocities and efficiency of energy transfer.

1. Case A: Jet Impingement on a Stationary Plate

Consider a circular jet of water striking a fixed flat plate inclined at angle θ to the axis of the jet, Fig. (1.2). The impinging jet is a free jet, that is, the pressure is atmospheric in any cross section of the jet and the ambient pressure is also atmospheric. The plate is assumed to be frictionless.

Consider a control volume as shown in Fig. 1.2. The velocity of the jet is V, its cross-sectional area = A and the discharge = Q. The pressure on the surfaces of the control volume is atmospheric. The jet strikes the plate and spreads radially (tangential to the plane of the plate) on the plate. Since there is no friction at the plate,

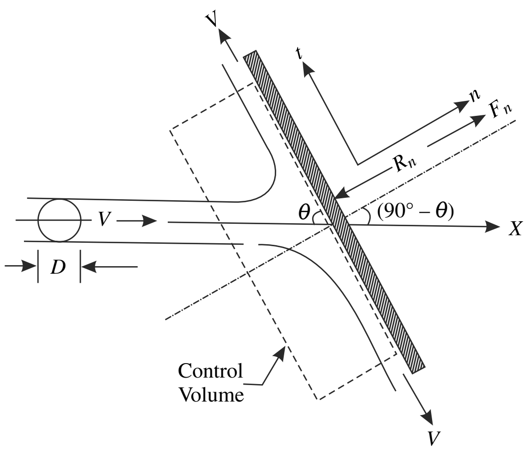


Fig. 1.2 *Jet impingement on an inclined plate*

the velocity of the flow at any tangential direction is constant at a value of V. Thus, in any tangential direction, there is no change in momentum flux. By linear momentum equation in any tangential direction, there is no reaction force on the control volume and hence there is no tangential force (shear force) on the plate.

Consider the normal direction; viz., n direction as shown in Fig. 1.2. Let R_n be the reaction force in the normal direction. The force on the plate in the normal direction will be F_n which is equal and opposite to R_n. While the velocity entering the control volume in the normal direction is $V \sin \theta$ the velocity leaving the control volume in n direction is zero. Thus, writing the linear momentum equation to the control volume in the normal direction:

$$\sum(\text{Forces in } n\text{-direction}) = (\text{Momentum flux in } n\text{-direction})_{out} - (\text{Momentum flux in } n\text{-direction})_{in}$$

$$0 - R_n = ((\rho Q) \times (0)) - (\rho Q V \sin \theta)$$

$$R_n = \rho Q V \sin \theta \tag{1.3}$$

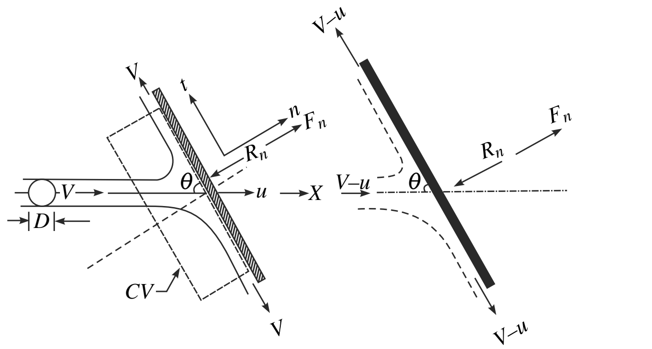
a) Thus, the normal force on the plate is $F_n = \rho Q V \sin \theta$ the positive n-direction, i.e., in the direction of the jet.
b) Since the tangential force on the control volume $F_t = 0$, the total force on the plate is

$$F = F_n = \rho Q V \sin \theta \tag{1.4}$$

c) The plate is stationary and as such, the force on the plate does no work.
d) When $\theta = 90°$, the case refers to the normal impingement of a free jet on a plate. In that case, the total force on the plate is $F = \rho Q V$ in the direction of the jet.

2. Case B: Jet Impingement on a Moving Plate

In this case, the plane of the plate is inclined at an angle θ to the jet and the plate is moving with a velocity u in the direction of the jet. Figure 1.3(a) is a definition sketch of the flow. Notice that in this flow situation, there is a relative motion between the jet of water and the plate. Relative to the plate, the jet has a velocity of $(V - u)$. Hence, to solve this problem, consider a velocity of magnitude $(-u)$, which is equal



(a) (b) Relative Stationary Plate

Fig. 1.3 *Jet impingement on a moving plate*

and opposite to the velocity of the plate, being superimposed on the flow system. The resulting flow situation, which can be called the *equivalent relative flow* over the plate, is shown in Fig. 1.3(b).

Here,

(a) the plate is relatively stationary, and

(b) the jet has an area A and relative velocity of $(V - u)$ and thus the relative discharge is

$$Q_r = A(V - u)$$

Now this relative flow situation is similar to Case A discussed earlier. There is no tangential force on the plate and the normal force on the plate F_n is given by

$$F_n = \rho Q_r\,(V - u) \sin\theta = \rho A\,(V - u)^2 \sin\theta \tag{1.5}$$

(c) The component of the velocity of the plate in the normal direction is $u_n = u \sin\theta$

The normal force $(= F_n)$ moves with a velocity u_n and hence the rate of work done on the plate = power transmitted to the plate =

$$P = F_n\, u \sin\theta = \rho A\,(V - u)^2\, u \sin^2\theta \tag{1.6}$$

Kinetic energy supplied by the jet per unit time = $\frac{1}{2}\rho(AV)V^2$

(d) Efficiency of power transmission = $\dfrac{\text{Power transmitted to the plate}}{\text{Kinetic energy supplied by the jet per unit time}}$

$$= \frac{\rho A(V-u)^2 u\ \sin^2\theta}{\frac{1}{2}\rho(AV)V^2} = \frac{2(V-u)^2 u \sin^2\theta}{V^3}$$

Observe that if θ is a variable, the efficiency is maximum for $\theta = 90°$, that is for normal impingement on the plate. Further, in this flow situation, the movement of the plate causes the distance between the nozzle and the plate to continuously increase with time. Hence, this is not a practically feasible flow situation.

3. Case C: Jet Impingement on a Set of Flat Plates Mounted on a Wheel

Consider a set of plates mounted radially on a wheel that is capable of rotating about its axis as in Fig. 1.4. A free jet of water strikes the plate and as the wheel moves, one plate or another intersects the jet continuously. If sufficient number of plates is properly assembled, these plates mounted on the rotating wheel can intercept the entire discharge from the jet. Let V be the velocity of the jet of cross sectional area A. Let u = peripheral velocity of the wheel. For a single plate, the flow situation is similar to the impingement of a jet on a moving plate as in Case-*B* discussed earlier. However, when all the plates on the wheel are considered, one or other vanes will intercept all the flow from the jet. Thus, the entire discharge issuing out of the jet is involved in the transfer of power to the wheel. While the relative velocity of plates is still $(V - u)$, the total discharge Q $(= AV)$ is involved in the momentum flux undergoing the change and not the relative discharge Q_r as in Case-*B*. The jet impinges normally on the plate ($\theta = 90°$) and the plate moves with a velocity u after the impact.

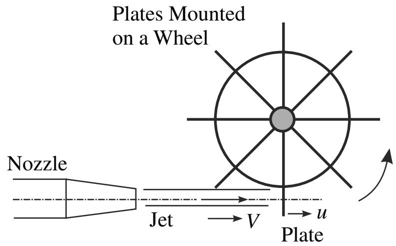


Fig. 1.4 *Jet impingement on plates mounted on a wheel*

The problem now reduces to the case of normal impingement of a free jet discharge of Q $(= AV)$ having a relative velocity $(V - u)$ on a relatively stationary plate. This is similar to Case - B with $\theta = 90°$.

Let $R_x = x$ – component of the reaction of the plate on the control volume.

$$R_x = [0 - \rho AV\,(V - u)] = -\,\rho AV\,(V - u)$$

The idealized force F_x is equal and opposite to R_x.

Thus, the idealized force on the plate in the x-direction $F_x = \rho AV\,(V - u)$

Assuming zero losses, rate of work done by the assembly of plates = Power transmitted to the wheel

$$P = F_x u = \rho AV\,(V - u)u \qquad (1.7)$$

Kinetic energy supplied by the jet per unit of time = $\frac{1}{2}\rho(AV)V^2$

$$\text{Efficiency of power transmission} = \frac{\text{Power transmitted to the wheel}}{\text{Kinetic energy supplied by the jet per unit time}}$$

$$= \eta = \frac{\rho A(V-u)u}{\frac{1}{2}\rho(AV)V^2} = \frac{2(V-u)u}{V^2} \qquad (1.8)$$

4. Case D: Jet Impingement on a Stationary Curved Plate

Figure 1.5 shows a two-dimensional free jet of cross-sectional area A impinging at the center of a symmetric stationary curved plate (vane) with a velocity V. The jet is at atmospheric pressure and the pressure on the boundaries of the control volume is

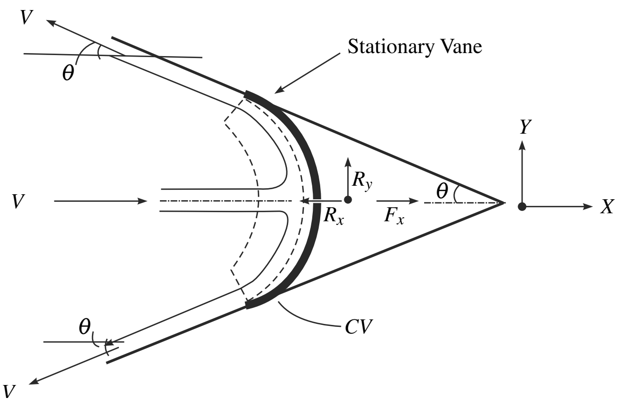


Fig. 1.5 *2D Jet impingement at centre of a stationary symmetric vane*

atmospheric. The flow is assumed frictionless and hence the jet leaving the vane will have the same velocity V as the impinging jet. The jet splits symmetrically on either side of the impingement plane. The flows exiting the vane make an angle θ with the negative direction of the x-axis as shown in Fig. 1.5. The deflection angle of the jet is $(180 - \theta)°$. The discharge impinging on the jet is $Q = A\ V$.

Let R_x = x-component of the reaction of the vane on the control volume

R_y = y-component of the reaction of the vane on the control volume

Consider the linear momentum equation in the x-direction on the control volume.

$\sum$(Forces in x-direction) =

(Momentum flux in x-direction)$_{out}$ – (Momentum flux in x-direction)$_{in}$

$$0 - R_x = (-\rho(AV)V\cos\theta) - \rho(AV)V$$

$$R_x = \rho(AV)^2\ (1 + \cos\ \theta)$$

Let F_x be the force on the vane in the x-direction. F_x is equal and opposite to R_x and hence is given by

$$F_x = \rho(AV)^2\ (1 + \cos\ \theta) \text{ acting in the positive } x\text{-direction.} \qquad (1.9)$$

By linear momentum equation in the y-direction:

$\sum$(Forces in y-direction) =

(Momentum flux in y-direction)$_{out}$ – (Momentum flux in y-direction)$_{in}$

$$0 - R_y = (\rho(AV)V\sin\ \theta) + (-\rho(AV)V\sin\theta) - 0 = 0$$

Hence, R_y, the force on the vane in the y-direction is zero.

(a) Note that this result is due to the symmetry of the plate. If the vane is asymmetrical with respect to the axis of the jet with different exit angles at its two exits, a finite value of R_y (and hence F_y) will result.

(ii) When $\theta = 90°$, the curved plate becomes a flat plate and $F_x = \rho AV^2$ as obtained earlier in Case A. Note that for $(0 \geq \theta < 90°)$ the force on a curved plate is always greater than that on a corresponding flat-plate case. The maximum of value of F_x is reached when $\cos\ \theta = 1$, that is, when the exit angle $\theta = 0°$. In this case, the deflected jet is in a direction opposite to the incoming jet, as in a semicircular vane. The corresponding value of force in x-direction is

$$F_x = (F_x)_{max} = 2\rho AV^2 \tag{1.10}$$

5. Case E: Jet Impingement on a Single Moving Symmetric Curved Plate

The flow is similar to Case D excepting for the fact the blade itself is moving along the jet with a velocity u. A two-dimensional jet impinges on the axis of the curved plate and splits symmetrically on either side. As in Case B, the flow situation relative to a stationary vane is considered. For this, a velocity equal and opposite to u is impressed on the jet as well as on the curved plate. The resulting flow situation is shown in Fig. 1.6. The deflection angle of the relative velocity of the jet is $(180 - \theta)°$.

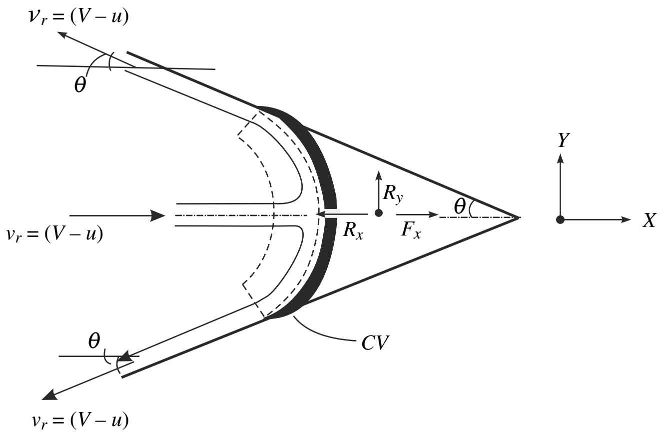


Fig. 1.6 *Jet impingement at centre of a symmetric moving vane—equivalent relative flow*

The curved plate is now relatively stationary, the incoming jet has a relative velocity of $v_r = (V - u)$ with a cross-sectional area of A. As there is no friction to the flow over the vane, the exit relative velocity from the vane is $v_r = (V - u)$, same as the impinging relative velocity of the jet. Each of the two relative flows exiting the vane makes an angle θ with the negative direction of the x-axis. The relative discharge is $Q_r = A(V - u)$.

It can now be observed that this relative flow situation is similar to that of Case D.

Let R_x = x-component of the reaction of the vane on the control volume

R_y = y-component of the reaction of the vane on the control volume

Consider the linear momentum equation in the x-direction.

$$\sum \text{(Forces in } x\text{-direction)}$$
$$= (\text{Momentum flux in } x\text{-direction})_{out} - (\text{Momentum flux in } x\text{-direction})_{in}$$
$$0 - R_x = [-\rho A\,(V - u)\,(V - u)\cos\theta\,] - [\rho A\,(V - u)\,(V - u)]$$
$$R_x = \rho A(V - u)^2\,(1 + \cos\,\theta)$$

If F_x be the force on the vane in the x-direction. F_x is equal and opposite to R_x and hence is given by

$F_x = \rho A(V-u)^2 (1 + \cos\theta)$ acting in the positive x-direction. (1.11)

By linear momentum equation in the y-direction:

$\sum$ (Forces in y-direction) =
(Momentum flux in y-direction)$_{out}$ – (Momentum flux in y-direction)$_{in}$

$0 - R_y = [\rho A(V-u)(V-u)\sin\theta] - [\rho A(V-u)(V-u)\sin\theta] - 0 = 0$

Hence R_y, the force on the vane in the y-direction, is zero.
Work done by the jet per second = Power transferred to the vane

$$= F_x.u = \rho A(V-u)^2 u(1+\cos\theta) \quad (1.12)$$

Kinetic energy supplied by the jet per unit time = $\frac{1}{2}\rho(AV)V^2$

$$\text{Efficiency of the system} = \frac{\text{Power transmitted to the vane}}{\text{Kinetic energy supplied by the jet per unit time}}$$

$$= \eta = \frac{\rho A(V-u)^2 u(1+\cos\theta)}{\frac{1}{2}\rho(AV)V^2} = \frac{2(V-u)^2 u(1+\cos\theta)}{V^3} \quad (1.13)$$

For a given vane (θ = constant) and fixed jet velocity (V = constant), the condition of maximum efficiency of the system can be obtained by putting $\frac{d\eta}{du} = 0$. Hence

$$\frac{d\eta}{du} = \frac{d}{du}\left[\frac{2(V-u)^2 u(1+\cos\theta)}{V^3}\right] = \frac{2(1+\cos\theta)}{V^3}\frac{d}{du}[V^2u + u^3 - 2Vu^2]$$

Thus for maximum (or minimum) efficiency, $\frac{d\eta}{du} = \frac{d}{du}[V^2u + u^3 - 2Vu^2] = 0$

$$V^2 + 3u^2 - 4Vu = 0$$
$$(V-u)(V-3u) - 0$$

which gives the condition $V = u$ or $V = 3u$. However, $V = u$ gives the minimum value of efficiency $\eta_{min} = 0$. Hence, for maximum efficiency $V = 3u$, i.e. the vane velocity is one-third the absolute velocity of the jet. The value of maximum efficiency corresponding to $u = \frac{V}{3}$ is

$$\eta_{max} = \frac{2\left(V - \frac{V}{3}\right)^2\left(\frac{V}{3}\right)(1+\cos\theta)}{V^3} = \frac{8}{27}(1+\cos\theta) \quad (1.14)$$

Further, if θ is varied, maximum most efficiency of $\eta_{max} = \frac{2\times 8}{27} = \frac{16}{27} = 59.26\%$ is obtained for $\theta = 0°$, that is for a semi-circular vane.

6. Case F: Jet Impingement on a Series of Curved Vanes Mounted on a Wheel

This case is similar to Case *C* except that a series of curved vanes are replacing the plates. Since one or other vanes will intercept all the flow from the jet, the entire discharge, Q, issuing out of the jet is involved in the transfer of power to the wheel. The relative velocity of the entering as well as exiting jet is $(V - u)$,

Consider the linear momentum equation in the x-direction.

$\sum$(Forces in x direction) =

(Momentum flux in x-direction)$_{out}$ – (Momentum flux in x-direction)$_{in}$

$$0 - R_x = [-\rho AV (V - u) \cos\theta] - [\rho AV (V - u)]$$

$$R_x = \rho AV(V - u)(1 + \cos\theta)$$

If F_x be the force on the vane in the x-direction, F_x is equal and opposite to R_x.

Thus, the idealised force on the vane in the x-direction is

$F_x = \rho AV(V - u)(1 + \cos\theta)$ acting in the positive x-direction. (1.15)

By linear momentum equation in the y-direction:

$\sum$ (Forces in y-direction) =

(Momentum flux in y-direction)$_{out}$ – (Momentum flux in y-direction)$_{in}$

$$0 - R_y = [\rho AV (V - u) \sin\theta] - [\rho AV(V - u) \sin\theta] - 0 = 0$$

Hence R_y, the force on the vane in the y-direction is zero.

Work done by the jet per second = Power transmitted to the wheel

$$= F_x \cdot u = \rho AV (V - u)\, u\, (1 + \cos\theta) \tag{1.16}$$

Kinetic energy supplied by the jet per unit time = $\frac{1}{2}\rho(AV)V^2$

$$\text{Efficiency of the system} = \frac{\text{Power transmitted to the wheel}}{\text{Kinetic energy supplied by the jet per unit time}}$$

$$= \eta = \frac{\rho AV(V-u)u(1+\cos\theta)}{\frac{1}{2}\rho(AV)V^2} = \frac{2(V-u)(1+\cos\theta)}{V^2} \tag{1.17}$$

(a) For a given vane (θ = constant) and fixed jet velocity (V = constant), the condition of maximum efficiency can be obtained by putting $\frac{d\eta}{du} = 0$.

$$\frac{d\eta}{du} = \frac{d}{du}\left[\frac{2u(V-u)(1+\cos\theta)}{V^2}\right] = \frac{2(1+\cos\theta)}{V^2}\,\frac{d}{du}\,[2Vu - u^2]$$

Thus for maximum efficiency, $\frac{d}{du}[2Vu - u^2] = 0$ which gives the condition

$$u = \frac{V}{2} \tag{1.18}$$

Substituting this result in Eq.(1.17)

$$\eta_{max} = \frac{2\frac{V}{2}\left(V - \frac{V}{2}\right)(1+\cos\theta)}{V^2} = \frac{(1+\cos\theta)}{2} \tag{1.19}$$

(b) If θ is varied, maximum most efficiency of η_{max} = 100% is obtained for $\theta = 0°$, that is for a semicircular vane.

This concept of Case F is made use of in Pelton turbine, where a jet of water impinges on a set of specially designed curved vanes (commonly called as *buckets*) mounted on the periphery of a wheel capable of rotating on its axis, Fig. 1.7.

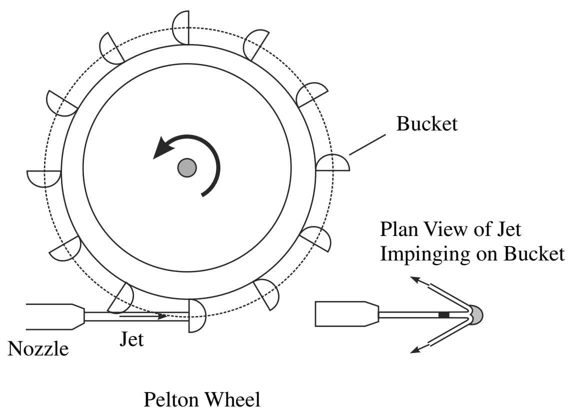


Fig. 1.7 *Schematic sketch of a Pelton wheel*

1.3 VELOCITY TRIANGLES

In the analysis of flow past a moving curved vane, three velocity vectors, viz. absolute velocity, velocity of the vane and the relative velocity of the flow, are involved in the transfer of energy. The vector diagrams of the velocity of the vane u, absolute velocity of the flow V and the relative velocity of flow v_r at the inlet and outlet of a vane in a rotodynamic system, such as a turbine or a pump, form triangles. These vector diagrams are known as *velocity triangles* and are used extensively in the analysis of flow across runners of turbines and impellers of pumps. These velocity triangles help in visualising the flow in a graphic notation and in the derivation or application of appropriate trigonometric relations relating various flow parameters. The notation used consistently in this book is: suffix 1 for the inlet and suffix 2 to the outlet conditions.

Consider a series of curved vanes mounted on a wheel. The wheel rotates about its axis at a constant speed. Water enters the vane in the wheel at inlet and exits at the outlet. The following notations are used:

V = Absolute velocity of water
v_r = Relative velocity of water with respect to the vane
u = Peripheral velocity of the vane

$\alpha =$ Angle made by the absolute velocity vector $\vec{V}$ with the direction of the peripheral velocity u, known as *guide vane angle*

$\beta =$ Angle made by the relative velocity vector $\vec{v}_r$ with the negative direction of the peripheral velocity u; known as *blade angle*

Construction of Velocity Triangles

Relative velocity v_r is the difference between the absolute velocity of flow V and the velocity of the vane, u. In vector notation, the absolute velocity vector $\vec{V}$ is related to the peripheral velocity vector $\vec{u}$ and the relative velocity vector $\vec{v}_r$ by the vector relationship

$$\vec{V} = \vec{u} + \vec{v}_r \tag{1.20}$$

The velocity triangle is a graphical relationship of this vector addition. The procedure is as follows:

Referring to Fig. 1.8,

(a) To represent the peripheral velocity of the wheel, draw $AB = \vec{u}$,

(b) From point B, draw BC to represent the relative velocity $\vec{v}_r$.

(c) Join AC, which represents the absolute velocity $\vec{V}$.

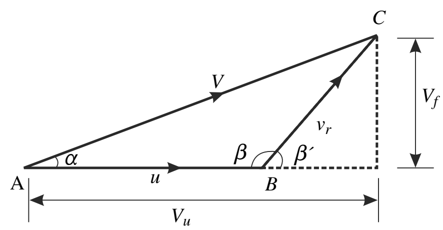


Fig. 1.8 *Velocity triangle, $\beta > \pi/2$*

The triangle ABC is known as the velocity triangle. If the velocity vectors refer to the inlet to the vane, then $\Delta\, ABC$ is known as *inlet velocity triangle*. If the velocity vectors refer to the outlet from the vane, then $\Delta\, ABC$ is known as *outlet velocity triangle*.

- In the triangle ABC of Fig. 1.8, the angle made by the absolute velocity with the peripheral velocity, $C\hat{A}B = \alpha$ is known as *guide vane angle.*
- Further, the angle made by the relative velocity with the peripheral velocity, $C\hat{B}A = \beta$ is known as *blade angle (*or *vane angle).*

Note that Fig. 1.8 represents an obtuse angle triangle ABC with $\beta > \dfrac{\pi}{2}$. The other possible shapes of the velocity triangles are Fig. 1.9 with $\beta = \dfrac{\pi}{2}$ and Fig. 1.10 with $\beta < \dfrac{\pi}{2}$.

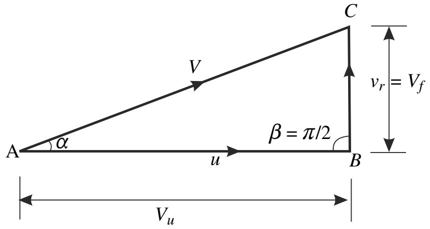


Fig. 1.9 *Velocity triangle, $\beta = \pi/2$*

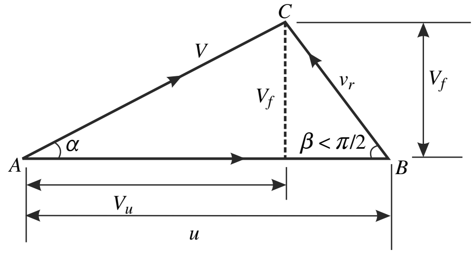


Fig. 1.10 *Velocity triangle, $\beta < \pi/2$*

The suffixes 1 and 2 are used to represent inlet and outlet conditions at all components and elements of the system, respectively. In general, the absolute velocity V can be resolved in to two components:

(i) Tangential component (the component in the direction of u) $V_u = V \cos \alpha$ known as *velocity of whirl* or *velocity of swirl*. The notation V_{u1} is used to represent the tangential component of inlet absolute velocity V_1.

(ii) Normal component (radial component) $V_f = V \cos \alpha$ known as *flow velocity (*or *velocity of flow)*. With the usual notations, V_{f1} represents velocity of flow of inlet absolute velocity V_1.

From the velocity triangles of Fig. 1.8, 1.9 and 1.10,

$$\tan \alpha = \frac{V_f}{V_u} \tag{1.21}$$

Further, it seen that for $\beta < \frac{\pi}{2}$, (Fig. 1.10),

$$V_f = V \sin \alpha = v_r \sin \beta \tag{1.22}$$

$$V_u = V \cos \alpha = u - v_r \cos \beta \tag{1.23}$$

Similar relationships as given in Table 1.1 exist for $\beta > \frac{\pi}{2}$ and for $\beta = \frac{\pi}{2}$.

Table 1.1 *Relationship for V_f and V_u for $\beta \geq \pi/2$*

Parameter	For $\beta > \frac{\pi}{2}$ (Let β' = (180–β))	For $\beta = \pi/2$
$V_f =$	$v_r \sin \beta' = V \sin \alpha$ (1.22-a)	$v_r = V \sin \alpha$ (1.22-b)
$V_u =$	$V \cos \alpha = u + v_r \cos \beta'$ (1.23-a)	$V \cos \alpha = u$ (1.23-b)

Velocity triangles are extensively used in the flow analysis of hydraulic turbines and centrifugal pumps in chapters 2 through 5. As an introduction to the use of velocity triangles, the following flow situation of a tangential free jet impingement on a curved vane is analyzed through use of velocity triangles.

1.4 TANGENTIAL JET IMPINGEMENT ON A MOVING VANE (CASE G)

The impingement of a free jet of water on a moving curved vane at the edge of the vane is treated under two categories as (i) single vane (Case $G1$) and (ii) series of vanes (Case $G2$)

1. Case G1: Tangential Impingement of a Free Jet on a Single Moving Vane

Figure 1.11 shows a curved unsymmetrical vane moving in x-direction with a velocity u. A free jet of water of area A and velocity V_1 impinges tangentially at the edge M of the vane. For shockless entry the direction of the tangent to the blade at the entry edge (M) shall be the same as the direction of the relative velocity at its entry at tip

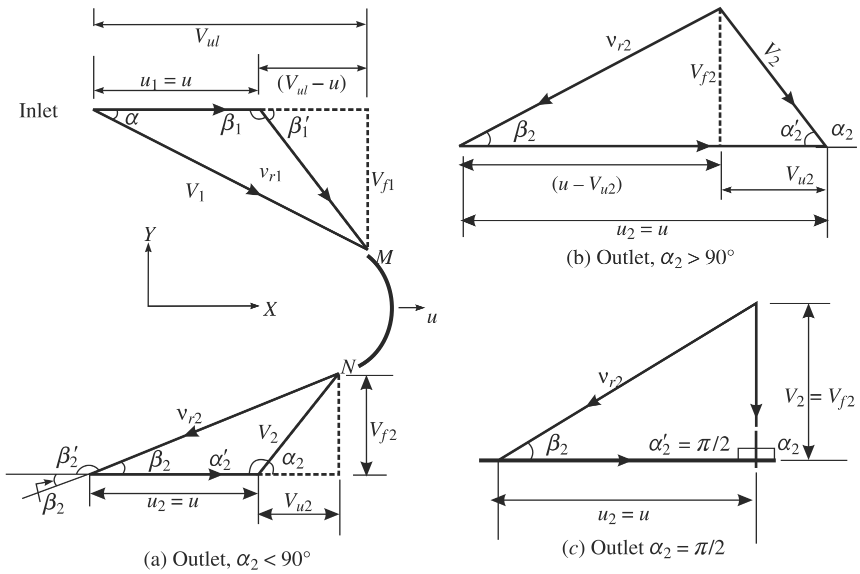


Fig. 1.11 *Velocity triangles for jet impingement on a moving vane for different outlet angle conditions*

M. In this case, this tangent makes an angle β_1 with the negative direction of the velocity *u* and this angle is known as inlet *blade angle*. In the present case angle β_1 is obtuse and the supplementary angle $\beta_1' = 180 - \beta_1$. The suffix 1 is used to indicate entry conditions at edge *M* and suffix 2 denotes conditions at exit (at edge *N*) of the jet from the vane. The flow over the vane is assumed to be frictionless. Vectorially, the absolute velocity of the jet V_1, the velocity of the vane *u* and the relative velocity v_{r1} are related as

$$\vec{V}_1 - \vec{u}_1 = \vec{v}_{r1} \tag{1.24}$$

The flow over the vane is assumed frictionless, i.e. $v_{r1} = v_{r2}$

The velocity triangles for the inlet and outlet are shown in Fig.1.11 (*a, b* and *c*). In this

V_1 = Absolute velocity of water at entry in to the vane

v_{r1} = Relative velocity of water with respect to the vane at entry

u = Velocity of the vane. In this case $u = u_1 = u_2$

α_1 = *Guide vane angle* at the entry. It is the angle made by the absolute velocity vector $\vec{V}$ with the direction of the velocity of vane *u* at the inlet

α_2 = *Guide vane angle* at the exit. It is the angle made by the absolute velocity vector $\vec{V}$ with the direction of the velocity of vane *u* at the exit

β_1 = *Blade angle* at the inlet. It is the angle made by the relative velocity vector $\vec{v}$ with the negative direction of the velocity of vane *u* at the inlet.

β_2 = *Blade angle* at the outlet. It is the angle made by the relative velocity vector $\vec{v}$ with the negative direction of the velocity of vane *u* at the outlet.

V_{u1} = *Velocity of whirl*, component of the absolute velocity V_1 in the direction of $u = V_1 \cos \alpha_1$

V_{f1} = *Velocity of flow*, component of the absolute velocity V_1 in the direction normal to that of $u = V_1 \sin \alpha_1$

The velocity triangle at the exit is shown for three conditions of angle α_2; specifically for $\alpha_2 < 90°$, $\alpha_2 = 90°$ and $\alpha_2 > 90°$. The relative velocity at the outlet v_{r2} is in the direction of the outward tangent to the vane at the edge N. Since the flow on the vane is assumed to be frictionless $v_{r1} = v_{r2}$

(a) When $\alpha_2 < 90°$

For the single vane moving with a velocity u, the force on the vane in the u-direction (x-direction) is given by the linear momentum equation as:

$F_x = \rho$ [relative discharge] × [change in the u-component of the velocity at inlet]
$= \rho$ [relative discharge] × $[-\Delta V_u]$

Considering the absolute velocities:

Initial u-component of absolute velocity (at inlet) = V_{u1}

Final u-component of absolute velocity (at exit) = V_{u2}

It is to be noted that while V_{u1} is in $(+x)$ direction, V_{u2} is in $(-x)$ direction. Hence,

$[-\Delta V_u] = (V_{u1} - (-V_{u2})) = (V_{u1} - V_{u2})$

Relative discharge = $Q_r = Av_{r1}$

$$F_x = \rho(Av_{r1})\,(V_{u1} + V_{u2}) \tag{1.25}$$

Work done per second = Power extracted = $F_x u$

$$P = \rho(Av_{r1})\,(V_{u1} + V_{u2})\,u \tag{1.26}$$

(b) When $\alpha_2 = 90°$

$V_{u2} = 0$ and hence Eq. (1.25) and (1.26) becomes

$$F_x = \rho(Av_{r1})\,V_{u1} \tag{1.25-a}$$

$$P = \rho(Av_{r1})\,V_{u1}u \tag{1.26-a}$$

(c) When $\alpha_2 > 90°$

V_{u2} is in the direction of $(+\,u)$ and hence $[-\Delta V_u] = (V_{u1} - V_{u2})$.

Eq. (1.25) and (1.26) would now read as

$$F_x = \rho(Av_{r1})\,(V_{u1} - V_{u2}) \tag{1.25-b}$$

$$P = \rho(Av_{r1})\,(V_{u1} - V_{u2})u \tag{1.26-b}$$

For all the three cases of guide vane angle α_2 studied above, the power transmitted to the vane can be represented as

$$F_x = \rho(Av_{r1})\,(V_{u1} \pm V_{u2}) \tag{1.25-c}$$

$$P = \rho(Av_{r1})\,(V_{u1} \pm V_{u2})u \tag{1.26-c}$$

(Use positive sign for $\alpha_2 < 90°$).

NOTE!

When there are no frictional and other losses

$(-\Delta V_u) = (-\Delta v_{ru})$ and hence Eq. (1.25 a, b and c) could also be written as

$$F_x = \rho\,(Av_{r1})\,(v_{ru1} \pm v_{ru2}) \tag{1.25-d}$$

and Eq. (1.26-a, b, and c) as $P = \rho(Av_{r1})\,(v_{ru1} \pm v_{ru2})u$ (1.26-d)

(Use positive sign for $\alpha_2 < 90°$).

See worked Example 1.9 for the derivation of this result.

(a) Efficiency of Power Transmission: Case G1 The efficiency of the jet-impingement system is the ratio of the power extracted to the total kinetic energy supplied to the vane by the jet per unit of time.

The total kinetic energy supplied by the free jet per unit time

$$= E_1 = \gamma(AV_1)\left(\frac{V_1^2}{2g}\right) \tag{1.27}$$

Hence, the efficiency of the system $\eta = \dfrac{P}{E_1} = \dfrac{\rho(Av_{r1})(V_{u1} \pm V_{u2})u}{\gamma(AV_1)\left(\dfrac{V_1^2}{2g}\right)}$

$$= \frac{2(V_{u1} \pm V_{u2})v_{r1}u}{V_1^3} \tag{1.28}$$

(with positive sign for $\alpha_2 < 90°$).

(b) Alternative Expression for the Efficiency of Case G1 when there is no Energy Loss If there is no friction on the vane surface an alternate form expression for the efficiency of the system could be obtained from the consideration of the kinetic energy of the jet.

Consider the case of a single vane.

The total power of the jet at inlet to the vane $= E_1 = \gamma Q_r \left(\dfrac{V_1^2}{2g}\right)$

Decrease in the power of the jet due to passage over the vane

= Power transferred to the vane

$$\Delta E = E_1 - E_2 = \gamma Q_r \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right)$$

The efficiency of the system $= \dfrac{\text{Power transmitted to the vane}}{\text{Kinetic energy supplied by the jet per unit time}}$

$$= \frac{\gamma Q_r\left[\dfrac{V_1^2}{2g} - \dfrac{V_2^2}{2g}\right]}{\gamma Q \dfrac{V_1^2}{2g}} = \frac{v_{r1}(V_1^2 - V_2^2)}{V_1^3} = \frac{v_{r1}}{V_1}\left[1 - \left(\frac{V_2}{V_1}\right)^2\right] \tag{1.29}$$

[**Note**: ***The expression of Eq. (1.29) is valid for the case of no energy loss only***.]

2. Case G2: Tangential impingement of a Free Jet on a Series of Vanes

If the free jet impinges tangentially on a series of vanes (as for example, vanes mounted on a wheel) the entire discharge is intercepted by one or other vanes and hence the discharge to be considered in Eq. (1.25-c) and Eq. (1.26-c) is the total discharge $Q = AV_1$ and **not** the relative discharge $Q_r = Av_{r1}$. The relevant equations for the total force in the x-direction and the power extracted by the vanes mounted on a wheel are, respectively,

$$F_x = \rho(AV_1)\,(V_{u1} \pm V_{u2}) \tag{1.30}$$

$$P = \rho(AV_1)\,(V_{u1} \pm V_{u2})u \tag{1.31}$$

(Use positive sign for $\alpha_2 < 90°$).

(a) Efficiency of Power Transmission in a Series of Vanes: Case G2 As defined earlier, the efficiency of the jet impingement system is the ratio of the power extracted to the total kinetic energy supplied to the vanes by the jet.

The total kinetic energy supplied by the free jet per unit time = $E_1 = \gamma(AV_1)\left(\dfrac{V_1^2}{2g}\right)$

Hence the efficiency of the system = $\eta = \dfrac{P}{E_1} = \dfrac{\rho(AV_1)(V_{u1} \pm V_{u2})u}{\gamma(AV_1)\left(\dfrac{V_1^2}{2g}\right)}$

$$= \frac{2(V_{u1} \pm V_{u2})u}{V_1^2} \tag{1.32}$$

(with positive sign for $\alpha_2 < 90°$).

(b) Alternative Expression for the Efficiency of Case G2 when there is no Energy Loss If there is no friction on the surface of the vanes, an alternate form expression for the efficiency of the system could be obtained from the consideration of the kinetic energy of the jet.

Consider the set of vanes in the series:

The total power of the jet at inlet to the vanes = $E_1 = \gamma Q\left(\dfrac{V_1^2}{2g}\right)$

Decrease in the power of the jet due to passage over the vanes

= Power transferred to the vane = $\Delta E = E_1 - E_2 = \gamma Q\left(\dfrac{V_1^2}{2g} - \dfrac{V_2^2}{2g}\right)$

The efficiency of the system = $\dfrac{\text{Power transmitted to the vane}}{\text{Kinetic energy supplied by the jet per unit time}}$

$$= \frac{\gamma Q\left[\dfrac{V_1^2}{2g} - \dfrac{V_2^2}{2g}\right]}{\gamma Q\dfrac{V_1^2}{2g}} = \frac{(V_1^2 - V_2^2)}{V_1^2} = \left[1 - \left(\frac{V_2}{V_1}\right)^2\right] \tag{1.33}$$

Note the difference in the expression for efficiency for frictionless cases in $G1$ and $G2$ cases as reflected in Equations (1.29) and (1.33).

[**Note**: The expression of Eq. (1.33) is valid for the case of *no energy loss* only.]

3. Summary of G1 and G2 Cases

The results of the above study of different variations in the basic flow problem of Case G is summarised in Table 1.2.

Table 1.2 *Summary of force and power in various situations of the jet impingement in Case G_1 and Case G_2*

Value of α_2	Single vane (Case G_1)	Series of vanes (Case G_2)
$\alpha_2 < 90°$	$F_x = \rho(Av_{r1})\,(V_{u1} + V_{u2})$ $P = \rho(Av_{r1})\,(V_{u1} + V_{u2})u$ $\eta = \dfrac{2(V_{u1}+V_{u2})v_{r1}u}{V_1^3}$ Also, when there is no energy loss $\eta = \dfrac{v_{r1}}{V_1}\left[1-\left(\dfrac{V_2}{V_1}\right)^2\right]$	$F_x = \rho(AV_1)\,(V_{u1} + V_{u2})$ $P = \rho(AV_1)\,(V_{u1} + V_{u2})u$ $\eta = \dfrac{2(V_{u1}+V_{u2})u}{V_1^2}$ Also, when there is no energy loss $\eta = \left[1-\left(\dfrac{V_2}{V_1}\right)^2\right]$
$\alpha_2 = 90°$	$F_x = \rho(Av_{r1})\,V_{u1}$ $P = \rho(Av_{r1})\,V_{u1}\,u$ $\eta = \dfrac{2V_{u1}v_{r1}u}{V_1^3}$ *Also, when there is no energy loss* $\eta = \dfrac{v_{r1}}{V_1}\left[1-\left(\dfrac{V_2}{V_1}\right)^2\right]$	$F_x = \rho\,(AV_1)\,V_{u1}$ $P = \rho(AV_1)\,V_{u1}u$ $\eta = \dfrac{2V_{u1}u}{V_1^2}$ *Also, when there is no energy loss* $\eta = \left[1-\left(\dfrac{V_2}{V_1}\right)^2\right]$
$\alpha_2 > 90°$	$F_x = \rho(Av_{r1})\,(V_{u1} - V_{u2})$ $P = \rho(Av_{r1})\,(V_{u1} - V_{u2})u$ $\eta = \dfrac{2\,(V_{u1} - V_{u2})\,v_{r1}u}{V_1^3}$ Also, when there is no energy loss $\eta = \dfrac{v_{r1}}{V_1}\left[1-\left(\dfrac{V_2}{V_1}\right)^2\right]$	$F_x = \rho(AV_1)\,(V_{ul} - V_{u2})$ $P = \rho(AV_1)\,(V_{ul} - V_{u2})u$ $\eta = \dfrac{2(V_{u1}-V_{u2})u}{V_1^2}$ Also, when there is no energy loss $\eta = \left[1-\left(\dfrac{V_2}{V_1}\right)^2\right]$

Note: When u is constant at inlet and outlet (i.e., $u_1 = u_2 = u$) the change in the absolute velocity in the u-direction is identical to the change in the relative velocity in that direction. Thus $(-\Delta V_u) = (-\Delta v_{ru})$ and hence the force equation and power equation can also be expressed as

$$F_x = \rho\,(Av_{r1})\,(v_{ru1} \pm v_{ru2}) \tag{1.25d}$$

$$P = \rho\,(Av_{r1})\,(v_{ru1} \pm v_{ru2})\,u \tag{1.26d}$$

(with positive sign for $\alpha_2 < 90°$).

1.5 MECHANISMS OF TRANSFER OF ENERGY FROM WATER TO A WHEEL

The impingement of a jet of water on a series of vanes mounted on a wheel is one of the mechanisms that has been exploited in the transfer of energy of flowing water to the rotation of a wheel. Such devices that extract the energy from the flowing water are called turbines. Turbines that work on this impact principle are known as *impulse turbines*. As indicated in the previous section (Sec. 1.2.3), Pelton turbine is an example of impulse turbine.

The energy transfer from water to a wheel can be achieved by another method. In this, a cascade of properly shaped vanes (blades) are mounted inside a wheel. Water is admitted under pressure at the entire periphery of the wheel. A set of fixed guide vanes help direct the water in to the vane cascade at a predetermined angle. The water under pressure enters the set of curved a vanes at the outer periphery, in a tangential direction to the vanes, and exits with reduced pressure and velocity at the inner circumference of the wheel. While doing so, the change of momentum and pressure causes a reaction force, which in turn causes a torque at the axis of the wheel. The moment of momentum principle is used calculate the torque and power transmitted by the water to the rotating wheel. The turbines that work under this principle are known as *reaction turbines.*

While an impulse turbine is very easy to understand, the mechanism of a reaction turbine is not intuitive. The principle of the reaction of flowing water causing the motion of a rotating element is illustrated in the following simple device. A revolving cylindrical tank with an arrangement for inflow of water is shown in Fig. (1.12). Two arms at the bottom of the tank, as shown in the figure, discharge the water from the tank to the atmosphere through nozzles fitted at the end of the arms. The change of pressure in the arms and the change in momentum of the flow cause a reaction force in the system, which results in a torque about the axis of the system. The tank with the arms revolves about the axis in a direction opposite to that of the discharging water. The rotating action of the common lawn sprinkler is similar to this example and forms another example the reaction causing a rotating motion.

The demonstration device of Fig. 1.12 is known as *Barker's mill* after its inventor Robert Barker (1740). Notice that in the Barker's mill or in the common lawn sprinkler, the jets do not impinge on the runner. In fact, they are leaving the runner. It is the reaction of the exit of the jets that produces forces that results in a torque to spin the runner. Note that the direction of the rotation of the runner is opposite to the direction of the jets of water issuing from the nozzles in the runner.

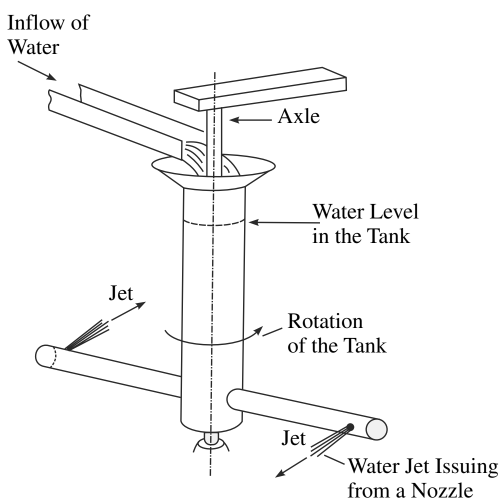


Fig. 1.12 *A rotating device based on reaction principle*

1.6 ILLUSTRATIVE EXAMPLES—SET 1.1

[All Illustrative Examples have been graded in three levels: Simple, Medium and Difficult. The markings for these are: Simple = *, Medium = **, Difficult = ***]

In this set of examples and in the rest of book, the following are the default values for the indicated physical constants and variables:

Density of water = ρ = 998 kg/m^3 Unit weight of water γ = 9.79 kN/m^3

Acceleration due to gravity g = 9.81 m/s^2

Impingement of Jets

*EXAMPLE 1.1

A free jet issuing from a nozzle of 0.0113 m^2 cross-sectional area impinges normally on a fixed vertical plate. The force on the plate due to this impingement is found to be 2.54 kN. (a) Estimate the discharge from the nozzle. (b) If the plate is allowed to move with a velocity of 3.0 m/s in the direction of the jet, what would be the (i) force on the plate and (ii) power transferred to the plate in this system?

Solution

(a) Figure 1.13(a) shows a control volume surrounding the impact zone. The pressure is atmospheric on all the surfaces of the control volume. If R_x is the reaction of the plate on the control volume, the force on the plate in x-direction F_x is equal and opposite to R_x. By linear momentum equation applied to the control volume:

$$\sum \text{(Forces in } x\text{-direction)} = \text{(Momentum flux in } x\text{-direction)}_{out} - \text{(Momentum flux in } x\text{-direction)}_{in}$$

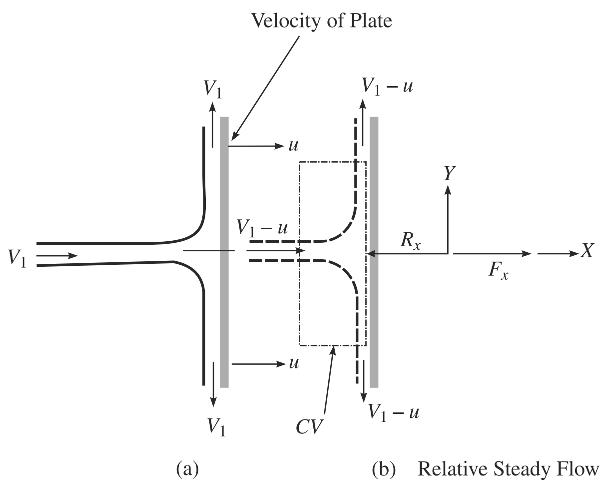


Fig. 1.13 *Example 1.1*

$0 - R_x = \rho Q\,(0 - V)$

$R_x = \rho QV = F_x$ in positive x-direction

Hence, $F_x = \rho QV = \dfrac{\rho Q^2}{A}$.

Here, $F_x = 2.54$ kN, area $A = 0.0113$ m^2 and hence

$$Q = \sqrt{\frac{AF_x}{\rho}} = \sqrt{\frac{0.0113 \times 2.54}{0.998}} = 0.1696 \text{ m}^3/\text{s}$$

(b) When the plate is moving, the relative velocity of the jet and also the relative discharge from the nozzle have to be considered. Figure 1.13 (b) shows the relative steady flow. Velocity of jet $V = \dfrac{Q}{A} = \dfrac{0.1696}{0.0113} = 15.0$ m/s.

Velocity of the plate = u = 3.0 m/s

Relative velocity $v_r = V - u = 15.0 - 3.0 = 12.0$ m/s

(i) Force on the plate in the x-direction = $F_x = \rho Q_r V_r$

$$= 0.998 \times (0.0113 \times 12) \times 12 = 1.624 \text{ kN}$$

(ii) Power transferred to the plate = $P = F_x u = 1.624 \times 3.0 = 4.872$ kW

*EXAMPLE 1.2

A 6.0 cm diameter free jet of water having a velocity of 10 m/s impinges on a plane, smooth plate at angle of 30° to the normal to the plate. (a) What will be the force due to jet impingement on the plate and work done per second when the plate is (i) stationary, and (ii) moving in the direction of the jet at 5.0 m/s velocity? (b) What will be the force due to jet on the plate when the plate is moving against the jet direction at 5.0 m/s velocity?

Solution

Consider Fig. 1.14.

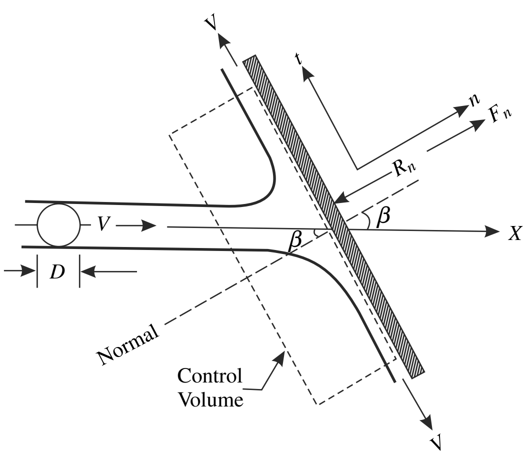


Fig. 1.14 *Jet impingement, Example 1.2*

(a)(i) *Stationary Plate:* Consider the normal and tangential directions and a control volume as in Fig. 1.14 . Let R_n = Normal reaction on the control volume. Let β = Inclination of the jet axis with the normal to the plate.

Since the pressure is atmospheric on all the control volume surfaces, by linear momentum equation:

$0 - R_n = \rho Q(0 - V \cos \beta)$ where V = Velocity of the jet, and β = Inclination of the jet to the normal of the plate.

$R_n = \rho QV \cos \beta$ and the normal force on the plate is equal and opposite to R.

Thus, $F_n = \rho QV \cos \beta$ in the inward normal direction as indicated in Fig. 1.14.

Here, $\beta = 30°$, Area $A = \frac{\pi}{4} \times (0.06)^2 = 0.002827$ m^2 and $V = 10$ m/s.

Discharge $Q = AV = 0.002827 \times 10 = 0.02827$ m^3/s.

Normal force on the plate, $F_n = 998 \times 0.2827 \times 10 \times \cos 30° = 244.4$ N

Since there is no friction, there is no tangential component of the force on the plate.

Further, since $u = 0$, work done on the plate is zero.

(ii) *Moving Plate:*

(a) When $u = 5$ m/s equivalent relative motion with respect to a stationary plate is to be considered. Relative velocity $v_r = (V - u) = (10 - 5) = 5$ m/s

Relative discharge $Q_r = Av_r = 0.002827 \times 5 = 0.01414$ m^3/s

Normal force on the plate, $F_n = \rho Q_r, v_r \cos \theta = 998 \times 0.01414 \times 5 \times \cos 30°$ $= 61.09$ N

Since there is no friction, there is no tangential component of the force on the plate.

Also, since $u = 5$ m/s, work done on the plate per second is = $P = F_n u \cos \theta$

$P = 61.09 \times 5 \times \cos 30° = 264.5$ W

(b) When the plate is moving in opposite direction to that of the jet, $u = -5$ m/s

Relative velocity $v_r = (V + u) = (10 + 5) = 15$ m/s

Relative discharge $Q_r = Av_r = 0.02827 \times 15 = 0.0424$ m^3/s

Normal force on the plate $F_n = \rho Q_r v_r \cos \beta = 998 \times 0.0424 \times 15 \times \cos 30°$ $= 549.8$ N

Since there is no friction, there is no tangential component of the force on the plate.

*EXAMPLE 1.3

A two-dimensional free jet of water of thickness B and discharge q per unit width of the jet strikes a stationary, frictionless, plate at angle β to the normal to the plate. Calculate (a) the force on the plate, (b) the ratio of discharges in the two streams that move on the plate on either side of the impact zone, and (c) the force on the plate when $\beta = 15°$, B = 10 cm and q = 1.5 m^3/s.

Solution

β is the angle made by the axis of the jet with the normal to the plate.

(a) Consider a unit width of the jet. In the direction normal to the plate let R_n be the reaction of the plate on the control volume enclosing the flow of water, (Fig. 1.15).

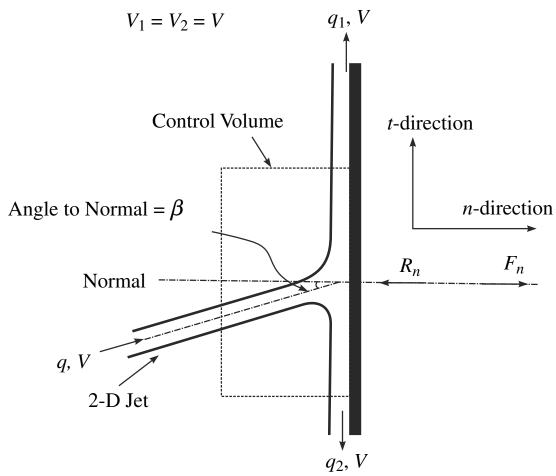


Fig. 1.15 *Impact of 2D Jet, Example 1.3*

Since the pressure is atmospheric throughout the control volume, in the normal direction the linear momentum equation can be written as

$0 - R_n = \rho q\,(0 - V\cos\beta)$ where V = Velocity of the jet.

$R_n = \rho qV\cos\beta$ and the normal force on the plate is equal and opposite to R_n.

Thus $F_n = \rho qV\cos\beta$ in the inward normal direction as indicated in Fig. 1.15

(b) Since there is no friction to the flow in the transverse direction, the velocity of the two streams in the transverse direction (V_1 and V_2) is the same as the velocity of the impinging jet V. Hence by referring to Fig. 1.15 $V = V_1 = V_2$. Since the pressure is atmospheric all over the control volume, linear momentum equation in the transverse direction is

$0 = [\rho q_1 V_1 + (-\rho q_2 V_2)] - \rho q\,V\sin\beta$

$qV\sin\beta = q_1V_1 - q_2V_2$

Since $V = V_1 = V_2$, $\quad q\sin\theta = q_1 - q_2$ (i)

Also, by continuity $\quad q = q_1 - q_2$ (ii)

Substituting ($q_2 = q - q_1$) in (i)

$q\sin\beta = q_1 - (q - q_1)$ leading to $q_1 = \dfrac{q}{2}(1+\sin\beta)$ (iii)

Similarly substituting ($q_1 = q - q_2$) in (i)

$q\sin\beta = q - q_2 - q_2$ leading to $q_2 = \dfrac{q}{2}(1-\sin\beta)$ (iv)

Thus the ratio $\dfrac{q_1}{q_2} = \dfrac{1+\sin\beta}{1-\sin\beta}$

(c) When $\beta = 15°$, $B = 10$ cm and $q = 1.5\ m^3/s$,

Considering unit width, velocity of jet $V = q/B = 1.5/(0.10) = 15$ m/s

Normal force on the plate $F_n = \rho qV\cos\beta = 998 \times 1.5 \times (15\cos 15°) = 21690$ N

The ratio of the discharges $\dfrac{q_1}{q_2} = \dfrac{1+\sin\beta}{1-\sin\beta} = \dfrac{1+\sin 15°}{1-\sin 15°} = 1.698$

*EXAMPLE 1.4

A 6 cm diameter jet of water impinges on a moving hemispherical cup in the middle of its hollow portion and is deflected by 180°. The jet has a velocity of 35 m/s and the cup has a velocity of 10 m/s in the same direction as the jet. Determine (a) the rate of work done by the jet on the cup and the efficiency of the system. (b) What would be the force on the cup if the cup were moving in a direction opposite to that of the jet? Assume frictionless flow.

Solution

Consider equivalent relative motion with respect to stationary cup, Fig. 1.16.
Since the flow is frictionless, Exit relative velocity v_{r2} = Inlet velocity $v_{r1} = v_r$.

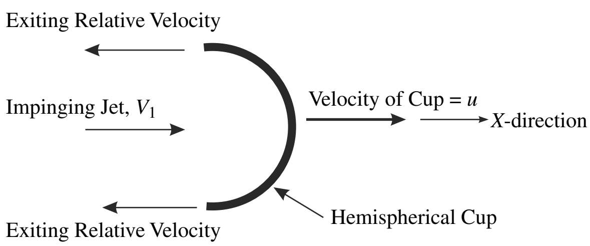


Fig. 1.16 *Jet impingement on a moving hemispherical cup*

Also, the geometry of the cup is such that the exit relative velocity is directed in the (-x) direction after undergoing a 180° defection angle in the cup.

Case (a): Cup moving along the direction of jet.

V_1 = absolute velocity of jet = 35 m/s, u = velocity of the cup = 10 m/s,

Relative velocity $v_r = (V_1 - u)$

Area of jet = $A = \dfrac{\pi}{4} \times (0.06)^2 = 0.002827 \text{ m}^2$

Relative discharge $Q_r = A(V_1 - u)$

$$= 0.002827 \times (35 - 10) = 0.07069 \text{ m}^3\text{/s}$$

By linear momentum equation, force on the cup $F_x = \rho Q_r [(V_1 - u) - (-(V_1 - u))]$

$$= 2\rho Q_r (V_1 - u)$$

Rate of work done on the cups = $P = 2\rho Q_r (V_1 - u)u$

$$= 2 \times \frac{998}{1000} \times 0.07069 \times (35\text{–}10) \times 10 = 35.27 \text{ kW}$$

Kinetic energy of the jet per unit time = power of the jet =

$$\frac{1}{2}\rho(AV_1)V_1^2 = \frac{1}{2} \times \frac{998}{1000} \times (0.00287 \times 35) \times (35)^2 = 61.40 \text{ kW}$$

Efficiency of power transmission $\eta = \dfrac{\text{Power transmitted to the vane}}{\text{Kinetic energy supplied by the jet per unit time}}$

$$= \frac{35.27}{61.40} = 0.5745 = 57.45\%$$

Case (b): *When the cup is moving in a direction opposite to that of the issuing jet*

Relative velocity at inlet $v_r = V_1 + u$ = Relative velocity at outlet

Relative discharge $Q_r = A(V_1 + u)$

$$= 0.002827 \times (35 + 10) = 0.1272 \text{ m}^3/\text{s}$$

Force on the cup $F_x = \rho Q_r\,[(V_1 + u) - (-(V_1 + u))]$

$$= 2\rho Q_r\,[V_1 + u] = 2 \times \frac{998}{1000} \times 0.1272 \times (35 \times 10) = 11.453 \text{ kN}$$

**EXAMPLE 1.5

A free jet of water of area A and velocity V strikes a vertical plate normally. The plate is moving with a velocity u in the direction of the jet. Obtain the value of the ratio u/V for maximum efficiency of this power transmission system. What is the value of corresponding maximum efficiency?

Solution

By linear momentum equation, force on the plate in the direction of the jet

$$= F_x = \rho Q_r\,(V - u) = \rho A\,(V - u)^2$$

This force F_x moves with a velocity u and as such the rate of work done on the plate is

$$P = F_x u = \rho A (V - u)^2\, u$$

$$\text{Efficiency of power transmission} = \frac{\text{Power transmitted to the plate}}{\text{Kinetic energy supplied by the jet per unit time}}$$

Kinetic energy supplied by the jet per unit time $= \dfrac{1}{2}\rho\,(AV)V^2$

Hence the efficiency $\eta = \dfrac{\rho A(V-u)^2 u}{\frac{1}{2}\rho(AV)V^2} = \dfrac{2(V-u)^2 u}{V^3}$

For a fixed jet velocity (V = constant), the condition of maximum efficiency can be obtained by putting $\dfrac{d\eta}{du} = 0$. Hence,

$$\frac{d\eta}{du} = \frac{d}{du}\left[\frac{2(V-u)^2 u}{V^3}\right] = \frac{2}{V^3}\frac{d}{du}[V^2 u + u^3 - 2Vu^2]$$

Thus for maximum efficiency, $\dfrac{d\eta}{du} = \dfrac{d}{du}[V^2 u + u^3 - 2Vu^2] = 0$

$V^2 + 3u^2 - 4Vu = 0$, i.e., $(V - u)\,(V - 3u) = 0$

which gives the condition $V = u$ or $V = 3u$. However, $V = u$ gives the minimum value of efficiency $\eta_{\min} = 0$. Hence, for maximum efficiency $\dfrac{u}{V} = \dfrac{1}{3}$. The value of maximum efficiency is

$$\eta_{\max} = \frac{2\left(V - \frac{V}{3}\right)^2\left(\frac{V}{3}\right)}{V^3} = \frac{8}{27}$$

**EXAMPLE 1.6

A 10 cm diameter jet of water strikes a curved vane with a velocity of 25 m/s. The inlet angle of the vane is zero and the outlet angle is 150° measured with respect to the impinging jet direction. Determine the resultant force on the vane (a) when the vane is stationary, and (b) when the vane is moving in the direction of the jet at 10 m/s velocity.

Solution

Let suffix 1 and 2 denote the inlet and outlet conditions respectively. Figure 1.17(a) is a definition sketch of the problem when the vane is stationary.

(a) *When the jet is stationary: $u = 0$.* Assuming no losses, $V_1 = V_2 = V = 25.0$ m/s. The direction of exit velocity, V_2 is (180° – 150°) = 30° with the negative x-direction.

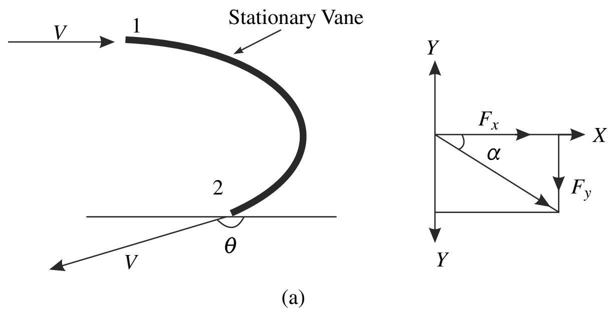


Fig. 1.17(a) *Example 1.6—stationary vane*

Discharge $Q = \frac{\pi}{4} \times (0.10)^2 \times 25 = 0.1963$ m^3/s

By linear momentum equation, the force on the vane in the x-direction is

$$F_x = \rho Q(V - (-V\cos 30°)) = \frac{998}{1000} \times 0.1963 \times (25 + 25\cos 30°) = 9.14 \text{ kN}$$

By momentum equation in y-direction, force on the plate in y-direction is

$$F_y = \rho Q(0 - V\sin 30°) = \frac{998}{1000} \times 0.1963 \times (0 - 25\sin 30°) = -2.45 \text{ kN}$$

acting in negative y-direction.

Resultant force $F = \sqrt{F_x^2 + F_y^2} = \sqrt{(9.14)^2 + (2.45)^2} = 9.46$ kN acting at an angle α to x-axis,

where α is given by $\alpha = \tan^{-1}\frac{F_y}{F_x} = \tan^{-1}\frac{(-2.45)}{9.14} = -15°$ to positive x-direction as shown in Fig. 1.17 (a)

(b) *When the vane is moving away from the jet with velocity u:*
Considering the equivalent flow relative to a stationary vane:
Relative velocity $v_r = V - u = (25 - 10) = 15$ m/s. Assuming no losses,
$v_{r1} = v_{r2} = v_r$.

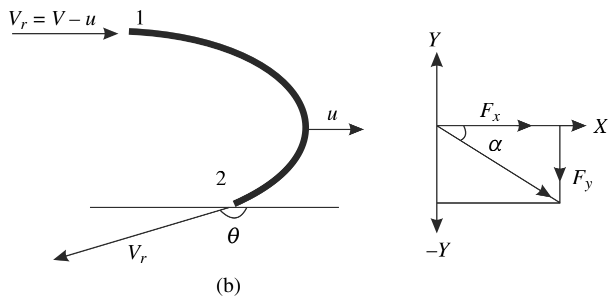


Fig. 1.17(b) *Relative flow, Example 1.6(b)*

The direction of exit relative velocity, v_{r2}, is (180° – 150°) = 30° with the negative x-direction.

Relative discharge $Q_r = A(V - u) = \frac{\pi}{4} \times (0.10)^2 \times (25 - 10) = 0.1178\ \text{m}^3/\text{s}$

The force on the plate in the x-direction is

$$F_x = \rho Q_r[v_r - (-v_r \cos 30°)] = \frac{998}{1000} \times 0.1178 \times (15 + 15 \cos 30°) = 3.29\ \text{kN}$$

By momentum equation in y-direction, force on the plate in y-direction

$$F_y = \rho Q_r(0 - v_r \sin 30°) = \frac{998}{1000} \times 0.1178 \times (0 - 15 \sin 30°)$$

$$= -0.882\ \text{kN acting in negative } y\text{-direction}$$

Resultant force $F = \sqrt{F_x^2 + F_y^2} = \sqrt{(3.291)^2 + (0.882)^2} = 3.407$ kN acting at an angle α to x-axis, where α is given by $\alpha = \tan^{-1} \frac{F_y}{F_x} = \tan^{-1} \frac{-0.882}{3.291} = -15°$ to positive x-direction as shown in Fig. 1.17 (b).

**EXAMPLE 1.7

A jet of water 15 cm in diameter strikes a curved blade at 20 m/s velocity. The inlet and outlet angles of the blade are 0° and 135° respectively measured with respect to the direction of motion of the initial jet velocity. Determine the resultant force exerted on the blade when (a) the blade is stationary, and (b) the blade moves against the direction of the jet at a velocity of 5 m/s. Neglect friction along the blade.

Solution

Let suffix 1 and 2 denote the inlet and outlet conditions respectively.

(a) *When the jet is stationary:* The definition sketch is similar to Fig. 1.17(a) of the previous problem (Example 1.6). Assuming no losses $V_1 = V_2 = V = 20.0$ m/s.

Discharge $Q = \frac{\pi}{4} \times (0.15)^2 \times 20 = 0.3534\ \text{m}^3/\text{s}$

By linear momentum equation, the force on the plate in the x-direction is

$$F_x = \rho Q(V - (-V \cos 135°)) = \frac{998}{1000} \times 0.3534 \times (20 + 20 \cos 45°) = 12.043\ \text{kN}$$

By momentum equation in y-direction, force on the plate in y-direction

$F_y = \rho Q(0 - V\sin 45°)) = \dfrac{998}{1000} \times 0.3534 \times (0 - 20\sin 45°) = -4.988$ kN acting in (–ve) y-direction.

Resultant force $F = \sqrt{F_x^2 + F_y^2} = \sqrt{(2.043)^2 + (-4.988)^2} = 13.035$ kN acting

at an angle α to x-axis, where α is given by $\alpha = \tan^{-1}\dfrac{F_y}{F_x} = \tan^{-1}\dfrac{-4.988}{12.043}$ $= -22.5°$ to positive x-direction.

(b) *When the vane is moving with velocity u against the jet:*
The relative motion is shown in Fig. 1.18

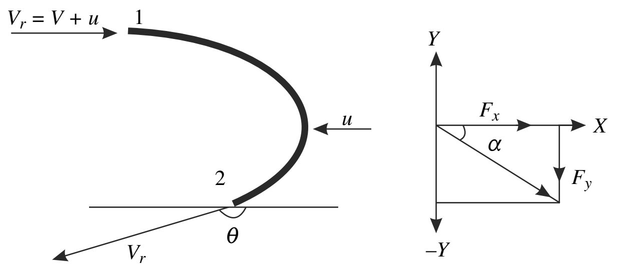


Fig. 1.18 *Relative motion, Example 1.7*

Considering the equivalent flow relative to a stationary vane:
Relative velocity $v_r = V + u = (20 + 5) = 25$ m/s.
Assuming no losses, $v_{r1} = v_{r2} = v_r$. This relative velocity v_r has a direction of (180° – 135°) = 45° with the negative x-direction.
Relative discharge $Q_r = A(V + u) = \dfrac{\pi}{4} \times (0.15)^2 \times (20 + 5) = 0.4418$ m³/s
$F_x = \rho Qr\,[v_r - (-v_r\cos 45°)]$

$$= \rho Q_r V_r (1 + \cos 45°) = \frac{998}{1000} \times 0.4418 \times 25\,(1 + \cos 45°) = 18.817 \text{ kN}$$

By momentum equation in y-direction, the force on the plate in y-direction

$F_x = \rho Q(0 - v_r\sin 45°) = \dfrac{998}{1000} \times 0.4418 \times (0 - 25\sin 45°) = -7.793$ kN acting in (-ve) y-direction.

Resultant force $F = \sqrt{F_x^2 + F_y^2} = \sqrt{18.817^2 + (-7.793)^2} = 20.367$ kN acting

at an angle α to x-direction. The angle α is given by

$\alpha = \tan^{-1}\dfrac{F_y}{F_x} = \tan^{-1}\left(\dfrac{-7.793}{18.817}\right) = 22.5°$ to positive x-direction as shown in Fig. 1.18

**EXAMPLE 1.8

A rectangular plate of weight W is hinged at its top edge to rotate about a horizontal axis. A free water jet of velocity V issuing from a nozzle of diameter D impinges *normally the plate at its center. (a) Obtain an expression for the deflection θ of the plate from its original vertical position. (b) When the jet is striking, what force R is needed at the bottom edge to keep the plate vertical? (c) Calculate the values of deflection angle θ and the retaining force R when D = 5.0 cm, V = 20 m/s and W = 2000 N.*

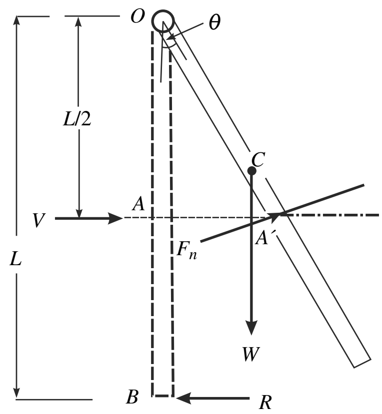


Fig. 1.19 *Jet impinging on a hinged plate, Example 1.8*

Solution

Figure 1.19 is the definition sketch of a hinged plate impacted by a jet as indicated in Example 1.8.

Let L be the vertical length of the plate. Referring to Fig. 1.19, the jet initially strikes the plate at point A and due to the deflection of the plate, the point of striking at the equilibrium state is A'. At equilibrium, the moment about the hinge O of the force due to impact of the jet at A' is balanced by the restoring moment of the weight W about the hinge.

Hence, moment of normal component of force F_n of jet impact about the hinge O is

$$M_1 = F_n \times \text{Lever arm } OA'$$

$$M_1 = \rho QV \cos\theta \times \left(\frac{L}{2\cos\theta}\right)$$
$$= \rho QVL/2$$

Moment of weight about the hinge = $M_2 = W \times$ Lever arm OC

$$M_2 = W \times \left(\frac{L}{2}\right)\sin\theta$$

Equating the moments M_1 and M_2, $\dfrac{\rho QVL}{2} = \dfrac{1}{2}\, WL \sin\theta$ and hence

$$\sin\theta = \frac{\rho QV}{W}$$

(ii) If R = Restraining force at the lower edge acting in a direction opposite to F_n

Then by equating the moments of F_n and R about the hinge O

$$R \times L = F_n \times \left(\frac{L}{2}\right) = \frac{\rho QVL}{2}$$

(iii) When D = 5 cm, V = 20 m/s and W = 2000 N

Discharge $Q = \left(\dfrac{\pi}{4} \times (0.05)^2\right) \times 20 = 0.3927\ \text{m}^3/\text{s}$

$$\sin\theta = \frac{\rho QV}{W} = \frac{998 \times 0.03927 \times 20}{2000} = \frac{783.8}{2000} = 0.3919$$

Angle $\theta = 23.07°$

$$R = \frac{\rho QV}{2} = \frac{998 \times 0.03927 \times 20}{2} = 392\,\text{N}$$

*** EXAMPLE 1.9

A free water jet of velocity V impacts a single curved vane at its top edge. The vane is moving with a velocity u in the x-direction. The jet moves over the vane and exits at its bottom edge. Show that the change in the component of absolute velocity in the u-direction is identical to the change in the component of relative velocity in the same u-direction.

Solution

Figure 1.20 shows the inlet and outlet velocity diagrams of a jet impinging on a moving curved vane.

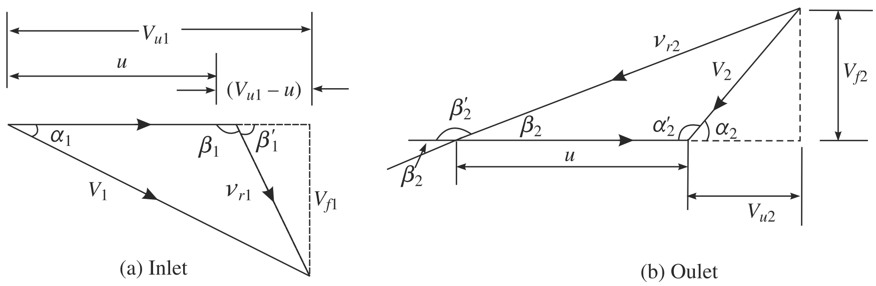


Fig. 1.20 *Velocity triangles at inlet and outlet, Examples 1.9 and 1.10*

In this, α = Angle made by the absolute velocity with the u-direction, and β is the angle made by relative velocity with the negative u-direction. Suffixes 1 and 2 denote the inlet and outlet conditions respectively. V = Absolute velocity, v_r = Relative velocity, V_u = Component of absolute velocity in u-direction and v_{ru} = Component of relative velocity in u-direction.

From inlet velocity triangle:

$$V_{u1} = V_1 \cos \alpha_1 = u + v_{r1} \cos \beta'_1 = u + v_{ru1} \quad \text{(i)}$$

$$V_{u2} = V_2 \cos \alpha_2 = v_{r2} \cos \beta_2 - u = v_{ru2} - u \quad \text{(ii)}$$

Considering the direction of the vectors and on subtracting (i) from (ii)

$$V_{u2} - (-V_{u1}) = (-u + v_{r\,u1} - ((-v_{ru2}) + u) = v_{ru2} - (-v_{ru1})$$

Thus $(\Delta V_u) = (\Delta v_{ru})$

***EXAMPLE 1.10

A jet of water with a velocity of 18 m/s impinges on a moving curved vane at 25° to the direction of motion. The velocity of the vane is 8 m/s. The vane angle at the outlet is 30°. Find (a) the blade angle at inlet so that the water enters without shock, (b) the rate of work done per unit weight of water entering the vane, and (c) the efficiency of the system.

Solution

Refer to Fig. 1.20 showing the inlet and outlet triangles of a free jet impinging on a curved blade. Suffix 1 denotes conditions at inlet and suffix 2 denotes the conditions at the outlet.

Given: $\alpha_1 = 25°$, $V_1 = 18$ m/s, $\beta_2 = 30°$, $u = 8.0$ m/s

Frictionless flow is assumed and as such the relative velocity at inlet $v_{r1} = v_{r2}$.

For the inlet velocity triangle: V_1 = absolute velocity of jet = 18 m/s

Flow velocity $V_{f1} = V_1 \sin \alpha_1 = 18 \sin 25° = 7.607$ m/s

Velocity of whirl $V_{u1} = V_1 \cos \alpha_1 = 18 \times \sin 25° = 16.314$ m/s

$$\tan \beta_1' = \frac{V_{f1}}{(V_{u1} - u)} = \frac{7.607}{(16.314 - 8.0)} = 0.915$$

$\beta'_1 = 42.46°$ and $\beta_1 = 180 - 42.46° = 137.54°$ = Blade angle at inlet

$$\text{Also } v_{r1} = v_{r2} = \frac{V_{f1}}{\sin \beta_1} = \frac{7.607}{\sin 42.46°} = 11.27 \text{ m/s}$$

From the outlet velocity triangle:

Blade angle $\beta_2 = 30°$. $u + V_{u2} = v_{r2} \cos \beta_2 = 11.27 \times \cos 30° = 9.76$

$V_{u2} = 9.76 - 8.0 = 1.76$ m/s in $(-x)$ direction

$$\text{Relative discharge } Q_r = Av_r = \left(\frac{\pi}{4} \times (0.10)^2\right) \times 11.27 = 0.08852 \text{ m}^3/\text{s}$$

$$\text{Discharge carried by the jet} = Q = AV_1 = \left(\frac{\pi}{4} \times (0.10)^2\right) \times 18 = 0.14137 \text{ m}^3/\text{s}$$

$$\text{Force on the plate } F_x = \rho Q_r (V_{u1} + V_{u2}) = \frac{998}{1000} \times 0.08852 \times (16.314 + 1.76)$$

$$= 1.597 \text{ in } x\text{-direction}$$

Power extracted = $P = \rho Q_r (V_{u1} + V_{u2})u$

Power per unit weight of water supplied by the jet

$$P_1 = \frac{\rho Q_r (V_{u1} + V_{u2})u}{\rho g Q} = \left[\frac{Q_r}{Q} \frac{(V_{u1} + V_{u2})u}{g}\right] = \left(\frac{0.08852}{0.14137}\right) \times \frac{(16.314 + 1.76)}{9.81} \times 8 = 9.229 \text{ m}$$

$$\text{Total kinetic energy supplied by the free jet per unit of time} = E_1 = \gamma Q \left(\frac{V_1^2}{2g}\right)$$

Kinetic Energy per unit weight of water = Energy head

$$= H_{E1} = \frac{V_1^2}{2g} = \frac{(18)^2}{2 \times 9.81} = 16.513 \text{ m}$$

$$\text{Efficiency of the system} = \eta = \frac{P_1}{H_{E1}} = \frac{9.229}{16.513} = 0.559$$

Check on Calculation of Efficiency:

This problem is of Case-$G1$ type with $\alpha_2 < 90°$. From Table 1.2

$$\eta = \frac{2v_{r2}(V_{u1}+V_{u2})u}{V_1^3} = \frac{2\times 11.27\times(16.314+1.36)}{(18)^3}\times 8 = \frac{37.59}{40} = 0.559$$

***EXAMPLE 1.11

A free jet of water with an initial velocity of 40 m/s impinges on a series of curved vanes moving at 20 m/s. The direction of the jet from the nozzle is at 20^o with the direction of motion of the vanes. Assuming the outlet relative velocity is 95% of the relative velocity at the inlet, compute (i) the angle made by the tangent to the relative velocity at its tips with the direction of motion of vanes, (ii) power transmitted per unit weight of water, and (iii) hydraulic efficiency of the system. Assume the absolute velocity of water at the outlet is normal to the direction of motion of vane.

Solution

Figure 1.21 shows the inlet and outlet velocity triangles. Given Data:

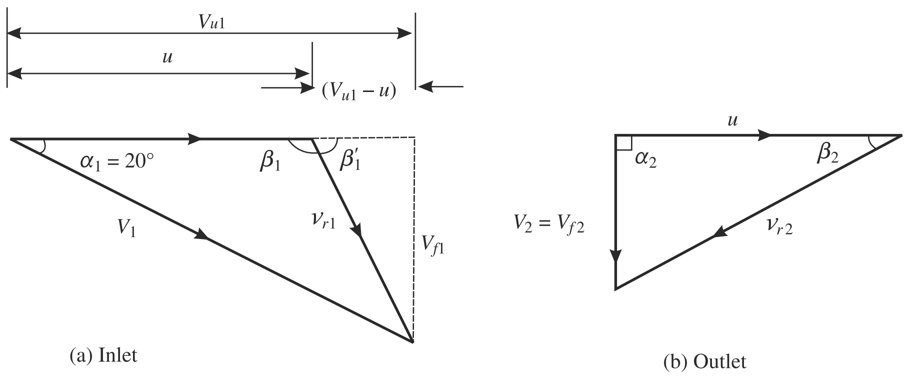


Fig. 1.21 *Velocity triangles of Example 1.11*

For the inlet velocity triangle: V_1 = Absolute velocity of jet = 40 m/s, u = 20 m/s and $\alpha_1 = 20°$.

For the outlet velocity triangle: V_2 = Absolute velocity of jet = V_{f2}, $v_{r2} = 0.95v_{r1}$, and $\alpha_2 = 90°$,

Flow velocity $V_{f1} = V_1 \sin \alpha_1 = 40 \times \sin 20° = 13.68$ m/s

Velocity of whirl $V_{u1} = V_1 \cos \alpha_1 = 40 \times \cos 20° = 37.59$ m/s

$$\tan \beta_1' = \frac{V_{f1}}{(V_{u1}-u)} = \frac{13.68}{(37.59-20.0)} = 0.778$$

$\beta_1' = 37.87°$ = Angle made by the tangent to the relative velocity with the direction of motion of vanes at the inlet.

$$v_{r1} = \frac{V_{f1}}{\sin \beta_1'} = \frac{13.68}{\sin 37.87°} = 22.28 \text{ m/s}$$

$v_{r2} = 0.95\, v_{r1} = 0.95 \times 22.28 = 21.166$ m/s

Also velocity of blades = u = 20 m/s

From the outlet velocity triangle:

$\alpha_2 = 90°$, $V_{f2} = V_2$ and $V_{u2} = 0$.

Also $V_2 = \sqrt{v_{r2}^2 - u^2} = \sqrt{(21.166)^2 - (20)^2} = 6.928$ m/s

$$\tan \beta_2 = \frac{V_{f2}}{u} = \frac{V_2}{u} = \frac{6.928}{20} = 0.346$$

$\beta_2 = 19{\cdot}106°$ = Angle made by the tangent to the relative velocity with the direction of motion of vanes at the outlet.

Since a series of vanes are used, the vanes capture all of the discharge from the nozzle. Hence, the full discharge Q is used in the calculation of the power.

Power per unit weight of water $P_1 = \dfrac{\rho Q(V_{u1} + V_{u2})u}{\rho g Q} = \dfrac{(V_{u1} + V_{u2})u}{g}$

$$= \frac{(37.59 + 0) \times 20}{9.81} = 76.636 \text{ m}$$

Kinetic Energy per unit weight of water = Energy head

$$= H_{E1} = \frac{V_1^2}{2g} = \frac{(40)^2}{2g} = 81.549 \text{ m}$$

Efficiency of the system = $\eta = \dfrac{P_1}{H_{E1}} = \dfrac{76.636}{81.549} = 0.9397 = 94\% = 94.0\%$

Check on Calculation of Efficiency:

This problem is of Case-G2 type with $\alpha_2 = 90°$. From Table 1.2

$$\eta = \frac{2V_{u1}u}{V_1^2} = \frac{2 \times 37.59 \times 20}{(40)^2} = \frac{37.59}{40} = 0.9397 = 94\%$$

***EXAMPLE 1.12

A free jet of water impinges on a series of vanes at an angle α_1 with the direction of motion of vanes. The blade angle at inlet and outlet, defined as angles made by the relative velocities with the negative direction of motion of the vane, are β_1 and β_2 respectively. If the relative velocity at exit is K times the relative velocity at entry and $\alpha_2 < 90°$, show that the maximum efficiency that can be achieved from the system is

$$\eta_{max} = \frac{\cos^2 \alpha_1}{2}\left[1 + \frac{K \cos \beta_2}{\cos \beta_1'}\right] \text{ where } \beta_1' = (180 - \beta_1).$$

Solution

Figure 1.22 shows the typical velocity diagrams for flow over a curved vane with $\alpha_2 < 90°$. The notations of the angles are as per the problem.

This vane is part of a series of such vanes undergoing impact from the jet. For $\alpha_2 < 90°$, it is known from linear momentum equation that the force on the vane in the direction of motion of the vane is (From Eq. 1.32 or from Table 1.2, Case G2, $\alpha_2 < 90°$).

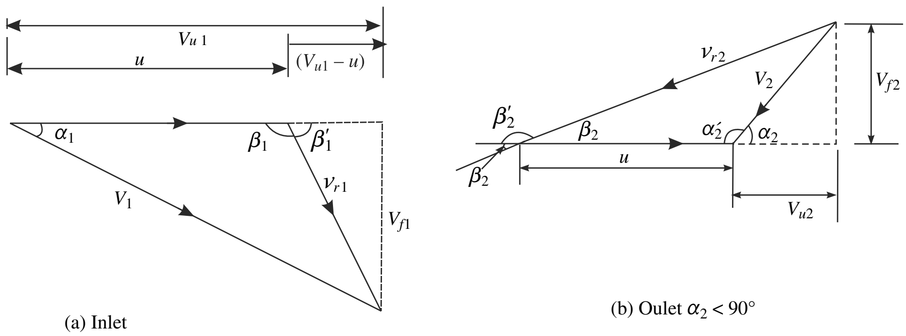


Fig. 1.22 *Velocity triangles of Example 1.12*

$F_x = \rho(AV_1)\,(V_{u1} \pm V_{u2})$ with positive sign for $\alpha_2 < 90°$. Thus,

$F_x = \rho(AV_1)\,(V_{u1} \pm V_{u2})$.

Since all the discharge from the jet will be intercepted by the series of vanes the full discharge ($Q = AV_1$)is used in the calculation of the momentum flux.

The power extracted by the series of vanes is $P = \rho(AV_1)\,(V_{u1} \pm V_{u2})u$

The efficiency of the system is $\eta = \dfrac{2(V_{u1} + V_{u2})u}{V_1^2}$

From inlet and outlet velocity triangles

$$V_{u1} = u + v_{r1} \cos \beta_1'$$

and $V_{u2} = v_{r2} \cos \beta_2 - u$

$$\text{Thus } \eta = \frac{2(v_{r1} \cos \beta_1' + v_{r2} \cos \beta_2)u}{V_1^2} = \frac{2v_{r1} \cos \beta_1' \left(1 + \dfrac{v_{r2} \cos \beta_2}{v_{r1} \cos \beta_1'}\right)u}{V_1^2}$$

Since $v_{r2} = Kv_{r1}$, the efficiency η can be written as

$$\eta = \frac{2v_{r1} u \cos \beta_1' \left(1 + K \dfrac{\cos \beta_2}{\cos \beta_1'}\right)}{V_1^2} \qquad \text{(i)}$$

Since $(V_{u1} - u) = v_{r1} \cos \beta_1' = V_1 \cos \alpha_1 - u$,

$$v_{r1} = \frac{(V_1 \cos \alpha_1 - u)}{\cos \beta_1'}$$

Substituting for v_{r1} and putting $\dfrac{u}{V_1} = \varepsilon$ in (i)

$$\eta = \frac{2(V_1 \cos \alpha_1 - u)u \left(1 + K \dfrac{\cos \beta_2}{\cos \beta_1'}\right)}{V_1^2} = 2(\cos \alpha_1 - \varepsilon)\varepsilon \left(1 + K \frac{\cos \beta_2}{\cos \beta_1'}\right)$$

For maximum efficiency $\dfrac{d\eta}{d\varepsilon} = 0$

Since $\left(1+K\frac{\cos\beta_2}{\cos\beta_1'}\right)\neq 0, \frac{d}{d\varepsilon}[\cos\alpha_1-\varepsilon]\varepsilon=0$

$[\cos\,\alpha_1-2\varepsilon]=0$ and hence $\varepsilon=\frac{\cos\alpha_1}{2}$.

Maximum efficiency $\eta_{max}=2\left(\cos\alpha_1-\frac{\cos\alpha_1}{2}\right)\left(\frac{\cos\alpha_1}{2}\right)\left(1+K\frac{\cos\beta_2}{\cos\beta_1'}\right)$

$$\eta_{max}=\frac{\cos^2\alpha_1}{2}\left(1+K\frac{\cos\beta_2}{\cos\beta_1'}\right)$$

NOTE:

It will be shown later (Chapter 2) that a particular case of this equation is used to get the maximum efficiency of Pelton turbine.

1.7 BASIC FEATURES OF HYDRAULIC TURBINES

1.7.1 Introduction to Turbines

A turbine is a device to convert the stored energy of a fluid mass in to mechanical energy. Depending upon the nature of the fluid involved, we have hydraulic turbine, gas turbine, steam turbine, etc. In a hydraulic turbine, the operating fluid is water and the energy of the fluid is in various combinations of potential energy and kinetic energy. The turbine transforms this stored energy of water in to the rotation of a shaft through the medium of a runner. Presently, a majority of turbines has an electric generator mounted on the shaft of the turbine and the energy of the water entering the turbine is thus ultimately converted in to electric energy.

The present-day hydraulic turbines are descendants of the water wheels of yore. Use of the energy of flowing water through water wheels as prime movers to run machinery for milling flour, pounding etc., was known to ancient India, China, Egypt, and several European countries. Earlier water wheels essentially worked on the principle of impact of water flow on blades mounted on a wheel. Many variations of this principle in the form of undershot wheels, breast wheels, etc., were in use. Development of an impulse turbine by LA Pelton in 1898 wherein a jet impacts on a series of specially formed cups mounted on a wheel was the forerunner of the present-day Pelton turbine. Use of reaction principle in utilising the energy of water efficiently was first successfully demonstrated in Furneyron turbine (1826). This turbine was an outward flow unit. While its efficiency was reasonably good, mechanical difficulties in its construction and operation led to its giving way to better inward flow turbines. In 1849, JB Francis successfully developed an efficient radial inward flow reaction turbine that effectively replaced the earlier forms. Now, all radial flow and mixed flow reaction turbines are classified as Francis turbines.

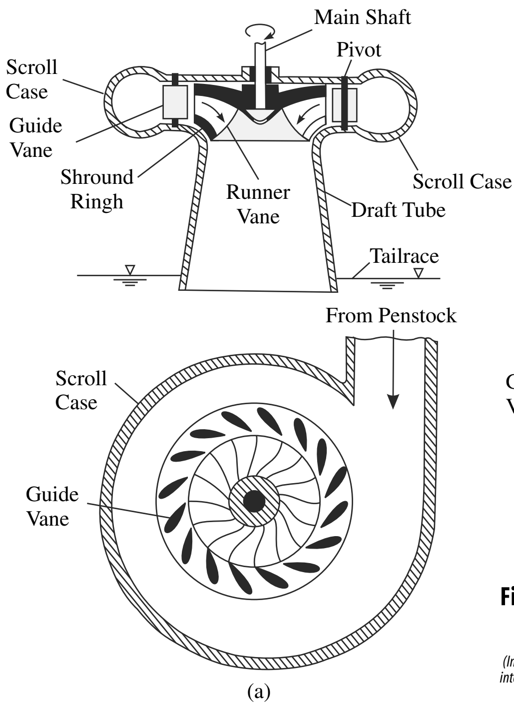


Fig. 1.23(a) *Schematic sketch of a Francis turbine*

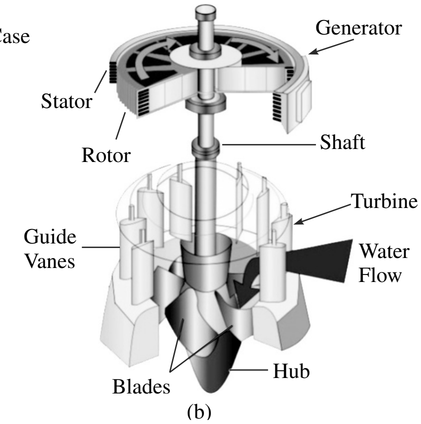


Fig. 1.23(b) *Schematic sketch of a Kaplan turbine and generator set.*

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Development of electric generators and fast hydroturbines as prime movers of these generators led to the obsolescence of slow water wheels. Very good descriptions of earlier water wheels along with relevant illustrations are available on the Internet, for example at ***http://en.wikipedia.org/wiki/Water_wheel#Greco-Roman_world***.

A modern hydraulic reaction turbine consists essentially of a rotating element, called runner, which is acted upon by water. The runner will have a series of blades. Appropriate casings (scroll casing) and guide mechanisms (guide vanes) are provided to have efficient entry of water to the rotor. The blades of the rotor are so shaped that they have efficient interaction with flowing water all the way from the inlet up to the exit. Figure 1.23(a) shows the basic layout and components of a reaction turbine known as *Francis turbine*. Water from a penstock enters the scroll casing, gets guided by the guide vanes and passes through the passage between the blades of the runner. While doing so the pressure of the water in the rotor changes. This type of turbine develops torque by reacting to the pressure of the fluid in a manner analogous to the rotation of a lawn sprinkler. The water that exits the runner passes through a divergent pipe known as *draft tube* and finally reaches the tailrace.

Another type of reaction turbine, known as *Kaplan turbine* consists essentially of a propeller acted upon by water under pressure. Figure 1.23(b) shows schematically the arrangements of such a turbine along with the generator.

The reaction turbines are very versatile in their capability to handle a wide range of discharges and heads and thus nearly 80% of water turbines in the world are of reaction type. The details of the reaction turbines are discussed in sufficient detail in chapters 2 and 3. The impulse turbines are described in detail in chapter 4.

1.7.2 Classification of Turbines

Turbines are classified in number of ways as follows;

1. Depending upon Nature of Interaction with Water Flow

They are broadly classified as (i) impulse turbines, and (ii) reaction turbines. The basic principles of impulse and reaction turbines are given in Sec. 1.5. The major differences in these two basic types of turbines can be summed up as below:

(i) Impulse Turbine In an impulse turbine, the pressure of water does not change while flowing through the rotor of the machine. In these turbines, the pressure change occur only in the nozzles of the machine and not in the rotor. The jet impingement takes place at atmospheric pressure. An example of impulse turbine is *Pelton Wheel* (Fig.1.7).

(ii) Reaction Turbine In a reaction turbine, the pressure of water changes while it flows through the rotor of the machine. The change in water velocity and reduction in its pressure causes a reaction on the turbine blades. The flow passages are under pressure and the rotor has to be encased in an airtight casing. Francis and Kaplan Turbines fall in the category of reaction Turbines, (Fig. 1.23(a) and 1.23(b)).

2. Depending upon the Nature of Flow in the Machine

The turbines are classified under this category as
(a) Radial flow (b) Axial flow, (c) Mixed flow, and (d) Tangential flow turbines.

(a) Radial Flow Turbine The flow from inlet to the outlet of the runner is along the radius. Inward radial flow is the most common feature. Early Francis turbines were of the inward radial flow type.

(b) Tangential or Peripheral Flow Turbine The flow direction is tangential to the runner. Pelton wheel where the path of the jet is tangential to the path of rotating wheel is an example of this type.

(c) Mixed Flow Turbine The water enters the rotor radially at the inlet and exits axially at the outlet. Present-day Francis turbine is a typical example of this category.

(d) Axial Flow Turbines In these, the flow enters and leaves the rotor axially. The rotor blades of these turbines are essentially in the form of a propeller, the flow enters and leaves in a direction parallel to axis of the shaft. Kaplan turbine, a turbine with adjustable propeller blades, is an example of axial flow turbine.

3. Depending upon the Head

The turbines are classified based on the gross head as follows:

(a) High-Head Units For gross heads greater than about 400 m are considered as high head. Pelton turbines are usually the first choice for high head, low discharge cases.

(b) Medium-Head Units Heads greater than about 60 m and less than 400 m are considered as medium head. Francis turbines are commonly used in this range.

(c) Low-Head Units Heads in the range 3 m to 60 m are classified as low head units. Classification based on head is not a rigorous classification and is subjective.

4. Depending on the Specific Speed

Classification depending upon the specific speed is a better method than the classification based on head only. In this, both head and discharge are accounted for in an objective way. Details of specific speed and its use are given Sec. 1.8.4. Table 1.3 is a brief summary of various types of turbines used under different ranges of specific speed.

Table 1.3 *Specific speeds of different kinds of turbines*

Type of turbine	Specific speed range (N_s in kW-rpm-m units)
Pelton (Single jet) (or per jet in a multi jet)	8–30
Francis	40–450
Kaplan	300–900

[Note these are approximate values]

5. Classification According to the Name of the Originator

Some of the names commonly associated with a turbine type originated by them are:

a) Francis turbine (after James B. Francis)
b) Kaplan Turbine (after Viktor Kaplan)
c) Pelton turbine (after Lester Allen Pelton)
d) Deriaz turbine (after Paul Deriaz)
e) Banki or Mitchel turbine (after Donat Banki; Anthony Mitchell)

1.7.3 Efficiencies of Turbines

Figure 1.24 is a definition sketch giving a schematic layout of a reaction turbine setup together with definition of various heads. Section 1 marks the entrance to the

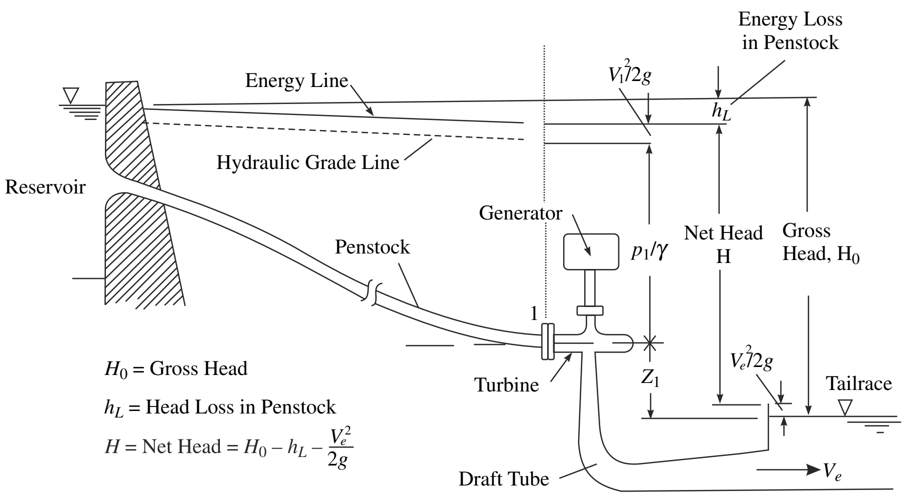

Fig. 1.24 *Schematic layout of a reaction turbine*

turbine. V_e is the velocity at the exit of the draft tube. The velocity head $\frac{V_e^2}{2g}$ represents the energy head not utilised by the turbine. For the turbine, the gross head H_0 is the difference between the upstream reservoir water level and the water level in the tailrace channel on the downstream. If h_L is the head loss in the *penstock* (water conductor system), the head available at entrance to the turbine minus the unutilized energy at the exit of the draft tube is called *net head*. The net head for a reaction turbine is $H = \left(H_0 - h_L - \frac{V_e^2}{2g}\right)$. For an impulse turbine, the definition of the net head is slightly different, (Sec.4.2.1).

If a discharge Q is admitted to the turbine under a net head H, the action of this water on the turbine causes the rotor to rotate at an angular velocity ω with a torque T being delivered to the shaft. In this set up the power taken from the water is γQH(known as *water power*) and the power delivered to the shaft, known as *brake power*, is $T\omega$. In practice, the brake power is always less than the power extractable from water and the ratio $\frac{T\omega}{\gamma QH} = \eta_0$ represents the *overall efficiency* of the machine. This efficiency can be considered in three components as below.

Consider a turbine unit with net head H and discharge Q available to the turbine at the inlet.

1. Volumetric Efficiency, η_v

It is possible that out of a total discharge of Q supplied to the turbine, some quantity of discharge (Q_L) supplied at the inlet of the turbine may exit the turbine in the form of various leaks without doing any work on the rotor. The volumetric efficiency of the turbine η_v is defined as

$$\eta_v = \frac{Q - Q_L}{Q} = \frac{\text{Discharge doing work on the rotor}}{\text{Discharge supplied to the rotor}} \tag{1.34}$$

Generally, leakage is a very small percentage of the discharge Q and is of the order of 0.5%.

2. Hydraulic Efficiency, η_h

While the net head H is available to the turbine, the energy head extracted by the turbine runner H_e will be less than H by an amount h_{fr} due fluid friction and form loss at the rotor including entrance and exit losses at the rotor. Thus,

$$H_e = H - h_{fr}$$

The hydraulic efficiency of the turbine is given by

$$\eta_h = \frac{\text{Head extracted by rotor}}{\text{Net head available to the rotor}} = \frac{H - h_{fr}}{H} = \frac{H_e}{H} \tag{1.35}$$

The discharge acting on the rotor is $(Q - Q_L)$ and as such the power produced by the runner is $\gamma(Q - Q_L)\, H_e$.

3. Mechanical Efficiency, η_m

Due to inevitable mechanical friction that occur between the rotor and other parts of the turbine unit such as bearings, glands, couplings, etc., and also due to presence of other mechanical losses such as windage losses (where applicable), the power available at the rotor shaft will be less than the power produced by the runner. Let the head lost in mechanical friction and other losses be h_m. The power transmitted by the turbine unit to the shaft is $P_s = \gamma(Q - Q_L)\,(H_e - h_m)$.
The mechanical efficiency η_m is defined

$$\text{as} \quad \eta_m = \frac{\text{Power available at the rotor shaft}}{\text{Power produced by the runner}} = \frac{\gamma(Q-Q_L)(H_e-h_m)}{\gamma(Q-Q_L)H_e} \tag{1.36}$$

Overall Efficiency, η_0 The overall efficiency (also known as *total efficiency*) of the turbine is defined as the ratio of power available at the rotor shaft to the power supplied by the water to rotor in the turbine unit.

$$\text{Thus} \quad \eta_0 = \frac{\text{Power available at the rotor shaft}}{\text{Power supplied by water to the runner}} = \frac{\gamma(Q-Q_L)(H_e-h_m)}{\gamma QH} \tag{1.37}$$

From the definitions of η_v, η_h and η_m in Equations (1.34, 1.35 and 1.36) respectively the total efficiency η_0 could be written as

$$\eta_0 = \frac{(Q-Q_L)}{Q} \times \frac{\gamma(Q-Q_L)H_e}{\gamma(Q-Q_L)H} \times \frac{\gamma(Q-Q_L)(H_e-h_m)}{\gamma(Q-Q_L)H_e}$$

$$\text{giving} \quad \eta_0 = \eta_v\,\eta_h\,\eta_m \tag{1.38}$$

The sequence of various losses and their locations in the system and components of the overall efficiency are shown in Fig. 1.25.

Note that the total head loss, h_{fr} and h_m take place in the machine.

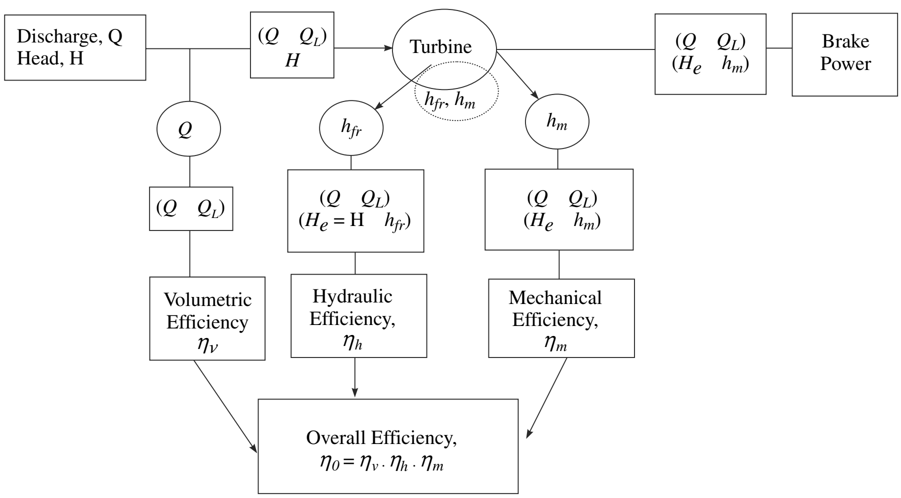

Fig. 1.25 *Schematic representation of turbine losses and efficiencies*

For a pump, the definitions of the efficiencies are analogous to the definitions given above for the turbine but with the essential difference that the power is supplied to the pump and a part of it is reflected as the power delivered to the fluid. The definitions of efficiencies for centrifugal pumps are given in chapter 5.

1.8 SIMILARITY LAWS AND SPECIFIC SPEED OF TURBINES

1.8.1 Similarity Laws for Turbines

Model studies are extensively used in professional practice to determine the performance characteristics of hydraulic machines through tests on similar scale models. Similarity laws, also known as *affinity laws*, enable one to estimate the prototype behavior from the data obtained from model studies. In addition, these laws can be used to estimate the performance of a turbine under different operating conditions. This section presents the salient scale ratios for modeling turbines and an important performance indicator known as *specific speed* that emanates from these modeling laws is highlighted.

A set of turbines are said to be *homologous* if they are geometrically similar and their velocity triangles are also similar. This assures that in homologous turbines, the flow lines are similar and their performance characteristics also bear a similarity relationship. Viscous effects are neglected and it is assumed that efficiencies of homologous turbines of differing sizes and operating conditions remain constant. Hence, the velocity triangles of a set of geometrically similar turbines will be similar making them belong to a set of homologous turbines. This leads to an important relationship that all velocities in this set will bear constant ratio to any selected velocity. Based on this, the following relationships for turbines are established.

1.8.2 Similarity Ratios for Practical Use

The discharge Q, reference diameter of the rotor D, rotational speed N, head H and power P are the most important physical variables governing the performance of a turbine. Considering the diameter D, speed N as repeating variables, a set of three ratios each containing one parameter out of Q, P and H is set out. Relationship is established between two similar turbines in terms of these ratios.

Consider a turbine set-up with an operating head H. In this, any velocity is proportional to $H^{1/2}$. Expressing the proportionality by the symbol $\propto$,

$$\text{Any velocity } V \propto \sqrt{H} \text{ and hence } \frac{V^2}{2g} \propto H \tag{1.39}$$

Discharge $Q = AV$ where A = Area of flow which is proportional to D^2.

$$\text{Hence } Q \propto D^2 \sqrt{H} \tag{1.40}$$

Power $P \propto QH$ and hence

$$P \propto H^{3/2} D^2 \tag{1.41}$$

The peripheral velocity of the turbine $u = \dfrac{\pi DN}{60}$ and hence $u \propto \sqrt{H}$

leading to $N \propto \dfrac{H^{1/2}}{D}$ (1.42)

or $H \propto N^2 D^2$ (1.42-a)

Using the relationships (1.40) through (1.42), the following affinity relations are established:

From (1.42-a), $\dfrac{ND}{H^{1/2}}$ = constant. Thus between two homologous turbines 1 and 2

$$\frac{N_1 D_1}{H_1^{1/2}} = \frac{N_2 D_2}{H_2^{1/2}} = \text{Constant} \tag{1.43}$$

The parameter $\dfrac{ND}{H^{1/2}}$ is called the *head coefficient*, C_H.

Replacing H in Eq. (1.40) by the result of Eq. (1.42-a),

$\dfrac{Q}{ND^3}$ = Constant. Thus, between two homologous turbines 1 and 2

$$\frac{Q_1}{N_1 D_1^3} = \frac{Q_2}{N_2 D_2^3} = \text{Constant} \tag{1.44}$$

The parameter $\dfrac{Q}{ND^3}$ is called the *flow coefficient*, C_Q.

Similarly, by replacing H in Eq. (1.41) by the result of Eq. (1.42-a)

$\dfrac{P}{N^3 D^5}$ = Constant. Thus between two homologous turbines 1 and 2

$$\frac{P_1}{N_1^3 D_1^5} = \frac{P_2}{N_2^3 D_2^5} = \text{Constant} \tag{1.45}$$

The parameter $\dfrac{P}{N^3 D^5}$ is called the *power coefficient*, C_P.

The similarity relations Eq. (1.43) through (1.45), covering C_H, C_Q and C_P, can be considered as the basic similarity ratios. They are of immense use in deriving other supplementary ratios as well as in comparing the performance of two homologous turbines.

1.8.3 Modeling of Efficiencies

In homologous turbines, through neglect of viscous effects it is assumed that the efficiencies to be the same for both model and prototype. However, it has been found in practice that large machines have smaller clearance to runner ratio and also smaller relative surface roughnesses. Further, larger machines have higher Reynolds number. This would give larger machines greater efficiencies when compared to a homologous smaller machine. Hence, modeling of efficiencies is done by use of empirical equations. *Moody's step-up formula* for turbines, which is also used for pumps, as given below is one such empirical equation which is in common use for modeling efficiencies due to size changes in two homologous machines 1 and 2:

Moody Step-up Formula for Efficiency

$$\frac{1-\eta_2}{1-\eta_1} = \left(\frac{D_1}{D_2}\right)^{1/5} \qquad (1.46)$$

In this equation, η is the overall efficiency and D is the diameter of the rotor. This formula is based on some theoretical arguments and has been found in practice to give satisfactory results. Equation 1.46, however, is not applicable to Pelton turbines, as the efficiencies of Pelton turbines are generally taken as independent of the size in preliminary computations.

1.8.4 Specific Speed of a Turbine

From the relationships Eq.(1.43) through (1.45) developed above, another factor involving P, N and H that finds extensive use can be developed.

From Eq. (1.43) $$D^2 = \frac{H}{N^2} \qquad (1.47)$$

Substituting Eq.(1.47) in Eq.(1.45), $\frac{PN^2}{H^{5/2}}$ = Constant. That is $N \propto \frac{H^{5/4}}{\sqrt{P}}$

If H = 1 unit and P is 1 unit, let the resulting speed $N = N_s$.

Now $N = \frac{N_s H^{5/4}}{\sqrt{P}}$ from which the term N_s is expressed as

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} \qquad (1.48)$$

This term N_s is known as *specific speed* and forms the most important factor to describe characteristics of a turbine.

A turbine can be run at varying speeds and efficiency of operation depends on the values of N, P and H. As such, it can have a range of $\frac{N\sqrt{P}}{H^{5/4}}$ values. However, as a convention, the value of $\frac{N\sqrt{P}}{H^{5/4}}$ calculated by using the values of N, P and H prevalent at the condition at the *maximum efficiency* of operation is called the specific speed of the turbine. Thus, if the value of N_s for a turbine is mentioned as 100, it indicates a turbine which has at *maximum efficiency* the relation $\frac{N\sqrt{P}}{H^{5/4}} = N_s = 100$ and in the derivation of N_s the value of speed N used is in rpm, the value of power P is in kW and the head H is in metres.

Thus, the specific speed represents the rotational speed of a homologous turbine that produces one unit of power at unit head, under conditions of maximum efficiency. Note that the specific speed is not dimensionless; it has the dimensions of $[F^{1/2}L^{-3/4}T^{-3/2}]$ but as a convention the units are not mentioned. Thus, the value of specific speed of a given machine depends on the system of units adopted. In this book the *SI* system of units (with *kW* for power, *rpm* for speed and *m* for head) are used.

The value of N_s remains the same for all turbines of the same design. Consequently, the specific speed is essentially related to the shape and type of the turbine and not on its size. Thus, it is a *type characteristic* of the turbine. The range of values of specific speed is distinctly different for different types of turbines. This fact makes the specific speed to be used extensively as an aid in selection of appropriate turbines for a specific hydropower development work.

An inspection of Eq. (1.48) that is used in the definition of N_s indicates that

i) For given P and H, turbines with low specific speed have low rpm and the speed N increase directly with the specific speed.

ii) Since $N_s \propto \dfrac{1}{H^{5/4}}$, for higher heads lower specific speed turbines are used.

iii) Diameter D of the runner is not explicitly involved in the derivation of the specific speed.

Nondimensional Specific Speed

The specific speed can be expressed in a nondimensional form to avoid the possible confusion resulting from the system of units used in its derivation. The nondimensional form of specific speed is known variously as *Shape factor/ Shape number/ Speed number* and is given as

$$S_p = \frac{N\sqrt{\dfrac{P}{\rho}}}{(gH)^{5/4}} = \frac{N\sqrt{P}}{\rho^{1/2}(gH)^{5/4}} \tag{1.49}$$

It should be noted that in calculating the shape number S_p (in revolutions), the units of N should be in revolutions/second; Power P in watts and Head H in metres. Thus, for water turbines using the value of ρ = 998 kg/m^3 and g = 9.81 m/s^2, the conversion relation of N_s to S_p in *revolutions*

$$S_p = 9.61 \times 10^{-4} N_s \tag{1.50}$$

Also, Specific speed in dimensionless form (shape factor) in *radians* is

$$S_{pr} = 2\pi \times 9.61 \times 10^{-4} N_s \tag{1.50-a}$$

NOTE

The reader is cautioned to be careful in interpreting **specific speed numbers** of turbines that he may come across in literature; in Internet; books and technical papers. There are many different systems of units in use for the specific speed. While most of recent literature from India uses *SI* units (with kW, rpm and m) there are many who use metric system with power in metric horse power, speed in rpm and head in meters. This is particularly so in old books and old technical material. Then there is the use of *FPS* system used in literature from *USA* and *UK*. In *FPS* system N is in rpm, P is in horsepower and H is in feet.

In this book, specific speed for turbines is derived by using kW for power, rpm for speed and meter for head. This practice (known as specific speed in *SI* units) is followed consistently throughout the book.

1.8.5 Similarity Ratios of Turbines by Dimensional Analysis

The three coefficients, viz., head coefficient (C_H), flow coefficient (C_Q), power coefficient (C_P), and the specific speed (N_s), discussed in the previous section were arrived at by logical reasoning and by necessity of simple ratios for practical use. In this section it is shown that these ratios can be derived by dimensional analysis procedure and modifying the ratios to suit the convenience of practical use.

NOTE

It is expected the reader is familiar with the basic aspects of *Dimensional Analysis* in an earlier exposure to basic course in Fluid Mechanics. However, for the benefit of those who are not exposed to this topic earlier, a brief introduction to dimensional analysis, along with a few illustrative examples, is given in Appendix-A1. This Appendix, titled *Dimensional Analysis and Similitude*, contains a brief introduction, Raleigh's Method and Buckingham Pi Theorem of performing dimensional analysis and basic aspects related to the similitude in Fluid Mechanics. A few worked examples to illustrate the analysis procedures are also included

The important physical variables governing the performance of a turbine can be listed as follows (Table 1.4):

Table 1.4 *Variables affecting performance of a turbine*

Symbol	Variable	Dimensions
D	Diameter of the runner, (Reference length parameter)	$[L]$
N	Rotational speed (*RPM*)	$[T^{-1}]$
H	Energy head (= Energy per unit weight)	$[L]$
Q	Discharge through the machine	$[L^3 T^{-1}]$
P	Power developed by rotor	$[ML^2 T^{-3}]$
g	Acceleration due to gravity	$[LT^{-2}]$
ρ	Density of water	$[ML^{-3}]$
μ	Coefficient of dynamic viscosity of water	$[ML^{-1} T^{-2}]$

Note that the fluid property K = Bulk modulus of elasticity, is not included as the water flow is incompressible. Similarly, the surface tension does not have any influence in the problem under consideration and as such is not included. Further, there is no direct gravity effect on the flow phenomenon as the flow in the machinery is essentially a closed conduit flow situation. It is thus usual practice to consider (gH)

as one variable instead of g and H separately. The term (gH) represents the energy per unit mass of the fluid flow and is sometimes called as *specific energy*. Thus, for a hydroturbine by considering the power developed by the rotor, P, as the dependent variable

$$P = f(D, N, gH, Q, \rho, \mu) \tag{1.51}$$

By using the Buckingham Pi theorem method of dimensional analysis, there are seven variables and three basic dimensions. Hence there will be four dimensionless Pi terms governing the phenomenon. Considering D, N and ρ as repeating variables it can be shown by dimensional analysis that the four dimensionless Pi terms can be expressed as

$$\pi_1 = Fn\,(\pi_2, \pi_3, \pi_4)$$

where the dimensionless terms are $\pi_1 = \dfrac{P}{\rho N^3 D^5}$, $\pi_2 = \dfrac{gH}{N^2 D^2}$, $\pi_3 = \dfrac{Q}{ND^3}$ and $\pi_4 = \dfrac{\rho ND^2}{\mu}$.

Thus,

$$\frac{P}{\rho N^3 D^5} = fn\left(\frac{gH}{N^2 D^2}, \frac{Q}{ND^3}, \frac{\rho ND^2}{\mu}\right) \tag{1.52}$$

Here the term $\pi_4 = \dfrac{\rho ND^2}{\mu}$ is the Reynolds number and in the present similitude its role is to ensure that model and the prototype are both in either turbulent mode or both in laminar mode. Hence, by making certain that the model and the prototype are both in the turbulent mode, the term $\pi_4 = \dfrac{\rho ND^2}{\mu}$ can be dropped in the functional relationship of Eq. (1.52), to obtain the functional relationship

$$\frac{P}{\rho N^3 D^5} = fn\left(\frac{gH}{N^2 D^2}, \frac{Q}{ND^3}\right) \tag{1.53 }$$

It is interesting to see that by considering $\dfrac{\pi_1^{1/2}}{\pi_2^{5/4}} = \dfrac{\left(N\sqrt{P}\right)}{H^{5/4}}\left(\dfrac{1}{\rho^{1/2} g^{5/4}}\right)$ one gets the nondimensional specific speed, mentioned earlier as shape number S_p in Eq. (1.49).

In practical use, since water is the operating fluid (with relatively very little change in its value of density), the density term ρ is dropped from the π terms. So also, the g term. Thus, the π terms get morphed for practical use as

$$\frac{P}{N^3 D^5} = fn\left(\frac{H}{N^2 D^2}, \frac{Q}{ND^3}\right) \tag{1.54}$$

For homologous turbines, each of these three ratios of Eq.(1.54) in the model must have the same value with the corresponding term in the prototype. Thus, between a model and its prototype

$$\left[\frac{P}{N^3 D^5}\right]_m = \left[\frac{P}{N^3 D^5}\right]_p = \text{Constant} \tag{1.55}$$

$$\left[\frac{Q}{ND^3}\right]_m = \left[\frac{Q}{ND^3}\right]_p = \text{Constant} \tag{1.56}$$

$$\left[\frac{ND}{\sqrt{H}}\right]_m = \left[\frac{ND}{\sqrt{H}}\right]_p = \text{Constant} \tag{1.57}$$

Note the expressions in Eq. (1.55 through 1.57) are the same as the corresponding expressions of power coefficient (C_P), flow coefficient (C_Q) and head coefficient (C_H) given in Equations (1.45, 1.44 and 1.43) respectively.

Also, by not considering ρ and g in the π–terms, the Shape number S_p would morph to $\left[\frac{N\sqrt{P}}{H^{5/4}}\right]_m = \left[\frac{N\sqrt{P}}{H^{5/4}}\right]_p$ = Specific speed = N_s (1.58)

1.8.6 Specific Quantities

Another set of *specific quantities*, which can be called as *double unit quantities*, are in use in analyzing performance parameters of homologous turbines. These are:

1. Specific Rate of Flow, Q_{11}

It is the discharge of a homologous turbine of unit reference diameter working on unit head. It is designated as Q_{11} and is given by $Q_{11} = \frac{Q}{D^2\sqrt{H}}$.

For similarity between homologous model and prototype turbines

$(Q_{11})_{\text{model}} = (Q_{11})_{\text{prototype}}$.

Thus between a model and its prototype

$$Q_{11} = \left[\frac{Q}{D^2\sqrt{H}}\right]_m = \left[\frac{Q}{D^2\sqrt{H}}\right]_p = \text{Constant} \tag{1.59}$$

2. Specific Power P_{11}

It is the power produced by a homologous turbine of unit reference diameter working on unit head. It is designated as P_{11} and is given by $\frac{P}{H^{3/2}D^2} = P_{11}$.

For similarity between homologous model and prototype turbines

$(P_{11})_{\text{model}} = (P_{11})_{\text{prototype}}$.

Thus between a model and prototype set

$$P_{11} = \left[\frac{P}{H^{3/2}D^2}\right]_m = \left[\frac{P}{H^{3/2}D^2}\right]_p = \text{Constant} \tag{1.60}$$

3. Defined Double Unit Speed, N_{11}

It is the unit speed of a homologous turbine of unit diameter. Since the term specific speed has already been used in another context, N_{11} does not have a defined name like other two specific quantities. It could, however, be called *defined speed*

$N_{11} = \frac{ND}{\sqrt{H}}$. For similarity between homologous model and prototype turbines $(N_{11})_{\text{model}} = (N_{11})_{\text{prototype}}$. Thus between a model and prototype

$$N_{11} = \left[\frac{ND}{\sqrt{H}}\right]_m = \left[\frac{ND}{\sqrt{H}}\right]_p = \text{Constant} \tag{1.61}$$

These three specific quantities find use in developing *iso-efficiency charts* and in other problems associated with model scaling up. (See Chap. 2, Sec. 2.10.3). Further, they are of use in determining model–prototype relations of various parameters in model studies on turbines. They are supplementary to the basic similarity ratios (C_H, C_Q and C_P) contained in equations (1.43 through 1.45).

1.8.7 Unit Quantities

It was shown that the specific speed N_s is the same for both model and prototype of a homologous hydraulic turbine set and Equations (1.43) through (1.45) enable the parameters, N, Q and P for a prototype to be established based on the model values of the corresponding items. However, when the diameter ratio is unity, it is easy to see from Equations ((1.43) through (1.45) that N, Q and P vary only on the head H as

$$N \propto H^{1/2} \tag{1.62}$$

$$Q \propto H^{1/2} \tag{1.63}$$

$$P \propto H^{3/2} \tag{1.64}$$

The constants of proportionality in equations (1.62), (1.63) and (1.64) are expressed in terms *unit quantities* as below:

1. The *unit discharge* Q_u is defined as the discharge of a geometrically similar turbine working under a unit head (of 1 m).
   Thus $Q_u = Q/H^{1/2}$ (1.65)
2. The *unit power* P_u is defined as the power of a geometrically similar turbine working under a unit head (of 1 m).
   Thus $P_u = P/H^{3/2}$ (1.66)
3. The *unit speed* N_u is defined as the speed of a geometrically similar turbine working under a *unit head* (of 1 m).
   Hence, $N_u = N/H^{1/2}$ (1.67)

Further, note that the unit quantities of Eq. 1.65, 1.66 and 1.67 are special cases of the respective specific quantities Q_{11}, P_{11} and N_{11} (Equations (1.59, 1.60 and 1.61) when $D_m = D_p$). Thus for two homologous turbines 1 and 2 having the same diameter ($D_1 = D_2 = D$), the discharge, power and speed, in these two turbines are related as

$$Q_{u1} = Q_{u2} \text{ that is } Q_{u1} = \left[\frac{Q}{\sqrt{H}}\right]_1 = \left[\frac{Q}{\sqrt{H}}\right]_2 = \text{Constant} \tag{1.68}$$

$$P_{u1} = P_{u2} \text{ that is } P_{u1} = \left[\frac{P}{H^{3/2}}\right]_1 = \left[\frac{P}{H^{3/2}}\right]_2 = \text{Constant} \tag{1.69}$$

$$N_{u1} = N_{u2} \text{ that is } N_{u1} = \left[\frac{N}{\sqrt{H}}\right]_1 = \left[\frac{N}{\sqrt{H}}\right]_2 = \text{Constant} \tag{1.70}$$

Alternatively, if a given turbine runs at two levels of operation, say speed N_1 and speed N_2 with corresponding discharges Q_1 and Q_2 and power P_1 and P_2, these parameters N, Q and P are related by the unit quantity relations given in Eqs. (1.68, 1.69 and1.70). Accordingly, the unit quantity relationships, Eq. (1.68) through Eq. (1.70), are particularly advantages when studying the performance of a given turbine under varying heads. Note that the unit quantities are not the basic similarity ratios but are only special cases corresponding to $D_1 = D_2$ in the cases under study.

1.8.8 Summary

Table 1.5 lists the basic and derived similarity ratios that should have the same value in the model and in the prototype turbines for similarity to exist. Observe that there are three basic ratios, one each, for calculating the respective dependent variable N, Q and P. The derived ratios are obtained from these basic ratios for convenience of calculations. Unit quantities being special cases of specific (double unit) quantities like P_{11}, Q_{11} and N_{11} under the condition $D_m = D_p$ are not included in the table.

Table 1.5 *Ratios for similarity in turbines*

Basic Similarity Ratios	Derived Similarity Ratios
$\frac{N_m D_m}{H_m^{1/2}} = \frac{N_p D_p}{H_p^{1/2}}$ = Constant = C_H	$N_{11} = \left[\frac{ND}{\sqrt{H}}\right]_m = \left[\frac{ND}{\sqrt{H}}\right]_p$ = Constant
$\frac{Q_m}{N_m D_m^3} = \frac{Q_p}{N_p D_p^3}$ = Constant = C_Q	$Q_{11} = \left[\frac{Q}{D^2\sqrt{H}}\right]_m = \left[\frac{Q}{D^2\sqrt{H}}\right]_p$ = Constant
$\frac{P_m}{N_m^3 D_m^5} = \frac{P_p}{N_p^3 D_p^5}$ = Constant = C_P	$P_{11} = \left[\frac{P}{H^{3/2}D^2}\right]_m = \left[\frac{P}{H^{3/2}D^2}\right]_p$ = Constant
	Specific Speed = N_s $\left[\frac{N\sqrt{P}}{H^{5/4}}\right]_m = \left[\frac{N\sqrt{P}}{H^{5/4}}\right]_p$ = Constant

The ratio of corresponding length parameters, such as the diameter, in the model and prototype is known as the scale ratio. Thus, a scale ratio of say 1/5 means $\frac{\text{Diameter of model}}{\text{Diameter of prototype}} = \frac{D_m}{D_p} = \frac{1}{5}$. Note that the ratio of heads in model and prototype is **not** the scale ratio. It represents the ratios of energy per unit weight of fluid that happens to have the dimension of length.

1.8.9 Role of Model Testing of Hydraulic Turbines

Just as the design of all major hydraulic structures like spillways, water intakes, energy dissipaters etc. which involve complex hydraulic behavior needs to be vetted through model testing, the design of all major hydraulic turbines are also tested through physical model studies for their performance characteristics. Model scaling laws as indicated in equations (1.43), (1.44) and (1.45) are used to scale various parameters in geometrically similar models and prototypes. It is interesting to note that physical model testing is currently the only way to guarantee the efficiency and performance of the prototype to be supplied by the manufacturer to the client. The similitude rules and procedures are strictly defined in International *IEC* standards 60193-1999(Ed.2)–*Hydraulic Turbines, Storage Pumps and Pump Turbines–Model Acceptance Tests*. These rules are of mandatory nature for all turbines and pumps of unit power more than 5 MW or with reference diameter greater than 3.0 m.

(For further details of model testing of hydraulic turbines, see Chapter 4, Sec. 4.10).

1.9 ILLUSTRATIVE EXAMPLES—SET 1.2

**EXAMPLE 1.13

In a hydroelectric project, the upstream reservoir water level is 200 m above the tailrace water level of the power plant. When a discharge of 3.0 m^3/s is supplied to the turbine, the frictional losses in the penstock are 20 m and the head utilised in the turbine is 160 m. The leakage loss is estimated at 0.1 m^3/s and the mechanical losses can be taken as 100 kW.

(a) Calculate the (i) volumetric efficiency, (ii) hydraulic efficiency (iii) mechanical efficiency, and (iv) overall efficiency of the system.

(b) If a homologous turbine having a runner of diameter ratio 0.8 were tested under similar conditions, what would be its overall efficiency?

Solution

Given: Gross head = H_0 = 200 m,

Penstock losses = h_f = 20 m,

Net head =H = 200 – 20 = 180 m,

Utilised head = H_e = 160

(i) Volumetric efficiency $\eta_v = \dfrac{Q - Q_L}{Q} = \dfrac{3.0 - 0.1}{3.0} = 0.967 = 96.7\%$

(ii) Hydraulic efficiency $\eta_h = \dfrac{H_e}{H} = \dfrac{160}{180} = 0.889 = 88.9\%$

Theoretical power extracted = $P_{th} = \gamma(Q - Q_L)\, H_e = 9.79 \times 2.9 \times 160$

$= 4542.6$ kW

Actual Brake power developed = 4542.6 – 100 = 4442.6 kW

(iii) Mechanical efficiency $\eta_m = \dfrac{4442.6}{4542.6} = 97.8\%$

(iv) Overall efficiency $\eta_0 = \eta_v \eta_h \eta_m = 0.967 \times 0.889 \times 0.978 = 84.1\%$

(b) By Moody's scale-up formula: $\frac{1-\eta_1}{1-\eta_2} = \left(\frac{D_2}{D_1}\right)^{0.2}$

$$\frac{1-0.841}{1-\eta_2} = (0.8)^{0.2} = 0.956$$

$$1 - \eta_2 = \frac{0.159}{0.956} = 0.166$$

$$\eta_2 = 1 - 0.166 = 0.834 = 83.4\%$$

**EXAMPLE 1.14

In a hydroelectric project, the available discharge is 60 m³/s with a net head of 35 m. Assuming a turbine efficiency of 92% and rotational speed of 300 rpm, determine the least number of turbines of the same size and having a specific speed of 275.

Solution

Given: Power potential $P = \eta_0\, \gamma Q\, H = 0.92 \times 9.79 \times 60 \times 35 = 16212$ kW

For a specific speed of 250, power produced per machine P_1 is given by:

$$N_s = \frac{N\sqrt{P_1}}{H^{5/4}} = 275 = \frac{300\sqrt{P_1}}{(35)^{5/4}}$$

and $$P_1 = \left(\frac{N_s}{N}\right)^2 H^{5/2} = \left(\frac{275}{300}\right)^2 \times (35)^{5/2} = 6090 \text{ kW}$$

Number of turbines required $n \approx (16212)/6090 = 3$

*EXAMPLE 1.15

A turbine is to operate under a head of 25 m at a speed of 300 rpm. The discharge is 12 m³/s. Assuming an efficiency of 0.85, calculate the power developed. What would be the specific speed, power, discharge and rotational speed at a head of 15 m?

Solution

$$P = \eta\, \gamma Q\, H = 0.85 \times 9.79 \times 12 \times 25 = 2496.5 \text{ kW}$$

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{300\sqrt{2496.5}}{(25)^{5/4}} = 268.1$$

At a head of 15 m:

N_s remains the same at a value of 268.1

Since the diameter is the same, i.e. $D_1 = D_2$, $\frac{N_1}{H_1^{1/2}} = \frac{N_2}{H_2^{1/2}}$

$$N_2 = N_1\sqrt{\frac{H_2}{H_1}} = 300 \times \sqrt{\frac{15}{25}} = 232.4 \text{ rpm}$$

By similarity unit relationships $Q_2 = Q_1\sqrt{\dfrac{H_2}{H_1}} = 12\times\sqrt{\dfrac{15}{25}} = 9.30\ \text{m}^3/\text{s}$

$$P_2 = P_1\left(\frac{H_2}{H_1}\right)^{3/2} = 2496.5\times\left(\frac{15}{25}\right)^{3/2} = 1160\ \text{kW}$$

*EXAMPLE 1.16

A turbine is to operate under a head of 25 m at a speed of 300 rpm. The discharge is 9.0 m³/s. If the efficiency is 90%, determine the performance of the turbine under a head of 20 m.

Solution

Given: $D_1 = D_2$, $N_1 = 300$ rpm, $H_1 = 25.0$ m, $H_2 = 20.0$ m, $Q_1 = 9.0\ \text{m}^3/\text{s}$, $\eta = 0.9$

$$P = \eta\,\gamma QH = 0.9\times 9.79\times 9.0\times 25 = 1982.5\ \text{kW}$$

Since the diameter is the same, i.e. $D_1 = D_2$, $\dfrac{N_1}{H_1^{1/2}} = \dfrac{N_2}{H_2^{1/2}}$

$$N_2 = N_1\sqrt{\frac{H_2}{H_1}} = 300\times\sqrt{\frac{20}{25}} = 268.3\ \text{rpm}$$

By similar unit relationships $Q_2 = Q_1\sqrt{\dfrac{H_2}{H_1}} = 9.0\times\sqrt{\dfrac{20}{25}} = 8.05\ \text{m}^3/\text{s}$

**EXAMPLE 1.17

A 1/5 scale model of a Kaplan turbine is designed to operate at a head of 25 m. The prototype produces 18.50 MW of power under a head of 49 m when operating at a speed of 250 rpm. Find the speed, discharge and power of the model. Assume the efficiency of the model and prototype is the same at a value of 88%.

Solution

Given: Scale ratio $D_r = 1/5$, $H_m = 25$, $H_p = 49$ m, $P_p = 18500$ kW, $N_p = 250$ rpm, $\eta_{0m} = \eta_{0p} = 0.88$

For prototype:

Power $\quad P_p = \eta_0\,\gamma Q_p H_p = 18500\ \text{kW}$

Discharge $\quad Q_p = \dfrac{P_p}{\eta_0\gamma H_p} = \dfrac{18500}{0.88\times 9.79\times 49} = 43.82\ \text{m}^3/\text{s}$

For a 1/5 Scale Model: $\dfrac{D_m}{D_p} = \dfrac{1}{5}$, Head ratio $\dfrac{H_m}{H_p} = \dfrac{25}{49}$

Speed: $\dfrac{N_m D_m}{\sqrt{H_m}} = \dfrac{N_p D_p}{\sqrt{H_p}}$

$$N_m = N_p \left(\frac{D_p}{D_m}\right)\sqrt{\frac{H_m}{H_p}} = 250 \times \left(\frac{5}{1}\right)\sqrt{\frac{25}{49}} = 892.9 \text{ rpm}$$

Hence, speed of model = 852.9 rpm

Discharge: $\frac{Q_m}{N_m D_m^3} = \frac{Q_p}{N_p D_p^3}$

$$Q_m = Q_p \left(\frac{D_m}{D_p}\right)^3 \left(\frac{N_m}{N_p}\right) = 43.82 \times \left(\frac{1}{5}\right)^3 \left(\frac{892.9}{250}\right) = 1.252 \text{ m}^3\text{/s}$$

Model discharge is 1.252 m^3/s

Power developed by the model = $P_m = \eta_0 \, \gamma Q_m H_m$

$$P_m = 0.88 \times 9.79 \times 1.252 \times 25 = 269.7 \text{ kW}$$

Alternate method for Q_m and P_m

Using the specific discharge relationships

$$Q_{11} = \left[\frac{Q}{D^2\sqrt{H}}\right]_m = \left[\frac{Q}{D^2\sqrt{H}}\right]_p$$

$$Q_m = Q_p \left(\frac{D_m}{D_p}\right)^2 \left(\frac{H_m}{H_p}\right)^{1/2} = 43.82 \times \left(\frac{1}{5}\right)^2 \left(\frac{25}{49}\right)^{1/2} = 1.252 \text{ m}^3\text{/s}$$

$$P_{11} = \left[\frac{P}{H^{3/2}D^2}\right]_m = \left[\frac{P}{H^{3/2}D^2}\right]_p$$

$$P_m = P_p \left(\frac{D_m}{D_p}\right)^2 \left(\frac{H_m}{H_p}\right)^{3/2} = 18500 \times \left(\frac{1}{5}\right)^2 \left(\frac{25}{49}\right)^{3/2} = 269.7 \text{ kW}$$

*EXAMPLE 1.18

A turbine is designed to produce 5000 kW of brake power under a speed of 200 rpm and head of 25 m. An overall efficiency of 88% has been estimated.

(a) What is the specific speed of this machine? (b) If it is proposed to test this turbine at a speed of 300 rpm, what should be the (i) head, and (ii) discharge to obtain similarity with the original design? (c) What brake power would be expected in this test if the efficiency remains unaltered?.

Solution

Given: $D_1 = D_2$, N_1 =200 rpm, N_2 =300 rpm, H_1 = 25.0 m, η = 0.88,
P_1 = 5000 kW

(a) $N_s = \frac{N_1\sqrt{P_1}}{H_1^{5/4}} = \frac{200\sqrt{5000}}{(25)^{5/4}} = 253$

(b) $P_1 = \eta\,\gamma Q_1\,H_1 = 0.88 \times 9.79 \times Q \times 25 = 5000$. Hence $Q_1 = 23.215$ m^3/s

Since the diameter is the same, i.e., $D_1 = D_2$, $\dfrac{N_1}{H_1^{1/2}} = \dfrac{N_2}{H_2^{1/2}}$

$$\frac{H_2}{H_1} = \left(\frac{N_2}{N_1}\right)^2$$

(i) $H_2 = 25 \times \left(\dfrac{300}{200}\right)^2 = 56.25$ m

By similarity unit relationship for discharge

(ii) $Q_2 = Q_1\sqrt{\dfrac{H_2}{H_1}} = 23.215 \times \sqrt{\dfrac{56.25}{25}} = 34.82$ m^3/s

(c) Brake power $P_2 = \eta\,\gamma Q_2\,H_2 = 0.88 \times 9.79 \times 34.82 \times 56.25 = 16{,}875$ kW

Alternatively,

$$P_2 = P_1\left(\frac{H_2}{H_1}\right)^{3/2} = 5000 \times \left(\frac{56.25}{25}\right)^{3/2} = 16875 \text{ kW}$$

** EXAMPLE 1.19

A Kaplan turbine operates under a head of 35 m, has a rotational speed of 420 rpm and develops 15 MW of power. The overall efficiency of the turbine is 90%. Calculate the specific speed, unit speed, and unit discharge and unit power.

Solution

Given: $H = 35$ m, $N = 420$ rpm, $P = 15{,}000$ kW, $\eta = 0.90$

Power $P = \eta\,\gamma QH$

Discharge $Q = \dfrac{P}{\eta\gamma H} = \dfrac{15000}{0.90 \times 9.79 \times 35} = 48.64$ m^3/s

Specific speed = $N_s = \dfrac{N\sqrt{P}}{(H)^{5/4}} = \dfrac{420\sqrt{15000}}{(35)^{5/4}} = 604.2$

Unit speed $N_u = \dfrac{N}{\sqrt{H}} = \dfrac{420}{\sqrt{35}} = 71.0$

Unit discharge $Q_u = \dfrac{Q}{\sqrt{H}} = \dfrac{48.64}{\sqrt{35}} = 8.22$

Unit power $P_u = \dfrac{P}{H^{3/2}} = \dfrac{15000}{(35)^{3/2}} = 72.44$

** EXAMPLE 1.20

A scale model of a 5 MW turbine working at 100 m head and having an operating speed of 500 rpm is desired. The model head is restricted to 5.0 m and the model *discharge is restricted to 0.5 m³/s. Calculate the scale ratio of the biggest sized model, its speed and power produced by the model. Assume the efficiency of the model and prototype to be the same at a value of 90%.*

Solution

Given: $P_p = 5000$ kW, $N_p = 500$ rpm, $H_p = 100$ m,

$Q_m = 0.5\ \text{m}^3/\text{s}$, $H_m = 5.0$ m

For the prototype

Power $P_p = \eta_0\,\gamma QH = 0.9 \times 9.79 \times Q_p \times 100 = 5000$ kW

Discharge $Q_p = 5000 / (0.9 \times 9.79 \times 100) = 5.675\ \text{m}^3/\text{s}$

Considering specific rate of flow $Q_{11} = \dfrac{Q_m}{D_m^2\sqrt{H_m}} = \dfrac{Q_p}{D_p^2\sqrt{H_p}}$

$$\frac{Q_m}{Q_p} = \left(\frac{D_m}{D_p}\right)^2 \left(\frac{H_m}{H_p}\right)^{1/2}$$

$$\frac{0.5}{5.675} = \left(\frac{D_m}{D_p}\right)^2 \left(\frac{5.0}{100}\right)^{1/2}$$

$$\left(\frac{D_m}{D_p}\right) = \left(\frac{0.0881}{0.2236}\right)^{1/2} = 0.6277 = \text{Scale ratio}$$

Considering the defined speed $N_{11} = \dfrac{N_m D_m}{\sqrt{H_m}} = \dfrac{N_p D_p}{\sqrt{H_p}}$

$$N_m = N_p\sqrt{\frac{H_m}{H_p}} \times \left(\frac{D_p}{D_m}\right) = 500 \times \sqrt{\frac{5}{100}} \times \left(\frac{1}{0.6277}\right) = 178 \text{ rpm}$$

Considering the specific power $P_{11} = \dfrac{P_m}{H_m^{3/2} D_m^2} = \dfrac{P_p}{H_p^{3/2} D_p^2}$

$$P_m = P_p\left(\frac{H_m}{H_p}\right)^{3/2} \times \left(\frac{D_m}{D_p}\right)^2 = 5000 \times \left(\frac{5}{100}\right)^{3/2} \times (0.6277)^2 = 22 \text{ kW}$$

**EXAMPLE 1.21

A model Francis turbine running at 1200 rpm had a discharge of 1/128 of the corresponding prototype discharge. If the prototype speed is 300 rpm, calculate the model scale ratio.

Solution

Given: $N_m = 1200$ rpm, $Q_r = 1/128$, $N_p = 300$ rpm

From speed relation $N_{11} = \dfrac{N_m D_m}{\sqrt{H_m}} = \dfrac{N_p D_p}{\sqrt{H_p}}$

$$\frac{N_m}{N_p} = \left(\frac{D_p}{D_m}\right)\left(\frac{H_m}{H_p}\right)^{1/2}$$

$$\frac{1200}{300} = \left(\frac{1}{D_r}\right)(H_r)^{1/2}$$

$$(H_r)^{1/2} = 4D_r$$

From discharge relation $Q_{11} = \dfrac{Q_m}{D_m^2\sqrt{H_m}} = \dfrac{Q_p}{D_p^2\sqrt{H_p}}$

$$\frac{Q_m}{Q_p} = \left(\frac{D_m}{D_p}\right)^2\left(\frac{H_m}{H_p}\right)^{1/2} = (D_r)^2(H_r)^{1/2}$$

$$\frac{1}{128} = (D_r)^2(4D_r) = 4D_r^3$$

$(D_r)^3 = \dfrac{1}{512}$ and hence scale ratio $D_r = 1/8$

**EXAMPLE 1.22

A reaction turbine has the following double unit values: Peak Specific Power P_{11} = 9.0, Peak Specific Discharge Q_{11} = 1.021, Peak Defined Speed N_{11} = 150. Estimate the specific speed, runner diameter, discharge, efficiency and speed of rotation of a homologous turbine working under a head of 25 m and developing 12 MW of power. Assume the efficiency to be constant for all sizes of turbines.

Solution

Given: At peak, [$P_{11} = 9$, $Q_{11} = 1.021$, $N_{11} = 150$], $H = 25$ m, $P = 12000$ kW

Specific speed at peak values = $N_s = \dfrac{N\sqrt{P}}{H^{5/4}} = \dfrac{ND}{\sqrt{H}} \times \dfrac{\sqrt{P}}{\sqrt{(H^{3/2}D^2)}} = \left(N_{11}\sqrt{P_{11}}\right)_{Peak}$

$$= 150 \times \sqrt{9} = 450$$

From the definition of the various parameters:

$$Q_{11} = \frac{Q}{D^2\sqrt{H}} \text{ giving } Q = Q_{11}D^2\sqrt{H}$$

$$P_{11} = \frac{P}{D^2H^{3/2}} \text{ giving } P = P_{11}D^2H^{3/2}$$

Power $\quad P = P_{11}D^2H^{3/2} = 9.00 \times D^2 \times (25)^{3/2} = 12000$ kW

$$D^2 = \frac{12000}{9 \times (25)^{3/2}} = 10.67, \text{ giving runner diameter } D = 3.27 \text{ m}$$

Speed $N = \dfrac{N_{11}\sqrt{H}}{D} = \dfrac{150\times\sqrt{25}}{3.27} = 229.4$ rpm

Discharge $Q = Q_{11}D^2\sqrt{H} = 1.021 \times (3.27)^2 \times \sqrt{25} = 54.6$ m^3/s

Efficiency $= \dfrac{P_{11}}{\gamma Q_{11}} = \dfrac{9.0}{9.79\times1.021} = 0.90$

**EXAMPLE 1.23

A turbine is to operate under a head of 150 m and uses 130 m^3/s of water. It has a speed of 140 rpm and the overall efficiency is 92%. Calculate the specific speed (a) in dimensional form N_s using SI units, and (b) in nondimensional form, S_p (revolutions). Use unit weight of water = 9.79 kN/m^3.

Solution

Power $P = \eta_0\,\gamma QH = 0.92 \times 9.79 \times 130 \times 150 = 175633$ kW

Specific speed in *SI* units $N_s = \dfrac{N\sqrt{P}}{H^{5/4}} = \dfrac{140\times\sqrt{175633}}{(150)^{5/4}} = 111.8$

Specific speed in nondimensional form

$$S_p = \frac{N\sqrt{P}}{(gH)^{5/4}}\times\frac{1}{\rho^{1/2}} = \frac{N\sqrt{P}}{(H)^{5/4}}\times\frac{1}{g^{3/4}}\times\frac{1}{\gamma^{1/2}}$$

In the above $g = 9.81$ m/s^2 and take given $\gamma = 9790$ N/m^3. Further the speed N is in *rps*, P is in *watts* and H in *metres*. Thus

$$S_p = \frac{N(\text{in rps})\sqrt{P(\text{in Watts})}}{[H(\text{in m})]^{5/4}}\times\frac{1}{g^{3/4}}\times\frac{1}{\gamma^{1/2}}$$

$$= \frac{\left(\dfrac{140}{60}\right)\times\sqrt{175633\times1000}}{(150)^{5/4}}\times\frac{1}{(9.81)^{3/4}}\times\frac{1}{(9790)^{1/2}}$$

$S_p = 0.1074$ (revolutions)

Alternately: Using the conversion equation Eq. (1.50)

$S_p = 9.61 \times 10^{-4}\, N_s$

$= 9.61 \times 10^{-4} \times 111.8 = 0.1074$ revolutions

NOTE!

Even in nondimensional form there is more than one way of representing the nondimensional specific speed (shape factor). While we calculated S_p in revolutions, some authors use radians as a measure of S_p. Obviously, S_{pr} (in radians) differs from S_p (in revolutions) by a factor of (2π) as given in Eq.(1.50) and Eq.(1.50-a).

Use the relationship

$$\frac{S_{pr}\text{ (radians)}}{2\pi} = S_p \text{ (revolutions)}$$

to convert shape factor in radians to shape factor in revolutions.

Review Questions

1.1 Explain briefly the concept of linear momentum equation for a control volume.
1.2 What do you understand by the term *velocity triangle* as used in the analysis of hydraulic machines? Write the velocity triangles for the inlet and outlet of a flow of a jet of water over a moving curved vane and explain the various notations used.
1.3 A jet of water from a nozzle impinges on a moving curved vane tangentially at one edge and exits at the other end. How is the efficiency of power transmission of this system defined?
1.4 How are hydroturbines classified?
1.5 Define volumetric efficiency, hydraulic efficiency, mechanical efficiency and overall efficiency of a water turbine. Describe their inter-relationships.
1.6 What are the affinity (*similarity*) laws of hydroturbines for speed, discharge and power of two homologous turbines?
1.7 What is specific speed of a turbine? How is it defined? Write an expression for the specific speed and indicate the units of various terms adopted in the common SI units.
1.8 What are the units of specific speed and how is it nondimensionalised to obtain *shape factor*?
1.9 What are unit quantities used in connection with turbines? Describe various unit quantities used in connection with turbines.
1.10 What are *specific quantities* (double unit quantities) used in connection with turbines? Describe different specific quantities commonly used.
1.11 Derive expressions for the following parameters related to a turbine:
(i) Specific speed, (ii) Unit speed, (iii) Unit power, (iv) Specific power
1.12 How are (i) velocity of whirl, and (ii) flow velocity defined? Explain these terms with the help of a velocity triangle related to the impact of a jet of water on a curved moving vane.

Problems

1. [All **Problems** and **Objective Questions** have been graded in three levels:– Simple, Medium and Difficult. The markings for these are: Simple = *, Medium = **, Difficult = ***]

Jet Impingement

P1.1 * A free jet of water issues out of a 4 cm diameter nozzle with a velocity of 15 m/s. It impinges normally on a stationary vertical plate. Estimate the force on the plate due to this impingement. (b) If the force on the plate is to be reduced by 25%, what should be the velocity of the jet?

[***Ans*:** F_x = 282.2 N, V_1 = 13.0 m/s]

P1.2 *A 15 cm diameter jet of water with a velocity of 15 m/s strikes a plate normally. If the plate is moving with a velocity of 6 m/s in the direction of the jet, (a) calculate the work done per second on the plate and the efficiency of energy transfer by the jet. (b) What would be the efficiency if the jet impact is on a series of blades mounted on a wheel?

[***Ans*:** (a) P = 8571 kW, η = 28.8% (b) η = 48%]

P1.3 **A 10 cm diameter free jet of water having a velocity of 30 m/s impinges on a plane, smooth plate at angle of 45° to the normal to the plate. (a) What will be the force due to jet impingement on the plate and work done per second when (i) the plate is stationary, and (ii) when moving in the direction of the jet at 6.0 m/s velocity. (b) What will be the force due to jet on the plate when the plate is moving against the jet direction at 6.0 m/s velocity?

[***Ans*:** (a) (*i*) 4988 N, P = 0, (ii) F_n = 3192.5 N, P = 13.544 kW, (b) F_n = 7183 N]

P1.4 **A two-dimensional jet of water impinges on a plane at an angle θ to the normal to the plane. If the jet splits in to two streams of discharges in the ratio 1: 2.5, calculate the angle θ. [***Ans*:** 25.376°]

P1.5 **A two-dimensional jet issuing from a long slot strikes a plate at angle of 50° with the plane of the plate. This causes the flow to divide in to two parts q_1 and q_2 on either side of the impact zone. Calculate the ratio q_1/q_2.

[***Ans*:** 4.6]

P1.6 **A square plate of weight 1500 N is hinged on its top edge about a horizontal axis. If a free circular jet of water of diameter 6 cm impinges at the centre of the plate normally and causes it to deflect by 15° from its initial vertical position, estimate the velocity of the jet. [***Ans*:** V = 11.73 m/s]

P1.7 **A rectangular plate 30 cm long has a weight of 200 N. It is hinged at its top edge to rotate freely about a horizontal axis. A free horizontal water jet of velocity 12 m/s issuing from a nozzle of 25 mm diameter impinges normally the vertical plate at its center. (a) Find the force needed to be applied at the middle of the lower edge to keep the plate vertical. If the plate is allowed to swing freely, find the inclination to the vertical that the plate will assume due to impact of the jet. [***Ans*:** (a) R = 35.27 kN, (b) θ = 20.65°]

P1.8 **A free water jet of 75 mm diameter and 30 m/s velocity impinges on a series of hemispherical cups on the hollow side and the jet is deflected through 180°. The cups have a velocity of 8 m/s in the same direction as the impinging jet. Determine (a) the power extracted by the series of cups, and (b) the efficiency of the system. [***Ans*:** (a) P = 46.55 kW, (b) η = 78.2%]

P1.9 **A 60 mm jet strikes a stationary frictionless curved vane, at 40 m/s velocity, tangentially. If the jet is deflected by an angle of 45° by the vane due to this impingement, determine the force exerted by the jet on the plate.

[***Ans*:** F = 3.45 kN acting at –67.5° to x-direction]

P1.10 *A jet of water, 10 cm in diameter, strikes a stationary curved blade with 20 m/s velocity and gets deflected by 30° with respect to the direction of the impinging jet. The exit velocity is reduced by 10% due to frictional effects. Calculate the component of force on the vane (i) in the direction of initial jet direction and (ii) in the normal direction to the jet.

[***Ans*:** F_x = 0.697 kN, F_y = –1.411 kN]

P1.11 *A stationary curved plate deflects a 10 cm diameter water jet through an angle of 120° with respect to the initial jet direction in the horizontal plane. Calculate the force required to hold the plate in position if the velocity of the jet is 15 m/s. [***Ans***: F = 3.054 kN at θ = –30° to x-direction]

P1.12***A jet of water 10 cm in diameter and having velocity of 18 m/s strikes a curved moving vane of velocity 8 m/s at 25° to the direction of motion of the vane. The vane angle at the outlet measured with respect to the direction of motion of the vane is 150°. Find the (i) vane angle at inlet so that the water enters without shock, and (ii) the component of force exerted by the jet on the vane in the direction of motion of the vane.

[***Ans***: (*a*) β'_1 = 42.46°, (*b*) F_x = 1.426 kN]

P1.13***A 75 mm diameter free water jet moving with a velocity of 25 m/s strikes a single vane moving in the same direction as the jet with a velocity of 12 m/s. The friction losses cause the relative velocity at exit to be 92% of the relative velocity at the inlet. The relative velocity at the exit has a total deflection angle of 165° with respect to the relative velocity at inlet. (a) Calculate the force exerted by the jet on the vane. (b) If the vane forms a part of a series of such vanes, calculate (i) the power extracted out of the jet, and (ii) power lost in friction.

[***Ans***: (a) P = 16.878 kW, (b) P = 32.484 kW, P_f = 1.44 kW]

Efficiency

P1.14 *A turbine has a net head of 15 m and produces 500 kW of power with an efficiency of 85%. If its volumetric efficiency is 98%, estimate the discharge supplied to the turbine unit. [***Ans***: Q = 4.097 m^3/s]

P1.15 *A turbine produces 6.0 MW of power at an overall efficiency of 85%. If mechanical friction losses are 150 kW and volumetric efficiency is 0.98, estimate the mechanical efficiency and hydraulic efficiency of this turbine.

[***Ans:*** η_m = 0.975, η_h = 0.889]

P1.16 *The gross head available to a hydroelectric power plant is 100 m. The utilised head in the runner of the hydraulic turbine is 72 m. If the hydraulic efficiency of the turbine is 90%, (a) estimate the head lost due to pipe friction in the penstock. (b) If the available discharge to a turbine unit is 10 m^3/s, calculate the specific speed of the turbine. Assume the mechanical efficiency as 0.95 and rotational speed of the turbine as 200 rpm. [***Ans***: h_f = 20 m, N_s = 68.4]

P1.17 *In a hydroelectric project, the available discharge is 45 m^3/s with a net head of 35 m. Assuming a turbine efficiency of 90% and speed of 300 rpm, determine the least number of turbines of the same size and having a specific speed of 300. [***Ans***: n = 2]

Similarity Laws

P1.18 **A turbine works at a head of 20 m and 500 rpm speed. Its 1: 2 scale model is to be tested at a head of 20 m. (a) Estimate the rotational speed of the model. (b) What would be the scale ratios of discharge and power in this model? (Assume the efficiency of the model and prototype is the same).

$$\left[\textbf{\textit{Ans}}\text{: } N_m = 1000 \text{ rpm}, \ Q_r = \frac{1}{4}, \ P_r = \frac{1}{4}\right]$$

P1.19 *A turbine is to operate under a head of 25 m at a speed of 300 rpm. The discharge is 12 m^3/s. Assuming an efficiency of 0.85, calculate the specific speed and power developed. What would be the specific speed, power, discharge and rotational speed at a head of 15 m?

[***Ans*:** N_s = 268.1, P_1 = 2496.5 kW, N_s = 268.1, P_2 = 1160 kW, Q = 9.30 m^3/s, N = 232.4 rpm]

P1.20 **A turbine develops 1.50 MW while running at 120 rpm under a head of 10.0 m. The diameter of the runner is 1.5 m. A 1: 3 scale model of this turbine is tested under a head of 3.0 m. Determine the speed and power developed in the model. Assuming an overall efficiency of 90% for both the model and prototype, calculate the discharge in the model and prototype.

[***Ans*:** N_m = 197.18, P_m = 27.39 kW, Q_m = 1.036 m^3/s, Q_p = 17.024 m^3/s]

P1.21 ***A 1: 2 model of a turbine is operated at 600 rpm under a net head of 15 m. The brake power of the model is 300 kW under a discharge of 2.3 m^3/s. The prototype is to work under net head of 90 m. Compute the (a) specific speed of the model, (b) overall efficiency of the model, (c) overall efficiency of the prototype, and (d) brake power of the prototype.

[***Ans*:** N_s = 352, N_{om} = 88.8%, N_{op} = 90.2%, P_p = 17910 kW]

P1.22 **A model of a Francis turbine has a runner of diameter 1.0 m and an overall efficiency of 0.885 at its optimum performance. (a) If a homologous prototype runner has a diameter of 2.0 m, estimate its maximum efficiency. (b) If another homologous model of the runner of diameter 0.4 is to be tested, what maximum efficiency can be expected in this new model?

[***Ans*:** η_p = 90.4%, η_2 = 86.2%]

P1.23 *Francis turbine produces 6750 kW at 300 rpm under a net head of 45 m with an overall efficiency of 85%. What would be the revolutions per minute, discharge and brake power of this turbine under a net head of 60 m and under homologous conditions?

[***Ans*:** N_2 = 346.41 rpm, Q_2 = 20.814 m^3/s, P_2 = 10392 kW]

P1.24 **A 1: 4 scale model of a turbine is tested in a laboratory under a head of 15 m. The prototype is required to work under a head of 35 m and to run at 420 rpm. At what speed must the model be run to develop 260 kW by using 2.0 m^3/s of water at this speed? If the prototype efficiency is 3% better than that of the model, what is the power expected from the prototype turbine?

[***Ans*:** N_m = 1100 rpm, η_p = 0.912, and P_p = 15272 kW]

P1.25 ***The following data is from a test conducted in a laboratory on a turbine model:

Head at test: 8.0 m, Unit power = 9.2,

Unit speed = 55, Unit discharge = 1.25

(a) Calculate the efficiency at this operation point. (b) If a homologous prototype turbine works under a head of 24 m, for an operation point corresponding to the above data, calculate (i) its speed, (ii) discharge, and (iii) power developed.

[***Ans*:** η_0 = 75.2%, N_p = 1082 rpm, P_u = 1082 kW, Q_p = 6.123 m^3/s]

P1.26 **The power developed by a turbine, P, is expressed as a function of six variables in the form

$$P = f(D, N, gH, Q, \rho, \mu).$$

In this the notations for the variables have the usual meaning. Perform dimensional analysis of these seven variables by using Buckingham's Pi theorem method and show that

$$\frac{P}{\rho N^3 D^5} = fn\left(\frac{gH}{N^2 D^2}, \frac{Q}{ND^3}, \frac{\mu}{\rho ND^2}\right)$$

OBJECTIVE-TYPE QUESTIONS

Jets

O1.1 * A water jet has an area of 0.03 m^2 and impinges normally on a plate. If a force of 1 kN is produced as a result of this impact, the velocity of the jet is
(a) 15 m/s (b) 33.4 m/s (c) 3.4 m/s (d) 5.78 m/s

O1.2 ** A free water jet has a velocity of 15 m/s and an area of 0.012 m^2. This jet impinges normally on a vertical plate moving with a velocity of 5 m/s in the direction of the jet. The force on the plate due to this impingement of the jet, by considering the density of water ρ = 1000 kg/m^3, is
(a) 2700 N (b) 1800 N (c) 1200 N (d) 120 N

O1.3 ** A jet of water with a velocity of 20 m/s impinges on a single vane moving at 5 m/s in the direction of the jet and transmits a power P_1. If the same jet drives a series of similar vanes mounted on a wheel under similar velocity conditions, the power transmitted by the jet is P_2. The ratio of P_1 to P_2 is
(a) 0.25 (b) 0.33 (c) 0.50 (d) 0.75

O1.4 A two-dimensional free jet of water strikes a fixed two-dimensional plate at 45° to the normal to the plate. This causes the jet to split in to two streams whose discharges are in the ratio
(a) 5.83 (b) 1.00 (c) 1.41 (d) 3.00

O1.5 * A free water jet has a velocity of 10 m/s and an area of 0.01 m^2. This jet impinges normally on a vertical plate that is moving with a velocity of 5 m/s against the direction of the jet. The force on the plate due to this impingement of the jet, by considering the density of water ρ = 1000 kg/m^3, is
(a) 2250 N (b) 1000 N (c) 500 N (d) 250 N

O1.6 ** A free water jet of 0.01 m^2 area and of 20 m/s velocity impinges normally on a stationary vertical plate. If the plate is to move in the direction of the jet at a velocity of 5 m/s, what increase in the discharge at the nozzle would keep the force on the plate unaltered?
(a) 0.005 m^3/s (b) 0. 05 m^3/s (c) 0.10 m^3/s (d) 0.25 m^3/s

O1.7 * A symmetrical stationary vane experiences a force *F* of 100 N as shown

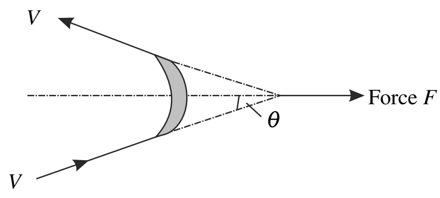

Fig. 1.26 *Objective Question No. O 1.7*

in Fig.1.26 when the mass flow rate of water over the vane is 5 kg/s with a velocity V of 20 m/s without friction. The angle θ of the vane is

(a) zero (b) 30° (c) 45° (d) 60°

O1.8 * A free water jet acts upon a water wheel that has semicircular vanes fitted on its periphery. The theoretical maximum efficiency of this wheel system is

(a) 50% (b) 67% (c) 75% (d) 100%

Efficiency

O1.9 ** A turbine with a hydraulic efficiency of 90% has a leakage of 0.1 m^3/s when the discharge supplied at the inlet is 2.0 m^3/s. If the overall efficiency of the turbine is 75%, the mechanical efficiency is

(a) 0.88 (b) 0.80 (c) 0.93 (d) 0.900

O1.10 * The hydraulic efficiency of a Pelton turbine is 80%. If the mechanical efficiency is 85%, what is its overall efficiency?

(a) 68% (b) 94.1% (c) 82.5% (d) 65%

O1.11 ** The gross head available to a hydroelectric power plant is 100 m. The utilized head in the runner of the hydraulic turbine is 72 m. If the hydraulic efficiency of the turbine is 90%, the pipe friction head is estimated to be

(a) 20 m (b) 18 m (c) 16.2 m (d) 1.8 m

Affinity Laws

O1.12 * The unit speed N_u of a turbine having a rotational speed of N and head H is equal to

(a) $N\sqrt{H}$ (b) $N/\sqrt{H}$ (c) $\sqrt{H}/N$ (d) $\sqrt{H/N}$

O1.13 * The unit power P_u of a turbine developing a power P under head H is equal to

(a) $\dfrac{P}{H^{5/2}}$ (b) $\dfrac{P}{H^{3/2}}$ (c) $PH^{3/2}$ (d) $PH^{5/2}$

O1.14 * In the model studies of a turbine the overall efficiency of the model

(a) is the same as that of the prototype
(b) is always larger than that of the prototype
(c) is always smaller than that of the prototype
(d) is smaller than that of the prototype if the prototype is having larger runner than the model

O1.15 * A turbine works under a head of 25 m, has a speed of 400 rpm and develops 625 kW of power. Its specific speed is

(a) 894 (b) 761 (c) 400 (d) 179

O1.16 ** The specific speed of the turbine N_s has the dimensions of

(a) $F^{1/2} L^{-3/4} T^{-3/2}$ (b) $F^{-1/2} L^{1/4} T^{-3/2}$
(c) $F^{1/2} L^{-5/4} T^{3/2}$ (d) $F^{1/2} L^{3/4} T^{-1/2}$

O1.17 * A turbine develops 400 kW of power under a net head of 40 m. If the overall efficiency of the turbine is 0.85, the discharge through the turbine is

(a) 0.85 m^3/s (b) 1.02 m^3/s (c) 1.20 m^3/s (d) 11.76 m^3/s

O1.18***In a two-jet Pelton wheel working at full load, each of the nozzles issues 10 cm diameter jet. The nozzles are so shaped as to avoid any further contraction of the jet. If the load is suddenly reduced to 36% of full load the altered jet diameter will be

(a) 1.8 cm (b) 3.0 cm (c) 3.6 cm (d) 6.0 cm

O1.19***In utilizing the scale models in the designing of hydraulic turbines, which of the following relationships must be satisfied?

(a) $\dfrac{H}{ND^3}$ = Constant; $\dfrac{Q}{N^2D^2}$ = Constant

(b) $\dfrac{Q}{\sqrt{H}D^2}$ = Constant; $\dfrac{H}{N^3D}$ = Constant

(c) $\dfrac{P}{QH}$ = Constant; $\dfrac{H}{N^2D^2}$ = Constant

(d) $\dfrac{NQ^{1/2}}{H^{3/2}}$ = Constant; $\dfrac{NP^{1/2}}{H^{3/4}}$ = Constant

O1.20 *A turbine has a discharge of 3 m^3/s when operating under a head of 15 m and speed of 500 rpm. If it is to operate under a head of 12 m, the rotational speed would be

(a) 600 rpm (b) 559 rpm (c) 447 rpm (d) 400 rpm

O1.21 *A turbine has a discharge of 3 m^3/s when operating under a heads of 14 m and speed of 400 rpm. If it is to operate under a head of 18 m, the discharge will be

(a) 3.86 m^3/s (b) 3.40 m^3/s (c) 2.65 m^3/s (d) 2.23 m^3/s

O1.22 **A turbine works at 400 rpm under a head of 30 m. If a 1:2 scale model of this turbine is to be tested at a head of 30 m, the model should have a rotational speed of

(a) 800 rpm (b) 566 rpm (c) 400 rpm (d) 200 rpm

O1.23 **A turbine model has a diameter of 400 mm. Tests show that it has an overall efficiency of 88%. What efficiency could be expected in the prototype having a diameter of 1000 mm?

(a) 0.855 (b) 0.900 (c) 0.925 (d) 0.880

O1.24***Two turbines *A* and *B* of the same type have the same specific speed and are working under the same head. Turbine *A* produces 400 k*W* at 1000 rpm. If the turbine *B* produces 100 kW then its rpm is

(a) 4000 (b) 2000 (c) 1500 (d) 1250

O1.25 *Specific speed of a turbine is defined as the speed of a member of same homologous series of such a size that it

(a) delivers unit discharge at unit head
(b) delivers unit discharge at unit power
(c) delivers unit power at unit discharge
(d) produces unit power under unit head

O1.26 ** The gross head on a turbine is 300 m. The length of penstock supplying water from the reservoir to the turbine is 400 m. The diameter of the penstock is 1.0 m and the velocity of water through penstock is 5 m/s. If the Darcy–Weiesbach friction coefficient $f = 0.01$, the net head on the turbine would be nearly

(a) 310 m (b) 295 m (c) 200 m (d) 150 m

O1.27 * Which of the following is the correct statement?

The specific speed of a hydroturbine

(a) refers to the speed of a turbine of unit dimensions
(b) is a type-number representative of its performance
(c) is specific to the particular turbine
(d) depends only upon the head under which the turbine operates

O1.28 ** Which of the following statements are correct with respect to hydraulic turbines?

1. Speed is inversely proportional to diameter of the turbine
2. Power is proportional to speed
3. Power is proportional to (3/2) power of head
4. Speed is proportional to square root of head

The correct answer is

(a) 2, 3 and 4 (b) 1, 2 and 3
(c) 3 and 4 only (d) 2 and 4 only

O1.29 ** A turbine works at 20 m head and 500 rpm speed. Its 1: 2 scale model that is to be tested at a head of 20 m should have a rotational speed of nearly

(a) 1000 rpm (b) 700 rpm (c) 500 rpm (d) 250 rpm

O1.30 * Tests on a turbine showed that a discharge of 12 m^3/s was being used under a head of 64 m. The unit discharge in this case is

(a) 0.1875 (b) 1.5 (c) 5.33 (d) 18.47

O1.31 * A hydraulic-reaction-type turbine working under a head of 16 m develops 640 kW of power. What is the unit power of the turbine?

(a) 10 kW (b) 40 kW (c) 60 kW (d) 160 kW

O1.32 ** A Francis turbine working at a speed of 400 rpm has a unit speed of 50 rpm and develops 500 kW of power. What is the effective head under which the turbine is working?

(a) 62.5 m (b) 64. 0 m (c) 40.0 m (d) 100 m

O1.33 ** A hydraulic turbine is to generate 300 kW of power at 400 rpm under a head of 40 m. For initial testing a 1: 4 scale model of the turbine operating under a head of 10 m is used. The power generated by the model will be

(a) 2.34 kW (b) 4.68 kW (c) 9.38 kW (d)18.75 kW

Two

Reaction Turbines-1: *Francis Turbine*

2.1 EULER EQUATION

2.1.1 Introduction

In this section, the basic equation of turbines, definitions and relationships are developed for a hydraulic turbine. Even though many of these equations and definitions can be adapted to a pump with appropriate changes of sign and nomenclature, separate equations and expressions will be developed for the use and analysis of rotodynamic pumps, in Chapter 5. It is believed that this arrangement is least confusing and helps in understanding the subject with clarity. In that sense, the relationships developed in this chapter are exclusively applicable to hydraulic turbines only.

2.1.2 General Vector Relationship of Fluid Flow

The linear momentum equation discussed in Sec. 1.2 of Chapter 1 is useful in obtaining relationship between the change in the momentum flux and resulting force in a flow stream. However, an entire class of problems dealing with rotational aspects of fluid streams and their impact on the rotating boundary as in the case of *turbomachines* are best analysed by considering the *moment-of-momentum equation*. First, it is necessary to realise that the linear momentum flux is a vector. In a rotating system, the moment of the linear momentum vector about a point O is called the *moment of momentum* and is also known as *angular momentum.* Angular momentum is also a vector that is obtained as a cross product of linear momentum vector and the *position vector* from the point O. Similarly, the moment about a reference

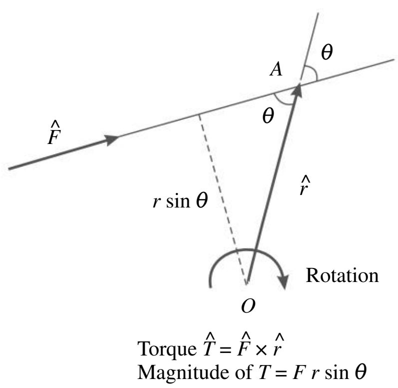

Fig. 2.1 *Definition sketch of torque*

point O of a force F acting at a point A that is at a radial distance r from the point O is the torque on O and is also a vector. It is the cross product of the force vector $\hat{F}$ and the position vector $\hat{r}$. The relationship between the torque, force and the moment arm (position vector) is shown in Fig. 2.1 (a).

The relationship between the torque and the moment of momentum acting in a fluid system is given by the *moment-of-momentum equation*. The moment-of-momentum principle is Newton's second law applied to a rotating fluid mass. This is similar to the linear momentum equation (Sec. 1.2.2) and is stated as follows:

In a steady-state flow, the rate of change of moment of momentum of a system is equal to the net torque acting on the system.

In analysing the mechanism of power transfer in turbomachines such as turbines and centrifugal pumps, we will be using the moment-of-momentum equation. In these machines, torques are more significant than the forces. Since all quantities involved are vectors, it is good to know the vector form of the moment-of-momentum relationship.

The steady-state moment-of-momentum equation for a two-dimensional rotating fluid flow states that:

If $\hat{r}$ is the position vector in a curvilinear motion of a fluid, $\hat{F}$ is the external force vector and $\hat{M}$ is the linear momentum vector, the moment-of-momentum principle states that

$$(\hat{r} \times \hat{F}) = \frac{d}{dt}(\hat{r} \times \hat{M}) \tag{2.1}$$

If the moment of external forces $(\hat{r} \times \hat{F})$ is replaced by a torque $\hat{T}$ then

$$\hat{T} = \frac{d}{dt}(\hat{r} \times \hat{M}) \tag{2.2}$$

The torque exerted by the fluid mass on the shaft is equal and opposite to $\hat{T}$.

This general vector relationship of torque and rate of change of moment of momentum, when applied to a rotodynamic machine rotating at constant speed, results in a very useful relationship known as *Euler equation.* The Euler equation is the basic equation relating fluid-flow velocities, their momentum and moment of momentum to the torque developed in the shaft. A simple form of the Euler equation is developed in the next section through one-dimensional method of analysis.

2.1.3 Euler Equation for Hydroturbines

Consider a hydro reaction turbine as shown in Fig. 2.2(a). This turbine is of the Francis-type reaction turbine where the entire circumference of a runner has water entry under pressure. The runner of this turbine essentially consists of a set of curved blades mounted symmetrically in the runner wheel. The flow is uniform along the circumference and is steady. Figure 2.2 (b) represents the details of velocity triangles at the inlet and outlet of the runner. Let the suffixes 1 and 2 refer to the inlet and outlet respectively. Observe that Figs. 2.2(*a* and *b*) represent an inward-flow turbine, in which the flow enters at a larger radius and exits at a lesser radius. Most present-day turbines are of this kind. The basic assumption in the derivation is that there are no

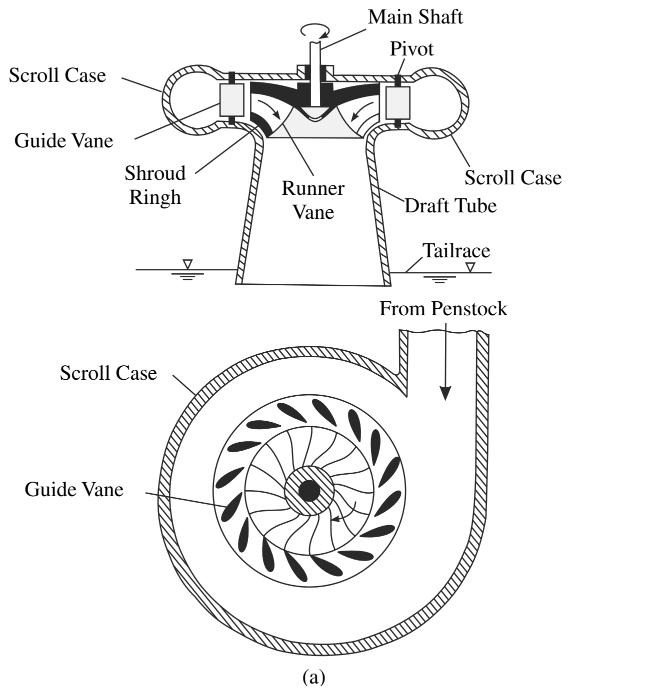

Fig. 2.2(a) *Schematic sketch of a Francis turbine*

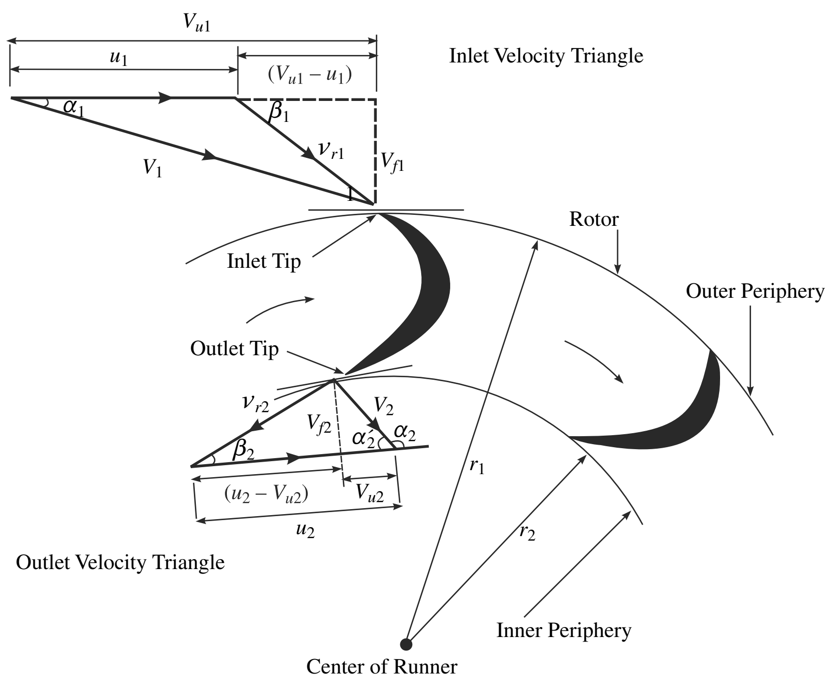

Fig. 2.2(b) *Velocity triangles in a Francis-turbine runner*

friction and other losses in the system. The fluid is assumed to have perfect guidance through the flow system. This implies infinite number of thin vanes.

The following symbols and notations are used in the derivation of the Euler's equation.

Notations

r_1 and r_2 = Radii of fluid element at entrance and exit

v_{r1} and v_{r2} = Relative velocities at entrance and exit

V_1 and V_2 = Absolute velocities at entrance and exit

α_1 and α_2 = *Guide-vane angle* = Angle made by the absolute velocity vector V with the positive direction of the peripheral velocity, u

$\alpha_2' = 180 - \alpha_2$ = Supplementary angle of α_2

β_1 and β_2 = *Vane angle* = Angle made by the relative velocity vector v_r with the negative direction of the peripheral velocity, u

ω = Angular velocity of the runner and

N = Revolutions per minute of the runner

The angular velocity $\omega = \frac{2\pi N}{60}$ and

u = Tangential velocity of the blade = ωr

$$u_1 = \omega r_1 = \frac{\pi D_1 N}{60} \quad \text{where } D_1 = \text{Outer diameter of the runner}$$

$$u_2 = \omega r_2 = \frac{\pi D_2 N}{60} \quad \text{where } D_2 = \text{Inner diameter of the runner}$$

For a steady, frictionless system, the moment-of-momentum equation in radial coordinates states:

Torque exerted by the fluid on the rotor

= Decrease in the rate of change of moment of momentum

= [rate of change of moment of momentum of fluid going *in to* the control volume] – [rate of change of moment of momentum of fluid going *out of* the control volume]

The set of blades are assumed to be mounted symmetrically on the runner and the flow is uniform along the perimeter of the runner at the inlet and outlet. Refer to Fig. 2.2(b). The torque exerted by the fluid on the runner, which is transmitted to the shaft without any loss due to friction and other losses, is

T = [Rate of mass flow through the rotor] × [Decrease of moment of momentum of the fluid in each blade]

$$= \dot{m} \times (r_1 V_1 \cos \alpha_1 - r_2 V_2 \cos \alpha_2') \tag{2.3}$$

where $\alpha_2' = 180 = \alpha_2$.

Since ρ = Density of water is constant,

$\dot{m}$ = Rate of mass flow $= \rho Q$

where Q = Total discharge entering the runner.

Substituting for $\dot{m}$, the torque exerted by the fluid on the runner is given by

$$T = \rho Q (r_1 V_1 \cos \alpha_1 - r_2 V_2 \cos \alpha_2') \tag{2.4}$$

Energy extracted = Power transmitted to the shaft = $P = T\omega$

$$= \rho\, Q\, \omega\, (r_1 V_1 \cos \alpha_1 - r_2\, V_2 \cos \alpha_2') \tag{2.5}$$

Since $\omega\, r = u$ = Tangential component of the runner at radius r;

$u_1 = \omega\, r_1$ and $u_2 = \omega\, r_2$. Hence,

$$P = \rho\, Q\, (u_1 V_1 \cos \alpha_1 - u_2\, V_2 \cos \alpha_2') \tag{2.6}$$

This equation (2.6) is known as *Euler equation for power in a hydroturbine*.

2.1.4 Alternate Forms of Euler Equation

Consider the velocity triangles at inlet and outlet, as shown in Fig. 2.2(b). The following additional notations are used:

Additional Notations

$V_1 \cos \alpha_1 = v_{u1}$ = Tangential component (also known as *whirl/swirl* component) of absolute velocity V_1

$V_2 \cos \alpha'_2 = v_{u2}$ = Tangential component (also known as *whirl/ swirl* component) of absolute velocity V_2. [Note that ($\alpha_2' = 180 - \alpha_2$)]

$V_1 \sin \alpha_1 = V_{f1}$ = *Flow* component of absolute velocity V_1

$V_2 \sin \alpha_2 = V_{f2}$ = *Flow* component of absolute velocity V_2

From the velocity triangle at inlet, Fig. 2.2(b):

$$V_{f1}^2 = v_{r1}^2 - (u_1 - V_{u1})^2 \tag{2.7}$$

$$V_1^2 = V_{u1}^2 + V_{f1}^2 = V_{u1}^2 + v_{r1}^2 - (u_1 - V_{u1})^2 \tag{2.8}$$

$$= V_{u1}^2 + v_{r1}^2 - u_1^2 - V_{u1}^2 + 2u_1 V_{u1}$$

$$2u_1 V_{u1} = V_1^2 - v_{r1}^2 + u_1^2$$

$$u_1 V_{u1} = (V_1^2 - v_{r1}^2 + u_1^2)/2 \tag{2.9}$$

Similarly from the velocity triangle at the outlet:

$$V_2^2 = V_{f2}^2 + V_{u2}^2 = [v_{r2}^2 - (u_2^2 - V_{u2}^2)] + V_{u2}^2$$

$$= v_{r2}^2 - u_2^2 + 2u_2 V_{u2}$$

$$u_2 V_{u2} = (V_2^2 - v_{r2}^2 + u_2^2)/2 \tag{2.10}$$

Thus from Eq. 2.6, the power transferred, which is equal to the energy removed from the fluid, is

$P = \rho\, Q\, (u_1 V_1 \cos \alpha_1 - u_2\, V_2 \cos \alpha_2')$ and this could be written as

$$P = \rho\, Q\, (u_1 V_{u1} - u_2 V_{u2}) \tag{2.6-a}$$

This (Eq. 2.6-a) is another popular form of the Euler equation for power.

Substituting the results of equations (2.9) and (2.10) in Eq. (2.6-a),

$$P = \frac{\rho Q}{2}\left(V_1^2 - v_{r1}^2 + u_1^2 - V_2^2 + v_{r2}^2 - u_2^2\right)$$

$$= \rho Q\left[\frac{\left(V_1^2 - V_2^2\right)}{2} + \frac{\left(v_{r2}^2 - v_{r1}^2\right)}{2} + \frac{\left(u_1^2 - u_2^2\right)}{2}\right] \quad (2.11)$$

Considering the energy head H_e = Energy per unit weight of fluid transferred from the fluid to the rotor

$$H_e = \frac{\rho Q}{\rho Q g}\left[\frac{\left(V_1^2 - V_2^2\right)}{2} + \frac{\left(v_{r2}^2 - v_{r1}^2\right)}{2} + \frac{\left(u_1^2 - u_2^2\right)}{2}\right]$$

$$H_e = \left[\frac{\left(V_1^2 - V_2^2\right)}{2g} + \frac{\left(v_{r2}^2 - v_{r1}^2\right)}{2g} + \frac{\left(u_1^2 - u_2^2\right)}{2g}\right] \quad (2.12)$$

Also by Eq. (2.9) and Eq. (2.10), $H_e = \dfrac{1}{g}(u_1 V_{u1} - u_2 V_{u2})$ (2.13)

Equation 2.13 is known as the *Euler equation for head extracted in a Hydroturbine*.

H_e is often called the *Euler head* and represents the *ideal head* extracted by the turbine and can be transmitted to the shaft under ideal conditions. However, the actual head extracted will be smaller than H_e due to friction and other losses in the system.

2.1.5 Nature of Energy Transfer

The expression for Euler head H_e in the form of Eq. (2.12) allows us to examine the nature of energy transfer. Equation (2.12) consists of three terms:

$$\frac{\left(V_1^2 - V_2^2\right)}{2g}, \frac{\left(v_{r2}^2 - v_{r1}^2\right)}{2g} \text{ and } \frac{\left(u_1^2 - u_2^2\right)}{2g}.$$

1. The term $\dfrac{V^2}{2g}$ represents the velocity head of the absolute velocity and hence the kinetic energy per unit weight of water at the designated section. Thus, the first term $\dfrac{\left(V_1^2 - V_2^2\right)}{2g}$ represents the change in the absolute kinetic energy between inlet and outlet of the wheel. This is called the *impulse effect* and represents a change in the dynamic head of the absolute flow.
2. Consider the energy equation in the form of Bernoulli equation for incompressible, frictionless flow between sections 1 and 2. Since an amount of energy head H_e has been extracted out of the flow between these two sections, the energy equation can be written as

$$\frac{V_1^2}{2g}+\frac{p_1}{\gamma}+z_1=\frac{V_2^2}{2g}+\frac{p_2}{\gamma}+z_2+H_e$$

Substituting the expression of Eq. (2.12) for H_e,

$$\frac{V_1^2}{2g}+\frac{p_1}{\gamma}+z_1=\frac{V_2^2}{2g}+\frac{p_2}{\gamma}+z_2+\left[\frac{\left(V_1^2-V_2^2\right)}{2g}+\frac{\left(v_{r2}^2-v_{r1}^2\right)}{2g}+\frac{\left(u_1^2-u_2^2\right)}{2g}\right]$$

$$\left[\frac{\left(v_{r2}^2-v_{r1}^2\right)}{2g}+\frac{\left(u_1^2-u_2^2\right)}{2g}\right]=\frac{(p_1-p_2)}{\gamma}+(z_1-z_2)=(h_1-h_2) \tag{2.14}$$

where (h_1-h_2) represents the change in the piezometric head between sections 1 and 2. However, in a turbine set-up, the term (z_1-z_2) is zero or negligibly small. Hence, the terms on the left-hand side of Eq. (2.14) represent essentially a change in the pressure head. Thus, the second and third terms of Eq. (2.12), viz., $\frac{\left(v_{r2}^2-v_{r1}^2\right)}{2g}$ and $\frac{\left(u_1^2-u_2^2\right)}{2g}$, represent change of energy in the form of pressure head.

- The second term $\frac{\left(v_{r2}^2-v_{r1}^2\right)}{2g}$ could now be interpreted as the energy change in the form of a pressure head due to acceleration in a convergent passage. If the passages are divergent, there will be deceleration. In an inward-flow runner, the flow passages decrease from the inlet to the outlet and usually $v_{r2}>v_{r1}$ and as such, the term $\frac{v_{r2}^2-v_{r1}^2}{2g}$ represents a decrease in pressure at 2 from that at 1 due to the acceleration of the relative velocity.
- Similarly, the third term $\frac{\left(u_1^2-u_2^2\right)}{2g}$ represents the pressure-head change due to centrifugal effect. The velocities u_1 and u_2 being peripheral velocities at inlet and outlet, they represent the velocity head of a *forced vortex*. Note that in an inward-flow reaction turbine, $u_1>u_2$ and $\frac{\left(u_1^2-u_2^2\right)}{2g}$ is a positive quantity. On the other hand, in an axial flow (propeller type) turbine, $u_1=u_2$ and $\frac{\left(u_1^2-u_2^2\right)}{2g}=0$.
- The sum of second and third terms of Eq. (2.12), viz. $\left[\frac{\left(v_{r2}^2-v_{r1}^2\right)}{2g}+\frac{\left(u_1^2-u_2^2\right)}{2g}\right]$, that represent the change in the static pressure head in the system is called the *reaction effect*.

- The relative values of the impulse effect and reaction effect in the energy head H_e of a turbine forms an important aspect in the design of the rotors. The ratio of reaction effect (energy transfer by static pressure) to the total energy transfer in the rotor is defined as the *Degree of reaction*. Thus, by considering unit weight of water flow in the turbine,

$$\text{Degree of reaction } R = \frac{\left[\dfrac{\left(v_{r2}^2 - v_{r1}^2\right)}{2g} + \dfrac{\left(u_1^2 - u_2^2\right)}{2g}\right]}{H_e} = \frac{H_e - \left(V_1^2 - V_2^2\right)/2g}{H_e} \quad (2.15)$$

A turbine rotor with a nonzero degree of reaction must necessarily be completely enclosed for development of pressure head. Such turbines are called *reaction turbines*. If the degree of reaction is zero, the rotor can be open to atmosphere. Such turbines are called *impulse turbines.* In a pure impulse turbine, such as a Pelton wheel, there is no reaction effect leading to $R = 0$. In these units, the rotor is not encased to contain water under pressure and the pressure is atmospheric everywhere.

2.1.6 Hydraulic Efficiency

The *hydraulic efficiency* of the turbine is given by

$$\eta_h = \frac{\text{Head Extracted by rotor}}{\text{Net Head available to the rotor}} = \frac{H - h_{fr}}{H} = \frac{H_e}{H}\,.$$

In this h_{fr} represents the total loss of head in the rotor.
Thus, by Eq. (2.13)

$$\eta_h = \frac{H_e}{H} = \frac{u_1 V_{u1} - u_2 V_{u2}}{gH} \quad (2.16)$$

NOTE

The difference between H and H_e

H = Net head available to the rotor = Difference between the total head just upstream of the turbine inlet minus the total head at the tailrace.

H_e = Euler head = (H – losses in the rotor and draft tube – unutilised energy head thrown out to tailrace as velocity head).

Consider the ideal power extracted by the turbine, in the form of Eq. (2.6-a) as

$$P = \rho Q(u_1 V_{u1} - u_2 V_{u2})$$

Since u_1 and u_2 are always positive, it is apparent that the maximum power is obtained when the term V_{u2} is negative. Negative values of V_{u2} mean that the swirl at the runner outlet is in the direction opposite to the direction of rotation of the runner. This situation is called *reverse swirl*. In practice, it has been found that except for a very small reverse swirl that may be advantageous in increasing the Euler head, presence of negative swirl in the outlet flow causes the efficiency to drop very rapidly. This is because large reverse swirl causes a large amount of kinetic energy in the exiting water and hence in the reduction in the utilisation of the available head. As such, it has been the practice to have no reverse swirl in a turbine and to have the term V_{u2} as zero. Since $V_2 \cos \alpha_2 = V_{u2}$, the term V_{u2} will be zero when $\cos \alpha_2 = 0$. That is, when $\alpha_2 = 90°$, the hydraulic efficiency η_h is maximum from practical considerations. Hence, in practice, the design of a reaction turbine will be such that the flow will be discharged radially giving $V_2 = V_{f2}$. This is achieved at the design stage of the reaction turbines and as such, unless otherwise stated, it is assumed that the exit flow from a reaction turbine is radial. This gives the following important exit conditions for reaction turbines:

At Exit $\alpha_2 = 90°$, $V_{u2} = 0$ and this leads to the widely used result

$$H_e = \frac{1}{g}(u_1 V_{u1}) \tag{2.17}$$

At the exit, the blades will have to be so shaped to achieve this condition of zero swirl velocity. Figure 2.3 shows the entry and exit velocity triangles of a Francis turbine. Observe that at the outlet velocity triangle, $\alpha_2 = 90°$ and $V_2 = V_{f2}$. The inlet velocity triangle depicts a blade angle $\beta_1 < 90°$. The inlet blade angle can also be obtuse ($\beta_1 > 90°$) or can be a right angle ($\beta_1 = 90°$). But, the outlet blade angle β_2 is always less than 90° as α_2 is always 90°.

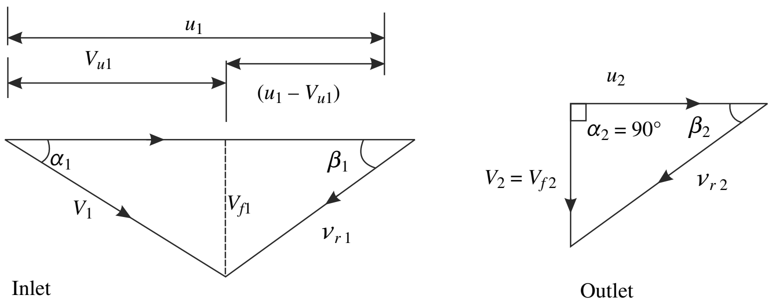

Fig. 2.3 *Velocity triangles at inlet and exit of a reaction turbine*

NOTE

The outlet swirl component V_{u2} will be negative if $\alpha_2 < 90°$. Example 2.16 deals with a flow situation where the exit discharge is not radial. This is a rare case. Figure 2.28 shows the outlet velocity triangle with $\alpha_2 < 90°$.

Parameters Affecting Hydraulic Efficiency

Using the expressions for the energy head developed in a reaction turbine, we now examine the parameters affecting the efficiency of energy transfer in a Francis turbine. Consider an ideal case of zero friction. Further, assume that the velocity of flow is constant, i.e. $V_{f2} = V_{f1}$ In a rotor, out of the total energy transferred to it by the water, only the kinetic energy of the water exiting the rotor is a loss to the turbine system and the rest of energy is expended in doing work on the shaft.

Energy utilised by the blades of the rotor per unit weight of water $= H_e = \frac{1}{g}(u_1 V_{u1})$

Loss of kinetic energy (as unutilised energy) per unit weight of water = $\dfrac{V_2^2}{2g} = \dfrac{V_{f2}^2}{2g}$
Hence, when friction and other losses are neglected,

$$\text{Hydraulic efficiency} = \eta_h = \frac{H_e}{H_e + (V_{f2}^2/2g)} \tag{2.18}$$

Referring to the inlet velocity triangle, Fig. 2.3,

$V_{u1} = V_{f1} \cot \alpha_1$ and $u_1 = V_{f1}(\cot \beta_1 + \cot \alpha_1)$

$$\text{Also } H_e = \frac{1}{g}(u_1 V_{u1}) = \frac{1}{g} V_{f1}^2 \cot \alpha_1 (\cot \beta_1 + \cot \alpha_1) \tag{2.19}$$

$$\text{Hydraulic efficiency } \eta_h = \frac{gH_e}{gH_e + (V_{f2}^2/2)} = \frac{V_{f1}^2 \cot\alpha_1(\cot\alpha_1 + \cot\beta_1)}{V_{f1}^2 \cot\alpha_1(\cot\alpha_1 + \cot\beta_1) + (V_{f2}^2/2)}$$

Since it is assumed that $V_{f2} = V_{f1}$,

$$\text{Hydraulic efficiency } \eta_h = \frac{2\cot\alpha_1(\cot\alpha_1 + \cot\beta_1)}{1 + 2\cot\alpha_1(\cot\alpha_1 + \cot\beta_1)} \tag{2.20}$$

This expression is also represented as

$$\eta_h = 1 - \frac{1}{1 + 2\cot\alpha_1(\cot\alpha_1 + \cot\beta_1)} \tag{2.20-a}$$

It is seen that when the friction is neglected, the hydraulic efficiency is a function of the guide vane angle α_1 and the blade angle β_1 and thus Eq. (2.20) underscores the importance of efficient design of these geometries.

If $V_{f2} = kV_{f1}$, Eq. 2.20 would assume the form as

$$\eta_h = 1 - \frac{2\cot\alpha_1(\cot\alpha_1 + \cot\beta_1)}{k^2 + 2\cot\alpha_1(\cot\alpha_1 + \cot\beta_1)} \tag{2.20-b}$$

2.1.7 Degree of Reaction

In Sec. 2.1.5, the degree of reaction was defined to be a measure of the degree of transfer of energy in a rotor through static pressure. Using the expressions for the energy head developed in a reaction turbine, we now obtain an expression for the degree of reaction of a Francis turbine in terms of the geometry of the blades at the inlet.

By Eq. (2.15), *degree of reaction* $R = \dfrac{H_e - \dfrac{\left(V_1^2 - V_2^2\right)}{2g}}{H_e}$.

From the velocity triangle at exit (Fig.2.3),

$V_2 = V_{f2}$ and hence $R = 1 - \dfrac{\left(V_1^2 - V_{f2}^2\right)}{2gH_e}$

It is assumed that the velocity of flow is constant and that the exit discharge is radial. From the inlet velocity triangle (Fig.2.3),

$$(V_1^2 - V_{f2}^2) = (V_1^2 - V_{f1}^2) = V_{u1}^2 = V_{f1}^2 \cot^2 \alpha_1$$

Also from Eq. (2.19), $H_e = \frac{1}{g}(u_1 V_{u1}) = \frac{1}{g} V_{f1}^2 \cot \alpha_1 (\cot \beta_1 + \cot \alpha_1)$

Thus, $$R = \frac{V_{f1}^2 \cot^2 \alpha_1}{2gH_e} = 1 - \frac{V_{f1}^2 \cot^2 \alpha_1}{2V_{f1}^2 \cot \alpha_1 (\cot \beta_1 + \cot \alpha_1)}$$

$$= 1 - \frac{\cot \alpha_1}{2(\cot \beta_1 + \cot \alpha_1)} \tag{2.21}$$

Thus, the degree of reaction is a function of α_1 and β_1 only. It is easy to see that for radial entry, as in medium-speed Francis turbines, $\beta_1 = 90°$ and $R = 1/2$.

2.2 FRANCIS TURBINE

The Francis turbine is probably the most widely used turbine for medium heads. It is named after Sir J B Francis who in 1849 developed the first adequately efficient inward-flow reaction turbine. In a Francis turbine, the water flow enters the runner radially from its outer perimeter and leaves the runner in an axial direction. Such turbines are classified as *mixed-flow turbines*. However, it is common to call all mixed-flow and radial-flow reaction turbines as Francis turbines.

Francis turbines are suitable for medium heads in the range of 40 m to 600 m. The discharge handled by a Francis turbine is relatively large. Present-day Francis machines have a very high overall efficiency that is about 90 to 95%. Figure 2.4 shows schematic views of a Francis turbine. The chief components of this turbine unit are

1. Spiral casing
2. Fixed vanes (stay vanes)
3. Adjustable guide vanes (wicket gates)
4. Runner
5. Draft tube
6. Governor

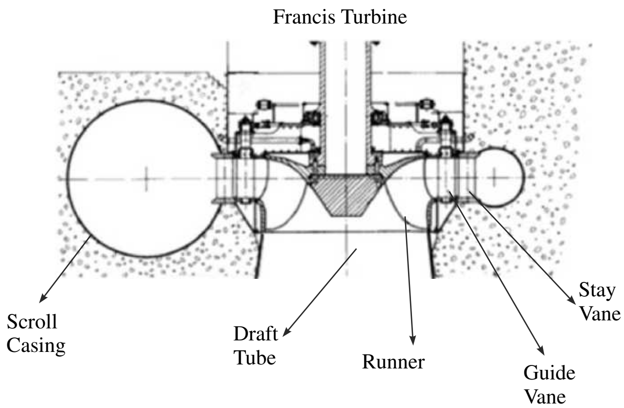

Fig. 2.4 *Sketch of a Francis turbine showing the essential components of the turbine*

Observe that the turbine shown in Fig. 2.4 is installed on a vertical shaft to have the runner in the horizontal plane. Depending upon the site conditions, horizontal shaft mountings (with the runner in the vertical plane) are also adopted. However, large installations are invariably of vertical shaft mounting. Francis turbines can be set in an open flume or attached to a penstock. For small heads and small power, open flumes are commonly employed.

2.2.1 Components of a Francis Turbine

1. *Spiral Casing*

Water from the source (reservoir/forebay) is fed to the turbine through a penstock. The flow from the penstock enters into a spiral casing surrounding the runner. The purpose of the spiral casing is to distribute water uniformly along the outer circumference of the guide vane ring (or stay-vane ring if it exists), at a constant velocity. To achieve this, the casing is provided with a diminishing cross section, with maximum cross section at the junction with the penstock and minimum at the other end. The shape of this casing, for a vertical shaft turbine, is such that the plan view appears like a snail-shaped spiral and hence the name. This casing is also known as *scroll casing*. A properly designed spiral casing produces uniform pressure all along the casing and the flow approaches all the guide vanes at constant velocity. Figure 2.5(a) shows a schematic layout of the scroll casing in a Francis-turbine unit. The radius of the spiral curve can be expressed as

$$r_s = r_i + \frac{\theta}{2\pi} d_i \tag{2.22}$$

where r_s = Radius of the spiral casing,

r_i = Outer radius of the stay-vane ring at any radius θ as in Fig. 2.5(a), and

d_i = Inside diameter of the inlet pipe.

A spiral with $\theta = 2\pi$ is called *full spiral* and if $\theta < 2\pi$, it is called a *half spiral* or *semi-spiral*.

For heads less than about 30 m in medium and large-sized units, it is economical to use a semi-spiral casing made of concrete. A typical semi-spiral casing is shown in Fig. 2.5(b). In this, the approach pipe is a rectangular conduit of concrete with the aspect ratio nearing unity. The baffle vanes are streamline shaped and are meant to divert the flow into the stay vanes. This type of semi-spiral casing is advantageously used in medium-head hydroelectric plants situated next to a dam. Reference 2.1 can be consulted for details of this type of semi-spiral casings.

The scroll casings are normally of welded steel-plate construction. Use of combination of cast steel and welded steel-plate construction is also adopted in some

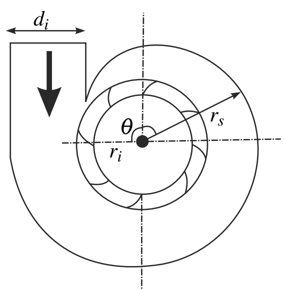

Fig. 2.5(a) *Definition sketch of spiral casing of a reaction turbine*

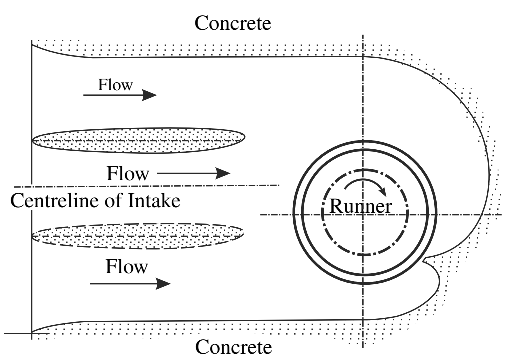

Fig. 2.5(b) *Semi-spiral casing made of concrete*

instances. The scroll casing is provided with draining and air-vent outlets and taps for pressure measurement. Further, an inspection port is often provided in the scroll casing. In large installations, the scroll casing is completely embedded in reinforced concrete.

The inlet valve, called the *Main Inlet Valve* (*MIV)* is provided at the junction of the scroll case and the penstock. *MIV* is used for normal operations and for isolating the unit during maintenance or for use in case of emergency. Usually, butterfly valves are used for heads up to 200 m and spherical valves for heads more than 200 m.

2. Stay Ring

The *stay ring* consists of upper and lower ring plates to which *stay vanes* are welded. Thus, the stay vanes are fixed in position and are non-adjustable. They are of streamline shape to have minimum losses in the flow. In addition to steering the flow to the *guide vanes*, stay vanes have another purpose of supporting the axial forces, mostly due to load of the generator mountings on the scroll case assembly. This aspect is particularly important in large vertical shaft installations. The number of stay vanes is the same as the number of adjustable guide vanes. The stay vanes are optional in nature and are mostly used in large installations.

3. Adjustable Guide Vanes (Wicket Gates)

Unlike the stay vanes that are optional in nature, *guide vanes* are essential components of a Francis-turbine unit. The adjustable guide vanes, also known as *wicket gates*, are usually even in number and are provided upstream of the runner assembly. Typically, a set of 8 to 20 blades is used in a turbine. Each of the guide vanes is mounted on a pinion and the entire set is mounted between two rings to form a wheel known as *guide wheel*. The vanes are of streamline shape and are positioned at equal angular spacing. The entire set of blades in the guide wheel moves as one unit through external linkages connected to the regulating ring so that all the flow-passages, which are formed between two adjacent vanes, have the same value. The regulating ring thus controls the flow passage area to the turbine. The motion of the regulating ring, in turn, is controlled by a servomotor of the governor system through linkages, (See Sec. 2.11.4). Figure 2.6 shows two positions of guide vanes. The passage between the vanes can be adjusted from a maximum value all the way down to zero opening. The purpose of the guide vanes is to guide the flow from the scroll case into the runner with a desired level of velocity and also in a tangential direction to the tip of the runner blade. This would achieve *shock-free entry* to the runner. Further, the ability to control the width of the passage between the guide vanes helps in controlling the total discharge into the runner. This also includes complete shutting off of the flow to the runner. Thus, the guide vanes form a part of the governing mechanism and are also used for starting and stopping the turbine. The guide vanes are usually made of cast steel. Since they are subjected to erosion due to action of sediment in the water, they require regular inspection and maintenance.

4. Turbine Runner

The runner consists of a set of warped blades arranged radially round an axis of rotation. The blades are held in position by a crown (hub) plate on the top and a shroud (band) ring at the bottom. The crown is fixed to the main shaft. The passages

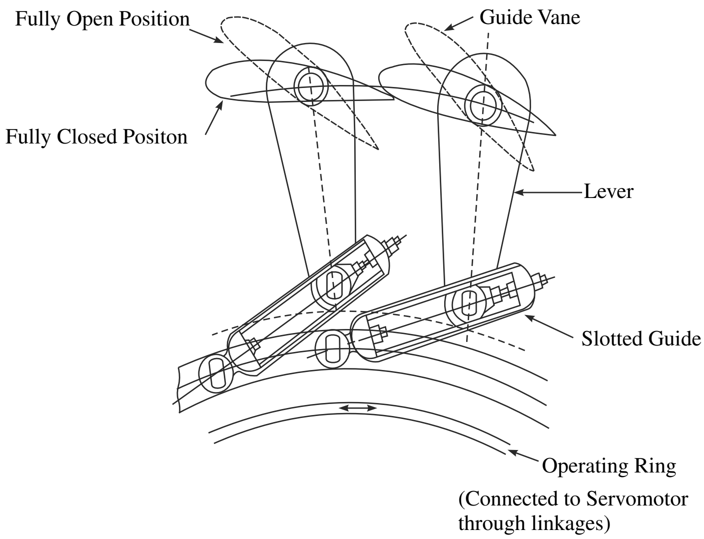

Fig. 2.6 *Schematic sketch of functioning of adjustable guide vanes*

formed between the crown, the shroud and the adjacent blades carry the flow from the guide vane passages. At the entrance, the flow in the passage is radial but at the exit, it will be in axial direction. Such a fluid passage is called *mixed flo*w and the runner is called mixed-flow Francis turbine runner. Purely radial flow runners are seldom used. The flow exiting the runner enters the draft tube (see Fig. 2.7 and 2.8).

The blades of the runner are invariably of stainless steel. Bronze blades are also in use, though not common. The runner blades are fixed in position and are usually odd in number. Typically, the number of blades lies in the range 7 to 13. The crown and shroud are generally of cast steel. Welded construction is adopted to join the plate blades, crown and shroud ring. The runner blades are thus fixed in position in the sense that they cannot be adjusted later on.

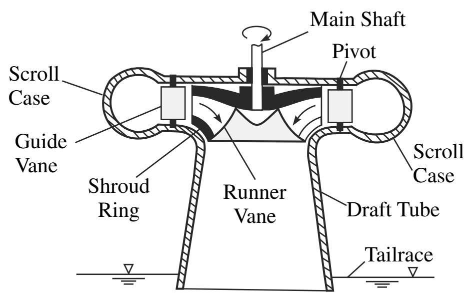

Fig. 2.7 *Francis-turbine runner*

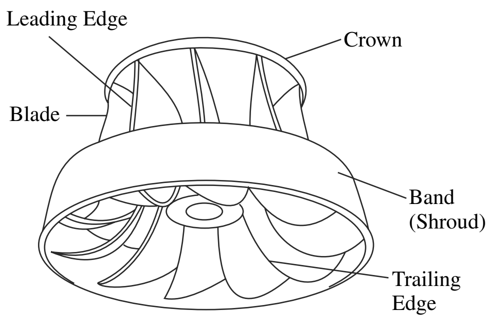

Fig. 2.8 *Francis-turbine runner (schematic sketch)*

5. Draft Tube

Reaction turbines work under pressure and hence the turbine system consisting of the runner assembly and the spiral casing are completely enclosed. The inlet and outlet will be through closed pipes flowing full. Any kinetic energy at the point of discharge of water to the tailrace is a waste of energy as far as the turbine is concerned. By minimising the kinetic energy at the outlet, the efficiency of the system can be improved. Towards this, a diverging tube that connects the runner end to the tailrace, called a *draft tube*, is used in reaction turbines.

The draft tube is thus a conduit attachment to the turbine exit to achieve the following benefits:

(a) To enable the turbine to be set up at an elevation higher than the tailwater level
(b) To utilise a major part of the kinetic energy of the water exiting the turbine

A draft tube is to be treated as an integral part of the reaction turbine and it plays a significant part in the installation of the turbine unit relative to the tailwater level. The hydraulic aspects governing draft tube installation are dealt in detail in Section 2.7.

6. Governor

The turbine governor is a system that regulates the inflow of water into a turbine. A hydroturbine is directly connected to the electric generator and hence is required to maintain the synchronous speed at all loads. Changes in the load necessitate speed adjustments. The required adjustments, in the form of control of flow into the turbine, are done by the servomotor of the governor. The governor is an essential, integral component of the present-day turbine unit and is described in detail in Section 2.11.

2.3 WORKING PROPORTIONS OF A FRANCIS TURBINE

2.3.1 Shape and Working Proportions

Consider a Francis turbine with H = Net head on the turbine, N = Rotational speed in rpm and P = Power developed by it. Let N_s = Specific speed of the turbine, D = Diameter and B = Width of the runner with suffixes 1 and 2 denoting inlet and outlet locations respectively.

Refer to Fig. 2.9, which shows schematically four Francis-turbine runners of different specific speeds sketched to a scale. An examination of this figure (Figs. 2.9*a* to *d*) would reveal that as the specific speed increases,

1. The flow of water from inlet to outlet in the runner change from nearly radial to mixed flow and then onwards to axial direction flow. The vanes of the runner become more warped in three-dimensions.
2. The width of the runner B_1 at the inlet increases.
3. Diameters D_1 and D_2 decrease.
4. The overall size of the runner decreases.

These geometrical changes occur because higher specific speed for a given *P*, *N* and *H* indicates higher discharge *Q* in the runner. The geometry of the runner passageways has to be such as to accommodate this increase in the discharge without choking. The last figure [Fig. 2.9 (*e*)] is that of an axial-flow turbine, which is the limiting stage of evolution of the shapes (*a*) through (*d*). The following relative proportions of a Francis turbine are based on the above geometrical changes as a function of the specific speed.

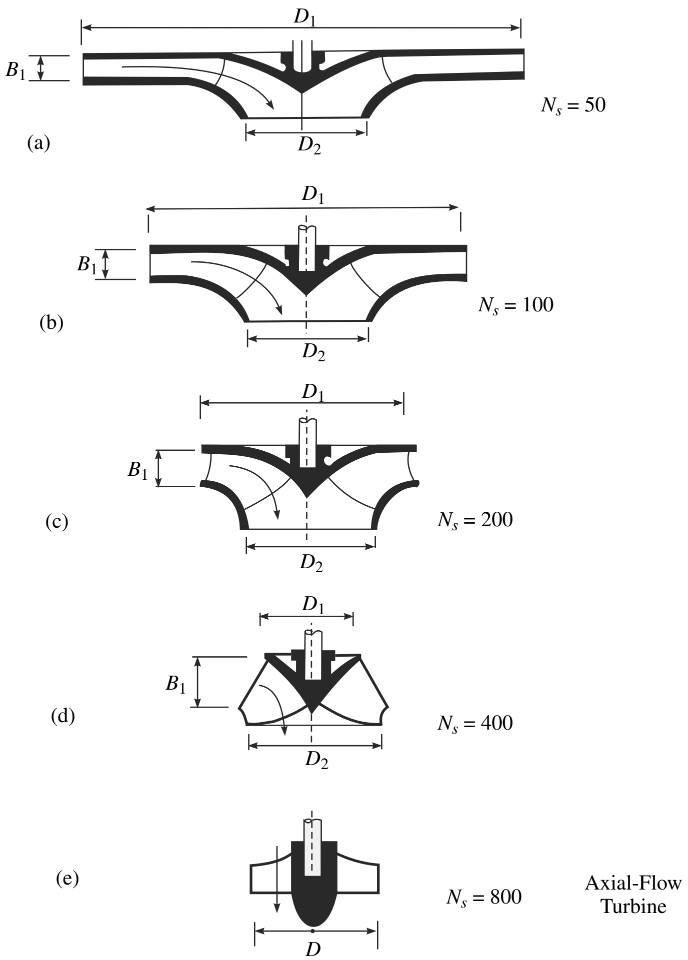

Fig. 2.9 *Shapes of Francis-turbine runners of different specific speeds and evolution of axial-flow turbine shape*

1. Relative Guide Vane Passage Size (n)

The relative passage size B_1/D_1 of the guide vanes is a function of the specific speed (N_s). The value of B_1/D_1 lies in the range 0.10 to 0.75 and its variation with the specific speed is shown in Table 2.1. In general, low-head turbines have high values of B_1/D_1.

Table 2.1 *Some working proportions of the Francis-turbine*

N_s	100	150	200	250	300	400
$n = B_1/D_1$	0.13	0.19	0.28	0.38	0.48	0.72
D_1/D_2	0.78	0.90	1.03	1.12	1.21	1.42
K_u	0.72	0.75	0.77	0.79	0.80	0.82
K_f	0.15	0.18	0.21	0.24	0.30	0.30

2. Speed Ratio, K_u

The peripheral velocity u_1 is related to the head H. The dependence of u_1 with H can be expressed as $u_1 = K_u \sqrt{2gH}$ where K_u = *speed ratio* and lies in the range 0.72 to 0.82. Speed ratio is a weak function of specific speed and its variation with N_s is indicated in Table 2.1.

3. Flow Ratio, K_f

Generally, unless otherwise stated, the velocity of flow is taken as constant. Thus, $V_{f1} = V_{f2} = K_f \sqrt{2gH}$ where K_f = *flow ratio*. K_f has a range 0.15 to 0.30 and it can be considered to vary linearly with the specific speed in the range shown in Table 2.1.

4. Vane Thickness Coefficient, K

The area at inlet $A_1 = (\pi D_1 B_1 - m t_1 B_1)$ where m is the number of vanes of thickness t_1 in the runner. However, it is convenient to represent the inlet area as $A_1 = (K_1 \pi D_1 B_1)$ where K = vane thickness coefficient ≈0.95, and suffix 1 indicating that it refers to the inlet.

5. No Swirl at the Exit

It is generally assumed that at the runner exit, there is no swirl velocity and the flow is purely radial. That is, referring to the outlet velocity triangle (Fig.2.10), α_2 = 90° and $V_{f2} = V_2$. This gives the relation, $\tan\beta_2 = \frac{V_{f_2}}{u_2}$. Further, β_2 is always less than 90°.

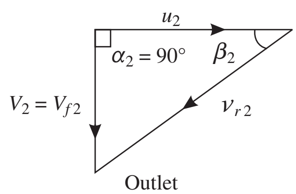

Fig. 2.10 *Outlet velocity triangle*

6. Guide Vane Angle and Blade Angle at Inlet

The entry of water to the runner should be shockless at design head, speed and discharge. The direction of the absolute velocity V_1 is guided by the guide-vane angle α_1. It is chosen in such a way that the relative velocity v_{r1} meets the runner blade tangentially, i.e. it makes an angle β_1 with the peripheral velocity at the blade inlet. Selection of the values of α_1 and β_1 is an important component of design of the turbine runner.

7. Classification Based on Speed

Figure 2.11(a, b, c and d) shows a definition sketch and three sets of velocity triangles at the outlet of a Francis runner. These are for $\beta_1 > 90°$, $\beta_1 \approx 90°$ and $\beta_1 < 90°$ respectively. These figures correspond to the turbine runner with low N_s, medium N_s and high N_s respectively. The main characteristics of these three categories of Francis turbine runners are listed in Table 2.2.

Table 2.2 *Classification of Francis-turbines*

Range of Specific Speed, N_s	Francis Turbine Category	Range of β_1	Range of α_1
40–120	Slow	120°–90°	15°–25°
120–200	Medium (/Normal)	≈90°	25°–32°
>200	Fast	45°–90°	32°–40°

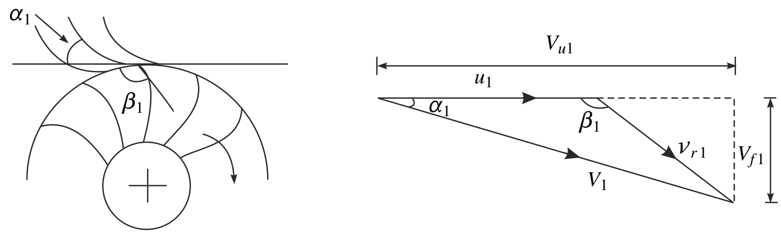

Fig. 2.11(a) *Definition sketch*

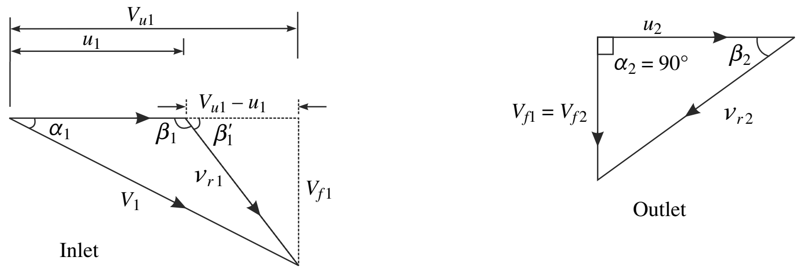

Fig. 2.11(b) $\beta_1 > 90°$, *Velocity triangles of slow Francis*

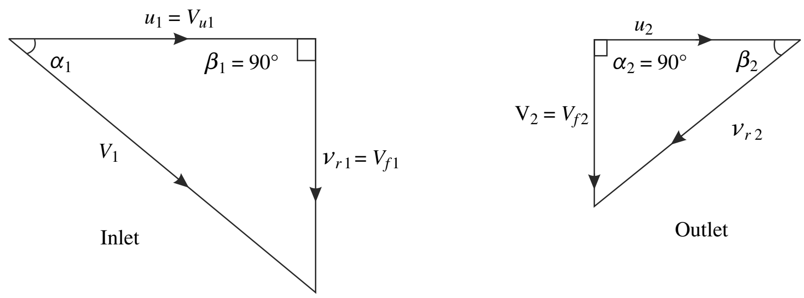

Fig. 2.11(c) $\beta_1 \approx 90°$, *Velocity triangles of normal Francis*

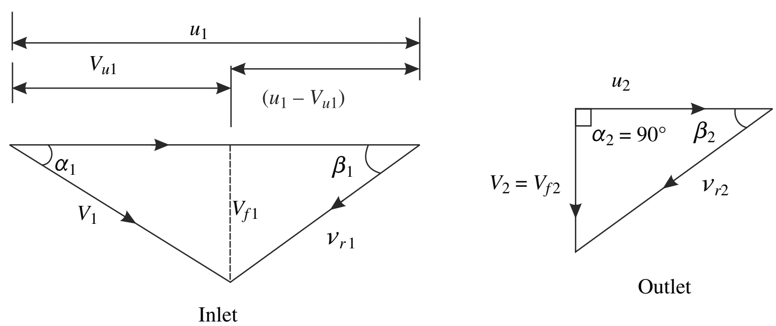

Fig. 2.11(d) $\beta_1 < 90°$, *velocity triangles of fast Francis*

It is to be observed that as the specific speed increases,

- Blade angle β_1 changes from obtuse angle at very low specific speed values to acute angle at high specific speeds. At medium (also known as *normal*) specific speeds, the angle β_1 has a value of about around 90°.
- The guide vane angle α_1 increases progressively from an angle of about 15° for very low specific speeds to a value around 40° for very high specific speeds.

8. Number of Guide Vanes, z_g

The number of guide-vane blades z_g is an even number and depends on the specific speed and diameter at inlet of the runner. Typical values of z_g are shown in Table 2.3.

Table 2.3 *Number of guide vanes, z_g*

z_g	8	10	12	14	16	18	20	24
For $N_s < 200$ D_1 in mm	Up to 200	250 to 400	400 to 600	600 to 800	800 to 1000	1000 to 1250	1250 to 1700	> 1700
For $N_s > 200$ D_1 in mm	Up to 300	300 to 450	450 to 750	750 to 1050	1050 to 1350	1350 to 1700	1700 to 2100	> 2100

The number of runner vanes is one more or one less than the number of guide vanes.

9. Basic Functional and Geometric Relationships

- From the definition, $u_1 = \dfrac{\pi D_1 N}{60}$, $u_2 = \dfrac{\pi D_2 N}{60}$
- Hydraulic efficiency for a practical reaction turbine, $\eta_h = \dfrac{u_1 V_{u1}}{gH}$
- Unless specifically stated otherwise, flow velocity $V_{f1} = V_{f2} = V_2$
- From velocity triangle at inlet:

$$\tan \alpha_1 = \frac{V_{f1}}{V_{u1}} \text{ and } \tan \beta_1 = \frac{V_{f1}}{V_{u1} - V_{u1}}$$

[Note that when $\beta_1 = 90°$, $u_1 = V_{u1}$; also when $\beta_1 > 90°$, $u_1 < V_{u1}$]

From velocity triangle at outlet:

$$\tan \beta_2 = \frac{V_{f2}}{u_2} = \frac{V_2}{u_2}$$

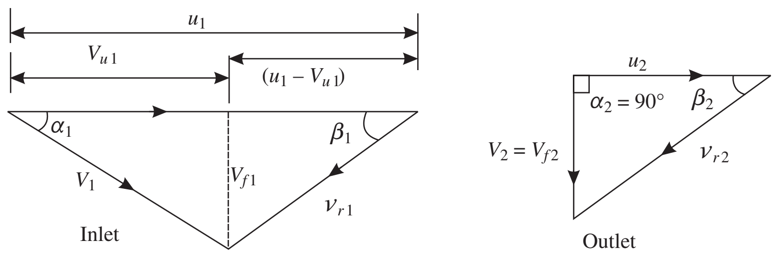

Fig. 2.12 *Inlet and outlet velocity triangles (for $\beta_1 < 90°$)*

10. Efficiencies

The efficiencies for a Francis turbine in the normal operating range are usually in a small range as indicted below:

- Overall efficiency η_0 lies in the range 0.80 to 0.90.
- The hydraulic efficiency η_h lies in the range 0.85 to 0.95.
- It is usual to assume volumetric efficiency as 1.00.
- Correspondingly, the mechanical efficiency is related as $\eta_m = \dfrac{\eta_0}{\eta_h}$.

11. Runaway Speed

Runaway speed is the speed attained by a turbine at full-gate position when the generator is disconnected from the system and the governor is inoperative. For a given type of turbine, runaway speed differs among manufacturers because of variations in design. The manufacturer supplies the runaway speed rating of a unit. All rotating parts of the turbine must be designed to withstand the stresses due to centrifugal force occurring due to the runaway speed. Generally, for a Francis turbine, the runaway speed is of the order of 175% of the design speed. For a slow Francis turbine (N_s in the range 70 to 180), the runaway speed is about 160 to 170% of the design speed. For fast Francis turbines ($N_s > 200$), the runaway speed is nearly 180% of design speed.

USBR (Ref. 2.1) recommends the following formula to calculate the runaway speed of reaction turbines. Thus, this formula is applicable to Francis and Kaplan turbines alike.

$$\frac{N_r}{N} = 0.63\,(N_s)^{1/5} \tag{2.23}$$

and

$$N_{r\,\max} = N_r \left(\frac{H_{\max}}{H_{\text{design}}}\right)^{0.5} \tag{2.24}$$

in these

N_r = Runaway speed at best efficiency head and full gate

N = Rotational speed

$N_{r\,\max}$ = Runaway speed at maximum head

$H_{\max}$ = Maximum head

H_{design} = Design head

12. Specific Speed

Normal range of specific speeds of Francis turbines is 40 to 450.

13. Range of Other Elements of the Turbine

Normal Head: 30 m – 600 m; Normal rotational speed: 90–1000 rpm,

Overall Efficiency: 90 – 94%

$\alpha_1 = 10° – 40°$; $\beta_1 = 45° – 120°$

Max. capacity as of 2012 = 805 MW

Using the relationships given in items (1.) through (12.) for the working proportions of a Francis turbine, the preliminary dimensions of a turbine for a given set of site conditions can be worked out. As an example, a procedure to estimate the basic dimensions of a Francis turbine for given values of H, P, η_0 and N is given below. These are further expanded in Illustrative Examples—Set 2.1 that follows.

2.3.2 A Procedure to Calculate Basic Dimensions of a Francis Turbine

Given data: Head H, power P, overall efficiency η_0, and rotational speed N

Required to find: Basic dimensions of a Francis turbine for the above data

- Find $\Rightarrow N_s = \dfrac{N\sqrt{P}}{H^{5/4}}$
- From $P = \eta_0 \rho QH$ — Find $\Rightarrow Q$
- From Table 2.1 — Estimate $\Rightarrow n = B_1/D_1$
- Since $A_1 = (K_1\, \pi\, D_1\, B_1) = K_1\, \pi\, n\, D_1^2$

  $V_{f1} = V_{f2} = K_f\sqrt{2gH}$ — Find $\Rightarrow V_{f1} = V_{f2}$
- $V_{f1} = V_{f2} = Q/A = \dfrac{Q}{K_1 \pi n D_1^2}$.

  Hence, $D_1 = \left[\dfrac{Q}{K_f\sqrt{2gH}\,K_1 \pi n}\right]^{1/2}$ — Find $\Rightarrow D_1$
- Assume η_m to get $\eta_h = \dfrac{\eta_0}{\eta_m}$ — Find $\Rightarrow \eta_h$
- Select any one of the following velocity triangle sets and modify as the analysis proceeds.
  - (a) $\beta_1 > 90°$

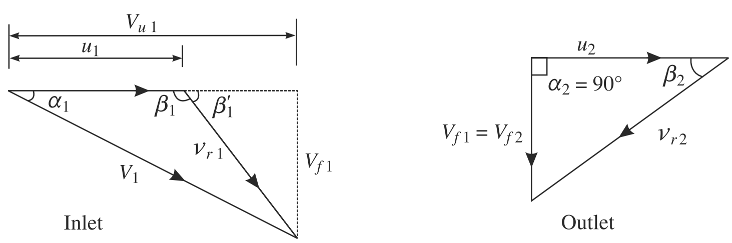

Fig. 2.13 *(a) Velocity triangles for $\beta_1 > 90°$*

- (b) $\beta_1 < 90°$

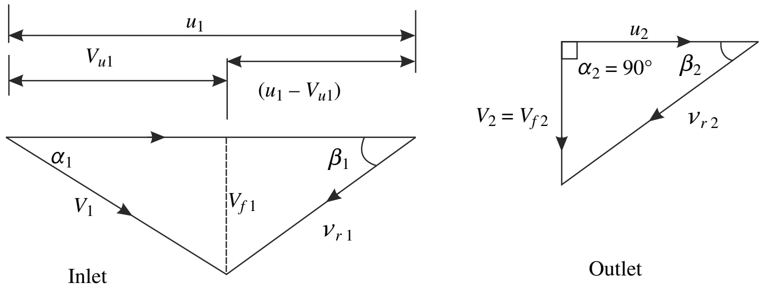

Fig. 2.13 *(b) Velocity triangles for $\beta_1 < 90°$*

- (c) $\beta_1 = 90°$

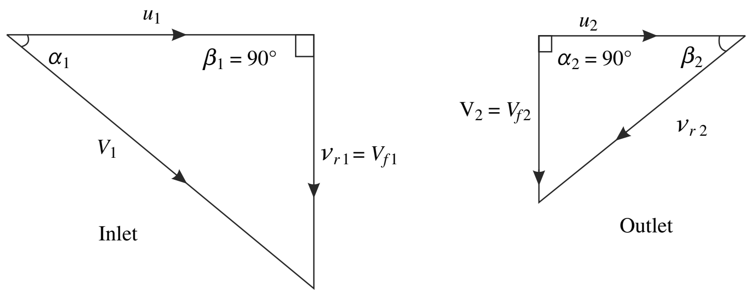

Fig. 2.13 *(c) Velocity triangles for $\beta_1 = 90°$*

- $u_1 = \dfrac{\pi D_1 N}{60}$ and $\eta_h = \dfrac{u_1 V_{u1}}{gH}$ — Find $\Rightarrow V_{u1}$
- From the inlet velocity triangle, $\tan a_1 = \dfrac{V_{f1}}{V_{u1}}$ — Find $\Rightarrow \alpha_1$
- $\tan \beta_1 = \dfrac{V_{f1}}{u_1 - V_{u1}}$ (for $\beta_1 < 90°$) — Find $\Rightarrow \beta_1$
- From Table 2.1 for known N_s — Estimate $\Rightarrow D_1 D_2$
- For the outlet triangle, take $\alpha_2 = 90°$
- $\tan \beta_2 = \dfrac{V_{f2}}{u_2}$ — Find $\Rightarrow \beta_2$

2.4 ILLUSTRATIVE EXAMPLES—SET 2.1

In all **Illustrative Examples**, the following are the defaults:

1. Exit flow is radial.
2. Velocity of flow is constant.
3. Unit weight of water, $\gamma = 9.79$ kN/m^3
4. Density of water, $\rho = 998$ N/m^3

2.4.1 Velocity Triangles

*EXAMPLE 2.1

A Francis turbine has a guide-vane angle of 15° and its inlet flow is radial. The ratio of inlet diameter to outlet diameter is 2.0. The flow velocity is 5.0 m/s and is constant. Considering the exit discharge as radial, determine the (a) the peripheral velocity at inlet, and (b) blade angle at outlet.

Solution

Given: $\alpha_1 = 15°$, $V_{f1} = V_{f2} = 5.0$ m/s, $\beta_1 = 90°$, $\alpha_2 = 90°$, $D_1/D_2 = 2$
Consider the velocity triangles at inlet and outlet (Fig.2.14).

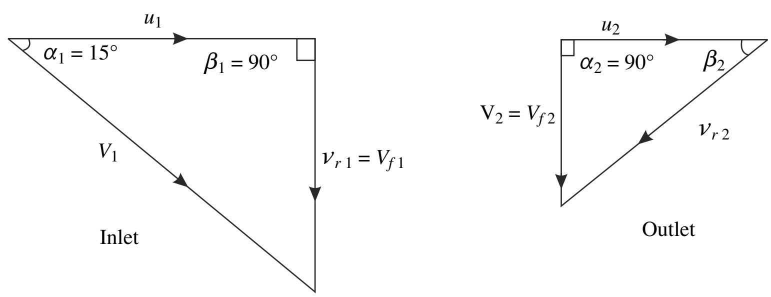

Fig. 2.14 *Inlet and outlet velocity triangles, Example 2.1*

From inlet velocity triangle: $\tan \alpha_1 = \dfrac{V_{f1}}{V_{u1}}$ and $V_{u1} = u_1$

$$u_1 = \frac{V_{f1}}{\tan \alpha_1} = \frac{5.0}{\tan 15°} = 18.66 \text{ m/s}$$

$$\frac{u_1}{u_2} = \frac{D_1}{D_2} = 2. \text{ Hence, } u_2 = u_1 \frac{D_2}{D_1} = \frac{18.66}{2} = 9.33 \text{ m/s}$$

Also $V_{f2} = V_{f1} = 5.0$ m/s

From the velocity triangle at the outlet: $\tan \beta_2 = \dfrac{V_{f2}}{u_2} = \dfrac{5.0}{9.33} = 0.5359$
$\beta_2 = 28.187°$ = Outlet blade angle.

*EXAMPLE 2.2

The following data are available for a Francis turbine:

Flow velocity = 4.0 m/s and is constant

Peripheral velocity at inlet = 30 m/s

Whirl velocity at inlet= 25 m/s

Assuming a hydraulic efficiency of 90% and zero whirl at the exit, determine (a) the net head available to the turbine, (b) the inlet blade angle, and (c) inlet guide vane angle.

Solution

Given: $V_{f1} = V_{f2} = 4.0$ m/s, $V_{u1} = 25$ m/s, $u_1 = 30.0$ m/s

Since $V_{u1} < u_1$, the inlet velocity triangle is an acute-angled triangle.

Refer to the inlet velocity triangle shown in Fig. 2.15.

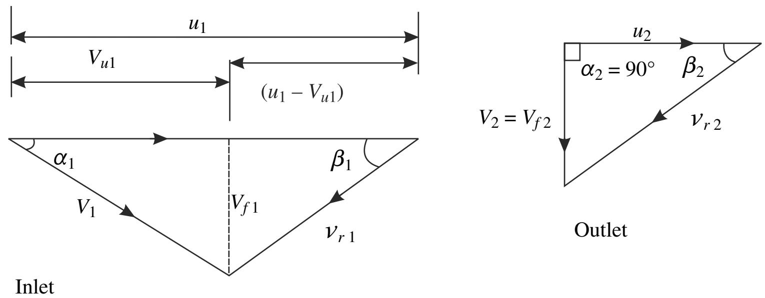

Fig 2.15 *Inlet and outlet velocity triangles, Example 2.2*

$$\tan \beta_1 = \frac{V_{f1}}{u_1 - V_{u1}} = \frac{4.0}{(30.0 - 25.0)} = 0.8$$

(a) Inlet blade angle $\beta_1 = 38.66°$

$$\tan \alpha_1 = \frac{V_{f1}}{V_{u1}} = \frac{4.0}{25} = 0.16$$

(b) Inlet guide vane angle $\alpha_1 = 9.09°$

Hydraulic efficiency $\eta_h = \dfrac{u_1 V_{u1}}{gH} = \dfrac{30 \times 25}{9.81 \times H} = 0.90$ $\quad (\because V_{u2} = 0)$

(c) Net head $H = \dfrac{30 \times 25}{9.81 \times 0.90} = 84.95$ m

*EXAMPLE 2.3

An inward-flow reaction turbine has inlet and outlet diameters of 1.2 m and 0.6 m respectively. The breadth at the inlet is 0.25 m and at the outlet, it is 0.35 m. At a speed of rotation of 250 rpm, the relative velocity at the entrance is 3.5 m/s and it is radial. Calculate the (a) absolute velocity at the entrance and its inclination to the tangent of the runner, (b) discharge, and (c) the velocity of flow at the outlet.

Solution

Given: $D_1 = 1.2$ m, $D_2 = 0.6$ m, $B_1 = 0.25$ m, $B_2 = 0.35$ m, $N = 250$ rpm, $v_{r1} = 3.5$ m/s, $\beta_1 = 90°$

Refer to the inlet velocity triangle in Fig. 2.16.

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1.2 \times 250}{60} = 15.7 \text{ m/s}$$

As $\beta_1 = 90°$ and $v_{r1} = V_{f1} = 3.5$ m/s

(a) $\tan \alpha_1 = \dfrac{V_{f1}}{u_1} = \dfrac{3.5}{15.7} = 0.228.$

Hence, $\alpha_1 = 12.56°$

Absolute velocity at entry

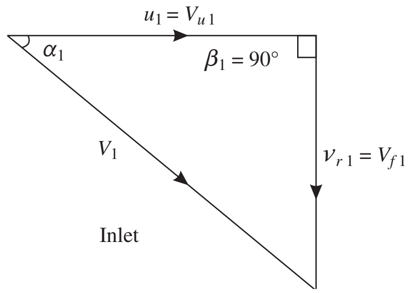

Fig 2.16 *Inlet velocity triangle, Example 2.3*

$$V_1 = \frac{V_{f1}}{\sin \alpha_1} = \frac{3.5}{\sin(12.56°)} = 16.095 \text{ m/s}$$

(b) Discharge $Q = \pi D_1 B_1 V_{f1} = \pi \times 1.2 \times 0.25 \times 3.5 = 3.299 \text{ m}^3/\text{s}$

(c) At outlet $Q = \pi D_2 B_2 V_{f2} = \pi \times 0.6 \times 0.35 \times V_{f2} = 3.299 \text{ m}^3/\text{s}$

V_{f2} = Velocity of flow at outlet = 5.0 m/s

*EXAMPLE 2.4

At what angle should the wicket gates of a Francis turbine be set to extract 8 MW of power from a flow of 30 m^3/s when running at a speed of 200 rpm? The diameter of the runner at the inlet is 3.0 m and the breadth of the openings at the inlet is 0.9 m. The flow can be assumed to leave the runner radially and the inlet blade angle is acute.

Solution

Given: $P = 8000$ kW, $Q = 30 \text{ m}^3/\text{s}$, $N = 200$ rpm, $D_1 = 3.0$ m, $B_1 = 0.9$ m, $\beta_1 < 90°$
The inlet and outlet velocity triangles are as shown in Fig. 2.17.

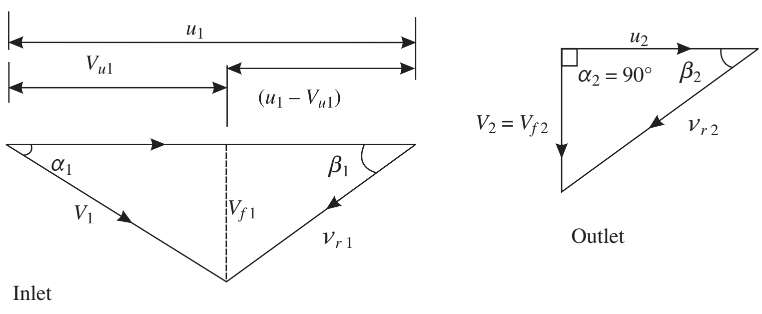

Fig 2.17 *Inlet and outlet velocity triangles, Example 2.4*

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 3.0 \times 200}{60} = 31.42 \text{ m/s}$$

Discharge $Q = \pi D_1 B_1 V_{f1} = \pi \times 3.0 \times 0.90 \times V_{f1} = 30.0 \text{ m}^3/\text{s}$

V_{f1} = velocity of flow at inlet = $\dfrac{30.0}{\pi \times 2.7}$ = 3.537 m/s

Since the flow is radial at the outlet $\alpha_2 = 90°$ and $V_{u2} = 0$

Power $P = \rho Q V_{u1} u_1 = \dfrac{998}{1000} \times 30.0 \times 31.42 \times V_{u1} = 8000$

$V_{u1} = 8.505$ m/s

From the inlet velocity triangle $\tan \alpha_1 = \dfrac{V_{f1}}{V_{u1}} = \dfrac{3.537}{8.505} = 0.4159$

$\alpha_1 = 22.58°$ = inlet guide vane angle.

*EXAMPLE 2.5

A reaction turbine works under a head of 115 m and its speed is 450 rpm. The diameter of the inlet is 1.2 m and the flow area is 0.4 m^2. At the inlet, the absolute and the relative velocities make angles of 20° and 60° respectively with the tangential velocity. Determine the power developed and the hydraulic efficiency. Assume the velocity of whirl at the outlet is zero.

Solution

Given: H = 115 m, N = 450 rpm, D_1 = 1.2 m, area of flow = 0.4 m^2, $\alpha_1 = 20°$, $\beta_1' = 60°$ [refer to the definition of blade angle β_1 in Sec. 2.1.3]

Refer to Fig. 2.18 that shows the inlet and outlet velocity triangles. At the inlet, $\alpha_1 = 20°$, $\beta_1' = 60°$, $\beta_1 = (180° - 60°) = 120°$. The inlet velocity triangle is obtuse-angled.

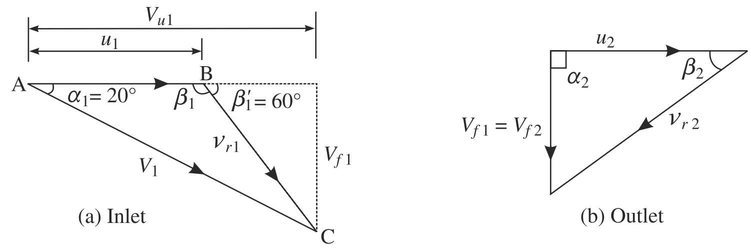

Fig. 2.18 *Inlet and outlet velocity triangles, Example 2.5*

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1.2 \times 450}{60} = 28.27 \text{ m/s}$$

From the inlet velocity triangle ABC, $A\hat{C}B = 40°$

$$\frac{V_1}{\sin 120°} = \frac{u_1}{\sin 40°}$$

$$V_1 = 28.27 \times \frac{\sin 120°}{\sin 40°} = 38.09 \text{ m/s}$$

Velocity of flow $V_{f1} = V_1 \sin 20° = 38.09 \sin 20° = 13.03$ m/s

Velocity of whirl $V_{u1} = V_1 \cos 20° = 38.09 \cos 20° = 35.79$ m/s

Discharge Q = Area × $V_{f1} = 0.4 \times 13.03 = 5.212$ m^3/s

Since the velocity of whirl is zero at the outlet,

Power developed $P = \rho Q V_{u1} u_1 = 998 \times 5.212 \times 28.27 \times 35.79 = (5263 \times 10^3)$

$= 5263$ kW

(ii) Hydraulic efficiency $\eta_h = \dfrac{u_1 V_{u1}}{gH} = \dfrac{28.27 \times 35.79}{9.81 \times 115} = 0.897 = 89.7\%$

**EXAMPLE 2.6

A Francis turbine has a speed of 300 rpm. The inlet diameter of the turbine is 1.20 m and its width is 280 mm at the inlet. The vane thickness coefficient can be taken as 0.95. If at the inlet, the guide vane angle is 30° and the blade angle is 90°, estimate the power produced by assuming an overall efficiency of 0.90. Assume radial discharge at the outlet.

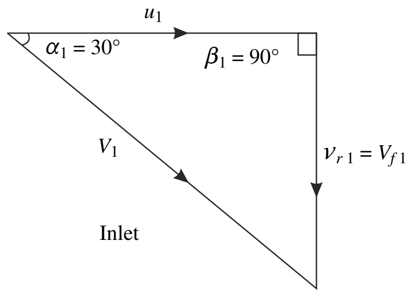

Fig 2.19 *Inlet velocity triangle, Example 2.6*

Solution

Given: Vane thickness coefficient

$K_1 = 0.95$, $N = 300$ rpm, $\eta_0 = 90\%$

At the inlet, $\alpha_1 = 30°$, $\beta_1 = 90°$,

$D_1 = 1.20$ m, $B_1 = 0.28$ m

From the inlet velocity triangle,

(Fig. 2.19),

$$V_{u1} = u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1.2 \times 300}{60} = 18.85 \text{ m/s}$$

Theoretical head extracted = $H_e = \dfrac{V_{u1} u_1}{g} = \dfrac{u_1^2}{g} = \dfrac{(18.85)^2}{9.81} = 36.219$ m

$$\tan \alpha_1 = \tan 30° = \frac{Vf_1}{u_1} = 0.5774$$

$$V_{f1} = 0.5774 \times u_1 = 0.5774 \times 18.85 = 10.88 \text{ m/s}$$

Area of flow $= A_1 = K_1 \pi D_1 B_1 = 0.95 \times \pi \times 1.2 \times 0.28 = 1.00$ m^2

Discharge $Q = A_1 V_{f1} = 1.00 \times 10.88 = 10.88$ m^3/s

Power produced $= P = \eta_0 \, \gamma Q H_e = 0.90 \times 9.79 \times 36.219 \times 10.88 = 3472$ kW

**EXAMPLE 2.7

A Francis turbine has the speed ratio of 0.80 and its flow ratio is 0.25. The width of the runner at the outer periphery is 1/4 times the outer diameter. Estimate the specific speed of the turbine. Consider the overall efficiency as 0.85.

Solution

Given: $K_{u1} = 0.80$, $K_f = 0.25$, $B_1 = D_1/4$, $\eta_0 = 0.85$

Peripheral velocity $u_1 = K_{u1}\sqrt{2gH} = 0.8\sqrt{2\times 9.81}\times H^{1/2} = 3.544\,H^{1/2}$

Since $u_1 = \dfrac{\pi D_1 N}{60}$, $\qquad N = \dfrac{3.544\times 60}{\pi D_1}\,H^{1/2} = \dfrac{67.677\,H^{1/2}}{D_1}$

Flow velocity $V_{f1} = K_f\sqrt{2gH} = 0.25\sqrt{2\times 9.81}\times H^{1/2} = 1.1074\,H^{1/2}$

Discharge $Q = \pi B_1 D_1\, V_{f1} = \pi \dfrac{D_1}{4} \times D_1 \times 1.074\, H^{1/2} = 0.8697\, D_1^2\, H^{1/2}$

Power $P = \eta_0\, \gamma Q H = 0.85 \times 9.79 \times 0.8697\, D_1^2 \times H^{1/2} \times H = 7.237\, D_1^2\, H^{3/2}$

Specific speed $N_s = \dfrac{N\sqrt{P}}{H^{5/4}} = \dfrac{67.677 H^{1/2}}{D_1}\times\sqrt{7.237 D_1^2 H^{3/2}}\times\dfrac{1}{H^{5/4}}$

$= (67.677 \times 2.690) = 182.1$

**EXAMPLE 2.8

Water leaves the guide vanes of a Francis turbine at angle α_1 to the tangent to the wheel. The blade angle at entry to the wheel is 90° and the velocity of flow at the exit is k times the flow velocity at the entry. The exit flow is radial. Prove that the peripheral velocity at entry is given by $u_1 = \sqrt{\dfrac{2gH}{2+k^2\tan^2\alpha_1}}$ where H = net head on the turbine.

Solution

$$H = \frac{V_{u1}u_1 - V_{u2}u_2}{g} + \text{losses} + \frac{V_2^2}{2g}$$

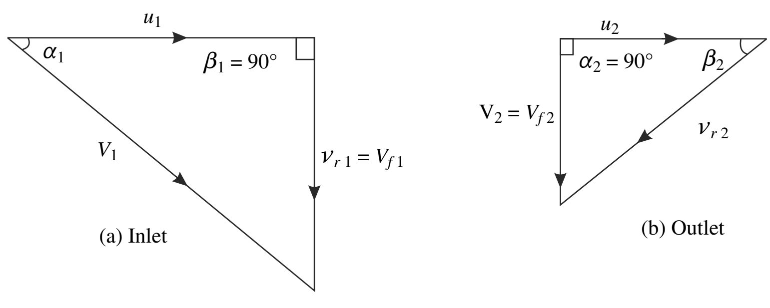

Fig 2.20 *Inlet and outlet velocity triangles of Example 2.8*

For radial flow at inlet $V_{u1} = u_1$ and $V_{f1} = u_1 \tan \alpha_1$

Losses are taken care of by $V_{f2} = kV_{f1}$. The exit flow is radial, $\alpha_2 = 90°$, i.e. $V_{u2} = 0$.

This leads to $V_{f2} = kV_{f1} = V_2$

Now $H = \dfrac{V_{u1}u_1}{g} + \dfrac{V_2^2}{2g}$

and $V_2 = kV_{f1} = ku_1 \tan \alpha_1$

Hence, $H = \dfrac{u_1^2}{g} + \dfrac{k^2 u_1^2 \tan^2 a_1}{2g}$

$$= \frac{u_1^2}{2g} + \left(2 + k^2 \tan^2 \alpha_1\right)$$

Hence, $u_1 = \sqrt{\dfrac{2gH}{2 + k^2 \tan^2 \alpha_1}}$

**EXAMPLE 2.9

An inward flow reaction turbine has, at inlet, a guide vane angle of 20° and blade angle of 30°. Determine the hydraulic efficiency and degree of reaction of the turbine. Assume the outflow to be radial and the velocity of flow to be constant.

Solution

Given: α_1=20°, β_1 = 30°, Radial discharge, $V_{f1} = V_{f2}$

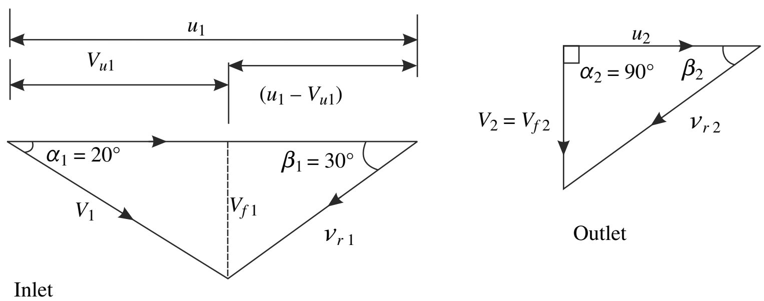

Fig. 2.21 *Inlet and outlet velocity triangles, Example 2.9*

(a) From inlet velocity triangle: $V_{f1} = V_{u1} \tan \alpha_1 = V_{u1} \tan 20° = 0.3640\, V_{u1}$

Also $\dfrac{V_{f1}}{u_1 - V_{u1}} = \tan \beta_1 = \tan 30° = 0.5773$

$0.5773\,(u_1 - V_{u1}) = V_{f1}$

$0.5773\, u_1 = 0.5773\, V_{u1} + V_{f1} = 0.5773\, V_{u1} + 0.3640\, V_{u1} = 0.9413\, V_{u1}$

$u_1 = 1.6304\, V_{u1}$

$$H = \frac{V_{u1}u_1}{g} + \frac{V_2^2}{2g} = \frac{V_{u1}u_1}{g} + \frac{V_{f1}^2}{2g} \qquad \ldots (\text{ as } V_{f1} = V_{f2} = V_2)$$

$$H = \frac{1.6304\,V_{u1}^2}{g} + \frac{(0.3640)^2\,V_{u1}^2}{2g} = 1.6966\frac{V_{u1}^2}{g}$$

Hydraulic efficiency $\eta_h = \dfrac{V_{u1}u_1}{gH} = \dfrac{1.6304\,V_{u1}^2}{1.6966\,V_{u1}^2} = 0.961 = 96.1\%$

(b) Degree of reaction: $R = \dfrac{H_e - \left(V_1^2 - V_2^2\right)/2g}{H_e}$

Refer to the velocity triangles at inlet and exit (Fig. 2.21).

Since $V_2 = V_{f2}$, $R = 1 - \dfrac{\left(V_1^2 - V_{f2}^2\right)}{2gH_e}$

It is assumed that the velocity of flow is constant. From velocity triangles,

$$\left(V_1^2 - V_{f2}^2\right) = \left(V_1^2 - V_{f1}^2\right) = V_{u1}^2$$

$$gH_e = V_{u1}\,u_1$$

Degree of reaction $R = 1 - \dfrac{1}{2}\left(\dfrac{V_{u1}}{u_1}\right) = 1 - \dfrac{1}{2\times 1.6305} = 0.693$

Alternate Method

(a) *Hydraulic Efficiency*: By Eq. (2.20-a),

$$\eta_h = 1 - \frac{1}{1 + 2\cot\alpha_1\left(\cot\alpha_1 + \cot\beta_1\right)}$$

$$\alpha_1 = 20°,\ \cot 20° = \frac{1}{0.3640} = 2.7475$$

$$\beta_1 = 30°,\ \cot 30° = \frac{1}{0.5774} = 1.7321$$

$$\eta_h = 1 - \frac{1}{1 + 2\cot\alpha_1\left(\cot\alpha_1 + \cot\beta_1\right)}$$

$$= 1 - \frac{1}{1 + \left[2\times 2.7475\times(2.7475 + 1.7321)\right]}$$

$$= 1 - \frac{1}{25.615} = 0.961$$

(b) *Degree of Reaction*: By using Eq. (2.21), $R = 1 - \dfrac{\cot\alpha_1}{2(\cot\beta_1 + \cot\alpha_1)}$

$$R = 1 - \frac{\cot 20°}{2(\cot 30° + \cot 20°)} = 1 - \frac{2.7475}{2\times(1.7321 + 2.7475)} = 0.693$$

***EXAMPLE 2.10

A Francis turbine has the degree of reaction of 0.6. The peripheral velocity at the inlet is 15 m/s and the velocity of flow is constant at 3.0 m/s. The rotor diameter at entry is twice that at exit. In addition, the discharge is radial. Assuming no frictional losses, determine the blade angles at entry and exit.

Solution

Given: $R = 0.6$, $u_1 = 15.0$ m/s, $V_f = 3.0$ m/s = Constant

[***Note***: *If no information is given, to start with, assume* $\beta_1 < 90°$.]

$$\text{Degree of reaction } R = \frac{H_e - \frac{1}{2g}\left(V_1^2 - V_2^2\right)}{H_e}$$

Since $R = 0.6$, $H_e - \frac{1}{2g}\left(V_1^2 - V_2^2\right) = 0.6\ H_e$

or $$\frac{1}{2g}\left(V_1^2 - V_2^2\right) = 0.4\ H_e$$

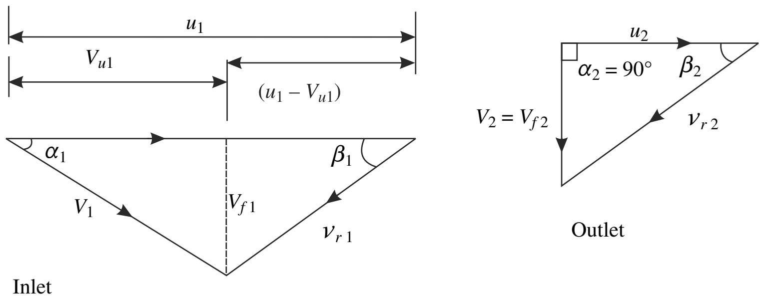

Fig 2.22 *Inlet and outlet velocity triangles, Example 2.10*

Since the outlet flow is radial, $V_{u2} = 0$.

$gH_e = V_{u1}u_1$

$$\frac{1}{2g}\left(V_1^2 - V_2^2\right) = 0.4\ H_e = 0.4\frac{V_{u1}u_1}{g}$$

$$\left(V_1^2 - V_2^2\right) = 0.8V_{u1}u_1$$

From outlet velocity triangle, $V_2 = V_{f2} = V_{f1}$

From inlet velocity triangle, $V_1{}^2 = V_{f1}^2 + V_{u1}^2$

$$V_1^2 - V_2^2 = V_{f1}^2 + V_{u1}^2 - V_{f1}^2 = V_{u1}^2$$

$$\therefore\ V_{u1}^2 = 0.8V_{u1}u_1 \quad \text{or} \quad V_{u1} = 0.8\ u_1$$

From inlet velocity triangle, $\tan \beta_1 = \dfrac{V_{f1}}{u_1 - V_{u1}} = \dfrac{V_{f1}}{0.2u_1} = \dfrac{3.0}{(0.2 \times 15)} = 1.0$

Hence blade angle at entry $\beta_1 = 45°$.

$$\frac{u_1}{u_2} = \frac{D_1}{D_2} = 2$$

$$u_2 = \frac{u_1}{2} = \frac{15}{2} = 7.5 \text{ m/s}$$

From outlet velocity triangle, $\tan \beta_2 = \dfrac{V_{f2}}{u_2} = \dfrac{3.0}{7.5} = 0.4$

Blade angle at outlet, $\beta_2 = 21.8°$

**EXAMPLE 2.11

An inward-flow reaction turbine has an inlet guide vane angle of 30° and inward edges of the runner blade at 115° to the direction of rotation. The breadth of the runner at the inlet is a quarter of the inlet diameter and there is no whirl velocity at the outlet. The gross head is 20 m and the speed is 1000 rpm. The hydraulic and overall efficiencies may be assumed to be 88% and 85% respectively. Estimate the runner diameter at inlet and the power developed.

Solution

Given: At inlet; $\alpha_1 = 30°$, $\beta'_1 = 115°$, $\beta_1 = (180° - 115°) = 65°$, $H = 20.0$ m,

$N = 1000$ rpm, $\eta_h = 0.88$, $\eta_0 = 0.85$, $B_1 = D_1/4$

Refer to Fig 2.23.

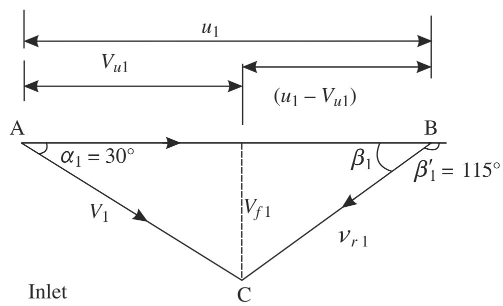

Fig 2.23 *Inlet velocity triangle, Example 2.11*

From the inlet velocity triangle ABC, $A\hat{C}B = (180° - 30° - 65°) = 85°$

$$\frac{V_1}{\sin 65°} = \frac{u_1}{\sin 85°} = \frac{v_{r1}}{\sin 30°}$$

$$V_1 = \frac{u_1 \sin 65°}{\sin 85°} = \frac{0.9063}{0.9962} u_1 = 0.91 u_1$$

$V_{u1} = V_1 \cot 30° = 0.866 \times 0.91\ u_1 = 0.788\ u_1$

Since the velocity of whirl is zero at the outlet,

Head extracted $H_e = \dfrac{V_{u1}u_1}{g} = \eta_h H$

$$H_e = \frac{0.788u_1^2}{9.81} = 0.88 \times 20$$

$$u_1^2 = 219.11$$

$$u_1 = 14.8 \text{ m/s}$$

If the diameter of the runner at inlet is D_1 then

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times D_1 \times 1000}{60} = 14.8 \text{ m/s}$$

Diameter $D_1 = 0.2827$ m

Discharge $Q = \pi D_1 B_1 V_{f1} = \pi D_1 \times \dfrac{D_1}{4} \times V_{f1}$

$V_{f1} = V_1 \sin 30° = 0.91\ u_1 \times 0.5 = 0.91 \times 14.8 \times 0.5 = 6.734$ m/s, and $D_1 = 0.2827$ m

Substituting in the expression for discharge,

$$Q = \pi \frac{(0.2827)^2}{4} \times 6.734 = 0.423 \text{ m}^3\text{/s}$$

Developed shaft power = $P = \eta_0\ \gamma Q H_e = 0.85 \times 9.79 \times 0.423 \times 20 = 70.4$ kW

***EXAMPLE 2.12

Two inward-flow reaction turbines have the same diameter of 0.70 m and the same efficiency. Both the runners work under the same head and they have the same velocity of flow of 5.8 m/s. One of the runners (A) revolves at a speed of 500 rpm and the inlet blade angle is 65°. If the other runner (B) has an inlet blade angle of 115° and the head developed is the same as that of the turbine A, what is its speed?

Solution

Given: **For Turbine *A*:** $D_a = 0.70$ m, $V_f = 5.8$ m/s = constant, $N_a = 500$ rpm, $\beta_{1a} = 65°$

For Turbine *B*: $D_b = D_a = 0.70$ m, $V_f = 5.8$ m/s, N_b = ?, $\beta_{1b} = 115°$

Consider Turbine A: $u_{1a} = \dfrac{\pi D_a N_a}{60} = \dfrac{\pi \times 0.7 \times 500}{60} = 18.326$ m/s

From inlet velocity triangle, $\tan \beta_{1a} = \tan 65° = \dfrac{V_{f1}}{u_{1a} - V_{u1a}} = 2.145$

$$\frac{V_{f1}}{u_{1a} - V_{u1a}} = \frac{5.8}{18.326 - V_{u1a}} = 2.145$$

$$V_{u1a} = \frac{33.509}{2.145} = 15.622 \text{ m/s}$$

Assuming the outlet discharge to be radial for both situations:

$$\text{Head developed } H_{ea} = \frac{V_{u1a}u_{1a}}{g} = \frac{15.622 \times 18.326}{9.81} = 29.183 \text{ m}$$

Now consider Turbine *B*: The head developed is the same as in Turbine *A*.

Hence, $H_{ea} = H_{eb}$

$$H_{eb} = \frac{V_{u1b}u_{1b}}{g} = 29.183 \text{ m}$$

$$V_{u1b}u_{1b} = 29.183 \times 9.81 = 286.29 \qquad \ldots\text{(i)}$$

From inlet velocity triangle of Turbine *B*,

$$\tan \beta'_{1b} = \tan (180° - 115°) = \tan 65° = \frac{V_f}{V_{ulb} - u_{1b}} = 2.145$$

$$V_{u1b} - u_{1b} = \frac{5.8}{2.145} = 2.704 \qquad \ldots\text{(ii)}$$

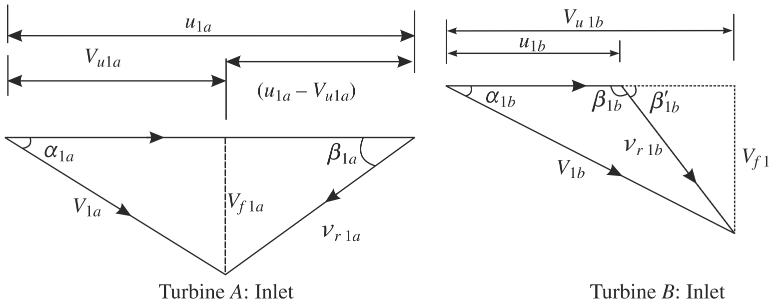

Fig 2.24 *Inlet and outlet velocity triangles of Example 2.12*

But from (i), $V_{u1b} = \dfrac{286.29}{u_b}$

Substituting this in Eq. (ii), $\dfrac{286.29}{u_b} - u_{1b} = 2.704$

$u_{1b}^2 + 2.704\, u_{1b} - 286.29 = 0$

On solving, $u_{1b} = 15.622$ m/s.

$$u_{1b} = \frac{\pi D_b N_b}{60} = \frac{\pi \times 0.7 \times N_b}{60} = 15.622 \text{ m/s}$$

$$\text{Speed of Turbine } B,\ N_b = \frac{15.622 \times 60}{\pi \times 0.7} = 426.2 \text{ rpm}$$

***EXAMPLE 2.13

A Francis turbine works under a head of 40 m and discharges 10 m^3/s of water. The speed of the runner is 300 rpm. At the inlet tip of the runner vane, the speed ratio K_u = 0.85 and the flow ratio K_f = 0.3. If the overall efficiency and hydraulic efficiency of the turbine are 80% and 90% respectively, Determine (a) the power developed in kW, (b) diameter and width of the runner at inlet (c) guide vane angle at the inlet, (d) blade angle of runner at inlet (e) specific speed of the turbine and (f) diameter of the runner at outlet. Assume the discharge at the outlet is radial and the velocity of flow to be constant.

Solution

Given: $Q = 10.0$ m^3/s, $H = 40.0$ m, $K_u = 0.85$, $K_f = 0.3$, $\eta_0 = 0.8$, $\eta_h = 0.9$

At inlet:

Peripheral velocity = $u_1 = 0.85\times\sqrt{2gH} = 0.85\sqrt{2\times 9.81\times 40} = 23.812$ m/s

Velocity of flow = $V_{f1} = 0.3\times\sqrt{2gH} = 0.3\sqrt{2\times 9.81\times 40} = 8.404$ m/s

(a) Power developed = $P = \eta_0\, \gamma Q H$

$= 0.8 \times 9.79 \times 10 \times 40 = 3132.8$ kW

Hydraulic efficiency $\eta_h = \dfrac{u_1 V_{u1}}{gH} = 0.90$

Swirl velocity at entry, $V_{u1} = \dfrac{\eta_h gH}{u_1} = \dfrac{0.9\times 9.81\times 40}{23.812} = 14.832$ m/s

(b) $u_1 = \dfrac{\pi D_1 N}{60}$

$23.812 = \dfrac{\pi D_1 \times 300}{60}$

$D_1 = \dfrac{23.812\times 60}{\pi\times 300} = 1.516$ m

Diameter at inlet = 1.516 m

But, discharge = (peripheral area) × $V_{f1} = \pi D_1 B_1 \times V_{f1}$

$10.0 = \pi \times 1.516 \times B_1 \times 8.404$, giving $B_1 = 0.250$ m = 25 cm

= Width of runner at inlet

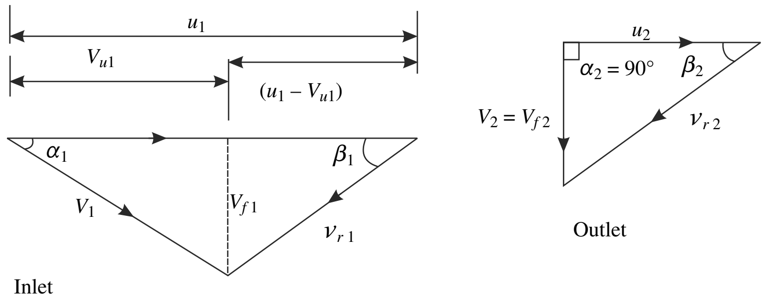

Fig. 2.25 *Inlet and outlet velocity triangles, Example 2.13*

(c) Guide vane angle at inlet, α_1:

From inlet velocity triangle at inlet: $\tan\alpha_1 = \dfrac{V_{f1}}{V_{u1}} = \dfrac{8.404}{14.832} = 0.5666$

$\alpha_1 = 29.536° =$ Guide vane angle

(d) Blade angle at inlet, β_1:

Since $V_{u1} < u_1$, the angle β_1 is acute and the velocity triangle at inlet will be as shown in Fig. 2.25.

From inlet velocity triangle:

$$\tan\beta_1 = \frac{V_{f1}}{u_1 - V_{u1}} = \frac{8.404}{(23.812 - 14.832)} = 0.9359$$

$\beta_1 = 43.10° =$ Blade angle at inlet

(e) Specific speed $N_s = \dfrac{N\sqrt{P}}{H^{5/4}} = \dfrac{300 \times \sqrt{3132.8}}{(40)^{5/4}} = 167$

(f) Diameter at outlet: Assume $V_{f1} = V_{f2}$. Since the discharge is radial,

$$Q = \frac{\pi}{4} D_2^2 V_{f1}$$

$$D_2 = \left[\frac{Q}{(\pi/4) \times V_{f1}}\right]^{1/2} = \left[\frac{10.0}{0.7854 \times 8.404}\right]^{1/2} = 1.23\,\text{m}$$

***EXAMPLE 2.14

A Francis turbine is to be designed to develop 3700 kW of power under a head of 70 m while running at a speed of 600 rpm. Following are some pertinent data of the turbine:

- *Ratio of width of runner to outer diameter of the runner = 0.1*
- *Ratio of outer diameter to inner diameter of the runner = 0.5*
- *Flow ratio = 0.25*
- *Hydraulic efficiency = 95%*
- *Mechanical efficiency = 80%*
- *Circumferential area occupied by thickness of vanes = 10%*

Assuming constant flow velocity, calculate (a) guide vane angle, (b) runner blade angle at inlet, and (c) blade angle at the outlet.

Solution

Given: $P = 3700$ kW, $H = 70$ m, $B_1/D_1 = 0.1$, $D_2/D_1 = 0.5$, $K_f = 0.25$, $\eta_h = 0.95$, $\eta_m = 0.80$, vane thickness coefficient $m = 0.1$

Assuming constant flow velocity and radial discharge at outlet, $V_{f1} = V_{f2} = V_2$

Flow ratio $= K_f = \dfrac{V_{f1}}{\sqrt{2gH}} = 0.25$

$V_{f1} = V_{f2} = 0.25 \times \sqrt{2 \times 9.81 \times 70} = 9.265$ m/s

Overall efficiency $\eta_0 = \eta_m \times \eta_n = 0.80 \times 0.95 = 0.798$

Now, power developed $P = \eta_0 \gamma Q H$

Hence, discharge $Q = \dfrac{P}{\eta_0 \gamma H} = \dfrac{3700}{0.798 \times 9.79 \times 70} = 6.75 \ \text{m}^3/\text{s}$

But discharge = (peripheral area) × V_{f1}

$6.75 = (1 - 0.1) \times \pi D_1 B_1 \times 9.265$

$D_1 B_1 = \dfrac{6.75}{\pi \times 0.90 \times 9.265} = 0.2577$

Since $B_1 = 0.1 D_1$, $D_1^2 = 2.577$ and $D_1 = 1.605$ m

$B_1 = 0.1 D_1 = 0.1605$ m

Also, since $D_2 = 0.5 D_1$, $D_2 = 0.803$ m

Peripheral velocity $u_1 = \dfrac{\pi D_1 N}{60} = \dfrac{\pi \times 1.605 \times 600}{60} = 50.423$ m/s

$$u_2 = \frac{\pi D_2 N}{60} = \frac{1}{2} \times \frac{\pi D_1 N}{60} = 25.212 \ \text{m/s}$$

Hydraulic efficiency $\eta_h = \dfrac{u_1 V_{u1}}{gH} = 0.95$

Hence, swirl velocity at entry $V_{u1} = \dfrac{\eta_h \, gH}{u_1} = \dfrac{0.95 \times 9.81 \times 70}{50.423} = 12.94$ m/s

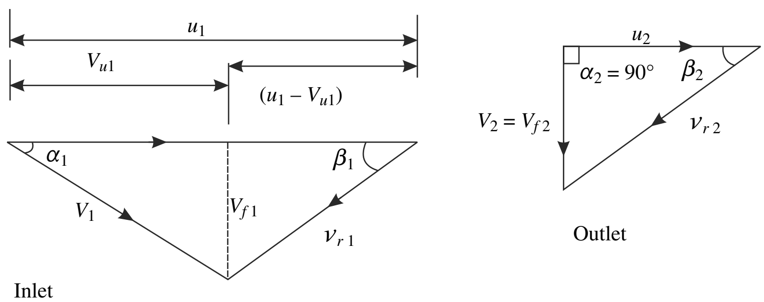

Fig. 2.26 *Inlet and outlet velocity triangles, Example 2.14*

(a) Guide vane angle at inlet, α_1:

From inlet velocity triangle at inlet: $\tan\alpha = \dfrac{V_{f1}}{V_{u1}} = \dfrac{9.265}{12.94} = 0.716$

$$\alpha_1 = 35.60°$$

(b) Blade angle at inlet, β_1:

Since $V_{u1} < u_1$, angle β_1 is acute and the velocity triangle at the inlet will be as shown in Fig. 2.26.

From inlet velocity triangle:

$$\tan\beta_1 = \frac{V_{f1}}{u_1 - V_{u1}} = \frac{9.265}{(50.423 - 12.94)} = 0.247$$

$$\beta_1 = 13.85°$$

(c) From outlet velocity triangle:

$$\tan\beta_2 = \frac{V_{f1}}{u_2} = \frac{9.265}{25.212} = 0.3675$$

$$\beta_2 = 20.18°$$

***EXAMPLE 2.15

A Francis turbine has a diameter of 1.5 m and rotates at 450 rpm. The velocity of flow is 10.0 m/s and the discharge at the outlet is radial. The turbine develops 13 MW of power with a discharge of 12 m^3/s of water. The difference of the piezometric heads between the entrance to and exit from the runner is 70.0 m. Find the (a) absolute velocity and the guide vane angle at the inlet, (b) angle of runner blades at the entry, and (c) Loss of head in the runner.

Solution

Given: Q =12.0 m^3/s, P = 13000 kW, N = 450.0 rpm,

V_{f1} = 10.0 m/s,

D_1 = 1.5 m

At entrance, peripheral velocity $u_1 = \dfrac{\pi D_1 N}{60}$

$$u_1 = \frac{\pi \times 450 \times 1.5}{60} = 35.34 \text{ m/s}$$

Power produced $P = \rho\, Q\, u_1 V_{u1}$

Swirl velocity at entry $V_{u1} = \dfrac{P}{\rho Q u_1} = \dfrac{3000}{0.998 \times 12 \times 35.34} = 30.716$ m/s

(a) Guide vane angle at inlet, α_1:

From inlet velocity triangle at inlet: $\tan\alpha_1 = \dfrac{V_{f1}}{V_{u1}} = \dfrac{10.0}{30.716} = 0.3256$

$$\alpha_1 = 18\ °$$

(b) Blade angle at inlet, β_1:

Since $V_{u1} < u_1$, the angle β_1 is acute.

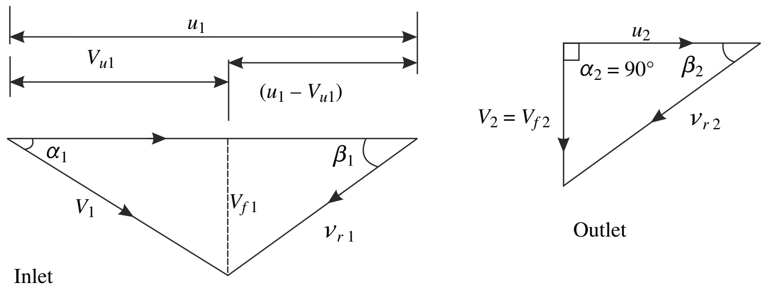

Fig. 2.27 *Inlet and outlet velocity triangles, Example 2.15*

From inlet velocity triangle:

$$\tan\beta_1 = \frac{V_{f1}}{u_1 - V_{u1}} = \frac{10.0}{(35.34 - 30.716)} = 2.163$$

$$\beta_1 = 65.18°$$

Ideal power developed by reaction of water flow $P = \gamma Q\, H_e$.

where H_e = Euler (ideal) head.

Hence, Euler head $H_e = \dfrac{P}{\gamma Q} = \dfrac{13\times10^6}{9790\times12} = 110.66$ m

Applying energy equation to the entrance to the runner (Section 1) and exit from the runner (Section2):

$$H_1 = H_2 + H_e + H_L$$

where H_1 = Total energy head at Section 1 = $\left[\dfrac{p_1}{\gamma} + Z_1\right] + \dfrac{V_1^2}{2g}$

H_2 = Total energy head at Section 2 = $\left[\dfrac{p_2}{\gamma} + Z_2\right] + \dfrac{V_2^2}{2g}$

H_e = Euler head and H_L = Energy head lost in the runner

V_1 = Absolute velocity at Section 1 = $\sqrt{V_{f1}^2 + V_{u1}^2} = \sqrt{(10.0)^2 + (30.716)^2}$

= 32.30 m/s

$$\frac{V_1^2}{2g} = \frac{(32.30)^2}{2\times9.81} = 53.17 \text{ m}$$

V_2 = Absolute velocity at Section 2 = Flow velocity = $V_{f2} = V_{f1}$ = 10.0 m/s

$$\frac{V_2^2}{2g} = \frac{(10.00)^2}{2\times9.81} = 5.10 \text{ m}$$

From given data, difference in piezometric heads between sections 1 and 2

$$= \left[\frac{p_1}{\gamma} + Z_1\right] - \left[\frac{p_2}{\gamma} + Z_2\right] = 70.0 \text{ m}$$

Energy equation is now $H_L = [H_1 - H_2] - H_e$

$$= \left\{\left[\frac{p_1}{\gamma} + Z_1\right] - \left[\frac{p_2}{\gamma} + Z_2\right]\right\} + \left\{\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right\} - H_e$$

$$H_L = 70.0 + (53.17 - 5.10) - 110.66 = 7.41 \text{ m}$$

***EXAMPLE 2.16

A Francis turbine has a runner of 60 cm diameter and the breadth at the inlet is 6 cm. The inner diameter is 45 cm and the breadth at the outlet is 8 cm. The vanes occupy 5% of the peripheral area. The guide vane angle at the inlet is 22° and the blade angles at the inlet and outlet are 80° and 25° respectively. The head on the turbine is 60 m and the hydraulic efficiency is known to be 90%. Assuming a mechanical efficiency of 95%, calculate the power delivered by the turbine.

Solution

Given: Head $H = 60$ m, At inlet: $\alpha_1 = 22°$, $\beta_1 = 80°$, $D_1 = 0.60$ m

At outlet; $\beta_2 = 25°$, $D_2 = 0.45$ m

Refer to Fig. 2.28.

Discharge $Q = \pi D_1 B_1 (1 - 0.05) V_{f1} = \pi D_2 B_2 (1 - 0.05) V_{f2}$

$$\frac{V_{f2}}{V_{f1}} = \frac{D_1 B_1}{D_2 B_2} = \frac{0.6 \times 0.06}{0.45 \times 0.08} = 1.0$$

Hence, $V_{f1} = V_{f2}$

From the inlet velocity triangle ABC, $A\hat{C}B = (180° - 22° - 80°) = 78°$

$$\frac{V_1}{\sin 80°} = \frac{u_1}{\sin 78°} = \frac{v_{r1}}{\sin 22°}$$

$$V_1 = \frac{u_1 \sin 80°}{\sin 78°} = \frac{0.9848}{0.9781} u_1 = 1.0068 u_1$$

$$\tan \alpha_1 = \frac{V_{f1}}{V_{u1}} = \tan 22° = 0.404$$

$$V_{u1} = 2.4751\, V_{f1}$$

Also $\tan \beta_1 = \dfrac{V_{f1}}{u_1 - V_{u1}} = \tan 80° = 5.671$

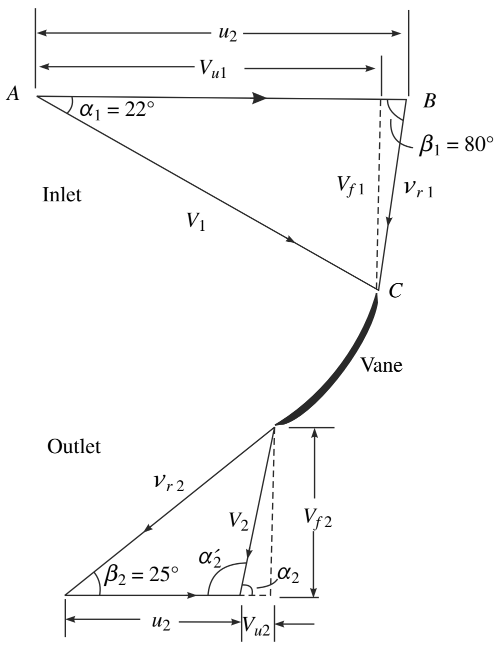

Fig 2.28 *Inlet and outlet velocity triangles, Example 2.16*

$$u_1 - V_{u1} = 0.1763\ V_{f1}$$

$$u_1 = V_{u1} + 0.1763\ V_{f1} = (2.4751 + 0.1763)V_{f1}$$

$$u_1 = 2.6514\ V_{f1}$$

At the outlet velocity triangle, $u_2 = \dfrac{D_2}{D_1}u_1 = \dfrac{0.45}{0.60}u_1 = 0.75u_1$

$$u_2 = 0.75 \times 2.6514\ V_{f1} = 1.9885\ V_{f1}$$

From outlet velocity triangle: $\tan\beta_2 = \dfrac{V_{f2}}{u_2 + V_{u2}} = \tan 25° = 0.4663$

$$\frac{V_{f1}}{1.9885V_{f1} + V_{f2}} = 0.4663 \qquad (\because V_{f1} = V_{f2})$$

Hence $V_{u2} = 0.156\ V_{f1}$

$$\tan\alpha_2 = \frac{V_{f2}}{V_{u2}} = \frac{V_{f1}}{0.156V_{f1}} = 6.41 \quad \text{giving } \alpha_2 = 81.13° \text{ or } \alpha'_2 = 98.87°$$

Since the angle α'_2 is obtuse, the direction of V_{u2} is opposite to the direction of rotation (i.e. opposite to the direction of peripheral velocity u) and hence V_{u2} is taken as negative.

Head extracted $= H_e = \dfrac{u_1 V_{u1} - u_2 V_{u2}}{g} = \left[\dfrac{(2.6514 \times 2.4751) - (-1.9885 \times 0.156)}{9.81}\right] V_{f1}^2$

$= 0.7\ V_{f1}^2$

Since the hydraulic efficiency $\eta_h = \dfrac{H_e}{H} = 0.90$

$0.7\ V_{f1}^2 = 0.90 \times 60$, yielding $V_{f1} = 8.783$ m/s

$u_1 = 2.6514\ V_{f1} = 2.6514 \times 8.783 = 23.288$ m/s

$u_1 = 23.288 = \dfrac{\pi D_1 N}{60} = \dfrac{\pi \times 0.6 \times N}{60}$

Speed of rotation, $N = 741.3$ rpm

Discharge $Q = \pi D_1 B_1 (1 - 0.05) V_{f1} = \pi \times 0.6 \times 0.06 \times 0.95 \times 8.783 = 0.944\ \text{m}^3/\text{s}$

Power delivered by the turbine

$= P = (\eta_h\ \eta_m)\ \gamma Q H = (0.90 \times 0.95) \times 9.79 \times 0.944 \times 60 = 474$ kW

2.5 CAVITATION

2.5.1 Cavitation Phenomenon

In the flow of water in a conduit, local pressures can attain sub-atmospheric (vacuum) values depending upon the local velocity, datum head and nature of flow boundary. Typical examples where local vacuum pressures can occur are a venturi tube, siphon, suction pipe of a pump and draft tube. In hydraulic machinery, due to high velocities that exist at the runner and impellers, opportunities for occurrence of high local velocities and vacuum pressures are high. When the local negative pressure reaches the vapour pressure of water, water vapour is released in the form of vapour bubbles. This phase is similar to boiling of water. When these bubbles move to regions of higher pressure in the flow, they get compressed, diminish in size and eventually may collapse. The collapse of a vapour bubble is an implosion (internal bursting) phenomenon and the surrounding water rushes to the void created by the collapsed bubble. It has been established that the bubbles collapse nonsymmetrically. When a bubble collapses, *microjets* are formed in the process of filling up of the void by the surrounding water. These microjets may impinge on the neighbouring boundary. Figure 2.29 is a schematic sketch of the formation of a microjet. When the bubbles collapse near a wall, the microjets, in turn, cause local high-pressure pulse on the boundary. This process of formation, travel and collapse of vapour bubbles in negative pressure zones of a liquid flow is known as *cavitation*.

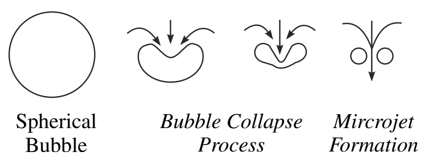

Fig. 2.29 *Mechanism of bubble collapse and formation of microjet near a wall*

In a cavitation phenomenon, the cycle of rapid formation, travel and collapse occurs in a tiny time scale (microsecond) and happens repeatedly.

The solid boundary surfaces that are impacted by microjets and consequent high-pressure pulses suffer material damage due to fatigue type of failure in a localised area. It is akin to repeated hammering by a ball peen hammer on a surface, but on a micro size. The affected boundary suffers characteristic erosionlike damage called *pitting*. In addition to damage, the cavitation phenomenon is also accompanied by a characteristic noise and vibration of the enclosing boundary. In fact, unusual vibration of the turbine unit accompanied by a characteristic noise similar to gravel being pumped/churned in the unit is an indication of the onset of cavitation. The milky appearance of the water at the exit of the draft tube is another sign of cavitation in a reaction turbine.

In reaction turbines, the blade suction side and exit location of the runner are regions highly susceptible to cavitation. In addition, cavitation may occur at the runner leading edge at off-design operation. The top portion of the draft tube is another region where cavitation has a high possibility of occurrence. In pumps, the suction pipe and inlet region of impellers are regions with cavitation possibility. The locations are specific to each type of machine and as such, in this book, they are indicated for each type of machine separately at appropriate locations.

The main effects of cavitation in a flow device, such as a hydraulic machine, are

1. Alteration of the performance of the system, for instance reduction of lift, increase in the drag, fall in the turbo-machine efficiency, etc.
2. Occurrence of noise and vibration of the component
3. Pitting damage in the wall region of the component undergoing cavitation
4. Significant shortened life of the machinery/equipment

Thus, it can be appreciated that cavitation is a harmful phenomenon and must be avoided in the design and installation of hydro machinery and devices. However, if cavitation is detected in an existing unit, the remedial measures are site-specific. In a general sense, for a cavitation problem in a turbine runner, reshaping of the blades or welding a layer of cavitation-resistant alloy at the affected part may sometimes be a remedial solution. Sometimes, modifying the operation of the unit, such as reduction of draft head through tailwater control and altering the range of the turbine operational parameters may be of some help. Of course, this will be at reduced production of power and efficiency.

2.5.2 Cavitation in Francis Turbines

In a Francis turbine, the common elements that may have cavitation damage are runner, discharge ring, draft tube liner and the wicket gates. For each rotating element, there will be a low-pressure side called *suction side* and the other high-pressure side called *pressure side*. Cavitation, being a local phenomenon, can occur on both suction and pressure sides. The major locations that are susceptible to cavitation damage can be listed as follows:

(a) *Runner*: (i) Leading edge of blade near band on suction side
(ii) Leading edge of blade near band on pressure side
(iii) Crown and at air vents in crown

(b) *Discharge ring* (opposite runner band)

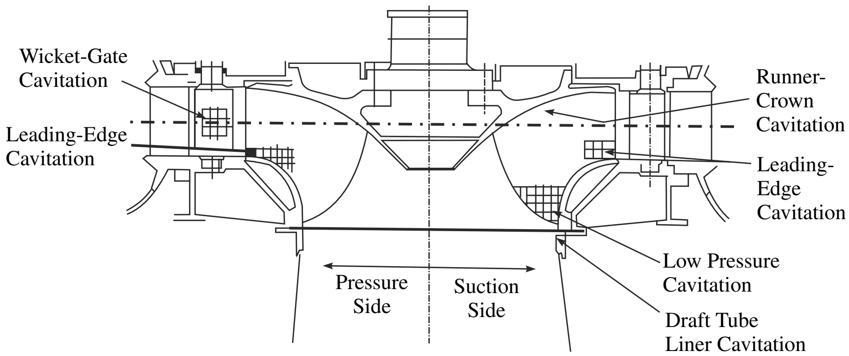

Fig. 2.30(a) *Areas of cavitation in a Francis turbine (Ref. 2.2)*

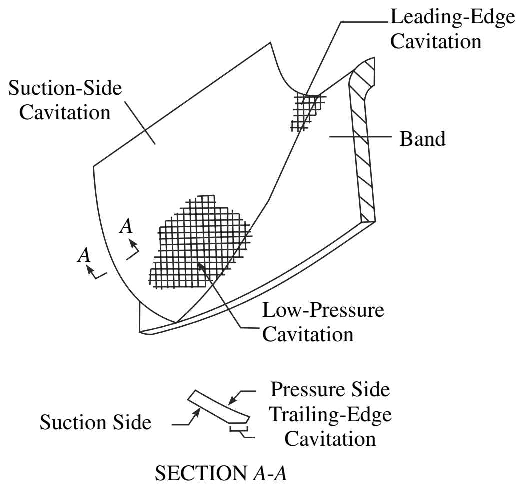

Fig. 2.30(b) *Areas of cavitation in a Francis runner (Ref. 2.2)*

(c) *Draft-tube liner* (under bottom of band; below band)

(d) *Wicket gates* (on the side of wicket gates)

Figures 2.30(a) and 2.30(b) indicate these areas very clearly.

2.5.3 Control and Repair of Cavitation Damage

1. Avoiding Cavitation

The ranges of different parameters within which the machine will run without cavitation must be identified by model studies/*CFD* studies. It should be understood by the operations group that operating the machine beyond the stipulated ranges of parameters leads to cavitation and consequent loss of efficiency and damage to the machine. Proper selection of operating regimes of the machines in a hydro plant is thus an important method of preventing cavitation.

2. Control

If an existing unit develops cavitation problems, the following aspects relating to control and repair are worth noting.

The most important step in any effort to control or minimise cavitation damage is to identify the cause. This requires careful examination of the turbine characteristics, turbine-operation pattern including the range of the chief variables in the operation and location and extent of damage. Obviously, these are site-specific. Basically, cavitation control aims at locating the cause, and correcting repairable features like faults in profile and contour. Table 2.4 (extracted from Ref. 2.2) indicates a few correlations of damage zone, possible causes and possible remedial measure for Francis turbines. Reference 2.2 contains detailed tables of this kind for Francis and Kaplan turbines.

Table 2.4 *Possible remedial measures for some cavitation damages in Francis turbines*

Location of Pitting	Possible cause	Possible Remedial Measure
Suction side of blade near trailing edge	Overall pressure too low	Reduce output
Leading edge of blade near band on suction side	Improper blade profile/ contour	- Correct contour profile - Correct leading edge profile if head range is normal
Crown or leading edge of blade on pressure side	Operation at extended period	Reduce maximum load
Draft tube liner below band	Seal leakage cavitation	Reduce maximum load
Wicket gate	Improper contour	Modify contour

3. Repair of Cavitation Damage

If damage in the form of pitting is noticed in a machine during inspection, four options of repairing are available. These are

(a) To fill the damage area with weld material

(b) To fill the damage area with non-fused material. Typical non-fused material in common use are (i) Epoxy, (ii) Ceramics, (iii) Metal spraying, and (iv) Urethane material.

(c) To weld plates over damaged parts. Obviously, this can be done on only selected parts in selected locations.

(d) To remove the damaged part and replace it with forced plates welded in place.

The basic feature of the repair procedures is that all new material used in the repair must be stronger and more resistive to cavitation damage than the material of the original part. Further, it should be remembered that if the damage is improperly repaired, the damage due to cavitation will not only continue but may continue at an increased rate, eventually leading to increased costs due to frequent outages.

These aspects of repair of cavitation damage are common to all hydraulic machines: turbines and pumps alike.

2.6 EROSION PROBLEM IN FRANCIS TURBINES

Erosion of surface of some parts due to action of silt in the water is a widely encountered problem, especially in geologically fragile areas. In Himalayan perennial rivers, the silt content is very high, especially in monsoon periods. Further, the quartz content of these silts is high and may reach as large a value as 60%. High velocities in certain passages of the turbine favoured by curvilinear flows and turbulence often lead to severe abrasion action leading to erosion of the affected boundaries. Erosion is a pure mechanical-action-based phenomenon. The severity of erosion problem is different in different types of turbines and geographies.

In Francis turbines, the parts most susceptible to erosion are (a) guide vane cascades, (b) labyrinth seals, (c) covers of guide vanes at the end of vanes, and (d) outlet and inlet edges of runners. Medium and high head units are more vulnerable to erosion problems as the velocities are generally on the high side in these units. The erosion process removes the material in select parts and causes uneven surfaces. These in turn would cause flow inefficiency and may cause conditions conducive to cavitation. Reduction of efficiency of the turbine unit, mechanical damage to parts, vibrations and frequent shutdown for maintenance are some consequences of erosion damage.

Proper control of entry of silt such as efficient settling chambers of adequate capacity; use of hard stainless steel like 13 Cr- 4%Ni or 16 Cr- 6% Ni for the components; use of hard ceramic coatings and regular frequent maintenance, are some of the possible remedial measures to mitigate erosion damage.

2.7 DRAFT TUBE

The draft tube is one of the important components of a reaction turbine and connects the outlet of the runner to the tailrace. It is essentially a closed conduit with gradual increase in the area of cross section from the inlet to the outlet. The main function of the draft tube is to enable installation of the turbine above the tailwater surface and to recover the substantial part of the kinetic energy from the flow exiting out of the runner, into pressure energy. It also helps to guide the vertical flow immediately after the runner to a horizontal flow of low velocity to enable the discharge to flow downstream in the tailrace channel. In Francis turbines, the kinetic energy at the outlet of the runner may amount to as much as 15% and in axial-flow turbines, it may go up to 50% of the total input of energy. The recovery of the kinetic energy is thus of great importance, especially in low-head installations.

2.7.1 Draft-Tube Principle

Analysis of flow through a draft tube consists principally of application of Bernoulli equation to the inlet and outlet ends of the tube along with appropriate boundary conditions.

Consider a reaction turbine with the flow from the runner exiting in the plane marked by Section 1 in Fig. 2.31. A draft tube with a *draft head* (also known as *static*

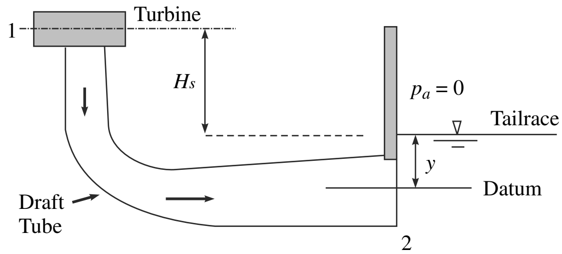

Fig. 2.31 *Draft tube in a reaction turbine*

head) of H_s connects the runner and leads the outflow to the tailrace. The centre line of the end of the draft tube is immersed in the tailrace to a depth y below the tailwater level. Section 2 marks a plane that is at a very infinitesimally short distance down the end of the draft tube. Here, the flow exits the draft tube, with a velocity V_2, as a free jet in to the tailwater.

Taking the centre line of Section 2 as the datum, we have the pressure p_2 at Section 2 given by

$$\frac{p_2}{\gamma} = \frac{p_a}{\gamma} + y \tag{2.25}$$

where $\frac{p_a}{\gamma}$ = Atmospheric pressure head (absolute) and $\frac{p_2}{\gamma}$ = Pressure head (absolute) at Section 2. With suffixes 1 and 2 to denote the sections 1 and 2 respectively, applying Bernoulli's theorem to sections 1 and 2,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + H_L \tag{2.26}$$

where V = Mean velocity,
p = Pressure (absolute)
Z = Datum head, and
H_L = Head loss in the draft tube due to friction and other causes between sections 1 and 2.

Substituting the value of $\frac{p_2}{\gamma}$ as given by Eq. (2.25) in Eq. (2.26) and noting that $Z_2 = 0$ and $Z_1 = H_s + y$, where H_s is the height of runner exit plane above the tail water (= static head), Eq. (2.26) would now read as

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + y + H_s = \frac{p_a}{\gamma} + y + \frac{V_2^2}{2g} + 0 + H_L$$

$$\frac{p_1}{\gamma} = \frac{p_a}{\gamma} - H_s - \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) + H_L \tag{2.27}$$

Putting the head loss $H_L = k\left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right)$ where k = a coefficient which is generally a small value of the order of 0.3,

$$\frac{p_1}{\gamma} = \frac{p_a}{\gamma} - H_s - (1-k)\left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) \tag{2.27-a}$$

From this equation, the following inferences relating to the pressure at the runner outlet and draft head could be derived.

Pressure at the Runner Outlet and Draft Head

Since $V_1 > V_2$ and H_s is positive, it is found that p_1 = the absolute pressure at Section 1 is less than the atmospheric pressure p_a. That is, the gauge pressure p_1 is negative. Thus, it is seen that it is possible to keep the turbine above the tailwater level by an amount H_s and this would result in negative gauge pressure at Section 1. Further, smaller values of V_2 and k and a larger value of H_s would decrease the value $p_{1.}$

The lowest value that p_1 can attain is controlled by the vapour pressure of water at the prevailing temperature. When p_1 approaches the vapour pressure, cavitation phenomenon sets in and vapour packets may form, resulting in mechanical damage, vibration and loss of efficiency. The maximum value of the draft head that can be used in a given situation depends on the cavitation susceptibility of the machine at the installation. This is discussed later.

Another interesting feature that could be observed from Eq. (2.27-a) is that all factors remaining the same, the larger the friction and other losses (that is, larger value of k), higher would be the maximum value of H_s for a given p_v.

2.7.2 Efficiency of the Draft Tube

- For the given draft tube, the kinetic energy head exiting the runner is $\frac{V_1^2}{2g}$. If there were no draft tube, this energy head was not recoverable and hence the kinetic energy wasted to the tailrace would have been $\gamma Q \frac{V_1^2}{2g}$, where Q = discharge from the runner.
- With the introduction of the draft tube, the wasted energy to the tailrace is $\gamma Q \frac{V_2^2}{2g}$

  Energy lost due to friction and other causes in the draft tube

  $= \gamma\, QH_L = \gamma\, Qk\left(\frac{\left(V_1^2 - V_2^2\right)}{2g}\right)$ where k is a coefficient.

  Thus, the recovery of kinetic-energy head due to introduction of draft tube is

  $$\gamma\, Q\left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g} - H_L\right) = \gamma Q(1-k)\left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right)$$

  Maximum possible recovery of kinetic energy, for a given V_2, is when there is no head loss H_L that is when $k = 0$. This would amount to $\gamma\, Q\left(\frac{V_1^2 - V_2^2}{2g}\right)$.

- The ratio of actual recovery of kinetic energy in the draft tube to the maximum possible recovery in the given draft tube is known as the *efficiency of the draft tube,* η_d. It is also sometimes called *pressure recovery factor*. Thus, the efficiency of the draft tube can now be expressed as

$$\eta_d = 1 - \frac{H_L}{\left(\frac{(V_1^2 - V_2^2)}{2g}\right)} = \frac{(1-k)\left(\frac{(V_1^2 - V_2^2)}{2g}\right)}{\left(\frac{(V_1^2 - V_2^2)}{2g}\right)} = (1-k) \tag{2.28}$$

This η_d gives a measure of frictional and other losses in the draft tube.

NOTE

Some authors define the efficiency of a draft tube as the ratio

$$\frac{\text{Regain of pressure head}}{\text{Velociy energy at the entrance to the draft tube}}.$$

Thus, according to this definition

$\eta_{d1} = 1 - \dfrac{\left(H_L + \frac{V_2^2}{2g}\right)}{\left(\frac{V_1^2}{2g}\right)}$. However, in this book the definition as in Eq. (2.28),

i.e. $\eta_d = 1 - \dfrac{H_L}{\left(\frac{V_1^2 - V_2^2}{2g}\right)}$, is preferred as logically sound and is used consistently throughout the book.

From the above, it is observed that the introduction of a draft tube helps

(a) the turbine to be set up at an elevation higher than the tailwater level, and
(b) utilisation of a major part of the kinetic energy of the water exiting the turbine for power production.

2.8 CAVITATION PARAMETER AND MAXIMUM HEIGHT OF TURBINE SETTING

1. Critical Sigma

As described earlier in Sec. 2.5, cavitation is a phenomenon wherein under specific conditions of sub-atmospheric pressure conditions in water flow, water vapour bubbles form, travel and then collapse in the flow causing material damage, vibration and loss of flow efficiency. In a Francis turbine, the exit region of the runner and the entrance region of the draft tube are regions of negative pressure and potential zones for cavitation. When the local pressures in a flow approaches the vapour pressure of the water at the prevailing temperature, cavitation is imminent. A *cavitation parameter* σ is defined for a reaction turbine as

$$\sigma = \frac{\left(\frac{p}{\gamma}\right)_{atm} - \left(\frac{p_v}{\gamma}\right) - H_s}{H} \tag{2.29}$$

In this, suffixes $\left(\frac{p}{\gamma}\right)_{atm}$ and $\left(\frac{p_v}{\gamma}\right)$ represent prevalent atmospheric pressure head and vapour pressure head respectively. H is the *net head* on the turbine and H_s is the *draft head* (height of turbine setting above tailwater level). From Eq.2.27, it is seen that $\left(\frac{p_{atm}}{\gamma} - H_s\right)$ is a measure of the pressure at the discharge end (Section 1) of the turbine. Thus, σ is a measure of how much the pressure at the discharge end of the turbine differs from the vapour pressure. Further, Eq. 2.29 indicates that for a given turbine, if the net head is held constant, the cavitation parameter σ decreases with increase in the value of draft head H_s.

The parameter σ is a function of the head H for a given speed and discharge of the turbine. When a turbine is tested at constant speed, fixed discharge and at values of lowered net head, it amounts to testing the turbine at lowering values of σ. Figure 2.32 represents the result of such a turbine test showing a schematic plot of the variation of efficiency with values of σ.

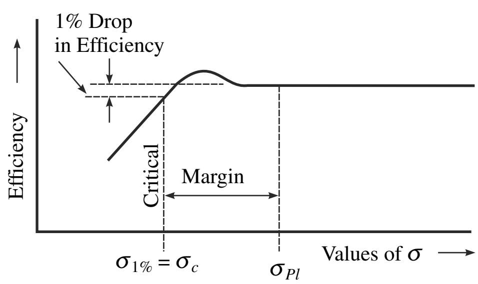

Fig. 2.32 *Definition of critical sigma*

Consider the curve as representing the behaviour of the efficiency of the turbine when the sigma of the turbine is reduced from a large initial value of sigma (say σ_a) towards zero (i.e. follow the curve from right to left). The curve is generally a constant-efficiency curve with a rapid decrease in the value of efficiency below a certain critical value of σ. The rapid fall in the efficiency of the turbine is due to the onset of cavitation. Before the onset of rapid fall, there can be a momentary rise followed by rapid fall. However, the curve suddenly breaks off representing a plunging efficiency for values of σ below the onset point of cavitation. The smallest value of σ at which incipient cavitation manifests at the turbine under study is identified by the criteria of 1% drop in efficiency. This point (marked $\sigma_{1\%}$ in Fig. 2.31) represents the lowest value of σ that can be allowed in the turbine without serious cavitation occurring in the turbine. This point is considered as the critical point and is called *critical cavitation parameter* (also known as *critical Thoma coefficient* or *critical sigma*) and is designated as σ_c. The parameter σ_c is a characteristic of the turbine under study. Thus, every turbine will have a unique σ_c and one can define the operation space of the turbine as follows:

(a) if $\sigma \geq \sigma_c$, there will be no cavitation danger, and
(b) if $\sigma < \sigma_c$, the turbine installation will suffer cavitation problem.

Using σ_c, the maximum draft head $H_{s\max}$ for a turbine setting can be now obtained from Eq.(2.29) as

$$H_{s\max} = \left(\frac{p}{\gamma}\right)_{\text{atm}} - \left(\frac{p_v}{\gamma}\right) - \sigma_c\ H \tag{2.30}$$

$$= H_{\text{atm}} - H_v - \sigma_c H$$

Note that the value of draft head given by Eq.(2.30), viz. $(H_s)_{\max}$, is the maximum value of H_s that could be adopted at the site and indicates the incipient cavitation conditions at that setting. To allow for unforeseen factors, it is usual to allow a margin (M) of up to about 0.5 m, and the actual H_s adopted will be less than or equal to ($H_{s\max} - M$). Sometimes, the factor of safety is incorporated into the value of σ_c by considering the operational critical sigma to be a certain percentage higher than σ_c obtained from 1% criteria. Such critical sigma is called as *plant sigma* (σ_{pl}) (See Fig. 2.31).

2. Variation of Critical Sigma

The critical cavitation parameter depends upon the specific speed of the turbine, and its normal variation with N_s for Francis and propeller-type turbines is given in Table 2.5.

Table 2.5 *Variation of critical Thoma coefficient with specific speed*

Parameter	Francis Turbines					Propeller Turbines		
N_s	80	160	240	320	400	400	600	800
σ_c	0.025	0.10	0.23	0.40	0.64	0.43	0.73	1.5

These are typical values and for intermediate values of the specific speed, they can be obtained by simple linear interpolation. Many empirical equations have been proposed for the estimation of critical cavitation parameter for turbines. These values can at best be used for preliminary studies. For final-stage designs and installations, more reliable and definitive results are needed. For each turbine, critical Thoma's coefficient is obtained by model tests and the turbine manufacturer furnishes its value.

Examination of typical values of σ given in Table 2.5 indicates a rapid rise in the value of σ with an increase in the value of the specific-speed. Study of Eq. (2.30) reveals that at some high specific-speed turbine settings (that is at low-head developments), the conditions may necessitate negative settings of the turbine. In such cases, the turbines will have to be below the tailwater level. In providing the requisite value of draft head, it is important to realise the need for correct estimation of the tailwater level as the draft head is provided relative to the tailwater level.

It is to be noted that atmospheric pressure decreases with altitude of a place. Roughly, the atmospheric pressure is 10.35 m of water at sea level and 6.6 m at an altitude of 3000 m above sea level. This should be kept in mind in the hydro developments in mountainous areas. The vapour-pressure head is a function of the temperature of water and Table 2.6 is a shortlist of vapour-pressure heads at various temperatures likely to be encountered in hydropower development. Note the difference of vapour-pressure head, of about 0.25 m, between a summer water temperature of 22°C and a severe winter temperature of about 5°C.

Table 2.6 *Variation of vapour-pressure head of water with temperature*

Temperature of water (°C)	0	5	10	15	20	25	30	40
Vapour pressure head $\frac{p_v}{\gamma}$ (m)	0.06	0.09	0.12	0.17	0.25	0.33	0.44	0.76

2.9 TYPES OF DRAFT TUBES

Turbine settings are decided based on site conditions, overall costs, tailwater elevations, turbine characteristics and cavitation susceptibility. Thus, they are site-specific to each installation. Since the draft tube is an integral part of the turbine, the turbine manufacturer proposes the basic design best suited to the turbine being used. Basically, the types of draft tube in use can be classified into four types as (a) vertical divergent pipe, (b) simple elbow-type, (c) bent tube with varying cross- sectional shape, and (d) Special types. Taking these in detail:

1. Vertical Divergent Type

This is the simplest kind and consists essentially of a vertical divergent pipe. The overall shape resembles a frustum of a cone with the smaller diameter at the top. The divergence is limited to a cone angle of 10°, as higher divergence would cause boundary-layer separation and consequent loss of energy (see Fig. 2.33). The flow characteristics will be highly non-uniform. While the efficiency of this draft tube is average being at around 60%, it suffers from the need of deep excavation to accommodate the divergent pipe satisfactorily. Hence, these are very rarely used nowadays.

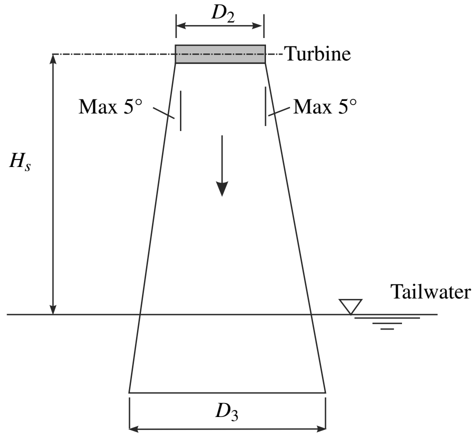

Fig. 2.33 *Vertical-type draft tube*

2. Simple Elbow-Type

In this kind, a divergent pipe is bent at 90° to create an elbow. The divergence of the flow is restricted to 10°, and the 90° bend helps in restricting the vertical distance covered by the draft tube. The overall geometry is not the best, hydraulically, and hence the overall efficiency is average being 50–60% (Fig. 2.34). This type of draft tube is seldom used these days.

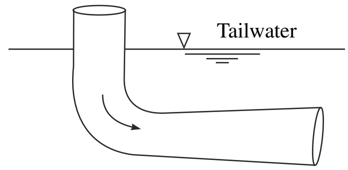

Fig. 2.34 *Simple elbow-type draft tube*

3. Elbow-type with Varying Cross-Sectional Shape and Size

This is a modification of the simple elbow type and has the cross-sectional form varying from circular at the top of the draft tube to a rectangular diverging conduit at the lower end. The change of shape allows the flow to be expanded considerably and gradually. Figure 2.35 shows a simple elbow-type draft tube in which the vertical circular pipe is gradually enlarged into a rectangular cross section in the horizontal portion.

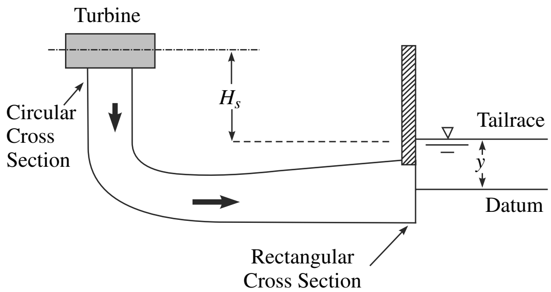

Fig. 2.35 *Elbow-type draft tube with varying cross section*

Figure 2.36 shows a well-designed, high-efficiency, elbow-type draft tube of varying cross-sectional shape. The change of the cross-section happens right at the top of the tube in a three-dimensional manner. The bottom rectangular section may

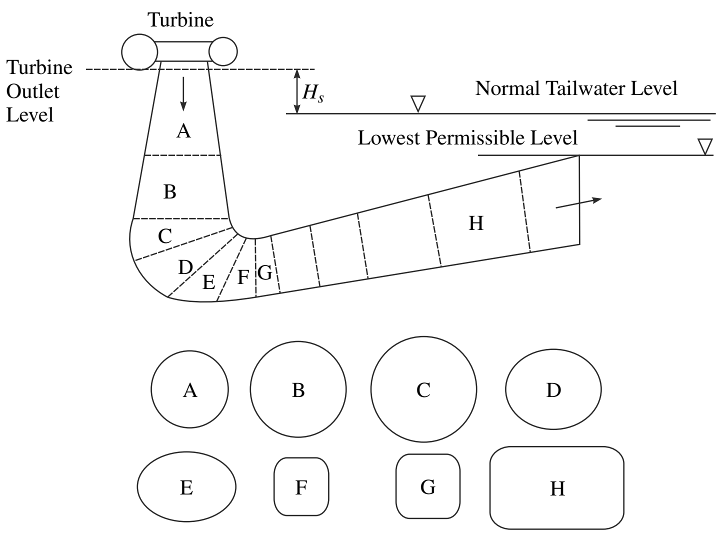

Fig. 2.36 *A well-designed elbow-type draft tube of varying cross section*

have one or more streamlined piers for controlling boundary-layer separation in the flow. The piers also aid in structural stability of the large conduit. At the end of the bend (Section *E* in Fig. 2.36), the height of the tube is minimum and beyond this section the roof of the conduit expands in the downstream direction. After the bend, the bottom of the tube may be horizontal or slightly raised in the downstream direction with a slope of the order of 1 Vertical: 10 Horizontal. The overall shape is a bent pipe with a pinched waist (called *Venturi design*) at the bottom of the bend. The shape of this type of draft tube is developed by analytical means to achieve minimum loss. The efficiency of this type of draft tube is high, being of the order of 90%. The shape change is done to achieve optimal hydraulic performance. The advantage of shallow construction and high efficiency make this type of draft tube very popular. The *IS* standard *IS*: 5496-1993 (First Revision) "*Guide for preliminary dimensioning and layout of elbow-type draft tubes for surface hydroelectric power stations*" may be consulted for preliminary dimensioning of elbow-type draft tubes.

4. Special Types

Many special types of draft tubes have been suggested and used, each type claiming some special advantages. Generally, shallow excavation, improved flow quality inside the draft tube, reduction in cavitation susceptibility, high divergence of flow in short length of the tube leading to higher efficiency are some of the goals aimed at in these special type draft tubes.

Moody's spreading draft tube (also sometimes called *hydrocone*) is an example of the special-type draft tube. This draft tube consists of a vertical diverging pipe with a solid cone occupying the lower part (see Fig. 2.37).

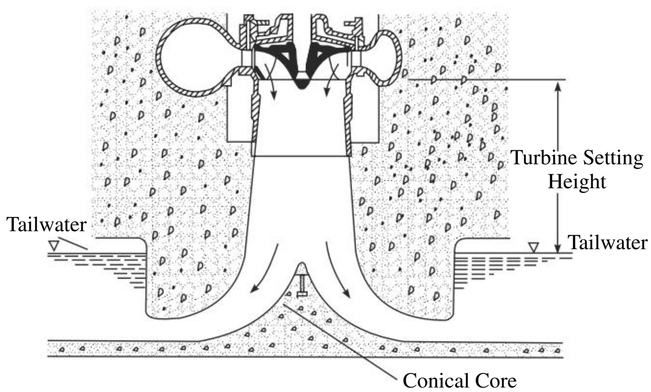

Fig. 2.37 *Moody's spreading draft tube*

The flow passes in an annular expanding ring-shaped passage in the lower parts of the draft tube. The divergence rate is very high and takes place in shallow depth. A special feature of this device is the effect of the solid cone in killing any swirl motion present in the turbine discharge. The efficiency of this device is claimed to be high with a value of about 85–90%.

2.10 CHARACTERISTICS OF FRANCIS TURBINES

The response of a turbine in the form of power or efficiency to various changes in the main parameters of the fluid flow, viz. head, discharge and speed, are studied under three categories as

1. Main characteristics
2. Operating characteristics and
3. Behaviour of the overall efficiency

2.10.1 Main Characteristics

Main characteristics are those that help in understanding the behaviour of a turbine towards selection of a turbine unit to meet specific requirements. Main characteristic curves are also known as *constant-head characteristics*. As such, in this category, for a constant head, the variations of discharge, power and efficiency with varying speed of the turbine are studied. The data for these curves comes from model testing of homologous turbine units in a laboratory and also, sometimes, from simulation studies. For the presentation of the results, the unit quantities (Sec.1.8.7) are commonly used. By using the concept of unit head, the unit speed is $N_u = \dfrac{N}{H^{1/2}}$, the unit discharge is $Q_u = \dfrac{Q}{H^{1/2}}$ and the unit power is $P_u = \dfrac{P}{H^{3/2}}$. The data set relating to the performance of a turbine is converted to unit quantities N_u, Q_u, and P_u and plotted as

1. Q_u *vs* N_u for a constant head for different gate openings (Fig. 2.38). In this figure, Q_u is the ordinate and N_u is the abscissa.
2. P_u *vs* N_u for a constant head for different gate openings (Fig. 2.39). In this figure, P_u is the ordinate and N_u is the abscissa.
3. η *vs* N_u for a constant head for different gate openings. In this, η is the efficiency of the homologous turbine unit (Fig. 2.40) and is plotted as the ordinate and N_u is the abscissa.

For ease of comparison of two or more sets, the ordinate can be percentages of maximum values of Q_u or P_u or η, as the case is, and the abscissa can be percentage of maximum value of N_u.

Figures 2.38, 2.39 and 2.40 are schematic sketches and indicate the trend of variation of the main parameters Q_u, P_u and η in a Francis turbine. A study of these three figures point out the following:

- The unit discharges at a constant head decrease nonlinearly with increase in the speed for all gate openings. In a given Francis turbine, with increase in speed, the peripheral velocity increases and the velocity of flow decreases. However, since

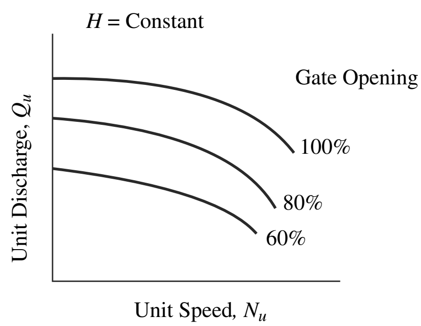

Fig. 2.38 *Schematic representation of $Q_u = F(N_u)$ for Francis turbine*

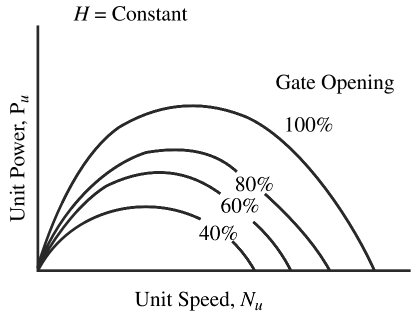

Fig. 2.39 *Variation of of unit power with unit speed*

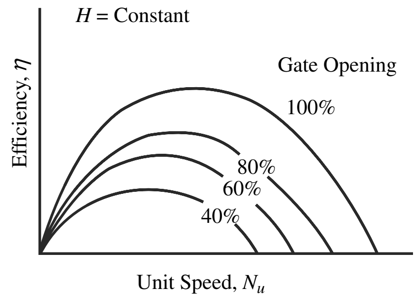

Fig. 2.40 *Variation of efficiency with unit speed*

the discharge depends upon on the velocity of flow, the discharge decreases with increase in speed.

- Power curves are parabolic in nature indicating occurrence of a maximum value at a particular value of speed that depends on the gate opening. At zero speed and at runaway speed, the power produced is zero and hence the nature of the curve with a maximum value.
- Maximum efficiency for different gate openings occurs at different speeds. The nature of the curve follows the trend of power *vs* speed curve.

2.10.2 Operating Characteristics

After a turbine is selected based on its main characteristics, it will be used by coupling to a generator for power production. The operating speed is essentially constant and the behaviour of the turbine at part load becomes important. In operational characteristics the speed is kept constant and the overall efficiency and power developed by the turbine are represented as functions of discharge for a given gate opening. The operating characteristic curves are also known as *constant-speed characteristics.* Figure 2.41 shows the variation of efficiency η and power P developed in a Francis turbine as a function of discharge Q at constant speed.

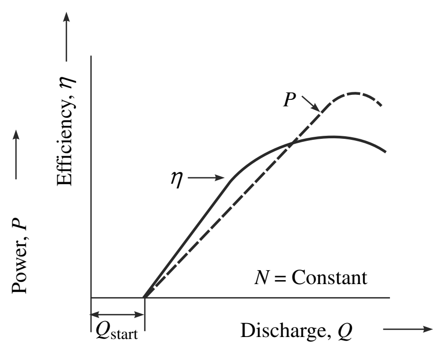

Fig. 2.41 *Variation of efficiency and power of Francis turbine at constant speed*

The power developed increases fairly linearly with the discharge in the initial part and it flattens at the peak and slightly decreases after the peak. The efficiency increases nonlinearly with the discharge. An interesting feature is that until a certain discharge Q_{start} is reached, both the power and the efficiency are zero. This occurrence is due to the inertia of the turbine unit, which requires a minimum torque to overcome it before it starts moving from an idle position. Thus, for starting a unit from idle position, the inertia has to be overcome and Q_{start} represents the condition when the critical torque is produced from the machine.

The overall efficiency being a very important parameter in the operation of a turbine, study of its variation has received maximum attention. Some of the commonly used representations of the variation of efficiency are presented below. In the characteristic curves, the overall efficiency is the default.

2.10.3 Efficiency Characteristics of Francis Turbine

1. Definitions

In a hydroelectric plant, the turbines are the prime movers and the object of the hydro plant is to achieve optimal power production. As invariably the turbine is coupled directly to a generator, the output of a station is in terms of electric units (kWh) of energy produced over a time period. In a general sense, the head and discharge of a hydro plant are variables. They are subjected to seasonal changes and are dependent on the hydrology of the catchment area of the water source. The terms *rated power (name plate power), installed power, rated head* and *design head,* as used in connection with the efficiency, have to be understood before proceeding with the discussion of efficiency of a turbine.

(a) Rated Power (Nameplate Power) It is the highest power at which a machine can be operated continuously without suffering damage. It is also known as *maximum continuous power*. The maximum rated output of the turbine (installed generator rated capacity) is usually indicated on a nameplate physically attached to the machine. The parameters used in relation with the rated power output, viz. H, N, Q, are called *rated head* (H_r) and *rated speed* (N_r) *rated discharge* (Q_r) respectively. The rated generating electrical power is expressed as

$$P_r \text{ (Electrical)} = P_r \text{ (Turbine)} \times \text{(Efficiency of Generator, } \eta_e) = \eta_e \, \eta_0 \, \gamma Q_r H_r \quad (2.31)$$

The *installed capacity* of a hydro-power plant is the sum of generator nameplate power ratings of all the units in that plant.

(b) Rated Head It is the net head at which full-gate output of the turbine produce the generator rated output of power. The turbine nameplate rating is given at this head. The rated head is different from the *design head*, which is defined as the net head at which peak efficiency is desired. Rated head is selected on the basis of yielding highest annual power output. Selection of rating head requires considerable deliberation and its relation to design head is site-specific. Generally, rated efficiency will be slightly lower than the peak efficiency of the turbine unit.

2. Efficiency Curve of a Francis Turbine

Figure 2.42 adapted from (Ref. 2.1) is a representation of the variation of efficiency of a Francis turbine. In this figure, the percentage design head is the ordinate and percentage of best efficiency power at design head is the abscissa. The figure shows the contours of iso-efficiency lines as third parameter and percentage of gate opening as fourth parameter. The figure is self-explanatory and clearly brings out the difference between *design point* (best efficiency point) and preferred generator rating point. Note that this figure is for a particular turbine but can be considered as typical

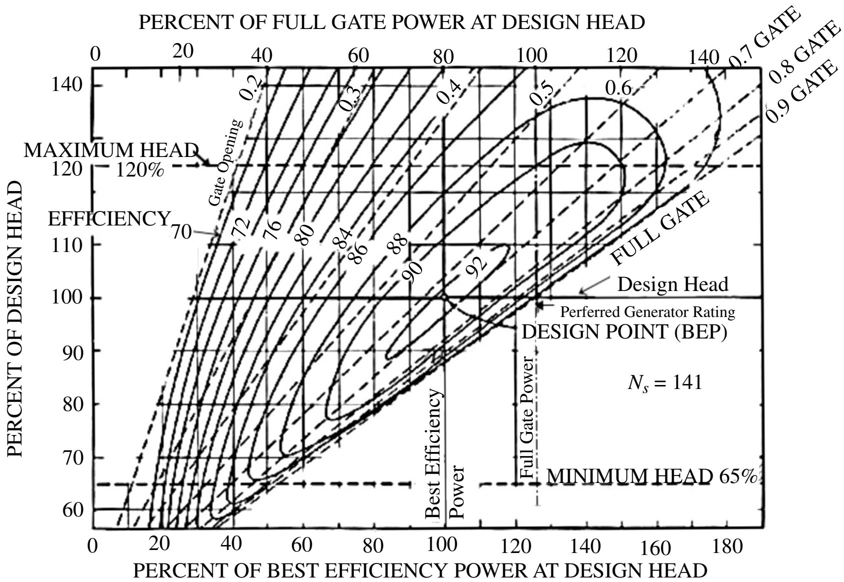

Fig. 2.42 *Efficiency curves of a Francis turbine (adapted from Ref. 2.1)*

of such plots. For this turbine, the rated efficiency is 88% and the design efficiency is 92%, which is the best efficiency. The rated power corresponds to full gate power and occurs at 120% of the design power. The turbine is rated to work in the range of 120% of design head as maximum head and 65% of design head as minimum head.

3. Francis Turbine Efficiency Curve for Preliminary Studies

While individual turbine designs may have slight variations in the absolute values of the efficiency at and near peak regions, for preliminary design the following commonly used chart of variation of efficiency vs. percentage rated capacity is recommended. Figure 2.43, which is adapted from IS: 26800 of 1991 and *IEC*-1116-1992 (Ref. 2.3, 2.4) shows the variation of expected overall efficiency η of a Francis turbine with percentage rated capacity of the turbine.

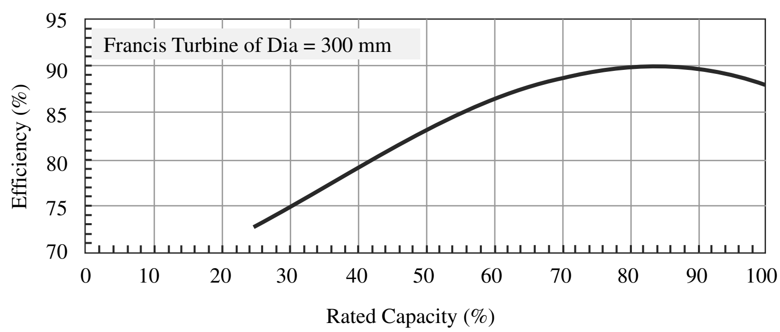

Fig. 2.43 *Rated efficiency of Francis turbine of 300 mm diameter for preliminary studies (adapted from Ref. 2.3, 2.4)*

The data has been adjusted for a turbine of 300 mm runner outer diameter and for any other size of the runner, the efficiency has to be stepped up (see Sec. 1.53). Table 2.7 gives the additive step values for various diameters.

Table 2.7 *Additive step-up values of efficiency*

Size of turbine in mm	300	600	900	1200	1800	2400	3000
Additive step-up value	1.00	1.10	2.10	2.50	3.30	3.70	4.00
The step-up value of this table is to be added to η value obtained from Fig. 2.43							

This figure gives the percentage rated efficiency and the *design peak efficiency* at design head is about 2–5% higher.

A study of Fig. 2.43 indicates that

- At 100% rated turbine capacity the rated efficiency = η_r is about 88%,
- Peak efficiency is 90% and occurs in the range 75 to 85% of the rated capacity,
- At 40% rated capacity, η is 80% and drops to about 72% at 25% rated capacity. Reaction turbines experience rough operation around 20%–40% of rated capacity with vibration and/or power surges. This is reflected in a rapid drop in the efficiency in this range.

The above features clearly indicate the suitable range of operation of a Francis turbine as between 70–110% of rated capacity. At part-loads, which are in the range of lower than 70% rated capacity, the efficiencies are considerably lower.

4. Efficiency Hill Chart

For a Francis turbine, the efficiency varies considerably when the gate opening and speed are varied in their respective range. In model tests, the head is kept constant, and the speed and load are varied for a particular gate opening. From this data, the variation of power and discharge with speed can now be obtained. In the plot of the results, the power or discharge will be the ordinate and speed would be the abscissa. The gate opening can be the third parameter. If for each data point the corresponding efficiency is calculated and marked on the same plot, the points of equal efficiency can be obtained by suitable interpolation and joined to get contour plots of equal efficiency. The resulting plot will have iso-efficiency curves. Such a plot of iso-efficiency is known as a *hill chart*. In such iso-efficiency curves one can imagine the hill shape of efficiency and identify its peak and slopes in the off-peak areas. From an iso-efficiency chart, it can be seen at a glance what the speed of a turbine should be, at any gate opening, in order to give the best efficiency for that gate opening. In addition, the chart gives the peak efficiency and the values of the parameters under which this peak would occur.

Different forms of the parameters for the ordinate and abscissa of the plot are in use to make the hill-chart useful in its various applications.

(a) One of the conventional ways is to use unit discharge (Q_u) or unit power (P_u) as ordinate and unit speed (N_u) as abscissa. Such an iso-efficiency plot is known as *Muschel chart* and the iso-efficiency curves are called *Muschel curves*.

Figure 2.44 shows a schematic Muschel chart of a Francis turbine. In this figure, the ordinate is unit power (P_u) and the *x*-axis is the unit speed (N_u). The amount of

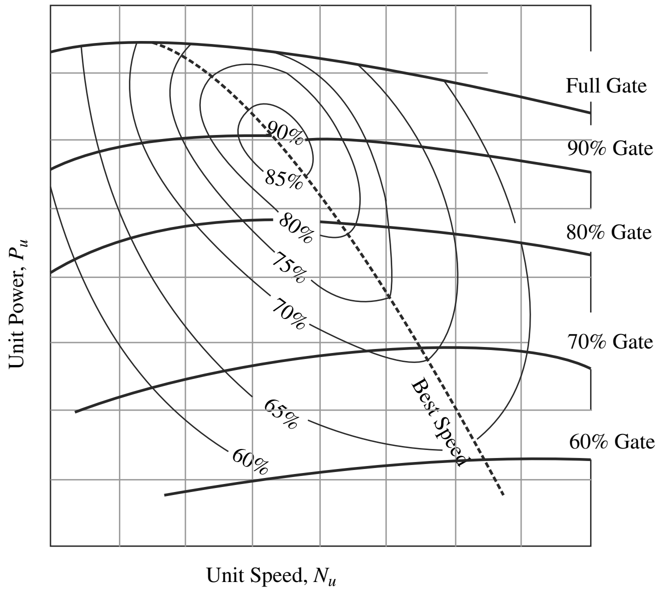

Fig. 2.44 *Schematic iso-efficiency curve (Muschel chart) of a Francis turbine*

gate opening is the third parameter. Based upon the experimental data on the model of the machine, iso-efficiency lines are drawn through appropriate interpolations. This chart shows the maximum efficiency for all conditions and gate openings. Also, the speed that produces maximum efficiency for any gate opening can be read off the curve.

It is worth mentioning that the term *Muschel chart* is hardly ever used these days and instead, the term is *hill chart* is regularly used to mean iso-efficiency curves in hydraulic machines. There are many variations in the use of parameters in the ordinate and abscissa parts of an efficiency hill chart; very often, defined ratios of unit discharge or unit power and unit speeds are used instead.

RECAP ***Unit quantities*** (Sec.1.8.7)

$Q_u = \left[\frac{Q}{\sqrt{H}}\right]$ = Unit discharge; $P_u = \left[\frac{P}{H^{3/2}}\right]$ = Unit power; $N_u = \left[\frac{N}{\sqrt{H}}\right]$ = Unit speed

Specific Quantities:

Specific power, $P_{11} = \frac{P}{H^{3/2}D^2}$

Specific discharge, $Q_{11} = \frac{Q}{D^2\sqrt{H}}$

Defined double unit speed, $N_{11} = \frac{ND}{\sqrt{H}}$

(b) Figure 2.45 shows a generalised hill diagram for a Francis turbine. In this, the ordinate is the percentage of design head and the abscissa is the percent discharge. Efficiency is the third parameter and the percentage gate opening is the fourth parameter. This figure can be considered as a typical characteristic of a Francis turbine. From the figure it is seen that a typical Francis turbine has high efficiencies ($\eta_0 > 70\%$) in a range 65% to 125% of design head and can have relatively high efficiencies down to about 25% of design flow.

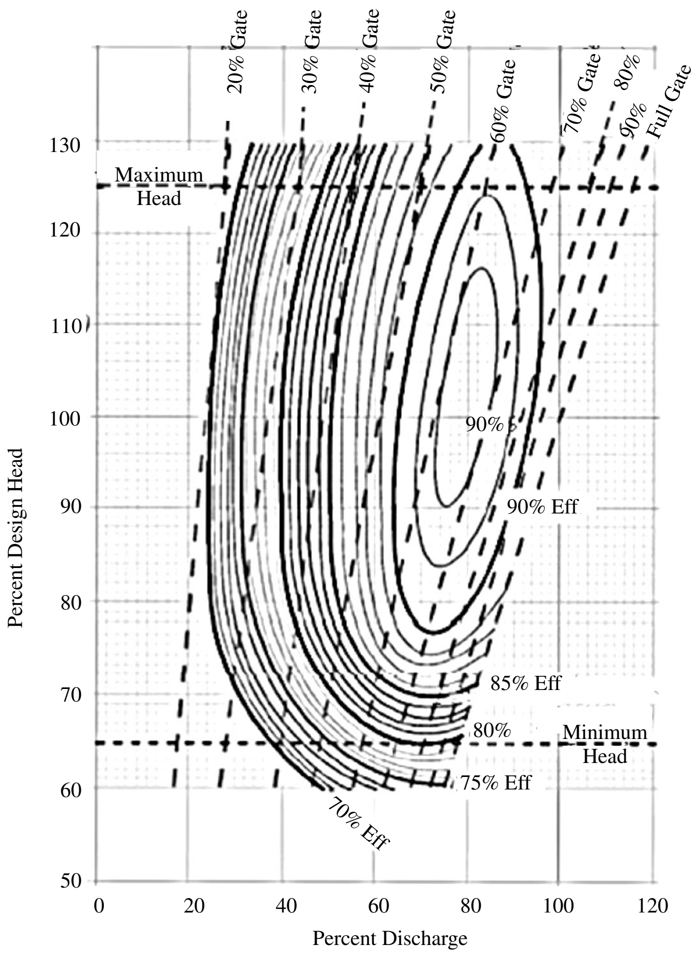

Fig. 2.45 *Generalised Hill diagram of a Francis turbine (adapted from Ref. 2.1)*

(c) Another method that is often adopted in numerical simulation of turbine performance is to use *specific rate of flow* Q_{11}, which is the discharge of a homologous turbine of unit head and unit diameter, as ordinate. The abscissa is $N_{11,}$ the defined unit speed of a homologous turbine of unit diameter. Thus, $Q_{11} = \dfrac{Q}{D^2\sqrt{H}}$ and $N_{11} = \dfrac{ND}{\sqrt{H}}$ form the basic parameters of the hill chart with iso-efficiency curves as the third parameter and sometimes the gate opening as the fourth parameter. If power is used instead of discharge in the ordinate, the corresponding parameter is specific power $P_{11} = \dfrac{P}{H^{3/2}D^2}$ which is the power produced by a homologous turbine of unit head and unit diameter. A typical hill chart of Q_{11} vs N_{11} as used in numerical studies can be viewed in the following links:

(i) *http //bit.ly/1D4ra*

(ii) *http //www.jstage.jst.go.jp/article/jfst/1/2/147/_pdf*

(d) In *CFD* studies and other forms of numerical simulation of the performance of a reaction turbine, the variation of efficiency is sometimes represented as

$$\eta = f(\phi, \psi, \alpha_1) \tag{2.32}$$

where ϕ = Non-dimensional discharge coefficient known as *flow coefficient* and is defined as $\phi = \dfrac{Q}{D^2\sqrt{2gH}}$

ψ = Non-dimensional energy coefficient known as *specific energy coefficient* and is defined as $\psi = \dfrac{2gH}{\omega^2 D^2}$

α_1 = Inlet guide vane angle and

ω = Angular velocity in radians/second.

It can be observed that $\phi = f(Q_{11})$ and $\psi = f(N_{11})$. The variation of the efficiency in the form of Eq. 2.32 is usually represented graphically in the form of a hill chart having the parameters ϕ, ψ and α_1.

2.11 GOVERNING OF TURBINES

2.11.1 Governing System

The turbine governor is a system that regulates the inflow of water into a turbine. The hydroturbine being a prime mover for a generator is directly coupled to the generator and its speed depends on the requirements of the load on the generator. In order to maintain the required generated frequency of 50 Hz, the speed of rotation of the generator and hence that of the turbine must be kept constant at the synchronous speed at all loads. The turbine governor receives information on the existing rotational speed of the turbine and adjusts the inflow of water to the turbine to maintain the speed at the correct desired level.

A governing system consists essentially of (a) a control system, and (b) a mechanical hydraulic actuating system. Figure (2.46) shows a schematic representation of a basic control system.

The control system may be (a) mechanical, (b) Electronic (Analogue) or (c) Digital. The actuator can be hydraulically controlled or mechanical motors or load actuator. While each of these has specific advantages, hydraulically controlled actuators are the most commonly used kind.

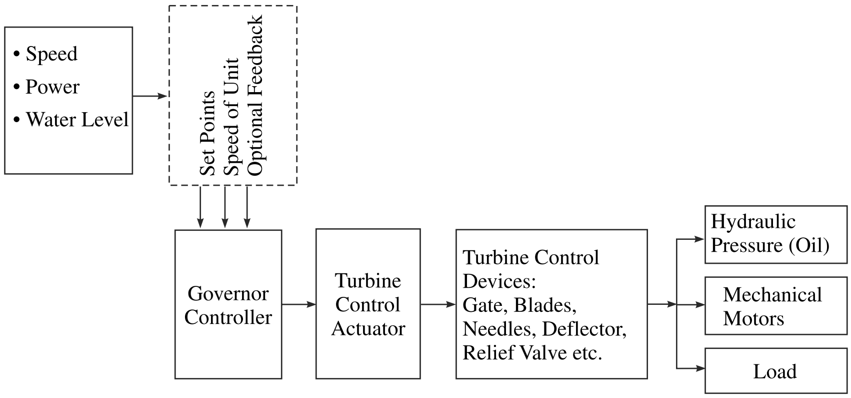

Fig. 2.46 *Schematic representation of a basic control system for a turbine*

Pure mechanical controllers (which can be called as first generation (1G) controllers) were in use up to the middle of 20^{th} century. Here, belt drive control devices were directly connected to the prime mover. Flyball type pendulum was the standard speed-sensing device.

In second generation (2G) mechanical controllers, speed sensing was achieved by pendulum motors and permanent magnet generators. Mechanical dashpots, springs and link mechanisms were used in achieving the desired settings. This type of controller belongs to the PI category.

The third generation (3G) controller is called *electro-hydraulic controller*. This consisted essentially in the reduction of mechanical components. Speed sensing was achieved by electrical/electronic devices. Analogue circuitry was used to develop set points signal that would help position the control actuators of hydraulic units. This type of controller belongs to the *PID* category.

The currently used fourth generation (4G) controllers are called *digital electro-hydraulic controller* (or more simply *digital controllers*). Digital control hardware running on application software realises required control function. Signals of speed, power, water level, etc., are acquired through electronic devices and transmitted in digital form. Hydraulic devices, controlled by digital signals perform the turbine control.

The present-day trend is to use digital controllers, as they are very flexible, highly reliable, have quick response and are adaptable to remote control. Further, the control process has reached a very high degree of sophistication, both in theory and in application and a variety of functions are handled by the controller. Self-diagnostic features and ability to change the control functions via software are some of the additional advantages. No wonder, 1G and 2G controllers are obsolete.

2.11.2 Theory of Controllers

Basically, the controller uses an algorithm that provides the control signal in a feedback loop. For every deviation from a set point value, there is a correction sent by the controller. There are three functions involved in calculating the correction.

1. Proportional Function, (P)

It deals with the present values, multiplying the current error by a set value *P* and subtracting the resultant value from the input of the process. The main problem of this type of proportional controller is that it will overreact and cause oscillations in the system.

2. Integral Stage (I)

It handles past values, integrating the errors over a period of time. This is then multiplied by a constant and subtracted from the input of the process. This helps reduce the oscillations of a proportional controller. In addition, the integral stage ensures that the stable error is reduced to zero. A control system that uses the *P* and *I* stages only is said to belong to *PI* category.

3. Derivative Term (D)

A system that uses only *P* and *I* terms (*PI* controller) reacts slowly to changes in the control variable. The derivative term attempts to overcome this by predicting the future performance of the system. The derivative of the error over time is the parameter. This is multiplied by a constant and subtracted from the input of the process. This enables the controller to respond to a change in the system much faster than a *PI* system.

Most present-day controller algorithms use the principle of *PID* functions and hence such controllers are called *Proportional Integral Derivative (PID) controllers*. Values of constants and other factors for use in a *PID* system are found by tuning the controller.

2.11.3 Elements of a Turbine Governor

Figure 2.47 shows a schematic representation of a turbine governor. The linkages of various units are self-explanatory. Definitions of some of the terms are given in the boxes below the figure. The important components of a turbine governor can be listed as below:

1. Definition: Servomechanism

An automatic control system in which the output is constantly/intermittently compared with the input through feedback so that the error or difference between two quantities can be used to bring about the desired amount of control. A servomechanism consists

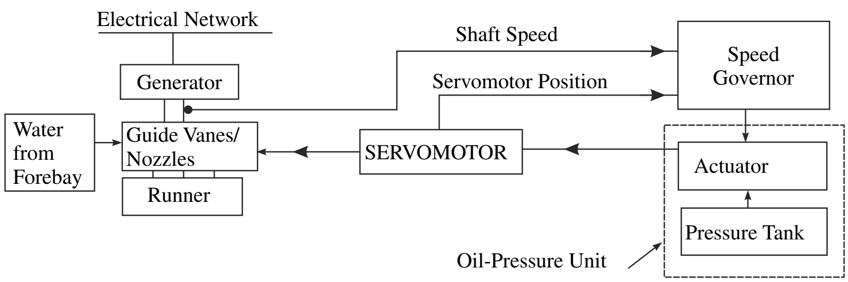

Fig. 2.47 *Schematic representation of a governing system*

of a sensing element, amplifier and servomotor. Speed control via a governor is a form of servomechanism.

2. Definition: Servomotor

A device such as a hydraulic piston/electrical motor that can be controlled by an amplified signal from a command device as in a servomechanism.

3. Definition: Actuator

A device responsible for actuating a mechanical device. A mechanical/electrical/ hydraulic device performs a mechanical motion in response to an input signal. It is operated by some source of energy in the form of electric/hydraulic pressure and converts the energy to some kind of motion.

(a) Speed-sensing Device A flyball-type pendulum was used as speed sensor in 1G and 2G governors. Speed sensing in later-generation governors is by electrical/ electronic devices.

(b) Oil-pressure System Consists of *actuators*, sump tank, pressure tank, compressed air station, sensors and safety systems. The actuator is the key hydraulic component. It consists of electromechanical devices that transform the electrical signal coming from the governor controller into a mechanical command, sending oil under pressure to the servomotor to control turbine rotation speed.

(c) Hydraulic Servomotors One or more motors are employed to move guide vanes (in Francis and Kaplan turbines)/needle (in Pelton turbine), blades (in Kaplan turbine) and relief valves.

(d) Over-speed Detector and Safety Device Usually a part of the speed governor.

(e) Stabilising/Compensating Elements Such as dashpots, springs, starting/ stopping gear, safety devices, load-limiting devices, stop-point settings, etc.

While realising that most of the present-day turbine governors are of digital type, a brief description of 2G (second generation) mechanical governor is given below as it helps one to intuitively understand the salient working of various components of the governor.

Figure 2.48 is a schematic representation of a 2G mechanical governor of a turbine. The speed sensing is through a flyball type pendulum that is connected to the main shaft of the turbine. The movement of the piston rod is the output from the servomotor.

Consider the case of a decrease in load on the generator: This causes the following sequential changes:

- Speed of the generator and hence that of the turbine increases.
- The flyball pendulum rotates faster and rises to higher position.
- The lever connected to a sleeve on the pendulum unit is balanced on a fulcrum and thus causes its lower end to move downwards.

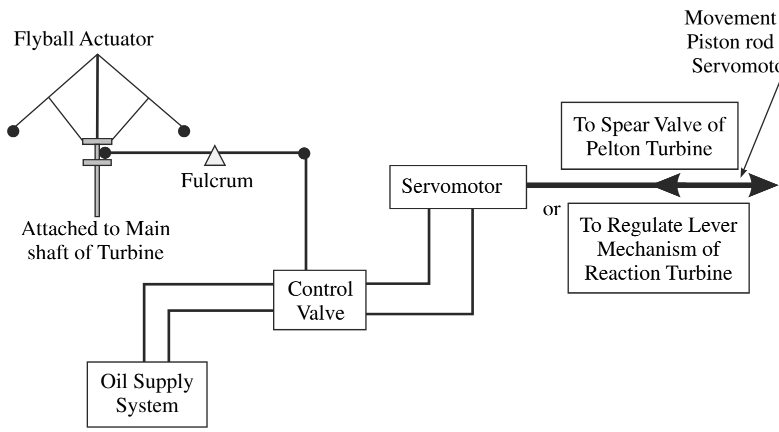

Fig. 2.48 *Schematic representation of 2G mechanical governor of a turbine*

- This causes the piston in the control valve to move down. High-pressure oil moves to the right chamber of the servomotor.
- The piston of the servomotor moves to the left, causing the output piston rod to move to the left.
- In a Francis turbine, the piston rod is connected to the ring that is connected to all the guide vanes. The pulling-in motion of the piston rod causes the guide vanes to close up and thereby reduce the passage available for the water to flow into the runner.
- Thus, less discharge enters the turbine and the speed of the turbine is reduced.
- This correction may or may not be adequate. There may be overcorrection also.
- The flyball pendulum senses the change in speed and corrective action is taken by the governor mechanism. The sequence of actions as indicated above is repeated. If increase in speed is required, the control valve moves up and high-pressure oil goes in to the left chamber of the servomotor and causes the piston rod to move to the right, thus opening up the guide vane passage.
- The entire sets of events are repeated until equilibrium is reached.

Based on this brief description, one can now list the desirable qualities of a good governor for a turbine as follows:

1. Sensitivity

It is a measure of the smallest change in a parameter that can be detected and corrected.

2. Response Time

This is a conflicting requirement. A very quick response and changing of the flow in to the turbine may set up *water-hammer* problems in the penstock. On the other hand, a very slow response may endanger the performance and operation of the electrical system. Provision of a bypass valve in reaction turbines and deflector for the jet in the Pelton turbine enables quicker responses without undue water-hammer problems.

3. Stability

The governing system must be stable and return to stable equilibrium after each episode of deviation from normal conditions. Further, quick convergence to set-point values is desired; that is, no hunting, no undue oscillations, etc. A typical stability value of a good governor is that speed oscillations does not exceed ± 0.15% of rated speed.

4. Reliability

The governor system should be reliable and safety devices incorporated to account for unforeseen events. Safety shut-off overrides should be available for emergencies. Adequate redundancy of essential components is usually provided to take care of this aspect.

It is seen that these requirements could be adequately addressed in an efficient manner in the present-day digital systems.

2.11.4 Governing of a Francis Turbine

In a Francis turbine, the governing action is essentially through regulation of the guide vanes. Each of the guide vanes is mounted on a pinion and the entire set is mounted between two rings to form a wheel that can move about its axis. A regulating ring is mounted on the top cover and it has pins bolted to the lower flange to which the guide vane links are connected (Fig. 2.6). The regulating ring itself is connected by a set of connecting rods and cam to the drive rod of the servomotor (Fig.2.49). Forward or backward motion of the piston rod causes the regulating ring to rotate and this, in turn, causes each of the guide vanes to rotate about its axis changing the opening available for flow. Covering the range of full gate opening to full closure, various levels of gate opening are possible. Depending upon the signal of speed variation, the servomechanism causes the motion of the regulating ring to close or open the flow passage between the guide vanes as per the need.

A relief valve is often provided in the supply line just next to the inlet into the runner. In case of sudden closures, the relief valve is opened to divert most of the flow from the runner and thereby help in reducing water-hammer pressures. Even though the relief valve is operated by the servomechanism of the governor, it is not operated routinely in the speed-control activity and its operation is reserved for certain contingencies only.

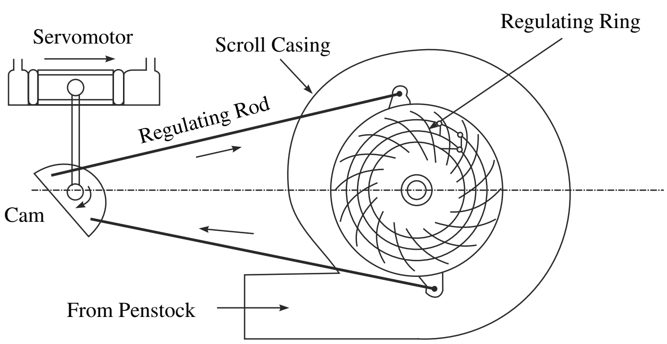

Fig. 2.49 *Governing of Francis turbine: regulating ring and cam arrangement (schematic)*

2.12 KEY FEATURES OF FRANCIS TURBINES

The Francis turbine is the most widely used turbine type and covers a vast range of discharge and head. The specific speed range is from 40 to 600. The head range is from 10 m to 350 m, the discharge range is from 0.4 m^3/s to 25 m^3/s and the output power range is from a few kW to about 800 MW. Francis turbines account for about 60% of global hydropower capacity. To get a quick idea of suitability of Francis turbine for a particular site, with given head and discharge values, Fig. 2.50 given below is in common use for small and medium-size hydro-power developments. In this figure, the region of suitability of *H–Q* space is indicated for normal Francis-turbine operation. Note that the power of the Pelton turbine in the high-head range and Francis and Kaplan turbines in the low-head ranges indicated in the figure is limited to 10 MW. Use of this figure in turbine selection for small hydro plants is described in Chapter 4 (Section 4.9).

It will be observed that there are overlap regions between different types of turbines. Further, the scales of the plot are logarithmic and keeping this in mind, it will be seen that the Francis turbine has the widest range of application among the various types of turbines. Highly flexible, it comes in a wide range of sizes that can operate under a wide range of heads: a fifteen-fold range from a low of about 40 m.

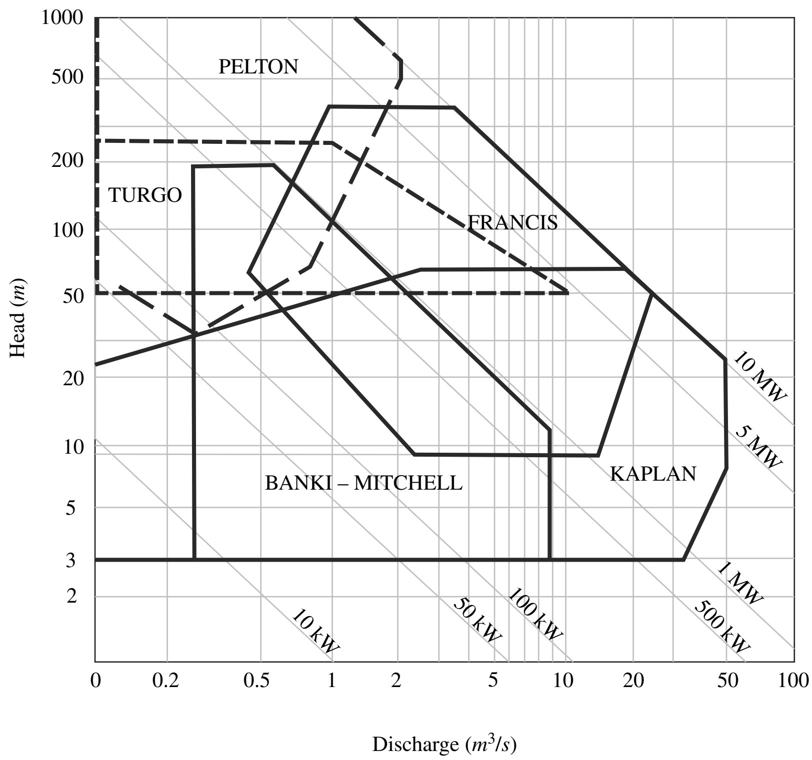

Fig. 2.50 *Normal range of operation of different turbine types in small hydro plants*

- The largest Francis turbines ever made are installed in the Three Gorges Project in China. The installation consists of 32 of 700 MW Francis turbines working under a head of 80.6 m. Each runner has a diameter of 10 metres and weighs 450 tons.
- Francis turbine generally is the most efficient solution for heads ranging from 40 to 600 metres. The runner design is very flexible and can be adapted to get the highest level of efficiency in its whole range of application.
- They are very robust and able to sustain the high mechanical stress resulting from high heads.
- Because they are adapted for high-head application, they often provide the lowest cost per installed MW.
- Table 2.8 gives a list of some major Francis-turbine installations in the world.

Table 2.8 *Some major Francis-turbine installations*

Project	Country	Installed power	Head (m)
Three Gorges	China, 2011	32 × 700 MW	80.6 m
Grand Coulee –III	USA, 1980, 1992	3 × 600 MW; 3 × 805 MW	160 m
Koyna IV	India, 1994	9 × 300 MW	525 m
La Grande 4	Canada, 1978	9 × 300 MW	117 m
Itaipu	Brazil, 1978	20 × 700 MW	118 m
Xiawan	China, 2010	6 × 700 MW	292 m
Jin Pin	China, 2007	8 × 600 MW	120 m

2.13 ILLUSTRATIVE EXAMPLES—SET 2.2

2.13.1 Draft Tube

*EXAMPLE 2.17

A conical draft tube of a reaction turbine has diameters of 1.5 m and 2.0 m at its ends. The turbine is set 7.0 m above the tailrace level. When the discharge from the turbine is 12 m³/s, calculate the (a) pressure head at the entrance of the draft tube, and (b) efficiency of the draft tube. Assume loss of energy in the draft tube as 0.35 times the velocity head at the draft-tube exit. Take atmospheric pressure head as 10.3 m of water.

Solution

Given: Discharge $Q = 12\ \text{m}^3/\text{s}$

$$A_1 = \frac{\pi}{4}(1.5)^2 = 1.767\ \text{m}^2 \qquad A_2 = \frac{\pi}{4}(2.0)^2 = 3.142\ \text{m}^2$$

$V_1 = \frac{12.0}{1.767} = 6.79$ m/s $\qquad V_2 = \frac{12.0}{3.142} = 3.82$ m/s

$\frac{V_1^2}{2g} = \frac{(6.79)^2}{2 \times 9.81} = 2.351$ m $\qquad \frac{V_2^2}{2g} = \frac{(3.82)^2}{2 \times 9.81} = 0.744$ m

(a) Loss of head $H_L = 0.35 \times \frac{V_2^2}{2g} = 0.35 \times 0.744 = 0.260$ m

By Bernoulli equation applied to sections 1 and 2,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + H_L$$

- Take the bottom of the draft tube as datum.
- Let the depth of draft tube below tailwater level = y (refer Fig. 2.51).
- Taking atmospheric pressure head = 10.3 m of water, $\frac{p_2}{\gamma} = 10.3 + y$

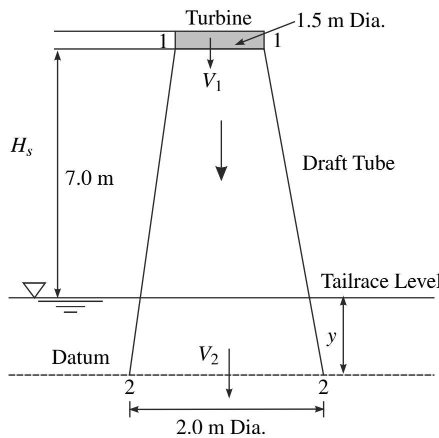

Fig. 2.51 *Draft tube set-up, Example 2.17*

The energy equation now reads as

$$\frac{p_1}{\gamma} + 2.351 + (7.0 + y) = (10.3 + y) + 0.744 + 0 + 0.260$$

$$\frac{p_1}{\gamma} + 10.3 + 0.744 + 0.260 - 2.351 - 7.0 = 1.953 \text{ m (abs)}$$

(b) Efficiency of the draft tube = $\eta_d = 1 - \frac{H_L}{\left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right)}$

$$= 1 - \frac{0.260}{(2.351 - 0.744)} = 0.838 = 83.8\%$$

*EXAMPLE 2.18

An elbow-type draft tube of a Francis turbine has an inlet diameter of 2.5 m and its area at outlet is 18 m^2. The inlet to draft tube is situated 5.0 m above the tailwater level and the velocity of flow at the inlet is 5.0 m/s. Assume the loss of head due to friction in the draft tube to be 80% of the velocity head at the draft-tube outlet. Calculate (a) the pressure head at the entrance to the draft tube, (b) power lost due to friction in the draft tube, (c) power wasted to the tailrace, and (d) the efficiency of the draft tube.

Solution

Let suffixes 1 and 2 denote the inlet and outlet of the draft tube respectively.

$V_1 = 5.0$ m/s, $A_1 = \frac{\pi}{4}(2.5)^2 = 4.908$ m^2, $A_2 = 18.0$ m^2,

$Q = 5.0 \times 4.908 = 24.54$ m^3/s, $V_2 = \frac{24.54}{18.0} = 1.363$ m/s, $V_1 = 5.0$ m/s

$$\frac{V_1^2}{2g} = \frac{(5.00)^2}{2 \times 9.81} = 1.274 \text{ m}$$

$$\frac{V_2^2}{2g} = \frac{(1.363)^2}{2 \times 9.81} = 0.0947 \text{ m}$$

Loss of head

$$H_L = 0.8 \times \frac{\left(V_2^2\right)}{2g} = 0.8 \times 0.0947 \text{ m} = 0.0758 \text{ m}$$

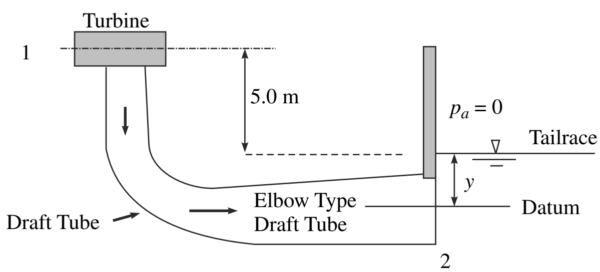

Fig. 2.52 *Draft tube set-up, Example 2.18*

(a) Considering the Bernoulli equation between sections 1 and 2,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + H_L$$

$$\frac{p_2}{\gamma} = \text{Atmospheric pressure head} = 0$$

$$\frac{p_1}{\gamma} + 1.274 + (5.0 + y) = (0 + y) + 0.0947 + 0 + 0.0758$$

$$\frac{p_1}{\gamma} = -6.1035 \text{ m (vacuum)}$$

(b) Power lost due to friction in draft tube = $P_{Lf} = \gamma Q H_L = 9.79 \times 24.54 \times 0.0758$
= 18.21 kW

(c) Power wasted to the tailrace = $P_L = \gamma Q \frac{\left(V_2^2\right)}{2g} = 9.79 \times 24.54 \times 0.0947$
= 22.75 kW

(d) Efficiency of the draft tube = $\eta_d = 1 - \dfrac{H_L}{\left(\dfrac{V_1^2}{2g} - \dfrac{V_2^2}{2g}\right)}$

$$= 1 - \frac{0.0758}{(1.274 - 0.0947)} = (1 - 0.0643)$$

$$= 0.936 = 93.6\%$$

**EXAMPLE 2.19

A Francis turbine develops 800 kW at efficiency of 90% under a net head of 12 m. The draft tube used in this set-up is a vertical cylindrical pipe of 2.0 m diameter. (a) What increase in power could be expected if a tapering vertical draft tube having an outlet diameter of 2.5 m replaces the existing cylindrical draft tube? (b) What would be the increase in the overall efficiency? Assume that the head, speed and discharge remain the same and there are no additional friction losses due to the new draft tube.

Solution

Power $P = \eta\,\gamma Q H_e = 0.90 \times 9.79 \times Q \times 12 = 800$

Discharge $Q = 7.566\ \text{m}^3/\text{s}$

V_c = Velocity of water at exit in cylindrical draft tube

$$= \frac{Q}{A} = \frac{7.566}{\frac{\pi}{4} \times (2)^2} = 2.408 \text{ m/s}$$

Velocity head = $\dfrac{V_c^2}{2g} = \dfrac{(2.406)^2}{2 \times 9.81} = 0.2956$ m

For the alternative conical draft tube:

V_a = velocity of water at exit in conical draft tube = $\dfrac{Q}{A} = \dfrac{7.566}{\frac{\pi}{4} \times (2.5)^2} = 1.5413$ m/s

Velocity head = $\dfrac{V_a^2}{2g} = \dfrac{(1.5413)^2}{2 \times 9.81} = 0.1211$ m

Velocity head gained = $\Delta H = \dfrac{V_c^2}{2g} - \dfrac{V_a^2}{2g} = 0.2956 - 0.1211 = 0.1745$ m

Power gained $P_g = \gamma Q\, \Delta H = 9.79 \times 7.566 \times 0.1745 = 12.93$ kW

Increase in efficiency = $\dfrac{\text{Gain in head}}{\text{Original head}} = \dfrac{0.1745}{12} = 0.01454 = 1.45\%$

*EXAMPLE 2.20

A Francis turbine operating under a net head of 20 m is tentatively proposed to be set up with a draft head of 2.0 m at a certain location . The local atmospheric head is 10.0 m and the vapour pressure head is 0.18 m. The critical cavitation parameter for the given turbine can be taken as 0.4. (a) Examine whether there is cavitation susceptibility. (b) If a safety margin of 0.5 m for the draft height is mandatory, what is the maximum height above the tailwater at which the turbine can be installed?

Solution

Maximum draft head $H_{s\max} = \left(\frac{p}{\gamma}\right)_{\text{atm}} - \left(\frac{p_v}{\gamma}\right) - \sigma_c\ H$

Here $\left(\frac{p}{\gamma}\right)_{\text{atm}}$ = 10.0 m, $\left(\frac{p_v}{\gamma}\right)$ = 0.18 m and σ_c = 0.4 and H = 20.0 m.

Substituting in the equation for $H_{s\max}$

$$H_{s\max} = 10.0 - 0.18 - (0.4 \times 20) = 1.82 \text{ m}$$

(a) Proposed turbine setting H_s = 2.0 m. Since $H_s > H_{s\max}$, the proposed turbine setting is vulnerable to cavitation.

(b) Theoretical maximum draft head H_{smax} = 1.82 m
Provide a safety margin M = 0.5 m
Maximum admissible draft height at plant = H_{ps} = ($H_{s\max} - M$) = (1.82 – 0.50)
= 1.32 m

2.13.2 Unit and Specific Quantities

**EXAMPLE 2.21

The following data is from a test on a 60 cm diameter Francis turbine working under a head of 10.0 m and full gate opening. Plot (a) the unit power vs unit speed curve, and the (b) efficiency vs unit speed curve for the data. Determine the maximum efficiency and the corresponding unit speed and unit power. If these maximum values were taken as design values, what would be the speed and power output from a 90 cm diameter homologous turbine running under a head of 30 m?

Unit Power	8.8	9.5	10.0	10.1	9.9	9.7	9.2	8.4
Unit Speed	55	65	75	84	95	102	115	140
Unit Discharge	1.25	1.24	1.23	1.20	1.15	1.14	1.12	1.11

Solution

Efficiency $\eta = \frac{P}{\gamma QH} = \frac{P_u\left(H^{3/2}\right)}{\gamma\left(Q_u\sqrt{H}\right)H} = \frac{P_u}{\gamma Q_u}$

Using this relationship, efficiency is calculated for each data set and the data for plotting is shown in tabular form below.

Data for Plotting:

Unit Power, P_u	8.8	9.5	10.0	10.1	9.9	9.7	9.2	8.4
Unit Speed, N_u	55	65	75	84	95	102	115	140
Unit Discharge, Q_u	1.25	1.24	1.23	1.20	1.15	1.14	1.12	1.11
Efficiency= $\eta = \dfrac{P_u}{\gamma Q_u}$	0.719	0.783	0.830	0.860	0.879	0.869	0.839	0.773

Figure 2.53 shows the curves of unit power vs unit speed and efficiency *vs* unit speed for the data. From an inspection of the two curves, the following values are noted:

Maximum efficiency = 87.9%. Corresponding unit power $P_u = 9.90$,

unit speed $N_u = 95$.

Using suffix 1 for the 60 cm diameter turbine:

$$D_1 = 0.60 \text{ m}, H_1 = 10.0 \text{ m}, \; N_1 = N_u\sqrt{H_1} = 95 \times \sqrt{10} = 300.4 \text{ rpm}$$

$$P_1 = P_u H_1^{3/2} = 9.9 \times (10)^{3/2} = 313.1 \text{ kW}$$

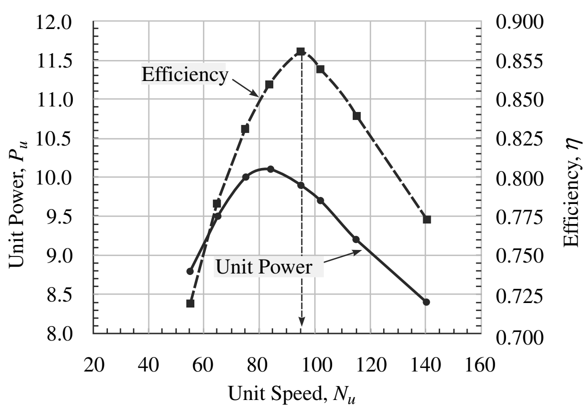

Fig. 2.53 *Variation of efficiency and unit power, Example 2.21*

For the 90 cm diameter turbine: Using the suffix 2 to represent 90 cm diameter turbine

For speed: $\dfrac{N_1 D_1}{\sqrt{H_1}} = \dfrac{N_2 D_2}{\sqrt{H_2}}$

$$N_2 = N_1\left(\frac{D_1}{D_2}\right)\sqrt{\frac{H_2}{H_1}} = 300.4 \times \left(\frac{0.60}{0.90}\right)\sqrt{\frac{30}{10}} = 347 \text{ rpm}$$

For power: $\dfrac{P_1}{N_1^3 D_1^5} = \dfrac{P_2}{N_2^3 D_2^5}$

$$P_2 = P_1 \left(\frac{N_2}{N_1}\right)^3 \left(\frac{D_2}{D_1}\right)^5 = 313.1 \times \left(\frac{347}{300.4}\right)^3 \times \left(\frac{0.90}{0.60}\right)^5 = 3665 \text{ kW}$$

***EXAMPLE 2.22

Noting that the data of Example 2.21 is for a runner of 0.6 m diameter, prepare a table of corresponding sets of specific quantities Q_{11}, P_{11}, N_{11} and efficiency for the turbine. Plot the graph of efficiency vs N_{11} and specific power P_{11} vs N_{11} and identify the peak values of the parameters P_{11}, N_{11} and efficiency. Taking the peak as the design point, obtain the power, discharge, efficiency, operating speed and specific speed of a turbine of 1.2 m size designed to work under a head of 25 m.

Solution

Specific (double unit) quantities: $N_{11} = \dfrac{ND}{\sqrt{H}}$, $Q_{11} = \dfrac{Q}{D^2\sqrt{H}}$ and $P_{11} = \dfrac{P}{H^{3/2}D^2}$

Unit quantities: $N_u = \left[\dfrac{N}{\sqrt{H}}\right]$, $Q_u = \left[\dfrac{Q}{\sqrt{H}}\right]$ and $P_u = \left[\dfrac{P}{H^{3/2}}\right]$

For a given turbine of diameter D: $N_{11} = N_u D$, $Q_{11} = \dfrac{Q_u}{D^2}$ and $P_{11} = \dfrac{P_u}{D^2}$ (i)

Efficiency $\eta = \dfrac{P}{\gamma QH} = \dfrac{P_{11}\left(H^{3/2}\right)D^2}{\gamma\left(Q_{11}D^2\sqrt{H}\right)H} = \dfrac{P_{11}}{\gamma Q_{11}}$ (ii)

Using relationships given by (i) and (ii), N_{11}, Q_{11} and P_{11} and efficiency are calculated for each data set and the data for plotting is shown in tabular form below. In converting unit quantities to P_{11}, Q_{11} and N_{11}, a value of 0.6 m for the diameter D is used.

Data for Plotting:

$P_{11} = \dfrac{P_u}{(0.6)^2}$	24.44	26.39	27.78	28.06	27.50	26.94	25.56	23.33
$N_{11} = N_u(0.6)$	33	39	45	50.4	57	61.2	69	84
$Q_{11} = \dfrac{Q_u}{(0.6)^2}$	3.472	3.444	3.417	3.333	3.194	3.167	3.111	3.083
$\eta = \dfrac{P_{11}}{(9.79)Q_{11}}$	0.719	0.783	0.830	0.860	0.879	0.869	0.839	0.773

Figure 2.54 shows the curves of specific power P_{11} vs defined double unit speed, N_{11} and efficiency vs defined double unit speed, N_{11} for the data.

From an inspection of the two curves of Fig. 2.54, the following values are noted:

Maximum efficiency = 87.9%.

Corresponding specific power P_{11} = 27.50 and N_{11}= 57.

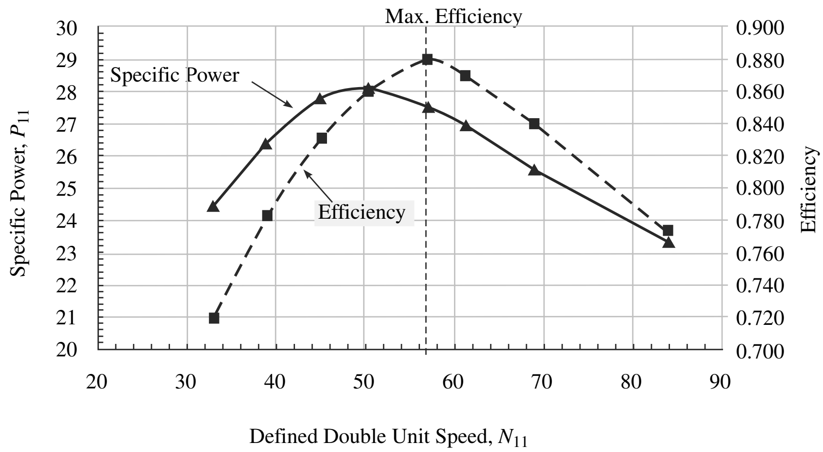

Fig. 2.54 *Plot of P_{11} and efficiency with N_{11}*

For the *1.2 m diameter turbine*:

D = 1.2 m, H = 25.0 m, Operating speed $N = \dfrac{N_{11}\sqrt{H}}{D} = \dfrac{57 \times \sqrt{25}}{1.2} = 237.5$ rpm

Power $P = P_{11}H^{3/2}D^2 = 27.50 \times (25)^{3/2}\,(1.2)^2 = 4950$ kW

Discharge $Q = Q_{11}D^2\sqrt{H} = 3.194 \times (1.2)^2 \times (25)^{1/2} = 23$ m^3/s

Specific Speed, $N_s = \dfrac{N\sqrt{P}}{H^{5/4}} = \dfrac{\left(\dfrac{ND}{\sqrt{H}}\right)\left(\sqrt{\dfrac{P}{DH^{3/2}}}\right)}{H^{5/4}} = \left[N_{11}\sqrt{P_{11}}\right]_{\text{design}}$

where the suffix *design* indicates design values.

$$N_s = 57 \times \sqrt{27.5} = 299$$

Check: Efficiency = $\dfrac{P}{\gamma QH} = \dfrac{4950}{9.79 \times 23 \times 25} = 0.879$

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Review Questions

2.1 Write the Euler equation for the head extracted in a reaction turbine in the form of velocity heads of absolute and relative velocities of flow and peripheral velocity heads. Explain the significance of each of the terms.

2.2 What are impulse effect and reaction effect in a hydroturbine?

2.3 What do you understand by degree of reaction of a turbine?

2.4 Describe briefly the components of a Francis turbine.

2.5 Draw neat sketches of a Francis-turbine installation and a Francis runner and label the following:
shroud ring; crown of the runner; blade; wicket gate; scroll casing and draft tube.

2.6 What is the role of stay vanes in a Francis turbine?

2.7 What is the role of scroll case in a Francis turbine?

2.8 What is the need of governing in a turbine?

2.9 Describe the general principles of governing of turbines.

2.10 What are the chief components of a turbine governor?

2.11 Write a brief note on governing of Francis turbines.

2.12 Distinguish between:
(a) Semi-scroll case and full-scroll case
(b) Guide vanes and stay vanes
(c) Net head and Euler head

2.13 What are the salient features of main characteristics curves of a Francis turbine?

2.14 Explain the characteristic features of Muschel curves (iso-efficiency curves) of a Francis turbine.

2.15 What is a hill chart? Describe a typical hill chart of a Francis turbine and indicate where it could be used advantageously?

2.16 Write a brief note on the efficiency characteristics of a Francis turbine. Include a list of at least three common ways of representing the efficiency hill chart of a Francis turbine.

2.17 Differentiate between
(a) Net head and gross head
(b) Rated head and design head
(c) Rated efficiency and design efficiency

2.18 What is the purpose of a draft tube in a reaction turbine?

2.19 Describe the commonly used high-efficiency, elbow-type draft tube for a Francis turbine.

2.20 Write a brief note on cavitation with special reference to a Francis turbine.

2.21 Discuss how cavitation affects the draft head of a draft tube.

2.22 Describe the erosion problem in Francis turbine.

2.23 What are the main features in classifying the runners of a Francis turbine as slow, medium and fast?

Problems

1. [All **Problems** and **Objective Questions** have been graded in three levels: – Simple, Medium and Difficult. The markings for these are: Simple = *, Medium = **, Difficult = ***]

Velocity Triangles

P2.1 * At what angle should the wicket gates of a Francis turbine be set to extract 8000 kW of power from a flow of 30 m^3/s when running at a speed of 200 rpm? The diameter of the runner at inlet is 3.0 m and the breadth of the openings at inlet is 0.9 m. The flow can be assumed to leave the runner radially. [***Ans:*** α_1 = 24.80°]

P2.2 * An inward-flow reaction turbine has inlet and outlet diameters of 1.2 m and 0.6 m respectively. The breadth at the inlet is 0.25 m and at the outlet, it is 0.35 m. At a speed of rotation of 250 rpm, the relative velocity at the entrance is 3.5 m/s and it is radial. Calculate the (a) absolute velocity at the entrance and its inclination to the tangent of the runner, (b) discharge, and (c) the velocity of flow at the outlet.
[***Ans:*** α_1 = 12.56°, Q = 3.299 m^3/s, V_{f2} = 5.0 m/s]

P2.3 * A reaction turbine works under a net head of 12.0 m. The guide-vane angle and the inlet-blade angle are 20° and 60° respectively. Assuming radial discharge at the outlet and constant flow velocity, determine the hydraulic efficiency of the turbine. [***Ans:*** η_h = 94.8%]

P2.4 * An inward flow reaction turbine has an outer diameter of 1.2 m and an inner diameter of 0.6 m. The breadth at the inlet is 0.20 m and it is 0.3 m at the outlet. Water enters the turbine with a flow velocity of 5.0 m/s. (a) Calculate the discharge and velocity of flow at the outlet. Assume vane thickness coefficient = 0.95 at both inlet and outlet. (b) If the speed of the turbine is 300 rpm, calculate the blade angle at the outlet. Assume the discharge to be radial at the outlet.
[***Ans:*** (a) Q = 3.5814 m^3/s, V_{f2}= 6.67 m/s, (b) β_2 = 19.486°]

P2.5 * For a Francis turbine with radial entry and radial exit, the ratio of diameter at inlet to the diameter at outlet is D_r. Show that the exit blade angle β_2 is related to the inlet guide vane angle α_1 as

$$\tan \beta_2 = D_r \tan \alpha_1.$$

P2.6 * An inward-flow reaction turbine has a runner of 1.2 m outer diameter and 0.6 m inner diameter. The blades occupy 5% of the peripheral area and the widths of the blades are 25 cm and 30 cm at the inlet and outlet respectively. If a discharge of 3.0 m^3/s enters radially, determine the flow velocities at the inlet and outlet of the runner. What is the inlet-vane angle required to have a speed of 300 rpm? [***Ans:*** V_{f1} = 3.35 m/s, V_{f2} = 5.58 m/s, α_1 = 10.078°]

P2.7 *A Francis turbine has an inlet diameter of 2.0 m and an outlet diameter of 1.2 m. The breadth of the blades is constant at 0.2 m. The speed of the runner is 250 rpm and the discharge is 8.0 m^3/s. The vanes are radial at the inlet and the discharge is radially outwards at the outlet. Calculate the angle of guide vanes at the inlet and the blade angle at the outlet.

[***Ans***: α_1 = 13.67°, β_2 = 34.04°]

P2.8 *A Francis turbine has a supply of 0.75 m^3/s of water under a head of 16 m at the inlet. The power developed is 100 kW with a normal speed of 375 rpm. The outer and inner diameters of the runner are 90 cm and 50 cm respectively. If the discharge is radial at the exit with an outlet velocity of 3.0 m/s, determine the (a) hydraulic efficiency and overall efficiency of the turbine, and (b) determine the guide-vane angle and blade angle at the inlet. Assume the width of the runner as constant.

[***Ans:*** η_h = 0.971, η_0 = 0.85, α_1 = 10.935°, β_1 = 10.44°]

P2.9 *A reaction turbine has a runner whose vanes are radial at the inlet and are inclined backwards to make a blade angle of 30° at the discharge end. The diameter of the runner at the entry is twice that at the exit and the width at the entrance is one-half of that at the exit. The guide-vane angle is 15° and the velocity of water leaving the guide vane is 25 m/s. Determine the peripheral velocity at the entrance of the runner and the absolute velocity of water at the exit. [***Ans:*** u_1 = 24.15 m/s, V_2 = 6.527 m/s]

P2.10 *A Francis turbine operating under a head of 30 m develops 4 MW of power while running at a speed of 300 rpm. The overall efficiency of the turbine is 85% and the hydraulic efficiency is 90%. The radial velocity of flow at the inlet is 7.0 m/s and the inlet guide vane angle is 30°. Calculate the diameter and width of the runner at inlet. Take blade thickness coefficient as 0.95.

[***Ans***: D_1 = 1.39 m, B_1 = 0.552 m]

P2.11 *Show that the specific speed of a Francis turbine can be expressed as

$$N_s = 987.4\ K_u \sqrt{K\, n\, \eta_0\, K_f}$$

where K_u = Speed ratio, K = Blade thickness coefficient, K_f = flow ratio, n = ratio of width to diameter of runner at inlet and η_0 = Overall efficiency. Take the unit weight of water γ = 9.79 kN/m^3.

P2.12 **A Francis turbine has a runner diameter of 1.0 m at the entrance and 0.5 m at the exit. At entrance, the blade angle is 90° and the guide-vane angle is 15°. The water at exit leaves the blades without any whirl velocity. The net head for the turbine operation is 30 m and the radial component of flow is constant. What is the (a) speed of runner in rpm, and (b) blade angle at exit? Is the speed calculated above is synchronous with 50 Hz frequency? If not, what speed would you recommend to directly couple the turbine with an alternator of 50 Hz? Assume a hydraulic efficiency of 0.95.

[***Ans***: N = 319.3 rpm, Synchronous speed = 300 rpm, β_2 = 28.18°]

P2.13 **Determine the (a) hydraulic efficiency and overall efficiency of a Francis turbine having the following data: Output power = 2500 kW, Operating head = 45 m, External diameter of runner = 1.5 m, Width of runner = 0.22 m, Inlet guide-vane angle = 20°, Runner-vane angle at inlet = 120° and Specific speed = 120. Assume the outlet discharge is radial.

[***Ans:*** η_h = 0.90, η_0 = 0.828]

P2.14 ** A Francis turbine has the speed ratio of 0.80 and its flow ratio is 0.25. The width of the runner at the outer periphery is ¼ the outer diameter. Estimate the specific speed of the turbine. [***Ans*:** N_s = 179.3]

P2.15 ** A medium-speed Francis turbine is having radial entry and radial discharge at the outlet. The velocity of flow at outlet is k times that at inlet. Show that the hydraulic efficiency is given by

$$\eta_h = \frac{2}{2 + k^2 \tan^2 \alpha_1}$$

where α_1 = Guide-vane angle at the inlet.

P2.16 ** An inward-flow reaction turbine develops 750 kW at 750 rpm under a net head of 100 m. The guide vanes are at angle of 15° with the tangent at the inlet. The breadth of the blade at inlet is 0.1 times the inlet diameter. The blade thickness blocks 5% of the inlet area. The hydraulic efficiency of the turbine is 88% and the overall efficiency is 84%. Determine the (a) wheel diameter at inlet, and (b) blade angle at inlet.

[***Ans*:** D_1 = 0.519 m, β_1 = 152.68°]

P2.17 ** A Francis turbine works under a total head of 25 m. The outer and inner diameters of the runner are 0.70 m and 0.35 m respectively. The vane tip is radial at the inlet and the flow leaves the turbine radially. The inlet guide-vane angle is 12°. Calculate the speed of the runner and the vane angle at the exit, if the velocity of flow is constant at 4.0 m/s. Assume the hydraulic efficiency as 0.90. [***Ans*:** N = 398.7 rpm, β_2 = 23.03°]

P2.18* ** A Francis turbine works under a head of 160 m. At the inlet, the rotor blade angle is 61°. The absolute velocity at the outlet is radially directed with a magnitude of 15.5 m/s and the radial component of velocity at the inlet is 10.3 m/s. If the discharge is 110 m^3/s, (a) find the power developed by the turbine, and (b) the degree of reaction. Assume 88% of the available head is converted in to work. [***Ans*:** P = 151.63 MW, R = 0.62]

P2.19* ** A Francis turbine having an overall efficiency of 76% is to produce 105 kW of power under a head of 12 m and speed of 150 rpm. The peripheral velocity at inlet is 10 m/s and the velocity of flow at inlet is 5.0 m/s. Assuming the hydraulic losses as 20% of available energy, calculate the (a) inlet guide vane angle, (b) wheel blade angle at inlet, and (c) width of wheel at inlet.

[***Ans*:** α_1 = 27.96°, β_1 = 83.36°, B_1 = 58.8 mm]

P2.20* ** A Francis turbine works under a head of 30 m and discharges 10 m^3/s of water. The speed of the runner is 300 rpm. At inlet tip of the runner vane, the peripheral velocity of the wheel is $0.9\sqrt{2gH}$ and radial velocity of flow is $0.3\sqrt{2gH}$ where H is the head on the turbine. If the overall efficiency and hydraulic efficiency of the turbine are 80% and 90% respectively, (a) determine the power developed in kW, (b) guide blade angle at the inlet, (c) inlet angle of runner vane (d) specific speed of the turbine and (e) diameter and width of the runner at inlet.

[***Ans:*** (a) P = 2349.6 kW, (b) α_1 = 30.98°, (c) β_1= 36.88°, (d) N_s = 207, (e) D_1= 1.39 m, B_1 = 31.46 cm]

Draft Tube

P2.21 ** A turbine has an exit velocity of 10 m/s and is provided with a straight conical draft tube. The velocity head at the exit of the draft tube is 1.0 m and loss of head in the draft tube is 1.5 m. To avoid cavitation, the minimum pressure head in the turbine is set at 2.0 m (abs). Taking atmospheric pressure as 10.3 m of water, estimate the maximum height of setting of the turbine above the tailwater level. [***Ans*:** H_s = 5.704 m]

P2.22 ** An elbow-type draft tube has a circular section of 1.5 m^2 at the top and a rectangular cross section of 12.5 m^2 at the exit section. The turbine is set at a height of 2.0 m above the tailrace level. If the velocity at inlet to the draft tube is 12.0 m/s, estimate the (a) negative pressure head at the inlet to the draft tube, (b) power thrown away into the tailrace, and (c) efficiency of the draft tube. Assume the frictional losses in the draft tube to be 15% of the inlet velocity head.

[***Ans*:** 2.168 m (abs), P_L = 18.68 kW, h_d = 84.8%]

P2.23 * A Francis turbine having a design power of 2.50 MW under a head of 35 m and speed of 200 rpm is selected for installation at a site. The atmospheric pressure head at the site is 10.0 m and the vapour pressure head is 0.24 m. Assuming the critical cavitation parameter σ_c is related to the turbine specific speed by the following empirical formula

$$\sigma_c = 0.0317\left(\frac{N_s}{100}\right)^2,$$

verify whether the proposed turbine setting at a draft height of 7.0 m at the site is safe from cavitation consideration. A safety margin of 0.5 m is mandatory in the draft height. [H_s = 7.00 m < ($H_{smax} - M$), safe].

Unit and Specific Quantities

P2.24 ** The following data is from a test on a Francis turbine model. The data is presented as the variation of specific discharge Q_{11}, specific power P_{11} and speed of a turbine of unit size under unit head N_{11}.

(a) Calculate the efficiency corresponding to all the data points and plot curves representing (a) Efficiency against defined speed N_{11} and (b) Specific power P_{11} against Defined Speed N_{11}.

(b) Identify the peak efficiency point and taking this as the design point, obtain the power, discharge, efficiency and operating speed of a turbine of 1.2 m size designed to work under a head of 20 m.

Specific Power P_{11}	18.1	21.0	23.2	25.1	22.5	21.0	20.2
Defined Speed N_{11}	65	75	80	85	90	95	100
Specific Discharge Q_{11}	2.294	2.619	2.772	2.933	2.985	3.004	3.145

[***Ans*:** η_{peak} = 0.87, P = 3233 kW, Q = 18.98 m^3/s, N = 316.8 rpm]

P2.25 *** The following data is from a test on a Francis turbine model. (a) Plot curves representing (i) efficiency against defined speed N_{11}, and (ii) specific power P_{11} against defined speed $N_{11.}$ (b) Identify the peak efficiency point and taking this as the design point, obtain the power, discharge and operating speed of a turbine of 0.9 m size designed to work under a head of 36 m. (c) What is the specific speed of the turbine?

Specific Power P_{11}	9.00	9.30	9.65	9.55	9.22	8.70
Defined Speed N_{11}	70.0	80.0	87.5	90.1	94.0	100
Efficiency (%)	89.0	93.3	93.2	92.8	92.1	90.1

[***Ans*:** η = 0.9355, $(N_{11})_{peak}$ = 83.0, $(P_{11})_{peak}$ = 9.41, N_s = 254.6, P = 1646 kW, N = 553 rpm, Q = 4.994 m^3/s]

Objective-Type Questions

O2.1 * Select the correct Euler equation of a turbine relating the energy transfer per unit weight $H_{e.}$ In the following equations u, V_u, v_r and V represents the peripheral, whirl, relative and absolute velocities respectively. Suffixes 1 and 2 refer to the turbine inlet and outlet respectively.

(a) $gH_e = u_1V_{u1} - u_2V_{u2}$ (b) $gH_e = u_1v_{u1} - u_2v_{u2}$
(c) $gH_e = u_1V_1 - u_2V_2$ (d) $gH_e = V_1V_{u1} - V_2V_{u2}$

O2.2 * What does Euler's equation of turbo machines relate to?

(a) Discharge and head (b) Discharge and velocities
(c) Head and power (d) Head and velocities

O2.3 ** A turbine develops 2516 kW at 240 rpm. The torque in the shaft is

(a) 400 kN.m (b) 3336 kN.m (c) 1000 kN.m (d) 100 kN.m

O2.4 * The moment of momentum of water is reduced by 15915 N.m in a turbine rotating at 600 rpm. The power developed in kW is

(a) 1000 (b) 1500 (c) 2000 (d) 5000

O2.5 ** A reaction turbine has a discharge of 30 m^3/s passing through it under a net head of 10 m. If the overall efficiency of the system is 0.851, the power developed is

(a) 3400 kW (b) 3450 kW (c) 2940 kW (d) 2500 kW

O2.6 ** A Francis turbine has a runner of 4.0 m outer diameter. The breadth at the inlet and also at outlet is 0.8 m and the velocity of flow is constant at 3.0 m/s. The discharge through the turbine is

(a) 15.08 m^3/s (b) 28.28 m^3/s (c) 30.16 m^3/s (d) 37.70 m^3/s

O2.7 ** In a Francis turbine, the discharge leaves the runner radially at the exit. For this turbine,

(a) the blade tip is radial at the outlet
(b) the blade tip is radial at the inlet
(c) the guide vane angle is 90°
(d) the absolute velocity is radial at the outlet

O2.8 ** In a Francis turbine, the runner blades are radial at the inlet and the discharge leaves the runner radially at the exit. For this turbine,
(a) the relative velocity is radial at the outlet
(b) the absolute velocity is radial at the outlet
(c) the guide-vane angle is 90° at inlet
(d) the velocity of flow is constant

O2.9 ** A Francis turbine has a runner of outer diameter 0.60 m and works under a head of 30 m. If the rotational speed of the runner is 600 rpm, the speed ratio K_u for this turbine is
(a) 0.39 (b) 0.78 (c) 1.56 (d) 20

O2.10 ** In a Francis turbine , for maximum efficiency
(a) the velocity of swirl at entrance must be zero
(b) the velocity of swirl at outlet must be zero
(c) the velocity of flow at outlet must be zero
(d) the velocity of flow at entrance must be zero

O2.11 *** Which one of the following is the correct statement?
The degree of reaction of an impulse turbine
(a) is greater than zero but less than unity
(b) is greater than unity
(b) is equal to zero
(d) is equal to unity

O2.12 *** For a reaction turbine with degree of reaction equal to 50% , if V is the absolute velocity at the inlet and α is the angle made by the absolute velocity to the tangent to the wheel, the blade speed is equal to
(a) $\frac{V\cos\alpha}{2}$ (b) $2V \cos \alpha$ (c) $V \cos^2 \alpha$ (d) $V \cos \alpha$

O2.13 ** A Francis turbine is coupled to an alternator to generate electricity with a frequency of 50 Hz. If the alternator has 12 poles then the turbine should be regulated to run at which one of the following constant speeds?
(a) 250 rpm (b) 500 rpm (c) 600 rpm (d) 1000 rpm

O2.14 ** Consider the following statements and identify the correct ones:
A hydroturbine governor
1. helps in starting and shutting down the turbine unit
2. controls the speed of the turbine unit set to match with the hydroelectric system
3. sets the amount of load that a turbine unit has to carry

The correct statements are:
(a) 1, 2 and 3 (b) 1 and 2 (c) 2 and 3 (d) 1 and 3

O2.15 *** For a Francis turbine running at a constant speed, the operating characteristic curves given in Fig. 2.55 show, that up to a certain discharge Q_{start}, both output power and efficiency remain zero. The discharge Q_{start} is required to
(a) overcome initial inertia
(b) overcome initial friction
(c) keep the hydraulic circuit full
(d) keep the turbine running at no load

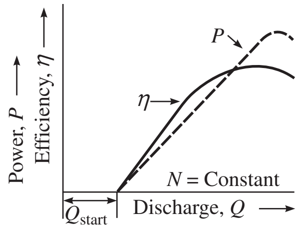

Fig. 2.55 *Objective Question O2.16*

O2.16 ** Muschel curves belong to the category of
(a) main characteristic curves of a turbine
(b) operating characteristic curves of a turbine
(c) constant efficiency curves of a turbine
(d) operating characteristics of a pump.

O2.17 ** Test on a model of a turbine revealed the peak specific discharge Q_{11} = 1.2 and peak specific power P_{11} = 10.0. Taking the unit weight of water as 9.8 kN/m^3, the peak efficiency of the model is
(a) 11.76 % (b) 85% (c) 91% (d) 98%

O2.18 ** The cavitation number of any fluid machinery is defined as $\sigma = \dfrac{p - p'}{\rho V^2 / 2}$ where p is absolute pressure, ρ = density and V = free stream velocity. The symbol p' denotes
(a) static pressure of the fluid
(b) dynamic pressure of the fluid
(c) vapour pressure of fluid
(d) shear stress in the fluid

O2.19 ** In a Francis turbine, the cavitation damage occurs under favorable conditions near
(a) inlet, on the convex side of blades only
(b) outlet, on the concave side of blades only
(c) inlet, on the concave side of blades only
(d) inlet and outlet, on both sides of the blades

O2.20 * Without residual whirl in the flow at the entrance to a draft tube, the best cone angle is
(a) < 10° (b) 12° (c) 18° (d) 24°

O2.21 * In reaction turbines, the draft tube is used
(a) for the safety of the turbine
(b) to convert kinetic energy of the flow by a gradual expansion of the flow cross section
(c) to destroy the undesirable eddies
(d) to convert datum head in to kinetic energy

O2.22 ** The level of the exit of the runner of a reaction turbine is 5 m above the tailrace and the atmospheric pressure is 10.3 m. The pressure at the exit of the runner for a divergent draft tube would be, in absolute units,
(a) 5.0 m (b) 5.3 m (c) 10.0 m (d) 10.3 m

O2.23 *A draft tube is used in a reaction turbine
(a) to guide water downstream without splashing
(b) to convert residual pressure energy into kinetic energy
(c) to convert residual kinetic energy into pressure energy
(d) to streamline the flow in the tailrace

O2.24* For a hydroelectric project with a reaction turbine, the lower end of the draft tube provided at the exit from the turbine is
(a) always immersed in water
(b) always above the water
(c) may be either above or below the water
(d) above or below the water depending upon the unit speed of the turbine

O2.25***The velocity heads of water at the inlet and outlet sections of a draft tube are 3.0 and 0.20 m respectively. The frictional and other losses in the draft tube are 0.4 m. What is the efficiency of the draft tube?
(a) 14.4% (b) 92.3% (c) 86.7% (d) 85.7%

Three

Reaction Turbines-2: *Propeller and Kaplan Turbines*

3.1 DESCRIPTION

A propeller turbine, which comes under the category of axial-flow turbine, is a reaction turbine that might be considered as a logical evolution of a Francis turbine to meet the particular needs of higher specific speeds. Consider Fig. 3.1 (which is the same as Fig. 2.9 of Chapter 2) showing schematically the configurations of a Francis turbine runner, to relative scale, at progressively increasing specific speeds. In Figs. 3.1 (*a*, *b, c* and *d),* the four runners belong to the Francis-turbine category. It is seen that as the specific speed increases, the size of the unit decreases, the flow changes from radial flow to mixed flow and then gradually becomes more and more axial. The fourth runner (Fig. 3.1-*d*) is a high-speed Francis where the flow is essentially diagonal and converts to almost axial flow as it passes the runner. It is to be noted that the shroud thickness has decreased considerably from Fig.3.1-*a* onwards and also the blade shape has been altered considerably. Figure 3.1(*e*) represents an axial-flow turbine as a limiting case of Francis runner at high-enough specific speed. The limiting case of a high-speed Francis turbine with the shroud removed and number of blades reduced is a propeller-shaped runner. The specific speed is higher and this meets the needs of a turbine for relatively large flows under smaller heads. The resulting runner shape is the propeller. The axial-flow turbine thus belongs to the group of reaction turbines and represents an extension of the specific speed range of the Francis turbine with a small overlap.

3.2 PROPELLER TURBINE

Figure 3.2 shows schematically a propeller turbine runner. Being a reaction turbine, the components are essentially the same as that of the Francis turbine and consist of

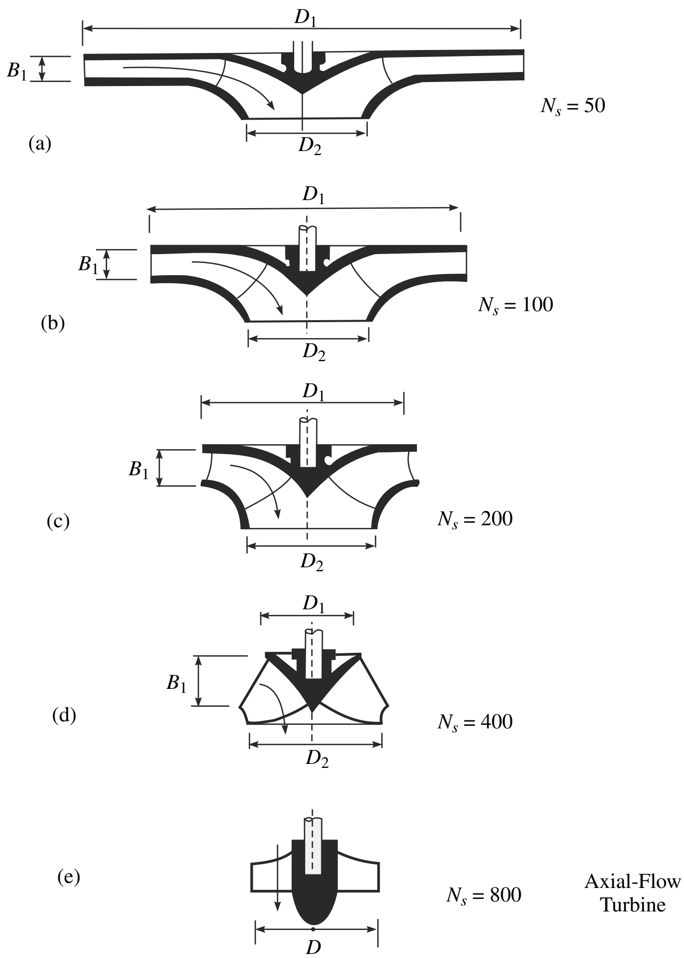

Fig. 3.1 *Evolution of axial-flow turbine profile*

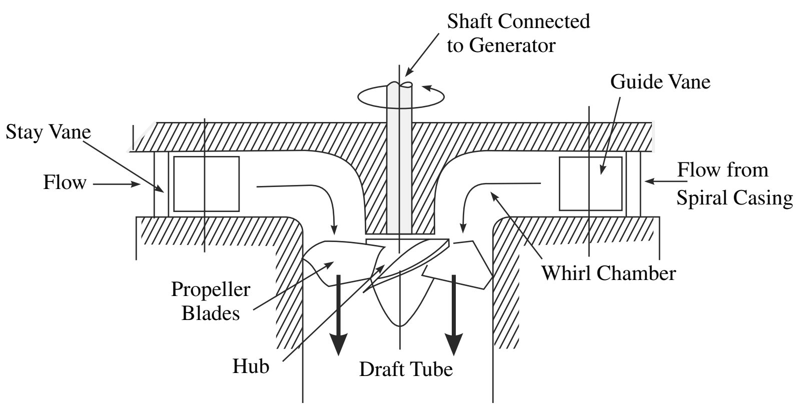

Fig. 3.2 *Schematic sketch of a propeller turbine*

- Scroll case (with or without stay vanes)
- Guide vanes
- Propeller-shaped runner (fixed blade)
- Governor (for control of guide-vane angle)
- Whirl chamber
- Draft tube

The basic feature of a propeller turbine is that the flow is axial. Hence, its generic name is *axial-flow turbine*. The number of blades is small and is in the range of 3 to 8. For a given diameter, an axial-flow turbine is capable of handling larger flow rate than a Francis turbine because the meridional velocities are higher. Another way of looking at it is that for a given discharge and head, propeller turbines have smaller size compared to an equivalent Francis turbine.

The propeller turbines are generally mounted on a vertical axis. Some variants such as *tubular turbines* and *bulb turbines*, which belong to the family of axial-flow turbines, have a horizontal or inclined axis. The blades are mounted on a hub on the axis. The blades are airfoil shaped in cross section and are suitably warped as per design to achieve shockless entry of water to blades at the design point. The guide vanes help in providing appropriate entry angle to the blades at the design point. Further, the guide vanes being mounted on pins, are controlled by a regulating ring, as in a Francis turbine, and are controlled by the governor mechanisms to adjust the discharge fed to the runner. For a propeller turbine, this is the only control mechanism available and as such, these turbines are said to be under *single control*.

A *Kaplan turbine* is a propeller turbine with additional feature of mechanisms to regulate the *pitch* (orientation of the blades to the flow direction) of the blades. This offers the flexibility of changing the blade angle at off-design points to achieve maximum efficiency. Corresponding governing mechanisms are incorporated in the turbine system. This feature of capability to change the blade disposition distinguishes the Kaplan turbine from the propeller turbine. Thus, the propeller turbine can be considered as a Kaplan turbine with fixed blades. Other hydraulic features of the Kaplan and propeller turbine units are essentially same and as such, further discussion on this topic will be confined basically to Kaplan turbines. Special features of propeller turbines, such as the performance characteristics that are different from those of the Kaplan turbine, will be indicated specifically.

3.3 KAPLAN TURBINE

In 1913, Victor Kaplan, an Austrian engineer, patented a variable-pitch propeller turbine that has since been known as Kaplan turbine. Since that time, this type of turbine has seen many design improvements that has enabled Kaplan turbines to be used is as high a head as 70 m. A Kaplan turbine is hydraulically similar to a propeller turbine except for its variable-pitch characteristic. Further, the hub is large to accommodate the mechanism for blade-angle changes. In most designs, the servomotor regulating the blades is located either in the hub or in the hub cone. The normal range of specific speed, N_s, of the Kaplan turbine is 260 to 900. The range of N_s from 260 to 450 is the overlap region with the Francis turbine.

The characteristic feature of a Kaplan turbine is the presence of adjustable propeller blades. A Kaplan turbine having adjustable blades and adjustable guide vanes is described as *double-regulated* and the word *Kaplan turbine* is generally used for this kind of turbine. However, if the guide vanes are fixed and the blades are adjustable, such turbines are to be described as *single-regulated* and this kind of turbine is commonly termed *Semi-Kaplan* to distinguish them from the double-regulated turbines. Axial-flow turbines with fixed runner blades are called *propeller turbines*.

Kaplan turbines have the following main components, (Fig. 3.3):

- Scroll casing and stay vanes
- Guide vanes
- Whirl chamber
- Propeller-shaped runner (with adjustable blades)
- Draft tube
- Governor mechanism (Consisting of main servomotor, runner-blade servomotor, regulating mechanism of runner blades and coordinating link for guide vanes and runner blades)

1. Scroll Casing

Scroll casings, in general, are similar to those used in a Francis turbine. For lower heads, i.e. up to 30 m head, concrete scroll cases are the general choice. The semi-spiral type [(Fig. 2.5(b)] concrete casings are highly preferred. For higher heads (30 to 60 m), where pressure heads are too high for concrete, steel plates are used in forming the scroll case. The cross section is normally circular. Stay rings with stay vanes anchored to concrete are provided in the scroll case. These stay vanes are generally of steel plates with infilling of reinforced concrete and conduct the water to guide vanes. They also act as structural members.

2. Guide Vanes

Guide-vane cascade that receives the water directed by stay vanes is similar to that of Francis turbine. The control of all the guide vanes is through a regulating ring actuated by a servomotor. Other features of the guide vane, regulating ring and control mechanism are essentially the same as in the Francis turbine described in Sec. 2.2 of Chapter 2.

3. Whirl Chamber

The space between the guide-vane outlet and the inlet of the runner is known as *whirl chamber*. In this chamber, the flow changes from radial orientation to axial direction. The lower portion of this chamber that is in the immediate vicinity of the blades is known as *runner chamber*.

The guide vanes impart a whirl component to the flow coming into the whirl chamber. This flow in the chamber could be approximated to a *free vortex* with whirl velocity (V_u) being inversely proportional to the radius. The product of V_u and radius r, called *circulation*, is a constant (that is, circulation $\Gamma = V_u\, r$ = constant) in the whirl chamber. Thus, we have maximum whirl velocity at the hub and minimum whirl velocity at the cylindrical boundary of the chamber. On the other hand, the rotating blade causes peripheral velocity u to vary directly with the radius, with minimum at the hub and maximum at the tip of the blade. The result is a complex velocity distribution in the blade passages. Further, to accommodate the complex velocity pattern all along the blade, the blades will have a twisted shape with airfoil cross section. Design of the shape of the blades is a very challenging task and the current practice relies heavily on advanced computation methods using *CFD* (Computational Fluid Dynamics) procedures.

The clearance between the outer blade ends and the *runner chamber ring* should be as small as possible for all possible blade angles of the runner. Generally, the gap is about 0.1% of the runner diameter D_1. Cavitation being a possibility in the region of the gap, the runner chamber is made of cast/welded stainless chromium–nickel steel. The ratio of hub diameter to blade diameter varies in the range 0.35 to 0.65 with higher values at lower specific speed. (See Table 3.2.)

4. Runner

The runner in a Kaplan turbine (and in a propeller turbine) is a set of specially designed blades that give the appearance of a ship's propeller. There is no shroud covering the tips of the blades. The blades are connected to the drive shaft at the hub (see Fig. 3.3). The hub itself is a small bulge in the shaft and has blades attached to it. In a Kaplan turbine, the pitch of the blades are adjustable and the link mechanism is housed inside the hub. In some models, servomotors are also housed in the hub. In propeller turbines, the blades are fixed rigidly to the hub. A nose cone projects from the hub below the fixture location of the blades to the hub. In a Kaplan turbine, the hollow of the nose cone and the hub is filled with oil under pressure to provide lubrication to moving parts of control mechanism packed in that portion. The outside of the hub has a spherical bulge to keep small clearances between the blades at all angles.

The number of blades varies from 3 to 8 and the blades are made of structural steel: stainless steel (for e.g., 16 Cr 5Ni). The hub is made of structural steel: carbon steel/high strength micro alloy/heat-treatment steel/stainless steel. Generally, four

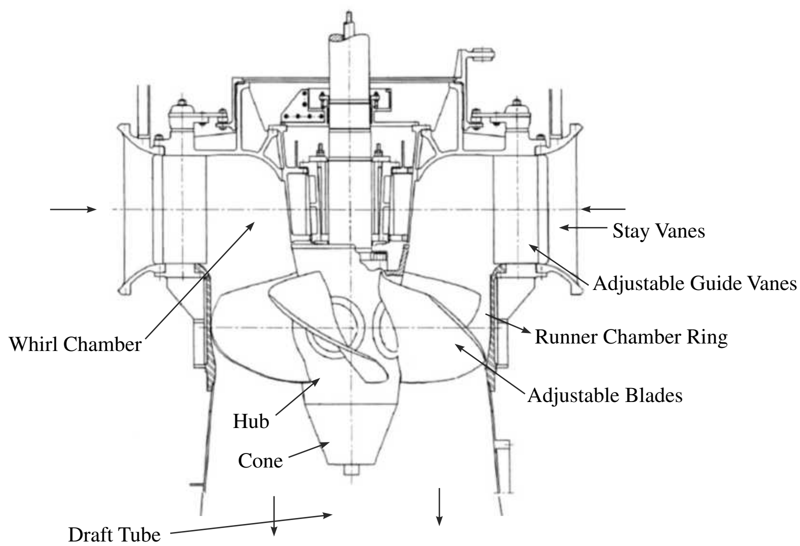

Fig. 3.3 *Definition sketch of a five-blade Kaplan turbine*

blades are used for heads up to 30 m and for every additional head slab of 10 m, an additional blade is needed. Very rarely eight blades are used. For heads less than about 15 m, sometimes three blades are used.

5. Draft Tube

A draft tube is an essential, integral part of a reaction turbine and plays a significant part in the installation of the turbine unit relative to the tailwater level. The hydraulic aspects relating to draft-tube installation are the same as described in connection with Francis turbines in Section 2.9. Kaplan turbines are generally associated with low heads and high discharges and the draft tube plays a very substantial role in the operation of the unit. In a low-head, high-discharge unit, the largest contribution of head loss is from the draft-tube. Thus, the height and geometry of the draft-tube has considerable influence on the efficiency and power output of a Kaplan turbine. Efficient elbow-type draft-tubes are of common choice. Cavitation in the draft-tube is another point of concern in draft tube design. In view of these, the design of optimal shape and dimensions of a draft-tube for an axial-flow turbine in a project receives extensive attention and study.

6. Governor

The governor is an essential, integral component of the Kaplan turbine. It does two functions:

(a) Controls the guide-vane openings and thereby regulates the discharge into the turbine.

(b) Regulates through servo control mechanism, the disposition of the blades to the flow and achieves high efficiencies at part loads

The aspect of blade control is absent in fixed-blade propeller turbines. Governing mechanisms, details of governing system and general control algorithm are the same as described in Section 2.11 in Chapter 2. Features specific to Kaplan turbine governing are described in detail in a subsequent section in this chapter (Section 3.6).

3.4 ANALYSIS

3.4.1 Analysis Procedure for Axial-flow Turbines

Figure 3.4(a) is a definition sketch of a Kaplan turbine. Figure 3.4 (b) shows inlet and outlet velocity triangles at any arbitrary radius r from the axis of the runner.

In this D = Outer diameter of the runner (= Diameter of the tip of the blades). D is commonly known as the *diameter of the runner*.)

D_h = Diameter of the hub (boss)

D_m = Diameter of the runner corresponding to midradius

u = Peripheral velocity at any radial distance

V = Absolute velocity at any radial section

v_r = Relative velocity

α = Guide-vane angle (= Angle made by the absolute velocity vector V with the positive direction of the peripheral velocity, u.)

β = Blade angle (Angle made by the relative velocity vector v_r with the negative direction of the peripheral velocity, u.)

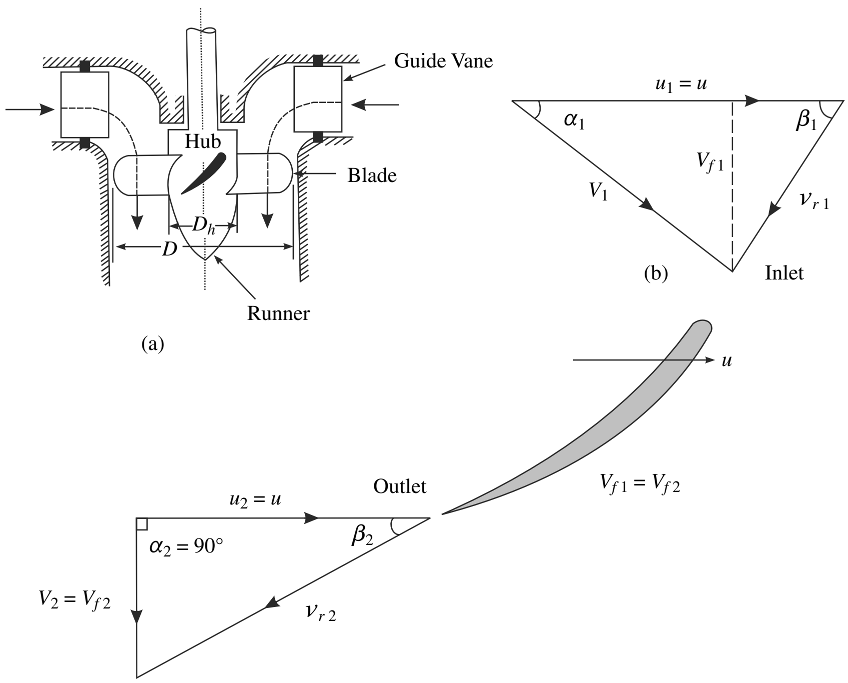

Fig. 3.4 *(a) Definition sketch of Kaplan runner (b) Inlet and outlet velocity triangles at any radius r from the axis of the runner*

Suffixes 1 and 2 denote inlet and outlet conditions respectively. Thus, for example, D_1 represents the inlet location at the tip of the runner. (Since for a Kaplan turbine, $D_1 = D = D_2$, in this book the diameter of the runner of a Kaplan propeller turbine is consistently referred to as D_1).

The flow is axial through the blades.

As such,

Area of flow = $A = \frac{\pi}{4}(D_1^2 - D_h^2)\,K_1$ where K_1 is the net area factor after deducting for the area occupied by the blades in the cross section. For preliminary studies, it is usual to take $K_1 = 1.0$.

1. Area of flow $A = \frac{\pi}{4}(D_1^2 - D_h^2)$ (3.1)
2. Discharge $Q = \frac{\pi}{4}(D_1^2 - D_h^2)\,V_{f1}$ (3.2)
3. $V_{f1} = V_{f2}$ = Velocity of flow which is taken as constant in the entire inlet–outlet space, i.e. all along the inlet radius as well as all along the outlet radius.
4. Let u_1 = Peripheral velocity at inlet at any radius r, and u_2 = Peripheral velocity at the outlet at the same radius r. Then

$$u_1 = u_2 = u = \frac{2\pi r N}{60} \tag{3.3}$$

5. The flow is assumed to leave the runner axially without any whirl component. Thus, at the outlet, α_2 = 90° and $V_2 = V_{f2}$, as used in the case of a Francis turbine
6. It is assumed that the flow entering the whirl chamber from the guide vanes creates a free vortex in the whirl and runner chambers where the velocity is inversely proportional to the radius. Thus, if r_h (= D_h/2) is the radial distance of the hub from the axis, r_t (= D_1/2) is the radial distance of the blade tip and r_m is the radial distance of the midpoint of the blades then

$$(V_{u1})_t\, r_t = (V_{u1})_m\, r_m = (V_{u1})_h\, r_h = \text{Constant} \tag{3.4}$$

In this, $(V_{u1})_x$ is the velocity of whirl at the given radius x on the inlet side of the blade.

7. The Euler equation is applicable giving the Euler head H_e as

$$H_e = \frac{V_{u1}u_1 - V_{u2}u_2}{g} = \frac{V_{u1}u_1}{g} \qquad (\because V_{u2} = 0) \tag{3.5}$$

Further, the hydraulic efficiency $\eta_h = \dfrac{H_e}{H} = \dfrac{V_{u1}u_1}{gH}$ where H = Net head.

8. Values of α_1, β_1 at inlet and β_2 at outlet change all along the blade length. The velocity triangle at the inlet at any radial location on the blade is an acute-angled triangle. The outlet velocity triangle is a right-angled triangle. Both are constructed in the same way as done in connection with the Francis turbine. The nature of variation of the flow angles of the velocity triangle, viz. α_1, β_1 at inlet and β_2 at outlet, could be described in a general way as below (Fig. 3.5):

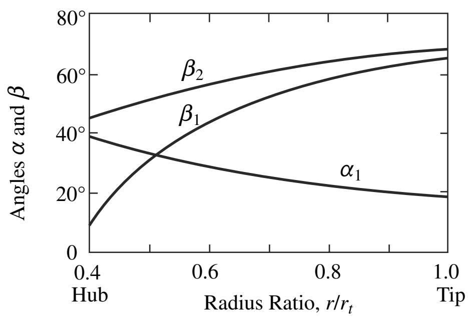

Fig. 3.5 *Typical nature of variation of blade angles in a Kaplan turbine*

- Angle β_1 is maximum at the blade tip and decreases along the radius to a minimum at the hub.
- Angle α_1 is minimum at the blade tip and increases along the radius to a maximum at the hub.
- Angle β_2 is maximum at the blade tip and decreases along the radius to a minimum at the hub. Generally, β_2 is larger than β_1 at all radial locations.

9. Mean radius $r_m = \dfrac{(D_1 + D_h)}{2}$ is used sometimes as a representative location for estimation of values of hydraulic efficiency, specific speed, power and overall efficiency of the turbine.
10. Following the same procedures and notations as adopted in Francis turbine, the velocity triangles at inlet and outlet at three locations, viz. hub, midradius and blade tip, are shown in Fig. 3.6.

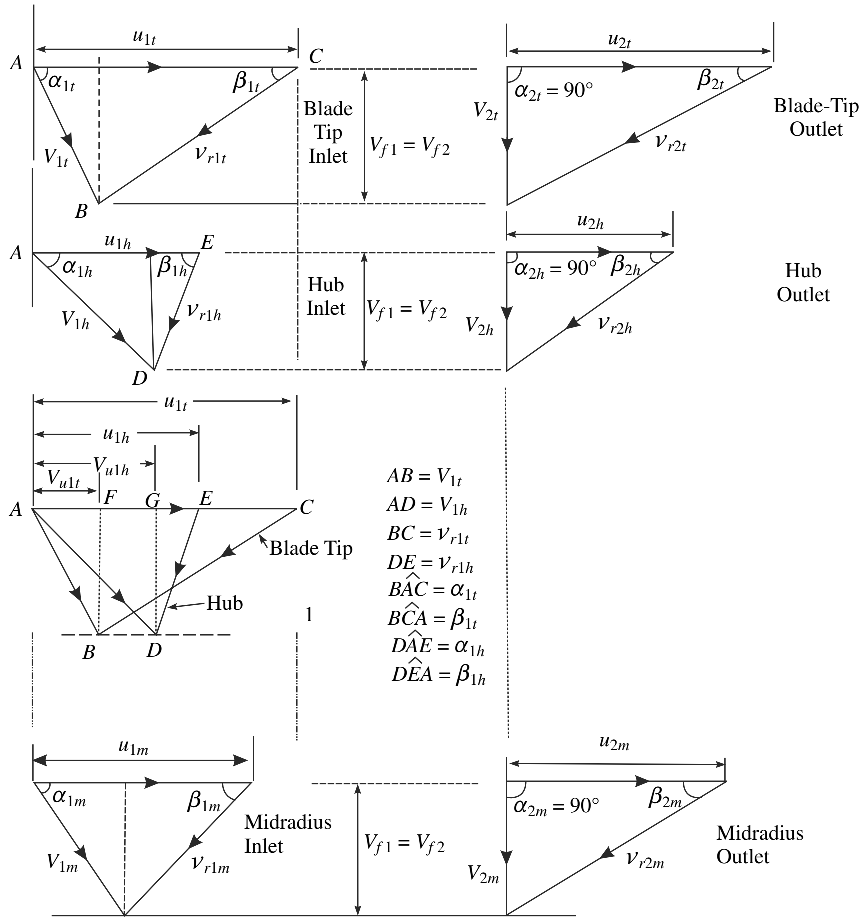

Fig. 3.6 *Salient velocity triangles in a Kaplan turbine (Schematic)*

Example 3.12 illustrates the calculation procedure of using the above relations and velocity triangles at various radii.

3.4.2 Application of Airfoil Theory

A blade surface that produces large lift and small drag is ideal for a propeller blade. An appropriate shape of such an airfoil suitable for use as a blade section can be identified from the available database of airfoil sections. Figure 3.7 shows the definition sketch of an airfoil-shaped section subjected to the action of a fluid flow. The relative flow makes an angle α_0 with the chord line of the airfoil. This type of airfoil section when used as the blade section of a Kaplan/propeller turbine will have a warped profile. However, a simplified one-dimensional analysis of the turbine blades can be made by analysing the flow past an appropriate airfoil section with proper orientation to estimate power and efficiencies of the turbine. This would be adequate for preliminary studies.

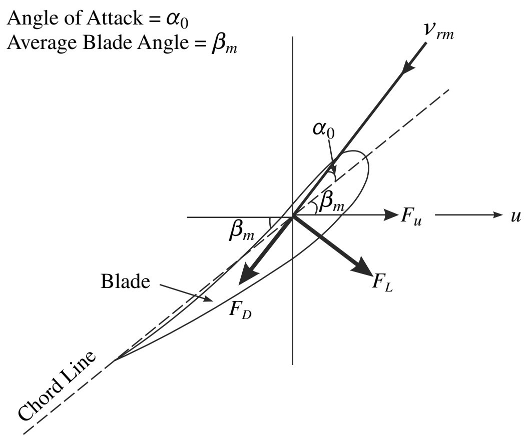

Fig. 3.7 *Airfoil theory applied to Kaplan turbines*

Consider the pitch circle of radius $r_m = \frac{(D_1 + D_h)}{2}$ of a Kaplan turbine. The turbine blade is considered as an airfoil section of chord length L, width (span) b and circumferential pitch t (Fig. 3.8). Let there be n such blades in the runner. V_1 and V_2 are the inlet and outlet absolute velocities at this section. The peripheral velocity at this section is $u_1 = u_2$ giving rise to relative velocities v_{r1} and v_{r2} at the inlet side and outlet side respectively (Fig. 3.9). The mean of v_{r1} and v_{r2} is designated as v_{rm} with a relative flow angle of β_m. This is considered as the average flow per unit width passing over the airfoil section of the turbine blade.

The airfoil section of Fig. 3.7 is now considered as being subjected to a water stream of velocity v_{rm} making an angle β_m with the direction of motion of the blades. This is one of n blades mounted on a pitch diameter D_m. The peripheral velocity at D_m is $u = u_1 = u_2$. Let the airfoil be set at an angle of incidence α_0 to the direction of v_{rm}. The stream of water impinging on the airfoil will cause a lift force F_L and drag force F_D on the blade as shown in Fig. 3.7. The lift force causes the runner blade to rotate while the drag force acts as resistance to the flow and causes loss of energy. It also causes an axial thrust on the blade.

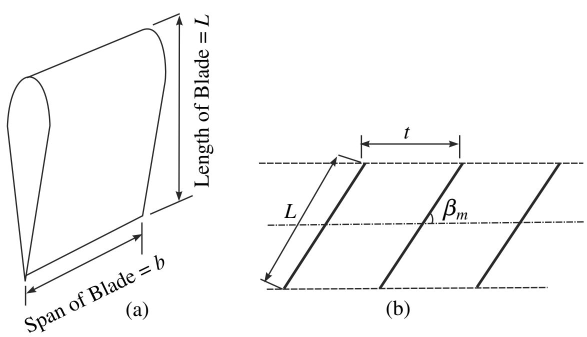

Fig. 3.8 *Definition sketch of blade geometry*

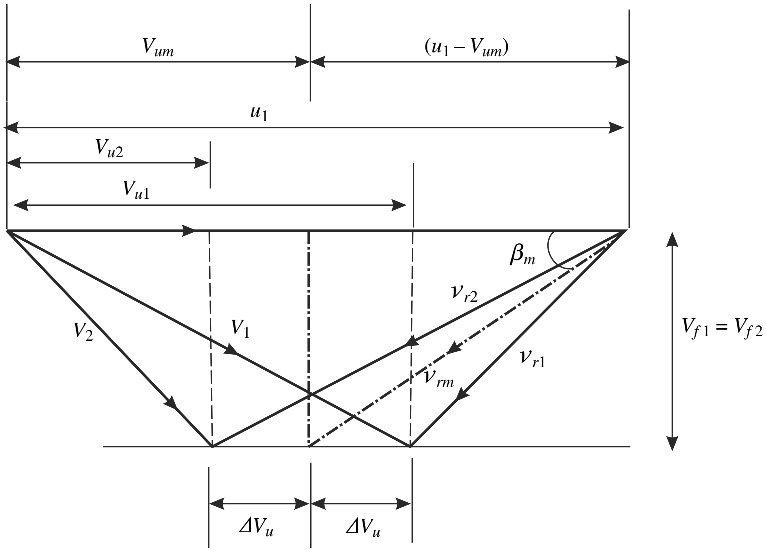

Fig. 3.9 *Inlet and outlet velocity triangles and definition of β_m*

Resolving the forces F_L and F_D in the direction of motion of the blade, the net force F_u in the direction of rotation is

$$F_u = F_L \sin \beta_m - F_D \cos \beta_m \tag{3.6}$$

The lift force is given by $F_L = C_L A\rho \dfrac{v_{rm}^2}{2}$ and the drag force is given by

$F_D = C_D A\rho \dfrac{v_{rm}^2}{2}$

In these, $A = L \times b$ = area of the blade

C_L = Appropriate lift coefficient of the airfoil corresponding to an angle of incidence of α_0.

C_D = Appropriate drag coefficient of the airfoil corresponding to an angle of incidence of α_0.

The power produced by n blades each of area A is

$$P = F_u\, u\, n = \rho\, A\, \frac{v_{rm}^2}{2}\, (C_L \sin \beta_m - C_D \cos \beta_m)\, u\, n \tag{3.7}$$

In this equation v_{rm} = Mean relative velocity, and

β_m = Average relative flow angle defined as

$\tan \beta_m = \left(\dfrac{\tan \beta_1 + \tan \beta_2}{2}\right)$ at a radial distance $r_m = (D_m/2)$ from the axis.

Figure 3.9 illustrates this definition graphically. Further, note that this definition of β_m defines the mean relative velocity v_{rm} also.

Standard airfoil tables give C_D and C_L for a given airfoil as function of the angle of incidence a_0. These values are for a single, isolated airfoil not affected by any interference. Let us designate these values as C_{D0} and C_{L0} respectively. In the present case, we have a *cascade of airfoils*. Each of these *n* airfoils is having interference due to the presence of other airfoils in the neighbourhood. Hence, a correction is needed to the single, isolated case values of C_{D0} and C_{L0}. The correction factor *K* to account for the interference effect in a cascade of airfoils is a function of pitch-to-chord length ratio, t/L. Such corrections are available in the literature on airfoils. The C_D and C_L to be used in Eq. (3.7) are the appropriately corrected values of drag and lift coefficients, respectively.

3.5 WORKING PROPORTIONS OF A KAPLAN TURBINE

Consider a Kaplan turbine with H = Net head on the turbine, N = Rotational speed in rpm and P = Power developed by it. Let N_s = Specific speed of the turbine, B_0 = Height of the flow passage at the guide vanes. Further, D_1 = Outer diameter of the runner, D_h = Outer diameter of the hub (boss).

- The number of guide vanes (Z_0) depends upon the diameter of the runner (D_1) and range between 8 and 24. The variation is shown in Table 3.1.

Table 3.1 *Number of guide vanes*

Z_0	8	10	12	14	16	18	20	24
D_1 in mm	< 300	300 to 450	450 to 750	750 to 1200	1200 to 1600	1600 to 2200	2200 to 4000	> 4000

1. Relative Guide Vane Passage Size, $\left(\frac{B_0}{D_1}\right)$

The relative passage size $\left(\frac{B_0}{D_1}\right)$ of the guide vanes is a function of the specific speed (N_s). Its value lies in the range 0.30 to 0.41 and its variation with specific speed is shown in Table 3.2.

2. Relative Hub Diameter, D_h/D_1

The relative hub size, D_h/D_1, varies with the specific speed and has a range of 0.40 to 0.64. Table 3.2 shows the variation covering the range of specific speeds of 260 to 770.

3. Flow Ratio, K_f

Flow ratio is defined as $K_f = \frac{V_{f1}}{\sqrt{2gH}}$ is a function of specific speed and has a range of 0.35 to 0.75 corresponding to a specific speed range of 260 to 860. The variation of K_f with N_s is shown in Table 3.2.

4. Speed Ratio, K_u

Speed ratio is defined as $K_u = \dfrac{u_1}{\sqrt{2gH}}$ where u_1 is the peripheral velocity at the blade tip. It is a function of specific speed and has a range of 1.3 to 2.3 corresponding to a range of specific speed 260 to 860. The variation of K_u with N_s is shown in Table 3.2. The following empirical equation is available for the estimation of speed ratio, K_u, of Kaplan turbines.

$$K_u = 0.0286\, N_s^{2/3} \tag{3.8}$$

Table 3.2 *Variation of some key parameters of Kaplan turbine with specific speed*

Specific Speed N_s	Speed Ratio K_u	Flow Ratio K_f	B_0/D_1	D_h/D_1
260	1.30	0.35	0.30	0.64
350	1.42	0.45	0.35	0.58
430	1.55	0.50	0.39	0.51
520	1.70	0.55	0.41	0.45
600	1.85	0.60	0.41	0.35 – 0.41
690	2.00	0.65	0.41	0.35 – 0.41
770	2.13	0.70	0.38	0.35 – 0.41
860	2.26	0.75		

NOTE

The ratios of Table 3.2 are only approximate values. Individual specialised designs of manufacturers often show considerable variations from the values of the table.

5. Size

The size of the runner (D_1) of a Kaplan turbine is generally large and is a function of the head under which the turbine operates and its rated power. For preliminary studies, the following empirical equation can be used to get a fair estimation of the diameter of the blades.

$$\frac{D_1}{\sqrt{P}} = C \tag{3.9}$$

where D_1 = Outer diameter of the turbine runner in metres, P = rated power of the turbine in kW and C is a function of the net head under which the turbine operates. This equation, being based on a large number of existing installations, is generally expected to give reasonably good results. The variation of the coefficient C with H for Kaplan and propeller turbines is shown in Table 3.3.

Table 3.3 *Variation of the coefficient C of Eq. (3.9)*

Net Head H (m)	3	4	5	6	8	10	20	30
C for Kaplan Turbine	0.10	0.095	0.080	0.052	0.045	0.040	0.027	0.021
C for propeller Turbine	0.15	0.13	0.12	0.11	0.10	0.063	0.053	0.050

It is seen that in general, under identical conditions, the propeller turbine has a larger diameter runner than the Kaplan turbine.

6. Efficiency

The overall efficiency of the Kaplan turbine is very high, being about 93% for a well-designed large unit. Further, its part-load efficiency is also very good. On the other hand, the overall efficiency of the propeller turbine is about 90% and its part-load efficiency is very poor. Aspects relating to the efficiency are discussed in detail under performance characteristics, in Section 3.8.

7. Runaway Speed

Equations (2.23) and (2.24) of Chapter 2, recommended by *USBR* for Francis turbines, are applicable to estimate the *runaway speed* of propeller/Kaplan turbines also. Generally, the runaway speed of a propeller turbine is in the range of 2.5 to 3.0 times the design speed.

$$\frac{N_r}{N} = 0.63\,(N_s)^{1/5} \qquad (3.10\ \&\ 2.23)$$

and

$$N_{r\,\max} = N_r \left(\frac{H_{\max}}{H_{\text{design}}}\right)^{0.5} \qquad (3.11\ \&\ 2.24)$$

in these N_r = Runaway speed at best efficiency head and full gate

N = Rotational speed

$N_{r\,\max}$ = Runaway speed at maximum head

$H_{\max}$ = Maximum head

H_{design} = Design head

The Kaplan turbine however, due to its adjustable blade provision, has a theoretical runaway speed approaching infinity at the closed or flat-blade position. In practice, the friction and windage of the connected generator normally will limit the runaway speed to 275 per cent of design speed. The specifications usually include a requirement for an adjustable stop on the blade rotation that will limit the runaway speed at 275 per cent to avoid excessive stresses in the generator.

8. Specific Speed

The normal range of specific speed of Kaplan turbine is 260 to 900.

9. Range of Other Elements of the Turbine

Normal Head: 1.5 m – 80 m; Normal speed: 70 – 600 rpm, Max. Capacity (2002) = 185 MW. α_1= 45 – 70°; β_1 = 20 – 60°.

3.6 GOVERNING OF KAPLAN TURBINES

The governing of a Kaplan turbine is different from that of the Francis turbine in the sense that it has double control of the turbine operation to meet the needs of part load. While the guide vanes control the flow and the inlet angle, the pitch of the blades are controlled in unison to achieve best results. The mechanism for controlling the guide vanes is exactly same as in the Francis turbine, (Sec. 2.11.4). The servomotor activates the regulating ring mounted on the top cover of the guide vanes. The movement of the ring causes the vanes to move about their individual axis to change the area of flow passage. Full opening to full closure of the passage is possible.

Similarly, the servomotor operating the runner blades causes the individual blades to rotate about their respective axis, in synchronisation, to achieve the desired position. Full opening to full closure of the blade passage is possible. Three locations are commonly adopted to house the blade servomotors. In the earlier designs, the blade servomotor used to be housed in a chamber surrounding the shaft and located above the turbine cover. Currently, a majority of designs include the blade servomotor either in the hub or in the hub cone.

Blade-Control Valve (Combinator)

Operation of both the guide vane set and the runner blades set simultaneously to achieve the best combination is indeed a complicated task. To meet the specific demands of a part load, special equipment is needed. The mechanism for controlling and maintaining the specified relationship between the guide vanes and the runner blade angle in a Kaplan turbine is called *blade-control valve* or *combinator*. Mechanical connection in the form of a cam with linkages connects the combinator to the blade servomotor and guide-vane servomotor. The cam and linkages are so proportioned that the correct relationship between the guide-vane angle and blade angle is obtained and the correct positioning is done by the hydraulic controls.

The ability to change both runner-blade angle and guide-blade angle simultaneously and in unison while the turbine is rotating has enabled designers to develop mechanisms to achieve optimum efficiency at all operating points. There are innumerable combinations of gate-angle opening α and blade angle β (at a specified location on the blade) to achieve requisite power to meet a given part-load. For any part-load operation, the efficiency is a function of gate opening α and blade angle β. Hence, there exists for each part load an optimum combination of α and β that gives the best efficiency at that part load. The entire operation space can thus be considered to be made up of sets of values of α and β for each part load and out of these, there is a unique set for each part load that gives the optimum efficiency. The unique relationship of α and β, covering the entire range of operation, that gives the optimum efficiency is known as *cam relationship*. The combinator transmits this information through the cam linkages to the servomotors. This cam relationship for a given turbine is obtained by the manufacturers through detailed model tests implemented on-site to the prototype. The governing system that is set to work on this optimum dual control system is said to be *on-cam*. If this dual control is disconnected, the system is said to be *off-cam*. Development and implementation of cam relationship is a complex and sophisticated task needing high expertise (Ref. 3.3).

3.7 VARIANTS OF PROPELLER TURBINES

3.7.1 Tubular Turbines

For very low heads and high discharges, a Kaplan-turbine installation involves costly civil construction. For units roughly less than about 20 MW, pure Kaplan-turbines work out to be rather costly. This has led to the search of alternatives that has led to a set of solutions.

Tubular turbine is one such successful alternative system. The scroll case has been dispensed with in this axial-flow turbine. The runner is placed directly in the main water conduit itself. Figure 3.10 shows a schematic layout of a tubular turbine. While the scroll case is absent, the guide vanes (wicket gates) are provided. These are mounted on the turbine housing and are connected to the water conduit. The water conduit is called the *tube*. The mounting of the turbine is generally horizontal; it could be in inclined position also. The generator is provided outside the tube. The number of bends in the conduit is minimised and the bends are smoothened to minimise the hydraulic losses. The runners are of propeller type with fixed or variable pitch (Kaplan type). The wicket gates are adjustable and hence single or dual type of control is adopted depending upon the type of propeller.

The tubular turbine has an efficiency that is about one to two per cent larger than vertical propeller turbine of same size. This is due to lesser hydraulic losses in the passageway. Standard tubular turbines are available off the shelf for a range of heads up to 20 m and power range up to about 7 MW.

Tubular turbines that have the generator outside the tube and the tube configuration in the shape of the alphabet *S* are sometimes called *S-turbines* (Fig. 3.10). The S-shape is arranged to facilitate the location of the generator outside the tube by taking the shaft of the turbine through the tube.

The salient features of the tubular turbine are.

Head range: 2 to 30 m

Specific speed: 500 to 1100

Maximum Power: 80 MW

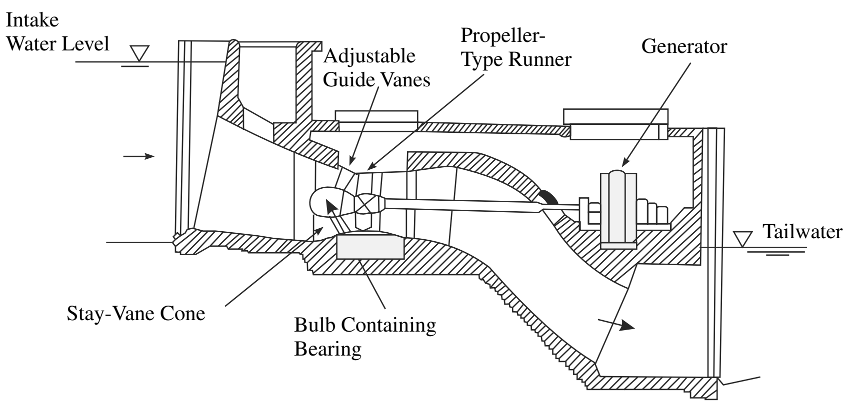

Fig. 3.10 *Schematic layout of an S-type tubular turbine*

3.7.2 Bulb Turbines

This is a variation of the tubular turbine wherein the generator is kept inside the tube. A bulb is provided upstream of the propeller blades and the generator is housed inside the bulb. The bulb is made watertight and houses the generator as well as the bearings of the turbine. The bulb is air cooled with air inside the bulb being pressurised to about two atmospheres. There is considerable savings in space and civil works in this compact arrangement. The efficiency is also high due to minimisation of the conduit losses. As in a tubular turbine,

- The orientation can be horizontal or inclined,
- The runner can be of fixed or variable-pitch propeller blades, and
- Guide vanes are provided in the annular space between the bulb and the tube. The servomotors control the guide vanes.

Figure 3.11 shows a schematic layout of a bulb turbine. Note that the guide vanes are provided in a conical portion of the tube. The vanes are adjustable through a ring mechanism as in a Francis or Kaplan turbine. The bulb is supported by stay supports connected to the tube. The support on the top of the bulb acts as access shaft to the generator. Further, air supply and ventilation to the generator chamber is through these supports. Bulb turbines have improved efficiency of about 2% over a vertical equivalent propeller unit and about 1% over an equivalent tubular turbine. This is possible due to straight passages and streamlined water flow past the runner. For very low heads, the generator speeds may have to be increased by means of gear arrangements. Standard bulb units are available off the shelf.

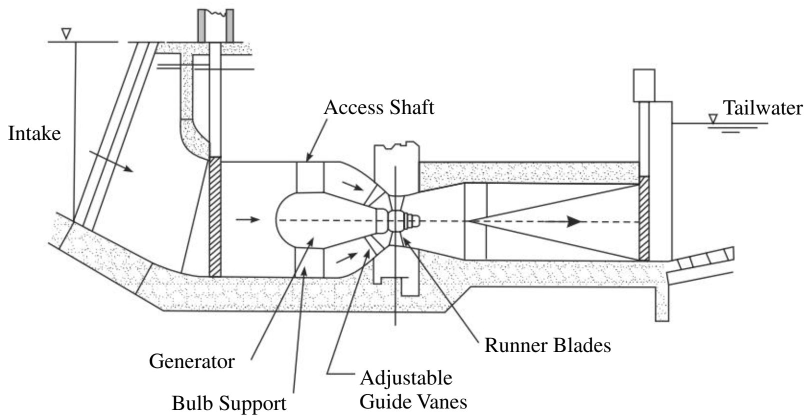

Fig. 3.11 *Schematic layout of a bulb turbine*

The salient features of the bulb turbines are:
Head range: 2 to 30 m
Specific speed: 500 to 1100
Maximum power: 80 MW

Key advantages of bulb turbines can be summarised as follows:

1. Bulb turbines are usually the most efficient solution for low heads of 2 to 30 metres. Bulb units have almost replaced Kaplan turbines for low-head situations.

2. Reduced size, cost and civil works requirements of up to 25% primarily due to the straight water passage in the draft tube that improves the hydraulic behaviour of the bulb unit and also results in a low need of excavation.
3. Bulb turbines could be designed to operate in both flow directions for tidal-plant applications.
4. While the bulb turbines are generally preferred for small hydro plants with capacities up to about 10 MW, there are many instances of its use in higher capacity plants, of up to 75 MW capacity, also. The following is a small list of high-capacity bulb-turbine installations to illustrate this aspect:
   - *Jirau and Santo Antonio (Brazil), 2008–2009*
     10 × 76.5 MW and 19 × 75.5 MW respectively. Both of 7.5 m dia.
   - *Rock Island (USA), 1978*
     8 × 58 MW—Head: 12 m.
   - *Ybbs-Persenberg, Austria, 1993*
     1 × 48 MW, 7.5 dia.
   - *Altenworth, Austria, 1973*
     1 × 44 MW, 6.0 m dia.
   - *Chang Zhou and Qiao Gong (China), 2007*
     3 × 42.9 MW—Head: 9.5 m and 4 × 58.5 MW—Head: 13.8 m
   - *Shingo-2, Japan, 1982*
     1 × 40.6 MW, 5.0 m dia.

3.7.3 Rim-type (Straflow) Turbine

The *rim-type turbine* is another variation of tubular turbine with a patented generator mechanism. The generator rotor is mounted on the periphery of the turbine runner itself. This type of turbine is also called as *straflow* and has been found to be suitable for small installations of 1–2 MW under 8–10 m head. The performance characteristics of rim-type turbines are similar to that of a bulb turbine. These turbines are available with or without wicket gates.

All the three variants of propeller turbines discussed above, viz. tubular turbine, bulb turbine and Straflow turbine, are suitable in tidal power development also. Further, the term *tubular turbine* is often used as a generic name to designate all the above three types. These three types, amongst them, cover a range from 2 m to 25 m of head, 200 kW to about 50 MW of power capacity and come in 0.8 to 6.0 m in diameter size. Reference 3.5 may be consulted for details regarding Straflow turbines.

3.7.4 Deriaz Turbine

The *Deriaz turbine* is a reaction turbine, named after its inventor, Paul Deriaz, who patented this device in 1947. This turbine can be described as diagonal, double regulated, mixed flow, adjustable blade, reaction turbine.

The Deriaz turbine occupies a space that is intermediate between Francis turbine and the Kaplan turbine. It is a mixed-flow turbine like the Francis turbine but has propellers with adjustable pitch, like a Kaplan turbine. In addition, the blade axis is not normal to the hub as in Kaplan turbine, but is inclined to the axis of the shaft. Figure 3.12 shows two schematic views of the Deriaz-turbine runner.

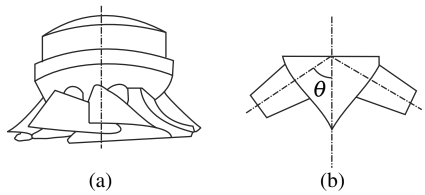

Fig. 3.12 *Runner of Deriaz turbine: (a) Schematic sketch of runner (b) Sketch showing inclined axis of the blades*

In a Deriaz turbine, the flow enters from the scroll case through guide vanes at an angle to the axis of the turbine shaft. It reacts with the blades of the turbine and leaves the runner axially. The blades are set at an acute angle θ to the axis of the turbine; the range of the angle being 30° to 60°. High values of the angle θ are used for lower values of design heads and correspondingly low values of θ are used for higher design heads. Like the Kaplan turbine, the blades of the Deriaz turbine are set to rotate about the blade axis. The mechanisms for pitch adjustments are located inside the hub. This ability to vary the pitch of the blades enables this turbine to have high part-load efficiencies when compared to an equivalent Francis turbine. A special feature of the Deriaz turbine is that the same machine can work as a turbine as well as a pump. Such machines that work both as a pump as well as a turbine are known as *reversible pump turbines*. [Francis-type and Kaplan-type reversible pump turbines are also available].

The governing mechanism is of dual type as in a Kaplan turbine. The governor has two sets of adjusting cams, one for pump operation and another for turbine operation, which ensure optimal cam relationship between the blades and wicket gates. The main advantage of a Deriaz turbine is that it has a head range of 25 m to 200 m and the specific speed range of N_s = 130 to 450 (S_{pr} = 0.12 to 0.43 revs.). Thus, the Deriaz turbine occupies a slot which has a large overlap with the Francis and Kaplan turbines. Further, this turbine could be operated as a pump also. Hence, it has large adoption in pumped storage hydroelectric schemes.

Invariably, the Deriaz turbine is used where pumping operation is also involved, as in the case of pumped storage schemes. Comparative efficiencies of Deriaz and Francis turbine pumps are shown in Fig. 3.13. In the range of 60% to 130% of rated capacity, the Deriaz turbine exhibits excellent efficiency characteristics. In this range,

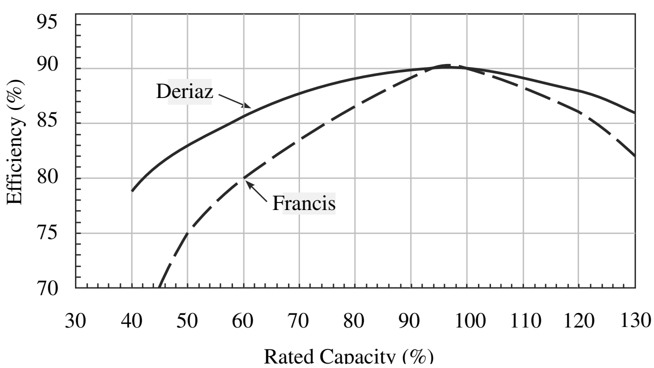

Fig. 3.13 *Efficiency curves of Deriaz and Francis turbine pumps*

the efficiencies of the Deriaz is well over 85% while that of the Francis turbine is considerably less except at the rated capacity.

Other key advantages of the Deriaz turbine can be listed as

1. Smooth and efficient operation characteristics over a wide range of head and load
2. Uniform distribution of pressure and load across the blades
3. Very good cavitation performance leading to freedom from cavitation problems over the entire operating range

Compared to an equivalent Kaplan turbine, Deriaz turbines are more challenging to design and manufacture. Consequently, they are more expensive. Generally, in the overlap region with a Kaplan turbine, at low heads Deriaz turbines are preferred only when the head is greater than about 35 m.

The ranges of salient characteristics of the Deriaz turbine are listed below:

- Maximum capacity ≈ 150 MW
- Head range = 25 to 200 m
- Speed = 90 to 300 rpm
- Specific speed N_s = 130 to 450 (Shape factor S_p= 0.12 to 0.43 rad.)
- Efficiency = 90 to 94%
- Runner diameter = 1.0 to 5.0 m
- Head variation (as percentage of rated head) = 130 to 60

3.8 PERFORMANCE CHARACTERISTICS OF KAPLAN TURBINES

3.8.1 Main Characteristics

Here, the characteristics of the turbine are studied under conditions of constant head under three headings, viz. variation of unit discharge Q_u with unit speed N_u, Variation of unit power P_u with N_u and Variation of efficiency with N_u. We now consider each one of these.

1. Variation of Discharge

Figure 3.14 shows the variation of unit discharge Q_u with unit speed N_u for four different gate openings in a Kaplan turbine. The head is kept constant in all the cases. It is seen that the discharge increases with the speed for all the gate openings. This is in contrast to the discharge–speed relationship of a Francis turbine (Fig. 2.38, Sec. 2.10.1) where it was shown that the discharge decreased with the speed.

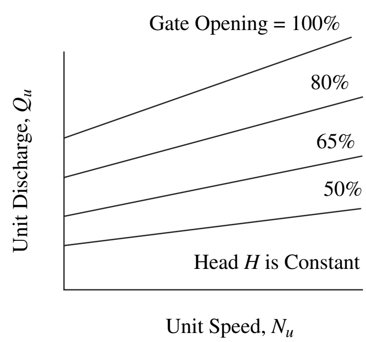

Fig. 3.14 *Variation of discharge with speed in a Kaplan turbine*

2. Variation of Power

Figure 3.15 shows schematically the variation of unit power P_u with unit speed N_u in a Kaplan turbine. Three different gate-opening situations are shown. The head is kept constant in all cases. The variation is parabolic for all the gate

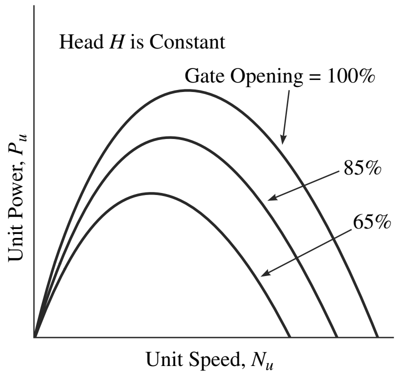

Fig. 3.15 *Variation of power with speed in a Kaplan turbine*

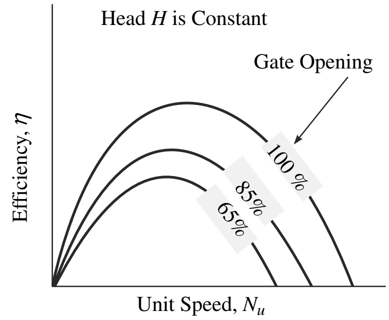

Fig. 3.16 *Variation of efficiency with speed in a Kaplan turbine*

openings and the general pattern is similar to that in the case of a Francis turbine, (Fig. 2.39, Sec. 2.10.1).

3. *Variation of Efficiency*

Figure 3.16 is a schematic depiction of the variation of efficiency of a Kaplan turbine with unit speed N_u under constant-head operation. The variation is parabolic in all the three gate openings considered. The peak efficiency occurs at different speeds for different gate openings. The nature of the curves is, in general, similar to the corresponding curves of a Francis turbine (Fig. 2.40, Sec. 2.10.1).

3.8.2 Operating Characteristics

1. *Kaplan Turbine Efficiency Curve for Preliminary Studies*

While individual turbine designs may have slight variations in the absolute values of the efficiency at and near peak regions, for preliminary designs, the following commonly used chart of variation of efficiency vs. percentage rated capacity is recommended. Figure 3.17, which is adapted from IS: 26800 of 1991 and *IEC*-1116-1992, shows the variation of expected overall *rated efficiency* η_0 of

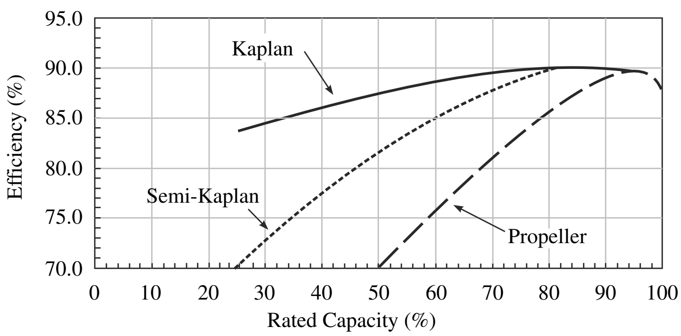

Fig. 3.17 *Rated efficiencies of axial-flow turbines, (Ref. 2.1, 3.1)*

Kaplan, semi-Kaplan and propeller turbines with percentage rated capacity of the turbine as abscissa. The data has been adjusted for a turbine of 300 mm runner outer diameter. For any other size of the runner, the efficiency has to be stepped up. Table 3.4 gives the additive step values for various diameters.

Table 3.4 *Additive step-up values of efficiency*

Size of turbine in mm	300	600	900	1200	1800	2400	3000
Additive step-up value	1.00	1.10	2.10	2.50	3.30	3.70	4.00

NOTE

The step-up value of this table (Table 3.4) is to be added to η value obtained from Fig. 3.17.

The values obtained from the table gives the percentage rated efficiency, and the *design peak efficiency* at design head is about 2 – 5% higher.

A study of Fig. 3.17 indicates the following:

- At 100% rated turbine capacity, the rated efficiency = η_r is about (88% + step-up value). The values of the graph (along with additive step-up value) will be called the *basic efficiency value* in further discussion.
- Peak basic efficiency is 90% and occurs in the range 78 to 95% of the rated capacity.
- The efficiency curve of a pure Kaplan turbine has a broad and flat humplike peak, thanks to the double regulation. The peak basic efficiency value of 90% is constant over the range 78 to 95% of rated capacity.
- Efficiency curve of a fixed-blade propeller turbine has a sharp peak and drops off very abruptly. The peak value occurs at 95% rated capacity.
- The semi-Kaplan has an efficiency curve midway between the above two types, viz. pure Kaplan and fixed-blade propeller turbines.
- It is clear that the Kaplan has the best efficiency curve, which even at 25% rated capacity has a basic efficiency value of 84%. Compared to this, the fixed-blade propeller turbine has a basic efficiency value of 70% at a rated capacity of 50%. Semi-Kaplan has a characteristic efficiency curve that lies midway between the above two types.

2. *Performance Curves*

Figure 3.18 is a performance diagram of the Kaplan turbine. It shows the discharge Q (as percentage of rated discharge, Q_r,) as the ordinate plotted against the generated capacity P_g (as percentage of rated generated capacity P_{gr}) in the abscissa. The generated capacity refers to the electrical power generated by the generator. The generator capacity is obtained by multiplying the turbine capacity by the generator efficiency. The third parameter is the operating head H (as percentage of rated head, H_r). The bounding lines of the head H, as for example, $H = 0.65\ H_r$ and $H = 1.40\ H_r$ in Fig. 3.18, represent the limits of satisfactory operation within normal industry guarantee standards. The top boundary line represents maximum recommended

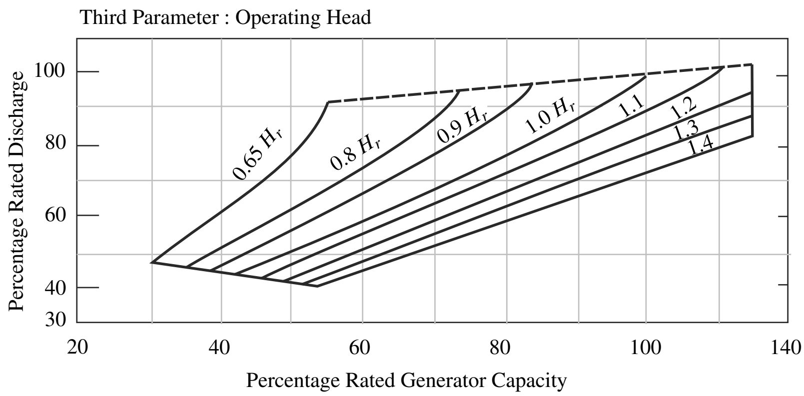

Fig. 3.18 *Performance diagram of Kaplan turbine (Ref. 2.1, 3.1)*

capacity as a percentage of rated capacity. The bottom boundary line represents the limit of stable operation. The right-hand boundary is established from a generator guarantee of 115% rated capacity. The area within the bounds represents the feasible operating space of these turbines. The head operation boundaries (0.60 H_r to 1.40 H_r) are typical values. These curves have been developed based on data from

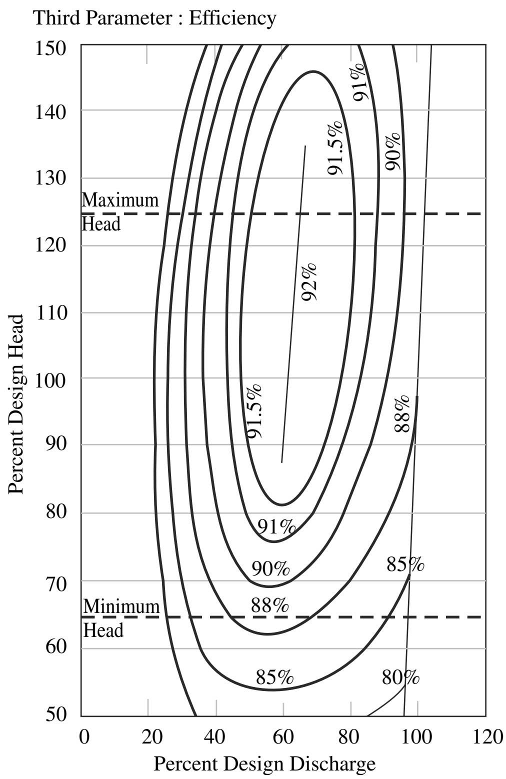

Fig. 3.19 *Efficiency hill chart of Kaplan turbine (Ref. 2.1)*

a large number of installations and thus are expected to represent a typical average performance.

Similar charts for semi-Kaplan and fixed blade propeller turbines are available in IS: 26800 of 1991.

3. *Efficiency Hill Chart*

Figure 3.19 is a typical generalized hill chart of a Kaplan turbine depicting iso-efficiency curves for a wide range of operating heads. The ordinate is the per cent design head and the abscissa is the per cent design discharge. Efficiency is the third parameter iso-efficiency curves form the hill chart. The percentage of gate opening, commonly used as fourth parameter (as in a Francis turbine, Fig. 2.42), is not relevant here due to the double regulation of the Kaplan turbine. Thanks to the double control, the range of maximum efficiency is spread over a wide range of head and discharge. It can be observed that the Kaplan turbine can operate in the range of 65% to 125% of design head and 80% to 100% of design discharge with good efficiencies. The peak efficiency of the depicted turbine is 92% and at the minimum head, it is in the range of 85%.

3.9 CAVITATION IN AXIAL-FLOW TURBINES

3.9.1 Locations Susceptible to Cavitation

The basic features of cavitation phenomenon, damages caused by cavitation and general aspects of cavitation in a Francis turbine were covered in Chapter 2, Section 2.5. Cavitation as related to draft tubes has been described in Sec. 2.8 of Chapter 2. In this section, specific aspects of cavitation in axial-flow turbines, viz. Kaplan turbine and its variants, are discussed briefly.

A Kaplan turbine is highly susceptible to cavitation in view of its high specific speed and low head usage. The critical Thoma number is high, being of the order of 1.0 for a specific speed of about 700. As such, cavitation protection is an important criterion in Kaplan-turbine design and installation. In Kaplan and bulb turbines, cavitation can occur at a number of different locations. Broadly, these locations are

1. Suction side of blade from centreline to trailing edge
2. Leading edge of blade on suction side

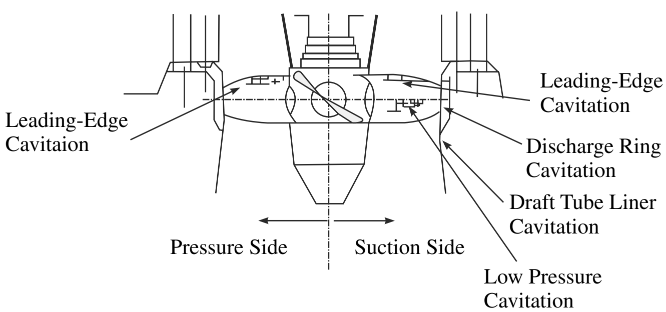

Fig. 3.20 *Locations of possible cavitation in a Kaplan turbine (Ref. 3.6)*

3. Leading edge of blade on pressure side
4. Trailing edge of blade on pressure side
5. Blade periphery on suction side
6. Hub
7. Discharge ring
8. Draft-tube liner

Cavitation can result in frosting or pitting in the above-mentioned locations. Figure 3.20 shows the locations of these cavitation possibilities.

3.9.2 Control and Repair of Cavitation Damage in Kaplan Turbines

The basic principles and procedures for avoiding, control and repair of cavitation in Kaplan turbines are the same as in the case of Francis turbines described in Sec. 2.5.3. Specific to the Kaplan/propeller turbines are the correlations of damage zone, possible causes and possible remedial measures; a few typical cases of which are indicated in Table 3.5. These are extracted from Ref. 3.6 that contains a detailed list of possible remedial measures for Kaplan turbines.

Table 3.5 *Possible remedial measures for some cavitation damages in a Kaplan turbine*

Location of Pitting	Possible cause	Possible Remedial Measure
Suction side of blade from centerline to trailing edge	- Setting is too high - Improper blade profile/ contour	Reduce maximum output Correct the profile/contour
Leading edge of blade on suction side	Operation at heads much higher than design	Modify leading-edge profile if head range is normal
Blade periphery on suction side of the peripheral edge of the blades	Leakage-ring cavitation	Add anti-cavitation fins
Hub	Operation for extended periods at low heads	Reduce low-load operation
Discharge ring	Leakage-ring cavitation	Add anti-cavitation fins
Anti-*cavitation fin* is a small finlike (riblike) device of height $h = 0.01\ D$, attached to the peripheral edge of the blades. It is a very simple device and easy to fix and has proven to be effective in specific types of cavitation Kaplan turbines. Details are of this fin are available in Ref. 3.6.		

3.10 DRAFT TUBE FOR AXIAL-FLOW TURBINES

The need for draft tubes in reaction turbines, types of draft tube and setting of draft height have been discussed in Chapter 2, Section 2.6. In this section, special needs of axial-flow turbines are highlighted.

Axial-flow turbines like Kaplan turbine and its variants, like bulb turbine and tubular turbines, are essentially high specific-speed units and hence are typically of high-speed, low-head and high-discharge installations. The kinetic energy leaving the runner is relatively very high, reaching to as high a value as 50% of the total input energy. The recovery of a substantial part of this kinetic energy is of great importance in Kaplan and other axial-flow units. This is particularly so in low-head and ultralow-head installations. Thus, for very low-heads where tubular and bulb turbines are most often used, the draft tube can be the singlemost important part of the plant. Further, in lowhead installation, the energy loss in the turbine system (including the draft tube) forms a significant part of the total available energy for conversion and as such, any reduction in energy loss goes a long way in improving the plant productivity. The friction and form loss in the draft tube is a significant part of total energy losses in the turbine system. In addition, the Kaplan turbine being highly susceptible to cavitation, the draft tube requires very careful planning and design. These factors highlight the need for very efficient draft tubes in Kaplan turbines in general and particularly in all very low-head installations.

The draft-tube is considered an integral part of the turbine and the turbine manufacturer does its design. The manufacturer furnishes the draft-tube shape and dimensions within the limitations of the specifications and it forms a part of the turbine-efficiency guarantee. Considerable design effort is spent at the manufacturer's design office in developing hydraulically efficient draft tubes. Until about 1960, the design of a draft tube was a trial-and-error process through model studies. Analytical methods using boundary-layer theory were first used around 1965 and highly efficient analytical procedures for studying the growth and separation aspects of boundary layers in draft tubes were available by 1985. However, currently, numerical simulations through advanced *CFD* form a significant part of analysis and very efficient design procedures are available. Model studies are adapted by the designers to the barest minimum necessary.

Generally, elbow-type draft tubes with varying cross sections are used in conjunction with Kaplan turbines. Figure 3.21 (a & b) shows a typical shape of the elbow-type draft tube for a Kaplan turbine. An isometric view of this draft tube is shown in Fig. 3.21(b).

It has been found that the best efficiency of the draft tube of this shape is achieved when height-to-inlet diameter ratio $h / D = 2.24$ and the length-to-diameter ratio

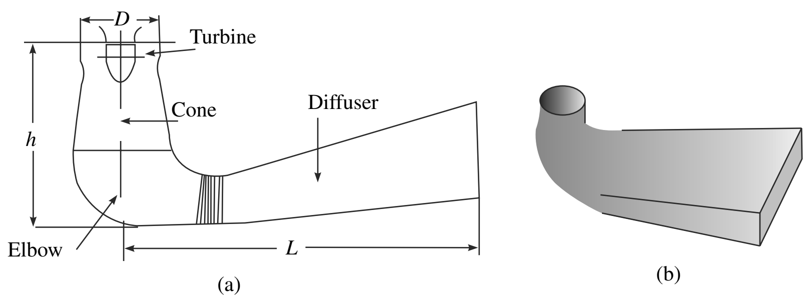

Fig. 3.21 *(a) Schematic sketch (b) Isometric view of elbow-type draft tube for Kaplan turbines*

L/D = 6.0. IS: 5496-1993 (First Revision),"*Guide for preliminary dimensioning and layout of elbow-type draft tubes for surface hydroelectric power stations*" can be consulted for preliminary dimensioning of elbow-type draft tube.

3.11 GENERAL FEATURES OF PROPELLER/KAPLAN TURBINE AND THEIR VARIANTS

3.11.1 Kaplan Turbines

1. Key Benefits

- Double-regulation system allows high performance over a large range of head and discharge
- Most preferred for economical operation for the range covering low-head, high-discharge applications: Best choice for head range from about 15 to 50 metres, and outputs from 20 MW up to 180 MW.
- Vertical-axis configuration of Kaplan turbines allows larger runner diameters (above 10 m) permitting to increase the unit power and thus minimising the number of units
- Kaplan turbines are certainly the machines that allow the most number of possible configurations. Hence, Kaplan and its variants cover the entire low-head, high-discharge range.
- Large Kaplan turbines are designed individually for each site at the highest possible efficiency at the available technology. The efficiencies are typically over 92%. They are very expensive to design, manufacture and install, but they operate for decades.

2. Concerns

- Low-head axial flow turbines are susceptible to cavitation.
- Kaplan turbines and bulb turbines are often installed on large rivers where fish ladders allow the fish to migrate upstream for spawning. Large amounts of fishes have to go through the turbines. Efficient designs to improve the survival of those species, while they are migrating, are needed and are being explored the world over.
- The hub of the Kaplan-type turbine contains oil under pressure. Accidental water pollution though oil spill would cause environmental pollution with many undesirable consequences.

3.11.2 Tubular/Bulb Turbines

1. Most efficient solution for low heads up to 30 metres and power output less than 20 MW. Bulb units have almost replaced Kaplan turbines for low-head situations.
2. For small and tiny hydro-development, tubular/bulb turbines and other variants are the preferred choice due to their efficiencies and reduced infrastructure cost.

3.11.3 Comparison of Kaplan and Francis Turbines

The following comparison of Francis and Kaplan turbines given in Table 3.6 is of interest in the overlap region of the specific speeds of these two kinds of turbines.

Table 3.6 *Comparison of Kaplan and Francis turbines in the overlap region of specific speeds*

	Francis Turbine	Kaplan Turbine
1	Has a scroll case and adjustable guide vanes	Has a scroll case and adjustable guide vanes
2	Runner blades encased in a shroud	Open, propeller-type runner blades
3	Inlet flow is radial and outlet flow is axial	Axial flow at inlet and at outlet
4	Fixed blades; number of blades relatively large (range of 9 to 25)	Variable pitch blades; number of blades relatively small (range of 3 to 8)
5	In each blade, one inlet edge is exposed to flow; only one inlet angle	The entire inlet edge from hub to tip is exposed to flow; inlet blade angle varies from hub to the tip
6	In each blade, one outlet edge is exposed to flow; only one outlet angle	The entire outlet edge from hub to tip is open to flow; outlet blade angle varies from hub to the tip
7	Has a draft tube	Has a draft tube; design of draft tube is critical from consideration of cavitation
8	Single-control governing of guide vanes only	Dual-control governing of guide vanes and blade pitch
9	Cavitation susceptibility	Greater cavitation susceptibility
10	Relatively higher frictional losses	Relatively smaller frictional losses
11	Overall efficiency is good	Overall efficiency is better than that of Francis turbine
12	Good part-load efficiency characteristics.	Very good part-load efficiency characteristics
13	Generally good choice for medium head and medium discharge	Generally good choice for low head and high discharge
14	Relatively large size and large civil infrastructure	Relatively compact and smaller civil works
15	Normal range of specific speed is in the range 40 to 450	Specific speed is in the range 260 to 900
16	Normal speed of the runner is in the range 90 to 1000 rpm	Normal speed of the runner is in the range 70 to 600 rpm
17	Normal range of head: 30 m–550 m	Normal range of head: 1.5 m–80 m
18	Runaway speed is in the range 160 to 200 % of design speed	Runaway speed is 200 to 300% of design speed

3.11.4 Some Major Kaplan-Turbine Installations

Given in Table 3.7 is a small list of major installations of Kaplan turbines in the world. This list gives an idea of maximum power and heads to which Kaplan turbines are being designed and adopted. Browsing through the Internet to get the images and other details of these projects is suggested as an exercise to obtain an idea of the scale of operations.

Table 3.7 *Some major Kaplan-turbine installations*

Project	Country	Installed power	Head (m)
Yacyreta	Argentina, 1988	1 × 155 MW	9.5 m
Nova Avanhandava	Brazil, 1979	1 × 112 MW	
Lajeado	Brazil, 2002	1 ×183 MW	34 m
Porto Primavera	Brazil, 2003	18 × 101 MW	10 m
Brilliant	Canada, 2004	1 × 120 MW	30 m
Grand Mere	Canada, 2004	3 × 77 MW	24 m
Ottari power Station, Unit 1	Japan, 1964	100 MW	51 m
Ottari power Station, Unit 2	Japan, 2003	89.5 MW	48 m
Akkatan	Sweden, 2008	2 × 75 MW	46 m

3.12 ILLUSTRATIVE EXAMPLES

NOTE

The peripheral velocity at the inlet to the tip of the blade, u_{t1}, which corresponds to diameter D_1 is designated as u_1 without the suffix *t*. Further note that in axial flow turbines $u_1 = u_2$. Peripheral velocity at any other location is designated with a letter symbol indicating the location. Thus, u_h represents the peripheral velocity at the hub and the peripheral velocity at mid-radius is u_m.

3.12.1 Basic Relations and Velocity Triangles

*EXAMPLE 3.1

A Kaplan turbine develops 15 MW of power at a head of 30 m. The diameter of the hub is 0.35 times the diameter of the runner. Assuming a speed ratio of 2.0, flow ratio of 0.65 and an overall efficiency of 90%, calculate the (a) diameter of the runner, (b) rotational speed, and (c) specific speed of the turbine in (i) kW, rpm and metre units, and (ii) in dimensionless form (using rps, W *and* m*).*

Solution

Given: P =15 MW, H = 30 m, D_h/D_1 = 0.35, K_u = 2.0, K_f = 0.65, η_0 = 0.90

(a) Speed ratio $K_u = \dfrac{u_1}{\sqrt{2gH}} = 2.0$

$$u_1 = 2.0 \times \sqrt{2 \times 9.81 \times 30} = 48.52 \text{ m/s}$$

Flow ratio $K_f = \dfrac{V_{f1}}{\sqrt{2gH}} = 0.65$

$$V_{f1} = 0.65 \times \sqrt{2 \times 9.81 \times 30} = 15.77 \text{ m/s}$$

Power $P = \eta_0\, \gamma QH$

$$15000 = 0.90 \times 9.79 \times Q \times 30$$

$$Q = 56.747 \text{ m}^3\text{/s}$$

Discharge $Q = \dfrac{\pi}{4}\left(D_1^2 - D_h^2\right) \times V_{f1}$

$$56.747 = \frac{\pi}{4}\left(D_1^2 - (0.35D_1)^2\right) \times 15.77 = 10.868\, D^2$$

$$D = 2.285 \text{ m and } D_h = 0.35 \times 2.285 = 0.80 \text{ m}$$

(b) Speed N is given by $u_1 = \dfrac{\pi D_1 N}{60}$

$$N = \frac{u_1 \times 60}{\pi D_1} = \frac{48.52 \times 60}{\pi \times 2.285} = 405.5 \text{ rpm}$$

(c) (i) Specific speed in *SI* units (kW, rpm and metre units):

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{405.5 \times \sqrt{15000}}{(30)^{5/4}} = 707.4$$

(ii) Specific speed in dimensionless form (*shape factor*) $S_p = 9.61 \times 10^{-4}\, N_s$

$$S_p = 9.61 \times 10^{-4} \times 707.4 = 0.68 \text{ revolutions.}$$

$$S_{pr} = 2\pi S_p = 2\pi \times 0.68 = 4.272 \text{ radians}$$

*EXAMPLE 3.2

A Kaplan turbine has a runner of 4.0 m diameter and its hub has a diameter of 1.2 m. The discharge through the turbine is 70 m³/s. The hydraulic and mechanical efficiencies can be assumed to be 0.9 and 0.93 respectively. Assuming the absence of whirl at the outlet and the discharge is free at the outlet, estimate the (a) net available head on the turbine and (b) power developed. If the speed ratio is 2.0, estimate the specific speed of the turbine.

Solution

Given: $D_1 = 4.0$ m, $D_h = 1.2$ m, $Q = 70$ m³/s, $\eta_m = 0.93$, $\eta_h = 0.90$, $K_u = 2.0$

Discharge $Q = \dfrac{\pi}{4}\left(D_1^2 - D_h^2\right) \times V_{f1}$

$$70.0 = \frac{\pi}{4}\left((4.0)^2 - (1.2)^2\right) \times V_{f1}$$

Velocity of flow $V_{f1} = V_{f2} = 6.121$ m/s

Since at the outlet $V_2 = V_{f1} = 6.121$ m/s, by energy equation,

$$\text{Head extracted } H_e = H - \frac{V_2^2}{2g} = \eta_h H$$

$$(H - 0.9\,H) = \frac{V_2^2}{2g}$$

$$(0.1\,H) = \frac{(6.121)^2}{2 \times 9.81} \text{ giving } H = 19.1 \text{ m}$$

$$\text{Power } P = (\eta_m\, \eta_h)\, \gamma QH$$

$$= (0.93 \times 0.90) \times 9.79 \times 70 \times 1.91 = 10955 \text{ kW}$$

$$\text{Speed ratio } K_u = \frac{u_1}{\sqrt{2gH}} = 2.0$$

$$u_1 = 2.0 \times \sqrt{2 \times 9.81 \times 19.1} = 38.72 \text{ m/s}$$

$$N = \frac{u_1 \times 60}{\pi D_1} = \frac{38.72 \times 60}{\pi \times 4} = 184.9 \text{ rpm}$$

$$\text{Specific speed } N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{184.9 \times \sqrt{10955}}{(19.1)^{5/4}} = 485$$

*EXAMPLE 3.3

A Kaplan turbine develops 23 MW of power at a head of 25 m with a rotational speed of 150 rpm. The outer diameter of the blades is 4.5 m and the hub diameter is 2.0 m. The overall efficiency of the turbine is 92% and the hydraulic efficiency is 95%. Calculate the (a) discharge, and (b) the following angles at the tip of the blades: (i) inlet guide vane angle, (iii) the inlet blade angle, and (iv) outlet blade angle.

Solution

Given: $P = 23$ MW, $H = 25$ m, $D_1 = 4.5$ m, $D_h = 2.0$ m, $\eta_0 = 0.92$, $\eta_h = 0.95$, $N = 150$ rpm

$$\text{Power } P = \eta_0\, \gamma QH$$

$$23000 = 0.92 \times 9.79 \times Q \times 25$$

$$Q = 102.1 \text{ m}^3\text{/s}$$

$$\text{Discharge } Q = \frac{\pi}{4}\left(D_1^2 - D_h^2\right) \times V_{f1}$$

$$102.1 = \frac{\pi}{4}\left((4.5)^2 - (2.0)^2\right) \times V_{f1}$$

Velocity of flow $V_{f1} = \dfrac{102.1}{12.76} = 8.0$ m/s

Consider a section at the tip of the blade.

NOTE

In this and in other problems that deal with the tip location of the blades only, the suffix t to denotes the tip location is not provided to avoid clutter in the symbols.

$$V_{f1} = V_{f2} = 8.0 \text{ m/s}$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 4.5 \times 150}{60} = 35.34 \text{ m/s}$$

Hydraulic efficiency $\eta_h = \dfrac{V_{u1} u_1}{gH}$

$$0.95 = \frac{V_{u1} \times 35.34}{9.81 \times 25} \text{ giving } V_{u1} = 6.592 \text{ m/s}$$

Whirl velocity at inlet at the tip of the blade $V_{u1} = 6.592$ m/s

Since $V_{u1} < u_1$, the inlet velocity triangle is an acute-angled triangle, as in Fig 3.22.

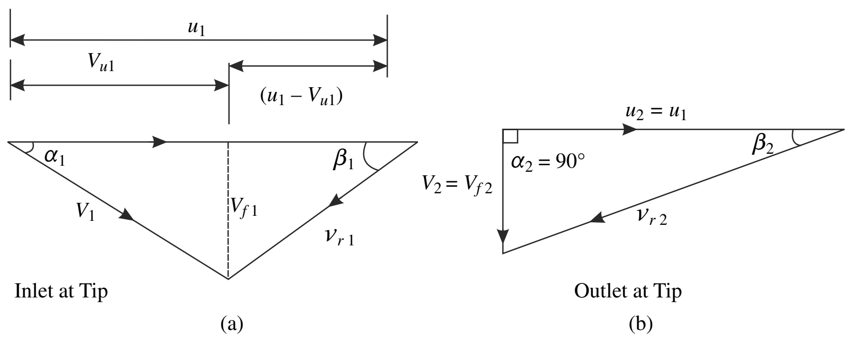

Fig. 3.22 *Velocity triangle: (a) Inlet (b) Outlet, Example 3.3*

From the inlet velocity triangle, $\dfrac{V_{f1}}{V_{u1}} = \dfrac{8.00}{6.592} = 1.2136$

$$\alpha_1 = 50.51°$$

$$\text{Further, } \tan \beta_1 = \frac{V_{f1}}{(u_1 - V_{u1})} = \frac{8.0}{(35.34 - 6.592)} = 0.2783$$

$$\beta_1 = 15.55°$$

Considering the outlet velocity triangle at the tip of the blade,

$$u_2 = u_1 = 35.34 \text{ m/s}$$

$$V_{f2} = V_{f1} = 8.0 \text{ m/s}$$

$$\tan \beta_2 = \frac{V_{f2}}{u_2} = \frac{8.0}{35.34} = 0.226$$

$$\beta_2 = 12.76°$$

*EXAMPLE 3.4

A Kaplan turbine develops 14.0 MW of shaft power under a net head of 8.0 m. The speed of rotation of the runner is 75 rpm. The speed ratio and flow ratio are 2.0 and 0.6 respectively. The ratio of hub diameter to runner diameter is 0.35. Calculate the overall efficiency of the turbine.

Solution

Given: P =14 MW, H = 8 m, D_h/D_1 = 0.35, K_u = 2.0, K_f = 0.6, N = 75 rpm

$$\text{Speed ratio } K_u = \frac{u_1}{\sqrt{2gH}} = 2.0$$

$$u_1 = 2.0 \times \sqrt{2 \times 9.81 \times 8.0} = 25.06 \text{ m/s}$$

Since $u_1 = \dfrac{\pi D_1 N}{60}$, the diameter of the runner

$$D_1 = \frac{60 \times u_1}{\pi N} = \frac{60 \times 25.06}{\pi \times 75} = 6.38 \text{ m}$$

The diameter of the hub $D_h = 0.35 \times 6.38 = 2.333$ m

$$\text{Area of flow} = A = \frac{\pi}{4}\left(D_1^2 - D_h^2\right)$$

$$= \frac{\pi}{4}\left((6.38)^2 - (2.33)^2\right) = 28.05 \text{ m}^2$$

$$\text{Flow ratio } K_f = \frac{V_{f1}}{\sqrt{2gH}} = 0.60$$

$$V_{f1} = 0.60 \times \sqrt{2 \times 9.81 \times 8.0} = 7.517 \text{ m/s}$$

$$\text{Discharge } Q = A \times V_{f1} = 28.05 \times 7.517 = 210.87 \text{ m}^3/\text{s}$$

$$\text{Power } P = \eta_0 \gamma QH$$

$$14000 = \eta_0 \times 9.79 \times 210.87 \times 8$$

$$\eta_0 = \frac{1400}{16515} = 0.8477 = 84.77\%$$

*EXAMPLE 3.5

A Kaplan turbine has a runner-diameter-to-hub-diameter ratio of 3.0. The speed ratio is 1.61. If this turbine produces 6.5 MW of power at a head of 15 m under a speed of 150 rpm, calculate (a) the specific speed, (b) discharge, and (c) flow ratio. Assume overall efficiency as 92%.

Solution

Given: $P = 6.5$ MW, $H = 15$ m, $D_h/D_1 = 1/3$, $K_u = 1.61$, $\eta_0 = 0.92$, $N = 150$ rpm

(a) Specific speed $$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{150 \times \sqrt{6500}}{(15)^{5/4}} = 410$$

(b) Speed ratio $$K_u = \frac{u_1}{\sqrt{2gH}} = 1.61$$

$$u_1 = 1.61 \times \sqrt{2 \times 9.81 \times 15.0} = 27.62 \text{ m/s}$$

Speed N is given by $$u_1 = \frac{\pi D_1 N}{60}$$

$$D_1 = \frac{u_1 \times 60}{\pi N} = \frac{27.62 \times 60}{\pi \times 150} = 3.52 \text{ m}$$

Diameter of hub $= D_h = D_1/3 = 3.52/3 = 1.172$ m

Power $P = \eta_0\, \gamma Q H$

$$6500 = 0.92 \times 9.79 \times Q \times 15$$

$$Q = 48.11 \text{ m}^3\text{/s}$$

Discharge $$Q = \frac{\pi}{4}\left(D_1^2 - D_h^2\right) \times V_{f1}$$

(c) $$48.11 = \frac{\pi}{4}\left((3.52)^2 - (1.17)^2\right) \times V_{f1}$$

Velocity of flow $$V_{f1} = \frac{48.11}{8.656} = 5.56 \text{ m/s}$$

Flow ratio $$K_f = \frac{V_{f1}}{\sqrt{2gH}} = \frac{5.56}{\sqrt{2 \times 9.81 \times 15}} = 0.324$$

*EXAMPLE 3.6

A Kaplan turbine is to be designed to develop 7,350 kW of power. The net available head is 5.5 m. Assume the speed ratio as 2.2, flow ratio as 0.68 and the overall efficiency as 90%. The diameter of the hub is one-third the diameter of the runner. Calculate the diameter of the runner. What is the outlet blade angle of the tip?

Solution

Given: $P = 7350$ kW, $H = 5.5$ m, $K_u = 2.2$, $K_f = 0.68$, $\eta_0 = 0.90$

The tip of the blade is considered. Suffix 1 refers to the inlet and the suffix 2 refers to the outlet.

$$V_{f1} = 0.68\ \sqrt{2 \times 9.81 \times 5.5} = 7.06 \text{ m/s} = \text{Constant}$$

Tangential velocity of the blade at the tip

$$u_1 = 2.2 \times \sqrt{2 \times 9.81 \times 5.5} = 22.8 \text{ m/s} = u_2$$

Power $P = \eta_0\ \gamma QH.$

$$\text{Hence } Q = \frac{P}{\eta_0 \gamma H} = \frac{7350}{0.90 \times 9.79 \times 5.5} = 151.67 \text{ m}^3\text{/s}$$

$$Q = \frac{\pi}{4}\left(D_1^2 - D_h^2\right) \times V_{f1} = \frac{\pi}{4}\left(D_1^2 - \left(\frac{D_1}{3}\right)^2\right) \times 7.06 = 151.67$$

$$4.9288\ D_1^2 = 151.67$$

$$D_1 = 5.5 \text{ m}$$

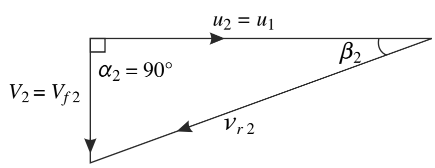

Fig. 3.23 *Outlet velocity triangle, Example 3.6*

From the outlet velocity triangle, $\tan \beta_2 = \dfrac{V_{f2}}{u_2} = \dfrac{V_{f1}}{u_1} = \dfrac{7.06}{22.85} = 0.309$

$$\beta_2 = 17.17°$$

**EXAMPLE 3.7

A Kaplan turbine runner has an outer diameter of 4.5 m and hub diameter of 2.0 m. It develops 20 MW of power while running at 140 rpm. The head is known to be 20 m. Assuming overall efficiency of 85%, find (a) the discharge through the turbine, (b) the angle between the absolute inlet velocity and the peripheral velocity at the tip, and (c) the inlet blade angle at midradius of the blades. [Assume hydraulic efficiency = 0.94].

Solution

Given: P =20 MW, H = 20 m, D_1 = 4.5 m, D_h = 2.0 m, N = 140 rpm, η_0 = 0.85

(a) Power $P = \eta_0\ \gamma QH$

$$20000 = 0.85 \times 9.79 \times Q \times 20$$

$$Q = 120.17 \text{ m}^3\text{/s}$$

$$\text{Discharge } Q = \frac{\pi}{4}\left(D_1^2 - D_h^2\right)\ V_{f1}$$

(b) $$120.17 = \frac{\pi}{4}\left((4.5)^2 - (2.0)^2\right) \times V_{f1}$$

$$\text{Velocity of flow } V_{f1} = \frac{120.17}{12.763} = 9.416 \text{ m/s} = \text{constant}$$

At the tip of the blades:

$$u_{1t} = \frac{\pi D_1 N}{60} = \frac{\pi \times 4.5 \times 140}{60} = 32.99 \text{ m/s}$$

$$\text{Hydraulic efficiency } \eta_h = \frac{\left(V_{u1} u_1\right)_{tip}}{gH}$$

$$0.94 = \frac{V_{u1t} \times 32.99}{9.81 \times 20} \text{ giving } V_{u1t} = 5.591 \text{ m/s}$$

Whirl velocity at inlet at the tip of the blade $V_{u1t} = 5.591$ m/s

Since $V_{u1t} < u_{1t}$, the inlet velocity triangle is an acute-angled triangle, as in Fig. 3.24.

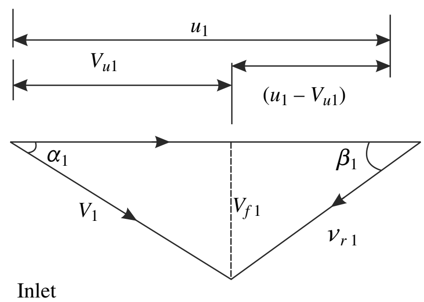

Fig. 3.24 *Inlet velocity triangle at any radius of the blade. (Consider appropriate suffix, t = tip and m = midradius, for the parameters), Example 3.7*

From the inlet velocity triangle, $\tan \alpha_{1t} = \left(\frac{V_{f1}}{V_{u1}}\right)_{tip} = \frac{9.416}{5.591} = 1.684$

Guide vane angle $\alpha_{1t} = 59.3°$

(c) *At midradius:*

Midradius = Radius of the midpoint of blades

$$= r_m = \frac{\left(D_1 + D_h\right)}{2 \times 2} = \frac{(4.5 + 2.0)}{4} = 1.625 \text{ m}$$

$$\text{Tangential velocity } u_{1m} = \frac{2\pi r_m N}{60} = \frac{2 \times \pi \times 1.625 \times 140}{60} = 23.823 \text{ m/s}$$

Since the whirl velocity varies inversely with the radius,

$$\text{Whirl velocity at } r_m = V_{u1m} = \frac{(V_{u1})_{tip} . (r)_{tip}}{r_m} = \frac{5.592}{1.625} \times \left(\frac{4.5}{2}\right) = 7.743 \text{ m/s}$$

From the velocity triangle of inlet at midradius which is an acute angled-triangle (Fig. 3.24),

$$\tan \alpha_{1m} = \frac{V_{f1m}}{V_{u1m}} = \frac{9.416}{7.743} = 1.216$$

Guide vane angle α_{1m} = 50.57°

$$\tan \beta_{1m} = \frac{(V_{f1})_m}{(u_{1m} - V_{u1m})}.$$

But velocity of flow is constant all along the radius at the inlet side of the blade, that is $(V_{f1})_{\text{tip}} = (V_{f1})_m = (V_{f1})_{hub}$ = 9.416 m/s. Hence,

$$\tan \beta_{1m} = \frac{(V_{f1})_m}{(u_{1m} - V_{u1m})} = \frac{9.416}{(23.823 - 7.743)} = 0.586$$

Inlet blade angle at midradius β_{1m} = 30.35°

**EXAMPLE 3.8

A Kaplan turbine works under a head of 22 m and has a rotative speed of 150 rpm. The diameters of the runner and the hub are 4.5 m and 1.2 m respectively. The flow ratio is 0.43. The inlet blade angle at the extreme edge of the runner is 16.5°. If the turbine discharges axially at outlet, determine the (a) hydraulic efficiency and (b) inlet guide vane angle and the outlet blade angle at the extreme edge of the runner.

Solution

Given: H = 22 m, N = 150 rpm, D_1= 4.5 m, D_h = 1.2 m, K_f= 0.43, $\beta_{1\text{tip}}$= 16.5°

(a) At the extreme edge of the runner

$$u_1 = u_2 = \frac{\pi D_1 N}{60} = \frac{\pi \times 4.5 \times 150}{60} = 35.34 \text{ m/s}$$

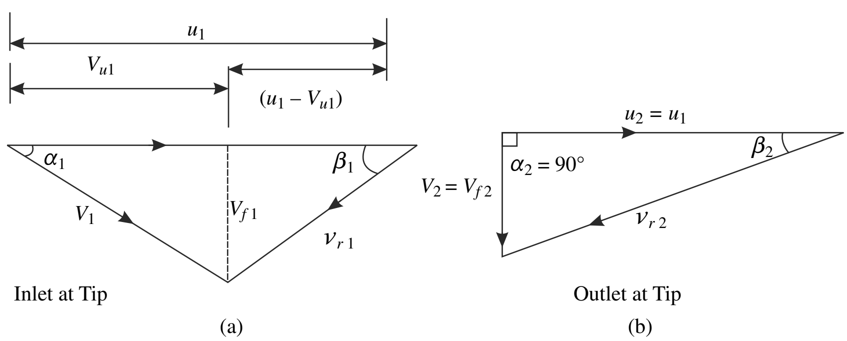

Fig. 3.25 *(a) Inlet velocity triangle (b) Outlet velocity triangle, Example 3.8*

From inlet velocity triangle, $V_{f1} = 0.43\sqrt{2 \times 9.81 \times 22} = 8.934$ m/s

$$\tan \beta_1 = \frac{V_{f1}}{(u_1 - V_{u1})}$$

$$\tan 16.5° = \frac{8.934}{(35.34 - V_{u1})} = 0.2962$$

$$V_{u1} = \frac{1.5337}{0.2962} = 5.178 \text{ m/s}$$

Hydraulic efficiency $\eta_h = \dfrac{V_{u1}u_1}{gH} = \dfrac{5.178 \times 35.34}{9.81 \times 22} = 0.848$

(b) $\tan \alpha_1 = \dfrac{V_{f1}}{V_{u1}} = \dfrac{8.934}{5.178} = 1.7253$

$\tan \alpha_1 = 59.90°$

From outlet velocity triangle, $\tan \beta_2 = \dfrac{V_{f2}}{u_2} = \dfrac{8.934}{35.34} = 0.2528$

$\beta_2 = 14.19°$

*EXAMPLE 3.9

In a propeller turbine, at a particular radial location, the inlet blade angle is 90°. The outlet flow is purely axial. Show that the outlet blade angle at that location is equal to the inlet guide-vane angle.

Solution

At the given radial location, from the inlet velocity triangle (Fig.3.26),

$$\tan \alpha_1 = \frac{V_{f1}}{u_1}$$

Since $V_{f1} = V_{f2}$ and $u_1 = u_2$, from the velocity triangle at the outlet

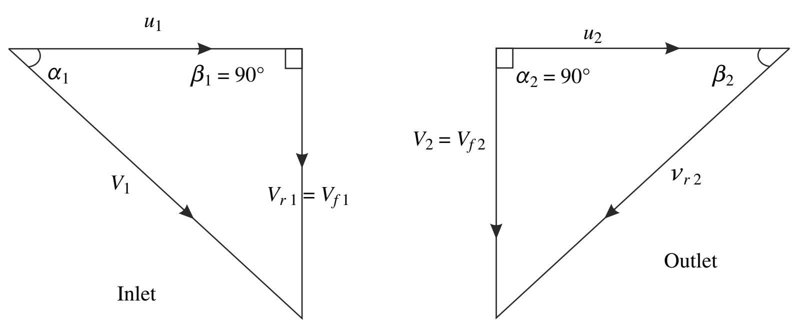

Fig. 3.26 *Inlet and outlet velocity triangles, Example 3.9*

$$\tan \beta_2 = \frac{V_{f2}}{u_2} = \frac{V_{f1}}{u_1} = \tan \alpha_1$$

Hence $\alpha_1 = \beta_2$

**EXAMPLE 3.10

A Kaplan turbine delivers 10 MW under a head of 25 m. The hub and tip diameters are 1.25 m and 3.0 m respectively. The hydraulic and overall efficiencies are 0.92 and 0.87 respectively. If at the tip, both the inlet and outlet velocity triangles are right-angled triangles, determine (a) the speed, and (b) inlet guide-vane angle and outlet blade angle at the tip of the blades.

Solution

Given: P =10 MW, H = 25 m, D_1 = 3.0 m, D_h = 1.25 m, η_0 = 0.87, η_h = 0.92

(a) Power $P = \eta_0 \gamma QH$

Hence $$Q = \frac{P}{\eta_0 \gamma H} = \frac{10000}{0.87 \times 9.79 \times 25} = 46.96 \text{ m}^3/\text{s}$$

$$Q = \frac{\pi}{4}\left(D_1^2 - D_h^2\right)V_{f1} = \frac{\pi}{4}\left((3)^2 - (1.25)^2\right) \times V_{f1} = 46.96$$

$$V_{f1} = \frac{46.96}{5.841} = 8.039 \text{ m/s}$$

Hydraulic efficiency $$\eta_h = \frac{V_{u1}u_1}{gH} = \frac{u_1^2}{gH} \text{ m/s}$$

$$\eta_h = 0.92 = \frac{u_1^2}{9.81 \times 25}$$

$$u_1 = \sqrt{225.63} = 15.021 \text{ m/s}$$

Speed of rotation $$= N = \frac{60 \times u_1}{\pi D_1} = \frac{60 \times 15.021}{\pi \times 3.0} = 95.63 \text{ rpm}$$

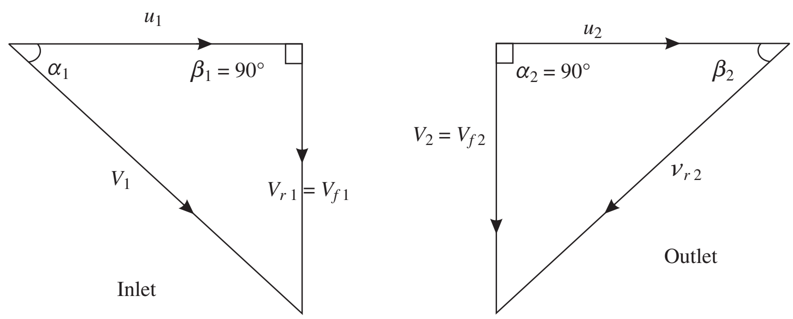

Fig. 3.27 *Inlet and outlet velocity triangles, Example 3.10*

(b) From inlet velocity triangle, $\tan \alpha_1 = \dfrac{V_{f1}}{u_1} = \dfrac{8.039}{15.021} = 0.535$

Guide-vane angle at the tip of blades $\alpha_1 = 28.16°$

From outlet velocity triangle, $\tan \beta_2 = \dfrac{V_{f2}}{u_2} = \dfrac{V_{f1}}{u_1} = \tan \alpha_1$

Hence, outlet blade angle at the tip of blades $\beta_2 = \alpha_1 = 28.16°$

**EXAMPLE 3.11

For a propeller turbine, show that the degree of reaction for maximum hydraulic efficiency is given by

$$R = 1 - \frac{V_1 \cos \alpha_{r1}}{2u_{r1}}$$

where V_1 = Absolute velocity at the entry periphery, u_{r1} = Peripheral velocity at radial distance r, α_{r1} = Angle between the absolute velocity and the peripheral velocity at any radius r.

Solution

Degree of reaction $R = 1 - \dfrac{\frac{1}{2g}\left(V_1^2 - V_2^2\right)}{H}$

Let suffixes 1 and 2 denote the inlet and exit regions. Consider the inlet region at a radial distance *r* from the axis of a propeller turbine. The exit flow is taken to be to be purely axial, $V_2 = V_f = V_{f1}$ = Constant.

Let V_1 = Absolute velocity at the inlet and α_{r1} = Angle between the absolute velocity and the peripheral velocity.

The hydraulic efficiency = $\eta_h = \dfrac{V_{u1r} u_{r1}}{gH}$

For maximum hydraulic efficiency of $\eta_h = 1.0$

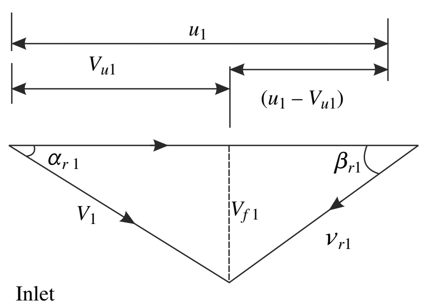

Fig. 3.28 *Inlet velocity triangle, Example 3.11*

Hence, $V_{u1r}u_{r1} = gH$

Thus degree of reaction for maximum hydraulic efficiency of a propeller turbine is

$$R = 1 - \frac{\frac{1}{2g}\left(V_1^2 - V_2^2\right)}{H} = 1 - \frac{1}{2}\frac{\left(V_1^2 - V_f^2\right)}{V_{ur1}u_{r1}}$$

From inlet velocity triangle, $V_{ur1} = V_1 \cos \alpha_{r1}$ and $V_{f1} = V_1 \sin \alpha_{r1}$

Hence, $R = 1 - \frac{1}{2}\frac{V_1^2\left(1 - \sin^2 \alpha_{r1}\right)}{V_1 u_{r1} \cos\alpha_{r1}} = \left[1 - \frac{V_1}{2u_{r1}}\cos\alpha_{r1}\right]$

NOTE

The degree of reaction R of a propeller turbine is highest at the hub where u_{r1} is least. Further, R decreases from the hub towards the tip.

***EXAMPLE 3.12

A Kaplan turbine develops 40 MW of power under a head of 35 m and a speed of 175 rpm. The tip diameter of the runner is 5.0 m and the hub diameter is 2.5 m. The overall efficiency and hydraulic efficiency of the turbine are 89% and 92% respectively. Determine the (a) speed ratio and flow ratio, and (b) the inlet guide vane angle, inlet and outlet blade angles at (i) hub, (ii) tip, and (iii) midradius.

Solution

Given: $P = 40$ MW, $H = 35$ m, $D_1 = 5.0$ m, $D_h = 2.5$ m, $\eta_0 = 0.89$, $\eta_h = 0.92$, $N = 175$ rpm

$$\text{Power } P = \eta_0 \gamma Q H$$

$$4000 = 0.89 \times 9.79 \times Q \times 35$$

$$Q = 131.17 \text{ m}^3/\text{s}$$

$$\text{Discharge } Q = \frac{\pi}{4}\left(D_1^2 - D_h^2\right) \times V_{f1}$$

$$131.17 = \frac{\pi}{4}\left((5.0)^2 - (2.5)^2\right) \times V_{f1}$$

$$\text{Velocity of flow } V_{f1} = \frac{131.17}{14.726} = 8.907 \text{ m/s}$$

$$u_1 = u_2 = \frac{\pi D_1 N}{60} = \frac{\pi \times 5 \times 175}{60} = 45.81 \text{ m/s}$$

$$\text{(a) Flow ratio } K_f = \frac{V_{f1}}{\sqrt{2gH}} = \frac{8.907}{\sqrt{2 \times 9.81 \times 35}} = 0.34$$

Table 3.8 *Calculation sheet of Example 3.12*

Term	Tip			Midradius			Hub		
Diameter	D_1		5.0 m	D_{1m}		3.75 m	D_{1h}		2.5 m
Dia. ratio	D_1/D_1		1.0	D_{1m}/D_1		0.75	D_1/D_{1h}		0.50
u_1	u_1	45.81 m/s	45.81 m/s	u_{1m}	45.81 × 0.75	34.350 m/s	u_{1h}	45.81× 0.5	22.905 m/s
V_{f1}	V_{f1}	8.907 m/s	8.907 m/s	V_{f1m}	8.907 m/s	8.907 m/s	V_{f1h}	8.907 m/s	8.907 m/s
V_{u1}	V_{u1}	6.895 m/s	6.895 m/s	V_{u1m}	6.895/ 0.75	9.193 m/s	V_{u1h}	6.895/0.5	13.79 m/s
$(u_1 - V_{u1})$	$(u_1 - V_{u1})$	(45.81– 6.895)	38.915 m/s	$(u_{1m} - V_{u1m})$	34.35 – 9.193	25.165	$(u_{1h} - V_{u1h})$	22.905 – 3.79	9.115 m/s
$\tan \beta_1$	$\tan \beta_1$	8.907/38.915	0.2289	$\tan \beta_{1m}$	8.907/25.165	0.354	$\tan \beta_{1h}$	8.907/9.115	0.977
β_1	β_1		12.89°	β_{1m}		19.49°	β_{1h}		44.33°
$\tan \beta_2$	$\tan \beta_2$	8.907/45.81	0.1944	$\tan \beta_{2m}$	8.907/34.35	0.259	$\tan \beta_{2h}$	8.907/22.905	0.3889
β_2	β_2		11.00°	β_{2m}		14.53°	β_{2h}		21.25°
$\tan \alpha_1$	$\tan \alpha_1$	8.907/6.895	1.292	$\tan \alpha_{1m}$	8.907/9.193	0.969	$\tan \alpha_{1h}$	8.907/13.79	0.646
α_1	α_1		52.26°	α_{1m}		44.09°	α_{1h}		32.86°

$$\text{Speed ratio } K_u = \frac{u_1}{\sqrt{2gH}} = \frac{45.81}{\sqrt{2\times 9.81\times 35}} = 1.748$$

$$\text{Hydraulic efficiency } \eta_h = \frac{V_{u1}u_1}{gH} = \frac{V_{u1}\times 45.81}{9.81\times 35} = 0.92$$

$$V_{u1} = \frac{0.92\times 9.81\times 35}{45.81} = 6.895 \text{ m/s}$$

(b) The calculations of blade angles and guide-vane angle at three radii (tip, midradius and hub) are shown in Table 3.8.

Note the blade angles (β_1 and β_2) and the angle between the absolute velocity and the tangential direction (inlet guide-vane angle) $\alpha_{1,}$ are calculated by inlet and outlet velocity triangles at an appropriate radius as below:

$$\tan\beta_1 = \frac{V_{f1}}{(u_1 - V_{u1})};\ \tan\beta_2 = \frac{V_{f2}}{u_2} = \frac{V_{f1}}{u_1};\ \text{ and }\ \tan\alpha_1 = \frac{V_{f1}}{V_{u1}}$$

**EXAMPLE 3.13

A Kaplan turbine has a net head of 25 m and runs at 150 rpm. The outer and hub diameters are 4.5 m and 2.0 m respectively. The guide-vane angle is 51° and the outlet blade angle at the tip of the blade is 13°. The exit flow is known to be purely radial. Estimate (a) the power, (b) specific speed, and (c) inlet blade angle at the tip of the blade.

Solution

Given: $H = 25$ m, $D_1 = 4.5$ m, $D_h = 2.0$ m, $\alpha_1 = 51°$, $\beta_2 = 13°$, $N = 150$ rpm.

From inlet velocity triangle at the tip, $\tan\alpha_1 = \frac{V_{f1}}{V_{u1}}$

From outlet velocity triangle at the tip, $\tan\beta_1 = \frac{V_{f2}}{V_{u2}}$

$$\frac{\tan\alpha_1}{\tan\beta_2} = \frac{u_2}{V_{u1}} = \frac{u_1}{V_{u1}} \qquad (\because u_1 = u_2)$$

$$u_1 = V_{u1} = \frac{\tan\alpha_1}{\tan\beta_2} = V_{u1}\frac{\tan 51°}{\tan 13°} = 5.349\ V_{u1}$$

But $$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi\times 4.5\times 150}{60} = 35.42 \text{ m/s}$$

$$V_{u1} = \frac{u_1}{5.349} = \frac{35.343}{5.349} = 6.607 \text{ m/s}$$

Hydraulic efficiency

$$\eta_h = \frac{V_{u1}u_1}{gH} = \frac{6.607 \times 35.342}{9.81 \times 25} = 0.952$$

$$\eta_0 = \eta_m \times \eta_h = 0.91 \times 0.952 = 0.866$$

$$V_{f1} = V_{u1} \tan \alpha_1 = 6.607 \times \tan 51° = 8.159 \text{ m/s}$$

$$A = \frac{\pi}{4}\left(D_1^2 - D_2^2\right) = \frac{\pi}{4}\left((4.5)^2 - (2.0)^2\right) = 12.763$$

$$Q = AV_{f1} = 12.763 \times 8.159 = 104.13 \text{ m}^3\text{/s}$$

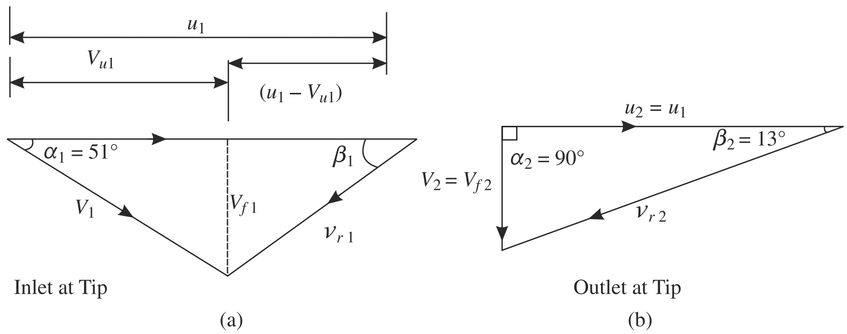

Fig. 3.29 *Schematic velocity triangles, Example 3.13*

(a) Power $P = \eta_0 \gamma QH$

$$P = 0.866 \times 9.79 \times 104.13 \times 25 = 22.071 \text{ kW}$$

(b) Specific speed $N_s = \dfrac{N\sqrt{P}}{H^{5/4}} = \dfrac{150 \times \sqrt{22071}}{(25)^{5/4}} = 398.6$

(c) $\tan \beta_1 = \dfrac{V_{f1}}{(u_1 - V_{u1})} = \dfrac{8.159}{(35.343 - 6.607)} = 0.2839$

$$\beta_1 = 15.85°$$

**EXAMPLE 3.14

A Kaplan turbine is to be designed to develop 7.5 MW, the net available head being 6.0 m. The other relevant data are the following Speed ratio = 2.05, Flow ratio = 0.66, Overall efficiency = 90%, Hydraulic efficiency = 95% and diameter of hub is 1/3 the diameter of the runner. Calculate (a) the diameter of the runner, (b) specific speed (c) inlet guide vane angle at the tip of the blades, and (d) inlet and outlet blade angles at the midradius.

Solution

Given: $P = 7.5$ MW, $H = 6$ m, $D_h/D_1 = 1/3$, $K_u = 2.05$, $K_f = 0.66$, $\eta_0 = 0.90$,

$\eta_h = 0.95$

(a) Speed ratio $K_u = \dfrac{u_1}{\sqrt{2gH}} = 2.05$

$$u_1 = 2.05 \times \sqrt{2 \times 9.81 \times 6} = 22.242 \text{ m/s}$$

Flow ratio $K_f = \dfrac{V_{f1}}{\sqrt{2gH}} = 0.06$

$$V_{f1} = 0.66 \times \sqrt{2 \times 9.81 \times 6} = 7.161 \text{ m/s} = \text{Constant}$$

Power $P = \eta_0 \, \gamma QH$

$$7500 = 0.90 \times 9.79 \times Q \times 6$$

$$Q = 141.87 \text{ m}^3\text{/s}$$

Discharge $Q = \dfrac{\pi}{4}\left(D_1^2 - D_h^2\right) \times V_{f1}$

$$141.87 = \frac{\pi}{4}\left(D_1^2 - \left(\frac{D}{3}\right)^2\right) \times 7.161$$

$$\left(D_1^2 - \frac{D_1^2}{9}\right) = 25.22$$

$$D_1^2 = 28.38,\ D_1 = 5.322 \text{ m and } D_h = 0.333 \times 5.322 = 1.776 \text{ m}$$

At the tip inlet: Tangential velocity of the blade = u_{1t}

Whirl velocity = V_{u1t}

Operational speed N is given by $u_{1t} = \dfrac{\pi D_1 N}{60}$

$$N = \frac{u_{1t} \times 60}{\pi D_1} = \frac{22.242 \times 60}{\pi \times 5.322} = 79.8 \text{ rpm}$$

(b) Specific speed $N_s = \dfrac{N\sqrt{P}}{H^{5/4}} = \dfrac{79.8 \times \sqrt{7500}}{(6)^{5/4}} = 736$

(c) Hydraulic efficiency $\eta_h = \dfrac{V_{u1t}u_{1t}}{gH} = \dfrac{V_{u1t} \times 22.242}{9.81 \times 6} = 0.95$

$$V_{u1t} = \frac{0.95 \times 9.81 \times 6}{22.242} = 2.514 \text{ m/s}$$

At the tip of blades:
From inlet velocity triangle,

$$\tan \alpha_{1t} = \frac{V_{f1}}{V_{u1t}} = \frac{7.161}{2.514} = 2.848$$

Inlet guide-vane angle at the tip of blades $\alpha_{1t} = 70.655°$

(d) *At midradius:*

$$V_{f1} = 7.161 \text{ m/s} = \text{constant} = V_{f1m}$$

$$D_m = \left(\frac{D_1 + D_h}{2}\right) = \left(\frac{5.32 + 1.776}{2}\right) = 3.548 \text{ m}$$

$$\text{Whirl velocity } V_{u1m} = \frac{V_{u1}D_1}{D_m} = \frac{2.514 \times 5.32}{3.548} = 3.77 \text{ m/s}$$

$$u_{1m} = u_{1t}\frac{D_m}{D_1} = 22.242 \times \frac{3.548}{5.32} = 14.834 \text{ m/s}$$

$$u_{1m} = u_{2m} = 14.834 \text{ m/s and } V_{f1m} = V_{f2m} = 7.161 \text{ m/s}$$

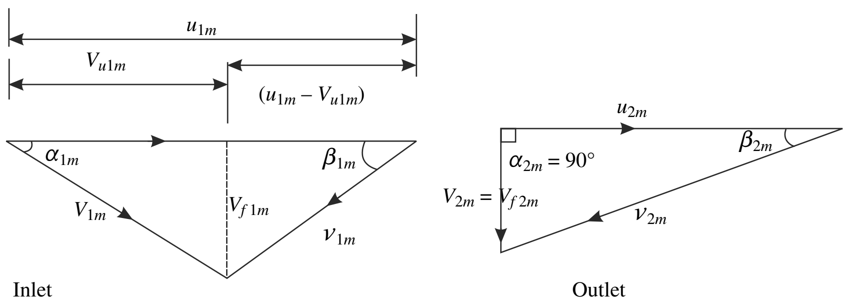

Fig. 3.30 *Inlet and outlet velocity triangles at midradius, Example 3.14*

From inlet velocity triangle at midradius:

$$\tan \beta_{1m} = \frac{V_{f1m}}{u_{1m} - V_{u1m}} = \frac{7.161}{14.834 - 3.77} = 0.6472$$

$$\beta_{1m} = 32.91°$$

From outlet velocity triangle at midradius

$$\tan \beta_{2m} = \frac{V_{f2m}}{u_{2m}} = \frac{7.161}{14.834} = 0.4827$$

$$\beta_{2m} = 32.97°$$

***EXAMPLE 3.15

A Kaplan turbine is designed with a shape factor (nondimensional power specific speed) of 3.0 (rad), a runner-tip diameter of 4.4 m and a hub diameter of 2.0 m. The net head is 20 m and the runner speed is 150 rpm. The hydraulic efficiency is 90% and the mechanical efficiency is found to be 99%. Determine (a) the shaft power output, (b) inlet and exit blade angles at the hub, and (c) the angle between the absolute velocity and the tangential velocity of the blade at the hub. The absolute flow at runner exit can be taken to be axial.

Solution

Given: S_{pr} = 3.0 rad., D_1 = 4.4 m, D_h =2.0 m, H = 20 m, N = 150 rpm, η_h = 0.90, η_m = 0.99

Shape factor S_{pr} = 3.0 (radians)

$$\text{Specific speed } N_s = \frac{S_{pr}}{2\pi \times 9.6 \times 10^{-4}} = \frac{3.0}{2\pi \times 9.6 \times 10^{-4}} = 497.4 \text{ (in SI units)}$$

$$\text{Specific speed } N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

$$P = \left[\frac{N_s H^{5/4}}{N}\right]^2 = \left[\frac{497.4 \times (20)^{5/4}}{150}\right]^2 = 19{,}667 \text{ kW}$$

$$\text{Power } P = \eta_0\, \gamma Q H$$

$$19667 = (0.90 \times 0.99) \times 9.79 \times Q \times 20$$

$$Q = 112.73 \text{ m}^3/\text{s}$$

$$\text{Area } A = \frac{\pi}{4}\left((4.4)^2 - (2.0)^2\right) = 12.064$$

$$V_f = \frac{Q}{A} = \frac{112.73}{12.064} = 9.344 \text{ m/s}$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 4.4 \times 150}{60} = 34.558 \text{ m/s}$$

$$\text{Hydraulic efficiency } \eta_h = \frac{V_{u1} u_1}{gH} = \frac{V_{u1} \times 34.558}{9.81 \times 20} = 0.90$$

$$V_{u1} = \frac{0.90 \times 9.81 \times 20}{34.558} = 5.110 \text{ m/s}$$

At the hub

$$V_{fh} = V_{f1} = 9.344 \text{ m/s} = \text{Constant}$$

$$u_{1h} = u_{2h} = \frac{u_1 D_h}{D_1} = \frac{34.558 \times 2.0}{4.4} = 15.708 \text{ m/s}$$

$$V_{u1h} = \frac{V_{u1} D_1}{D_h} = \frac{5.110 \times 4.4}{2.0} = 11.242 \text{ m/s}$$

$$(u_{1h} - V_{u1h}) = (15.708 - 11.242) = 4.406 \text{ m/s}$$

At hub inlet

$$\tan \beta_{1h} = \frac{V_{f1h}}{(u_{1h} - V_{u1h})} = \frac{9.344}{4.406} = 2.121$$

Hence, inlet blade angle at the hub, $\beta_{1h} = 64.75°$

$$\tan \alpha_{1h} = \frac{V_{f1h}}{V_{u1h}} = \frac{9.344}{11.242} = 0.831$$

$\alpha_{1h} = 39.73°$

At hub exit

$$\tan \beta_{2h} = \frac{V_{f2}}{u_{2h}} = \frac{V_{f1}}{u_{1h}} = \frac{9.344}{15.708} = 0.595$$

Exit blade angle at the hub, $\beta_{2h} = 30.75°$

**EXAMPLE 3.16

A Kaplan turbine with a four-blade runner has airfoil section for blades. The diameter of the blade circle is 3.25 m and the blade length in the radial direction is 0.75 m. The runner speed is 120 rpm. The airfoil section has a chord length of 2.20 m and the inclination of the chord to the direction of motion is 25°. For a head of 8.25 m and a velocity of flow of 5.0 m/s, calculate the power produced and the theoretical efficiency of the turbine. Take C_L = 0.80 and C_D = 0.04.

Solution

Diameter of runner, $D_1 = 3.25$ m

Diameter of the hub, $D_h = 3.25 - (2 \times 0.75) = 1.75$ m

Diameter at middle of the blade $D_m = \frac{(D_1 + D_h)}{2} = \frac{(3.25 \times 1.75)}{2} = 2.5$ m

Peripheral velocity at midradius = $u_{1m} = \frac{\pi D_m N}{60} = \frac{\pi \times 2.5 \times 120}{60} = 15.708$ m/s

Flow velocity at midradius = $V_{f1} = V_{f1m} = 5.0$ m/s

Consider the velocity triangle on the inlet side at the midradius:

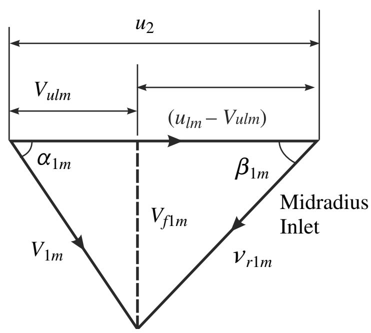

Fig. 3.31 *Inlet velocity triangle at midradius, Example 3.16*

From the inlet velocity triangle,

$$\frac{V_{f1m}}{v_{r1m}} = \sin \beta_{1m} = \sin 25°$$

where v_{r1m} = Relative inlet velocity at the midradius.

$$v_{r1m} = \frac{V_{f1m}}{\sin\beta_{1m}} = \frac{5.0}{\sin 25°} = 11.831 \text{ m/s}$$

Area of blade = Chord length × Blade length = 2.20 × 0.75 = 1.65 m^2.

Power, by airfoil theory, is given by Eq. 3.7 as

$$P = F_u\, u\, n = \rho\, A\frac{v_{rm}^2}{2}\left(C_L \sin\beta_{1m} - C_D\cos\beta_{1m}\right)un$$

where n = Number of blades

$$P = \frac{998}{1000}\times 1.65\times\frac{(11.831)^2}{2}\times\left((0.8\times\sin 25°)-(0.04\times\cos 25°)\right)\times 15.708\times 4$$

$$= 115.25 \times 0.3018 \times 62.832 = 2185 \text{ kW}$$

$$\text{Discharge } Q = \frac{\pi}{4}\left(D_1^2 - D_h^2\right)\times V_{f1}$$

$$= \frac{\pi}{4}\left((3.25)^2 - (1.75)^2\right)\times 5.0 = 29.45 \text{ m}^3/\text{s}$$

Let η_{th} = theoretical efficiency (which is same as hydraulic efficiency)

$$P = \eta_0\,\gamma QH$$

$$2185 = \eta_{th} \times 9.79 \times 29.45 \times 8.25$$

$$\eta_{th} = \frac{2185}{2379} = 0.918 = 91.8\%$$

**EXAMPLE 3.17

Show that the specific speed of a Kaplan turbine can be expressed as

$$N_s = 493.7\, K_u\sqrt{(1-n^2)\eta_0 K_f}$$

where K_u = Speed ratio, K_f = Flow ratio, n = Ratio of hub diameter to tip diameter of runner and η_0 = Overall efficiency. Take the unit weight of water γ = 9.79 kN/m^3.

Solution

$$u_1 = K_u\sqrt{2gH} = \frac{\pi D_1 N}{60}$$

$$N = \frac{60\times K_u\sqrt{2gH}}{\pi D_1}$$

$$\text{Discharge } Q = \frac{\pi}{4}D_1^2\left(1-\left(\frac{D_h}{D_1}\right)^2\right)u_1 = \frac{\pi}{4}D_1^2(1-n^2)K_f\sqrt{2gH}$$

$$\text{where } n = \left(\frac{D_h}{D_1}\right) = \text{Ratio of hub diameter to tip diameter}$$

$$\text{Power } P = \eta_0 \gamma Q H$$

$$= \eta_0 \gamma \frac{\pi}{4} D_1^2 (1-n^2) K_u \sqrt{2g} \; H^{3/2}$$

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{\left(60 \times K_u \sqrt{2g}\right)\left[\left(\eta_0 \gamma \frac{\pi}{4}\right) D_1^2 \left(1-n^2\right) K_f \sqrt{2g}\right]^{1/2} H^{5/4}}{H^{5/4}}$$

$$= \left[60 \times (2g)^{3/4} \left(\frac{1}{4\pi}\right)^{1/2}\right]\left[\eta_0^{1/2} \gamma^{1/2}\right]\left(K_u ((1-n^2) K_f)^{1/2}\right)$$

$$= 493.7 K_u \left[\eta_0 \left(1-n^2\right) K_f\right]^{1/2}$$

Hence, $$N_s = 493.7 K_u \sqrt{\eta_0 \left(1-n^2\right) K_f}$$

3.12.2 Draft Tube and Cavitation

**EXAMPLE 3.18

A Kaplan turbine develops 4.0 MW of power under a net head of 6.0 m. A vacuum gauge connected to the entrance of the draft tube indicates a reading of 5.0 m suction pressure head. The elbow-type draft tube of the turbine has an inlet diameter of 3.2 m and an outlet area of 25 m^2. The draft-tube efficiency is known to be 75% and the draft head is 2.0 m. Calculate (a) the overall efficiency of the turbine, (b) the head lost in the draft tube, and (c) power thrown in to the tailrace.

Solution

Given: $P = 4.0$ MW, $H = 6.0$ m

Refer to Fig. 3.32.

$$A_1 = \frac{\pi}{4} \times (3.2)^2 = 8.04 \text{ m}^2 \qquad A_2 = 25 \text{ m}^2$$

$$\text{Velocity at 1} = V_1$$

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{8.04}{25} V_1 = 0.3217 V_1$$

$$\frac{V_2^2}{2g} = \frac{(0.3217)^2}{2g} V_1^2 = 0.1035 \frac{V_1^2}{2g}$$

$$\frac{V_1^2}{2g} - \frac{V_2^2}{2g} = (1-0.1035)\frac{V_1^2}{2g} = 0.8965 \frac{V_1^2}{2g}$$

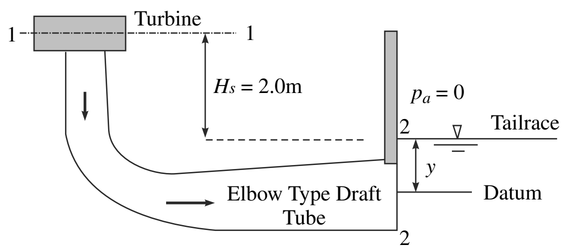

Fig. 3.32 *Draft tube set-up, Example 3.18*

(a) Efficiency of the draft tube = $\eta_d = 1 - \dfrac{H_L}{\left(\dfrac{V_1^2}{2g} - \dfrac{V_2^2}{2g}\right)} = 0.75$

$$\eta_d = 1 - \frac{H_L}{0.8965\left(\dfrac{V_1^2}{2g}\right)} = 0.75$$

Loss of head $H_L = (0.25 \times 0.8965)\dfrac{V_1^2}{2g} = 0.224\dfrac{V_1^2}{2g}$

Considering the Bernoulli equation between sections 1 and 2,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + H_L$$

$$-5.0 + \frac{V_1^2}{2g} + (2.0 + y) = (0 + y) + \frac{V_2^2}{2g} + 0 + 0.224\frac{V_1^2}{2g}$$

$$-3.0 + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) = 0.224\frac{V_1^2}{2g}$$

$$\frac{V_1^2}{2g} = \frac{3.0}{0.6725} = 4.46 \text{ m}$$

$$V_1 = 9.355 \text{ m/s}$$

Discharge $Q = A_1V_1 = 8.04 \times 9.355 = 75.21 \text{ m}^3/\text{s}$

Power $P = \eta_0\, \gamma Q\, H$

$4000 = \eta_0 \times 9.79 \times 75.21 \times 6.0$

Overall efficiency $\eta_0 = \dfrac{4000}{4418} = 0.905 = 90.5\%$

(b) Head loss in the draft tube = $H_L = 0.224\dfrac{V_1^2}{2g} = 0.224 \times 4.46 = 1.00 \text{ m}$

(c) Power wasted to the tailrace =

$$P_{LT} = \gamma Q \frac{V_2^2}{2g} = 9.79 \times 75.21 \times (0.1035 \times 4.46) = 340 \text{ kW}$$

**EXAMPLE 3.19

A Kaplan turbine with a runner diameter of 1.25 m is to have a discharge of 12 m³/s. The outlet area of the draft tube is 6 times the inlet area. The pressure head at the entry to the draft tube is limited to 1.5 m of suction pressure head. Estimate the maximum height above the tailwater level, at which the turbine can be set. The efficiency of the draft tube can be taken as 0.70 m.

Solution

Given: $Q = 12.0$ m³/s, $D_1 = 1.25$ m

Efficiency of the draft tube = $\eta_d = 1 - \dfrac{H_L}{\left(\dfrac{V_1^2}{2g} - \dfrac{V_2^2}{2g}\right)} = 0.70$

Loss of head $H_L = 0.30 \left(\dfrac{V_1^2}{2g} - \dfrac{V_2^2}{2g}\right)$

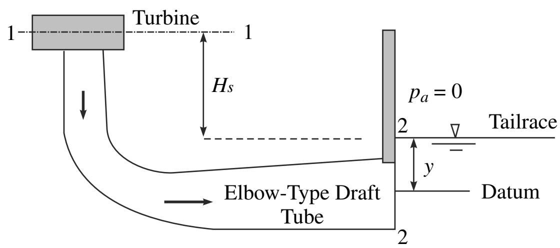

Fig. 3.33 *Draft tube set-up, Example 3.19*

Considering the Bernoulli equation between sections 1 and 2

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + H_L$$

$$-1.5 + \frac{V_1^2}{2g} + (H_s + y) = (0 + y) + \frac{V_2^2}{2g} + 0 + H_L$$

$$\text{H}_s = 1.5 + H_L - \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) = 1.5 - 0.7\left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) \qquad \text{(i)}$$

Discharge $Q = 12$ m³/s.

$$A_1 = \frac{\pi}{4} \times (1.25)^2 = 1.227$$

$$V_1 = \frac{Q}{A} = \frac{12}{1.227} = 9.778 \text{ m/s} \qquad V_2 = \frac{V_1}{6} = 1.63 \text{ m/s}$$

$$\frac{V_1^2}{2g} = 4.873 \text{ m} \qquad \frac{V_2^2}{2g} = 0.135 \text{ m}$$

$$\frac{V_1^2}{2g} - \frac{V_2^2}{2g} = (4.873 - 0.135) = 4.738 \text{ m}$$

Substituting in Eq. (i), maximum draft head = H_{smax} = 1.5 – (0.7 × 4.738) = 3.317 m

**EXAMPLE 3.20

At a site on a river, the power potential is 225 MW under a net head of 15 m. It is desired to operate the turbines at a speed of 60 rpm. Two choices of turbines, as follows, are available:

(a) Francis turbine having specific speed not exceeding 300 (b) Kaplan turbine having specific speed not exceeding 600

(i) How many units of each type, all of same size, would be required?

(ii) The atmospheric pressure is 9.9 m (abs) and vapour pressure head is 0.30 m. Calculate the maximum draft head in each case. The critical Thoma number can be taken as 0.36 for Francis turbine and 0.73 m for Kaplan turbine.

Solution

Power potential P_T = 225,000 kW

Specific speed $N_s = \dfrac{N\sqrt{P}}{H^{5/4}}$

For Francis turbine, power of one turbine unit

$$= P_f = \left[\frac{N_s H^{5/4}}{N}\right]^2 = \left[\frac{300 \times (15)^{5/4}}{60}\right]^2 = 21785 \text{ kW}$$

Number of Francis turbines required = $\dfrac{225000}{21785}$ = 11 units

For Kaplan turbine, power of one turbine unit =

$$P_k = \left[\frac{N_s H^{5/4}}{N}\right]^2 = \left[\frac{600 \times (15)^{5/4}}{60}\right]^2 = 87143 \text{ kW}$$

Number of Kaplan turbines required = $\dfrac{225000}{87143}$ = 3 units

Critical Thoma number $\sigma_c = \dfrac{H_a - H_v - H_s}{H}$

Hence, draft head $H_s = H_a - H_v - \sigma_c H$

For Francis turbines $H_s = 9.9 - 0.30 - (0.36 \times 15) = 4.2$ m

For Kaplan turbines $H_s = 9.9 - 0.30 - (0.73 \times 15) = -1.35$ m

(The Kaplan turbine is to be installed below the tailwater level by an extent of not less than 1.35 m.)

**EXAMPLE 3.21

An axial-flow turbine having a specific speed of 700 is installed 1.5 m below the tailwater. The atmospheric pressure at the site is 95 kN/m^2 and the vapour pressure is 3.4 kN/m^2. What is the maximum permissible head under which the turbine could operate? Take critical cavitation coefficient as 0.90 and the unit weight of water $\gamma = 9.79$ kN/m^3.

Solution

Atmospheric pressure head $H_a = \dfrac{p_a}{\gamma} = \dfrac{95}{9.79} = 9.70$ m (absolute)

Vapour pressure head $H_v = \dfrac{p_v}{\gamma} = \dfrac{3.4}{9.79} = 0.347$ m (absolute)

Critical Thoma number $\sigma_c = \dfrac{H_a - H_v - H_s}{H}$

Hence net head $H = \dfrac{H_a - H_v - H_s}{\sigma_c}$

Maximum net head $H = \dfrac{9.70 - 0.347 - (-1.5)}{0.90} = 12.06$ m

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Review Questions

3.1 With the help of neat sketches, describe the components of a Kaplan turbine.

3.2 What are the main variants of the Kaplan turbine that are in common use? Describe their range of operation.

3.3 Describe briefly: (a) Tubular turbine, (b) Bulb turbine, and (c) Straflow turbine

3.4 Describe the main differences between a regular propeller turbine and a Kaplan turbine.

3.5 Describe the variation of the following velocities in a Kaplan turbine runner:
(a) Velocity of flow
(b) Tangential velocity
(c) Whirl velocity

3.6 Draw a neat sketch of velocity triangles at inlet and outlet of a propeller turbine blade at the following locations:
(a) Tip of blade
(b) Hub of the blade
(c) Midradius location

3.7 Compare and contrast the salient features of a Francis turbine and a Kaplan turbine.

3.8 Compare and contrast the salient features of a common propeller turbine and a Kaplan turbine.

3.9 Describe how the airfoil theory can be used to theoretically estimate the power and hydraulic efficiency of a propeller turbine.

3.10 Write a brief note on the working proportion of a Kaplan turbine. Indicate the functional relationships of the variation of various key parameters and approximate range of variation of the values of the key parameters.

3.11 Compare the efficiency characteristics of a propeller turbine and a Kaplan turbine.

3.12 Write a brief note on different types of axial-flow reaction turbines that are essentially the variants of a Kaplan turbine. In addition to the material given in Chapter 3, consult the Internet for your answer.

3.13 Describe briefly the salient features of (a) Bulb turbine, (b) Tubular turbine (c) Deriaz turbine, and (d) Straflow turbine.

3.14 Write a note on the governing mechanism of a Kaplan turbine.

3.15 Write a brief note on the cavitation problem in axial-flow turbines.
3.16 Describe the importance of draft tubes in Kaplan turbines.

Problems

Basic Relations and Velocity Triangles

P3.1 *A Kaplan turbine has a speed ratio of 2.0 and specific speed of 450. Determine the diameter of the runner in order that it will develop 10,100 kW under a head of 20 m. [***Ans:*** D_1 = 4.0 m]

P3.2 *A Kaplan turbine develops 6 MW of power at a net head of 5 m. Taking an overall efficiency of 90%, speed ratio of 2.0, flow ratio of 0.6 and ratio of hub diameter to outer diameter of the runner as 0.40, determine the diameter and specific speed of the turbine. Also calculate the shape factor in revolutions as well as in radians.
[***Ans:*** D_1 = 5.893 m, N_s = 665, S_p = 0.639 revolutions, S_{pr} = 4.01 radians]

P3.3 * A Kaplan turbine develops 1.3 MW of power under a head of 20 m. Considering the following ratios; speed ratio = 2.1, flow ratio = 0.62, hub-diameter-to-blade-diameter ratio = 0.34 and overall efficiency = 89%, find the diameter of the runner, speed of rotation and specific speed of the runner.
[***Ans:*** D_1 = 0.935 m, N = 849.7 rpm, N_s = 724.4]

P3.4 *A Kaplan turbine develops 1.0 MW of power under a head of 4.5 m. Considering the following ratios; speed ratio = 1.8, flow ratio = 0.5, hub-diameter-to-blade-diameter ratio = 0.35 and overall efficiency = 90%, find the diameter, speed of rotation and specific speed of the runner.
[***Ans:*** D_1 = 2.79 m, N = 115.8 rpm, N_s = 558.7]

P3.5 **A Kaplan turbine has the following features:
Outer diameter of the runner = 4.5 m
Hub diameter of the runner = 2.0 m
Guide-vane angle = 59°
Inlet-blade angle at the tip = 19°

The hydraulic and overall efficiencies are 0.92 and 0.90 respectively. If the discharge through the turbine is 120 m^3/s, determine (a) Speed and (b) power developed by the turbine. The discharge at the outlet can be taken as purely axial. [***Ans:*** N = 139.8 rpm, P = 21.8 MW]

P3.6 ** A propeller turbine has midblade diameter of 3.75 m and the length of the blades along the radius is 1.25 m. The guide-vane angle is 50° and the inlet-blade angle at the tip of the blades is 14°. The overall and hydraulic efficiencies are 86% and 90% respectively. If the speed of the runner is 150 rpm, estimate the (a) discharge through the turbine (b) power developed, and (c) specific speed of the turbine.
[***Ans:*** Q = 119.25 m^3/s, P = 30.335 MW, N_s = 368.8]

P3.7 **A Kaplan turbine working under a head of 25 m develops 16,000 kW of power. The outer diameter of the runner is 4 m and hub diameter is 2 m. The guide-vane angle is 35°. The hydraulic and overall efficiency are 90%

and 85% respectively. If the velocity of whirl is zero at outlet, determine (a) the inlet and outlet blade angles at the midradius, and (b) specific speed of the turbine. [***Ans:*** β_{1m} = 48.18°, β_{2m} = 23.30°, N_s = 273]

P3.8 ** A Kaplan turbine develops 20 MW of power at a head of 35 m with a rotational speed of 420 rpm. The outer diameter of the blades is 2.5 m and the hub diameter is 0.85 m. If the overall efficiency of the turbine is 85% and the hydraulic efficiency is 88%, calculate the (a) discharge, (b) inlet guide-vane angle, and (c) the inlet blade angle at the tip of the blades.
[***Ans:*** Q = 68.67 m^3/s, α_1 = 70.85°, β_1 = 17.73°]

P3.9 ** A Kaplan turbine develops 11.5 MW of shaft power under a net head of 10.0 m. The speed of rotation of the runner is 96.3 rpm. The speed ratio and flow ratio are 1.8 and 0.55 respectively. The radial length of blades is 1.5 m. Calculate the overall efficiency of the turbine.
[***Ans:*** η_0 = 92.5%]

P3.10* ** A Kaplan turbine operating under a net head of 20 m develops 15 MW with an overall efficiency of 90%. The diameter of the runner is 4.25 m and the hub diameter is 2.0 m. The specific speed of the turbine is 380. Taking the hydraulic efficiency as 93%, calculate the inlet and exit angles of the runner blades at the hub and at the tip of blades. The flow leaving the runner can be assumed to be completely axial.
[***Ans:*** β_{1h} = 86.57°, β_{2h} = 29.29°, β_{1t} = 18.56°, β_{2t} = 14.79°]

P3.11 ** A large Kaplan turbine installed in a hydroelectric station has the following relevant data:

Speed = 72 rpm; Runner diameter = 8.0 m; Hub-diameter-to-tip-diameter ratio = 0.4

During a test run it was found that for a head of 10.0 m and a discharge of 300 m^3/s, the measured power output was 24.7 MW. The mechanical efficiency of the set is 97% and the efficiency of the generator is 96%. Determine the hydraulic efficiency and the angles of the inlet and outlet velocity triangles at the tip and hub of the runner. The outlet flow may be assumed to be axial. [***Ans:*** α_{1t} = 67.54°, β_{1t} = 14.63°, β_{2t} = 13.26°, α_{1h} = 44.06°, β_{1h} = 56.40°, β_{2h} = 30.5°]

P3.12 ** A propeller turbine is designed to develop 22 MW of power under a head of 21 m with a speed of 150 rpm. The outer and hub diameters have been arrived at as 4.5 m and 2.0 m respectively. The hydraulic efficiency is taken as 95% and the overall efficiency is 90%. Determine the runner blade angles at the hub and at the outer periphery. Assume the turbine discharges without whirl. [***Ans:*** β_{1t} = 17.36°, β_{2t} = 14.77°, β_{1h} = 70.77°, β_{2h} = 30.67°]

P3.13 ** The runner and hub diameters of a propeller turbine are 4.0 m and 2.0 m respectively. This turbine works under a net head of 22 m and develops a shaft power of 14 MW. The guide-vane angle is 40° and the hydraulic and overall efficiencies can be taken as 90% and 85% respectively. Calculate the speed of the runner and the inlet and outlet blade angles at the mid-radius of the runner. [***Ans:*** N = 95.9 rpm, β_{1m} = 64.99°, β_{2m} = 28.30°]

Application of Airfoil Theory

P3.14 ** An axial-flow propeller turbine has four blades. The blades have cross sections of an airfoil. The mean radius of the blade circle is 2.20 m and length in radial direction is 0.9 m. The chord length of airfoil is 2.30 m. The appropriate lift and drag coefficients can be taken as 0.8 and 0.05 respectively. The inclination of the chord to the direction of motion can be taken as 35°. The turbine works under a head of 19 m and revolves at 150 rpm. Assuming a velocity of flow of 6.0 m/s, calculate the power developed and the hydraulic efficiency of the turbine.

[***Ans:*** P = 13065 kW, η_{th} = 0.941 = 94.1%]

P3.15 ** A Kaplan turbine has a runner with four blades. The turbine works under a head of 5.0 m and has a speed of 75 rpm. The blades have an airfoil section of 2.6 m chord length. The chord makes an angle of 30° to the direction of motion. The blade length in the radial direction is 0.6 m and the mean radius of the blade circle is 1.6 m. The velocity of flow is 5.5 m/s. Calculate the power developed and the hydraulic efficiency of the turbine. Take appropriate lift and drag coefficients of the airfoil as 0.75 and 0.05 respectively.

[***Ans:*** P = 1570 kW, η_{th} = 96.7%]

Draft Tube and Cavitation

P3.16 ** A draft tube has an inlet area of 25 m^2 and an outlet area of 112.5 m^2. The inlet velocity is 8.0 m/s and the efficiency of the draft tube is 80%. The inlet is 0.9 m above the tailwater level. Calculate the (a) pressure at the inlet of the draft tube, (b) the power lost in the draft tube, and (c) power lost to the tailrace.

[***Ans:*** $\frac{p_1}{\gamma}$ = – 3.381 m, P_{dL} = 1214 kW, P_{LT} = 315 kW]

P3.17 ** A propeller turbine develops 7.0 MW of power under a net head of 25.0 m. A vacuum gauge connected to the entrance of the draft tube indicates a reading of 3.5 m suction pressure head. The elbow-type draft tube of the turbine has an inlet diameter of 3.0 m and an outlet area of 38 m^2. The draft-tube efficiency is known to be 80% and the draft head is 2.5 m. Calculate (a) the overall efficiency of the turbine, (b) the power lost in the draft tube, and (c) power lost to the tailrace.

[***Ans:*** η_d = 0.80, P_{LD} = 62.4 kW, P_W =11.21 kW]

P3.18 * A Kaplan turbine has a design power of 40 MW at a net head of 20 m and speed of 100 rpm. At the site, the atmospheric pressure head is 10.0 m of water and the relevant vapour pressure head is 0.33 m of water. The variation of critical Thoma number with specific speed for Kaplan turbines is given below:

Specific speed, N_s	400	600	800
Critical Thoma number, σ_c	0.43	0.73	1.50

Calculate the maximum draft head for the installation of the turbine at the site. Take safety margin as 0.30 m. [***Ans:*** H_s = 0.81 m]

Use of Specific Speed

P3.19 *In a hydroelectric project, the power potential is estimated to be a water discharge of 350 m^3/s and a net head of 20 m. Assuming a turbine efficiency of 90% and speed of 120 rpm, estimate the least number of turbines, all of the same size, that are needed if

(a) a Francis turbine of specific speed not exceeding 240 is selected

(b) a Kaplan turbine of specific speed not exceeding 520 is selected.

[***Ans:*** Francis = 9 units, Kaplan = 2 units]

Objective-type Questions

O3.1 *A Kaplan turbine is

(a) a high-head mixed flow turbine

(b) a low-head axial-flow turbine

(c) an outward-flow reaction turbine

(d) in inward-flow impulse turbine

O3.2 **Consider the following types of water turbines:

1. Bulb 2. Francis 3. Kaplan 4. Straflow

The correct sequence of order in which the median value of the normal head range is

(a) 2–3–1–4 (b) 3–4–1–2 (c) 2–1–4–3 (d) 1–3–2– 4

O3.3***What is the range of the speed ratio K_u for a Kaplan turbine for its most efficient operation?

(a) 0.10 to 0.30 (b) 0.43 to 0.65

(c) 0.85 to 1.20 (d) 1.40 to 2.25

O3.4 *Which of the following types of turbines are suitable for tidal plants?

1. Tubular turbine 2. Deriaz turbine
3. Bulb turbine 4. Francis turbine

Select the correct answer using the codes given below:

(a) 1 only (b) 1 and 3 (c) 2 and 4 (d) 4 only

O3.5***Which of the following four graphs correctly represents the relationship between head and specific speed for Kaplan and Francis turbines?

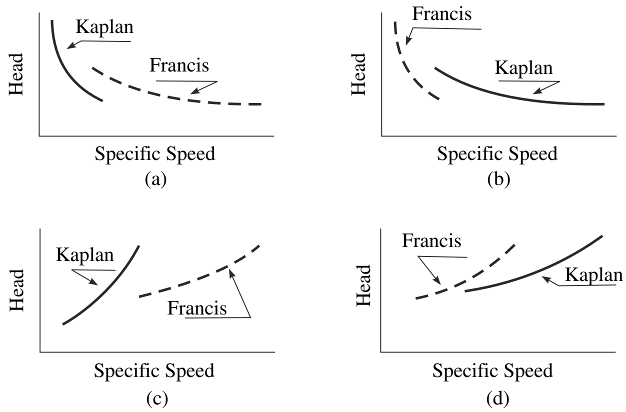


Fig. 3.34 *Objective Question 3.6*

(a) Fig. (*a*) (b) Fig. (*b*) (c) Fig. (*c*) (d) Fig. (*d*)

O3.6 ** Which of the following advantages is/are possessed by a Kaplan turbine over a Francis turbine?
(1) Low frictional losses
(2) Considerably high part-load efficiency
(3) More compact and smaller in size
The correct answer is
(a) Only 1 (b) 1 and 2 only (c) 2 and 3 only (d) 1, 2 and 3

O3.7 ** In a Kaplan turbine, the diameter of the runner is 3.0 m. The diameter of the hub is one-third the diameter of the runner. If the velocity of flow and the peripheral velocities at the inlet side of the blade at its tip are 5.0 m/s and 40 m/s respectively, the discharge through the runner is
(a) 235.6 m^3/s (b) 31.4 m^3/s (c) 125.7 m^3/s (d) 251.3 m^3/s

O3.8 * In a Kaplan turbine working under a head of 35 m, the speed ratio is 2.0. If the outer diameter of the runner is 2.0 m and the hub diameter is 0.6 m, the rotational speed is
(a) 500 rpm (b) 125 rpm (c) 150 rpm (d) 250 rpm

O3.9***A Kaplan turbine has a runner of 4.0 m diameter. The diameter of the hub is 1.6 m. If the velocity of flow and the swirl velocity at the inlet side of the blade at the hub are 6.0 m/s and 10.0 m/s respectively, the flow and swirl velocities at the inlet side of the tip are, respectively,
(a) 15.0 m/s and 10 m/s (b) 6 m/s and 25 m/s
(c) 6.0 m/s and 4.0 m/s (d) 2.5 m/s and 10 m/s

O3.10***A Kaplan turbine has a runner of 5.0 m diameter. The diameter of the hub is 2.0 m. If the peripheral velocity and the swirl velocity at the inlet side of the blade tip are 40.0 m/s and 5.0 m/s respectively, the peripheral velocity and swirl velocities at the inlet side of the midradius section are, respectively,
(a) 40.0 m/s and 7.1 m/s (b) 57.1m/s and 3.5 m/s
(c) 28.0 m/s and 5.0 m/s (d) 28.0 m/s and 7.1 m/s

O3.11 ** A bulb turbine has a normal range of net head of
(a) 1.5 m to 30. m (b) 20 m to 75 m
(c) 100 m to 1500 m (d) 8.0 m to 58 m

O3.12** In a propeller turbine, the swirl velocity on the inlet side of the blades
(a) is constant all along the radius
(b) varies directly with the radius
(c) varies inversely with the radius
(d) is maximum at the mid radius

O3.13 * In a Kaplan turbine, the flow velocity
(a) is constant all along the radius
(b) varies directly with the radius
(c) varies inversely with the radius
(d) is maximum at the mid radius

O3.14 ** In a propeller turbine, the relative inlet velocity is
(a) is constant all along the radius
(b) varies directly with the radius
(c) varies inversely with the radius
(d) none of the above

O3.15*** Consider the following statements:

The usual assumptions in one dimensional analysis of flow in a propeller turbine include:

1. Flow velocity is constant all along the radius in both inlet and outlet side of the blades.
2. In frictionless, ideal flow, the relative velocity is constant all along the radius.
3. The swirl velocity varies inversely with the radius.
4. The peripheral velocity varies directly with the radius.

The correct statements are

(a) 1, 2 and 3 (b) 2, 3 and 4 (c) 1, 3 and 4 (d) only 2

O3.16 ** In a propeller turbine, at any radius, the angle between the absolute inlet velocity and the peripheral velocity at that radius

(a) is largest at the inlet
(b) is largest at the hub
(c) is constant all along the radius
(d) is largest at the midradius

O3.17 * Which of the following is not a variant of the Kaplan turbine?

(a) Bulb turbine (b) Straflow turbine
(c) Tubular turbine (d) Turgo turbine

O3.18 ** The ranges of speed ratio and flow ratio of Kaplan turbines are respectively

(a) (0.35 to 0.75) and (1.30 to 2.25)
(b) (1.30 to 2.25) and (0.35 to 0.75)
(c) (0.60 to 0.90) and (1.0 to 1.30)
(d) (0.4 to 0.8) and (0.1 to 0.4)

O3.19 ** Runaway speeds of a propeller turbine expressed as percentage of the design speed *N* are in the range

(a) 1.1 *N* to 3.5 *N* (b) 2.0 *N* to 3.75 *N*
(c) 1.8 *N* to 1.9 *N* (d) 2.0 *N* to 3.0 *N*

O3.20 ** In a Kaplan turbine, the absolute flow angle α_1, viz. the angle between the peripheral velocity at any radial point and the absolute flow velocity at that point,

(a) increases from the tip of the blade towards the hub
(b) decreases from the tip of the blade towards the hub
(c) remains constant all along the radius
(d) is maximum at the midradius

O3.21 ** In a Kaplan turbine, the relative flow angle β_1, viz. the blade angle at inlet,

(a) increases from the tip of the blade towards the hub
(b) decreases from the tip of the blade towards the hub
(c) remains constant all along the radius
(d) is always greater than β_2

O3.22 ** In a Kaplan turbine, the blade angle at exit β_2

(a) increases from the tip of the blade towards the hub
(b) decreases from the tip of the blade towards the hub
(c) remains constant all along the radius
(d) is maximum at the midradius

FOUR

Impulse Turbines—Pelton Turbine, and Cross-flow and Banki Turbines

4.1 INTRODUCTION

Hydroturbines come in two basic types: reaction turbines and impulse turbines. Reaction turbines are described in detail in chapters 2 and 3. This chapter is devoted to the detailed description of impulse turbines. In contrast to reaction turbines, an impulse turbine has an interaction of free jet at atmospheric pressure, on a series of curved buckets mounted on a wheel. The pressure is atmospheric throughout the runner. The *Pelton turbine*, named after the inventor, is probably the most widely used kind of impulse turbine; so much so that the term Pelton turbine has become a generic name to imply impulse turbines.

The Pelton turbine was invented in 1889. After various improvements and applications of modern technologies in various phases of its manufacture and operation, this turbine has achieved a dominant position in the field of hydroturbines. A schematic layout of a Pelton turbine set-up is shown in Fig. 4.1. In a Pelton turbine, water from a high-head location is lead through a penstock to a powerhouse built

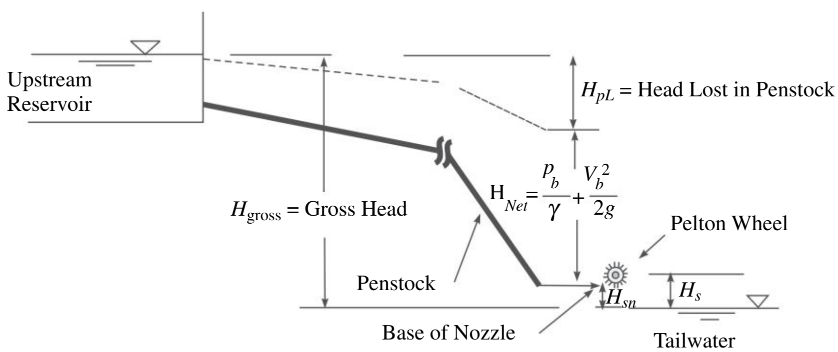


Fig. 4.1 *Layout and definition sketch*

near the tailwater stream. The potential energy of water in the upstream reservoir is converted into kinetic energy by fitting a well-designed nozzle to the end of the penstock. The nozzle produces a coherent, circular free jet of water that impinges on a series of suitably designed curved vanes (called *buckets*) fitted on the rim of a wheel. The arrangement, called a Pelton wheel, is mounted on a vertical or horizontal axis and is coupled directly to a generator. The impinging high-velocity, high-momentum jet suffers nearly 180° of deflection in the buckets and thus undergoes a change in the momentum. The resulting force acting tangentially to the rotating wheel imparts a torque on the shaft of the wheel and conveys power to the shaft. Controls for regulating the flow, governing mechanism to meet varying loads on the generator, emergency shut-off mechanisms, and safe disposal of spent water to the tailrace, all form parts of the Pelton-turbine system. The entire transfer of energy from the high-velocity water jet to the wheel shaft takes place under ambient atmospheric pressure conditions. Consequently, the draft tube that forms the integral part of reaction turbines, is conspicuously absent in the Pelton turbine set-up. Other components of reaction turbines not found in Pelton turbines are scroll case, stay vanes and wicket gates. Thus, the Pelton turbine set-up is relatively simpler with fewer components and consequently is easy to manage.

The Pelton turbine is ideal for high-head and low-discharge situations. The number of jets in a turbine is usually one. However, where additional power is needed and adequately large water supply is available; multiple jets ranging from one to six are used. As multijet units are expensive in view of additional components and control mechanisms, the choice between more units of single-jet turbine and smaller units of multi-jet units is based on economic considerations. Generally, vertical axis mountings are used when more than two jets are employed.

The regulation of speed is achieved through servomechanisms, which control the discharge in the nozzle. Towards this, a needle valve is used as a part of the jet nozzle. A spearlike device moves inside the nozzle and controls the area of nozzle opening and thus the discharge of the jet. A servomotor controls the motion of the spear needle.

The casing of the turbines does not have any role in the hydraulics of the flow in the turbine. The turbine is housed in a steel casing essentially to confine the spent water and to divert it to the tailrace. Further, the casing serves as a safety measure against accidents and prevents the splashing and related nuisance of spent water.

4.2 BASIC THEORETICAL ANALYSIS

4.2.1 Definitions and Working Ratios

1. Diameter of the Wheel

In a Pelton turbine, the diameter D that is used in equations and calculations is the diameter of the *pitch circle* (Fig. 4.2). Pitch circle is the centreline of the circle of buckets to which the jet axis is tangential.

2. Net Head

The *net* or *effective head* is the head available at the base of the nozzle (Fig. 4.1).

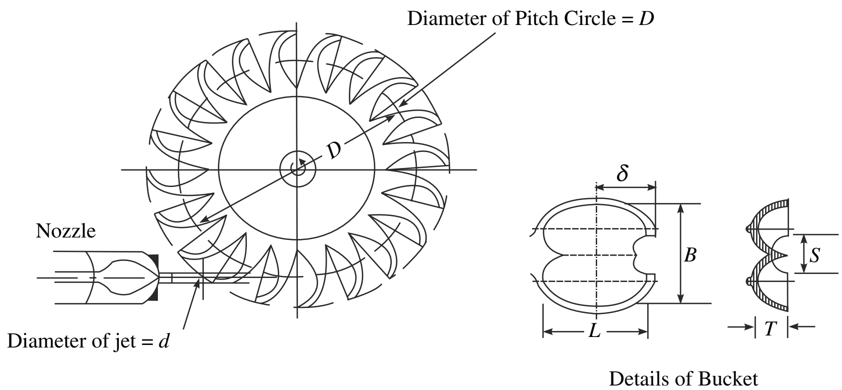


Fig. 4.2 *Pelton wheel*

The nozzle is considered as the integral part of the turbine. Consider Fig. 4.1. Here, net head = $H = H_{\text{gross}} - H_{\text{pL}} - H_{\text{sn}}$ (4.1)

where H_{gross} = Gross head = Difference in water surface elevation of upstream reservoir and tailwater level.

H_{pL} = Head loss in the penstock

H_{sn} = Height of the lowest nozzle above the tailwater level

Also,
$$H = \frac{p_b}{\gamma} + \frac{V_b^2}{2g} \tag{4.2}$$

where $\frac{p_b}{\gamma}$ = Pressure head at the base of the nozzle and $\frac{V_b^2}{2g}$ = Velocity head at the base of the nozzle.

NOTE

If H_s = Turbine setting height = Difference in elevation of the centre of wheel and tailwater level

- For Pelton turbines mounted on a vertical axis, $H_{\text{sn}} = H_s$
- For Pelton turbines mounted on a horizontal-axis, $H_{\text{sn}} < H_s$, and the difference between these two terms may not be significant in most of the installations.

NOTE

The term head H as used in hydraulic engineering (especially in India) represents energy per unit weight of fluid. In European countries the energy is commonly represented as E = energy per unit mass and is called *specific energy*. *IEC* (International Electrotechnical Commission) uses the specific energy E (in lieu of H) in all its codes and standards related to hydraulic machines. The head H and specific energy E differ by the factor g. $(E = gH)$.

3. Kinetic Energy of Jet

Let the nozzle of Fig. 4.1 produce a jet of area A_1, diameter d and velocity V_1 at *vena contracta*. Ideally, if there were no losses, the entire net head H would be converted to kinetic energy of the jet. However, there are three kinds of losses in this energy.

(a) Loss in the Nozzle If C_v is the velocity coefficient of the nozzle then the velocity of the jet is given by

$$V_1 = C_v \sqrt{2gH} \tag{4.3}$$

The energy loss in the nozzle (H_{Ln}) is given by the relation:

$$H_{Ln} = \left(\frac{1}{C_v^2} - 1\right)\left(1 - \left(\frac{A_1}{A_b}\right)^2\right)\frac{V_1^2}{2g} \tag{4.4}$$

where A_b = Cross-sectional area at the base of the nozzle, and

A_1 = Cross-sectional area of the jet.

(b) Loss of Energy in the Bucket During the transit of the jet over the bucket, from inlet to the exit, there is a certain amount of loss of energy H_{Lb}, which could be described by

$$H_{Lb} = \frac{v_{r1}^2}{2g} - \frac{v_{r2}^2}{2g} = (1 - K^2)\frac{v_{r1}^2}{2g} \tag{4.5}$$

where $v_{r1}$and v_{r2} are relative velocity (i.e. relative to the bucket velocity, u) of water entering and exiting the bucket respectively. K = Coefficient to account for frictional losses, called *bucket friction coefficient*. It is defined as $K = \dfrac{v_{r2}}{v_{r1}}$.

(c) Kinetic Energy not used in Momentum Change in the Bucket The kinetic energy of the jet is reduced to a minimum after passing through the bucket. However, the water that leaves the bucket and goes to the tailwater will still have some residual energy that is going waste, so far as the turbine is concerned. This is equivalent to the energy thrown to the tailwater in the draft tube of the reaction turbines. This part of energy, H_{Le}, is represented as

$$H_{Le} = \frac{V_2^2}{2g} \tag{4.6}$$

where V_2 is the absolute velocity of water leaving the bucket.

If H_e is the actual energy head transmitted to the buckets of the turbine then

Net head (H) = Energy transmitted to the turbine (H_e) + Losses in [nozzle (H_{Ln})+ buckets (H_{Lb})] + Energy going waste to tailwater (H_{Le})

$$H = H_e + [H_{Ln} + H_{Lb} + H_{Le}]$$

$$= H_e + H_{Lke} \tag{4.7}$$

where H_{Lke} = Total loss of kinetic energy in the nozzle - bucket system

In addition to the above three kinds of kinetic energy losses, there will be mechanical losses at the bearings and air friction of rotating elements (called *windage* losses) including the wheel. All the losses are generally represented under various categories of efficiencies such as nozzle efficiency, wheel efficiency, hydraulic efficiency, mechanical efficiency, overall efficiency, etc.

The *shaft power P* developed by the interaction of a jet with the buckets in the Pelton wheel is given by

$$P = \eta_0 \gamma QH \tag{4.8}$$

where η_0 is the overall efficiency, H = Net head, and Q = Discharge through the nozzle.

4. Speed Ratio

The ideal velocity that can be generated from the net head H is equal to $\sqrt{2gH}$. This is known as *spouting velocity*. The ratio of the peripheral velocity of the buckets at the pitch circle, u , to the spouting velocity is known as *speed ratio*, K_u. Thus,

$$\text{Speed ratio} = K_u = \frac{u}{\sqrt{2gH}} \tag{4.9}$$

where K_u is a coefficient with a value which is less than 0.5 and usually lying in the range 0.43 to 0.47.

5. Specific Speed

As in the reaction turbines, the specific speed of the Pelton turbine is defined as

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} \tag{4.10}$$

In this H = Net head in metres, N = Peripheral speed of the wheel in rpm and P = Brake power in kW developed per jet. Thus, for example, in a multijet turbine with five jets if the total power developed by the turbine is P_5 then the power per jet is $P_5/5$.The specific speed is calculated as $N_s = \frac{N\sqrt{P_5/5}}{H^{5/4}}$. The impulse turbines have specific speed in the range 8 to 30 (calculated on power per jet basis). The best performance is obtained at around a value of N_s = 17. The range of 13 to 23 for N_s is acceptable. However, values of N_s beyond this range show a markedly sharp drop in the efficiency. This is valid for each of the jets in a multiple-jet (with a maximum of six jets) turbine also. In effect, a multijet turbine of say five jets is equivalent to five single-jet separate turbines. This concept is made use of in the selection of either the number of turbines or the number of jets in a multijet turbine or combinations of these in a hydropower development project of high power potential.

By comparing with the specific speeds of Francis and Kaplan turbines (Table 4.1), it can be seen that the Pelton turbine is a low-specific-speed turbine. It implies that for these machines, the discharge involved is relatively small and the available head is relatively large.

Table 4.1 *Specific speeds of different kinds of turbines*

Type of turbine	Specific-speed range (N_s in kW- rpm-m units)
Pelton (Single jet or for each jet in a multijet unit; maximum of six jets)	8–30 (on power per jet basis)
Francis	40–450
Kaplan	300–900

NOTE

These are approximate values. The ranges are likely to be stretched as a majority of turbines in medium and large sized projects are individually designed and manufactured to meet specific project situations.

6. Rotational Speed, N

The rotational speed in rpm is related to the tangential velocity of the wheel u as

$$u = \frac{\pi DN}{60} \text{ and hence}$$

$$N = \frac{60u}{\pi D} = \frac{60 \times K_u \sqrt{2gH}}{\pi} \times \frac{1}{D} \tag{4.11}$$

Since K_u varies in a small range, the speed N is essentially a function of the wheel diameter D and varies inversely with D. Pelton turbines have a wide range of speeds, covering 75 to 1000 rpm. However, a range 300 rpm to 750 rpm is generally preferred. Unless the speed is very low, the generators are directly coupled to the turbines and the speed is adjusted to the synchronous speed. The synchronous speed of a generator is given by

$$N_{sy} = \frac{120f}{p} \tag{4.12}$$

where f = Frequency in Hz (in India, f = 50 Hz), and p = Number of magnetic poles in the generator. If the speed of the turbine is very low, a *speed increaser* (a geared mechanical device) is used between the turbine and the generator.

Generally, the highest feasible speed is preferred. The advantages of a higher speed for a turbine are as follows:

All other factors remaining same, a higher speed implies a higher specific speed. In general, higher specific speed implies

(a) Smaller size of turbine and hence lesser cost
(b) Smaller size of generator and reduction in cost of generator
(c) The overall efficiency of a single jet Pelton turbine is relatively high in a small range of specific speeds, viz.13 to 23, with a peak around 17. If by changing the speed, a specific speed in this range is obtained, one gets the additional benefit of increased efficiency.

In some situations, having two or more jets enables a smaller runner to be used for a given flow with increased rotational speed.

7. Coefficient of Velocity of the Nozzle

The jet issuing out of the nozzle has the least diameter d and velocity V_1 at the vena-contracta. This least diameter d is known as *jet diameter* and the corresponding area $A_1 = \frac{\pi}{4} d^2$ is called *jet area*. The velocity V_1 at the vena contracta is known *jet velocity*. The ratio of the jet velocity to the spouting velocity is known as *coefficient of velocity*, C_v. Thus

$$C_v = \frac{V_1}{\sqrt{2gH}}$$

or $$V_1 = C_v \sqrt{2gH}$$

The discharge from the nozzle = Discharge at the vena contracta = $Q = A_1 V_1$

$$Q = \left(\frac{\pi}{4} d^2\right) V_1 = \left(\frac{\pi}{4} d^2\right) C_v \sqrt{2gH} \tag{4.13}$$

The value of C_v for a well-designed nozzle is in the range of 0.98 to 0.99.

8. Jet Ratio

The ratio of the pitch diameter of the runner to the diameter of the jet at vena contracta is known as *jet ratio*, m. Thus

$$\text{Jet ratio} = m = \frac{D}{d} \tag{4.14}$$

For a Pelton turbine, the jet ratio is a very important parameter influencing the characteristics of the turbine. If the value of $\left(\frac{D}{d}\right)$ is relatively large, the cost of the wheel becomes correspondingly large and also the bearing friction and windage losses increase. On the other hand, if $\left(\frac{D}{d}\right)$ is relatively small, at some critical value of m, the bucket dimensions become unreasonable relative to the wheel diameter. However, before this point is reached, there will be increased departure from the tangential impact of the jet on the buckets. This would result in reduction of the efficiency of the wheel. Thus, proper selection of the value of jet ratio is crucial to the economic and efficient design of the Pelton turbine. As indicated earlier, for a Pelton wheel, the specific speed range is 8 to 30 and the corresponding *D/d* ratio is 26 to 7. Variation of the overall efficiency with the specific speed for a Pelton turbine indicates that the efficiency is maximum at around $N_s = 17$ and hence this happens to be the preferred N_s value. The corresponding preferred *D/d* ratio is 12. Having two or more jets enable a smaller runner to be used for a given flow with increase in the rotational speed.

The jet ratio is a strong function of the specific speed. An expression for the dependence of specific speed on the jet ratio and other salient coefficients of the wheel can be derived as shown below.

9. Specific Speed in Terms of Defined Ratios

Specific speed $N_s = \dfrac{N\sqrt{P}}{H^{5/4}}$ where P is the power per jet.

Speed of rotation $N = \dfrac{60 \times u}{\pi D} = \dfrac{60 \times K_u \sqrt{2gH}}{\pi D}$ (4.15)

Considering the power per jet, $P = \eta_0\, \gamma QH = \eta_0\, \gamma \times \left(\dfrac{\pi d^2}{4}\right) \times V_1 H$

$$= \eta_0\, \gamma \times \left(\frac{\pi d^2}{4}\right) \times H \times C_v\sqrt{2gH} \tag{4.16}$$

Substituting Eq. (4.15) and Eq.(4.16) in the expression for the specific speed,

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{1}{H^{5/4}} \times \frac{60 \times K_u\sqrt{2gH}}{\pi D} \times \sqrt{\left[\eta_0 \gamma \left(\frac{\pi}{4}\right) d^2 \times H \times C_v\sqrt{2gH}\right]}$$

$$N_s = \frac{60 \times (\gamma)^{1/2}\left(\dfrac{\pi}{4}\right)^{1/2}}{\pi} \times K_u\sqrt{C_v} \times \left(\frac{d}{D}\right) \times (2g)^{3/4} \times (\eta_0)^{1/2}$$

$$N_s = \frac{60}{\sqrt{\pi}} \times \frac{1}{2} \times (\gamma)^{1/2}(\eta_0)^{1/2}(2g)^{3/4} K_u\sqrt{C_v} \times \left(\frac{d}{D}\right) \tag{4.17}$$

Substituting $\gamma = 9.79$ m^3/s and $g = 9.81$ m/s^2,

$$N_s = 493.7 K_u \left(\frac{d}{D}\right)\sqrt{\eta_0 C_v} \tag{4.18}$$

This could be written as $N_s = \dfrac{K_1}{\left(\dfrac{D}{d}\right)}$ (4.19)

where K_1 is a coefficient which varies slightly. Hence, it can be said that the specific speed N_s of a Pelton turbine primarily depends on, and varies inversely as, $\left(\dfrac{D}{d}\right)$. Using normal values of coefficient of velocity, $C_v = 0.985$, speed ratio $K_u = 0.45$, overall efficiency $\eta_0 = 0.90$ and unit weight of water $\gamma = 9.79$ kN/m^3, Eq. (4.18) becomes

$$N_s = \frac{209}{\left(\dfrac{D}{d}\right)}$$

or

$$m = \frac{D}{d} = \frac{209}{N_s} \tag{4.20}$$

This gives a value of $m = 26$ for $N_s = 8$, $m = 14$ for $N_s = 15$ and $m = 7$ for $N_s = 30$, thus establishing the range of (26 to 7) for m values. Further, Eq. (4.17) is supported by experimental data also.

4.2.2 Basic Theory of Pelton Wheel

The basic one-dimensional flow analysis of the energy transfer by the water jet to the buckets is the adoption of the principle of jet impingement on a series of vanes mounted on a wheel described in Section 1.2.3, Case *F*.

1. Notations

The following notations are used in the derivation of the basic equations for power and efficiency of a Pelton turbine. Some of the notations are explained once again, for purpose of clarity, in the course of the derivations that follow.

u = Peripheral velocity of the wheel

V_1 = Jet velocity = absolute velocity of jet entering the bucket

v_{r1} = Relative velocity of jet at the inlet = $(V_1 - u)$

D = Pitch diameter of the wheel

d = Diameter of the free jet

β = Bucket angle = Deflection angle of the relative velocity of the jet

β' = $(180 - \beta)$ = Supplementary angle of bucket angle β.

K = Bucket friction coefficient

N = Speed of rotation

Q = Discharge carried by the jet

H = Net head on the turbine

NOTE

The slight difference in the definition of the bucket angle of the Pelton turbine. Since in the bucket of a Pelton turbine the impinging jet is turned by an angle of about 170° (and the ideal angle is 180°), it is usual practice to stipulate the bucket angle as an obtuse angle. Hence the definition of bucket angle as: β = Bucket angle = Deflection angle of the relative velocity of the jet.

Consider a coherent, circular, free jet of diameter d and velocity V_1 impinging on a curved bucket mounted on a wheel of pitch diameter D. The bucket is one of a series of buckets on the wheel.

Refer to Fig. 4.3. Let N = Speed of rotation of the wheel, in rpm, and $u = \dfrac{\pi DN}{60}$ = Peripheral velocity of the wheel at the pitch diameter. The bucket has a sharp splitter in the middle where the jet impacts on it tangentially to the pitch circle. After the impact, the jet splits into two symmetrical parts and exits at a *deflection angle* of β with the original direction of the jet (see Fig. 4.3). The deflection angle of the jet at the bucket is usually obtuse. At the inlet, the velocities involved in the velocity triangle are

u = Peripheral velocity of the wheel

V_1 = Jet velocity = Absolute velocity of jet entering the bucket

v_{r1} = Relative velocity of jet at the inlet = $(V_1 - u)$

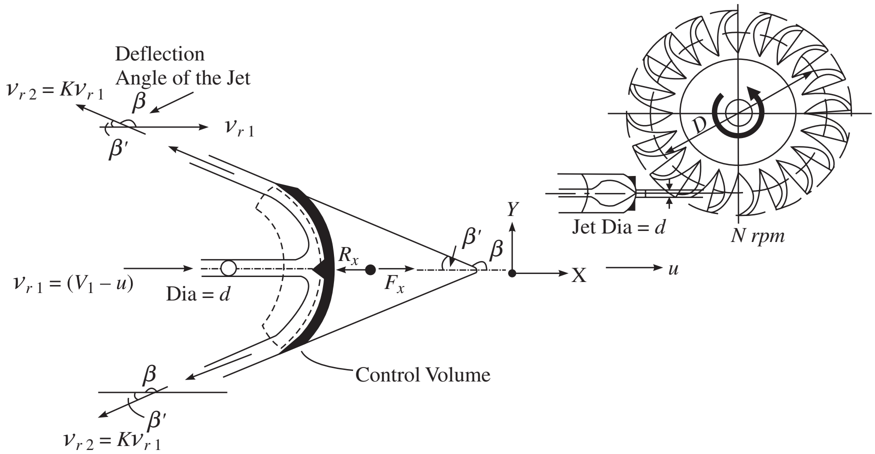


Fig. 4.3 *Jet impingement at the centre of a of a symmetric moving vane*

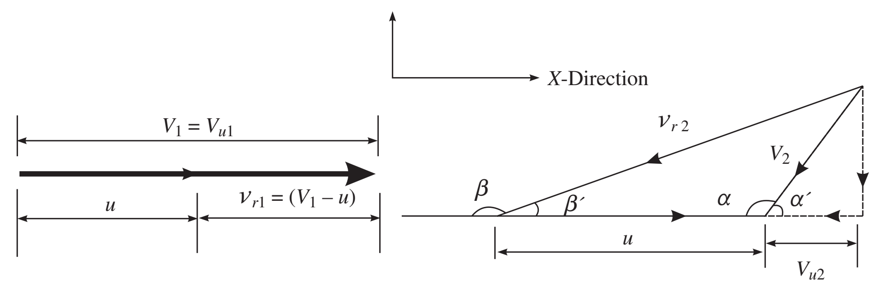
The velocity triangle at the inlet degenerates into a straight line and is shown in Fig. 4.4(a). At the outlet of the bucket, the velocity triangle is shown in Fig. 4.4 (b). Note that in this triangle, β is the bucket angle = deflection angle of the relative velocity of the jet.

Consider the tangential direction of the bucket wheel (i.e. direction of the impinging jet) as the positive x-direction. An examination of the velocity triangles at inlet and outlet reveals that

At the inlet to the bucket: $v_{r1} = (V_1 - u)$ in $(+\,x)$ direction

At the outlet of the bucket: The relative velocity v_{r2} makes an angle $\beta' = (180 - \beta)$ to $(-x)$ direction. Hence the component of the relative velocity in $(+\,x)$ direction $= -\,v_{r2} \cos \beta'$.

Consider $v_{r2} = Kv_{r1}$ where K is *bucket friction coefficient*, the component of the relative velocity in $(+\,x)$ direction $= -\,Kv_{r1} \cos \beta'$. All other losses due to mechanical or other causes are assumed to be zero. To that extent, the following analysis presents a quasi-ideal situation. Note that the flow situation is similar to the case of a free jet impinging on a series of buckets mounted on the periphery of a wheel (Case F, Section 1.2.3). Hence, even though it is a relative flow situation, one bucket or more



(a) Degenerated Velocity Triangle at Inlet (b) Outlet Velocity Trianlge

Fig. 4.4 *Velocity triangle at the inlet and outlet of a Pelton turbine bucket* buckets will intercept all the discharge in the jet. The total mass of fluid undergoing momentum change per second in the buckets is (ρQ). Hence, by linear momentum equation, the force on the bucket in the initial direction of the jet (i.e. in the ($+x$) direction), is

$$F_x = \rho\, Q\, [(v_{r1}) - (-\, K v_{r1} \cos \beta')]$$

Substituting $v_{r1} = (V_1 - u)$ in the above equation,

$$F_x = \rho\, Q\, [(V_1 - u)\,(1 + K \cos \beta')] \tag{4.21}$$

The torque acting on the wheel is

$$T = F_x \frac{D}{2} = \rho\, Q\big[(V_1 - u)(1 + K \cos \beta')\big]\frac{D}{2} \tag{4.22}$$

Hence, the power transmitted to the buckets mounted on the wheel that is moving with a tangential velocity u is

$$P = F_x u = \rho\, Q\, u\, [(V_1 - u)\,(1 + K \cos \beta')] \tag{4.23}$$

From this, the head extracted is obtained as

$$H_e = \frac{P}{\gamma H} = \frac{1}{g}\, u\, [(V_1 - u)\,(1 + K \cos \beta')] \tag{4.24}$$

This head H_e is called as *Euler head.*

Equation 4.23 can now be written in terms of H_e as, $P = \gamma Q H_e$ (4.23-a)

Hence, the hydraulic efficiency of the nozzle – wheel system is

$$\eta_h = \frac{H_e}{H} = \frac{u\big[(V_1 - u)(1 + K \cos \beta')\big]}{gH} \tag{4.25}$$

2. Alternate Method of Deriving Eq. (4.25)

From the velocity triangle at inlet (Fig. 4.4 (a)): $v_{r1} = (V_1 - u)$

V_{u1} = component of absolute velocity V_1 in the direction of $u = V_1$

From the outlet velocity triangle,

$V_{u2} = v_{r2} \cos \beta' = K v_{r1} \cos \beta' = K\,(V_1 - u) \cos \beta'$

Further, V_{u2} is negative as it is in the direction of ($-\,x$).

Hence, by Euler equation $H_e = \dfrac{V_{u1}u_1 - (-V_{u2}u_2)}{g}$

Since $u_1 = u_2 = u$, Euler head

$$H_e = \frac{(V_{u1} + V_{u2})u}{g} = \frac{\big[V_1 + K(V_1 - u)\cos \beta' - u\big]u}{g}$$

$$H_e = \frac{u\big[(V_1 - u)(1 + K \cos \beta')\big]}{g}$$

Hydraulic efficiency

$$\eta_h = \frac{H_e}{H} = \frac{u\left[(V_1 - u)(1 + K\cos\beta')\right]}{gH} \tag{4.25a}$$

It is usual convention to indicate the inlet conditions at a bucket with the suffix 1 and exit conditions with the suffix 2. In a Pelton turbine, the inlet velocity triangle degenerates to a straight line and only the exit velocity triangle is available for analysis. Two angles of this exit velocity triangle, viz. the *bucket angle* β and the angle of the absolute velocity with the tangent to the wheel α, are denoted by suffixes 2 in the analysis of the velocity triangle as well as in the application of the trigonometrical relations in the calculations. In illustrative examples, this notation is adopted for the sake of clarity.

4.2.3 Analysis of Power, Torque and Efficiency

The following analysis continues with the analysis of the quasi-ideal set-up of Sec. 4.2.2 and also considers the other possible losses in the jet-wheel system. Conditions for maximum power, hydraulic efficiency and torque are established.

1. Power

If η_0, η_h, η_m *and* η_v are the overall, hydraulic, mechanical and volumetric efficiencies of the turbine then they are related as

$$\eta_0 = \eta_h\,\eta_m\,\eta_v = \frac{P}{\gamma QH}$$

Substituting the expression for η_h from Eq. (4.25),

$$\eta_0 = \frac{u\left[(V_1 - u)(1 + K\cos\beta')\right]}{gH} = \frac{\eta_0}{\eta_m\eta_v} = \frac{P}{\eta_m\eta_v\gamma QH}$$

$$P = \eta_m\,\eta_v\,\rho\,Q\,u\,(V_1 - u)(1 + K\cos\beta') \tag{4.26}$$

For a given Pelton turbine having a known discharge under a net head H, the power output is given by

$$P = C_1\,u\,(V_1 - u) = P = C_2\,\frac{u}{V_1}\left(1 - \frac{u}{V_1}\right)$$

where C_1 and C_2 are constants for a given turbine unit. Noting that $\frac{\pi DN}{60}$, the power transferred to the turbine varies parabolically with u and hence with the rotative speed of the unit. Thus, P has a maximum value at a certain value of u/V_1. Putting $\varepsilon = u/V_1$. The power is given by

$$P = C_2\,\varepsilon\,(1 - \varepsilon) \tag{4.27}$$

The condition for maximum power is obtained by putting $\frac{dP}{d\varepsilon} = 0$. Differentiating Eq. (4.27) with respect to ε and equating it to zero,

$$\frac{dP}{d\varepsilon} = C_2\,(1 - \varepsilon) = 0$$

This gives the condition for maximum power as $\varepsilon = 0.5$, that is $u = \dfrac{V_1}{2}$.

Substituting $u = K_u\sqrt{2gH}$ and $V_1 = C_v\sqrt{2gH}$, the condition of maximum power reduces to

$$K_u = \frac{C_v}{2} \tag{4.28}$$

In an ideal case, C_v= 1.0 and the ideal K_u value for maximum power is 0.5.

Note that $P = 0$ when $u = 0$, that is at static condition of the runner and further $P = 0$ at $u = V_1$. The latter case corresponds to the *runaway condition* of the runner; a condition that occurs when the load is completely removed from a running turbine.

Variation of power with the speed ratio for an ideal Pelton turbine (i.e. a Pelton having no resistance and with β_2 =180°) is shown in Fig.4.5. The variation is parabolic. In actual turbines, β_2 < 180° and due to inevitable resistance to flow and windage losses, the peak power occurs at values of u/V_1 smaller than 0.5. Further, the power is zero at value of u/V_1 at around 0.9 (Fig. 4.5).

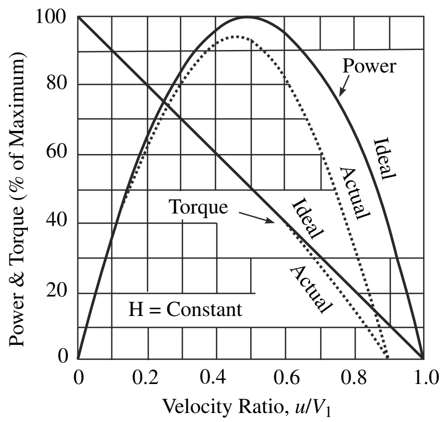


Fig. 4.5 *Variation of power and torque with speed ratio*

Hydraulic Efficiency under Quasi-Ideal Conditions The mechanical and volumetric efficiencies are connected with manufacture and installation aspects of the turbine and only the hydraulic efficiency is connected with the design and operational aspects of the turbine.

From Eq. (4.25), hydraulic efficiency $\eta_h = \dfrac{H_e}{H} = \dfrac{u\left[(V_1 - u)(1 + K\cos\beta')\right]}{gH}$

Since $V_1 = C_v\sqrt{2gH}$, the denominator of Eq. (4.25) $gH = \dfrac{V_1^2}{2C_v^2}$

Substituting this relationship in Eq. (4.25),

$$\eta_h = \frac{2C_v^2 u\left[(V_1 - u)(1 + K\cos\beta')\right]}{V_1^2} = 2C_v^2\left(\frac{u}{V_1}\right)\left[\left(1 - \frac{u}{V_1}\right)(1 + K\cos\beta')\right] \tag{4.25-b}$$

Hence, by putting $\varepsilon = u/V_1$, the above expression for the hydraulic efficiency could be expressed as

$$\eta_h = C_3\,\varepsilon(1 - \varepsilon) \tag{4.29}$$

where C_3 is another constant for a given turbine set-up. It is easy to see that the variation of η_h is of the same form as that of the power that was given in Eq. (4.27).

The condition for maximum hydraulic efficiency is obtained by putting $\frac{d\eta_h}{d\varepsilon} = 0$. Differentiating Eq. (4.29) with respect to ε and equating it to zero,

$$\frac{d\eta_h}{d\varepsilon} = C_3\, \varepsilon(1-\varepsilon) = 0$$

From this, the condition for maximum hydraulic efficiency is obtained as $\varepsilon = 0.5$, that is $u = \frac{V_1}{2}$. Thus, under ideal conditions, the characteristics of the variation of hydraulic efficiency with speed ratio are similar to that of the variation of power and the significant points are as follows:
For an ideal Pelton wheel

- The variation of hydraulic efficiency with $\frac{u}{V_1}$ is parabolic.
- The maximum hydraulic efficiency occurs at $\varepsilon = 0.5$, that is at $u = \frac{V_1}{2}$. Hence, the maximum hydraulic efficiency and maximum power occur simultaneously.
- Substituting $u = \frac{V_1}{2}$ in the expression for η_h (Eq. (4.25) or (4.25-a), the value of maximum hydraulic efficiency is obtained as

$$\eta_{h\max} = \frac{V_1^2}{4}\frac{(1+K\cos\beta')}{gH} = \frac{C_v^2}{2}(1+K\cos\beta') \tag{4.30}$$

2. Torque Power is related to torque as $T = \frac{P}{\omega}$, in which ω is the angular velocity.

Since $u = \frac{\pi DN}{60}$ and angular velocity $\omega = \frac{2\pi N}{60}$ radians/second, $u = \frac{\omega D}{2}$.

Consider the expression for power as in Eq. (4.26),

$$P = \eta_m\, \eta_v\, \rho\, Q u\, (V_1 - u)(1 + K\cos\beta')$$

From this, the torque could be expressed as

$$T = \frac{P}{\omega} = \frac{\eta_m \eta_v \rho\, Q D}{2}\,(V_1 - u)(1 + K\cos\beta') \tag{4.31}$$

Also, in terms of H_e,

$$T = \frac{\gamma Q H_e}{\omega} = \gamma Q H_e\left(\frac{D}{2u}\right) \tag{4.31-a}$$

$$= C_4\,(V_1 - u) \tag{4.31-b}$$

Note that the expression for the torque in Eq. (4.31) differs from that of Eq. (4.22) to the extent that the mechanical losses and volumetric losses are considered in

Eq. (4.31). It is easy to see from Eq. (4.31-b) that the torque varies linearly with the tangential velocity *u,* i.e. with the speed of the runner. The torque is maximum at $u = 0$, that is when the wheel is at rest. Further, the torque reaches a minimum value of zero at $u = V_1$. As already indicated, this condition of $u = V_1$ corresponds to the *runaway speed*.

Variation of torque with the velocity ratio for ideal Pelton turbine (i.e. with no resistance and $\beta_2 = 180°$) is shown in Fig.4.5. In an ideal Pelton wheel, the torque varies linearly and reaches a value of zero at $u/V_1 = 1.0$. In actual turbines, due to various resistances, the torque varies slightly nonlinearly from a value of u/V_1 around 0.6 onwards and becomes zero at a value of u/V_1 of about 0.9.

Runaway Speed Consider Eq. (4.26), which is the equation for actual power output from a Pelton wheel:

$$P = \eta_m \, \eta_v \, \rho \, Q u \, (V_1 - u)(1 + K \cos \beta')$$

It is seen that the power $P = 0$ when $u = 0$ and when $u = V_1$. In the first case, that is when $u = 0$, the wheel is stationary and hence no power is produced. In the second case of $u = V_1$, the wheel is running at maximum velocity of $u = \dfrac{\pi DN}{60} = V_1$ and no power is produced. This corresponds to the runaway speed of the turbine with sudden removal of the entire load. Let the normal wheel velocity be u_0 and let it be u_R under runaway conditions.

Under normal conditions $u_0 = K_u\sqrt{2gH}$ and $V_1 = C_v\sqrt{2gH}$.

Under runaway conditions, the theoretical runaway speed is

$$u_R = V_1 = C_v\sqrt{2gH} = \left(\frac{C_v}{K_u}\right)u_0 \tag{4.32}$$

Since $u \propto N$, theoretical runaway speed in rpm is

$$N_R = \left(\frac{C_v}{K_u}\right)N_0 \tag{4.32-a}$$

where N_0 = Normal operative speed.

Considering average values of $C_v = 0.985$ and $K_u = 0.45$, the runaway speed for a Pelton turbine is $N_R \approx 2.19\, N_0$. In practice, however, the runaway speed is found to be about 1.87 times the normal speed due to windage and bearing friction that cause resistance and consequent loss of energy.

3. Wheel Efficiency, η_w

The hydraulic efficiency η_h was defined by considering the nozzle as an integral part of the nozzle–bucket wheel system. However, by considering only the wheel, the *wheel efficiency* is defined as

$$\text{Wheel efficiency} = \eta_w = \frac{\text{Power transmitted to the wheel by the water jet}}{\text{Power input to the wheel as kinetic energy}}$$

$$\eta_w = \frac{\rho Q u\left[(V_1 - u)(1 + K\cos\beta')\right]}{\rho Q \dfrac{V_1^2}{2}} = 2\left(\frac{u}{V_1}\right)\left(1 - \frac{u}{V_1}\right)(1 + K\cos\beta') \qquad (4.33)$$

It is seen that the wheel efficiency depends on the velocity ratio $\left(\frac{u}{V_1}\right)$, bucket resistance coefficient K and the bucket angle $\beta = (180 - \beta')$. As in the case of hydraulic efficiency, wheel efficiency also has a maximum at $\left(\frac{u}{V_1}\right) = 0.5$. It should be noted that the wheel efficiency does not include the losses in the nozzle and as such, the value of η_w is larger than η_h for the same machine. Note that the hydraulic efficiency and the wheel efficiency are related as

$$\eta_w = \frac{\eta_h}{(C_v^2)} \qquad (4.33\text{-a})$$

Substituting $u = \frac{V_1}{2}$ in the expression for η_w given by Eq. 4.33, the value of maximum wheel efficiency is obtained as

$$\eta_{w\max} = 2 \times \frac{1}{2} \times \frac{1}{2} \times (1 + \text{K}\cos\beta')$$

$$\eta_{w\max} = \frac{1}{2}(1 + \text{K}\cos\beta') \qquad (4.34)$$

Nozzle Efficiency The *nozzle efficiency* is defined as the ratio of the energy at the nozzle outlet to the energy at the nozzle inlet (i.e. base of the nozzle). Thus

$$\text{Nozzle efficiency} = \eta_n = \frac{\text{Energy at nozzle outlet}}{\text{Energy at nozzle inlet}}$$

$$\eta_n = \frac{\left(\dfrac{V_1^2}{2g}\right)}{H} = \frac{V_1^2}{2gH} = \frac{\left(C_v^2 \times 2gH\right)}{2gH} = C_v^2 \qquad (4.35)$$

Note that the hydraulic efficiency η_h and the wheel efficiency η_w are related as

$$\eta_h = \eta_w \cdot \eta_n \qquad (4.36)$$

4.3 COMPONENTS OF PELTON TURBINE

4.3.1 Wheel Assembly and Buckets

1. Wheel

The runner of the Pelton turbine, commonly called the *wheel*, has evolved over a century into the efficient system of buckets mounted on the wheel, as it exists today. The present-day highly efficient shape of the buckets has been arrived at through knowledge pool of large amount of model testing, field-performance studies and of experience.

The Pelton wheel is a large disc with spoon-shaped buckets mounted on it at even spacing. Two kinds of wheel fabrication are in common use. The preferred and common practice in modern power-equipment manufacturing plants is to use the monocast system: disc and the buckets are cast in one piece. This has been found to be particularly advantageous in bigger and high-power units. The other method, which is adopted by many small and medium fabricators, is to make the disc and the buckets separately and to fix the bucket to the disc through suitable bolt-and-nut arrangement. The material for the runners and buckets are chosen depending upon the stresses, fatigue hazards and possibility of erosion due to silt content in the water and pitting due to cavitation. While for small units, high-quality alloy steel is used, very high-head and large size units are invariably made of stainless steel of 13% Cr 4% Ni composition. A main quality requirement of the material used is that it should not only be of high strength but also be adequately weldable.

Installation Two types of Pelton turbine mountings are in use. If the number of jets is one or two, horizontal-axis mounting can be used. In this type of mounting, the fouling of lower level buckets due to spent water of other jets normally precludes the use of more than two jets. Though rare, there are instances of three-jet Pelton turbines having horizontal-axis installation; and three-jets appear to be the upper limit of horizontal-axis installation. Since the turbine is directly coupled to the generator, when large power is to be generated by a single generator, two horizontal-axis turbines can be mounted on either side of a generator on the same horizontal shaft. Such an arrangement is called *double overhung installation*. Another arrangement for a horizontal-axis turbine is to mount one turbine on the extended shaft of the generator. Such an arrangement is known as *single overhung installation*.

If the turbine is mounted on a vertical shaft, it is possible to use more than two nozzles. Up to a maximum of six nozzles can be used depending on the need. The nozzles are spaced at equal angular increments around the runner. The number of nozzles in a turbine depends upon the equivalent specific speed of the unit. Since it is customary to calculate the specific speed per nozzle, an *equivalent specific speed* is the product of single-jet specific speed and square root of number of jets in the turbine. With six jets, a maximum equivalent specific speed of about70 is feasible for a Pelton turbine.

The turbine, nozzle and the associated components should be installed well above the maximum water level of the tailrace channel so that the turbine is not inundated by floods. Even though, the setting height of the lowest nozzle H_{sn} of the turbine above the tailwater level represents non-utilisation of available head (and hence a loss), it would normally be a small fraction of the net head as Pelton turbines are used in high-head developments only.

2. Buckets

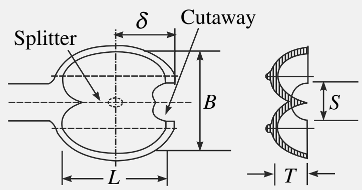
The shape of the buckets used in present-day Pelton turbines can be described as a combination of two semi-ellipsoidal shaped cups joined at the sides. The joining edge is shaped so that it acts as a splitter of the impinging jet (see Fig. 4.2 and inset in Table 4.1). The splitter is of sharp-ridge type and splits the impinging jet into two halves. This helps avoid the formation of any dead spots in the centre of the cups and aids lateral flow of the jet. From ideal theoretical considerations, for best momentum change, the jet must turn on itself, i.e. have an 180° deflection angle. However, from practical considerations of avoiding the interference of the oncoming jet and the return jet, the buckets are shaped to have a deflection angle of about 160° to 165°. The jet is deflected by this extent by each half of the bucket.

Generally, the buckets are made of stainless steel to perfect shape and surfaces of the buckets are milled, ground and polished. The correctness of the shape is checked with templates. The shape and surface characteristics of the bucket have an important bearing on the efficiency of the turbine.

There is a *cutaway* (a shaped notch) provided in the lower lip of the bucket in the central portion, symmetrically, on either side of the ridge. This is to help prevent fouling of the jet by the back of the next bucket coming into the jet. The cutaway permits a smooth entrance of a bucket into the jet while allowing gradual cutting off of water to the previous bucket.

The main dimension of the bucket is the axial width B. If it is too large, the surface friction will be unnecessarily large. While the exact geometrical shape of the bucket is a proprietary item of manufacturers, the range of basic dimensions of the geometry of the bucket is given in Table 4.2.

Table 4.2 *Basic geometrical dimensions of the Pelton bucket*



Radial length = L	Axial width = B	Depth of bucket = T	Axial width of cutaway = S	Radial length of cutaway = δ	Inlet bucket angle = β_1	Outlet bucket angle = β_2
L/d	B/d	T/d	S/d	δ/d	β_1	β_2
2.0 to 3.0	3.0 to 5.0	0.8 to 1.2	1.1 to 1.2	0.18 to 0.20	5° to 8°	165° to 170°

The number of buckets, Z, is another important parameter of the wheel. It depends on the jet ratio m (= D/d) and the following empirical relationship has been arrived at through experimental observations:

$$Z = \frac{m}{2} + 15 \tag{4.37}$$

4.3.2 Nozzle with Control Arrangement

1. Nozzle

The *nozzle* (also known as *injector*) is one of the very important components of a Pelton-turbine system and forms an integral part of the turbine system. The jet of water necessary to drive the Pelton wheel is produced in the nozzle that is connected to the end of the *penstock* through suitable connecting pipes. A spear-shaped needle moves inside the nozzle, being controlled by a servomotor. When the needle is fully withdrawn, the nozzle is fully open and emits the maximum discharge as a free jet. A small advance of the needle from its fully withdrawn position closes the opening partially and correspondingly the discharge of the jet from the nozzle is reduced. At the limit of the full forward motion, the needle completely closes the nozzle opening and the discharge is reduced to zero. A servomotor controls the motion of the needle along the axis of the nozzle. It should be noted that in a nozzle, the effective head being essentially constant, the velocity of jet remains constant and the change in the area of flow due to movement of the needle causes the change in the discharge.

The nozzle must be designed and manufactured to have minimum of head loss. The shape of the nozzle and the spearhead needle are designed to achieve perfect streamlined flow path at all needle openings. The exact internal geometry of the nozzle–needle unit has been arrived at by the individual manufacturers as a result of long experience supported by analysis using highly sophisticated analytical techniques (e.g. multifluid *CFD* analysis) of the present-day. The nozzle–needle system is made out of stainless steel (13% Cr – 4% Ni alloy).It is finished to a very smooth surface to achieve the least amount of frictional losses. Experimental studies have shown that the efficiency of the Pelton-wheel assembly is very sensitive to the quality of water jet that impacts on the buckets. The ideal jet of water would be solid, with uniform velocity distribution and free from surface spray. The character of the jet depends not only in the nozzle shape but also on the approach flow to the nozzle. The approach flow, namely the shapes, sizes and alignment of connections of the nozzle from the penstock, should be such as to cause minimum of turbulence and vortices at the base of the nozzle.

Figure 4.6 shows a nozzle and needle-valve control. The servomotor controlling the motion of the needle is outside the nozzle. A *flow straighter* surrounding the

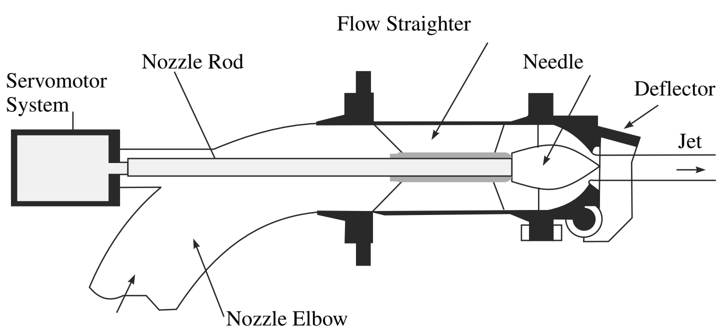


Fig. 4.6 *Typical nozzle assembly of a Pelton turbine*

needle shaft is provided to create good approach flow at the commencement of the nozzle contraction. Sometimes the servomotors are housed in a streamlined bulb provided inside the nozzle approach.

In Fig. 4.6, observe that the nozzle exit diameter is larger than the jet diameter, *d*. This is due to the contraction of the jet issuing from the nozzle–needle assembly. The coefficient of contraction is of the order of 0.8. The value of the coefficient of velocity C_v of a well-designed nozzle varies from 0.95 at 50% discharge to 0.99 at maximum discharge at fully open condition.

NOTE

That in all calculations and formulae related to a Pelton turbine, it is the jet diameter *d* that is used. The nozzle-exit diameter is not brought into the hydraulic calculations explicitly.

2. Deflectors

A *deflector* is a device provided at the tip of the nozzle to deflect the path of a part or the full jet issuing from the nozzle in such a manner that the deflected part of the jet does not impact the bucket and falls away from the runner. The deflector serves another purpose of protecting the jet from the exit water spray from the runner, to some extent. The deflector servomechanism is connected by link mechanisms with the control valve of the needle servomotor.

The need for deflector action arises during the governing action related to sudden load drop. A reduction in load would necessitate the discharge to be reduced and the servomotor pushes the spear-shaped needle in the nozzle forward towards the nozzle outlet to reduce the area of the jet and hence the discharge. However, it is a question of the time span within which this is to be accomplished. The quick response of the governor needs the spear to be pushed into position as fast as possible but such a rapid closure in the high velocity nozzle would cause *water hammer* problems in the penstock. A slow closure while minimising the water-hammer action would have a very slow and sluggish governing action. To help in this dilemma, the deflector is employed. When a quick reduction in the water flow is desired, the deflector responds quickly and deflects a part of the jet away from the original path. The servomotor adjusts the spear in the nozzle at sufficient and appropriate speed to the correct position. While this is being done, the deflector is simultaneously withdrawn from the jet. The deflector has the capacity to completely deflect the jet out of the wheel and the needle valve follows the closure at the requisite closure speed. This feature is essential for quick disconnection of the jet without causing water-hammer effects in the penstock pipe.

Figure 4.7 shows the deflector in action. There are two kinds of deflectors in use: (a) *push-out* deflector, and (b) *cut-in deflector.* The push-out deflector presses the entire jet to a path which skips the bucket circle. It bends and deforms at the portion of the jet that touches the bucket first. However, the inlet part of the bucket is very sensitive to jet characteristics and any deviation from the designed normal condition may cause drop erosion and even cavitation. Frequent use of the push-out deflector may lead to serious problems in these fronts.

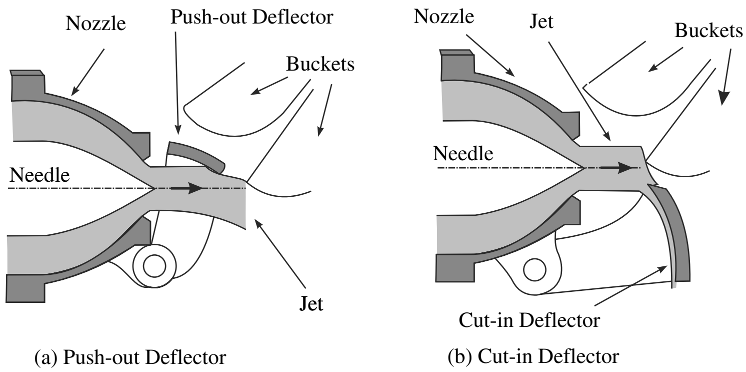


Fig. 4.7 *Deflectors: (a) Push-out deflector (b) Cut-in deflector*

The cut-in deflector does not alter the jet characteristics in the impact zone where the jet first touches the bucket. There is no adverse change of jet shape and hence the deflector action does not lead to any negative consequences. As such, presently the cut-in deflector being superior, is generally preferred to the push-out variety.

4.3.3 Manifold, Braking Jet and Auxiliary Jet

1. *Manifold*

When more than one jet is used in a Pelton turbine, the flow from the penstock has to be divided into separate pipes to feed each of the nozzles. The connecting pipe assembly that is used to connect each nozzle to the main penstock is known as *manifold*. The main feature of the manifold is to distribute the total flow to all the nozzles evenly, with minimum of losses. The geometry of the manifold, viz. shape, size of components and the alignment, depend upon the number of nozzles and the flow quantity. The main components are *wye-pieces* (bifurcations), connecting conduits between the wye-pieces. The nozzles are symmetrically distributed around the wheel. The *Main Inlet Valve* (*MIV*), usually a spherical valve, is provided in the penstock at the junction with the manifold.

Figure 4.8 shows a Pelton turbine fed by six jets. The wheel is mounted on a vertical axis. The pipe system that connects the penstock to all the six nozzles is the manifold. Note that this manifold has five *wye-joints* and diameter of the connecting conduit after each wye-joint decreases. The main objective of efficient design of a manifold is to get even distribution of discharge, minimisation of losses and minimisation of secondary currents and turbulence at the base of the nozzle. Current practice is to make preliminary design based on in-house experience and fine-tune the design through analysis of the flow through advanced *CFD* techniques and finalise the same after model studies.

The manifold with nozzles at the end of each limb is a very costly component. It is easy to see that in a multijet turbine the manifold, nozzles and needle valves, servomotors and governing mechanisms to handle all the needle valves simultaneously all add up to the cost of the unit and make it very expensive.

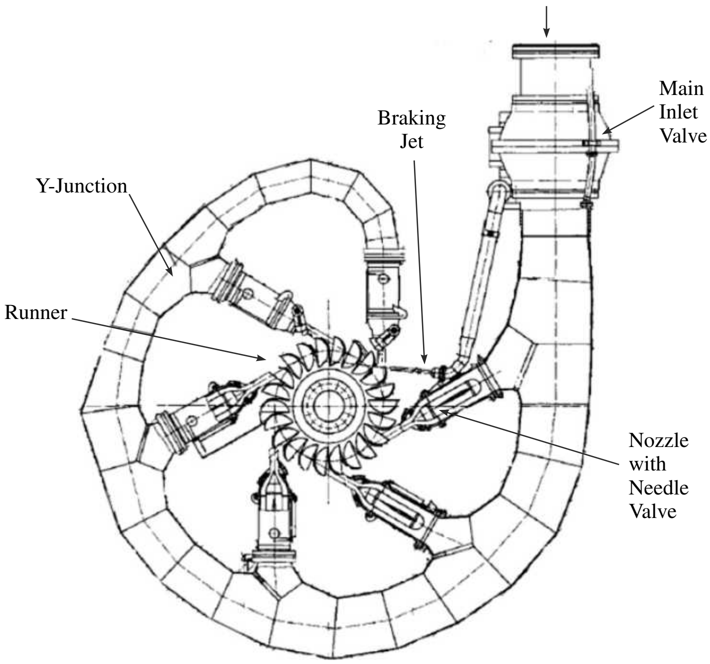


Fig. 4.8 *Schematic sketch of the manifold and jet arrangements in a six-jet Pelton turbine*

2. *Braking Jet*

When a turbine is to be stopped, the discharge from the nozzle is to be fully cut off. However, even after the stoppage of the jet flow the wheel will be rotating due to inertia and comes to rest only due to air and bearing resistance. For hastening the stoppage, a jet from a smaller nozzle is provided to act on the back of the buckets. The impact of this jet on the back of the buckets causes a restraining torque on the shaft by acting in a direction opposite to the direction of rotation due to power jets. The additional jet that causes the braking action on the wheel is called *braking jet*. Water-jet braking systems are used for emergency stoppage and fast reduction of rotational speed. Observe the *braking jet* in Fig. 4.8.

3. *Auxiliary Nozzle (Relief Nozzle)*

An *auxiliary nozzle,* also known as *relief nozzle,* does the same task as that of a deflector, viz. coping with sudden reduction of load. The auxiliary nozzle is an additional nozzle–needle valve combination connected in parallel with the main nozzle (also known as *power nozzle*). The servomotors of the power nozzle control the auxiliary nozzle. The position of the needle valve is such that when the power nozzle is open, the auxiliary (relief) nozzle is closed and vice versa. When due to an emergency unloading, the servomotors of the power nozzle begins the closing motion of the needle of the main nozzle, it simultaneously causes the relief nozzle to open proportionately. The relief nozzle discharges the water to tailwater without intercepting the wheel. After the complete closure of the main nozzle, the auxiliary nozzle also closes in slow closure mode.

It is obvious that when the auxiliary nozzle is installed, there is no need for deflectors in the set-up. Presently, auxiliary nozzles are not in much use.

4.3.4 Enclosing Chamber (Casing)

Since the Pelton turbine works under atmospheric pressure ambience, the enclosing chamber of the turbine (also known as *casing*) has no hydraulic functions. It is needed primarily to prevent splashing of water, for reduction of noise level and as a safety device against accidents. In addition, it provides a support and housing for the bearings of the turbine shaft. Arrangements for collecting splash water and provision of baffles for the reduction of windage losses is made inside the casing. There is considerable amount of air entrainment by high velocity-jets and splashing water in the turbine system and as such, a regular supply of air into the enclosing chamber through air vents of appropriate size is a necessity. In very large units, the air demand is indeed very large running into the order of a cubic meter per second.

4.3.5 Working Proportions of Pelton Turbine

The salient characteristics and working proportions of a common Pelton turbine are listed in tabular manner in Table 4.3. The values given in Table 4.3 are indicative values only and actual values in practice are likely to vary around these values due to manufacturers' special designs.

Table 4.3 *Salient working proportions and ranges of important parameters of a Pelton turbine*

S.No	Item	Range	Normal value
1	Head, H	100–1870 m	
2	Speed	75–1000 rpm	
3	Maximum capacity	420 MW	
4	Speed ratio, K_u	0.43–0. 47	0.46
5	Coefficient of velocity, $C_v = \frac{V_1}{\sqrt{2gH}}$	0.98 to 0.99 for full valve opening	0.985
6	Jet ratio = $m = \frac{D}{d}$	$N_s = 209\left(\frac{d}{D}\right)$	7 to 26
7	Runaway speed, N_R	$1.85\ N – 1.90\ N$	$1.87\ N$
8	Bucket angle, β_2	170–165°	165°
9	Number of buckets	$Z = \frac{m}{2} + 15$	
10	Specific Speed, N_s	Single jet: N_s = 8–30 Also per jet in multijet Pelton. (Max. of 6 jets)	

4.4 PERFORMANCE CHARACTERISTICS OF PELTON TURBINES

The performance characteristics of a turbine are developed through experimental studies on homologous models in the laboratory. Results are valid to the prototype through model laws. The study of the performance characteristics is done under two categories: main characteristics and operational characteristics.

4.4.1 Main Characteristics

Here, the characteristics of the turbine are studied under conditions of constant head under three headings: variation of unit discharge Q_u with unit speed N_u, Variation of unit power P_u with N_u and variation of efficiency with N_u. We consider each one of these:

1. Variation of Discharge

Figure 4.9 shows the variation of unit discharge Q_u with unit speed N_u for four different-needle-valve openings in a Pelton turbine. The head is kept constant in all the cases. It is seen that the discharge is constant and does not vary with the speed. This is true for all the four valve openings depicted. The velocity of the jet remains constant at any valve opening. The needle-valve setting to any value changes only the area and not the velocity. The discharge thus remains constant for all speeds.

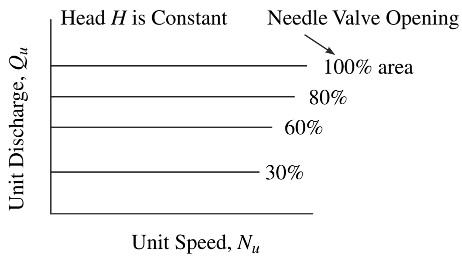


Fig. 4.9 *Variation of discharge with speed in a Pelton turbine*

2. Variation of Power

Figure 4.10 (a) shows the variation of unit power P_u with unit speed N_u in a Pelton turbine. Three different needle-valve opening situations are shown schematically. The head is kept constant in all cases. The variation is parabolic for all the valve openings shown in the figure. It was shown in Fig. 4.5 that the power P varies parabolically with u/V_1. For a constant head, the jet velocity V_1 is constant. Thus, P is a function of N only for this case. The location of peak is essentially constant and varies within a narrow range of N_u. The small variation in the speed location of the peak is due to mechanical resistance of bearings and windage.

3. Variation of Efficiency

Figure 4.10 (b) is a schematic depiction of the variation of efficiency of a Pelton turbine with unit speed N_u under constant-head operation. The variation of the efficiency is seen to be parabolic in all the three valve openings considered. However, the peak efficiency occurs in a narrow band of speed for all the different valve openings considered. The small variation in the location of the peak efficiency is due mechanical resistance of bearings and windage.

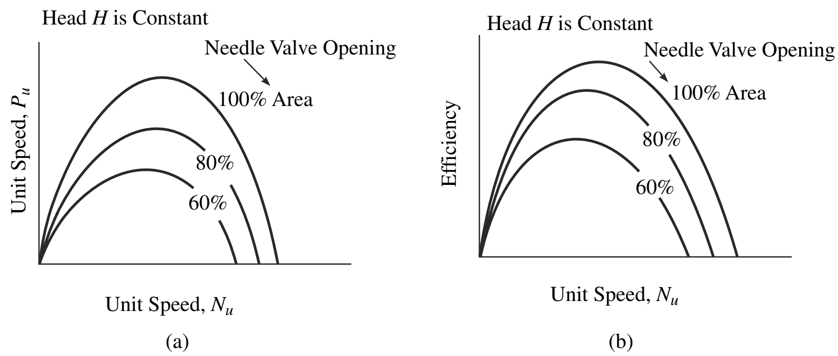


Fig. 4.10 *Variation of (a) power, and (b) efficiency with speed in a Pelton turbine*

4.4.2 Operating Characteristics

In the study of operational characteristics, the speed is kept constant and the overall efficiency and power developed by the turbine are represented as functions of discharge. The operating characteristic curves are also known as *constant-speed characteristics.*

1. Variation of Efficiency and Power with Discharge

Figure 4.11 shows the variation of overall efficiency η and power as a function of discharge Q in a Pelton turbine in constant-head operation. The ordinate is *relative efficiency* defined as the efficiency at any discharge as a ratio of the maximum efficiency. The ratio is expressed as percentage. Similarly, the abscissa is *relative discharge*, which is the ratio of operating discharge to the maximum discharge expressed as a percentage.

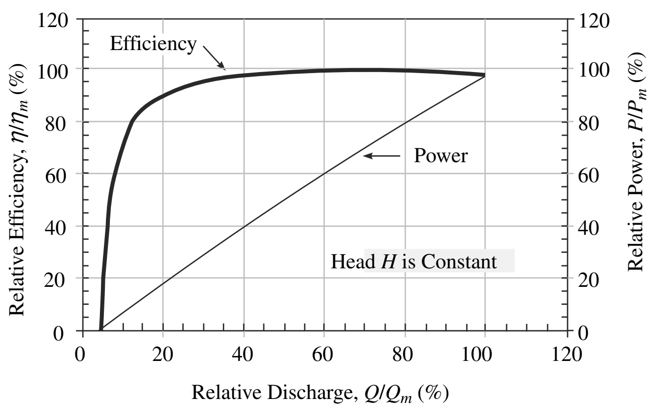


Fig. 4.11 *Variation of power and efficiency with discharge in a Pelton turbine*

The efficiency increases nonlinearly with the discharge. An interesting feature is that until a certain discharge Q_{start} is reached, the efficiency is zero. This is due to the inertia of the Pelton-wheel unit, which requires a minimum torque to overcome it before start moving from an idle position. Thus, for starting a turbine unit from idle position, the inertia has to be overcome and Q_{start} represents the condition when the critical torque is produced from the machine.

The power is represented in a nondimensional way as a ratio of power P at any discharge to the maximum power $P_{max.}$ It is seen that the power developed increases essentially linearly with the discharge all the way till Q/Q_m = 100%. Any variation in power due to change in the efficiency is too small to be noticed in the graph.

2. *Pelton Turbine Efficiency Curve for Preliminary Studies*

While individual turbine designs may have slight variations in the absolute values of the efficiency at and near peak regions, for preliminary designs, the following commonly used chart of variation of efficiency vs. percentage rated capacity is recommended. Figure 4.12, which is adapted from IS: 26800 of 1991 and *IEC*-1116-1992, shows the variation of expected overall efficiency η of a single-jet Pelton turbine. As indicated in Sec. 1.8.3, the efficiencies of Pelton turbines are generally taken as independent of size for preliminary studies, that is in model studies the efficiency of the model is taken as the efficiency of the prototype. However, for accurate work or when dealing with major installations, a procedure for correcting for scale effect in model studies as given in *IEC* 60193-1999 (2^{nd} Ed.) is used.

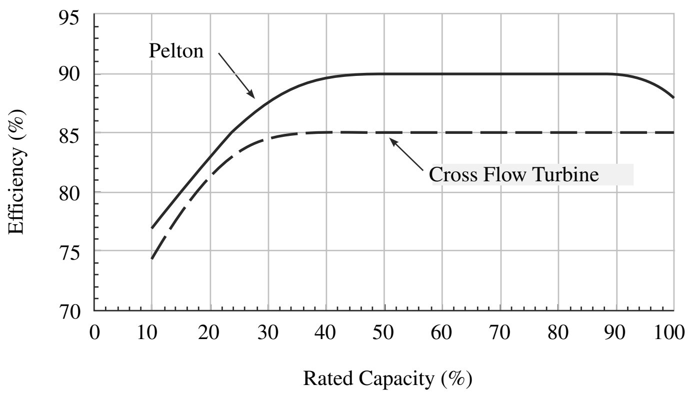


Fig. 4.12 *Variation of efficiency of a single-jet Pelton turbine*

A study of Fig. 4.12 indicates the following:

- At 100% rated turbine capacity, the rated efficiency (η_r) is about 90%.
- Peak efficiency is 90% and occurs in the range 40 to 95% of the rated capacity.
- The single-nozzle Pelton turbine has a broad, flat humplike peak. The part-load efficiency value is thus good, and the turbine may be operated down to loads of 20% of rated capacity with good efficiency.
- The efficiency characteristic of a multijet Pelton is even broader than that of a single-jet turbine. Figure 4.13 shows the characteristics of a six-jet unit. By cutting off some of the jets and operating only the optimum number of jets in the lower loads, the good part-load efficiency characteristic of a single-jet turbine can be extended up to nearly 10% of the rated capacity. For comparison purposes, the characteristic of a low-speed Francis turbine, which works in the overlap region of specific speeds of Pelton and Francis units, is shown in Fig. 4.13. The superior part-load performance of a six-jet Pelton is evident.

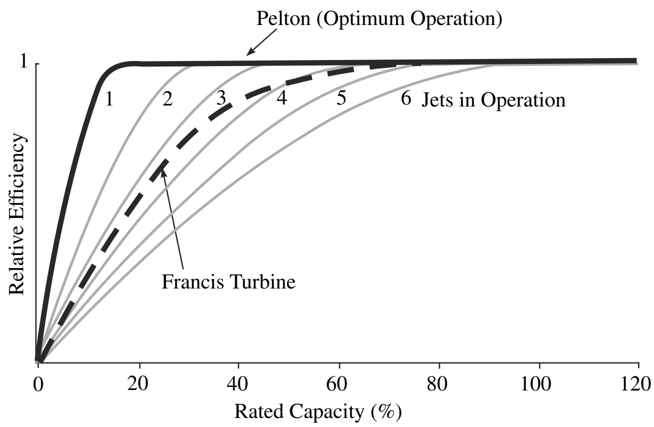


Fig. 4.13 *Optimum efficiency pattern of a six-jet Pelton turbine*

4.5 GOVERNING OF PELTON TURBINES

4.5.1 General Principles

Components and operation of a turbine governor was described in Sec. 2.11.3. The general description of the system given there is applicable to the Pelton turbine also. The features of governing which are specific to the Pelton turbine are described here.

In a Pelton turbine, the governing action is essentially through regulation of the needle valve and deflector acting in unison. On receipt of signals from the speed governor, the servomotor (see Fig.2.47–Sec. 2.11.3) directs the actuating mechanisms of the needle valve and the deflector to act according to the nature of adjustments required.

1. Consider the case of a partial-load rejection in a single-jet Pelton turbine:
   - On the receipt of the signal of partial-load rejection, the action to be taken is signalled as reduction of discharge.
   - The actuating mechanism of the deflector acts first and deflects a part of the jet so that only the requisite discharge from the jet is impinging on the buckets. This action is almost instantaneous.
   - The needle in the nozzle starts moving forward to cause a reduction in the area of the nozzle opening. This motion is in slow closure mode so that water-hammer pressures do not occur in the penstock.
   - Simultaneously, the deflector is also pulled back at a regulated speed so that at any time only the requisite discharge impinges on the buckets.
   - When the needle in the nozzle is in the final position, the deflector will be in the fully pulled-out position. The regulation operation is now complete for this load change.
2. Now consider the case of a change from a part-load to full-load acceptance:
   In this case, the signal is for the increase in the discharge of the nozzle.
   - Based on the signal from the sensor, the servomotor causes the actuating mechanisms of the needle in the nozzle to pull back to cause requisite enhancement of the area of the jet. The jet velocity being constant, increased jet area causes increased discharge in the jet.

- The pull back of the needle will stop as soon as the requisite jet area is achieved.
- Note that in this phase, there is no role of the deflector. The deflector will be in fully pulled-out condition.

4.5.2 Governing in a Multijet Pelton

The governing operation in a multijet Pelton turbine is a complicated task. At the basic level of an individual jet, the actuating procedure of the needle-in-the-nozzle and the deflector is the same as described above. However, the exact operation schedule covering all the nozzles in the turbine is complex due to the number of alternatives available.

Consider the case of a six-jet Pelton turbine having a sudden partial load shedding. At this instant, the options are

- To have all the six nozzles working but at reduced discharge,
- To have a few nozzles cut off completely, a few nozzles working on reduced jet area and to have the remaining jets operating at rated capacity. Obviously, the number of alternatives in this option is very large.
- Depending on the efficiency characteristic of the turbine, an optimisation pattern is developed for a variety of possible partial-load conditions and the corresponding operating instructions are fed to the controller to follow. Figure 4.13 shows the efficiency curve of a typical six-jet Pelton and the optimised number of jets to be put on service to meet partial loads. A multijet Pelton turbine that is being run on optimisation pattern in an automatic program through governor system is said to be *on-cam*.

4.6 VARIANTS OF PELTON TURBINE

Two variants of Pelton turbine are in popular use for a niche segment of specific speeds. These are (a) *turgo turbine,* and (b) *cross-flow turbine*.

4.6.1 Turgo Turbine

This variant of Pelton turbine was developed in 1919 and since then has undergone many refinements and now it is a recognised alternative in a specific range of head and discharge. Figure 4.14 (a) and (b) show schematic sketches of a *turgo wheel*. A turgo turbine runner [Fig. 4.14(a)] looks like a Pelton turbine split into two along the

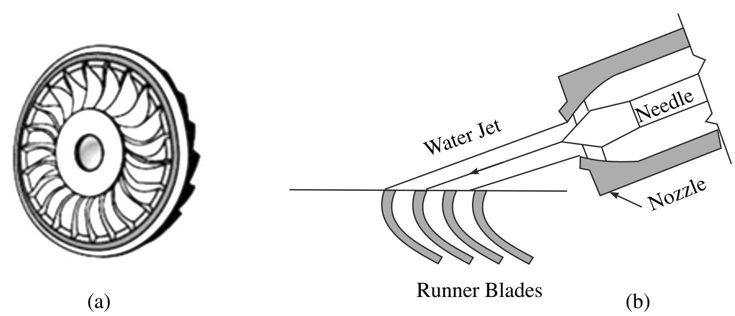


Fig. 4.14 *Schematic sketch of a turgo wheel*

splitters. Its cups are bowl-shaped, shallow, single-depression units. These cups look like half of a Pelton bucket. Water from a jet enters the turbine from the side at an angle of about 20° and traverses across the cup before exiting on the other side. The shape of the cup is such that the exiting water from one cup does not interfere with the flow in other cups. The wheel being an impulse turbine, the entire energy transfer in the wheel is done at atmospheric pressure.

The rotational speed of a turgo wheel is higher than that of an equivalent Pelton wheel for the same discharge and head. Consequently, its specific speed is higher being in the range of 10 to 80. For the same power, a turgo wheel is about 50% smaller than that of an equivalent Pelton wheel. It is classified as medium head and medium discharge turbine. The range of head is from 50 m to 250 m. A turgo turbine can have one or two jets and can be mounted either on a horizontal shaft or on a vertical shaft.

The peak efficiency is about 85% and the efficiency *vs* load curve has a flat hump and thus exhibits very good part-load performance. This flat efficiency curve and the ability to handle larger volumes of flow make this turbine a preferred choice when highly variable flows are involved. The turgo wheel is a good choice in *Small Hydro Projects (SHP)* that fall in the overlap specific speed region of Pelton and Francis turbines. Standard units of capacity up to 5 MW are available.

NOTE

Different countries are following different norms for classifying small hydro projects. It is generally agreed, in many countries, that a hydro project of rated capacity of 10 MW or less is an *SHP*. This definition is accepted by *ESHA* (the European Small Hydro Association). *SHP* is further subdivided into *mini hydro* (< 500 kW) and *micro hydro* (< 100 kW).

The classification in India is, however, slightly different. Hydropower projects are broadly classified into two categories: Large and small hydro projects. Hydro projects up to 25 MW of station capacity are categorised as Small Hydro power projects (*SHP*). *SHPs* are further classified as *micro hydro* (< 100 kW); *mini hydro* (101 kW to 2 MW) and *small hydro* (2 MW to 25 MW). Projects larger than 25 MW are classified as *large hydro*.

4.6.2 Cross-flow Turbine

The *cross-flow turbine*, also known variously as *Mitchell turbine*, *Banki turbine* or *Ossberger turbine*, is another variant of the Pelton turbine. Being an impulse turbine, the pressure is atmospheric throughout the interaction of the jet on the wheel. Further, this turbine can be considered as a 2-D avatar of the Pelton turbine. The runner is cylindrical (drum) shaped and is mounted on a horizontal shaft [Fig. 4.15 (b)]. The blades are approximately arcs of a circle in cross section and span the full length of the cylindrical runner. In appearance, the blades are similar to a section of a pipe obtained by slitting it axially. The jet is a flat two-dimensional jet capable of acting on the full or part length of the runner. The number of blades in the runner is fairly large, being of the order of 35.

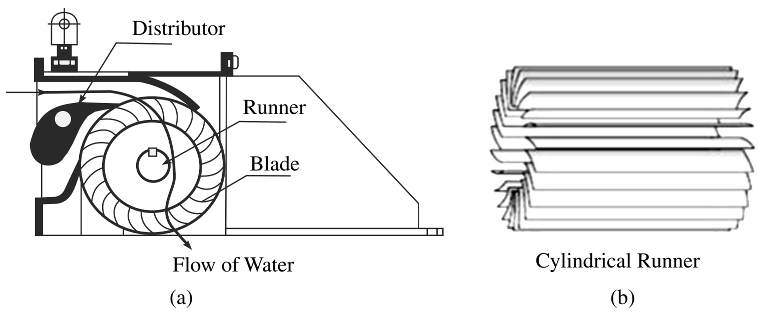


Fig. 4.15 *Schematic sketch of cross-flow turbine layout and components*

NOTE

The Cross-flow turbine is associated with three independent inventors:

1. Anthony Mitchell, of Austria, patented a cross-flow turbine in 1903.
2. Donat Banki, of Hungary, independently developed a cross-flow turbine in 1917–19.
3. Fritz Ossberger, of Germany, patented a cross-flow turbine in 1933.

Figure 4.15(a) shows the schematic view of a cross-flow turbine set-up. The flow from the nozzle enters the runner from the top on to the buckets at an angle of about 45°. After its reaction with the buckets the flow exits into the hollow central space of the runner. Then, it crosses over to the opposite part of the runner, enters another set of buckets and exits to the region outside the runner after losing most of its kinetic energy. Thus, the jet reacts with the buckets two times in its path through the runner. It is estimated that nearly 2/3 of the kinetic energy of the jet is transferred in the first pass and the remaining part of energy is transferred in the second pass. The guide vane provided at the entry at the top guides the flow onto the runner. Usually, the runner length is divided into three parts and the flow can be admitted into 1/3, or 2/3 or full length of the runner drum depending on the load.

The range of heads of the cross-flow turbine is 5 m to 200 m. This head overlaps the Francis and Kaplan turbines. The efficiency characteristic in part-load operation is flat and is similar to that of a Pelton turbine (Fig. 4.12). However, the peak efficiency is lower than that of the Pelton turbine. The efficiency vs. rated capacity curve of a well-made turbine exhibits a peak value of about 85%. However, indigenous makes exhibit peak efficiency of the order of 60%. The efficiency curve is flat and the part-load performance is very good. The peak value occurs over a range of 30 to 100% of the rated capacity. The cross-flow turbine is easily made indigenously and is found suitable for power up to about 300 kW, thus making it suitable for low-cost *Small Hydro Plants* (*SHP*). This turbine is classified as low-to medium-head and low-discharge turbine and has a specific speed range of 10 to 80; the same range as Pelton and turgo wheels.

The value of power to be used in the calculation of specific speed of cross-flow turbine is the power per unit (L/D) ratio, where L = Length of the turbine drum and D = Pitch diameter of the runner. Thus, if total power developed by a cross-flow turbine of length L and diameter D is P_t then the power used in the calculation of specific speed is $P_{1c} = \dfrac{P_t}{(L/D)}$. The specific speed is calculated as $N_s = \dfrac{N\sqrt{P_{1c}}}{H^{5/4}}$.

4.7 CAVITATION AND EROSION PROBLEMS

4.7.1 Cavitation in Pelton Turbines

The basic features of cavitation phenomenon, damages caused by cavitation and general aspects of cavitation in Francis turbine were covered in Chapter 2, Section 2.5. Specific aspects of cavitation in axial-flow turbines, viz. Kaplan turbine and its variants, are discussed in Sec. 3.9. In this section, the specific aspects of cavitation problems in a Pelton turbine are described.

Relative to Kaplan and Francis turbines, cavitation is not a very serious concern in Pelton turbines, in the sense that it does not impose limits to operation. Pelton turbines work under atmospheric pressure. The pressures in the system are either atmospheric, as in the case of the wheel or above atmospheric as in closed conduits like the nozzle and manifold. Hence, cavitation due to negative ambient pressure does not arise. However, the velocities involved are very high and possibilities of local vacuum pressures and hence cavitation is indeed real. If cavitation prevails at any location, loss of efficiency and damage to the boundary in the form of pitting occur. Some of the locations where cavitation phenomenon and damage do occur are (a) the tip of the needle, (b) inner side of nozzle end, (c) seal rings in the nozzle, and (d) buckets. In the needle, streaklike corrugations appear and affect the performance of the needle. In the buckets, the erosion may result in erroneous splitter geometry and hence impact the efficiency of the wheel. Mostly, imperfections in the geometrical boundaries are responsible for the cavitation in these locations.

The presence of silt in the water is sometimes attributed to cavitation initiation. The silt particles erode the surfaces in the region of high velocity due to abrasion. This, in turn, makes these surfaces lose their fine finish and become uneven. The uneven surfaces may initiate cavitation and consequent damage to the boundary in the neighbourhood of cavitation source.

4.7.2 Erosion Problems in Pelton Turbines

Pelton turbines are used in high-head developments and the suitable sites are invariably in hilly and mountainous regions. Due to geological reasons, some of the regions produce high amount of silt and these are carried down in the rivers. Typically, in Himalayan regions of India and Nepal, and in north-east regions of India, the rivers carry considerable amount of sediment, particularly during the monsoon period. These silt particles that may get into the supply line of the nozzles cause very heavy damage. The damage is higher if the percentage of quartz in the silt is high.

The nature of damage is erosion of the surfaces due to abrasive action of the silt transported under high-velocity flow conditions. The common locations of silt damage in Pelton turbines are the same where cavitation damages occur, viz. tip of the needle, inner side of nozzle tip, seal rings in the nozzle and buckets. The damages are in the form of erosion of the boundary material leading to uneven surfaces. Consequences are loss of efficiency leading to decreased output, abetting of cavitation, pulsation of pressures, vibrations and frequent shutdown for maintenance.

Proper control of entry of silt, use of hard stainless steel like 13 Cr-4%Ni or 16 Cr-6% Ni for the components, use of hard coatings and frequent maintenance are some of the possible remedial measures to mitigate this damage.

4.8 MISCELLANEOUS ASPECTS

4.8.1 Comparison of Impulse and Reaction Turbines

Table 4.4 gives the comparison of salient features of impulse and reaction turbines

Table 4.4 *Comparison of salient features of impulse and reaction turbines*

	Impulse Turbine	Reaction Turbine
1	All the available energy is converted to kinetic energy at fixed nozzles.	Only a portion of the total energy is converted to kinetic energy before the water enters into the runner.
2	Fixed nozzles emit free jet. Atmospheric pressure prevails in the impact zone and all over the wheel.	Water is under pressure. Positive pressure exists in the scroll case and runner and negative pressure in the draft tube.
3	Only a part of the wheel receives water from the jet at any time.	The entire runner with its entire bucket assembly is under the action of flowing water at all times.
4	The casing has no hydraulic role.	The scroll case, runner and the draft tube are all under pressure. The casing has to be of proper geometry to ensure proper recovery of pressure. Each unit has a hydraulic function.
5	Turbine unit is installed above tailwater. No need for draft tube. The height of setting represents a loss of net energy.	Draft tube enables higher setting above tailwater. Loss of energy is only due to friction in the draft tube.
6	Regulation of flow at the nozzle does not cause any extra loss of energy.	Flow regulation at the wicket gate causes energy loss depending on the amount of gate closure.
7	Relative velocity undergoes a very small reduction in the bucket due to friction.	Relative velocity in the blades may undergo considerable changes depending on the geometry of the runner.

Table 4.5 gives the comparison of specific salient properties of Pelton and Francis turbines in the overlap region of specific speeds.

Table 4.5 *Comparison of Pelton and Francis turbines in the overlap region*

Feature	Pelton Turbine	Francis Turbine
Performance	• Higher part-load efficiency • Loss of head due to setting is a small percentage of net head.	• 3% higher peak efficiency • Low outlet loss
Operational	No major concern regarding cavitation and part load	• High part load vibrations • Cavitation and variation of axial thrust
Abrasion sensitivity to silt-laden water	Affected by erosion action. Drop in efficiency due to erosion.	More affected. More parts get affected. Runners are difficult to coat properly. Resultant drop in efficiency is more than that of Pelton.

Table 4.5 *Contd.*

Lifetime cost	Relatively low. Thanks to technology of forged runners and coatings of buckets.	Higher lifetime cost, due to effect of sand erosion and cavitation.
Overall assessment	Investment cost is generally high.	Lower investment cost except in the case of small unit in the higher head range.

4.8.2 Important Standards

Some important standards relating to hydroturbines are listed in Appendix-B. Consultation of these and such other related standards are necessary in connection with tenders and contract documents relating to procurement in a hydropower project.

4.8.3 List of Major Pelton Installations in the World

Table 4.6 *List of some major Pelton installations in the world*

Project	Country, Year	Installed Power	Head (m)
Bieudron Hydroelectric Power Station	Switzerland, 2010	3 × 423 MW	1869 m
Akkoy-II	Turkey, 2008	2 × 117 MW	1220 m
Bissorte 3	France, 1981	1 × 156 MW	1,186 m
Vishnuprayag	India, 2001	4 × 103 MW	915 m
Daï Ninh	Vietnam, 2004	2 × 153 MW	627 m
Yele	China, 2000	2 × 122.5 MW	580 m
Zamang-I	Russia, 2008	2 × 176.5 MW	63.5

4.9 SELECTION OF TURBINE TYPE

In a hydropower project, after basic surveys of topography of the area, hydrology of the basin, power aspects and economic aspects are performed, one will be able to have basic information needed to go ahead with the selection of turbine. Towards this, the basic data needed include the following:

(a) Gross Head and Net Head Magnitude and variation in a water year

(b) Available Discharge Average daily/ weekly magnitude and probabilities of occurrence in a water year

(c) Power Potential Daily/weekly over a year

From this information, one would be able get design head H, design discharge Q and the corresponding design power P (with an assumed efficiency). The task of the designer is to select the type, geometry, size and number of units for the project. This is a very important phase of the project and takes considerable study and iteration before the final design is made for preliminary report. The details of the selection process and preliminary design are beyond the scope of this book. Only the tools that are available to arrive at the type of the turbine based on the head, discharge and power are indicated in this section.

Figure 4.16 shows a set of envelope lines that demarcate the zones of efficient operation of various turbines. This figure, which was briefly introduced as Fig. 2.50 in Chapter 2, is based on data from a large number of installed turbines

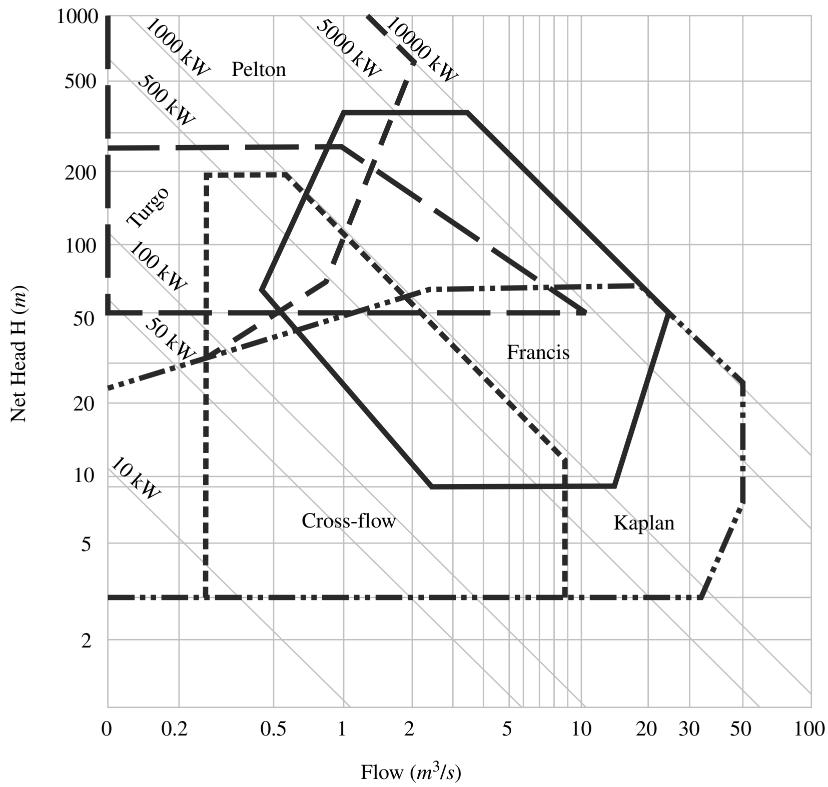


Fig. 4.16 *Turbine selection chart for small hydro projects (SHP)*

and thus represents some kind of an industry status. Considering the rapid growth of technology, the data is obviously dated. Further, all medium- and large-sized turbines are individually designed and manufactured for specific site conditions. In view of this, the charts based on industry data are only approximate. The ordinate of Fig. 4.16 is the head H in metres and the abscissa is the discharge Q in m^3/s. The third parameter is the power calculated by assuming an overall efficiency of 0.8. Lines of constant power are shown in Fig. 4.16. Note that this figure depicts the zones of not only the three basic types of turbines, viz. Pelton, Kaplan and Francis, but also includes the zones of some of the variants like cross-flow and turgo. Observe the zone of usefulness of the cross-flow and turgo wheel in *SHP* in Fig. 4.16. The overlapping zones of Kaplan, Francis, Pelton and its variants makes it clear that the selection of an appropriate turbine for an *SHP* requires some detailed deliberations of the technical and economical features of the choices available.

The limits of variables depicted in the chart are H = 1000 m, Q = 100 m^3/s and P = 10 MW. This range covers essentially all of the small hydro projects.

To use this chart, the given head and discharge are entered and the point representing the site in question is located in the chart. There can be only one choice or in most of the cases there can be two or more choices of turbine types. Now with an idea of preferred speed, the specific speed N_s is calculated. Table 4.6 shows a summary of the salient features of various turbine types. Using this table, the speed

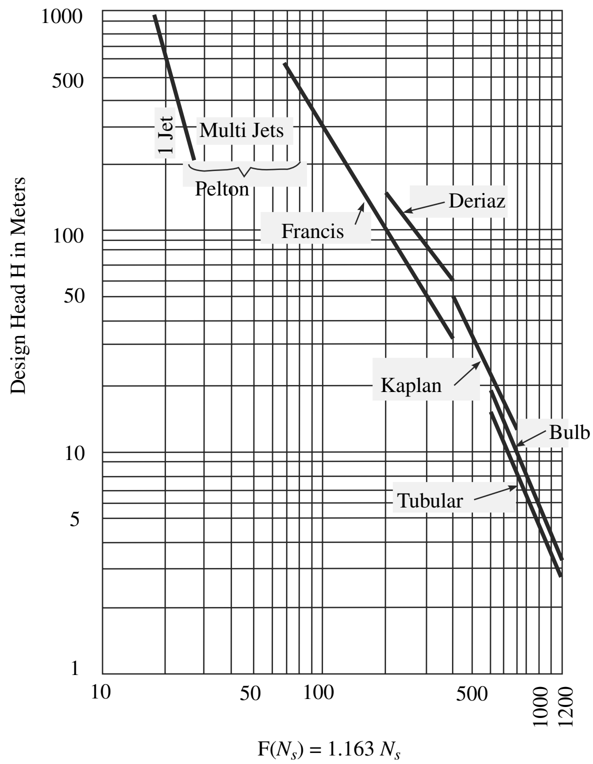


Fig. 4.17 *Chart for selection of turbine based on specific speed*

is tweaked within permissible ranges to get the preferred turbine type. Figure 4.17 is a log–log plot of design head H against a function of specific speed $F(N_s)$ given by

$$F(N_s) = 1.163\, N_s \tag{4.38}$$

where N_s = Specific speed of turbine = $\dfrac{N\sqrt{P}}{H^{5/4}}$ in (kW, m and rpm units).

In this graph, a set of six turbine types are plotted .The plotting position of the data of project under study is to be marked on this chart to verify the suitability of speed to achieve the requisite specific speed. If necessary, the operational speed is further tweaked. Use of this chart helps in narrowing the alternatives available to a given project.

It may happen that more than one type of turbine is found suitable for the project under study. In that case, the following aspects are considered as additional input:

1. Efficiency vs. Part-load Characteristics of the Turbines Under Study

Efficiency characteristic is one of the most important property used in the selection of a suitable turbine type out of two or more competing candidate types. A comparative plot of the part-load efficiency characteristics of various types of turbines is shown in Fig. 4.18. Five types of turbines, viz. Kaplan, propeller, Francis, Pelton and cross-flow turbines have been compared. Both Kaplan turbine, with its adjustable blades and adjustable wicket gates, and Pelton turbine tops the list with efficiency of around 90% and very good part-load efficiency characteristic. At less than 50% part-load

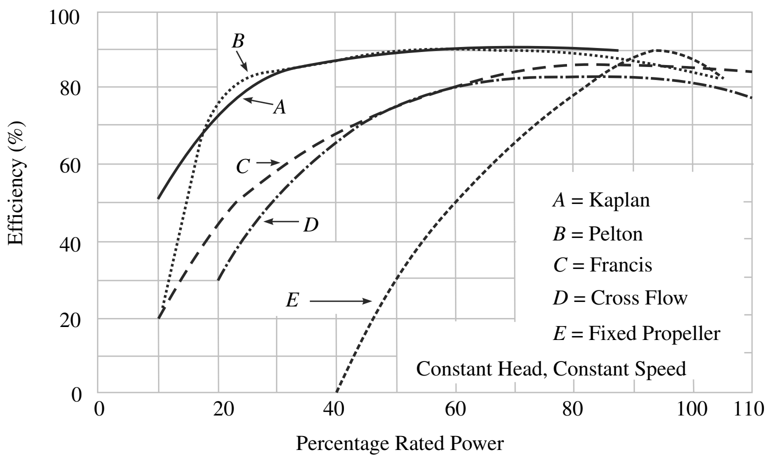


Fig. 4.18 *Comparative efficiencies of various turbine types*

values, best efficiency is by Pelton followed by Kaplan, Francis and cross-flow, in that order. Propeller turbine has the worst part-load efficiency characteristic. Obviously, given a choice the turbine with a flatter efficiency curve is more advantageous.

Figure 4.19 is a plot of peak efficiency against specific speed for the three main types of turbines. Since the curves in the figure are based on rather dated data, the values are indicative of relative values only. This plot is of help in tweaking the specific speed to achieve the best efficiency of the chosen turbine type. Note that best specific speed for Pelton turbine is around 17 and for the Francis turbine it is around 200.

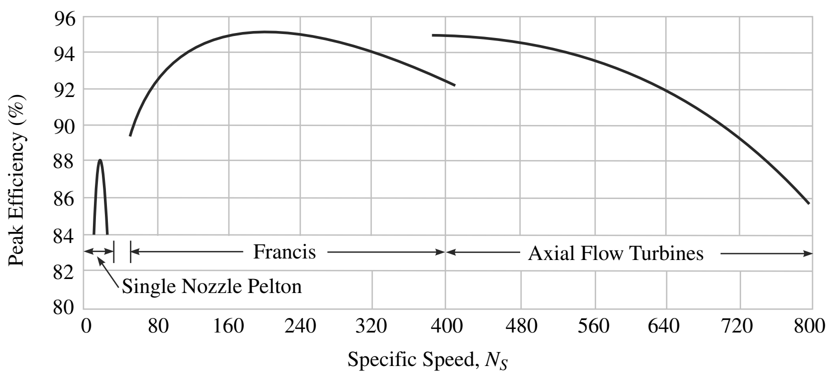


Fig. 4.19 *Variation of turbine peak efficiency with specific speed*

2. Response to Range of Discharges and Range of Heads Anticipated at the Site

Table 4.7 lists the nature of acceptance of variation of discharges and head by various turbine types. Here also, between candidate turbine types, the one that has high acceptance of both the head and discharge variations is preferred.

3. Cavitation

Cavitation problems in the turbine types is a factor worth considering seriously.

Table 4.7 *Salient Features of various Turbine types to aid selection*

Sl. No	Turbine	Specific Speed Range	Head Range	Acceptance of Flow Variation	Acceptance of Head Variation	Part Load (% of Rated Capacity)
1	Pelton: Single-jet	8–30	50–1500	High	Low	30–100
2	Pelton: multijet	8–30 (per jet)	50–1800	High	Low	10–100
3	Kaplan	300–900	2–40	High	High	15–100
4	Propeller	300–900	2–40	Low	Low	80–100
5	Francis	40–400	25–350	Medium	Low	70–100
6	Deriaz	130–450	25–200	High	High	25–125
7	Cross-flow	10–80	5–200	High	Medium	30–100
8	Turgo	10–80	50–200	High	Low	30–100
9	Bulb and tubular	500–1100	2–30	High	High	15–100

4. Relative Costs Including Total Capital Cost and Operation and Maintenance Costs of the Project

Some additional features to be considered are the following:

- In some cases to have more than one unit may be beneficial. Using 2 or 3 smaller units than one large unit may be more cost-effective.
- In situations where there is an overlap of Pelton and Francis, a low-speed Pelton with multijet configuration may turn out to be more advantageous.
- Similarly, in the overlap region of Francis and Kaplan types, the available alternatives are many and they all must be explored.

The factors mentioned in this section are only a rough guide to turbine-type selection of best turbine type in a given project. In actual practice, considerable iteration and comparative study will be required. Sophisticated commercial softwares (for e.g., TURBNPRO, ***http //www.turbnpro.com***) are available for comprehensive calculations of all aspects and alternatives to aid decision on the type of the turbine for specified field data. Usually, these softwares will have capabilities to make preliminary design and drawings for a detailed project report.

4.10 MODEL STUDIES OF HYDRAULIC TURBINES

4.10.1 Advantages of Model Studies

Large-size turbines, generally of unit power greater than 5 MW, are individually designed and produced in sophisticated factories to suit the specific needs of the project. Prototype testing of these large-size turbines is very difficult, expensive and is of restricted nature. Hence, the usual practice is to test the scale models of the turbines for obtaining performance characteristics of the prototype. In addition, some of the specific advantages of model testing are the following:

- In a model, the machine can be subjected to extreme conditions such as prolonged extreme cavitation and maximum runaway speed. Obviously, these are neither possible nor permissible in connection with a prototype.

- Model tests can be performed over a wide range of the variables such as the discharge, head and speed. This feature is also not always possible with a prototype.
- Model testing is conducted under fully controlled condition and sensitive, sophisticated instrumentation can be deployed. Data acquisition, analysis and report generation in modern laboratories have reached a very high level of sophistication. For example, system design software like *LabVIEW* (short for Laboratory Virtual Instrumentation Engineering Workbench) is commonly used in modern laboratories for data acquisition, instrument control and automation. Typically, *LabVIEW* will record performance parameters like pressures, flow, power and efficiency, and draw performance curves of the machine under test.
- In tests for development or remedial measures (such as for cavitation or silt erosion problem), a wide choice of alternatives is possible.
- Model tests on a turbine model can provide the performance characteristics of the turbine for various end uses. Some of the common uses are
  (a) Confirmation of manufacturer's compliance with contractual guarantee
  (b) Comparing models from several competitive bidders for specific aspects
  (c) Rehabilitation of old machines to present-day standards of efficiency

In view of the above-mentioned advantages of model studies, it is an accepted practice to test the compliance of the machine to the contractual specifications in a model. Vendors for most of the large-size machines being international companies; it is the normal practice to have compliance to international standards in contract documents. For turbines and large pumps, the standards prescribed by the International Electrotechnical Commission (*IEC*) are followed. The similitude rules and procedures are strictly defined in *IEC* standards 60193-1999 (Ed.2)–*Hydraulic Turbines, Storage Pumps and Pump Turbines—Model Acceptance Tests*. These rules are of mandatory nature for all machines of unit power more than 5 MW or with reference diameter larger than 3.0 m. In addition, these rules form the part of tender/contract documents and no guarantees are accepted if not complying with these standards and rules. *IEC*-60193 covers all types of turbines, pumped turbines and large pumps. All types of turbines, viz. impulse (Pelton), reaction (Francis, Kaplan and bulb) are covered under *IEC*-60193.

4.10.2 Standard Hydroturbine Testing Laboratory

Present-day standard hydraulic turbine-testing laboratories ordinarily would have the following facilities:

- Ample sump and discharge circulation capability
- Adequately large uninterrupted power supply
- Adequate fresh water supply and storage; also, a good filtration plant with ozoniser to ensure clean fresh water with longer recirculating time
- Adequate size of test beds and facility to test horizontal- and vertical-axis machines
- Normally a battery of flowmeters (usually magnetic flow meters are preferred choice) of different ranges for flow measurement
- Pressure transducers for pressure measurement
- Dynamometers, torque meters and devices to measure the speed of rotation of shafts

- Supply pumps with sophisticated control mechanisms (the pump must be capable of continuous adjustment of head and discharge in the operating range and capability to hold any operation point for sufficient length of time.)
- Sophisticated data-acquisition system and computer facility with appropriate software
- A good laboratory, generally, not only meets the needs of routine testing but also would be capable of meeting the needs of research and developmental activities. This feature would require some features and equipment that may not be directly be related to standard tests.
- Further, all laboratories posses appropriate certification for various mandatory quality, safety and other local statutory requirements.

All the major hydroturbine-manufacturing organisations have their own standard turbine-testing laboratories. In addition, several independent private enterprises, public-sector organisations and universities and higher education technical institutions have established hydroturbine testing laboratories for commercial as well as for research and training purposes.

4.10.3 Types of Tests

The normal types of tests performed on a model can be classified as follows:

1. Determination of the performance characteristics of the turbine covering the full operating range. In short, it means the development of the hill chart of the turbine. The efficiency is scaled up to prototype conditions. *IEC* 6913(Ed.2) gives a scaling-up procedure for reaction turbines based on the model and prototype Reynolds number.
2. Performing cavitation tests to develop plant sigma covering the operating range of the turbine
3. Determination of the runaway speed at plant sigma and other critical operating points
4. Tests for the performance of control systems

The parameters to be measured in the above tests are the pressures at key points, head H, discharge Q, torque T and speed of rotation N. All the metering devices should be capable of independent calibration of required accuracy over the entire range of measurement.

The temperature of water is monitored continuously. Density of water is estimated from standard tables using the temperature data. For tests involving cavitation, the air content at key locations are monitored continuously. Additional parameters like the ambient barometric pressure, temperature and humidity are recorded and the environmental conditions of the tests are controlled to the extent possible.

4.11 ILLUSTRATIVE EXAMPLES

*EXAMPLE 4.1

The water jet in a Pelton wheel is 8 cm in diameter and has a velocity of 93 m/s. The rotational speed of the wheel is 600 rpm and the deflection angle of the jet at the bucket is 170°. If the ratio of bucket velocity to jet velocity is 0.47, determine the (a) *diameter of the wheel, and (b) power transferred to the wheel by the jet. Take bucket friction coefficient K = 0.96.*

Solution

Given: $d = 0.08$ m, $V_1 = 93$ m/s, $N = 600$ rpm, $\beta_2 = 170°$, $\dfrac{u}{V_1} = 0.47$, $K = 0.96$

Since, $u/V_1 = 0.47$,

$$u = 0.47 \times 93 = 43.71 \text{ m/s}$$

(a) $u = \dfrac{\pi DN}{60}$

Diameter of the wheel $D = \dfrac{60 \times u}{\pi N} = \dfrac{60 \times 43.71}{\pi \times 600} = 1.391$ m

$\beta'_2 = 180 - 170 = 10°$

Discharge $Q = \dfrac{\pi}{4}(0.08)^2 \times 93 = 0.4675 \text{ m}^3/\text{s}$

(b) Power transmitted to the runner by the jet = Theoretical power =

$P = \rho Qu (V_1 - u)(1 + K \cos \beta'_2)$

$P = 0.998 \times 0.4675 \times 43.71 \times (93 - 43.71)(1 + 0.96 \times \cos 10°)$

$P = 1955$ kW

*EXAMPLE 4.2

A Pelton turbine has a net head of 425 m. Assuming: $C_v = 0.97$, speed ratio $K_u = 0.46$, jet deflection angle at the bucket $\beta_2 = 165°$ and bucket friction coefficient $K = 0.9$, calculate the (a) hydraulic efficiency (b) wheel efficiency, and (c) nozzle efficiency of the turbine.

Solution

Given: $H = 425$ m, $C_v = 0.97$, $K_u = 0.46$, $\beta_2 = 165°$, $K = 0.9$

(a) $V_1 = C_v \sqrt{2gH} = 0.97 \times \sqrt{2 \times 9.81 \times 425} = 88.58$ m/s

$u = K_u \sqrt{2gH} = 0.46 \times \sqrt{2 \times 9.81 \times 425} = 42.0$ m/s

$\dfrac{u}{V_1} = \dfrac{42.00}{88.58} = 0.4742$. By Eq. (4.25-b)

$$\eta_h = 2C_v^2 \left(\frac{u}{V_1}\right)\left(1 - \frac{u}{V_1}\right)(1 + K \cos \beta'_2)$$

$$= 2 \times (0.97)^2 \times (0.4742) \times (1 - 0.4742)(1 + (0.9) \times \cos 15°)$$

Hydraulic efficiency $\eta_h = 0.877$

(b) Wheel efficiency $\eta_w = \dfrac{\eta_h}{C_v^2} = \dfrac{0.877}{(0.97)^2} = 0.922$

(c) Nozzle efficiency $\eta_h = C_v^2 = (0.97)^2 = 0.941$

*EXAMPLE 4.3

The following data pertains to a Pelton turbine:

Speed ratio = 0.46	*Coefficient of velocity of the nozzle = 0.97*
Mechanical efficiency = 0.92	*Speed = 300 rpm*
Pitch diameter of runner = 2.0 m	*Bucket friction coefficient K = 0.97*
Jet deflection angle at bucket = 170°	*Discharge = 0.5 m^3/s*

Calculate the (a) power transmitted by the jet to the wheel, and (b) net head.

Solution

Power transmitted by the jet to the wheel (Euler power) is given by

$$P = \rho\, Qu\, (V_1 - u)\, (1 + K \cos \beta_2')$$

$$u = \frac{\pi DN}{60} = \frac{\pi \times 2.0 \times 300}{60} = 31.423 \text{ m/s}$$

$$\frac{u}{V_1} = \frac{K_u}{C_v} \text{ and hence } V_1 = \frac{uC_v}{K_u} = \frac{31.426 \times 0.97}{0.46} = 66.26 \text{ m/s}$$

$$P = 0.998 \times 0.5 \times 31.423 \times (66.26 - 31.423)(1 + 0.97 \cos 10°)$$

$$= 15.68 \times 34.84 \times 1.9553 = 1068 \text{ kW}$$

Since, $u = K_u \sqrt{2gH}$

$$\text{Net head } H = \frac{u^2}{2gK_u^2} = \frac{(31.423)^2}{2 \times 9.81 \times (0.46)^2} = 237.8 \text{ m}$$

*EXAMPLE 4.4

A Pelton-wheel installation that receives water from a reservoir through a penstock has a gross head at the wheel of 495 m. A nozzle supplies 2.0 m^3/s of water to the turbine where the jet undergoes a deflection of 165°. Loss of head in the penstock could be taken as one-third of the gross head. Considering the speed ratio = 0.45, C_v = 0.98 and overall efficiency as 0.85, determine the (a) power generated by the Pelton wheel, and (b) hydraulic efficiency. Take bucket friction coefficient K = 0.95.

Solution

Given: $H = 495$ m, $C_v = 0.98$, $K_u = 0.45$, $\beta_2 = 165°$, $K = 0.95$, $\eta_0 = 0.85$,

$Q = 2.0 \text{ m}^3/\text{s}$

Gross head = 495 m

Loss of head at penstock = $\frac{1}{3} \times 495 = 165$ m

Net head at turbine = $H = 495 - 165 = 330$ m

(a) Power generated by the wheel

$$P = \eta_0\, \gamma QH = 0.85 \times 9.79 \times 2.0 \times 330 = 5492 \text{ kW}$$

(b) $V_1 = C_v \sqrt{2gH} = 0.98 \times \sqrt{2 \times 9.81 \times 330} = 78.86$ m/s

$u = K_u \sqrt{2gH} = 0.45 \times \sqrt{2 \times 9.81 \times 330} = 36.21$ m/s

Head Extracted (Euler head) $H_e = \frac{1}{g} u (V_1 - u)(1 + K \cos \beta_2')$

$= \frac{1}{9.81} \times 36.21 \times (78.86 - 36.21)(1 + 0.95 \times \cos 15°) = 301.89$ m

Hydraulic efficiency $\eta_h = \frac{H_e}{H} = \frac{301.89}{330} = 0.915$

*EXAMPLE 4.5

The following data pertains to a Pelton turbine:

Speed ratio = 0.46	*Coefficient of velocity of the nozzle = 0.98*
Mechanical efficiency = 0.94	*Hydraulic efficiency = 0.95*
Pitch diameter of runner = 1.50 m	*Jet diameter = 15 cm*

Calculate the specific speed of the turbine.

Solution

Specific speed of Pelton turbine is given by Eq. 4.18 as

$$N_s = 493.7\ K_u \left(\frac{d}{D}\right)\sqrt{\eta_0 C_v}$$

$$\frac{d}{D} = \frac{0.15}{1.50} = 0.1$$

$$\eta_0 = \eta_m\ \eta_h = 0.94 \times 0.95 = 0.893$$

$$C_v = 0.98,\ K_u = 0.46$$

$$N_s = 493.7 \times 0.46 \times (0.1) \times \sqrt{0.893 \times 0.98}$$

$$= 21.25$$

**EXAMPLE 4.6

A Pelton wheel has two jets and is designed to produce a power of 7500 kW with a net head of 400 m. The buckets deflect the jet by an angle of 165°. The reduction of the relative velocity due to friction in the buckets can be taken as 15%. Calculate the (a) total discharge through the turbine, (b) diameter of each jet, and (c) the total force exerted by the jets on the wheel in the tangential direction. Assume overall efficiency = 80%, coefficient of velocity = 0.98, and speed ratio = 0.47.

Solution

Given: $P = 7500$ kW, $H = 400$ m, $C_v = 0.98$, $K_u = 0.47$, $\beta_2 = 165^\circ$, $\eta_0 = 0.80$

Since there are two jets, for each jet:

Power $P = 7500/2 = 3750$ kW.

(a) Speed ratio $K_u = \frac{u}{\sqrt{2gH}} = 0.47$

Hence, $u = 0.47 \times \sqrt{2 \times 9.81 \times 400} = 41.64$ m/s

$P = 3750 = \eta_0 \, \gamma QH$

$$Q = \frac{3750}{0.80 \times 9.79 \times 400} = 1.197 \text{ m}^3\text{/s.}$$

Total discharge through the turbine $Q_t = 2\,Q = 2 \times 1.197 = 2.394$ m^3/s

(b) $V_1 = C_v \sqrt{2gH} = 0.98 \times \sqrt{2 \times 9.81 \times 400} = 86.82$ m^3/s

Discharge per jet $Q = 1.197 = \frac{\pi}{4} d^2 \, V_1$

Hence, $d^2 = \dfrac{1.197}{\frac{\pi}{4} \times 86.82} = 0.01755,$

Diameter of each jet = d = 0.1325 m = 13.25 cm

(c) Total force exerted by each jet in the tangential direction, by Eq. 4.21

$$F_{x1} = \rho \, Q \, (V_1 - u)(1 + K \cos \beta_2')$$

$$\beta_2' = (180 - \beta_2) = 15°; \quad \cos \beta_2' = \cos 15° = 0.9659$$

$$K = (1 - 0.15) = 0.85$$

$$F_{x1} = \frac{998}{1000} \times 1.197 \times (86.82 - 41.64) \times (1 + 0.85 \times 0.9695) = 98.286 \text{ kN}$$

Total tangential force on the wheel due to two jets

$$F_{xT} = 2F_{x1} = 2 \times 98.286 = 196.57 \text{ kN}$$

**EXAMPLE 4.7

A Pelton wheel of 2.5 m diameter operates under the following conditions:

Net available head = 300 m	*Jet deflection angle in the bucket = 165°*
Coefficient velocity of the jet = 0.98	*Diameter of jet = 20 cm*
Bucket friction coefficient = 0.95	*Mechanical efficiency = 0.95*
Speed **=** *300 rpm*	

Determine (a) the shaft power (b) hydraulic efficiency, and (c) specific speed.

Solution

$$\beta_2' = 180 - 165 = 15°$$

Velocity of jet $V_1 = C_v \sqrt{2gH} = 0.98 \times \sqrt{2 \times 9.81 \times 300} = 75.186$ m/s

Discharge $Q = A_1 V_1 = \frac{\pi}{4} (0.2)^2 \times 75.186 = 2.362$ m^3/s

$$u = \frac{\pi DN}{60} = \frac{\pi \times 2.5 \times 300}{60} = 39.27 \text{ m/s}$$

Euler head $H_e = \frac{1}{g} u(V_1 - u)(1 + K \cos \beta_2')$

$$= \frac{1}{9.81} \times 39.27\ (75.186 - 39.27)\ (1 + 0.95 \cos 15°)$$
$$= 275.7 \text{ m}$$

Hydraulic efficiency $\eta_0 = \frac{H_e}{H} = \frac{275.7}{300} = 0.919$

Overall efficiency $\eta_0 = (\eta_m \eta_h) = 0.919 \times 0.95 = 0.873$

Shaft power $P = \eta_0\ \gamma QH = 0.873 \times 9.79 \times 2.362 \times 300 = 6057$ kW

Specific speed $N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{300\sqrt{6057}}{(300)^{5/4}} = 18.7$

**EXAMPLE 4.8

A double-jet Pelton wheel has specific speed of 26 and is required to deliver 10 MW of power. The turbine is supplied through a pipeline from a reservoir whose level is 400 m above the nozzles. Allowing 5% for friction loss in the pipe, calculate (a) speed in rpm, (b) diameter of the jets and (c) pitch diameter of the wheel,
[Assume $K_u = 0.46$, $C_v = 0.98$ and $\eta_0 = 0.85$].

Solution

Net available head $H = 400\ (1 - 0.05\ ) = 380$ m

Specific speed $N_s = \frac{N\sqrt{P}}{H^{5/4}} = 26$

NOTE

That for multijet Pelton turbines the specific speed is calculated on power produced per jet.

Power per jet $P = 10{,}000/2 = 5000$ kW

$$N_s = 26 = \frac{N\sqrt{5000}}{(380)^{5/4}}$$

Operating speed $N = 616.9$ rpm

$$u = K_u\sqrt{2gH} = 0.46 \times \sqrt{2 \times 9.81 \times 380} = 39.72 \text{ m/s}$$

Since $u = \frac{\pi DN}{60}$, the pitch diameter $D = \frac{60 \times u}{\pi N} = \frac{60 \times 39.72}{\pi \times 616.9} = 1.23$ m

Total power developed $= P_t = \gamma\, \eta_0\, Q_t\, H$

$$1000 = 9.79 \times 0.85 \times Q_t \times 380$$

Total discharge $Q_t = 3.162$ m^3/s

Discharge per jet $Q = 3.162/2 = 1.581\ m^3/s$

Velocity of jet $V_1 = C_v\sqrt{2gH} = 0.98 \times \sqrt{2\times 9.81\times 380} = 84.62$ m/s

Since $Q = \frac{\pi}{4}d^2V_1$, the jet diameter $d = \sqrt{\frac{4Q}{\pi V_1}} = \sqrt{\frac{4\times 1.581}{\pi\times 84.62}} = 0.1542\ \text{m} = 15.42\ \text{cm}$

**EXAMPLE 4.9

A Pelton turbine is to produce 20 MW of power under a net head of 500 m. A turbine of specific speed N_s = 17 is proposed. If the ratio of bucket velocity to jet velocity is 0.47 and the jet ratio is 16, determine the (a) operational speed, (b) wheel diameter, and (c) diameter of the jet. Assume overall efficiency = 0.85.

Solution

Given: P = 20,000 kW, H = 500 m, u/V_1= 0.47, η_0= 0.85, N_s = 17, D/d = 16

Rotational Speed $N = \frac{N_s H^{5/4}}{\sqrt{P}} = \frac{17\times(500)^{5/4}}{\sqrt{20000}} = 284.2$ rpm

Discharge $Q = \frac{P}{\eta_0 \gamma H} = \frac{20000}{0.85\times 9.79\times 500} = 4.807\ m^3/s$

$$\frac{u}{V_1} = \frac{\pi DN}{60}\times\frac{\pi d^2}{4Q} = \frac{\pi^2(Dd^2)N}{240Q}$$

Since $\frac{D}{d}$ = 16 substituting $d = \frac{D}{16}$ in the expression for u/V_1

$$\frac{u}{V_1} = \frac{\pi^2(D)^3N}{(256)\times 240Q} = 0.47\cdot$$

Substituting the values of N and Q,

$$D^3 = \frac{256\times 240\times 4.807\times 0.47}{\pi^2\times 284.2} = 49.488$$

$$D = 3.67\ \text{m and}\ d = \frac{D}{16} = \frac{3.67}{16} = 0.229\ \text{m}$$

**EXAMPLE 4.10

A Pelton turbine has jet velocity of 90 m/s and peripheral velocity of 40 m/s. If the deflection angle of the jet in the bucket is 160°, find the (a) head loss due to bucket friction, and (b) kinetic energy head of exit discharge from the buckets. Take bucket friction coefficient K = 0.9.

Solution

Given: V_1 = 90 m/s, u = 40 m/s, β_2 = 160°, K = 0.9

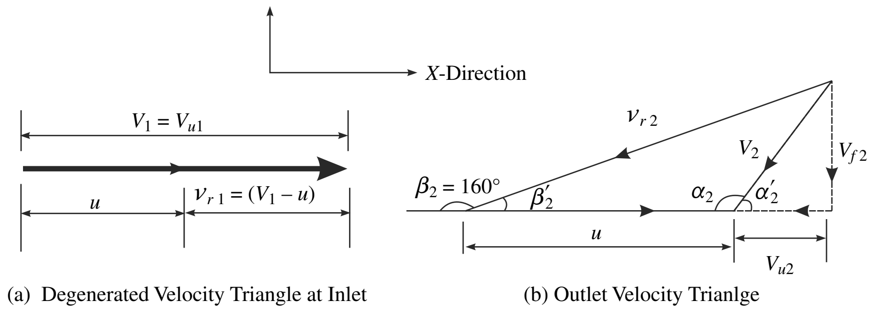


Fig. 4.20 *Velocity triangles, Example 4.10*

Figure 4.20 shows the inlet and outlet velocity triangles. $\beta_2' = 180 - 160 = 20°$

Since $K = 0.9$, relative velocity $v_{r2} = 0.9v_{r1} = 0.9\,(V_1 - u)$.

$$v_{r2} = 0.9 \times (90 - 40) = 45 \text{ m/s}$$

Let α_2' be the direction of the absolute velocity V_2 with the peripheral velocity.

$$V_{f2} = V_2 \sin \alpha_2' = v_{r2} \sin \beta_2'$$

$$V_{f2} = 45 \sin 20° = 15.39 \text{ m/s}$$

$$V_{u2} = V_2 \cos \alpha_2' = v_{r2} \cos \beta_2' - u$$

$$V_{u2} = 45 \cos 20° - 40 = 2.29 \text{ m/s}$$

$$\tan \alpha_2' = \frac{V_{f2}}{V_{u2}} = \frac{15.39}{2.29} = 6.72$$

Angle $\alpha_2' = 81.537°$

$$V_2 = \frac{V_{f2}}{\sin \alpha_2'} = \frac{15.39}{\sin 81.537°} = 15.56 \text{ m/s}$$

(a) Energy loss at the bucket $= \dfrac{v_{r1}^2}{2g} - \dfrac{v_{r2}^2}{2g} = \dfrac{(50)^2 - (45)^2}{2 \times 9.81} = 24.2\,\text{m}$

(b) Kinetic energy head of exit discharge from the buckets $= \dfrac{V_2^2}{2g} = \dfrac{(15.56)^2}{2 \times 9.81} = 12.34\text{ m}$

**EXAMPLE 4.11

The following data pertains to a Pelton turbine:

Bucket angle at bucket outlet = 165°	*Coefficient of velocity of the nozzle = 0.98*
Net head = 340 m	*Jet diameter = 12 cm*
Pitch diameter of runner = 2.25 m	*Speed ratio = 0.46*

Due to bucket friction, the relative velocity at bucket exit is found to be 5% less than the relative velocity at inlet to the bucket. Calculate (a) water power, (b) speed of the turbine, and (c) power lost in discharge.

Solution

$$V_1 = C_v\sqrt{2gH} = 0.98 \times \sqrt{2\times 9.81\times 340} = 80.04 \text{ m/s}$$

$$u = K_u\sqrt{2gH} = 0.46 \times \sqrt{2\times 9.81\times 340} = 37.57 \text{ m/s}$$

(a) Since $u = \dfrac{\pi DN}{60}$, speed $N = \dfrac{60u}{\pi D} = \dfrac{60\times 37.57}{\pi\times 2.25} = 318.9$ rpm

(b) $A_1 = \dfrac{\pi d^2}{4} = \dfrac{\pi(0.12)^2}{4} = 0.01131\text{ m}^2$

Discharge $Q = A_1V_1 = 0.01131 \times 80.04 = 0.9053\text{ m}^3/\text{s}$

Water power = $P_{ideal} = \gamma QH = 9.79 \times 0.9053 \times 340 = 3013$ kW

(c) Relative velocity $v_{r1} = (V_1 - u) = (80.04 - 37.57) = 42.47$ m/s

Since friction coefficient $K = (1 - 0.05) = 0.95$, $v_{r2} = 0.95v_{r1}$

$= 0.95 \times 42.47 = 40.347$ m/s

From outlet velocity triangle: $v_{r2}\cos\beta_2' = u + V_{u2}$, (Fig. 4.21)

Velocity of whirl at the outlet = $V_{u2} = v_{r2}\cos\beta_2' - u$

$= 40.\ 347\cos 15° - 37.57 = 1.4017$ m/s

$V_{f2} = v_{r2}\sin\beta_2' = 40.37\sin 15° = 10.443$ m/s

Absolute velocity of exit flow

$$V_2 = \sqrt{V_{f2}^2 + V_{u2}^2} = \sqrt{(10.443)^2 + (1.4017)^2} = 10.536 \text{ m/s}$$

Power lost in discharge $= P_d = \gamma Q\dfrac{V_2^2}{2g} = 9.79\times 0.9053\times\dfrac{(10.536)^2}{2\times 9.81} = 52.79$ kW

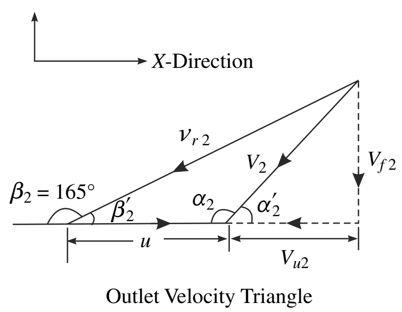


Fig. 4.21 *Velocity triangle, Example 4.11*

**EXAMPLE 4.12

A Pelton wheel having a diameter of 1.25 m operates under a net head of 200 m. The other characteristics of the installation are the following:

C_v = 0.98	*Operational speed = 250 rpm*	*Diameter of the jet = 10 cm*
Bucket angle at outlet = 162°	*Bucket friction coefficient = K = 0.96*	*Mechanical efficiency = 0.96*

Determine the (a) hydraulic efficiency (b) power delivered to the shaft, and (c) specific speed in SI units and nondimensional specific speed in radians.

Solution

$$\beta_2' = 180 - 162 = 18°$$

Velocity of jet $V_1 = C_v \sqrt{2gH} = 0.98 \times \sqrt{2 \times 9.81 \times 200} = 61.39$ m/s

Discharge $Q = \frac{\pi}{4} d^2 V_1 = \frac{\pi}{4} (0.10)^2 \times 61.39 = 0.482$ m^3/s

$$u = \frac{\pi DN}{60} = \frac{\pi \times 1.25 \times 250}{60} = 16.36 \text{ m/s}$$

Euler head $H_e = \frac{1}{g} u (V_1 - u)(1 + K \cos \beta_2')$

$$= \frac{1}{9.81} \times 16.36 (61.39 - 16.36)(1 + 0.96 \cos 18°)$$

$$= 143.8 \text{ m}$$

Hydraulic efficiency $\eta_h = \frac{H_e}{H} = \frac{143.6}{200} = 0.718$

Overall efficiency $\eta_0 = (\eta_m \eta_h) = 0.718 \times 0.96 = 0.689$

Shaft power $P = \eta_0 \gamma QH = 0.689 \times 9.79 \times 0.482 \times 200 = 650.5$ kW

Specific speed (in *SI* units) = $N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{250 \times \sqrt{650.5}}{(200)^{5/4}} = 8.48$

Specific speed in dimensionless form: Shape factor S_{pr} (in radians) is given by [Eq. 1.50(a)] as

$S_{pr} = 2\pi \times 9.61 \times 10^{-4} \times N_s = 2\pi \times 9.61 \times 10^{-4} \times 8.48 = 0.0512$ radians

**EXAMPLE 4.13

A double overhung impulse turbine installation is to develop 15 MW at 260 rpm under a net head of 350 m. (a) Determine the specific speed and wheel pitch diameter when one jet is used in each turbine. (b) What would these values be when instead of two turbines (i) a single wheel with a single jet is used, and (ii) a single wheel with four jets is used? Assume speed ratio = 0.46. (c) Taking the overall efficiency as 0.90 in all cases, calculate the discharge and jet diameter in all the three cases referred above. Assume C_v = 0.98 in all the cases.

Solution

Case-1. In this double overhung system, there are two single jet turbines.

Power per turbine P = 15000/2 = 7500 kW

Specific speed $N_s = \dfrac{N\sqrt{P}}{H^{5/4}} = \dfrac{260\times\sqrt{7500}}{(350)^{5/4}}$ = 14.87

$u = K_u \sqrt{2gH}$ = 0.46 × $\sqrt{2\times 9.81\times 350}$ = 38.12 m/s

Since $u = \dfrac{N\sqrt{P}}{H^{5/4}}$,

Diameter of wheel $D = \dfrac{60\times u}{\pi N} = \dfrac{60\times 38.12}{\pi\times 260}$ = 2.8 m

Discharge per wheel $Q = \dfrac{P}{\eta_0 \gamma H} = \dfrac{7500}{0.9\times 9.79\times 350}$ = 2.432 m^3/s

Total discharge = Q_t = 2 × 2.432 = 4.864 m^3/s,

Velocity of jet $V_1 = C_v \sqrt{2gH}$ = 0.98 × $\sqrt{2\times 9.81\times 350}$ = 81.21 m/s

$$Q = A\times V_1 = \frac{\pi}{4} d^2 \times 81.21 = 2.432$$

$$A = 0.02994 \text{ m}^2$$

Diameter of jet $d = \sqrt{\dfrac{4A_1}{\pi}} = \sqrt{\dfrac{4\times 0.02994}{\pi}}$ = 0.195 m

Case-2: Single wheel with a single nozzle:

Diameter of wheel remains the same at 2.8 m.

Velocity of jet remains same at V_1 = 81.21 m/s

P = 15,000 kW, H = 350 m and N = 260 rpm

$N_s = \dfrac{N\sqrt{P}}{H^{5/4}} = \dfrac{260\times\sqrt{15000}}{(350)^{5/4}}$ = 21.03

Discharge in the nozzle = Q_t = 4.864 m^3/s

$A_1 = \dfrac{Q_t}{V_1} = \dfrac{4.864}{81.21}$ = 0.0599 m^2

Diameter of jet $d_1 = \sqrt{\dfrac{4A_1}{\pi}} = \sqrt{\dfrac{4\times 0.0599}{\pi}}$ = 0.2762 m

Case-3: Single wheel with four nozzles (the suffix 4 is used to represent this case)

Power per nozzle P_4 = 15000/4 = 3750 kW

$N_s = \dfrac{N\sqrt{P_4}}{H^{5/4}} = \dfrac{260\times\sqrt{3750}}{(350)^{5/4}} = 10.52$

Discharge per nozzle Q_4 = 4.864/4 = 1.216 m^3/s

$$A_4 = \frac{Q_4}{V_1} = \frac{1.216}{81.21} = 0.015 \text{ m}^2$$

Diameter of jet $d_4 = \sqrt{\frac{4A_4}{\pi}} = \sqrt{\frac{4 \times 0.015}{\pi}} = 0.138$ m

**EXAMPLE 4.14

A Pelton wheel is working under a head of 180 m with a discharge of 0.8 m³/s. The following data are available for the turbine:

Coefficient of velocity = 0.985	*Angle of deflection of jet = 165°*
Speed ratio = 0.46	*Relative velocity at exit = Relative velocity at inlet*
Jet ratio = 12	

Compute the (a) hydraulic efficiency, (b) velocity of whirl at inlet and outlet, and (c) mean bucket speed. Assume zero frictional loss in the bucket.

Solution

$\beta_2' = 180 - 165 = 15°$, $H = 180$ m.

Velocity of jet $V_1 = C_v \sqrt{2gH} = 0.985 \times \sqrt{2 \times 9.81 \times 180} = 58.54$ m/s

$u = K_u \sqrt{2gH} = 0.46 \times \sqrt{2 \times 9.81 \times 180} = 27.34$ m/s

$\frac{u}{V_1} = \frac{27.34}{58.34} = 0.467$

(a) Hydraulic efficiency by Eq. (4.25-b), $\eta_h = 2C_v^2 \left(\frac{u}{V_1}\right)\left(1 - \frac{u}{V_1}\right)(1 + K \cos \beta_2')$

Since there are no frictional losses in the buckets $K = 1.0$

Hydraulic efficiency $\eta_h = 2 \times (0.985)^2 \times (0.467) \times (1 - 0.467)\ (1 + \cos 15°)$

$= 1.940 \times (0.467) \times (0.533) \times (1.966) = 0.9495$

(b) Figure 4.22 shows the velocity diagrams for the inlet and out of the bucket.

For the inlet: Velocity of whirl = $V_{u1} = V_1 = 58.54$ m/s

Relative velocity $v_{r1} = (V_1 - u) = (58.54 - 27.34) = 31.20$ m/s

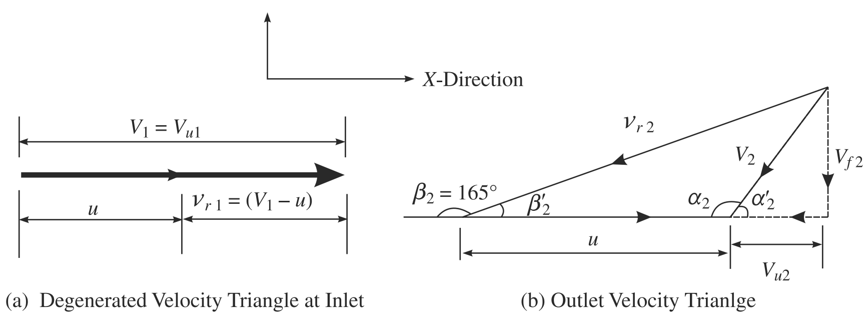


Fig. 4.22 *Velocity triangles, Example 4.14*

Since there is no friction in the buckets, relative velocities $v_{r1} = v_{r2}$

From outlet velocity triangle: $v_{r2} \cos \beta_2' = u + V_{u2}$

Velocity of whirl at the outlet = $V_{u2} = v_{r2} \cos \beta' - u$

V_{u2} = 31.20 cos 15° – 27.34 = 2.797 m/s (in the direction of (– u))

(c) Q = Discharge = 0.8 m^3/s,

Area of jet $A_1 = \dfrac{Q.}{V_1} = \dfrac{0.8}{58.54} = 0.01367\ \text{m}^2$

Diameter of the jet $d = \sqrt{\dfrac{4A_1}{\pi}} = \sqrt{\dfrac{4 \times 0.01367}{\pi}} = 0.132$ m

$D/d = 12$

Hence, $D = 12\ d = 12 \times 0.132 = 1.583$ m

Rotational speed $N = \dfrac{60 \times u}{\pi D} = \dfrac{60 \times 27.34}{\pi \times 1.583} = 330$ rpm

**EXAMPLE 4.15

A three-jet Pelton turbine is required to generate 10 MW under a net head of 400 m. The bucket angle at the outlet is 165° and the decrease in the relative velocity while passing over the bucket is 5%. Given that:

Overall efficiency = 80%	*Coefficient of velocity = 0.98*	*Coefficient of velocity = 0.98*

(a) Determine (i) the diameter of the jet, and (ii) the force exerted by the jet on the buckets. (b) If the jet ratio is not to be less than 10, calculate the highest speed of the wheel for a frequency of 50 Hz and the corresponding wheel diameter.

Solution

Given: Number of jets = 3, H = 400 m, β_2' = 15° and bucket friction coefficient $K = 0.95$, $C_v = 0.98$, $K_u = 0.46$

(a) Power of 3 jets = P_{total} = 10 MW = 10,000 kW

Velocity of jet $V_1 = C_v \sqrt{2gH} = 0.98 \times \sqrt{2 \times 9.81 \times 400} = 86.82$ m^3/s

$u = K_u \sqrt{2gH} = 0.46 \times \sqrt{2 \times 9.81 \times 400} = 40.75$ m/s

Let the suffix 1 refer to properties of one jet.

(i) P_1 = power per jet = 10000/3 = 3333 kW. Also, $P_1 = \eta_0\, \gamma Q_1 H$

Hence $P_1 = 3333 = 0.8 \times 9.79 \times Q_1 \times 400$

$Q_1 = 1.064$ m^3/s and total discharge $Q_t = 3 \times 1.064 = 3.192$ m^3/s

Area of one jet $A_1 = \dfrac{Q_1}{V_1} = \dfrac{1.064}{86.82} = 0.01226\ \text{m}^2$

Diameter of jet $d = \sqrt{\dfrac{4A_1}{\pi}} = \sqrt{\dfrac{4 \times 0.01226}{\pi}}$ = 0.125 m

(ii) Force on the buckets due to three jets in the tangential direction, Eq. (4.21):

$F_x = \rho Q\,(V_1 - u)(1 + K \cos \beta_2')$

$F_x = 0.998 \times 3.192 \times (86.82 - 40.75) \times (1 + 0.95 \cos 15°) = 281.4$ kN

(b) Let D/d = 10, then $D = 10\,d = 10 \times 0.125 = 1.25$ m

Since $u = \dfrac{\pi DN}{60}$, speed $N = \dfrac{60u}{\pi D} = \dfrac{60 \times 40.75}{\pi \times 1.25}$ = 622.6 rpm

Synchronous speed $N_g = \dfrac{120 f}{p}$

Hence, number of poles $p = \dfrac{120 \times 50}{622}$ = 9.646. Select p = 10 to get

$N = N_g = \dfrac{120 f}{p} = \dfrac{120 \times 50}{10}$ = 600 rpm

Revised diameter of the wheel $D = \dfrac{60u}{\pi N} = \dfrac{60 \times 40.75}{\pi \times 600}$ = 1.30 m

New D/d = 1.30/0.125 = 10.4 > 10, hence OK.

**EXAMPLE 4.16

A Pelton turbine is to produce 20 MW under a head of 480 m and speed of 500 rpm. If the jet ratio is to be in the range 9 to 12, determine the number of jets required. Take overall efficiency = 85%, Coefficient of velocity = 0.97 and speed ratio = 0.46.

Solution

$u = K_u \sqrt{2gH} = 0.46 \times \sqrt{2 \times 9.81 \times 480}$ = 44.64 m/s

Diameter of wheel $D = \dfrac{60u}{\pi N} = \dfrac{60 \times 44.64}{\pi \times 500}$ = 1.705 m

Select $\dfrac{D}{d}$ = 10, then $d = \dfrac{1}{10} D$ = 0.1705 m/s

$V_1 = C_v \sqrt{2gH} = 0.97 \times \sqrt{2 \times 9.81 \times 480}$ = 94.133 m/s

Discharge per jet = $Q_1 = A_1 V_1 = \dfrac{\pi}{4} (0.1705)^2 \times 94.133 = 2.150$ m^3/s

$P = \eta_0\, \gamma QH = 0.85 \times 9.79 \times Q \times 480 = 20000$

Discharge Q = 5.007 m^3/s

Number of jets required = $Q/Q_1 = \dfrac{5.007}{2.150} \approx 2$. Hence Select 2 nozzles

Revised Calculations:

New discharge per jet Q_1= 5.007/2 = 2.5036 m^3/s

$$A_1 = \frac{Q_1}{V_1} = \frac{2.5036}{94.133} = 0.0266$$

Diameter of the jet $d = \sqrt{\frac{4A_1}{\pi}} = \sqrt{\frac{4\times 0.0266}{\pi}}$ = 0.184 m = 18.4 cm

Specific speed (on power per jet basis)

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{500\times\sqrt{20000/2}}{(480)^{5/4}} = 22.25$$

This is within the usual limits and hence OK.

Actual $\frac{D}{d} = \frac{1.705}{0.184} = 9.27$. This is within limits prescribed in the problem and hence OK.

***EXAMPLE 4.17

A Pelton turbine has the following data:

Diameter of the wheel = 3.30 m	*Speed = 250 rpm*
Coefficient of velocity = 0.97	*Nozzle diameter = 150 mm*
C_c (Coefficient of contraction) of the nozzle = 0.85	

When the turbine is operating at the theoretical condition of maximum efficiency, find the head required at the base of the nozzle. Also, if the specific speed of the turbine is 9.2, estimate the maximum overall efficiency.

Solution

Diameter of nozzle d_n = 150 mm

$$A_n = \frac{\pi}{4}d_n^2 = \frac{\pi}{4}(0.15)^2 = 0.01767\ m^2$$

Since C_c = 0.85, area of jet A_1= 0.85 × 0.01767 = 0.015 m^2

Ideal theoretical condition for maximum efficiency: $\frac{u}{V_1} = 0.5$

$$u = \frac{\pi DN}{60} = \frac{\pi\times 3.3\times 250}{60} = 43.97\ m/s$$

$$V_1 = 2u = 2\times 43.197 = 86.394\ m/s$$

Since $V_1 = C_v\sqrt{2gH}$ the net head H is given by

$$H = \frac{V_1^2}{2gC_v^2} = \frac{(86.394)^2}{2\times 9.81\times(0.97)^2} = 404.3\ m$$

Specific speed $N_s = \frac{N\sqrt{P}}{H^{5/4}} = 9.2$

$$\sqrt{P} = \frac{9.2 \times (404.3)^{5/4}}{250} = 66.72$$

Power $P = 4451$ kW

Discharge $Q = A_1 V_1 = 0.015 \times 86.394 = 1.296$ m^3/s

Overall efficiency $\eta_0 = \dfrac{P}{\gamma QH} = \dfrac{4451}{9.79 \times 1.296 \times 404.3} = 0.868 = 86.8\%$

***EXAMPLE 4.18

In a Pelton turbine, V_1 = absolute velocity of the jet, C_v = coefficient velocity of the nozzle, u = bucket speed and β' = (180 – β) where β is the bucket angle, defined as deflection angle of the relative velocity of the jet at the bucket. If friction in the buckets reduces relative velocity by a factor K and if the windage losses reduce the effectiveness of the affected parameters by a factor k_1

(a) show that the resulting overall efficiency is

$$\eta_0 = \frac{2C_v^2\left[u(V_1 - u)(1 + K\cos\beta') - k_1 u^2\right]}{V^2}$$

(b) Further, show that the condition for maximum efficiency is given by

$$\varepsilon = \frac{1 + K\cos\beta'}{2(1 + k_1 + K\cos\beta')} \text{ where } \varepsilon = \frac{u}{V_1}$$

Solution

Rate of work done by the deflection of the jet at the moving buckets considering the bucket friction only,

P_1 = (Change of momentum flux in the tangential direction) × (Velocity of the buckets)

$$P_1 = \rho\, Q\, u\, [(V_1 - u) + (V_1 - u)\, K \cos\beta']$$
$$= \rho\, Q\, u\, [(V_1 - u)\,(1 + K\cos\beta')]$$

Loss of energy per unit weight of water due to bearing friction and windage = $F(u) = k_1 u^2$

Power lost due to windage and friction = $\rho Q k_1 u^2$

Net power produced $P = \rho\, Q\, [u\, (V_1 - u)(1 + K\cos\beta') - k_1 u^2]$

Ideal power (water power) = $P_{ideal} = \gamma QH$

Overall efficiency =

$$\eta_0 = \frac{\rho Q\left[u(V_1 - u)(1 + K\cos\beta') - k_1 u^2\right]}{\gamma QH} = \frac{u\left[(V_1 - u)(1 + K\cos\beta') - k_1 u^2\right]}{2gH}$$

Since $V_1 = C_v\sqrt{2gH}$, the net head is given by $H = \dfrac{V_1^2}{2gC_v^2}$.

Substituting in the expression for η_0,

$$\eta_0 = \frac{2C_v^2\left[u(V_1-u)(1+K\cos\beta')-k_1u^2\right]}{V_1^2}$$

$$\eta_0 = 2C_v^2\left[\frac{u}{V_1}\left(1-\frac{u}{V_1}\right)(1+K\cos\beta')-k_1\left(\frac{u}{V_1}\right)^2\right]$$

Putting $\varepsilon = \frac{u}{V_1}$, $\eta_0 = 2C_v^2\,[\varepsilon(1-\varepsilon)(1+K\cos\beta')-k_1\varepsilon^2]$

For maximum efficiency $\frac{d\eta_0}{d\varepsilon} = 0$

$$\frac{d\eta_0}{d\varepsilon} = 2C_v^2\,[(1+K\cos\beta')(1-2\varepsilon)-2k_1\varepsilon] = 0$$

$$(1+K\cos\beta')(1-2\varepsilon) = 2k_1\varepsilon$$

$$2\varepsilon\,[k_1+(1+K\cos\beta')] = 1+K\cos\beta'$$

$$\varepsilon = \frac{1+K\cos\beta'}{2\left[1+k_1+K\cos\beta'\right]}$$

**EXAMPLE 4.19

The following data pertains to a single-jet Pelton turbine:

Jet diameter = 10 cm	*Jet ratio = 12*
Coefficient of velocity of the nozzle = 0.98	*Speed ratio = 0.46*
Bucket friction coefficient = 0.95	*Jet deflection angle at the bucket = 165°*
Net head = 150 m	

Calculate the torque (a) at the start, and (b) at normal speed.

Solution

$D/d = 12$. Hence, $D = 12\,d = 12 \times 0.10 = 1.20$ m

Area of jet $A_1 = \frac{\pi}{4}d^2 = \frac{\pi}{4} \times (0.10)^2 = 0.007854\ \text{m}^2$

Velocity of jet = $V_1 = C_v\sqrt{2gH} = 0.98 \times \sqrt{2\times 9.81\times 150} = 53.16$ m/s

Discharge $Q = A_1V_1 = 0.007857 \times 53.16 = 0.4176\ \text{m}^3/\text{s}$

By Eq. (4.22),

Torque $T = \frac{\rho QD}{2}(V_1-u)(1+K\cos\beta_2')$

$$\beta_2' = (180-\beta_2) = 180-165 = 15°$$

$$T = \frac{0.998\times 0.4176\times 1.2}{2}(53.164-u)\,[1+(0.95)(\cos 15°)]$$

$$T = 0.4794(53.164-u)$$

At start, $u = 0$ and hence

Torque $T = 0.4794\,(53.164 - 0) = 25.49$ kNm

At normal speed, $u = K_u \sqrt{2gH} = 0.46 \times \sqrt{2 \times 9.81 \times 150} = 24.955$ m/s

Torque $T = 0.4794\ (53.164 - 24.955) = 13.52$ kN.m

**EXAMPLE 4.20

A Pelton turbine of 1.5 m diameter has a speed of 450 rpm. The net head is 300 m and the discharge is 0.5 m^3/s. Following are the other relevant parameters;

Coefficient of velocity = 0.985	*Bucket angle = 165^o*
Mechanical efficiency = 0.90	*Bucket friction factor K = 1.0*

Calculate the (a) Torque at the start and at normal speed, and (b) Theoretical runaway speed.

Solution

$$V_1 = C_v \sqrt{2gH} = 0.985 \times \sqrt{2 \times 9.81 \times 300} = 75.6 \text{ m/s}$$

$$\text{Tangential velocity of wheel } u = \frac{\pi DN}{60} = \frac{\pi \times 1.5 \times 450}{60} = 35.34 \text{ m/s}$$

$$K_u = \frac{u}{\sqrt{2gH}} = \frac{35.34}{\sqrt{2 \times 9.81 \times 300}} = 0.46$$

(a) Torque $T = \eta_m\, \rho\, Q\, \dfrac{D}{2}(V_1 - u)(1 + K\cos\beta_2')$ (From Eq. 4.31)

$$K = 1.0 \text{ and } \beta_2' = 180 - 165 = 15°$$

$$T = 0.90 \times 0.998 \times 0.5 \times \frac{1.5}{2} \times (75.6 - u)\ (1 + \cos 15°)$$

$$= 0.662\ (75.6 - u)$$

At start, $u = 0$ and $T = 0.662 \times 75.6 = 50.06$ kN.m

At normal speed , $u = 35.34$ m/s and $T = 0.662\ (75.6 - 35.34) = 26.65$ kN.m

(b) Theoretical runaway speed $u_R = \left(\dfrac{C_v}{K_u}\right) u$ (by Eq. 4.32)

or $$N_R = \left(\frac{C_v}{K_u}\right) N$$

$$N_R = \left(\frac{0.985}{0.46}\right) \times 450 = 963.6 \text{ rpm}$$

**EXAMPLE 4.21

A Pelton turbine is to develop 13,500 kW of power under a net head of 820 m while running at a speed of 600 rpm. If the coefficient of velocity of the nozzle is 0.98 and the jet ratio is 16, calculate the (a) quantity of water supplied to the turbine per second (b) diameter of the pitch circle, and (c) the number of jets required. Take overall efficiency of the turbine as 0.85.

Solution

Total power = 13500 kW

$$\text{Specific speed} = N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{600 \times \sqrt{3500}}{(820)^{5/4}} = 15.9$$

This is in the good range of 13 to 23 and near the best value of 17 and hence one jet would suffice.

$$V_1 = C_v \sqrt{2gH} = 0.98 \times \sqrt{2 \times 9.81 \times 820} = 124.3 \text{ m/s}$$

$$\text{Power } P = \eta_0\, \gamma QH = 0.85 \times 9.79 \times Q \times 820 = 13500$$

$$\text{Discharge } Q = 1.978 \text{ m}^3/\text{s}$$

$$\text{Area of jet } A_1 = \frac{Q}{V_1} = \frac{1.978}{124.3} = 0.0159 \text{ m}^2$$

$$\text{Diameter of the jet } d = \sqrt{\frac{4A_1}{\pi}} = \sqrt{\frac{4 \times 0.0159}{\pi}} = 0.142 \text{ m} = 14.2 \text{ cm}$$

Given D/d = 16.

Hence, diameter of pitch circle $D = 16\, d = 16 \times 0.142 = 2.278$ m

**EXAMPLE 4.22

A hydro project has a potential of 90 MW to be developed under a head of 350 m. A speed of 300 rpm is preferred. Identify a few alternatives of combinations of Pelton turbines that can be used in this project.

Solution

Let the preferred specific speed of each of the jets be 17. Also, let there be a total of x number of jets in the turbines that will be deployed.

Total power potential = 90,000 kW

Power per jet = 90,000/x

$$\text{Specific speed per jet} = N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{300 \times \sqrt{(90000/x)}}{(350)^{5/4}} = 17$$

$$\left(\frac{90{,}000}{x}\right)^{1/2} = 85.78 \text{ and } x = 12.22$$

Select 12 jets, all of them being of the same size. Consider them as 12 equivalent single jet turbines. Then the specific speed of each single jet turbine would be

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{300 \times \sqrt{(90000/12)}}{(350)^{5/4}} = 17.16$$

This number being in the acceptable range of 13 to 23 and close to the peak efficiency point of $N_s = 17$, the selection of 12 equivalent single jets for the given power development is OK.

Any of the following combinations of multijet homologous Pelton turbines would be acceptable. All the jets must be of the same size and should contribute to the same specific speed of 17.16 per jet.

(a) 6 Nos of 2 jet units

(b) 4 Nos of 3 jet units

(c) 3 Nos of 4 jet units

(d) 2 Nos of 6 jet units

**EXAMPLE 4.23

A Pelton turbine is producing a shaft power of 1.0 MW while working under a net head of 150 m. Following are some relevant data of the turbine:

Speed = 480 rpm	*Bucket angle = 165°*
Coefficient of velocity = 0.97	*Friction and windage losses = 5% of velocity head of the jet*
Bucket friction coefficient = 0.9	*Pitch diameter of runner = 1.2 m*

Draw the outlet velocity triangle and prepare an energy balance of the jet and calculate (a) hydraulic efficiency, (b) mechanical efficiency, and (c) overall efficiency of the turbine.

Solution

(a) $V_1 = C_v \sqrt{2gH} = 0.97 \times \sqrt{2 \times 9.81 \times 150} = 52.62$ m/s

$$\frac{V_1^2}{2g} = \frac{(52.62)^2}{2 \times 9.81} = 141.13 \text{ m}$$

Head loss in the nozzle H_N = 150.00 − 141.13 = 8.87 m

(b) Friction and windage losses = $H_{fw} = \frac{5}{100} \times 141.13 = 7.06$ m

(c) $u = \frac{\pi DN}{60} = \frac{\pi \times 1.2 \times 480}{60} = 30.16$ m/s

Relative velocity at inlet = $v_{r1} = (V_1 - u)$ = 52.62 – 30.16 = 22.46 m/s

Relative velocity at exit = $v_{r2} = K\, v_{r1}$ = 0.9 × 22.46 = 20.215 m/s

Energy loss in the buckets

$$= H_b = \frac{v_{r1}^2}{2g} - \frac{v_{r2}^2}{2g} = \frac{1}{(2 \times 9.81)}\left[(22.46)^2 - (20.215)^2\right] = 4.88 \text{ m}$$

From outlet velocity triangle: (Fig. 4.23)

$v_{r2} \cos \beta_2' = 20.215 \cos 15° = 19.526$ m

$V_{f2} = v_{r2} \sin\beta_2 = 20.215 \times \sin 15° = 5.232$ m/s

Since $v_{r2} \cos \beta_2' < u$, the outlet velocity triangle is an acute-angled triangle (Fig. 4.23).

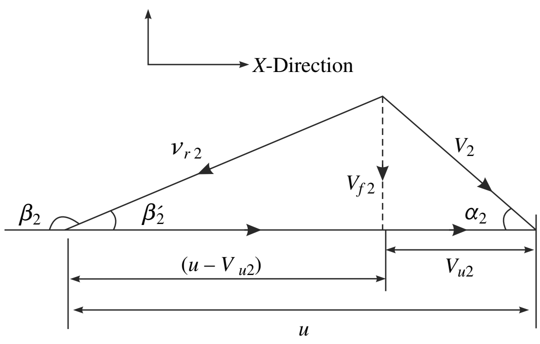


Fig. 4.23 *Outlet velocity triangle, Example 4.23*

Hence, $(u - V_{u2}) = v_{r2} \cos \beta_2' = 19.52$ m

$$V_{u2} = u - v_{r2} \cos \beta_2'$$

$$V_2^2 = (V_{u2})^2 + (V_{f2}^2) = (30.16 - 19.526)^2 + (5.232)^2 = 140.45$$

$$V_2 = 11.85 \text{ m/s}$$

Velocity head of discharge at the exit $= \dfrac{V_2^2}{2g} = \dfrac{140.45}{2 \times 9.81} = 7.16$ m

Energy Balance and Efficiencies

Input head = Net head = 150.00 m	Losses:	(m)
	At nozzle	8.86
	Friction and windage	7.06
	At buckets	4.88
	Discharge energy head	7.16
	Total losses	**27.96**
Shaft energy = Input energy – Total losses = 150.00 – 27.96 = **122.04 m**		

Hydraulic efficiency	[150.0 – (8.86 + 4.88 + 7.16)]/150.0 = (129.10/150)	0.8606
Mechanical efficiency	(129.10 – 7.06)/129.10 = (122.04/129.1)	0.9453
Overall efficiency	(122.04/150.00)	81.36

**EXAMPLE 4.24

The following data were collected from a Pelton turbine:

Head at the base of the nozzle = 200 m	*Discharge from the nozzle = 0.5 m^3/s*
Area of the jet = 8300 mm^2	*Power available at the shaft = 820 kW*
Mechanical efficiency = 93%	

(a) Calculate the power loss (i) in the nozzle, (ii) in the runner (including the energy of discharge from buckets), and (iii) in mechanical friction and windage.
(b) Calculate (i) nozzle efficiency, (ii) wheel efficiency, and (iii) overall efficiency.

Solution

Power at base of nozzle (water power) = γQH

$= 9.79 \times 0.5 \times 200 = 979$ kW

Velocity of jet $= V_1 = \dfrac{Q}{A_1} = \dfrac{0.5}{0.0083} = 60.25$ m/s $\qquad \dfrac{V_1^2}{2g} = \dfrac{(60.24)^2}{2 \times 9.81} = 185.0$ m

(i) Kinetic energy of the jet = $\gamma Q \dfrac{V_1^2}{2g} = 9.79 \times 0.5 \times 185 = 905.6$ kW

Power loss in the nozzle = 979.0 – 905.6 = 73.4 kW

Nozzle efficiency = $\eta_n = \dfrac{905.6}{979} = 0.925 = 92.5\%$

(ii) Power input into the runner = Power of jet = 905.6 kW

Power developed by the runner = $\dfrac{\text{Power at shaft}}{\text{Mechanical efficiency}} = \dfrac{820.0}{0.93} = 881.7$ kW

Power loss in runner (including the energy of discharge from bucket)

= 905.6 – 881.7 = 23.9 kW

Wheel efficiency $\eta_w = \dfrac{881.7}{905.6} = 0.975 = 97.4\%$

(iii) Power lost in mechanical friction = 881.7 – 820.0 = 61.7 kW

(iv) Hydraulic efficiency = $\eta_h = \eta_n \, \eta_w = 0.925 \times 0.974 = 0.90 = 90\%$

(v) Overall efficiency = $\eta_0 = \eta_m \, \eta_h = 0.93 \times 0.90 = 0.837 = 83.7\%$

Check: Overall efficiency $\eta_0 = \dfrac{\text{Shaft power}}{\text{Water power}} = \dfrac{820}{979} = 0.837 = 83.7\%$

*EXAMPLE 4.25

A Pelton wheel develops power P_1 under a head H_1. (a) If the power is reduced to P_2 by needle control valve, estimate the percentage reduction in the discharge and head on the turbine when $P_2 = 0.75\ P_1$. (b) If the reduction in power to $P_2 = 0.75\ P_1$ is achieved through partial closure of the main control valve in the penstock without any change in the needle valve setting, determine the percentage reduction in the discharge and head on the turbine. Assume that the overall efficiency of the turbine and the coefficient of velocity of the valve remain unchanged in both the operations.

Solution

Case-1: In the needle valve operation, only the discharge is changed due to reduction in the area of the jet while the head on the turbine remains unchanged. Let the suffix 2 denote the conditions after the reduction of power.

$P_2 = 0.75\ P_1$

$P_2 = \eta_0\ \gamma Q_2\ H_2$ and $P_1 = \eta_0\ \gamma Q_1\ H_1$

$$\therefore \quad \frac{Q_2}{Q_1}\frac{H_2}{H_1} = \frac{P_2}{P_1} = 0.75$$

But $H_2 = H_1$ and hence $\dfrac{Q_2}{Q_1} = 0.75$

Thus, in needle valve control $Q_2 = 0.75\ Q_1$ and $H_2 = H_1$.

Case-2: In the operation of main control valve, throttling of flow takes place and the reduction of power is achieved through reduction of discharge and corresponding reduction of head. Thus, both the head and the discharge would undergo change.

$P_2 = 0.75\ P_1$

$P_2 = \eta_0\ \gamma Q_2\ H_2$ and $P_1 = \eta_0\ \gamma Q_1\ H_1$

$$\therefore \quad \frac{Q_2}{Q_1}\frac{H_2}{H_1} = \frac{P_2}{P_1} = 0.75$$

But $Q = A_j V_j = A_j\ C_v\sqrt{2gH}$, where A_j refers to jet area.

Hence, $Q \propto \sqrt{H}$ and thus $\dfrac{H_2}{H_1} = \left(\dfrac{Q_2}{Q_1}\right)^2$

$$\therefore \quad \frac{P_2}{P_1} = \frac{Q_2}{Q_1}\frac{H_2}{H_1} = \left(\frac{Q_2}{Q_1}\right)^3 = 0.75 \text{ and } \frac{Q_2}{Q_1} = 0.9085$$

$$\frac{H_2}{H_1} = \left(\frac{Q_2}{Q_1}\right)^2 = (0.9085)^2 = 0.953$$

Thus, in main valve control $Q_2 = 0.9085\ Q_1$ and $H_2 = 0.953\ H_1$.

**EXAMPLE 4.26

For a Pelton turbine the bucket friction coefficient $K = 0.95$, coefficient of velocity $C_v = 0.93$ and the speed ratio $K_u = 0.46$. Find the deflection angle β_2 of the relative velocity of the jet at the exit that would cause zero velocity of whirl at the exit of the bucket.

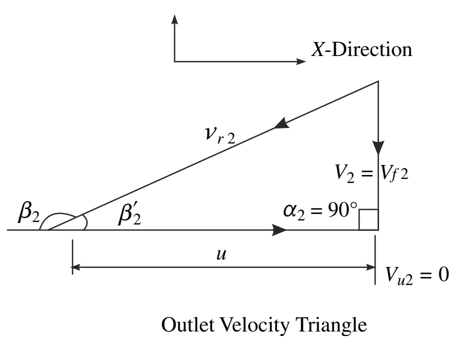


Fig. 4.24 *Velocity triangle, Example 4.26*

Solution

Condition for zero velocity of whirl at the exit of the bucket is $\alpha_2 = 90°$.

At this condition, $v_{r2} \cos \beta_2' = u$ where $v_{r2} = K(V_1 - u)$ = Relative velocity at the exit of the bucket, (See Fig. 4.24). In this, β_2 = Bucket angle = Deflection angle of the relative velocity of jet at the exit of bucket and $\beta_2' = 180° - \beta_2$.

Hence, $K(V_1 - u) \cos \beta_2' = u$

$$\cos \beta_2' = \frac{u}{v_{r2}} = \frac{u}{K(V_1 - u)} = \frac{1}{K\left(\frac{V_1}{u} - 1\right)}$$

$$= \frac{1}{K\left(\frac{C_v}{K_u} - 1\right)} = \frac{1}{0.95\left(\frac{0.98}{0.46} - 1\right)} = 0.93162$$

$$\cos \beta_2' = 21.31°$$

$$\beta_2 = 180 - \beta_2' = 158.69°$$

**EXAMPLE 4.27

Show that the hydraulic efficiency of a Pelton turbine whose buckets are such that it causes zero whirl velocity at the bucket exit is given by the expression

$$\eta_h = 2K_u C_v$$

where K_u = speed ratio and C_v= coefficient of velocity at the nozzle.

Solution

Condition for zero velocity of whirl at the exit of the bucket is $\alpha_2 = 90°$.

At this condition, $v_{r2} \cos \beta_2' = u$ where $v_{r2} = K(V_1 - u)$ = relative velocity at the exit of the bucket, (See Fig. 4.25).

In this, β_2 = deflection angle of the relative velocity of jet at the bucket exit and $\beta_2' = 180° - \beta_2$.

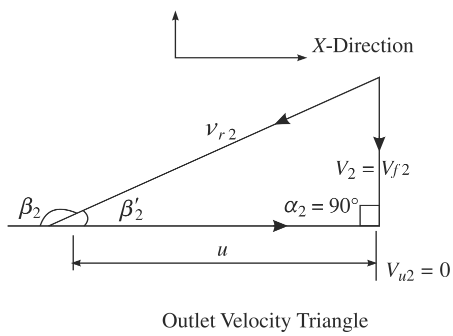


Fig. 4.25 *Velocity triangle, Example 4.27*

Hence, $K(V_1 - u) \cos \beta_2' = u$

$$\text{Cos } \beta_2' = \frac{u}{K(V_1 - u)} = \frac{\left(\frac{u}{V_1}\right)}{K\left(1 - \frac{u}{V_1}\right)} = \frac{\varepsilon}{K(1 - \varepsilon)}$$

where $\varepsilon = \frac{u}{V_1}$.

The hydraulic efficiency is given by Eq. (4.25b) as

$$\eta_h = 2C_v^2 \left(\frac{u}{V_1}\right)\left(1 - \frac{u}{V_1}\right)(1 + K \cos \beta_2')$$

$$\eta_h = 2C_v^2\varepsilon(1-\varepsilon)\left(1+K\frac{\varepsilon}{K(1-\varepsilon)}\right)$$

$$= 2C_v^2\varepsilon(1-\varepsilon)\left(1+K\frac{\varepsilon}{K(1-\varepsilon)}\right) = 2C_v^2\varepsilon$$

But $\varepsilon = \dfrac{u}{V_1} = \dfrac{K_u\sqrt{2gH}}{C_v\sqrt{2gH}} = \dfrac{K_u}{C_v}$

Hence, $\eta_h = 2C_v^2\dfrac{K_u}{C_v} = 2C_v K_u$.

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Review Questions

4.1. Differentiate between the following with the help of neat sketches:
(a) Net head in a Pelton turbine
(b) Net head in a reaction turbine

4.2. Give at least two major functional differences between a Pelton turbine and a reaction turbine.

4.3. Sketch the velocity triangles for the inlet and outlet of the buckets of a Pelton turbine and label all the salient velocities and angles. Indicate clearly the velocity of whirl at inlet and outlet. Include a list of description of the symbols used in the diagram.

4.4. Differentiate the following items with respect to a Pelton turbine:
(a) Power transmitted to the wheel by the jet, (Euler power)
(b) Water power
(c) Shaft power

4.5. Differentiate the following efficiencies in a Pelton turbine:
(a) Hydraulic efficiency
(b) Nozzle efficiency
(c) Wheel efficiency

4.6. What is the function of a casing in a Pelton turbine?

4.7. Explain briefly the concept of specific speed of Pelton turbines.

4.8. Describe the variation with speed of (a) Power, and (b) Torque in an ideal Pelton turbine.

4.9. Describe briefly the governing operation of a Pelton turbine.

4.10. Describe the complexities involved in governing of a multijet Pelton turbine.

4.11. What is the role played by the cut-away in the bucket of a Pelton turbine?

4.12. Describe briefly the cavitation problem in a Pelton turbine.

4.13. Describe briefly the erosion problem in a Pelton turbine.

4.14. Write brief notes on the following components of a Pelton turbine:
(a) Manifold (b) Deflector
(c) Auxiliary nozzle (d) Brake jet

4.15. Describe briefly (a) turgo wheel, and (b) cross-flow turbine.

4.16. List the comparative features of a Pelton turbine and a Francis turbine in the overlap region of specific speeds.

4.17. Compare and contrast the salient features of a turgo turbine and a Pelton turbine in the overlap region of specific speeds.

4.18. Describe the salient operating characteristics of a Pelton turbine.

4.19. Compare and contrast the efficiency characteristics of Francis and Pelton turbines, both belonging to the same overlap region of specific speeds.

4.20. Compare the main characteristics of Pelton turbine and Francis turbine.

4.21. Compare the salient features of the variation of unit discharge with unit speed, at constant head, of Pelton, Francis and Kaplan turbines.

4.22. Write a brief note on model testing of hydraulic turbines.

4.23. Describe briefly the essential components of a standard Turbine testing laboratory.

Problems

P4.1 *A Pelton wheel develops 3500 kW under a head of 220 m with an overall efficiency of 85%. Find the diameter of the jet if the coefficient of velocity of the nozzle is 0.98. [***Ans:*** d = 19.4 cm]

P4.2 *A Pelton wheel is working under a head of 45 m and the discharge is 0.8 m^3/s. The mean bucket speed is 14 m/s. Find the overall efficiency and the shaft power produced if the jet is deflected by the buckets through an angle of 165°. Assume the coefficient of velocity = 0.985 and mechanical efficiency = 0.95. The bucket friction coefficient can be assumed as 0.96. [***Ans:*** η_0 = 0.887, P = 312.6 kW]

P4.3 *The following data pertains to a Pelton turbine:

Speed ratio = 0.46	Coefficient of velocity of the nozzle = 0.97
Mechanical efficiency = 0.92	Hydraulic efficiency = 0.93
Pitch diameter of runner = 1.45 m	Jet diameter = 12 cm

Calculate the specific speed of the turbine. [***Ans:*** N_s = 17.10]

P4.4 *A double overhung 1.5 m diameter impulse turbine installation is to develop 3000 kW at 400 rpm under a net head of 270 m. If the overall efficiency is 0.90, determine the (a) diameter of the jet, (b) speed ratio, and (c) specific speed. Take C_v = 0.95. [***Ans:*** d = 0.1075 m, K_u = 0.432, N_s = 14.15]

P4.5 *A Pelton turbine is to operate under a net head of 500 m and at a speed of 420 rpm. If a single jet with 18 cm diameter is used, find the specific speed of the machine. Take C_v = 0.98, speed ratio = 0.46 and overall efficiency = 0.85. [***Ans:*** N_s = 18]

P4.6 *A Pelton wheel has a pitch-circle of 90 cm diameter and has one jet of diameter 8 cm with a coefficient of velocity of 0.97. The wheel has a speed ratio of 0.45 and a bucket angle of 170°. The bucket friction coefficient can be taken as 0.93. If the net head available on the wheel is 500 m, calculate (a) the power transferred to the wheel by the jet, (b) hydraulic efficiency and (c) specific speed of the wheel. Take mechanical efficiency of the wheel as 0.90. [***Ans:*** P_t = 2120, η_h = 0.897, N_s = 17.47]

P4.7 *A Pelton wheel in a laboratory has a mean bucket speed of 10 m/s with a jet of water flowing at a rate of 0.7 m^3/s under a head of 30 m. The bucket deflects the jet through an angle of 160°. Assuming C_v = 0.98 and bucket friction coefficient K = 0.95, calculate the shaft power and overall efficiency of the turbine. Assume mechanical efficiency = 0.93.
[***Ans:*** P = 169.4 kW, η_0 = 0.886]

P4.8 *Find the hydraulic efficiency of a Pelton turbine for which: C_v = 0.98, speed ratio = 0.46, bucket angle = 165°, ratio of outlet relative velocity to inlet relative velocity = 0.99. [***Ans:*** η_h = 0.936]

P4.9 *In a Pelton wheel the overall efficiency is 0.90, C_v = 0.95, speed ratio = 0.45 and the ratio of the wheel diameter to the jet diameter is 12. Calculate the specific speed of the wheel in *SI* units and in nondimensional form with revolutions as unit. [***Ans:*** N_s= 17.12, S = 0.01645 revolutions]

P4.10 *A Pelton wheel has two jets of 15 cm diameter each and develops 6500 kW of power. If the net available head on the turbine is 300 m determine the overall efficiency of the turbine. Also, if the rotational speed is 375 rpm, calculate the specific speed. Take C_v = 0.98. [***Ans:*** N_s = 17.12]

P4.11 *The water available to a powerhouse is 3.0 m^3/s and the total head from the reservoir to the nozzle is 250 m. There are three Pelton wheels of each having two jets. All the six jets have the same diameter and are supplied water from a single penstock of 500 m length. The efficiency of power transmission through the pipe is 90% and the overall efficiency of the turbine is 85%. The C_v for each nozzle is 0.95 and the Darcy–weisbach friction factor

f = 0.02. Determine the (a) Shaft power, (b) diameter of jet, and (c) diameter of the Penstock. [***Ans:*** P = 5617 kW, d = 10 cm, D_0 = 0.785 m]

P4.12 ** A Pelton turbine is to produce 6 MW of power under a net head of 300 m. A turbine of specific speed N_s = 18.5 is proposed. If the ratio of bucket velocity to jet velocity is 0.48 and the jet ratio is 13, determine the (a) speed, (b) wheel diameter, and (c) diameter of the jet. Assume overall efficiency = 0.88 [***Ans:*** N = 298.2, D = 2.485 m and d = 191 mm]

P4.13 * A multijet Pelton is to develop 46 MW of power at a speed of 300 rpm and net head of 360 m. Determine the number jets needed for this task. [***Ans:*** 6 jets]

P4.14 ** A Pelton turbine is to work at the foot of a dam whose reservoir level is 220 m. The head at the base of the nozzle at full nozzle opening is 200 m. The coefficient of velocity of the nozzle can be taken as 0.98. The turbine is to operate at 200 rpm and develop a power of 3.70 MW. Assuming the bucket to jet velocity ratio as 0.46, estimate the wheel diameter. If the bucket outlet angle is 16°, determine (a) the wheel efficiency, and (b) the hydraulic efficiency of the turbine. Neglect friction losses in the buckets. [***Ans:*** η_h = 0.936, η_w = 0.947]

P4.15 ** A Pelton wheel is working under a head of 250 m. Following are some relevant data of the turbine:

Coefficient of velocity = 0.98	Angle of deflection of jet = 170°
Speed ratio = 0.46	Bucket friction coefficient, K = 0.95
Mechanical efficiency = 0.96	

Compute the (a) hydraulic efficiency, (b) Wheel efficiency, (c) nozzle efficiency, and (d) Overall efficiency of the turbine. [***Ans:*** η_h = 0.926, η_w = 0.964, η_n = 0.960, η_0 = 0.889]

P4.16 ** A Pelton wheel is working under a net head of 200 m. Relevant data are as given below:

Coefficient of velocity = 0.98	Angle of deflection of relative velocity of jet = 165°
Relative velocity at exit = 0.95 times relative velocity at inlet	Speed ratio = 0.45

Compute the (a) velocity of whirl at inlet and outlet, and (b) absolute velocity of exit flow at the bucket. [***Ans:*** V_{u1} = 61.38 m/s, V_{u2} =2.266 m/s, V_2 = 8.47 m/s]

P4.17 ** A Pelton wheel produces 300 kW of power when working under a head of 180 m with a discharge of 0.2 m^3/s. Compute the (a) hydraulic efficiency, (b) mean bucket speed, and (c) velocity of whirl at inlet and outlet. Take C_v = 0.985, angle of deflection of jet = 165° and assume that relative velocity at exit remains unchanged till the exit at the bucket. Assume mechanical efficiency =1.0.
[***Ans:*** η_h = 0.851, u = 19.665 m/s, V_{u1} = 58.54 m/s, V_{u2} = 17.89 m/s]

P4.18 ** In a Pelton wheel the jet is deflected by170°. The initial jet velocity is 96 m/s and the peripheral velocity of the wheel at the pitch circle is 44 m/s. Calculate the kinetic energy of the jet leaving the bucket and the magnitude direction of the absolute velocity at the exit of the jet from the bucket. Assume bucket friction coefficient $K = 1.0$

[***Ans:*** $\frac{V_2^2}{2g} = 6.81$ m, $V_2 = 11.155$ m/s, $\alpha_2' = 51.39°$]

P4.19*** The following data pertains to a single-jet Pelton turbine:

Jet diameter = 12 cm	Jet ratio = 12
Coefficient of velocity of the nozzle = 0.985	Speed ratio = 0.45
Bucket friction coefficient = 0.96	Bucket angle =170°
Mechanical efficiency = 96%	Net head = 150 m

Calculate the (a) overall efficiency, (b) torque at the start, and (c) torque at normal speed.

[***Ans:*** $\eta_0 = 0.899$, $T_0 = 45.14$ kN.m, $T = 24.52$ kN.m]

P4.20 ** A double-jet Pelton wheel develops 1.0 MW of power under a head of 60 m. Following are the data of this turbine:

Speed ratio = 0.46	Overall efficiency = 0.90
Coefficient of velocity of the nozzle = 0.97	Jet ratio = 12

Calculate the (a) pitch diameter of the wheel, (b) total discharge, and (c) specific speed of the wheel.

[***Ans:*** $D = 2.28$ m, $Q_t = 1.892$ m^3/s, $N_s = 17.7$]

P4.21 ** A Pelton wheel designed to produce 5 MW of power has a diameter of 2.0 m and a speed of 600 rpm. Following are some relevant data of this turbine:

Bucket deflection angle = 165°	Bucket friction coefficient $K = 0.95$
Mechanical efficiency = 92%	Coefficient of velocity of the nozzle = 0.98
Speed ratio $K_u = 0.45$	

Estimate the (a) overall efficiency and (b) discharge through the turbine.

[***Ans:*** $\eta_0 = 0.863$, $Q = 0.596$ m^3/s]

P4.22 ** (a) Show that the hydraulic efficiency of a Pelton wheel is given by

$$\eta_h = 2K_u\,(C_v - K_u)\,(1 + K\cos\beta')$$

where K_u = speed ratio, C_v = coefficient of velocity of the nozzle, K = bucket friction coefficient and $\beta' = (180 - \beta)$ where β = jet deflection angle at the bucket.

(b) Considering that K_u can be varied while keeping C_v constant, show that maximum hydraulic efficiency is obtained when $K_u = C_v/2$ and the maximum value of the hydraulic efficiency is given by $\eta_{h\max} = 2K_u^2\,(1 + \cos\beta')$.

P4.23*** Obtain an expression for the ratio of Euler head (H_e) to the energy head of the exit discharge (H_{exit}) in a Pelton turbine when there is no velocity of whirl at the bucket exit as

$$\frac{H_e}{H_{exit}} = \frac{2\varepsilon(1-\varepsilon)}{1-2\varepsilon} \quad \text{where } \varepsilon = \frac{u}{V_1}\cdot$$

Taking speed ratio = 0.46 and coefficient of velocity of nozzle = 0.98, obtain the value of $\frac{H_e}{H_{exit}}$. The bucket friction coefficient can be assumed to be 1.0 in the derivation. [***Ans:*** $\frac{H_e}{H_{exit}}$ = 8.032]

P4.24*** A Pelton turbine works under a head of 400 m and has 2.2 m diameter wheel. The normal operating speed is 350 rpm and the discharge is 0.75 m^3/s. Other relevant data of the turbine are:

Coefficient of velocity = 0.98; bucket angle = 170°; and mechanical efficiency = 0.93. Bucket friction factor K = 1.0.

Calculate the (a) power developed, (b) overall efficiency, (c) torque at start and at normal speed, and (d) theoretical runaway speed.

[***Ans:*** η_h = 0.948, P = 2590 kW, T_{start} = 132 kN.m, T_{normal} = 70.7 kN.m, N_R = 754 rpm]

Objective-type Questions

O4.1. *Efficiency of an ideal Pelton wheel will be maximum if the ratio of jet velocity to tangential velocity of the wheel is

(a) 1/2 (b) 1 (c) 2 (d) 4

O4.2. *If the angle of deflection of the jet in a frictionless Pelton turbine is β, the maximum efficiency of this ideal Pelton wheel is given by

(a) $\frac{1-\cos\beta}{2}$ (b) $\frac{1+\cos\beta}{2}$ (c) $\frac{\cos\beta}{2}$ (d) $\frac{1-\cos\beta}{4}$

O4.3. **A Pelton wheel with a single jet rotates at 600 rpm. The velocity of the jet from the nozzle is 100 m/s. If the ratio of the bucket velocity to jet velocity is 0.44, what is the diameter of the Pelton wheel?

(a) 0.7 m (b) 1.4 m (c) 2.1 m (d) 2.8 m

O4.4. *The net head H of a vertical axis impulse turbine is given by

(a) H = Gross head – Head lost due to friction in the penstock – Head loss in the nozzle
(b) H = Gross head – Head lost due to friction in the penstock – Height of turbine setting above tailwater
(c) H = Gross head + Velocity head at the nozzle – Head lost due to friction in the penstock
(d) H = Gross head – Head loss due to friction in penstock

O4.5. *The net head H in a Pelton turbine installation is the

(a) kinetic energy head of the jet issuing from the nozzle
(b) difference in elevation between Forebay water level and the nozzle outlet
(c) head at the base of the nozzle
(d) difference in level between water levels at the Forebay and the tailwater level.

O4.6. *A Pelton wheel is ideally suited for
(a) high-head and low discharge
(b) high-head and high discharge
(c) low head and low discharge
(d) medium head and medium discharge

O4.7. **Consider the following types of water turbines:
1. Bulb 2. Francis 3. Kaplan 4. Pelton

The correct sequence of order in which the operating head decreases while developing the same power is
(a) 4–2–3–1 (b) 3–4–1–2 (c) 2–1–4–3 (d) 1–3–2–4

O4.8. **Which of the following turbines maintain a high efficiency over a long range of the part load?
1. Francis turbine 2. Kaplan turbine
3. Pelton turbine 4. Propeller turbine
Select the correct answer using the code given below:
(a) 1 and 4 (b) 2 and 3 (c) 1, 2 and 3 (d) 2, 3 and 4

O4.9. **The runaway speed of a Pelton turbine is in the range of
(a) 3.5 to 4.0 N (b) 2.0 to 2.6 N (c) 1.8 to 1.9 N (d) 1.3 to 1.6 N

where N = normal speed of the turbine.

O4.10. **The speed ratio of a Pelton turbine operating under a head of 900 m is 0.45. The peripheral velocity of the turbine wheel is nearly
(a) 28 m/s (b) 96 m/s (c) 42 m/s (d) 60 m/s

O4.11. **In connection with selection of turbines by specific speed or head, which one of the following statements is NOT correct?
(a) For specific speeds in the range 10–35, Kaplan turbines are the best.
(b) For specific speeds in the range 60–300, Francis turbines are the best.
(c) For heads in the range 50–150 m, Francis turbines are suitable.
(d) For heads above 300 m, Pelton turbines are suitable.

O4.12. ***In a two-jet Pelton wheel working at full load, each of the nozzles are issuing 10 cm diameter jets. The nozzles are so shaped as to avoid any further contraction of the jet. If the load is suddenly reduced to 36% of full load, the altered jet diameter will be
(a) 1.8 cm (b) 3.0 cm (c) 3.6 cm (d) 6.0 cm

O4.13. **The hydraulic efficiency of a Pelton turbine is 80%. If the mechanical efficiency is 85%, what is its overall efficiency?
(a) 68% (b) 94.1% (c) 82.5% (d) 65%

O4.14. **A Pelton wheel with single jet rotates at 600 rpm. The velocity of the jet from the nozzle is 100 m/s. If the ratio of the bucket velocity to jet velocity is 0.44 and the speed ratio is 0.43, what is the coefficient of velocity of the nozzle?
(a) 0.87 m (b) 0.922 (c) 0.95 (d) 0.977

O4.15. **In the inlet part of the jet impinging on a Pelton bucket, the velocity of whirl is equal to
(a) V_1 (b) v_{r1} (c) u (d) zero

where V_1 = velocity of jet, v_{r1} = relative velocity of jet at the inlet and u = tangential velocity of the bucket.

O4.16. *Consider the following statement concerning efficiency of nozzle η_n, hydraulic efficiency η_h, wheel efficiency η_w, mechanical efficiency η_m and the overall efficiency η_0:

1. $\eta_h \cdot \eta_n = \eta_0$ 2. $\eta_h = \eta_n \cdot \eta_w$ 3. $\eta_h = \eta_m \cdot \eta_n$ 4. $\eta_0 = \eta_w \cdot \eta_m \cdot \eta_h \cdot \eta_n$

The correct statements are
(a) 1 and 2 (b) 1 only (c) 3 and 4 (d) 2, 3 and 4

O4.17. **In a Pelton turbine, the degree of reaction is
(a) one (b) zero
(c) in the range 0 to 0.5 (d) in the range 0.5 to 1.0

O4.18. *In Pelton turbines, the flow from the nozzle is controlled by a needle valve which works inside a nozzle. Typical value of coefficient of velocity C_v for a needle valve controlled nozzle, for 50% opened condition is
(a) 0.63 (b) 0.95 (c) 0.75 (d) 0.15

O4.19. **Consider the specific speed ranges of the following types of turbines:
1. Francis 2. Kaplan 3. Pelton

The sequence of their specific speeds in increasing order is
(a) 1,2,3 (b) 3,1,2 (c) 3,2,1 (d) 2,3,1

O4.20. *In a multijet Pelton turbine, the flow from the penstock passes through the following unit before entering the nozzles.
(a) Spiral case (b) Volute chamber
(c) Manifold (d) draft tube

O4.21. *If a turbine has a specific speed of 17, this turbine would be a
(a) Francis turbine (b) Bulb turbine
(c) semi-Kaplan turbine (d) impulse turbine

O4.22. *** If a 1/3 scale model of a Pelton turbine is tested, the efficiency of the prototype will be taken for preliminary studies as
(a) less than that of the model
(b) greater than that of the model
(c) same as that of the model
(d) greater than that of the model if the diameter of the model runner is greater than 50 cm

O4.23. *** A Pelton turbine has a diameter of 2.5 m and its speed is 300 rpm. Further, it has coefficient of velocity = 0.975, speed ratio = 0.45 and jet ratio = 16. The theoretical runaway speed of this turbine is
(a) 4800 rpm (b) 750 rpm (c) 669 rpm (d) 650 rpm

FIVE

Centrifugal Pumps

5.1 INTRODUCTION

5.1.1 General

Pumps are the most widely used class of device in the industry worldwide and they account for nearly 25% of electrical energy in the world. Further, pumps being an important component of equipment in the manufacturing industry, the pump industry itself is commercially very important and vast in all parts of the world. The knowledge base of pumps is indeed vast and many excellent treatises and industry-oriented handbooks are available. This book deals with the topic of pumps at an introductory level in chapters 5 through 7. In this chapter, various basic and salient aspects of centrifugal pumps are described.

A pump is a device that transfers energy to aid transportation of a liquid from one location to another. A typical example is the pumping of water from a well or a sump to an overhead reservoir. Another related use of pumps is to circulate a liquid in a closed system. Typical examples of this are the circulation of cooling waters in machines and circulation of lubricating oil to various moving parts of a machine. While in their vast range of usages, the sizes and shapes of pumps vary very widely. The pump types can be broadly classified as (a) *rotodynamic pump,* and (b) *positive-displacement pump*.

In a rotodynamic type of pump, also called *dynamic pump*, a rotary element known as *impeller,* imparts energy to the liquid. The impeller itself is driven by a prime mover; electric motor, *IC* engine or steam engine. The kinetic energy of the specially designed rotating impeller is transferred to the liquid in terms of pressure energy and kinetic energy and the differential pressure causes the liquid to flow out to the delivery point. The rotodynamic pumps are popularly known as *centrifugal pumps* and form the most common type of pumps for pumping water in domestic water supply and in many industrial applications. All centrifugal pumps are of radially outward flow type. Nearly 75% of the pumps installed in industries are of centrifugal type.

Positive displacement pumps transfer a captive volume of liquid successively through action of a device such as a piston, vane, screw, etc., in a closed chamber. Further classification of displacement pumps are shown in Fig. 5.1. Note that

Fig. 5.1 also includes a category of pumps as "miscellaneous". This class of pumps works on principles of physics that are neither of pure rotodynamic type nor of the pure positive-displacement type. The classification listed in Fig. 5.1 is of the first-order type and subclassification of each type (if any) is not shown. Centrifugal pumps form the subject matter of the rest of the portions of this chapter. While the pure (radial-flow) centrifugal pump is described in detail, the mixed-flow pumps and axial-flow pumps are dealt at the introductory level. Reciprocating pumps (conventional piston/plunger type) are described in Chapter 6. A major part of Chapter 7 is devoted to the description of the rotary pumps and the miscellaneous pumps listed in Fig. 5.1.

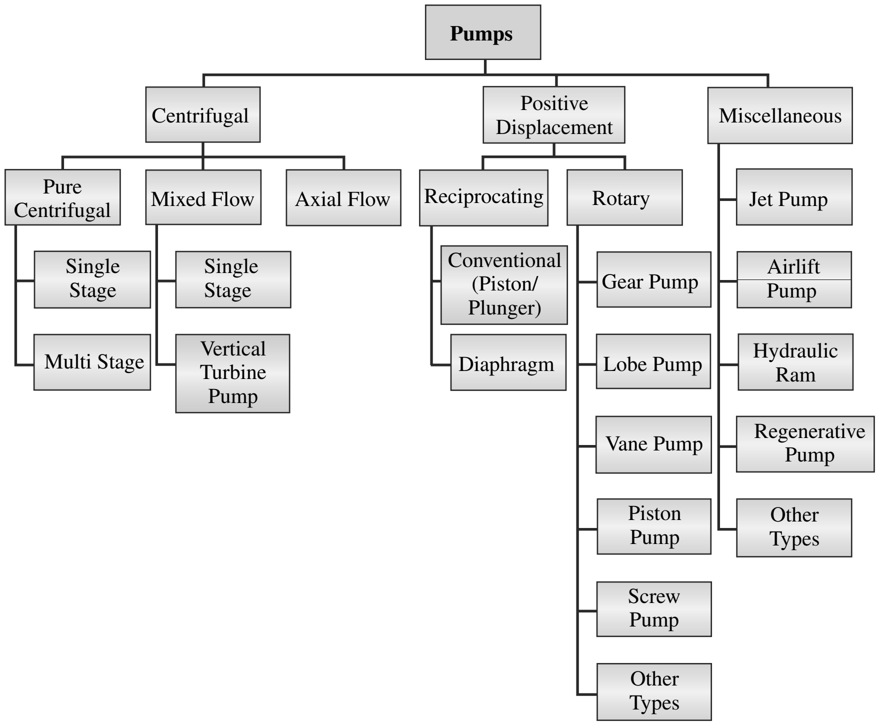


Fig. 5.1 *Classification of pumps*

5.1.2 Energy-Transfer Mechanism of a Centrifugal Pump

Centrifugal pumps are dynamic machines that impart energy to liquids. In these pumps, the energy of a prime mover causes an impeller to rotate inside a casing. The casing itself is filled with liquid and has a connection to the source through a pipe called *suction pipe*. The liquid enters the casing axially at its centre, called the *eye*. The rotating impeller with its curved blades whirls the liquid radially towards the circumference of the casing. In this process, the impeller imparts velocity and pressure to the liquid. The space between the outer impeller edge and the inner surface of the casing is shaped to act as a diffuser to the pumped liquid. The passage takes the form of a gradual expanding conduit, something like the reverse action of the spiral case of a reaction turbine. In this passage, known as *volute chamber*, the velocity head of the liquid is gradually converted into pressure head. The liquid enters the delivery pipe at the end of the volute chamber. Figure 5.2 shows a schematic sketch of a centrifugal pump and its components.

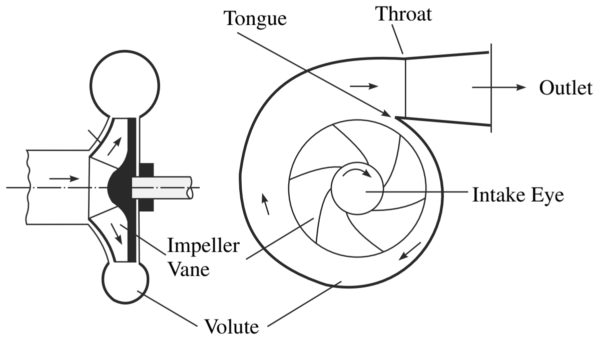


Fig. 5.2 *Schematic sketch of a centrifugal pump*

The pump must be full of liquid before starting. At the start of the pumping of liquid, a partial vacuum will be formed at the centre of the impeller. Liquid from the suction pipe flows into the pump inlet, at the eye of the casing. The process is continuous so long as the impeller is rotating at the requisite speed.

5.2 COMPONENT PARTS OF A CENTRIFUGAL PUMP

A centrifugal pump is a relatively simple machine and consists of one rotating part. Its main components are (a) impeller, (b) casing, (c) suction pipe, (d) delivery pipe, (e) delivery control valve, and (f) inlet strainer and foot valve.

5.2.1 Impeller

An impeller is the equivalent of a rotor in a turbine. It is the main rotating component and consists of an assembly of a set of curved blades mounted on the main shaft. The shaft is driven by a prime mover, which in usual cases will be an electric motor or *IC* engine. The blades come in three types:

1. Shrouded/Fully Enclosed Impellers

Shrouded impellers are the most common type and in this, the blades are covered on both sides by *shrouds* (cover plates). Figure 5.3(a) shows a sketch of a shrouded impeller. The impeller is essentially a set of two circular discs sandwiching the blades between them. One of the discs (discs are called shrouds) has a circular co-axial opening in the centre to provide inlet to the liquid. The other shroud is solid and has a connection on the outer side to the driving shaft. The flow from the suction pipe enters through the central circular opening and then moves radially outwards and ejected at the periphery of the impeller. This type of impeller gives better guidance to the liquid in its passage through the impeller. There is no slippage of the liquid that is acted upon by the impeller. Shrouded impellers are used for all clear liquids, i.e., those liquids that do not have suspended solids.

2. Semi-enclosed Impellers

This type, shown in Fig. 5.3 (b), has a shroud on only one side; the side that is connected to the shaft. This type of impeller is used when the liquid to be pumped contains some solids in suspension. The absence of one shroud gives some leeway for the solids to negotiate the path from the inlet to outlet through the impeller and prevents clogging of impeller passages.

3. Fully Open Impeller

In this type [Fig. 5.3(c)], the blade assembly is open in the sense that there are no shrouds. This type of open impeller is meant to handle highly solid-laden liquids like concrete, slurry, sewage and water containing sand and silt. They are also good for handling highly viscous liquids. To provide strength to the blade assembly, ribs or partial shrouds are sometimes used. Liquid slippage in the clearances between the casing and the impeller is a possibility in these impellers.

5.2.2 Casing

The pump casing houses the impeller assembly in an airtight chamber. The casing includes suction and delivery openings and arrangements for the drive shaft to be connected to the impeller. The space between the impeller and the inside boundary of the casing is shaped to provide a diffuser for the flow emanating from the impeller. This is to convert a considerable part of the kinetic energy of the flow that comes out of the impeller, into pressure energy and thereby reduce friction and other losses. Two types diffuser arrangements in the casing are in common use.

(a) Shrouded Impeller (b) Semi-enclosed Impeller (c) Fully Open Impeller

Fig. 5.3 *Types of impellers*

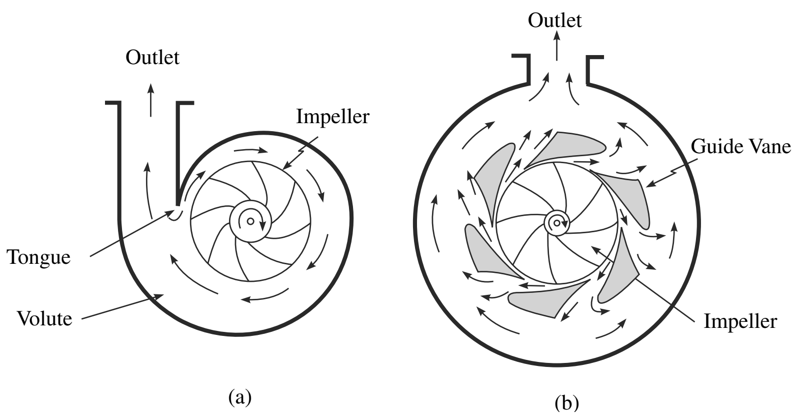


Fig. 5.4 *Types of centrifugal pumps: (a) Volute pump (b) Turbine pump*

1. Volute Casing

In this, the impeller discharges into a spiral form of casing known as *volute casing*. The cross-sectional area of the volute is least at the junction with the outlet pipe (known as *tongue*) and gradually increases towards the delivery pipe exit (Fig. 5.2, 5.4). The idea behind having this shape is to keep the velocity of flow constant at any cross section. This unit is like the reverse of the spiral chamber in a reaction turbine. Volute casings are very widely used in single-stage centrifugal pumps. A centrifugal pump with volute casing is known as *volute pump*.

A variant of this arrangement is to have an annular space between the impeller and the volute. This annular space is known as *whirl chamber* or *vortex chamber* and helps formation of a free vortex in the flow from the impeller to the volute.

2. Double-volute Casing

In high-capacity, high-head, centrifugal pumps, the shape of a single volute chamber causes substantial unbalanced radial forces on the shaft. To overcome this problem, a double-volute casing was developed. Figure 5.5 shows a schematic view of a double-volute casing. In this, a divide partitions the main volute into two parts so that the radial forces on the shaft are balanced. This means, smaller shaft sizes and reduction in bearing costs. At the same time, the main purpose of the volute in acting as a gradual diffuser of the velocity is achieved.

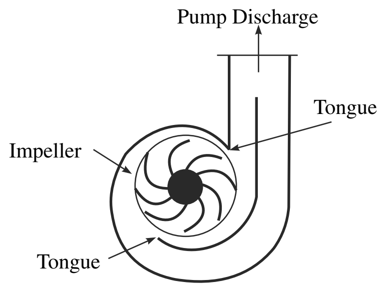


Fig. 5.5 *Double-volute casing (schematic)*

3. Turbine Pump (Casing with Guide Vanes)

A casing in which a stationary set of guide vanes are provided to diffuse the flow out of the impeller on its way to the outlet is known as *turbine-type casing* [Fig. 5.3(b)]. The casing is usually circular in cross section and concentric with the impeller. Centrifugal pumps of this kind are sometimes called *turbine pumps*. Guide vanes are employed in almost all multistage pumps while volute casings are extensively adopted in single-stage pumps.

5.2.3 Suction Pipe with Strainer and Foot Valve

Suction pipe denotes the pipe leading from the intake to the pump. If the intake is a sump situated below the level of the pump, the suction pipe will be subjected to vacuum pressures and hence airtight fittings is a necessity. At the end of the suction pipe, a strainer is fitted to prevent debris from chocking the suction pipe inlet. Further, a nonreturn foot valve is fitted at the downstream end of the suction pipe to prevent the suction pipe draining out when the pump is stopped. Also, the foot valve helps in priming of the pump.

5.2.4 Delivery Pipe with Control Valve

The pipe that carries the liquid from the pump to its destination is known as a delivery pipe. In the delivery pipe, adjacent to the pump, a control valve (commonly known as *delivery valve* and is usually a gate valve) is provided for purposes of regulating the flow. All centrifugal pumps are started with the delivery valve in closed position. Further, before stopping the pump, the delivery valve is closed first and then the pump is stopped. This operation is to prevent possible backflow from the delivery pipe and consequent damage to the pump assembly.

5.2.5 Classifications

Centrifugal pumps come in a very wide variety of types. They could be broadly classified into following types on different classification criteria as follows:

Table 5.1 *Classification of centrifugal pumps*

Basis of Classification	Types
Type of casing	• Volute (single volute, double volute)
	• Turbine type
Number of stages	• Single
	• Multistage
Type of suction inlet	• Single suction
	• Double suction
Impeller types	• Closed
	• Semi-Open
	• Open
Construction of casing	• Vertical split
	• Horizontal split
Axis of rotation	• Horizontal
	• Vertical
	• Inclined
Basis of flow direction	• Radial flow
	• Mixed flow
	• Axial flow

5.3 EULER'S EQUATION FOR CENTRIFUGAL PUMP

5.3.1 General Vector Relationship

The basic equation relating the velocities, their momentum and the moment of momentum of the flow to the torque developed in the shaft of a constant-speed roto-dynamic machine is known as *Euler equation*. The moment of momentum principle is Newton's second law applied to a rotating fluid mass. The steady-state moment-of-momentum equation for two-dimensional rotating fluid flow states that:

> *If $\hat{r}$ is the position vector in a curvilinear motion of a fluid, $\hat{F}$ is the external force vector and $\hat{M}$ is the linear momentum vector, the moment-of-momentum principle states that*

$$\left(\hat{r}\times\hat{F}\right)=\frac{d}{dt}\left(\hat{r}\times\hat{M}\right) \tag{5.1}$$

If the moment of external forces ($\hat{r} \times \hat{F}$) is replaced by torque $\hat{T}$ then

$$\hat{T} = \frac{d}{dt}(\hat{r} \times \hat{M}) \tag{5.2}$$

The torque exerted by the fluid mass on the shaft is equal and opposite to $\hat{T}$.

This steady-state general vector relationship of torque and rate of change of moment of momentum, when applied to a rotodynamic machine, results in a simple relationship known as *Euler equation.* In the following sections, the derivation of Euler equation and its variants for a pump are presented.

5.3.2 Euler Equation for Centrifugal Pump

Consider a centrifugal pump as shown in Fig. 5.2. The flow is uniform along the circumference and is steady. Further, it is assumed that there are no mechanical or hydraulic friction and other eddy losses in the system. The impeller is considered to have an infinite number of blades having zero friction and all the flow from the inlet is assumed to be guided with uniform velocity at the exit. This assumption assures that there is no circulation-induced crossflows in the system. Since the flow is steady, the rotational speed of the impeller is constant. The impeller as above is known as an *ideal impeller*. A schematic sketch of the impeller blade and the velocity triangles at the inlet and outlet are shown in Fig. 5.6.

Referring the suffixes 1 and 2 to the inlet and outlet respectively, consider in detail the flow along one blade of the impeller:

Let

r_1 and r_2 = Radii of fluid element at entrance and exit
v_{r1} and v_{r2} = Relative velocities at entrance and exit
V_1 and V_2 = Absolute velocities at entrance and exit
ω = Angular velocity of the impeller
N = Revolutions per minute of the impeller

Note that the angular velocity of the impeller $\omega = \dfrac{2\pi N}{60}$

u = Tangential velocity of the blade at any radius $r = \omega r$

$u_1 = \omega r_1 = \dfrac{\pi D_1 N}{60}$ where D_1 = Outer diameter of the impeller

$u_2 = \omega r_2 = \dfrac{\pi D_2 N}{60}$ where D_2 = Inner diameter of the impeller

For a steady, frictionless system,

Torque exerted by the impeller on the fluid = Increase in the rate of change of moment of momentum

= [Rate of change of moment of momentum of fluid going **out of** the control volume] – [rate of change of moment of momentum of fluid going **in to** the control volume]

Since the set of blades are assumed to be mounted symmetrically on the impeller and the flow is uniform along the perimeter of the impeller at the inlet and outlet, the torque exerted by the impeller on the fluid and which is transmitted to the shaft without any loss due to friction and other losses is given by

Torque T = [Rate of mass flow through the impeller] × [Increase of moment of momentum of the fluid in each blade]

$$= \dot{m} \times (r_2 V_2 \cos \alpha_2 - r_1 V_1 \cos \alpha_1) \quad (5.3)$$

Since ρ = Density of water, is constant,

$\dot{m}$ = Rate of mass flow through the impeller = ρQ

where Q = Total discharge entering the impeller.

Now $T = \rho Q(r_2 V_2 \cos \alpha_2 - r_1 V_1 \cos \alpha_1)$ (5.4)

Energy transferred in unit time = Power transmitted by the impeller to water

$$P = T\omega = \rho Q \omega (r_2 V_2 \cos \alpha_2 - r_1 V_1 \cos \alpha_1) \quad (5.5)$$

Since $\omega r = u$ = Tangential component of the impeller at radius r,

$u_1 = \omega r_1$ and $u_2 = \omega r_2$

$$P = \rho Q (u_2 V_2 \cos \alpha_2 - u_1 V_1 \cos \alpha_1) \quad (5.6)$$

This equation (5.6) is known as *Euler's equation relating power in a centrifugal pump.*

NOTE

Compare the Euler's equation for power for a turbine given by Eq. (2.6) and the Euler equation relating power for a pump as given by Eq. (5.6) and observe the difference. Note that the definition of suffixes 1 and 2 are consistent.

5.3.3 Alternate Forms of Euler Equation

Consider the generalised velocity triangles at inlet and outlet of an impeller, as shown in Fig. 5.6. In this, the suffixes 1 and 2 refer to inlet and outlet respectively. The following notations are used consistently in relation to centrifugal pumps:

Notations

V_1 = Absolute velocity at inlet, V_2 = Absolute velocity at outlet

u_1 = Tangential velocity of the inner periphery of the impeller, and

u_2 = Tangential velocity of the outer periphery of the impeller

v_{r1} = Relative velocity at inlet, v_{r2} = Relative velocity at outlet

D_1 and D_2 = Inner and outer diameters of the impeller, respectively.

α_1 and α_2 = Direction of absolute velocity at inlet and outlet respectively. α is the angle made by the absolute velocity vector V with the positive direction of the peripheral velocity, u.

β_1 and β_2 = *Vane angle* = Angle made by the relative velocity vector v_r with the negative direction of the peripheral velocity, u at inlet and outlet respectively.

$V_1 \cos \alpha_1 = V_{u1}$ = Tangential component (also known as *Whirl* component) of absolute velocity V_1.

$V_2 \cos \alpha_2 = V_{u2}$ = Tangential component (also known as *Whirl* component) of absolute velocity V_2.

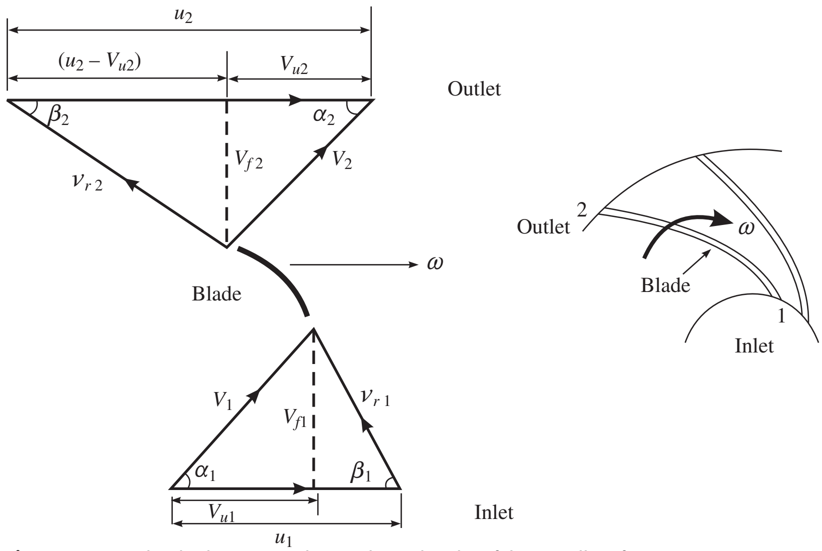


Fig. 5.6 *Generalized velocity triangles at inlet and outlet of the impeller of a pump*

$V_1 \sin \alpha_1 = V_{f1}$ = Flow component of absolute velocity V_1

$V_2 \cos \alpha_2 = V_{f2}$ = Flow component of absolute velocity V_2

N = Rotational speed of the impeller in rpm

Referring to Fig. 5.6, from the velocity triangle at outlet,

$$V_{f2}^2 = v_{r2}^2 - (u_2 - V_{u2})^2 \tag{5.7}$$

$$V_2^2 = V_{u2}^2 + V_{f2}^2 = V_{u2}^2 + v_{r2}^2 - (u_2^2 - V_{u2}^2) \tag{5.8}$$

$$= V_{u2}^2 + v_{r2}^2 - u_2^2 - V_{u2}^2 + 2u_2 V_{u2}$$

$$2u_2 V_{u2} = V_2^2 - v_{r2}^2 + u_2^2$$

$$u_2 V_{u2} = (V_2^2 - v_{r2}^2 + u_2^2) / 2 \tag{5.9}$$

Similarly, from the velocity triangle at the inlet,

$$V_1^2 = V_{u1}^2 + V_{f1}^2 = V_{u1}^2 + v_{r1}^2 - (u_1^2 - V_{u1}^2)$$

$$V_1^2 = V_{r1}^2 - u_1^2 + 2u_1 V_{u1}$$

$$u_1 V_{u1} = (V_1^2 - v_{r1}^2 + u_1^2) / 2 \tag{5.9-a}$$

Thus, from Eq. 5.6 the power transferred = energy transmitted by the impeller to water per unit time is

$$P = \rho Q\,(u_2 V_2 \cos \alpha_2 - u_1 V_1 \cos \alpha_1) = \rho Q\,(u_2 V_{u2} - u_1 V_{u1}) \tag{5.10}$$

Equation 5.10 is a very commonly used form of *Euler equation for power* in a pump.

Substituting the results of equations (5.9 and 5.9-a) in Eq. (5.10),

$$P = \frac{\rho Q}{2}\left(V_2^2 - v_{r2}^2 + u_2^2 - V_1^2 + v_{r1}^2 - u_1^2\right)$$

$$= \rho Q\left[\frac{\left(V_2^2 - V_1^2\right)}{2} + \frac{\left(v_{r1}^2 - v_{r2}^2\right)}{2} + \frac{\left(u_2^2 - u_1^2\right)}{2}\right] \tag{5.11}$$

Considering the energy head H_e = energy per unit weight of fluid transferred to the fluid by the impeller,

$$H_e = \frac{\rho Q}{\rho Q g}\left[\frac{\left(V_2^2 - V_1^2\right)}{2} + \frac{\left(v_{r1}^2 - v_{r2}^2\right)}{2} + \frac{\left(u_2^2 - u_1^2\right)}{2}\right]$$

$$H_e = \left[\frac{\left(V_2^2 - V_1^2\right)}{2g} + \frac{\left(v_{r1}^2 - v_{r2}^2\right)}{2g} + \frac{\left(u_2^2 - u_1^2\right)}{2g}\right] \quad (5.12)$$

Also by Eq. (5.9) $H_e = \frac{1}{g}(u_2 V_{u2} - u_1 V_{u1})$ (5.13)

Equation 5.13 is the *Euler equation for head* in a centrifugal pump.

H_e is called *Euler head* and represents the head transferred to water by the impeller. In this,

- The first term $\frac{\left(V_2^2 - V_1^2\right)}{2g}$ represents the increase in kinetic energy.
- The second term $\frac{\left(u_2^2 - u_1^2\right)}{2g}$ represents the increase in static pressure due to centrifugal action.
- The third term $\frac{\left(v_{r1}^2 - v_{r2}^2\right)}{2g}$ indicates the change in the kinetic energy due to retardation of flow.

It is a common practice to have the flow enter the impeller radially at the design discharge. This implies that $\alpha_1 = 90^o$ and $V_{f1} = V_1$. This is similar to having radial exit in Francis turbines. This feature of radial entry gives optimal efficiency and stability in operation. As such, in all further parts of this chapter, radial entry is assumed as the default condition, Fig. 5.7.

From Eq. (5.13), the Euler head, for practical use, is thus

$$H_e = \frac{1}{g}(u_2 V_{u2}) \quad (5.14)$$

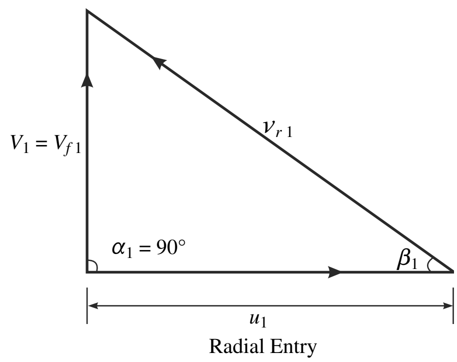


Fig. 5.7 *Radial entry at inlet*

5.4 ANALYSIS BASED ON EULER'S EQUATION

5.4.1 Definitions of Heads used in Pump Analysis

1. Static Head

Static head refers to the vertical distance from the water level of the sump to the water level of the receiving reservoir on the delivery side, (See Fig.5.8). Static head (H_{stat}) is made up of *static suction lift* h_s and *static delivery lift* h_d as

$$H_{stat} = h_s + h_d$$

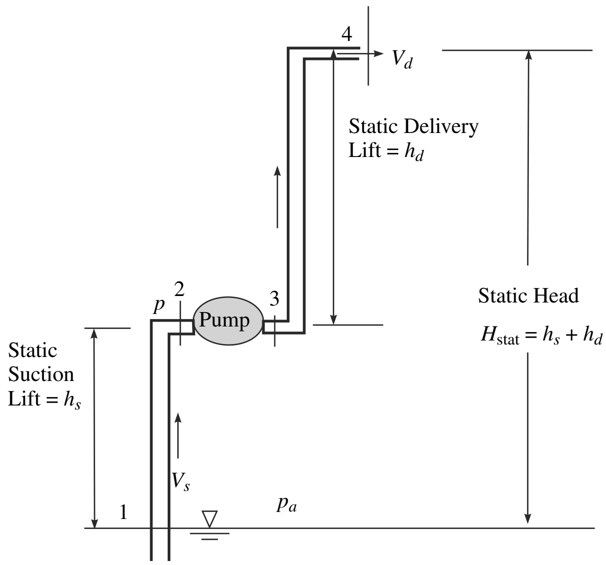


Fig. 5.8 *Definition sketch of a pump set-up*

Thus, the static head represents the total vertical height, and hence the change in the datum heads, to be accomplished in the pumping of the liquid.

2. *Euler Head* (*Theoretical Head*) of a Pump

Euler head H_e is the energy per unit weight of liquid supplied by the impeller in the pump. It is thus the work done by the impeller per second on unit weight of the liquid. By considering radial entry at the inlet, the Euler head is represented by Eq. 5.14 as

$$H_e = \frac{V_{u2}u_2}{g}$$

If there were no losses, H_e would represent the total lift from the sump that could be achieved by the pump. That is why it is also called *theoretical head* as certain basic assumptions such as non-existence of circulatory flow and absence of energy losses are made in its derivation. Euler head can thus be defined as the theoretical head of a pump with an infinite number of blades and total absence of any loss, hydraulic or mechanical, in the system. Note that the theoretical head is independent of the liquid characteristics.

However, losses are inevitable and as such the actual lift would be smaller than H_e.

3. *Manometric Head*

Manometric head (H_m) is the actual total head that could be achieved by the pump. It is smaller than the Euler head H_e by an extent of h_{fi} representing the energy loss in the impeller and casing. These losses are known as *hydraulic losses* and include fluid frictional losses in the blade passage, circulatory flow between the blades due to finite number of blades in the impeller and shock losses at the entrance to the impeller. Manometric head H_m is given by the relation

$$H_m = H_e - h_{fi}$$

The ratio of manometric head to Euler head is known as *manometric efficiency*. Thus, manometric efficiency

$$\eta_{ma} = \frac{H_m}{H_e} = \frac{gH_m}{V_{u2}u_2} \tag{5.15}$$

Expressions for Manometric Head Consider a pump set-up as in Fig. 5.8. The static suction head is h_s and the static delivery head is h_d. Let the pressure at the sump liquid surface (Point 1) be atmospheric pressure. The pump is considered as discharging to the atmosphere and hence at the exit point 4, the pressure is p_a. An energy head H_m is added by the pump to the liquid to lift the liquid from the sump (Section 1) to the delivery point (Section 4).

Apply Bernoulli's theorem to Section 1 and Section 4 by taking the liquid level in the sump as datum.

$$\left(\frac{p_a}{\gamma} + 0 + 0\right) + H_m = \left(\frac{p_a}{\gamma} + (h_s + h_d) + (h_{fs} + h_{fd}) + \frac{V_d^2}{2g}\right) \tag{5.16}$$

where h_{fs} = Frictional losses including minor losses in the suction pipe

h_{fd} = Frictional losses including minor losses in the delivery pipe

V_d = Discharge velocity at the delivery pipe

Hence, $H_m = (h_s + h_d) + (h_{fs} + h_{fd}) + \dfrac{V_d^2}{2g} = H_{stat} + \dfrac{V_d^2}{2g} + \Sigma$ Losses (5.17)

In most applications, $\dfrac{V_d^2}{2g}$ is usually neglected as too small or is included in the minor losses. The manometer head H_m is then taken as

$$H_m = (h_s + h_d) + (h_{fs} + h_{fd}) = H_{stat} + \Sigma \text{ Losses} \tag{5.17-a}$$

Thus, an alternative definition for the manometer head is given:

Manometric head of a pump is the gross head that must be provided by the impeller for the liquid to flow from the sump to the delivery point.

The difference of energy heads between the outlet flange and the inlet flange of the pump should also be equal to the manometer head, H_m. Thus, considering the sections 2 and 3 to be at the same elevation,

$$\left(\frac{p_3}{\gamma} + \frac{V_d^2}{2g}\right) - \left(\frac{p_2}{\gamma} + \frac{V_s^2}{2g}\right) = H_m$$

$$\left(\frac{p_3}{\gamma} - \frac{p_2}{\gamma}\right) = H_m - \left(\frac{V_d^2}{2g} - \frac{V_s^2}{2g}\right)$$

If $V_d = V_s$ or if the difference between the two velocity heads is negligibly small then

$$\left(\frac{p_3}{\gamma} - \frac{p_2}{\gamma}\right) = H_m \tag{5.17-b}$$

Thus, the manometric head is also essentially equal to the difference in piezometric heads across the pump. If a differential manometer is connected to the pump at sections 3 and 2 then the head corresponding to the manometer reading would be essentially equal to the manometer head, H_m. Hence the name manometer head for this parameter.

5.4.2 Ideal Increase in Pressure Head in the Impeller

Consider the case of an ideal impeller in which there are no losses. The inflow is assumed to be radial. The energy balance between the inlet and outlet of the impeller can be written, per unit weight of liquid, as follows:

Energy of liquid at inlet + Energy added externally by the pump to the liquid = Energy of liquid at outlet

$$\left(\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}\right)+\frac{u_2V_{u2}}{g}=\left(\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}\right)$$

Increase in piezometric head at the pump =

$$\left(\frac{p_2}{\gamma}+z_2\right)-\left(\frac{p_1}{\gamma}+z_1\right)=\Delta H_p=\left(\frac{V_1^2}{2g}-\frac{V_2^2}{2g}+\frac{u_2V_{u2}}{g}\right) \qquad (5.18)$$

Since the average values are considered in this one-dimensional analysis, for the impeller inlet and outlet, Z_1=Z_2. Hence, the piezometric difference

$$\Delta H_p=\frac{\Delta p_i}{\gamma}=H_m=$$

Difference between pressure head between the outlet and inlet of the impeller.

Thus, for $Z_1 = Z_2$, for the ideal case of no losses, the relationship between H_m and H_e can be expressed as

$$\left(\frac{p_2}{\gamma}-\frac{p_1}{\gamma}\right)=H_m=\left(\frac{V_1^2}{2g}-\frac{V_2^2}{2g}\right)+\frac{u_2V_{u2}}{g}$$

But by Eq. (5.14), $H_e=\frac{1}{g}(u_2V_{u2})$.

Hence, $H_e = H_m + \Delta KE$ (5.18-a)

or $\Delta Hp = H_m = H_e - \Delta KE$

where $\Delta KE=\left(\frac{V_2^2}{2g}-\frac{V_1^2}{2g}\right)$

= (Kinetic energy head leaving the impeller

– Kinetic energy head entering the impeller)

= Excess of kinetic energy head leaving the impeller

Since V_2 is larger than V_1, ΔKE is a positive quantity.

With the above definitions, an expression for ΔH_p $\left(=\frac{\Delta p_i}{\gamma}\right)$ under ideal conditions is obtained as below:

1. Expression for ΔH_p under Ideal Conditions

Consider the outlet velocity triangle of a pump as shown in Fig. 5.9. In this,

$$\frac{V_{f2}}{(u_2 - V_{u2})} = \tan \beta_2$$

$$V_{u2} = u_2 - \frac{V_{f2}}{\tan \beta_2} = u_2 - V_{f2} \cot \beta_2$$

$$V_2^2 = V_{u2}^2 + V_{f2}^2$$

$$= (u_2^2 + V_{f2}^2 \cot^2 \beta_2 - 2u_2 V_{f2} \cot \beta_2 + V_{f2}^2)$$

$$= V_{f2}^2 (1 + \cot^2 \beta_2) + u_2^2 - 2u_2 V_{f2} \cot \beta_2$$

$$= V_{f2}^2 \text{cosec}^2 \beta_2 + u_2^2 - 2u_2 V_{f2} \cot \beta_2 \quad (5.19)$$

Further, from inlet velocity triangle for radial entry (Fig. 5.9), $V_1 = V_{f1}$.

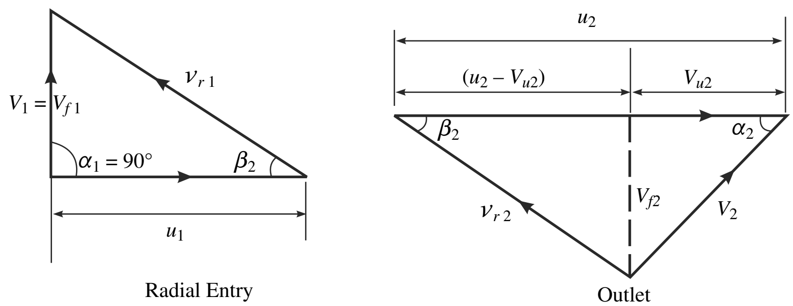


Fig. 5.9 *Radial entry and outlet velocity triangle of a pump*

Substituting the above in the expression for ΔH_p in Eq. (5.18),

$$\Delta H_p = \frac{\Delta p_i}{\gamma} = \frac{1}{2g}\left[V_1^2 - V_2^2 + 2V_{u2}u_2\right]$$

$$\frac{\Delta p_i}{\gamma} = \frac{1}{2g}\left[V_{f1}^2 - V_{f2}^2 \text{cosec}^2 \beta_2 - u_2^2 + 2u_2 V_{f2} \cot \beta_2 + 2(u_2 - V_{f2} \cot \beta_2) u_2\right]$$

$$\frac{\Delta p_i}{\gamma} = \frac{1}{2g}\left[V_{f1}^2 - V_{f2}^2 \text{cosec}^2 \beta_2 - u_2^2\right] \quad (5.20)$$

2. Increase in Pressure Head for Real Pumps

First, the basic aspects of increase in pressure head are summarized.

(a) The Euler head H_e for the pump given by Eq. (5.14) represents the total energy per unit weight transferred to the fluid by an impeller under ideal conditions of no loss of energy in the system. This head H_e is made up of two components [Eq. (5.18–a)] as

(i) Difference in pressure head across the impeller $\left(\frac{\Delta p_i}{\gamma}\right)$, and

(ii) Excess kinetic energy per unit weight of liquid leaving the impeller,

$$\Delta KE = \left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g} \right).$$

(b) The role of the volute casing and other form of diffusers in the pump is to utilise this *excess kinetic energy* (ΔKE), by converting a major part of it to pressure energy. Any recovered kinetic energy will be reflected in the increase in the difference of pressure heads across the pump. In real pumps, there will be some loss in the impeller itself and in the diffuser. The recovery of kinetic energy in the diffuser would also be partial.

3. Real Pump-Volute Systems

For the real pump–volute systems, the losses in the impeller, if any, can be lumped and assumed to occur just outside the impeller. That is, all losses are assumed to occur in the volute casing or any other form of diffusers in the pump assembly. With such an arrangement, the hydrodynamic behaviour of the impeller is governed by Euler equation applicable to the geometry of the impeller. In addition, we can relate the manometric head to the actual difference in pressure head across the pump as below:

$$H_{\text{ma}} = \left(\frac{\Delta p_i}{\gamma} \right)_{\text{Actual}} = \left(\frac{\Delta p_i}{\gamma} \right)_{\text{theo}} + \Delta KE_r \tag{5.21}$$

$$\text{and } H_e = \left(\frac{\Delta p_i}{\gamma} \right)_{\text{Actual}} + \Delta KE_{\text{unrec}} + [\text{Losses in impeller } (= h_{Li}), \text{ if any}] \tag{5.22}$$

$$\text{and } \frac{H_{\text{ma}}}{H_e} = \eta_m = \text{Actual manometric efficiency}$$

In the above,

$\left(\frac{\Delta p_i}{\gamma} \right)_{\text{theo}}$ = Difference in pressure head given by Eq. 5.20,

ΔKE_r = Kinetic energy per unit weight recovered in the volute casing, (= recovered part of ΔKE), and

ΔKE_{unrec} = Unrecovered kinetic energy at the exit, (= unrecovered part of ΔKE).

The actual manometric efficiency reflects the unrecovered kinetic energy of exit flow as well as the losses in the impeller, if any. The term ΔKE_r is the part of (ΔKE) that is recovered. The ratio $\frac{\Delta KE_r}{\Delta KE}$ represents the fraction (percentage) recovery of the kinetic energy in the volute casing and approximately is a measure of the efficiency of the volute casing. The velocity of flow out of the volute casing into the discharge pipe, V_d, marks the lower limit up to which the exit velocity V_2 can be lowered in the process of recovery of kinetic energy in the volute.

Examples 5.20 and 5.21 illustrate the use the above concepts to estimate the pressure recovery in the diffuser and in the calculation of the efficiency of the diffuser.

5.4.3 Circulatory Flow

In the derivation of Euler head H_e, an ideal impeller that has infinite number of vanes of zero thickness is assumed. However, in real life, the number of vanes is finite and each of the vanes has finite thickness. This fact has an effect on the characteristics of the flow, especially the velocity distribution of the flow, emanating from the impeller. This change in the velocity distribution from the ideal causes the actual head produced to be lower than the Euler head.

Consider the space between two vanes of an impeller shown schematically in Fig. 5.10(a). Due to the flow taking place between a finite-sized passage between the two adjacent vanes, relatively high pressure is formed along the leading edge of the vane. This is indicated by symbol (++) in Fig. 5.10(a). On the trailing edge of the same blade, relatively low pressure is formed and this is indicated as (– –) in the figure. Notice that in the sectorlike space formed by two adjacent vanes, one side has relatively low pressure and another side has relatively higher pressure. The other two sides are without solid boundaries: one side has inflow and another has outflow of liquid. The difference in pressure across the main flow sets up a crossflow from high-pressure region to low-pressure region. Due to continuity considerations, this crossflow results in a circulatory motion as shown in Fig. 5.10(b). The interaction of this circulatory flow with the main flow results in the exit velocity distribution being non-uniform, shown as resultant flow in Fig.5.10(c).

The impact of the non-uniform flow in the passage between impeller blades is reflected in the exit velocity triangle. The ideal exit velocity triangle at any point in

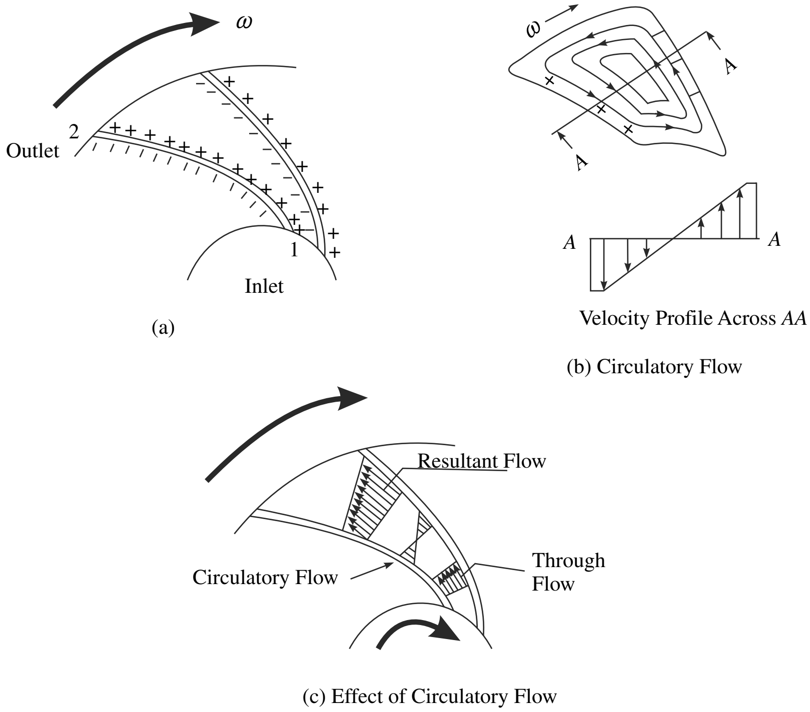


Fig. 5.10 *Circulatory flow in inter-vane passages*

the perimeter of the impeller between two adjacent vanes is shown as ΔABC in Fig. 5.11. The relative velocity makes an angle β_2 with the peripheral velocity u_2. The exit absolute velocity is V_2 and the tangential component of the exit velocity is V_{u2}. The average value of the angle β_2' of the actual flow leaving the impeller is greater than the vane angle β_2 of the ideal impeller.

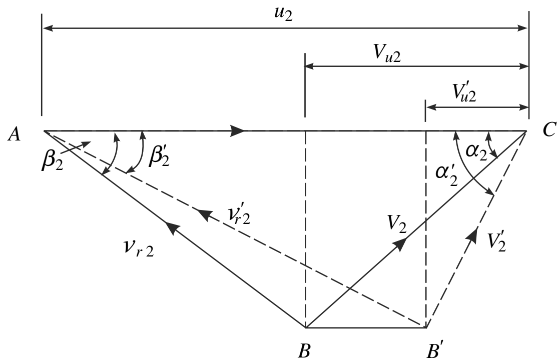


Fig. 5.11 *Definition sketch for slip factor*

The mean velocity of the actual flow, in this sectorlike space, is V_2' which will be smaller than the ideal velocity V_2. The actual tangential component of the velocity is V_{u2}'. The actual exit velocity triangle is $\Delta AB'C$. Dotted lines AB' and $B'C$ in Fig. 5.11 represent the actual relative velocity v'_{r2} and absolute velocity V_2' respectively. Notice that the actual whirl component V_{u2}' is smaller than the ideal whirl component V_{u2}.

Thus, the presence of finite number of vanes of finite thickness causes the actual velocity triangle at the exit of the impeller to be different from the ideal one with less value of tangential component than that of the ideal impeller. The actual head developed is $H_e' = \dfrac{u_2 V_{u2}'}{g}$ that is smaller than the ideal Euler head $H_e = \dfrac{u_2 V_{u2}}{g}$. The difference in these two whirl velocities $\Delta V_{u2} = (V_{u2} - V'_{u2})$ is called the *slip of the impeller*. The ratio

$$\mu_s = \frac{V_{u2}'}{V_{u2}} \tag{5.23}$$

is called the *slip factor of the impeller*. Thus, the actual head produced by the real-life impeller is

$$H_e' = \mu_s H_e = \mu_s \frac{u_2 V_{u2}}{g} \tag{5.24}$$

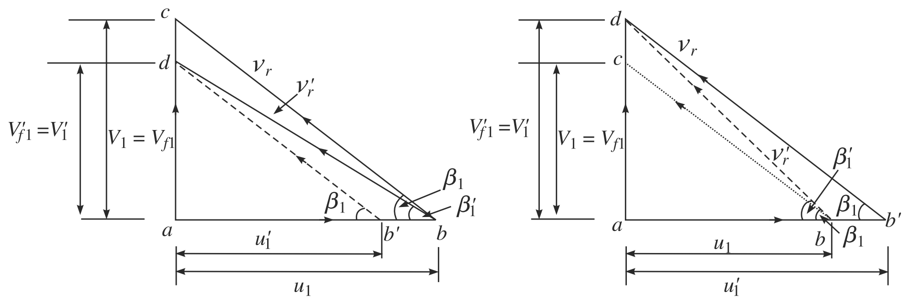
The difference $(H_e - H'_e) = (1-\mu_s)H_e$ is taken as the loss in the impeller due to circulatory flow. This loss is considered under the category of hydraulic losses. Typically, the value of μ_s is about 0.9. The slip factor is a function of number of vanes, diameter ratio of the impeller (D_2/D_1) and the exit blade angle β_2.

NOTE

The slip and slip factor of the impeller discussed in this section should not be confused with the loss (sometimes called the *slip*) of the discharge that leads to volumetric efficiency less than unity.

5.4.4 Energy Loss due to a Change from Normal Discharge

A pump is designed to function with maximum efficiency at the designed discharge against the designed head. This discharge is known as *normal discharge*. At this design discharge at the design speed, the inlet-velocity is radial and the relationship between the inlet blade angle β_1 and the relative velocity is such that the direction of the relative velocity and the tangent to the vane at the tip are parallel. Consider the inlet velocity triangle shown in Fig.5.12. The inlet-vane angle is β_1. At the design speed and discharge, the peripheral velocity is u_1 and is shown by the line *ab*. The flow velocity $V_{f1} = V_1$ (= absolute velocity) and is represented by the line *ac*.



(a) Reduction in Normal Discharge (b) Increase from Normal Discharge

Fig. 5.12 *Energy loss due to change in normal discharge*

1. Reduction from the Normal Discharge

Consider the situation where the discharge is reduced from the normal value through throttling while the pump continues to run at the same speed, Fig. 5.12(a). Let the reduced flow velocity be $V'_{f1} = V'_1$ (= absolute velocity). This is represented by the line *ad*. Line *ab* representing the peripheral velocity remains unchanged. The new velocity triangle is *abd*. In this, the line *bd* representing the new relative velocity v'_{r1} makes an angle β'_1 that is smaller than β_1. The new relative velocity approaches the vane of the impeller at an angle that is not parallel to the tangent at its tip and hence there is a mismatch. This results in the *shock* at the entrance and, consequently, there is an energy loss. There will be an adjustment of the relative velocity such that its direction is along the vane angle β_1 and the flow velocity is V'_1. Thus, the relative velocity will change to the new position represented by the line *b'd*. This line *b'd* being parallel to *bc* satisfies all the necessary conditions for the flow, viz. entrance angle of β_1, and velocity of flow equal to V'_1. Line *b'b* represents the loss of velocity $(u_1 - u'_1)$. The velocity u'_1 could be considered *equivalent peripheral velocity*. Noting that $u'_1 = V_1' \cot \beta_1$ the loss of head due to shock as a consequence of reduction of discharge from the normal value is given by

$$h_{qr} = \frac{(u_1 - u'_1)^2}{2g} = \frac{(u_1 - V'_1 \cot \beta_1)^2}{2g}$$

$$= \frac{(u_1 - V'_{f1} \cot \beta_1)^2}{2g} \qquad (5.25\text{-a})$$

2. Increase from the Normal Discharge

The analysis procedure is similar to the one described for the reduction of discharge. Figure 5.12 (b) shows the changes in the relative velocity and the loss of velocity. The loss of head due to shock as a result of increase in the discharge is

$$h_{qi} = \frac{(u_1' - u_1)^2}{2g} = \frac{(V_1' \cot\beta_1 - u_1)^2}{2g}$$

$$= \frac{(V_{f1}' \cot\beta_1 - u_1)^2}{2g} \qquad (5.25\text{-b})$$

Note that the numerical value of head loss calculated by Eq.(5.25-a) will be the same if Eq. (5.25-b) is used instead. Thus, Eq.(5.25-a) and (5.25-b) can be combined into a general relationship as

Loss of head owing to shock occurring due to discharge being different from the normal discharge =

$$h_q = \frac{(\text{Difference in peripheral velocity and equivalent peripheral velocity})^2}{2g} \qquad (5.26)$$

This energy loss due to departure from the normal discharge is called *shock loss* at entrance to the impeller.

3. Estimation of Shock Loss

A simplified expression for estimating shock loss by using Eq. 5.26 is obtained as below;

Let Q = Normal discharge,

Q' = Changed discharge while keeping the speed unchanged

A_1 = Area of flow in the impeller at the inlet.

Consider the inlet section.

At normal flow: $V_1 = V_{f1}$ and $\cot\beta_1 = \dfrac{u_1}{V_{f1}} = \dfrac{u_1}{(Q/A_1)}$

After change in the discharge: Flow velocity = $V_1' = V_{f1}' = \dfrac{Q'}{A_1}$

$$u_1' = V_{f1}' \cot\beta_1 = \frac{Q'}{A_1}\frac{A_1 u_1}{Q} = u_1 \frac{Q'}{Q}$$

Thus, shock loss $= h_q = \dfrac{(u_1 - u_1')^2}{2g} = \dfrac{\left(u_1 - u_1 \dfrac{Q'}{Q}\right)^2}{2g}$

$$= \frac{u_1^2}{2g}\left(1 - \frac{Q'}{Q}\right)^2 \qquad (5.26\text{-a})$$

5.4.5 Minimum Speed of Pump

When the pump is started by switching on the motor, the flow will take place only when the rise in pressure due to impeller action is large enough to overcome monometer head. Consider the Euler equation in the form of Eq. 5.12 given as

$$H_e = \left[\frac{\left(V_2^2 - V_1^2\right)}{2g} + \frac{\left(v_{r1}^2 - v_{r2}^2\right)}{2g} + \frac{\left(u_2^2 - u_1^2\right)}{2g}\right]$$

and $H_e = \dfrac{V_{u2}u_2}{g}$

At the time of start the fluid velocities are zero and the only head that is operative is the centrifugal head $\dfrac{\left(u_2^2 - u_1^2\right)}{2g}$. This centrifugal force must overcome the manometric head and hence

$$\frac{\left(u_2^2 - u_1^2\right)}{2g} \geq H_m. \tag{5.27}$$

Since manometric efficiency $\eta_m = \dfrac{H_m}{H_e} = \dfrac{gH_m}{(V_{u2}u_2)}$, the condition governing the minimum speed of the pump is

$$\frac{\left(u_2^2 - u_1^2\right)}{2g} \geq \eta_{ma} H_e \tag{5.28}$$

Alternatively, $$\frac{\left(u_2^2 - u_1^2\right)}{2g} \geq \eta_{ma}\frac{V_{u2}u_2}{g} \tag{5.28-a}$$

Let N_m be the minimum speed required to start the pumping action. Then

$u_2 = \dfrac{\pi D_2 N_m}{60}$ and $u_1 = \dfrac{\pi D_1 N_m}{60}$

Thus, at the minimum required speed, $\dfrac{\pi^2 N_m^2}{(60)^2}[D_2^2 - D_1^2] = 2gH_m$

Hence, $$N_m = \frac{60}{\pi\sqrt{(D_2^2 - D_1^2)}}\sqrt{2gH_m} \tag{5.29}$$

5.4.6 Effect of Outlet-Vane Angle on Manometric Efficiency

Consider a pump with radial entry. At the exit of the impeller,

$$\frac{V_{u2}u_2}{g} = H_m + \frac{V_2^2}{2g}$$

Hence, $H_m = \dfrac{V_{u2}u_2}{g} - \dfrac{V_2^2}{2g}$

Consider the outlet velocity triangle as in Fig. 5.13. α_2 is the blade angle at the outlet.

$$V_2^2 = V_{u2}^2 + V_{f2}^2$$

$$\frac{V_{f2}}{(u_2 - V_{u2})} = \tan\beta_2$$

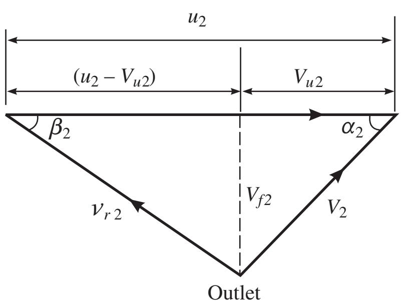


Fig. 5.13 *Outlet velocity triangle of a pump*

$$V_{u2} = u_2 - \frac{V_{f2}}{\tan\beta_2} = u_2 - V_{f2}\cot\beta_2$$

Substituting in the expression for H_m,

$$H_m = \frac{V_{u2}u_2}{g} - \frac{V_2^2}{2g} = \frac{(u_2 - V_{f2}\cot\beta_2)u_2}{g} - \left[\frac{(u_2 - V_{f2}\cot\beta_2)^2 + V_{f2}^2}{2g}\right]$$

$$= \frac{1}{2g}\left[2(u_2^2 - u_2V_{f2}\cot\beta_2) - (u_2^2 + V_{f2}^2\cot^2\beta_2 - 2u_2Vf_2\cot\beta_2 + V_{f2}^2)\right]$$

$$\frac{1}{2g}[u_2^2 - V_{f2}^2(1+\cot^2\beta_2)] = \frac{(u_2^2 - V_{f2}^2\text{cosec}^2\beta_2)}{2g}$$

Manometer efficiency $\eta_{ma} = \frac{H_m}{H_e} = \frac{gH_m}{V_{u2}u_2} = \frac{(u_2^2 - V_{f2}^2\text{cosec}^2\beta_2)}{2u_2(u_2 - V_{f2}\cot\beta_2)}$ (5.30)

Thus, the manometric efficiency $\eta_{ma} = f(u_2, V_{f2}, \beta_2)$ (5.31)

To examine the effect of blade angle on the efficiency, let us consider the extreme values.

1. First consider the case with $\beta_2 = 90°$:
   Then, cosec 90° = 1 and cot 90° = 0.

   $\eta_{ma} = \frac{1}{2}\left(1 - \frac{V_{f2}^2}{u_2^2}\right)$ which is always less than 0.5.

2. Next, let us consider $\beta_2 \to 0$. Then cosec $\beta_2 \to$ cot β_2

   $$\eta_{ma} \to \frac{u_2 + V_{f2}\cot\beta_2}{2u_2}$$

   Further, as $\beta_2 \to 0$, $V_{u2} \to 0$ and $(V_{f2}\cot\beta_2 = (u_2 - V_{u2})) \to u_2$

and $\left(\frac{u_2 + V_{f2}\cot\beta_2}{2u_2}\right) \to 1.0$ Hence, the manometric efficiency tends to the highest possible value of 1.0. It is thus clear that smaller the value of vane angle β_2, higher would be the value of efficiency. From practical considerations, very small value of vane angles mean very long blades and hence excessive resistance to flow inside the impeller. Hence, it is usual practice to limit the blade angle to a value of not less than about 20° and use a value of β_2 in the neighbourhood of 20° to achieve high manometric efficiency.

5.4.7 Theoretical Head—Discharge Relationship of a Pump

Consider the theoretical head (Euler head) relationship as given by Eq. (5.14) as

$$H_e = \frac{u_2V_{u2}}{g}.$$

The basic assumption involved in the derivation of this equation is that the entry at the inlet is radial. This implies that $\alpha_1 = 90^o$ and $V_{f1} = V_1$.

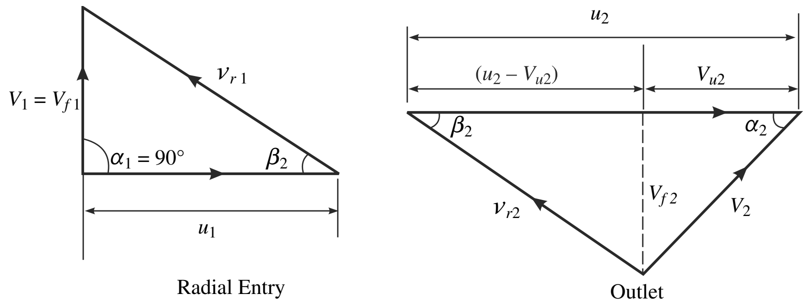


Fig. 5.14 *Velocity triangles at entry and outlet (radial entry at inlet)*

Consider the velocity triangle at outlet (Fig. 5.14).

$$\frac{V_{f2}}{u_2 - V_{u2}} = \tan\beta_2$$

$$V_{u2} = u_2 - V_{f2}\cot\beta_2$$

By neglecting the blade thickness the discharge at the impeller outlet,

$$Q = \pi b_2 D_2 V_{f2}$$

where b_2 = Width of the impeller at the outlet and D_2 = outlet diameter of the impeller.

$$V_{f2} = \frac{Q}{\pi b_2 D_2} \text{ and } V_{u2} = u_2 - \frac{Q}{\pi b_2 D_2}\cot\beta_2 .$$

Hence $$H_e = \frac{u_2}{g}\left(u_2 - \frac{Q}{\pi b_2 D_2}\right)\cot\beta_2 \quad (5.32)$$

For a given pump running at constant speed, Eq.(5.32) can be written as

$$H_e = A - BQ\cot\beta_2 \quad (5.33)$$

where A and B are constants for a given impeller at a constant speed.

Equation (5.33) shows that for a given vane angle β_2, the theoretical head H_e varies linearly with the discharge Q. When $Q = 0$, there is a finite positive value of H_e. This indicates that if the delivery valve of a running pump is completely closed shut, and the pump is kept running, a positive pressure head is produced by the pump. This is known as *shut-off head*. From Eq.(5.32), the ideal value of shut-off head is

$$H_{e-shutoff} = \frac{u_2^2}{g} \quad (5.32\text{-a})$$

The actual value will however be much smaller, roughly about 60% of this value, due to nonrecoverable losses in the impeller.

Further, from Eq. (5.33) for a given speed, b_2 and D_2, Euler head H_e is a linear function of $Q\cot\beta_2$. It shows that for a given β_2, the Euler head H_e varies linearly with Q. Further, depending upon whether (a) β_2 is less than 90°, or (b) equal to 90°, or (c) greater than 90°, the term cot β_2 can be negative, unity or positive. This in turn would be reflected in the head H_e decreasing, remaining constant or increasing with the discharge in the respective ranges of β_2. The variation of head with discharge in three kinds of pumps, viz. pumps with backward curved blades, radial blades and forward curved blades is shown in Fig 5.15 (a) for ideal situation.

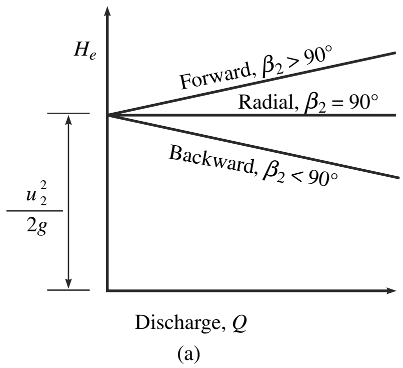


Fig. 5.15(a) *H_e-Q relationship for ideal pumps*

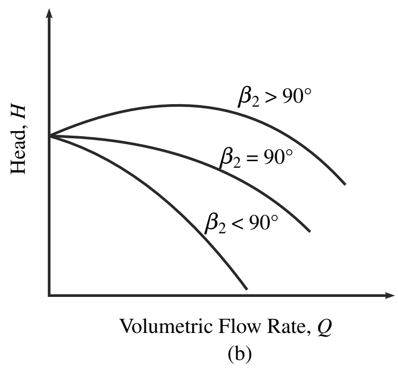


Fig. 5.15(b) *H_e-Q relationship for actual pumps*

It is seen that the slope of $H_e - Q$ line is positive for forward curved vanes and H_e is constant for all discharges (Q) in the radial vane pumps. For pumps with backward curved vanes, the Euler head H_e decreases with increase in the discharge.

1. Forward and Radial Curved Vanes

At the outset, it appears from the above that the forward-curved and radial blades have a very favorable *H-Q* relationship. However, it is found that in forward-curved vane pumps and in radial vane pumps, the efficiencies are very poor. In addition, forward-curved vane impellers produce larger absolute velocities that require very efficient diffusers to convert the exit kinetic energy into pressure energy. The energy losses are normally high and hence the efficiency is poor. Further, in forward-curved vane pumps, an instability called *pump surging* is likely to occur in certain regions of operation.

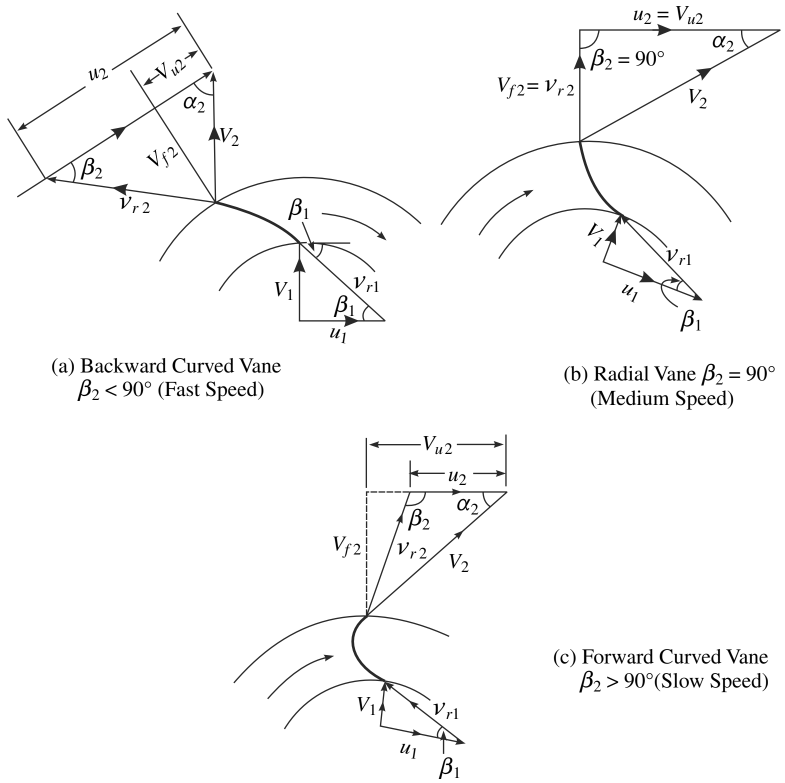
2. General Practice

In view of the adverse factors enumerated above, forward curved vanes or radial vanes are seldom used in pumps. They find application in certain specialty designs. Normally, backward-curved vanes with β_2 in the range of 20°–40° are of common use. Due to losses which are present in actual pumps the H_e–Q relationship of actual pumps take the shapes as shown in Fig. 5.15(b) for various ranges of angle β_2.

The outlet velocity triangles that result in different vane configurations are shown in Fig. 5.16 (a, b and c). It is usual to subclassify the radial flow centrifugal pumps on the basis of the range of value of β_2 as below:

Table 5.2 *Subclassification of centrifugal pumps*

Range of Blade Angle β_2	Description of the Blade Tip	Classification
$\beta_2 < 90°$	Backward curved blade. Outlet tip of the blade curves in a direction opposite to that of the rotation of the impeller. [Fig. 5.6(a)].	Fast speed
$\beta_2 = 90°$	Radial blade. The relative velocity at outlet is in a radial direction. [Fig. 5.6(b)].	Medium speed
$\beta_2 > 90°$	Forward curved blade. Outlet tip of the blade curves in the direction of the rotation of the impeller. [Fig. 5.6(c)].	Slow speed



(a) Backward Curved Vane $\beta_2 < 90°$ (Fast Speed)

(b) Radial Vane $\beta_2 = 90°$ (Medium Speed)

(c) Forward Curved Vane $\beta_2 > 90°$(Slow Speed)

Fig. 5.16 *Velocity triangles for different vane configurations*

5.4.8 Energy Losses in the Impeller

The expression for Euler head as $H_e = \dfrac{u_2 V_{u2}}{g}$ has been developed based on idealised flow conditions and with the assumption of radial entry. The real-world pumps are obviously different from the ideal conditions in many ways. Significant differences include (a) the number of vanes in a pump is finite and not infinite as in an ideal impeller, (b) the presence of frictional resistance to flow in the passages of the impeller, (c) occurrence of non-ideal entry and exit conditions, and (d) non-ideal diffuser performance of volute casings. Because of these, real pumps have many sources of energy losses that make the energy efficiency to be less than unity. The energy losses can be broadly classified as hydraulic losses and nonhydraulic losses. The latter consist of volumetric losses due to leakage and mechanical losses due to friction at rotating parts.

1. Hydraulic Losses

The hydraulic losses stem from the following:

a) *Circulatory flow*, h_c, at the passages of the impeller as described in Sec.5.4.3. This component of the hydraulic losses is practically independent of the discharge.
b) *Fluid friction at the flow passages*, h_{fp}. This loss depends on the area in contact with the fluid flow and the roughness magnitude of the surface. The loss can be considered to vary as the square of the velocity and hence $h_{fp} = K_1Q^2$ where K_1 is a coefficient.
c) *Shock losses at the entrance to impeller*, h_q. This loss occurs due to improper entry angle of the flow with respect to the blade angle. At design conditions this loss is practically zero and increases at reduced or increased flow from normal values.

2. Nonhydraulic Losses

Nonhydraulic losses can be considered under two types:

(a) Leakage Losses Due to improper clearances between the impeller and the casing, some water after receiving the energy at the impeller may not exit from of the pump outlet. A part of it may leak down to the suction side and a part may also leak out of the casing to waste. Such leakage represents energy loss and affects the efficiency of the pump.

(b) Mechanical Losses These include mechanical losses due to friction at bearings and packings. Also the friction loss due to interaction of the rotating impeller shroud with the fluid surrounding it (called *disk-friction loss*) is considered as mechanical friction. Disk-friction loss is equivalent of windage losses in the turbine.

Figure 5.17 is a schematic representation of various losses in the form of a plot of power *vs* discharge in the pump. This figure shows the variation of head *vs* discharge for an impeller with backward facing blades. Note that the leakage is not represented here, as leakage of discharge in a pump represents a loss of power but does not affect the head. Observe that the shock loss is zero at normal discharge and increases as the departure from normal discharge increases.

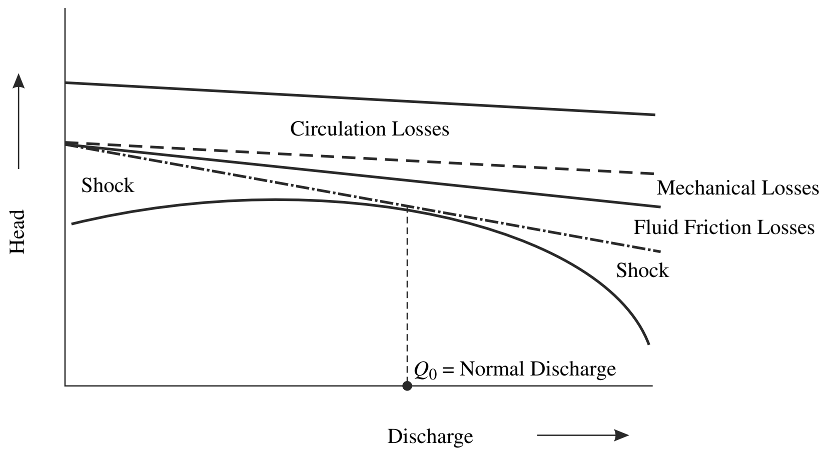


Fig. 5.17 *Schematic representation of pump losses*

5.4.9 Efficiencies

Consider a pump unit with manometric head H_m and outlet discharge Q . The power supplied to the shaft from the prime mover is P. It is assumed that the entry to impeller at inlet is radial and as such, the Euler head is taken as $H_e = \frac{u_2 V_{u2}}{g}$.

1. Volumetric Efficiency, η_v

It is possible that out of a total discharge of water discharged by the pump, some quantity of discharge (Q_L) might not have been delivered at the discharge end of the pump due to leaks in the clearances between the impeller and the casing. These leakages include those that flow back to the suction side and those that go waste outside the casing. The volumetric efficiency of the pump η_v is defined as

$$\eta_v = \frac{\text{Discharge reaching the pump outlet}}{\text{Discharge entering the eye of the impeller}} = \left(\frac{Q}{Q + Q_L}\right) \tag{5.34}$$

Generally, leakage is a very small percentage of the discharge Q.

2. Manometric Efficiency, η_{ma}

While the theoretical head (Euler head) that should have been developed by the pump is H_e, the actual manometric head developed is less than H_e by an amount h_{fi} representing the total hydraulic head loss in the flow through impeller. These losses include fluid frictional losses in the blade passage, circulatory flow between the blades due to finite number of blades in the impeller and shock losses at the entrance to the impeller. Manometric efficiency is defined as the ratio of manometric head developed by the pump to the head imparted by the impeller to the liquid in the pump. Thus,

$$\eta_{ma} = \frac{\text{Manometric head developed}}{\text{Head imparted to the liquid by the impeller}} = \frac{H_m}{H_e} = \frac{gH_m}{(V_{u2} u_2)} \tag{5.35}$$

Since $H_m = H_e - h_{fi}$,

$$\eta_{ma} = \frac{H_m}{H_m + h_{fi}} \tag{5.35-a}$$

where h_{fi} = Hydraulic losses in the impeller and casing.

3. Mechanical Efficiency, η_{mech}

In a pump there are inevitable mechanical frictions that occur: (a) between the impeller and other parts of the pump unit such as bearings, glands, couplings, etc., and (b) due to disc friction on the outside of the impeller shroud. Due to this, the power transmitted to the liquid by the impeller is smaller than the power supplied to the shaft by the prime mover. Let the head lost in mechanical friction and other losses be h_{mech}. The mechanical efficiency η_{mech} is defined as

$$\eta_{mech} = \frac{\text{Power actually delivered by the impeller}}{\text{Power supplied to the shaft}} = \frac{\gamma(Q+Q_L)H_e}{P} \tag{5.36}$$

$$\eta_{mech} = \frac{H_e}{H_e + h_{mech}} \tag{5.36-a}$$

4. Overall Efficiency, η_0

The overall efficiency (also known as *total efficiency*) of the pump is defined as the ratio of actual power output from the pump to the power supplied by prime mover to the shaft.

Thus $$\eta_0 = \frac{\text{Power output from the pump}}{\text{Power supplied by prime mover to the shaft}} = \frac{\gamma Q H_m}{P} \tag{5.37}$$

The power input in to the pump by the prime mover =

$$\text{Brake Power} = P = \frac{\gamma Q H_m}{\eta_0} \tag{5.37-a}$$

From the definitions of η_Q, η_h and η_{ma} in equations (5.35, 5.36 and 5.37) respectively, the total efficiency η_0 could be written as

$$\eta_0 = \frac{H_m}{H_e} \times \frac{Q}{(Q+Q_L)} \times \frac{\gamma(Q+Q_L)H_e}{P}$$

giving $\eta_0 = \eta_{ma}\,\eta_v\,\eta_{mech}$ (5.38)

NOTE

The expression for brake power input into the pump is given by Eq.(5.37-a) as

$$P = \frac{\gamma Q H_m}{\eta_0}$$

The efficiency term is in the denominator, as the actual energy supplied to the shaft will be higher than the power ultimately transmitted to the liquid by an amount equal to the sum of hydraulic, mechanical and volumetric energy losses.

5.5 WORKING RATIOS OF CENTRIFUGAL PUMPS

The following are the default values of various key parameters:

- Inlet velocity is assumed to be radial. This implies that $\alpha_1 = 90°$ and $V_{f1} = V_1$.
- There is no leakage of flow. Volumetric efficiency = 1.0
- The blades are backward curved, $\beta_2 < 90^o$.

1. Relative Inlet Diameter, D_1/D_2

The relative inlet size, D_1/D_2, where D_2 is the outside diameter of the impeller, varies in the range 1/3 to 2/3. The ratio $D_1/D_2 = 0.5$ is a common choice.

2. Speed Ratio, K_u

Speed ratio covering the peripheral velocity at the blade tip is defined as $K_u = \dfrac{u_2}{\sqrt{2gH_m}}$ and the value of K_u varies from 0.95 to 1.25.

3. Flow Ratio, K_f

The flow ratio covering the flow velocity V_{f2} is defined as $K_f = \dfrac{V_{f2}}{\sqrt{2gH_m}}$ and the value of K_f varies from 0.1 to 0.25

4. Normal Discharge, Q

Normal discharge is the discharge for which the pump is designed to deliver against design head and design speed of rotation. It is given by

$$Q = C\,\pi\, bD_2V_{f2} \tag{5.39}$$

where D_2 = Outer diameter of the impeller

b = Breadth (distance between the shrouds) of the impeller

C = Factor called *contraction factor* to account for the space taken by the blades of the impeller. The value of C lies in the range 0.8 to 0.9.

V_{f2} = Flow velocity at the outlet of the impeller

5. Inlet Blade Angle, β_1

The usual practice is to have the inlet blade angles in the range 10^o to 25^o.

6. Outlet Blade Angle, β_2

Use of backward-curved blades is the norm. The range of the outlet-blade angle is 20^o to 40^o. Generally, β_2 is made slightly larger than β_1.

7. Number of Vanes

The number of vanes must be sufficient to impart the energy to the flow and at the same time it should not choke the flow. Further, it should offer least amount of resistance to flow. A rule of thumb due to Stepanoff is $Z = \dfrac{\beta_2}{3}$ where Z = Number of vanes, β_2 = Vane angle at outlet in degrees. For low-to-medium specific-speed pumps, the usual range for Z is 6 to 12.

8. Suction and Delivery Pipes

For small pumps, both the suction and delivery pipes will have the same diameter. However, for large pumps the suction pipes should have a slightly larger diameter than the delivery pipe.

9. Efficiency

Efficiency is generally large for large-sized pumps. Thus pumps with capacity larger than about 600 lps are likely to have overall efficiencies in the range 90 to 92% as their design and manufacture are likely to have been invested with greater care. However, smaller pumps may have overall efficiencies in the range 65 to 85%. Typically, a 30 lps pump from a reputed manufacturer may have an efficiency of about 80%.

10. Operating Range

Preferred operating range is 40% to 110% of design value.

11. Specific Speed, N_{sq}

$$N_{sq} = \frac{N\sqrt{Q}}{H^{3/4}} = 10 \text{ to } 220, \text{ and}$$

***Shape factor*, S_q** $S_q =$ 0.03 to 0.66 (*Details in Sec. 5.8.5*)

Nameplate Every pump should have a permanent metal nameplate that is securely attached and easy to read. The nameplate should contain the manufacturer's name and address, model and serial number of pump, date and specification number, head, capacity, speed and hydrostatic test pressure. The direction of rotation of the pump impeller should be clearly indicated by an arrow, cast or stamped on to the casing.

Definitions and Synonyms

Head against which the machine has to work: Manometric head, net head, net useful head, net pump head, rated net head, manometric delivery head, total manometric head, total dynamic head (*TDH*),

Power delivered to the fluid: Water power

Power required to drive the pump: Brake power

5.6 ILLUSTRATIVE EXAMPLES—SET 5.1

The following are the default values of parameters in the illustrative examples.

- The inlet to the pump is radial
- Water is the liquid that is being pumped.
- Unit weight of water $\gamma = 9.79$ kN/m^3
- Head of a pump = Manometric head of the pump

Velocity Triangles and Relationships

*EXAMPLE 5.1

A centrifugal pump has an impeller of 30 cm outer diameter. The vane tips are radial at the outlet. For a rotative speed of 1450 rpm, calculate the manometric head developed. Assume a manometric efficiency of 82%.

Solution

Given: D_2 = 0.30 m, N = 1450 rpm, $\eta_o = 0.82$

Consider the outlet velocity triangle shown in Fig. 5.18. From this,

$V_2 \cos \alpha_2 = V_{u2} = u_2$

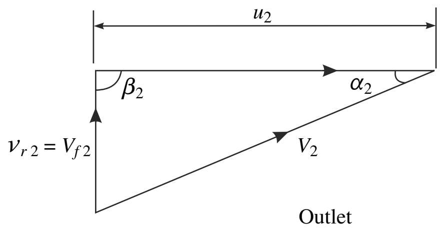


Fig. 5.18 *Outlet velocity triangle, Example 5.1*

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.30 \times 1450}{60} = 22.78\,\text{m/s}$$

$$\text{Manometric efficiency} = \eta_{ma} = \frac{gH_m}{u_2 V_{u2}} = \frac{gH_m}{u_2^2}$$

$$0.82 = \frac{9.81 \times H_m}{(22.78)^2}$$

H_m = Manometric head developed = 43.38 m

*EXAMPLE 5.2

A centrifugal pump delivers water against a total head of 10 m at a design speed of 1000 rpm. The vanes are curved backwards and make an angle of 30^o with the tangent at the outer periphery of the impeller. The impeller diameter is 30 cm and has a width of 5 cm at the outlet. (a) If the manometric efficiency is 0.95%, estimate the discharge of the pump. (b) Assuming an overall efficiency of 76%, estimate the power required to drive the pump.

Solution

Given: H_m = 10.0 m, N = 1000 rpm, $\beta_2 = 30^o$, D_2 = 0.30 m, b_2 = 0.05 m, η_{ma} = 0.95, $\eta_0 = 0.76$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.30 \times 1000}{60} = 15.708\,\text{m/s}$$

(a) Manometric efficiency $\eta_{ma} = \dfrac{gH_m}{u_2 V_{u2}}$

$$0.95 = \frac{9.81 \times 10.0}{15.708 \times V_{u2}}$$

V_{u2} = 6.574 m/s

From the outlet velocity triangle, since $\beta_2 < 90°$

$$\tan \beta_2 = \frac{V_{f2}}{(u_2 - V_{u2})}$$

$$\tan 30° = \frac{V_{f2}}{(15.708 - 6.574)}$$

$$V_{f2} = 5.274 \text{ m/s}$$

$$\text{Discharge} = Q = \pi D_2 b_2 V_{f2}$$

$$= \pi \times 0.30 \times 0.05 \times 5.274$$

$$Q = 0.249 \text{ m}^3\text{/s} = 249 \text{ L/s}$$

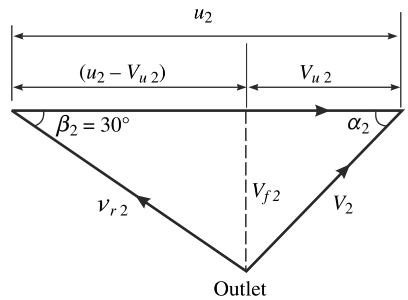


Fig. 5.19 *Outlet velocity triangle, Example 5.2*

(b) Power required to drive the pump

$$= P_s = \frac{\gamma Q H_m}{\eta_0} = \frac{9.79 \times 0.249 \times 10}{0.76} = 32.0 \text{ kW}$$

*EXAMPLE 5.3

A centrifugal pump with 40 cm impeller diameter delivers 75 L/s of oil of relative density 0.85 at a tip speed of 25.1 m/s. The flow velocity is constant at 2.0 m/s and the outlet blade is curved backwards at an angle of 35°. The overall efficiency is 0.88. (a) Calculate the brake power and torque applied to the pump shaft. (b) If the inlet diameter is 25 cm, calculate the inlet-blade angle.

Solution

Given: $Q = 0.075$ m^3/s, $D_2 = 0.40$ m, $u_2 = 25.1$ m/s, $V_{f1} = V_{f2} = 2.0$ m/s, relative density $S = 0.85$, $\eta_0 = 0.88$, $\beta_2 = 35°$

(a) From the outlet velocity triangle, Fig.5.20:

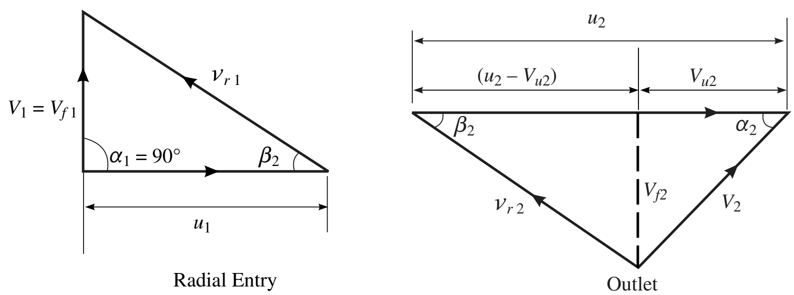


Fig. 5.20 *Inlet and outlet velocity triangles, Example 5.3*

$$\tan \beta_2 = \frac{V_{f2}}{(u_2 - V_{u2})} = \tan 35° = 0.70$$

$$25.1 - V_{u2} = \frac{2.0}{0.70} = 2.857$$

$$V_{u2} = 25.1 - 2.857 = 22.243 \text{ m/s}$$

$$\text{Euler head} = H_e = \frac{u_2 V_{u2}}{g} = \frac{25.1 \times 22.243}{9.81} = 56.91 \text{ m}$$

Brake power applied to the shaft =

$$P_e = \frac{\gamma Q H_e}{\eta_0} = \frac{1}{0.88}(0.85 \times 9.79) \times 0.075 \times 56.91 = 40.36 \text{ kW}$$

Angular velocity at outlet = $\omega_2 = \dfrac{u_2}{(D_2/2)} = \dfrac{25.1}{(0.40/2)} = 125.5$ rad/s

Torque applied to the shaft = $T_e = \dfrac{P_e}{\omega_2} = \dfrac{40.36}{125.5} = 0.322\,\text{kN.m}$

(b) From the inlet velocity triangle, Fig. 5.20, for radial entry:

$$u_1 = \frac{u_2 D_1}{D_2} = \frac{25.1 \times 0.25}{0.40} = 15.69\,\text{m/s}$$

$V_{f1} = V_{f2} = 2.0$ m/s,

$$\tan\beta_1 = \frac{V_{f1}}{u_t} = \frac{2.0}{15.69} = 0.1275$$

Inlet-vane angle $\beta_1 = 7.26^o$

*EXAMPLE 5.4

A centrifugal pump has an impeller of 80 cm diameter and delivers a discharge of 1.1 m^3/s against a head of 70 m. The speed of the impeller is 1000 rpm and the width of the impeller is 8 cm at the outlet. (a) If the leakage loss is 4% of the discharge, external mechanical loss is 10 kW and manometric efficiency is 82%, calculate the blade angle at the outlet. (b) What is the value of overall efficiency of this pump?

Solution

Given: $D_2 = 0.80$ m, $Q = 1.10$ m^3/s, $H_m = 70$ m, $N = 1000$ rpm, $b_2 = 0.08$ m, $\eta_{ma} = 0.82$

Leakage loss = 4%

$Q_{th} = (1.10 \times 1.04) = 1.144$ m^3/s

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.8 \times 1000}{60} = 41.89\,\text{m/s}$$

Since $Q_{th} = \pi D_2 b_2 V_{f2}$

$$V_{f2} = \frac{1.144}{\pi \times 0.8 \times 0.08} = 5.69\,\text{m/s}$$

Since $\eta_{ma} = \dfrac{gH_m}{u_2 V_{u2}}$,

$$V_{u2} = \frac{9.81 \times 70}{41.89 \times 0.82} = 20.0\,\text{m/s}$$

From velocity triangle at the outlet, Fig. 5.21,

$$\tan\beta_2 = \frac{V_{f2}}{(u_2 - V_{u2})} = \frac{5.69}{(41.89 - 20.0)} = 0.26$$

Outlet blade angle $\beta_2 = 14.57^{\circ}$

Power required to drive the pump (Brake power): $P = \left[\dfrac{V_{u2}u_2}{g}\gamma Q_{th}\right] + 10.0$

$$= \left(\frac{20.0 \times 41.89}{9.81}\right) \times 9.79 \times 1.144 + (10.0)$$

= 966.5 kW

Mechanical efficiency $\eta_{mech} = \dfrac{956.5}{966.5} = 0.99$

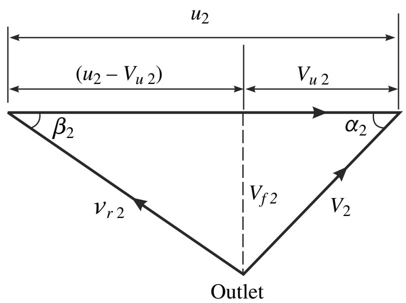


Fig. 5.21 *Outlet velocity triangle—Example 5.4*

Overall efficiency $\eta_0 = \eta_{mec} \times \eta_{ma} = 0.99 \times 0.82 = 0.812$

$\eta_0 = 81.2\%$

*EXAMPLE 5.5

A centrifugal pump discharges 0.2 m³/s of water at a head of 25 m when running at a speed of 1400 rpm. The manometric efficiency is 80%. If the impeller has an outer diameter of 30 cm and a width of 5 cm, determine the vane angle at the outlet.

Solution

Given: $D_2 = 0.30$ m, $Q = 0.2$ m^3/s, $H_m = 25$ m, $N = 1400$ rpm, $b_2 = 0.05$ m, $\eta_{ma} = 0.80$

Let β_2 be the blade angle at the outlet.

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.3 \times 1400}{60} = 22.0\,\text{m/s}$$

Since $\eta_{ma} = \dfrac{gH_m}{u_2 V_{u2}}$,

$$V_{u2} = \frac{9.81 \times 25}{22.0 \times 0.80} = 13.94\,\text{m/s}$$

Since $Q = \pi D_2 b_2 V_{f2}$

$$V_{f2} = \frac{0.2}{\pi \times 0.3 \times 0.05} = 4.244 \text{ m/s}$$

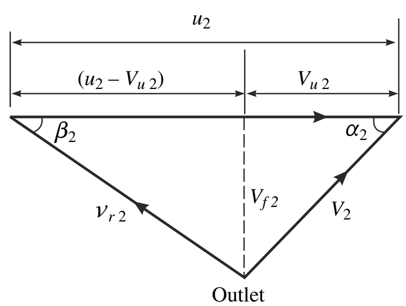


Fig. 5.22 *Outlet velocity triangle, Example 5.5*

From velocity triangle, Fig. 5.22, at the outlet,

$$\tan\beta_2 = \frac{V_{f2}}{(u_2 - V_{u2})} = \frac{4.244}{(22 - 13.94)} = 0.527$$

Outlet vane angle $\beta_2 = 27.77^{\circ}$

*EXAMPLE 5.6

A centrifugal pump has an overall efficiency of 70%, delivers 1500 L/min of water against a static head of 20 m. The suction and delivery pipes each are of 20 cm diameter and have a combined length of 1000 m. Assuming the Darcy–Weisbach friction *factor f = 0.02, estimate the required power input to the pump. Assume minor losses to be 15 times the velocity head in the pipe.*

Solution

$$Q = \frac{1500}{1000 \times 60} = 0.025\ \text{m}^3/\text{s},\ H_{stat} = 20\ \text{m},\ \eta_0 = 0.70,$$

Velocity in the pipes = $V_p = \dfrac{Q}{\frac{\pi}{4} D_p^2} = \dfrac{0.025}{\frac{\pi}{4}(0.20)^2} = 0.796$ m/s

Velocity head in the pipe = $\dfrac{V_p^2}{2g} = \dfrac{(0.796)^2}{2 \times 9.81} = 0.0323$m

Minor losses = $h_{ml} = 15\ \dfrac{V_p^2}{2g}$ = 15 × 0.0323 = 0.484 m

Head loss in the pipes = $h_f = \dfrac{flV_p^2}{2gD_p} = \dfrac{0.02 \times 1000 \times (0.796)^2}{2 \times 9.81 \times 0.2} = 3.23$m

Total head H_m = 20.0 + 3.23 + 0.484 + 0.0323 = 23.746 m

Brake power = $P_s = \dfrac{\gamma Q H_m}{\eta_0} = \dfrac{1}{0.70} \times 9.79 \times 0.025 \times 23.746 = 8.303$ kW

***EXAMPLE 5.7

A centrifugal pump lifts water from a sump to an overhead reservoir. The static lift is 40 m out of which 3 m is the suction lift. The suction and delivery pipes are both of 35 cm diameter. The friction loss in suction pipe is 2.0 m and in delivery pipe it is 6.0 m. The impeller is 0.5 m in diameter and has a width of 3 cm at the outlet. The speed of the pump is 1200 rpm. The exit blade angle is 20°. If the manometric efficiency is 85%, determine the pressures at the suction and delivery ends of the pump and the discharge. Assume that the inlet and outlet of the pump are at the same elevation.

Solution

D_2 = 0.50 m, b_2 = 0.03 m, h_s = 3.0 m, h_d = 37 m, h_{fs} = 2.0 m, h_{fd} = 6 m, N = 1200 rpm, β_2 = 20°, η_{ma} = 0.85. D_p = 0.35 m.

Net head = Static lift + Friction loss = 40.0 + 2.0 + 6.0 = 48.0 m

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 1200}{60} = 31.42\ \text{m/s}$$

By assuming radial flow at inlet, manometric efficiency $\eta_{ma} = \dfrac{gH_m}{u_2 V_{u2}}$

$$0.85 = \frac{9.81 \times 48.0}{31.42 \times V_{u2}}$$

V_{u2} = 17.63 m/s

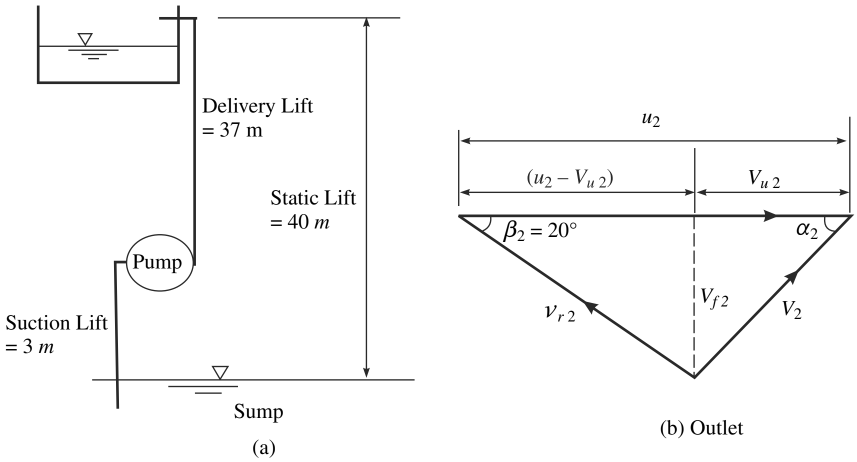


Fig. 5.23 *(a) Schematic layout of the pump; (b) Outlet velocity triangle, Example 5.7*

From outlet velocity triangle, Fig. 5.23(b), $\tan\beta_2 = \dfrac{V_{f2}}{(u_2 - V_{u2})}$

$$\tan 20° = \frac{V_{f2}}{(31.42 - 17.63)} = 0.3639$$

$V_{f2} = 5.02$ m/s

Discharge $Q = \pi D_2 b_2 V_{f2} = \pi \times 0.5 \times 0.03 \times 5.02 = 0.2366\ m^3/s$

Velocity in delivery pipe $= V_d =$ Velocity of suction pipe $= V_s$

$$V_s = \frac{Q}{\frac{\pi}{4}D_p^2} = \frac{0.2366}{\frac{\pi}{4}\times(0.35)^2} = 2.459 \text{ m/s}$$

$$\frac{V_s^2}{2g} = \frac{V_d^2}{2g} = \frac{(2.459)^2}{2\times 9.81} = 0.308 \text{ m}$$

Delivery side of pump: Let the pressure on delivery side = p_d

$$\frac{p_d}{\gamma} + \frac{V_d^2}{2g} = h_d + h_{fd} + \frac{V_d^2}{2g}$$

$$\frac{p_d}{\gamma} = h_d + h_{fd} = 37 + 6 = 43.0 \text{ m}$$

$p_d = 43.0 \times 9.79 = 421$ kPa (gauge)

Suction side of pump:

Let the pressure on suction side = p_s and atmospheric pressure = p_{atm}. Then

$$\frac{p_{atm}}{\gamma} = h_s + h_{fs} + \frac{p_s}{\gamma} + \frac{V_s^2}{2g}$$

Taking atmospheric pressure as datum pressure, $0 = 3 + 2 + \dfrac{p_s}{\gamma} + 0.308$

$\dfrac{p_s}{\gamma} = -5.308$ m (vacuum pressure)

$= -(5.308 \times 9.79) = -51.97$ kPa (vacuum)

**EXAMPLE 5.8

A centrifugal pump while running at 1000 rpm is required to discharge 65 L/s of water against a total head of 16 m. The manometric efficiency of the pump is 0.85. If the vane angle at the outlet is 35° and the velocity of flow is 1.5 m/s, estimate the outer diameter of the impeller and its width at the exit.

Solution

Given: $N = 1000$ rpm, $Q = 0.065$ m^3/s, $\eta_{ma} = 0.85$, $\beta_2 = 35°$, $V_{f1} = V_{f2} = 1.5$ m/s, $H_m = 16.0$ m

Manometric efficiency $\eta_{ma} = \dfrac{gH_m}{u_2 V_{u2}}$,

$$0.85 = \frac{9.81 \times 16}{u_2 V_{u2}}$$

$u_2 V_{u2} = 184.66$ (i)

From the outlet velocity triangle,

$$\tan\beta_2 = \frac{V_{f2}}{u_2 - V_{u2}} = \tan 35° = 0.700$$

$$u_2 - V_{u2} = \frac{1.50}{0.70} = 2.142$$

$V_{u2} = (u_2 - 2.142)$

Substituting in Eq. (i), $u_2^2 - 2.142\, u_2 - 184.66 = 0$

Taking the positive root $u_2 = 14.702$ m/s

$$V_{u2} = \frac{184.66}{14.702} = 12.56 \text{ m/s}$$

Since $u_2 = \dfrac{\pi D_2 N}{60}$, Impeller diameter $D_2 = \dfrac{60 \times u_2}{\pi N} = \dfrac{60 \times 14.702}{\pi \times 1000} = 0.280$ m

Discharge $Q = \pi D_2 b_2 V_{f2}$

$0.065 = \pi \times 0.280 \times b_2 \times 1.5$

b_2 = Width of the impeller at exit = 0.0493 m = 4.93 cm

**EXAMPLE 5.9

Assuming no loss of energy, show that the increase in piezometric head across the impeller of a centrifugal pump can be expressed as

$$\left(\frac{p_2}{\gamma}+Z_2\right)-\left(\frac{p_1}{\gamma}+Z_1\right)=\frac{(u_2^2-u_1^2)}{2g}-\frac{(v_{r2}^2-v_{r1}^2)}{2g}$$

where u = Peripheral velocity, v_r = Relative velocity, Z = Datum height and p = pressure. Suffix 1 and 2 refer to inlet and outlet parameters respectively.

Solution

For an ideal centrifugal pump that does not have any losses, either hydraulic or mechanical,

Euler head = $H_e = \dfrac{(u_2V_{u2}-u_1V_{u1})}{g}$

Noting that $V_{u2} = V_2 \cos \alpha_2$

From the velocity triangle at outlet, Fig. 5.24,

$$V_2^2 + u_2^2 - 2u_2V_{u2} = v_{r2}^2$$

Similarly, for the inlet velocity triangle in a general case,

$$V_{u1} = V_1 \cos \alpha_1, \text{ and}$$

$$V_1^2 + u_1^2 - 2u_1V_{u1} = v_{r1}^2$$

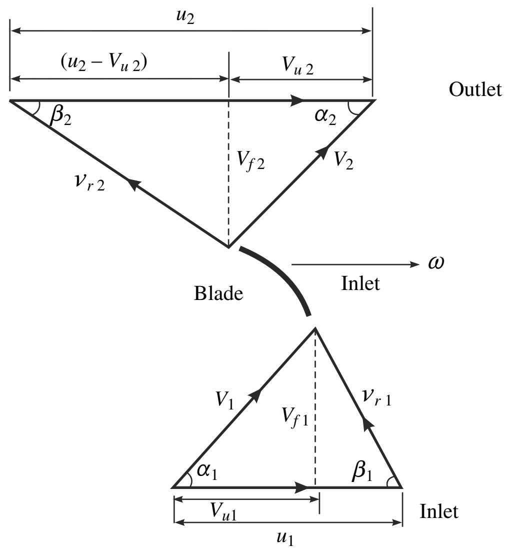


Fig. 5.24 *General inlet and outlet velocity triangles, Example 5.9*

Thus,

$$u_2V_{u2} = \left(\frac{1}{2}(V_2^2+u_2^2-v_{r2}^2\right)$$

and $$u_1V_{u1} = \left(\frac{1}{2}(V_1^2+u_1^2-v_{r1}^2\right)$$

Hence, $$H_e = \frac{(u_2V_{u2}-u_1V_{u1})}{g} = \frac{1}{2g}[(V_2^2-V_1^2)+(u_2^2-u_1^2)-(v_{r2}^2-v_{r1}^2)]$$

By Bernoulli theorem applied to inlet (suffix 1) and outlet (suffix 2) of the pump,

$$H_e = \left[\frac{p}{\gamma}+Z+\frac{V^2}{2g}\right]_2 - \left[\frac{p}{\gamma}+Z+\frac{V^2}{2g}\right]_1$$

Hence, $$\left[\left(\frac{p_2}{\gamma}-\frac{p_1}{\gamma}\right)+(Z_2-Z_1)\right] = H_e - \left(\frac{V_2^2}{2g}-\frac{V_1^2}{2g}\right) = \frac{1}{2g}[(u_2^2-u_1^2)-(v_{r2}^2-v_{r1}^2)]$$

Increase in piezometric head =

$$\left[\left(\frac{p_2}{\gamma}-\frac{p_1}{\gamma}\right)+(Z_2-Z_1)\right] = \left[\frac{(u_2^2-u_1^2)}{2g}-\frac{(v_{r2}^2-v_{r1}^2)}{2g}\right]$$

NOTE

The above expression for increase in piezometric head could be put in the following form of the familiar Bernoulli equation with the addition of a new term:

Adding $\frac{1}{2g}\left[(V_2^2 - V_1^2)\right]$ to both sides of the equation

$$\left[\left(\frac{p_2}{\gamma} - \frac{p_1}{\gamma}\right) + (Z_2 - Z_1)\right] = \left[\frac{(u_2^2 - u_1^2)}{2g} - \frac{(v_{r2}^2 - v_{r1}^2}{2g}\right]$$

$$\left[\frac{p}{\gamma} + Z + \frac{V^2}{2g}\right]_2 - \left[\frac{p}{\gamma} + Z + \frac{V^2}{2g}\right]_1 = H_e = \frac{1}{2g}\left[(V_2^2 - V_1^2) + (u_2^2 - u_1^2) - (v_{r2}^2 - v_{r1}^2)\right]$$

$$\left[\frac{p}{\gamma} + Z + \frac{v_{r2}^2}{2g}\right]_2 - \left[\frac{p}{\gamma} + Z + \frac{v_{r1}^2}{2g}\right]_1 = \left(u_2^2 - u_1^2\right)$$

$$\left[\frac{p}{\gamma} + Z + \frac{v_r^2}{2g} - \frac{u^2}{2g}\right]_2 - \left[\frac{p}{\gamma} + Z + \frac{v_r^2}{2g} - \frac{u^2}{2g}\right]_1$$

$$\left[\frac{p}{\gamma} + Z + \frac{v_r^2}{2g} - \frac{u^2}{2g}\right]_2 - \left[\frac{p}{\gamma} + Z + \frac{v_r^2}{2g} - \frac{u^2}{2g}\right]_1 = \text{constant}$$

This equation is known as ***Bernoulli equation in rotating coordinates*** and applies to ideal incompressible flow.

*EXAMPLE 5.10

In a centrifugal pump having a manometric efficiency of 0.7 and having the outer diameter equal to twice the inner diameter, show that the minimum outer diameter D_{2m} of the impeller which will enable it to just start pumping water to a total head of H metres when rotating at a speed of N rpm is given by

$$D_{2m} = \frac{81.7\sqrt{H}}{N} \text{ meters.}$$

Solution

For a centrifugal pump to just start pumping, if there are no losses, the centrifugal head must be equal to the actual head. Hence, under ideal conditions, the total head H is related to the speed as $H = \frac{(u_2^2 - u_1^2)}{2g}$. However, for actual pumps to account for losses in the suction and delivery pipes, the manometer head is used to represent total head as $H = H_m$ and in the pump, the manometer efficiency is used to account for losses in the pump as

$$\frac{(u_2^2 - u_1^2)}{2g} \geq \frac{H_m}{\eta_{ma}}$$

Since $u_2 = \dfrac{\pi D_2 N}{60}$ and $u_1 = \dfrac{\pi D_2 N}{60}$.

$$H = H_m = \frac{1}{\eta_{ma}} \frac{\pi^2 D_2^2 N^2}{2g(60)^2}\left(1-\left(\frac{D_1}{D_2}\right)^2\right)$$

Since $D_2 = 2\,D_1$ and $\eta_{ma} = 0.7$,

$$H = \frac{1}{(0.7)} \frac{\pi^2 D_2^2 N^2}{2g(60)^2}\left(1-\left(\frac{1}{2}\right)^2\right) = 0.00014971\,D_2^2 N^2$$

Indicating the minimum diameter for a given speed by suffix m,

$$D_{2m}^2 = \frac{6679.4H}{N^2}$$

$$D_{2m} = \frac{81.7\sqrt{H}}{N}$$

where N = Speed of the pump in rpm and $H = H_m$ = Manometric head of the pump.

**EXAMPLE 5.11

A centrifugal pump discharges 0.20 m^3/s of water at a rotational speed of 750 rpm. The outlet and inlet diameters of the impeller are 0.50 m and 0.20 m respectively. (a) Calculate the loss of head due to shock at the impeller when the discharge is reduced by 30% through throttling of the outlet valve while the speed of rotation remains unchanged. (b) Determine the minimum starting speed of the pump against a head of 8.0 m.

Solution

Given: $D_2 =$ 0.50 m, $D_1 = 0.20$ m, $H_m = 8$ m, $Q = 0.2$ m^3/s, $N = 750$ rpm

(a) Shock loss due to change of discharge from the normal is given by (Eq. 5.26-a)

$$h_q = \frac{u_1^2}{2g}\left(1-\frac{Q'}{Q}\right)^2$$

u_1 = Peripheral velocity at the inlet = $\dfrac{\pi D_1 N}{60} = \dfrac{\pi \times 0.2 \times 750}{60}$ = 7.854 m/s

$Q = 0.20$ m^3/s and $Q' = 0.20\,(1 - 0.3) = 0.14$ m^3/s

$$\text{Shock loss} = h_q = \frac{u_1^2}{2g}\left(1-\frac{Q'}{Q}\right)^2 = \frac{(7.854)^2}{2\times 9.81}\left(1-\frac{0.14}{0.20}\right)^2 = 0.283 \text{ m}$$

(b) Minimum starting speed = N_m is given by (Eq. 5.29),

$$N_m = \frac{60}{\pi\sqrt{(D_2^2 - D_1^2)}}\sqrt{2gH_m} = \frac{60}{\pi\sqrt{((0.50)^2-(0.20)^2)}}\sqrt{2\times 9.81\times 8.0} = 522 \text{ rpm}$$

***EXAMPLE 5.12

A centrifugal pump has an impeller of 0.5 m outer diameter and when running at 600 rpm discharges 9000 Lpm against a head of 11.0 m. The water enters the impeller radially without whirl or shock. The inner diameter is 0.15 m. The vanes are set back at an angle of 28° to the tangent at the periphery of the outlet. The area of flow is constant from inlet to outlet of the impeller and is 0.05 m^2. Determine the (a) vane angle at inlet, (b) manometric efficiency of the pump, and (c) minimum speed at which the pump commences to work.

Solution

Given: $Q = \dfrac{9000}{1000 \times 60} = 0.150$ m^3/s, $H_m = 11.0$ m, $N = 600$ rpm, $D_2 = 0.50$ m, $D_1 = 0.15$ m, $\beta_2 = 28°$, area of flow = 0.05 m^2

Figure 5.25 shows the velocity triangles at the inlet and out of the pump.

$$V_{f1} = V_{f2} = \frac{Q}{\text{area}} = \frac{0.15}{0.05} = 3.0 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 600}{60} = 15.71 \text{ m/s}$$

and $$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.15 \times 600}{60} = 4.71 \text{ m/s}$$

(a) From the inlet velocity triangle, Fig. 5.25, $\tan \beta_1 = \dfrac{V_{f1}}{u_1} = \dfrac{3.0}{4.71} = 0.6366$

Vane angle at inlet $\beta_1 = 32.48^{\text{o}}$

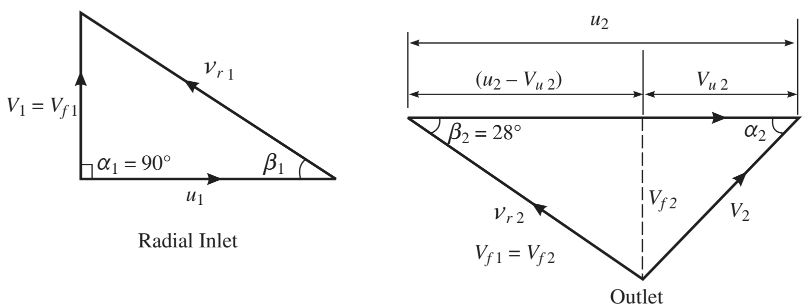


Fig. 5.25 *Velocity triangles, Example 5.12*

(b) From outlet velocity triangle, Fig. 5.25: $\tan \beta_2 = \dfrac{V_{f2}}{(u_2 - V_{u2})} = \tan 28° = 0.5317$

$(15.71 - V_{u2}) = 3.0/0.5317 = 5.642$

$V_{u2} = 15.71 - 5.642 = 10.068$ m/s

Manometric efficiency $\eta_{ma} = \dfrac{gH_m}{u_2 V_{u2}} = \dfrac{9.81 \times 11}{15.71 \times 10.068} = 0.682$

(c) Minimum starting speed = N_m is given by (Eq. 5.29)

$$N_m = \frac{60}{\pi\sqrt{(D_2^2 - D_1^2)}}\sqrt{2gH_m} = \frac{60}{\pi\sqrt{((0.5)^2 - (0.15)^2)}}\sqrt{2\times 9.81\times 11} = 588 \text{ rpm}$$

*EXAMPLE 5.13

Show that the manometric head H_m of a centrifugal pump having a discharge Q and speed N can be expressed as

$$H_m = AN^2 + BNQ + CQ^2$$

where A, B and C are constants

Solution

The manometric head $H_m = K_1 \dfrac{u_2 V_{u2}}{g} - K_2 \dfrac{V_2^2}{2g}$

where

(a) K_1 is a coefficient to account for slip factor and other losses in the impeller, and

(b) $K_2 \dfrac{V_2^2}{2g}$ is the fraction of the discharge head not converted into pressure energy and lost in eddies in the volute casing.

For a given impeller $u_2 = \dfrac{\pi D_2 N}{60} = aN$ where a is a constant.

From outlet velocity triangle,

$V_{f2} = \dfrac{Q}{A_2} = bQ$ where b is a constant.

$V_{u2} = u_2 - V_{f2} \cot \beta_2 = aN - cQ$ where c is a constant.

$V_2 = V_{f2} \operatorname{cosec} \alpha_2 = dQ$ where d is a constant.

Hence, $H_m = K_1 \dfrac{u_2 V_{u2}}{g} - K_2 \dfrac{V_2^2}{2g} = K_1((aN)(aN - cQ) - K_2 d Q^2$

$= AN^2 + BNQ + CQ^2$ where A, B and C are constants.

**EXAMPLE 5.14

A four-stage centrifugal pump has impellers each of 36 cm diameter and 2 cm width at the outlet. The outlet vane angle is 45^o and the vanes occupy 8% of the outlet area. The manometric efficiency is 84% and the overall efficiency is 72%. (a) Determine the head generated when the impeller is rotating at 900 rpm and discharging 60 litres/s. (b) Also, determine the power required to drive the pump.

Solution

Given: $D_2 = 0.36$ m, $b_2 = 0.02$ m, $\beta_2 = 45°$, Vane area = 8% of outlet area, $\eta_{ma} = 0.84$, $\eta_0 = 0.72$, $N = 900$ rpm, $Q = 0.06$ m^3/s, Number of stages = 4

Impeller of each stage is considered as identical.

For any one impeller: Area at outlet = $(1 - 0.08)\ \pi D_2 b_2$

$$= 0.92 \times \pi \times 0.36 \times 0.02 = 0.0208 \text{ m}^2$$

$$V_{f2} = \frac{Q}{A} = \frac{0.06}{0.0208} = 2.883 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.36 \times 900}{60} = 16.965 \text{m/s}$$

From velocity triangle at the outlet,

$$\tan\beta_2 = \frac{V_{f2}}{(u_2 - V_{u2})} = \tan 45° = 1.0$$

$u_2 - V_{u2} = V_{f2} = 2.883$

$V_{u2} = u_2 - 2.883 = 16.965 - 2.883 = 14.082$ m/s

$$\text{Manometric head} = H_m = \eta_{ma}\frac{u_2 V_{u2}}{g} = \frac{0.84 \times 16.965 \times 14.082}{9.81} = 20.46\text{m}$$

$$\text{Brake power per stage } P_s = \frac{\gamma Q H_m}{\eta_0} = \frac{9.79 \times 0.06 \times 20.46}{0.72} = 16.7 \text{ kW}$$

Power required to drive the 4-stage pump = $P_{4s} = 4 \times P_s = 4 \times 16.7 = 66.77$ kW

**EXAMPLE 5.15

A centrifugal pump has vanes which are radial at the outer periphery. The impeller has an outer diameter of 20 cm and a width of 3 cm at that diameter. If the discharge is 1800 L/min and the total head produced is 3.5 m, calculate the (a) rotational speed of the impeller, and (b) the magnitude and direction of absolute velocity at the exit.

Assume manometric efficiency as 0.85.

Solution

Given: $D_2 = 0.20$ m, $Q = 1800$ L/min, $\eta_{ma} = 0.85$, $b_2 = 0.03$ m, $H_m = 3.5$ m

$$Q = \frac{1800}{1000 \times 60} = 0.03 \text{ m}^3\text{/s.}$$

Consider the outlet velocity triangle shown in Fig. 5.26.

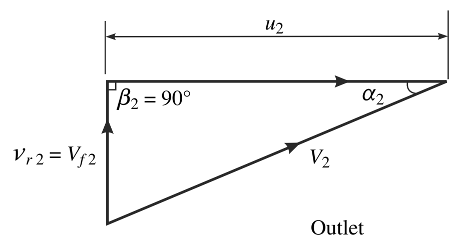


Fig. 5.26 *Outlet velocity triangle, Example 5.15*

Discharge $Q = \pi D_2 b_2 V_{f2}$

$= \pi \times 0.2 \times 0.03 \times V_{f2} = 0.03$ m^3/s

$V_{f2} = 1.5915$ m/s

Since $\beta_2 = 90^o$, $V_{u2} = u_2$

$$\text{Manometric efficiency} = \eta_{ma} = \frac{gH_m}{u_2 V_{u2}} = \frac{gH_m}{u_2^2}$$

$$0.85 = \frac{9.81\times3.5}{(u_2)^2}, \text{ giving } u_2 = 6.3556 \text{ m/s. Also, } V_{u2} = u_2 = 6.3556 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi\times0.20\times N}{60} = 6.3556 \text{ m/s, giving } N = 607 \text{ rpm}$$

(b) Magnitude of absolute velocity

$$V_2 = \sqrt{V_{u2}^2 + V_{f2}^2} = \sqrt{(6.3556)^2 + (1.5915)^2} = 6.552 \text{ m/s}$$

$$\tan\alpha_2 = \frac{V_{f2}}{u_2} = \frac{1.5915}{6.3556} = 0.2504$$

Direction of absolute velocity $\alpha_2 = 14.06°$ with the direction of u_2 as shown in Fig. 5.26.

**EXAMPLE 5.16

A centrifugal pump impeller is 40 cm in outer diameter and 2.5 cm wide at the exit. Its blade angle at the outlet is 30°. When run at a speed of 1500 rpm, the flow rate through the pump is 80 liters/s. (a) Calculate the radial, relative and absolute velocities at the impeller exit. (b) If there is no inlet whirl, what would be the theoretical head added to the water by the impeller?

Solution

Given: $D_2 = 0.40$ m, $N = 1500$ rpm, $b_2 = 0.025$ m, $Q = 0.08$ m^3/s $\beta_2 = 30°$

(a) $u_2 = \dfrac{\pi D_2 N}{60} = \dfrac{\pi\times0.40\times1500}{60} = 31.415 \text{ m/s}$

Discharge $Q = \pi D_2 b_2 V_{f2} = \pi \times 0.4 \times 0.025 \times V_{f2} = 0.08$ m^3/s

$V_{f2} = 2.546$ m/s

From outlet velocity triangle, Fig. 5.27,

$$\tan\beta_2 = \frac{V_{f2}}{u_2 - V_{u2}} = \tan 30° = 0.577$$

$$31.415 - V_{u2} = \frac{2.546}{0.577} = 4.113$$

$V_{u2} = 27.0$ m/s

$$\sin\beta_2 = \frac{V_{f2}}{v_{r2}} = \sin 30°$$

$$v_{r2} = \frac{V_{f2}}{\sin 30°} = \frac{2.546}{0.5} = 5.092 \text{ m/s}$$

$$V_2 = \sqrt{V_{u2}^2 + V_{f2}^2} = \sqrt{(27.0)^2 + (2.546)^2} = 27.12 \text{ m/s}$$

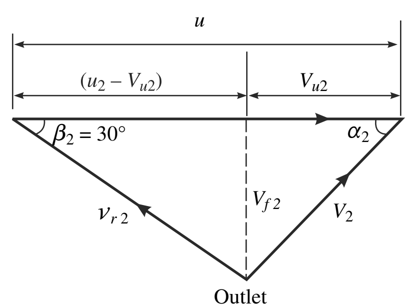


Fig. 5.27 *Outlet velocity triangle, Example 5.16*

$$\tan\alpha_2 = \frac{V_{f2}}{V_{u2}} = \frac{2.546}{27.0} = 0.0943$$

Direction of absolute velocity $\alpha_2 = 5.39^{\circ}$

(b) Euler head $H_e = \frac{u_2 V_{u2}}{g} = \frac{31.415 \times 27.0}{9.81} = 86.5\,\text{m}$

= theoretical head added.

*EXAMPLE 5.17

The impeller of a centrifugal pump has an outer diameter of 50 cm and inner diameter of 25 cm. If the discharge pipe is closed and the pump is full of water, what would be the theoretical difference in pressure between the outer and inner periphery when the pump rotates at 600 rpm?

Solution

Given: $D_1 = 0.25$ m, $D_2 = 0.50$ m, $N = 600$ rpm

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 600}{60} = 15.708 \text{ m/s}$$

Ideal shut-off head = Head developed when the machine is running and the outlet valve is shut off = Theoretical difference in pressure head between the inlet and the periphery of the impeller

$$= H_{\text{shut off}} = \frac{u_2^2}{g}$$

$$= \frac{(15.708)^2}{9.81} = 25.15\,\text{m}$$

***EXAMPLE 5.18

A centrifugal pump is required to discharge 600 L/s of water and develop a head of 15 m when the impeller rotates at 750 rpm. The manometric efficiency is 0.80. The loss of head in the pump due to fluid resistance can be assumed to be 0.027 V^2 where V = velocity with which the water leaves the impeller. Water enters the impeller without shock and whirl and the velocity of flow is 3.2 m/s. Determine the (a) impeller diameter, (b) blade angle at outlet, and (c) outlet area.

Solution

Given: $V_{f2} = 3.2$ m/s, $Q = 0.60$ m^3/s, $H_m = 15$ m, $N = 750$ rpm, $\eta_{ma} = 0.80$.

Loses = $0.027\, V_2^2$

(a) $H_m = 15.0 = \dfrac{u_2 V_{u2}}{g} - 0.027 V_2^2$ (i)

Since $\eta_{ma} = \dfrac{gH_m}{u_2 V_{u2}} = 0.8$

Since $\dfrac{u_2 V_{u2}}{g} = \dfrac{H_m}{0.8} = \dfrac{15}{0.8} = 18.75$ (ii)

From (i), $15.0 = 18.75 - 0.027 V_2^2$

$$V_2 = 11.785 \text{ m/s}$$

From velocity triangle at the outlet (Fig. 5.28), $\dfrac{V_{f2}}{V_2} = \sin \alpha_2$

$$\sin \alpha_2 = \frac{3.2}{11.785} = 0.2715$$

$\alpha_2 = 15.76^o$

Also $V_{u2} = V_2 \cos \alpha_2$

$= 11.785 \cos 15.76° = 11.342$

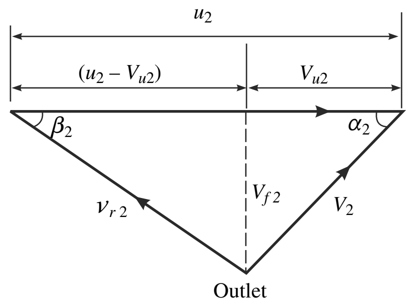


Fig. 5.28 *Outlet velocity triangle, Example 5.18*

Substituting in (ii),

$$u_2 = \frac{g \times 18.75}{V_{u2}} = \frac{9.81 \times 18.75}{11.342} = 16.22 \text{ m/s}$$

Also $u_2 = \dfrac{\pi D_2 N}{60}$, hence $D_2 = \dfrac{u_2 \times 60}{\pi N} = \dfrac{16.22 \times 60}{\pi \times 750} = 0.413$ m

D_2 = Diameter of impeller = 41.3 cm

(b) Blade angle at outlet β_2 is given by

$$\tan \beta_2 = \frac{V_{f2}}{(u_2 - V_{u2})} = \frac{3.2}{(16.22 - 11.342)} = 0.656$$

Outlet-blade angle $\beta_2 = 33.27°$

(c) Outlet area $= \dfrac{\text{Discharge}}{\text{Flow velocity}} = \dfrac{Q}{V_{f2}} = \dfrac{0.60}{3.2} = 0.1875 \text{ m}^2$

**EXAMPLE 5.19

A centrifugal pump with impeller of 0.25 m diameter has backward-curved blades having outlet blade angle of 35^o. The pump is running at 1500 rom. Estimate the theoretical total head and exit velocity (a) when the pump is running as designed, and (b) when the pump is running in reverse direction at 1500 rpm. Assume the flow velocity at outlet as 15% of peripheral velocity in both cases.

Solution

Given: V_{f2} = 15% of u_2, N = 1500 rpm, Designed value $\beta_2 = 35°$, $D_2 = 0.25$ m

(i) *Designed condition:* Backward-curved blades with $\beta_2 = 35°$.

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.25 \times 1500}{60} = 19.63 \text{ m/s}$$

$V_{f2} = 19.63 \times 0.15 = 2.945$ m/s

From Fig. 5.29, showing the outlet velocity triangle:

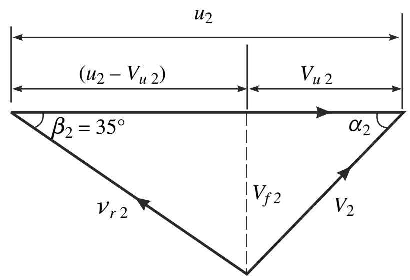


Fig. 5.29 *Outlet velocity triangle of backward-curved blades, Example 5.19*

$$\tan\beta_2 = \frac{V_{f2}}{u_2 - V_{u2}} = \tan 35° = 0.70$$

$$u_2 - V_{u2} = \frac{V_{f2}}{0.70} = \frac{2.945}{0.7} = 4.207$$

$V_{u2} = u_2 - 4.207 = 19.63 - 4.207$
$= 15.423$ m/s

Euler head = ideal total head =

$$H_e = \frac{u_2 V_{u2}}{g} = \frac{19.63 \times 15.423}{9.81} = 30.86 \text{ m}$$

Absolute Exit velocity $V_2 = \sqrt{V_{f2}^2 + V_{u2}^2} = \sqrt{(2.945)^2 + (15.423)^2} = 15.70$ m/s

(b) *When running in reverse direction*:

The blades are curved forward (in the direction of rotation). Hence, blade angle $\beta_2 = 180 - 35 = 155^{\circ}$, $\beta'_2 = 35°$. The outlet velocity triangle is as in Fig. 5.30.

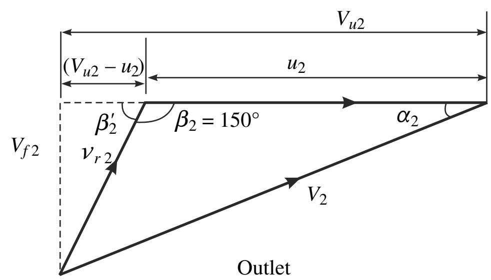


Fig. 5.30 *Outlet velocity triangle when running in reverse direction, Example 5.19*

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.25 \times 1500}{60} = 19.63 \text{ m/s}, \; V_{f2} = 19.63 \times 0.15 = 2.945 \text{ m/s}$$

$$\tan\beta'_2 = \frac{V_{f2}}{V_{u2} - u_2} = \tan 35° = 0.70$$

$$V_{u2} - u_2 = \frac{V_{f2}}{0.70} = \frac{2.945}{0.7} = 4.207$$

$V_{u2} = u_2 + 4.207 = 19.63 + 4.207 = 23.84$ m/s

Euler head = Ideal total head = $H_e = \dfrac{u_2 V_{u2}}{g} = \dfrac{19.63 \times 23.84}{9.81} = 47.70$ m

Absolute exit velocity $V_2 = \sqrt{V_{f2}^2 + V_{u2}^2} = \sqrt{(2.945)^2 + (23.84)^2} = 24.02$ m/s

**EXAMPLE 5.20

A centrifugal pump installation to pump water has the following values of parameters:

Impeller diameter at outlet = 40 cm	*Velocity of flow = constant = 2.0 m/s*
Outlet blade angle of backward-facing blades = 35°	*Speed = 1000 rpm*

(a) Calculate the theoretical rise in pressure head across the impeller and the kinetic energy of the flow leaving the impeller (b) If a volute casing can convert 50% of the kinetic energy of the exit discharge into pressure energy, calculate the theoretical total pressure head difference between the outlet and inlet of the pump.

Solution

Given: $D_2 = 0.40$ m, $V_{f1} = V_{f2} = 2.0$ m/s, $\beta_2 = 35^{o}$, $N = 1000$ rpm

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.40 \times 1000}{60} = 20.944 \text{ m/s}$$

From outlet velocity triangle, Fig. 5.31, $\tan\beta_2 = \dfrac{V_{f2}}{u_2 - V_{u2}} = \tan 35° = 0.70$

$$u_2 - V_{u2} = \frac{V_{f2}}{0.70} = \frac{2.0}{0.7} = 2.856$$

$V_{u2} = u_2 - 2.856 = 20.944 - 2.856 = 18.09$ m/s

Euler head (theoretical total head) = $H_e = \dfrac{u_2 V_{u2}}{g} = \dfrac{20.944 \times 18.09}{9.81} = 38.62$ m

Absolute velocity at exit = $V_2 = \sqrt{V_{f2}^2 + V_{u2}^2} = \sqrt{(2.0)^2 + (18.09)^2} = 18.20$ m/s

Kinetic energy per unit weight of liquid at impeller exit (Exit velocity head) =

$$\frac{V_2^2}{2g} = \frac{(18.20)^2}{2 \times 9.81} = 16.88 \text{ m}$$

$$\frac{V_1^2}{2g} = \frac{V_{f1}^2}{2g} = \frac{(2.0)^2}{2 \times 9.81} = 0.204 \text{ m}$$

ΔKE = 16.88 – 0.20 = 16.68 m

(a) *Theoretical rise in pressure:*
Since there are no losses, across the impeller:

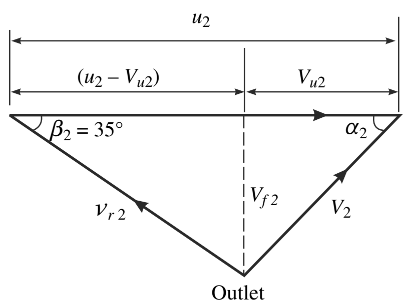


Fig. 5.31 *Outlet velocity triangle, Example 5.20*

Equation 5.18(a), $H_e = H_m + \Delta KE$

Here, $\Delta KE = \left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right) = 16.68\,\text{m}$

$$H_e = \frac{\Delta p_i}{\gamma} + \Delta\text{KE} = 38.62 \text{ m}$$

$$\frac{\Delta p_i}{\gamma} = 38.62 - 16.68 = 21.94 \text{ m}$$

$\frac{\Delta p_i}{\gamma}$ = Difference in pressure head between the outlet and inlet of the impeller = 21.94 m

(b) *When there is recovery of kinetic energy head:*
Head recovery = 50% of kinetic energy head at outlet

$$= \Delta KE_r = 0.5 \times \frac{V_2^2}{2g} = 0.5 \times 16.88 = 8.44 \text{ m.}$$

This recovered energy gets added as pressure head. Hence, actual pressure head difference between outlet and inlet of pump

$$\left(\frac{\Delta P_i}{\gamma}\right)_{Actual} = \left(\frac{\Delta p_i}{\gamma}\right)_{theo} + \Delta KE_r$$

$$= 21.94 + 8.44 = 30.38 \text{ m}$$

NOTE

The change in piezometric head ΔH_p can also be calculated independently by Eq. (5.21) as $\frac{\Delta p_i}{\gamma} = \frac{1}{2g}\left[V_{f1}^2 - V_{f2}^2 \text{cosec}^2 \beta_2 + u_2^2\right]$.

$$\frac{\Delta p_i}{\gamma} = \frac{1}{2 \times 9.81}\left[(2.0)^2 - \frac{(2.0)^2}{\sin^2 35°} + (20.944)^2\right] = 21.94 \text{ m}$$

***EXAMPLE 5.21

The impeller of a centrifugal pump has an outer diameter of 25 cm and rotates at a speed of 1500 rpm. The impeller has 10 blades, each of 5 mm thickness. The blades are backward facing at 30° to the tangent. The breadth of the flow passages at the outlet is 12.5 mm. Pressure gauges are fitted close to the pump at the suction and discharge pipes and both are 2.5 m above the water level of the supply sump. When the discharge is 26 L/s, the pressure readings are 4 m water (vacuum) in the suction end and 16.5 m of water (gauge) at the delivery end of the pump. If 50% of velocity is recovered as static head in the volute, estimate (a) theoretical head, (b) manometric *efficiency, (c) losses in the impeller, and (d) capacity of the motor to drive the pump, if the mechanical efficiency of the pump is 0.9. Assume that there is no whirl at inlet and no whirl slip.*

Solution

Given: $D_2 = 0.25$ m, $\beta_2 = 30^\circ$, $N = 1500$ rpm, $Q = 0.026$ m^3/s, $b_2 = 0.0125$ m, $\eta_{mech} = 0.9$

(a) $u_2 = \dfrac{\pi D_2 N}{60} = \dfrac{\pi \times 0.25 \times 1500}{60} = 19.63\,\text{m/s}$

Area of flow passage at exit $= A = (\pi \times 0.25 - (10 \times 0.005)) \times 0.0125$
$= 0.00919$ m^2.

Discharge $Q = AV_{f2}$

$$V_{f2} = \frac{Q}{A} = \frac{0.026}{0.00919} = 2.828 \text{ m/s}$$

Actual difference in pressure head across the impeller =

$$\left(\frac{\Delta p_i}{\gamma}\right)_{\text{actual}} = 16.5 + 4.0 = 20.5 \text{ m}$$

Also, $H_{ma} = \left(\dfrac{\Delta p_i}{\gamma}\right)_{\text{actual}} = 20.5$ m

From outlet velocity triangle: $\tan\beta_2 = \dfrac{V_{f2}}{u_2 - V_{u2}} = \tan 30^\circ = 0.5773$

$$u_2 - V_{u2} = \frac{V_{f2}}{0.70} = \frac{2.828}{0.5773} = 4.899$$

$$V_{u2} = u_2 - 4.899 = 19.63 - 4.899 = 14.731 \text{ m/s}$$

Euler head (theoretical total head) = $H_e = \dfrac{u_2 V_{u2}}{g} = \dfrac{19.63 \times 14.73}{9.81} = 29.48\,\text{m}$

(b) Actual manometric efficiency =

$$\eta_{ma} = \frac{\text{Net rise in pressure head}}{\text{Euler head}} = \frac{H_{ma}}{H_e} = \frac{20.5}{29.48} = 0.695$$

(iii) Absolute velocity at exit =

$$= V_2 = \sqrt{V_{f2}^2 + V_{u2}^2} = \sqrt{(2.828)^2 + (14.73)^2} = 15.0\,\text{m/s}$$

Kinetic energy per unit weight of liquid (velocity head) at impeller exit

$$= \frac{V_2^2}{2g} = \frac{(15.0)^2}{2 \times 9.81} = 11.47\,\text{m}$$

$$\Delta KE = \left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right) = \left(11.47 - \frac{V_1^2}{2g}\right)$$

Head recovery = 50% of kinetic energy head at outlet

$$= 0.5 \times \frac{V_2^2}{2g} = 0.5 \times 11.47 = 5.735 \text{ m.}$$

Unrecovered kinetic energy head at outlet

$$\Delta KE_{\text{unrec}} = \left(11.47 - \frac{V_1^2}{2g} - 5.735\right) = \left(5.735 - \frac{V_1^2}{2g}\right)$$

$$H_e = \left(\frac{\Delta p_i}{\gamma}\right)_{\text{Actual}} + \Delta KE_{\text{unrec}} + [\text{Losses in impeller } (= h_{Li}), \text{ if any}]$$

$$29.48 = 20.5 + 5.735 - \frac{V_1^2}{2g} + \text{losses in impeller}$$

Considering $\left(-\frac{V_1^2}{2g} + \text{losses in impeller}\right)$ as total losses in the impeller

$29.48 = 20.5 + 5.735 + H_{iL}$

H_{iL} = Losses in the impeller = 29.48 – 26.325 = 3.245 m

(d) Overall efficiency $= \eta_0 = \eta_{\text{ma}} \times \eta_{\text{mech}}$

$= 0.695 \times 0.9 = 0.6255$

Brake Power $P_p = \frac{\gamma Q H_m}{\eta_0} = \frac{9.79 \times 0.026 \times 29.48}{0.6255} = 12.0 \text{ kW}$

= Output Capacity of the required motor.

5.7 CHARACTERISTICS OF ACTUAL CENTRIFUGAL PUMPS

Centrifugal-pump characteristic curves are graphical representation of the interdependency of prime variables (head, discharge, power, efficiency and speed) in a pump. Since there are five variables, necessarily some of the variables are kept constant and the interdependency of the others are represented in the form of charts. Major pump manufacturers have their own testing laboratories and each pump type is tested in these laboratories to provide data for the development of characteristic curves for the pump type tested. Pump characteristics are usually studied under four categories: (a) main characteristics, (b) iso-efficiency (hill chart/Muschel) curves, (c) operating characteristics, and (d) constant-head and constant-discharge curves. In this section, the characteristics of actual radial-flow centrifugal pumps are presented.

5.7.1 Main Characteristics

To prepare main-characteristics charts, laboratory data obtained for a particular pump type, having a fixed geometry, is plotted with head, power and efficiency as ordinates and discharge as a common abscissa, with speed as the third variable. Characteristic curves are usually prepared for a certain identified impeller size; in which case the speed N (in rpm) is the third variable. This is the practice followed in academic circles including research publications. These characteristic curves can also be made applicable for a certain fixed nominal speed (N rpm): in which case the size of the impeller is the third variable representing the effect of speed. This practice of using impeller diameters is adopted in pump manufacturers' literature and instruction material. It should be remembered that both speed (N rpm) and diameter of the impeller (D_2) together define the various velocities, such as peripheral velocities and relative velocities used in the analysis.

Figure 5.32(a) is a schematic representation of a typical actual head-discharge variation of a certain centrifugal pump (Type-X) of impeller size D_2. The head (H) here refers to the manometric head developed by the pump. Each curve represents H-Q relation for a constant value of speed (N) noted against each curve. In these figures, H-Q curves for three values of speeds are indicated. It is seen that for a given speed (say N_1), the variation of head with discharge is nonlinear, and the variation is approximately parabolic: the general trend being a decrease in head with increasing in discharge. At $Q = 0$, there is a finite intercept with the H-axis for each speed, indicating a nonzero shut-off head. This shut-off head is much smaller than the theoretical shut-off head given by Eq. (5.32-a). Generally, the actual shut-off head is of the order of 0.6 times the theoretical value of shut-off head. Another point to be noted is that while the theoretical $H–Q$ relationship for a given speed is linear as per Eq. 5.33, due to the non-linear variation of hydraulic and mechanical losses the actual ($H–Q$) curve is of parabolic type.

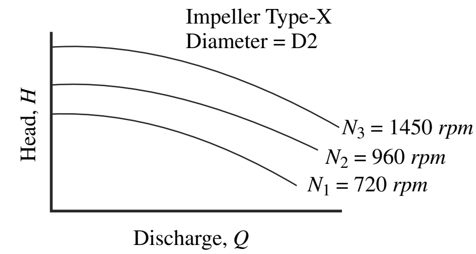


Fig. 5.32(a) *Head—discharge characteristics*

Figure 5.32(b) shows schematically the variation of actual power supplied (P_s) with the discharge for various constant speeds. The power increases with discharge. The variation of P_s with Q is nonlinear. Further, it is seen that higher the speed, greater would be the power required for pumping a given discharge Q.

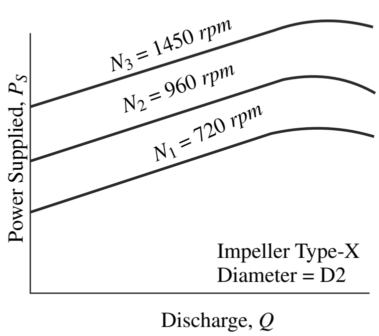


Fig. 5.32(b) *Variation of power with discharge*

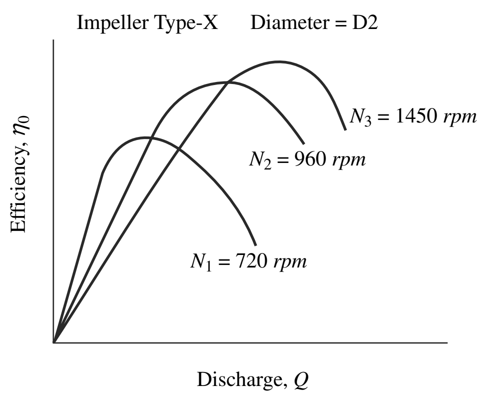


Fig. 5.32(c) *Variation of efficiency with discharge*

Figure 5.32 (c) is a schematic representation of the variation of the overall efficiency of centrifugal pump with the discharge for different speeds. Each $\eta_0 - Q$ curve of a constant N starts from zero, reaches a maximum and then decreases with further increase in the value of Q. The point at which the efficiency reaches the maximum is known as *Best Efficiency Point* (*BEP*). The peak efficiency values (*BEP*) themselves increase with the increase in the speed of operation.

5.7.2 Iso-efficiency Curves

The main characteristics depicted in Figs. 5.32 (a, b, c) can be combined to produce H–Q curves for different speeds with iso-efficiency curves drawn appropriately on the chart. To draw such curves, consider a horizontal line in Fig. (5.32-c) representing a known efficiency value $\eta_{01.}$ This line gives discharges corresponding to different speeds having the designated efficiency $\eta_{01.}$ Note that due to the nature of the efficiency-discharge curve, each horizontal line gives two discharge values for each speed. Through similar intercepts with different horizontal lines, a set of points $f(Q, N, \eta_0)$ can be obtained. These are then transferred to Fig. (5.32-a) to get points $f(Q, N, \eta_0, H)$. Points having same efficiency are joined (with interpolation where necessary) to obtain iso-efficiency curves in the appropriate H–Q reference axes. Figure (5.33) represents one such schematic chart. Iso-efficiency curves (also sometimes known as hill charts/*Muschel curves*) represent the entire characteristics of the pump graphically in one chart and help in locating best regions of operation in a given problem. Figure 5.33 represents three impellers of diameter $D1$, $D2$ and $D3$ all running at a nominal speed $N1$.The figure depicts iso-efficiency lines as well as lines corresponding to power $P1$, $P2$ and $P3$. This chart is typical of manufacturers' way of representing the data of a class of pumps in a compact manner. The chart of Fig. 5.33 also contains *NPSHR* values of the particular class of pumps.

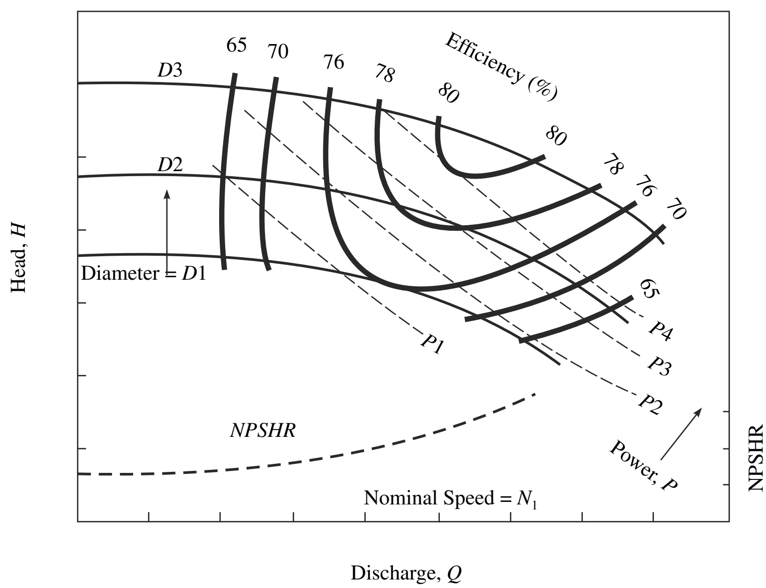


Fig. 5.33 *Typical centrifugal pump performance iso-efficiency curve (schematic)*

5.7.3 Operating Characteristics

Operating characteristics of a pump are the constant-speed characteristics of a given type of pump of fixed geometry. This is essentially the set of curves consisting of (H–Q) curve, (P_s–Q) curve and ($\eta_0 - Q$) curve all belonging to a pre-fixed value of speed N; all extracted from the main-characteristics curve. The pumps are normally designed for the maximum efficiency condition (*BEP*) and the head corresponding to maximum efficiency is known as *normal head*. Similarly, values of brake power, discharge and speed corresponding to maximum efficiency point (*BEP*) are known respectively as *normal brake power*, *normal discharge* and *normal speed*. Operating characteristics curves are drawn for the normal speed. A typical operating characteristics curve of a radial centrifugal pump is shown schematically in Fig. 5.34. This figure depicts the variation of H, P_s and η_0 with discharge when the pump is running at normal speed. The *NPSHR*, being an essential characteristic of the pump, is also represented in the figure as a function of discharge. The following points are worth noting:

- The brake power is a finite value at no-discharge condition and this represents the power spent in creation of actual shut-off head of the pump. In addition, the shut-off power is smaller than the power consumed at normal operating conditions.
- The difference between the maximum power and normal operating power is not much. Thus, there is no danger of overloading the motor of the pump at any operating point.

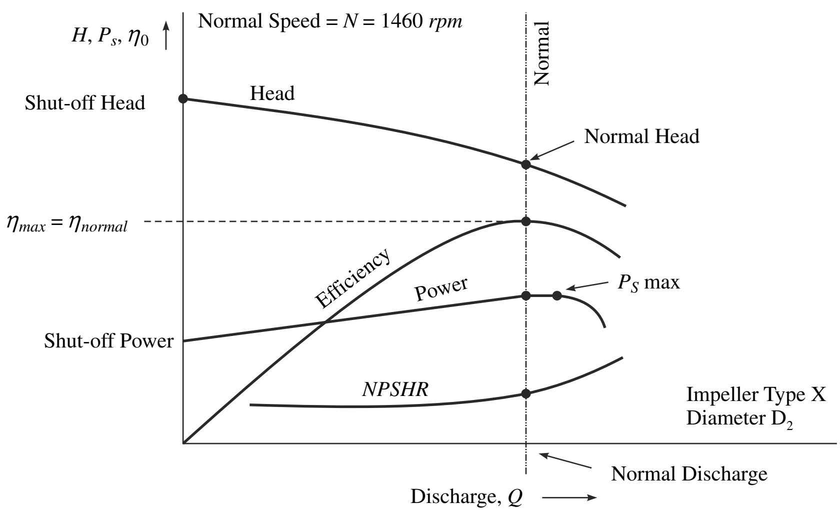


Fig. 5.34 *Typical operating characteristics of a centrifugal pump (schematic)*

5.8 SIMILARITY RATIOS (AFFINITY LAWS) FOR CENTRIFUGAL PUMPS

Laboratory testing of pumps is an integral part of pump industry. Scaling up of laboratory results to prototype, conversion of performance characteristics to similar pumps belonging to the same homologous series are some of the functions that are routine in pump studies and design. This section deals with dimensional analysis of pump parameters to obtain coefficients and to establish similarity ratios that are applicable to geometrically and dynamically similar pumps.

5.8.1 Dimensional Analysis

The important physical variables governing the performance of a rotodynamic pump can be listed as follows:

Table 5.3 *Variables affecting performance of a rotodynamic pump*

Symbol	Variable	Dimension
D	Reference diameter (a length parameter)	[L]
N	Rotational speed (rpm)	$[T^{-1}]$
H	Energy head (Energy per unit weight)	[L]
Q	Discharge through the pump	$[L^3T^{-1}]$
P	Power transferred by the impeller	$[ML^2T^{-3}]$
g	Acceleration due to gravity	$[LT^{-2}]$
ρ	Density of liquid pumped	$[ML^{-3}]$
μ	Coefficient of dynamic viscosity of liquid	$[ML^{-1}T^{-1}]$

Note that the fluid property K = bulk modulus of elasticity, is not included in the above list as the liquids involved in the pumping are incompressible in the practical range. Similarly, the surface tension does not have any influence in the problem under consideration and as such is not included. Further, there is no direct gravity effect on the flow phenomenon because the flow in the pump is essentially a closed conduit flow and no free surface effects are involved. It is thus a normal practice to consider (gH) as one variable instead of g and H separately. The term (gH) represents the energy per unit mass of the fluid flow and is known as specific energy. Thus, for a centrifugal pump by considering the power P as the dependent variable,

$$P = f(D, N, gH, Q, \rho, \mu) \tag{5.40}$$

Considering D, N and ρ as repeating variables, it can be shown by dimensional analysis that

$$\frac{P}{\rho N^3 D^5} = fn\left(\frac{gH}{N^2D^2}, \frac{Q}{ND^3}, \frac{\rho ND^2}{\mu}\right) \tag{5.41}$$

The dimensionless terms are $\pi_1 = \dfrac{P}{\rho N^3 D^5}$, $\pi_2 = \dfrac{gH}{N^2D^2}$, $\pi_3 = \dfrac{Q}{ND^3}$, and $\pi_4 = \dfrac{\rho ND^2}{\mu}$

Here, the term $\pi_4 = \dfrac{\rho ND^2}{\mu}$ is the Reynolds number and in the present similitude, its role is to see that the model and the prototype are both in the same mode; in turbulent or in transition or in laminar mode. Hence, by ensuring that the model and the prototype are both in the fully developed turbulent flow mode, the term $\pi_4 = \dfrac{\rho N D^2}{\mu}$ can be dropped in the functional relationship of Eq. (5.41) to get

$$\frac{P}{\rho N^3 D^5} = fn\left(\frac{gH}{N^2 D^2}, \frac{Q}{ND^3}\right) \tag{5.42}$$

NOTE

Appendix A-1 contains details of dimensional analysis and two procedures to conduct dimensional analysis of a set of variables of a phenomenon to express them as a function of nondimensional terms.

5.8.2 Similarity Ratios (Affinity Laws)

While Eq. (5.42) presents the general functional relationship between the three dimensionless terms governing the performance of a pump, this equation needs to be simplified for practical application. In practical use, since the acceleration due to gravity, *g,* is a constant, it is removed from the π_2 term. The density ρ of the liquid can be dropped as we are dealing with incompressible liquid. However, as a note of caution, the density term should be included when the liquids used in two homologous systems under study are of different densities. As an example, one may use in the model study a liquid whose relative density may be different from that of the prototype. Thus, subject to the above limitation, the π terms get morphed for practical use as

$$\frac{P}{N^3 D^5} = fn\left(\frac{gH}{N^2 D^2}, \frac{Q}{ND^3}\right) \tag{5.43}$$

Note that the term D can be replaced by any reference length parameter of the impeller. Similarly, the head H can be replaced by any reference pressure head involved in pumping operation. It is usual to take the outer diameter D_2 of the impeller, and the manometric head H_m as length and head parameters respectively for writing the similarity ratios of centrifugal pumps.

If the efficiency remains the same, for fully developed turbulent flow, geometrically similar family of pumps (*homologous series*) obey the following similarity equations. For any two homologous centrifugal pumps designated as A and B, having different sizes (D) and rotational speeds (N), the power (P), discharge (Q) and head (H) are related as shown below:

$$\left(\frac{P}{\rho N^3 D^5}\right)_A = \left(\frac{P}{\rho N^3 D^5}\right)_B = C_p = \textit{Power coefficient} \tag{5.44}$$

When the density of liquid being pumped is the same for both pumps A and B,

$$\left[\frac{P}{N^3 D^5}\right]_A = \left[\frac{P}{N^3 D^5}\right]_B = \text{Constant} \tag{5.44-a}$$

$$\left[\frac{Q}{ND^3}\right]_A = \left[\frac{Q}{ND^3}\right]_B = \text{Constant} = C_Q = \textit{Capacity coefficient} \tag{5.45}$$

$$\left[\frac{H}{N^2D^2}\right]_A = \left[\frac{H}{N^2D^2}\right]_B = \text{Constant} = C_H = \textit{Head coefficient} \tag{5.46}$$

A homologous family of pumps will have constant capacity, head and power coefficients. These relationships are known as *similarity ratios* (also known as *affinity laws*) for centrifugal pumps.

For quick use, they can be considered in three special cases as below:

1. For the ***same pump*** (impeller diameter D is the same in pumps A and B)

$$\frac{Q_B}{Q_A} = \frac{N_B}{N_A} \qquad \frac{H_B}{H_A} = \left(\frac{N_B}{N_A}\right)^2 \qquad \frac{P_B}{P_A} = \left(\frac{N_B}{N_A}\right)^3 \tag{5.47}$$

   [In the above, suffixes A and B refer to states A and B respectively]

2. For the ***same speed*** (speed is fixed at a constant value = N in pumps A and B)

$$\frac{Q_B}{Q_A} = \left(\frac{D_B}{D_A}\right)^3, \qquad \frac{H_B}{H_A} = \left(\frac{D_B}{D_A}\right)^2, \quad \frac{P_B}{P_A} = \left(\frac{D_B}{D_A}\right)^5 \tag{5.48}$$

   Note that in Eqs. (5.47) and (5.48), the power ratio is the product of the Q and H ratios.
   [In the above D_A and D_B refer to the impeller diameter of pump A and pump B respectively.]

3. ***Different Liquids*** If the liquid used in pump A is of different density than that used in pump B:

The scaling ratio for the powers is given by Eq. (5.44-a) as

$$\left(\frac{P}{\rho N^3 D^5}\right)_A = \left(\frac{P}{\rho N^3 D^5}\right)_B = C_p = \text{Power coefficient}$$

The affinity laws are of immense importance in model studies and in comparing the performance of two similar pumps. It is also of use in predicting the performance of homologous pumps of different geometries based on the available characteristics of one of the pumps of the series. Further, based on the available set of performance data of one pump, the effect of change in the speed, size and fluid on the same pump as well as on geometrically similar pumps can be established.

It is usual for pump manufacturers to offer a choice of a set of 3 to 4 impellers for a single-volute casing design. Strictly speaking, the pumps belonging to this set are not geometrically similar as the relative sizes of volute casing are different. However, in practice, this kind of pump sets are taken to be geometrically similar and affinity laws are assumed to be applicable if the impeller sizes do not change by more than 20%. A similar situation arises where an impeller is *trimmed* (i.e. impeller diameter is reduced) to achieve a specific operational requirement. In this case also, a change in diameters up 20% range is assumed to come under the affinity laws.

5.8.3 Modeling of Efficiency

In the development of affinity laws, it was assumed that the efficiency of the two pumps that were compared (for example, a model and prototype) had the same efficiency. However, it is a proven fact that larger the pump, higher is the attainable efficiency. Larger pumps have higher Reynolds numbers, lower relative roughness and lower clearance ratios. Smaller size models, even with micropolish, tend to have higher relative roughness and the clearances are relatively large. Therefore, the model efficiency will be smaller than that of the larger prototype. To account for this difference a *step-up formula* is commonly used. Two empirical efficiency step-up formulae in use are

1. Due to Moody (1926): $$\frac{1-\eta_B}{1-\eta_A} = \left(\frac{D_A}{D_B}\right)^{1/5} \quad (5.49)$$

Hence, η is the overall efficiency and D is the diameter of the impeller. This formula is the same as has been recommended for the reaction turbines (see Eq. 1.42).

2. Due to Anderson (1980): $$\frac{0.94-\eta_B}{0.94-\eta_A} = \left(\frac{Q_A}{Q_B}\right)^{0.32} \quad (5.50)$$

In the above equations, suffixes A and B refer to two pumps of differing sizes belonging to the same homologous series. Thus, A can be the prototype and B can be its model. Example 5.25 illustrates the use of Eqs. (5.49 and 5.50).

In some cases the difference in the efficiency between the model and the prototype may not be significant and hence can be ignored.

5.8.4 Similitude Scale Ratios

A compact way to express the model–prototype relationship of similarity laws for pumps is in terms of ratios of different parameters. Using the suffix r to refer to the ratio of a parameter in model to the value in the corresponding prototype, the length ratio of a pump model is the ratio of diameters. Let the suffixes m and p refer to the model and prototype, respectively. Then

Length ratio = Ratio of diameters = $$D_r = \frac{D_m}{D_p} \quad (5.51)$$

Since the discharge coefficient $$\left[\frac{Q}{ND^3}\right]_m = \left[\frac{Q}{ND^3}\right]_p;\ \frac{Q_m}{Q_p} = Q_r = N_r D_r^3 \quad (5.52)$$

Similarly for head, $$\left[\frac{H}{N^2D^2}\right]_m = \left[\frac{H}{N^2D^2}\right]_p;\ \frac{H_m}{H_p} = H_r = N_r^2 D_r^2 \quad (5.53)$$

For power, $$\left[\frac{P}{N^3D^5}\right]_m = \left[\frac{P}{N^3D^5}\right]_p;\ \frac{P_m}{P_p} = P_r = N_r^3 D_r^5 \quad (5.53\text{-a})$$

Eliminating N_r from discharge, head and power ratios,

$$Q_r = H_r^{1/2} D_r^2 \quad (5.54)$$

$$H_r = \frac{Q_r^2}{D_r^4} \quad (5.55)$$

$$P_r = H_r^{3/2} D_r^2 = \frac{Q_r^3}{D_r^4} \tag{5.56}$$

These scaling ratios are useful in establishing the ratios of various parameters through known scale ratios. Table 5.4 shows all the scaling ratios used in modeling pumps in a tabular manner for ready reference.

Table 5.4 *Similitude scale ratios for pumps*

Parameter	Scale Ratio	Expressions		
Diameter (Length)	$D_r =$	$\left(\frac{Q_r}{N_r}\right)^{1/3}$	$\left(\frac{H_r^{1/2}}{N_r}\right)$	
Speed	$N_r =$	$\left(\frac{H_r^{1/2}}{D_r}\right)$	$\frac{Q_r}{D_r^3}$	
Discharge	$Q_r =$	$N_r D_r^3$	$H_r^{1/2} D_r^2$	
Head	$H_r =$	$N_r^2 D_r^2$	$\frac{Q_r^2}{D_r^4}$	
Power	$P_r =$	$N_r^3 D_r^5$	$H_r^{3/2} D_r^2$	$\frac{Q_r^3}{D_r^4}$

5.8.5 Specific Speed

Specific speed of a pump is a parameter involving the speed, discharge and head obtained by combining two scaling ratios (discharge and head coefficients) and is defined as

$$N_{sq} = \frac{N\sqrt{Q}}{H_m^{3/4}} \tag{5.57}$$

The values of parameters N, Q and H_m used in computing N_{sq} correspond to the maximum efficiency point (*BEP*) of the pump characteristic. The head H_m is the manometric head of the pump at *BEP*. Thus, the specific speed N_{sq} of a pump could be defined as the speed of a pump running at maximum efficiency under a unit head and delivering unit discharge. The specific speed as defined by Eq. 5.57 is **not** dimensionless and hence depends upon the units used for Q, H_m and N. In *SI* units, speed N is in rpm (revolutions per minute), head H_m is in metres and discharge Q is in m^3/s. In this book, *SI* units are used in defining specific speeds of pumps as well as turbines.

Note that the specific speed does not involve the size of the impeller explicitly. Thus N_{sq} is a size-independent parameter. A single value of N_{sq} represents a family of homologous pumps of a given shape of impeller. Change of impeller shape changes the specific speed. As in turbines, specific speed of a pump is the most important single parameter used in selection of type and number of pumps for a particular job. It is also useful in comparing the characteristics, such as best efficiency, of different types of centrifugal pumps on a common base. Figure 5.35 shows the approximate range of optimum efficiency of centrifugal pumps as a function of the specific speed. Observe the transition from the radial flow to axial flow in the pumps as the specific speed increases. The corresponding changes in the impeller shapes are also indicated schematically in the figure. The radial-flow type impellers are seen to have smaller specific speeds and the axial-flow pumps have the highest range of N_{sq} values with the mixed-flow pumps occupying the region in between. Further, the efficiency curves of the three types of pumps shown in the figure have overlap regions. This feature is similar to the overlap regions of the three types of turbines exhibited in Fig.4.19 (Sec. 4.9, Chap 4).

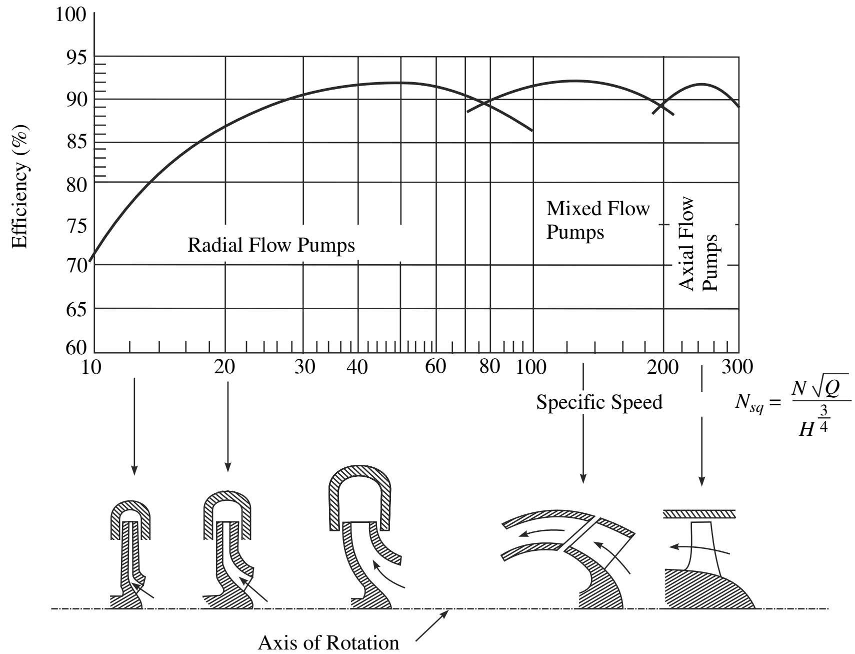


Fig. 5.35 *Variation of optimum efficiency of centrifugal pumps with specific speed*

High value of specific speed implies relatively high discharge and low heads. Axial flow pumps are seen to be the best choice for this situation. Similarly, relatively low discharges and high heads imply low specific speeds and the best choice for this situation is the radial-flow centrifugal pump. In the overlap regions, there exists a choice of the type of pumps. From the shapes of the impellers for various ranges of specific speeds, it is seen that low N_{sq} impellers have outlet diameters much larger than the inlet diameter and relatively narrow flow passages. Similarly, high N_{sq} pump impellers have outlet diameters that are practically same size as the inlet diameter and large open flow passages.

NOTE

The commonly used form of specific speed N_{sq} being dimensional, there is quite a wide range of units used world over. For example the *US* customary units are *US* gallons per minute, feet and rpm; while the British customary units are cubic feet per second, feet and rpm.[Since 1995: revolutions per minute (rpm), flow in cubic metres per hour (m^3/h) and head in metres (m) are used in *UK*]. *SI* units used in India are: cubic metres per second, meter and rpm. There are many (organisations and individuals) who use nonconventional units also. As such, the following caution should be kept in mind:

When comparing or interpreting pump documentation data be aware of the units used.

In this book *SI* units are the **default** units for specific speeds.

Figure 5.36 shows the typical optimum efficiency of normal commercial pumps as a function of the specific speed and the capacity as the third parameter. At any given specific speed, the optimum efficiency increases with increase in the capacity. However, the rate of increase in efficiency becomes smaller as the discharge increases. Beyond about 600 L/s capacity, the increase in efficiency with discharge is marginal and further, only one curve could represent all values of capacity beyond 600 L/s. Explanations for models having smaller efficiency than the prototype as given in Sec. 5.8.3 are valid here also. The optimum efficiency *vs* specific-speed curve given in Fig. 5.35 refers to the high discharge pumps ($Q > 600$ L/s) of each type. Another feature to note is that the optimum efficiency drops off rapidly for low discharges and specific speeds less than 15.

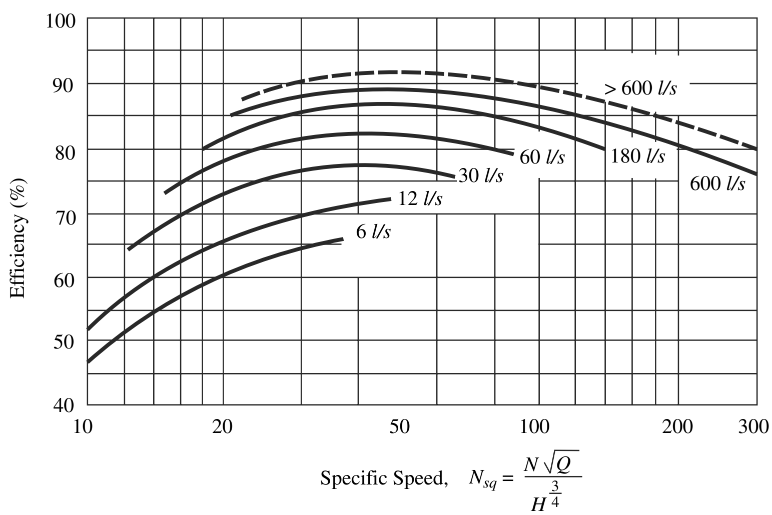


Fig. 5.36 *Variation of optimum pump efficiency as a function of specific speed and capacity*

While calculating specific speeds, the following common conventions are in use:

- For double-suction pumps, the specific speed is based in one half of the total capacity of the pump. This is because the double suction pump is treated as two pumps connected back to back.

- For a multistage centrifugal pump of m stages, all the impellers are of the same size and the pump is considered as m number of single-stage pumps connected in series. Thus in the calculation of N_{sq} the head per stage (that is, the total head divided by the number of stages) is used.

Typical ranges of specific speeds of different types of centrifugal pumps are as follows:

Table 5.5 *Typical ranges of specific speeds of various types of centrifugal pumps*

Type of Pump	Subclassification	Range of Specific Speed, N_{sq}	Range of Shape Factor S_q in (revs)
Radial flow pumps	Slow speed	10 to 30	0.03 to 0.09
	Median speed	30 to 50	0.09 to 0.15
	High speed	50 to 100	0.15 to 0.30
Mixed flow pumps		75 to 220	0.225 to 0.66
Axial flow pumps		180 to 450	0.54 to 1.35

It is interesting to see that by considering C_Q and C_H one gets the nondimensional specific speed:

$$\frac{C_Q^{1/2}}{C_H^{3/4}} = \frac{(\sqrt{Q})}{\sqrt{N}D^{3/2}}\left(\frac{N^{3/2}D^{3/2}}{g^{3/4}H^{3/4}}\right) = \frac{N\sqrt{Q}}{(gH)^{3/4}}$$

Nondimensional specific speed (*shape factor*) for pumps = $S_q = \dfrac{N\sqrt{Q}}{(gH)^{3/4}}$ (5.58)

The nondimensional specific speed of a centrifugal pump is sometimes called as *shape factor (shape number)*. In calculating S_q the speed should be in revolutions per second (rps); the discharge Q should be in m^3/s and the head H in metres. Further, to avoid confusion, if the speed is in radians per second, the shape factor will be designated as S_{qr}.

Conversion factors

The following conversion factors are useful while dealing with specific speed N_{sq} or shape factor S_q of pumps:

S_q (revolutions) = 0.003 × N_{sq} (SI units)
S_{qr} (radians) = $2\pi S_q$ (revolutions)
N_{sq} (US gallons, feet, rpm) = 51.64 N_{sq} (SI units)

5.9 CAVITATION IN CENTRIFUGAL PUMPS

5.9.1 Cavitation Phenomenon

Cavitation is a dangerous phenomenon that haunts the entire gamut of hydraulic machines. The basic features of cavitation phenomenon, damages caused by cavitation and general aspects of cavitation in a Francis turbine were covered in Chapter 2, Section 2.5. Cavitation phenomenon in draft tubes is described in Sec. 2.8 of Chapter 2. In Section 3.9 of Chapter 3, specific aspects of cavitation in axial-flow turbines are discussed briefly. Specific aspects of cavitation problems in a Pelton turbine is described in Section 4.7, Chapter 4. In this section, some aspects of the cavitation phenomenon with special reference in centrifugal pumps is described. The emphasis of this section is the cavitation phenomenon with specific reference to centrifugal pumps only and as such for any details relating to the basic aspects of the cavitation phenomenon, the reader is referred to Sec. 2.5 of Chapter-2.

When the absolute local pressure at any point in a conduit carrying a liquid approaches the vapour pressure p_v of the liquid, the dissolved gases and liquid vapour come out of the liquid as bubbles. The phenomenon of formation, travel and collapse of vapour pressure is known as cavitation. Cavitation phenomenon, in general, has a strong negative impact on the performance of hydraulic machinery. In a centrifugal pump, the suction end of the pump and blade passages are most susceptible to cavitation. The low-pressure regions of the vane tips and the junction area of the vanes with the shroud at the inlet are common spots where cavitation can occur. In a vaned diffuser, the inlet region of the vanes is also susceptible. The impact of cavitation in a centrifugal pump could be listed as follows:

- ***Performance Impairment*** (reduction of head and efficiency)
- ***Mechanical Damage*** (reduction in useful life)
- ***Noise*** Low-frequency noise up to about 10 kHz with nil or low damage and high frequency in the 10 to 200 kHz range with significant damage to the parts. Cavitation noise is a typical crackling sound as if gravel is being pumped along with water. An experienced mechanic usually is able to diagnose cavitation by its unique sound signature.
- ***Pressure Pulsation*** Torque fluctuation and vibration of casing and bearing house.
- ***Vapour Lock*** A situation where the impeller is trying to operate with a fluid that is very much lighter due to presence of vapour bubbles. This results in complete loss of generation of head.

Figure 5.37 shows the variation of head and discharge of a pump subjected to cavitation. The ordinate is the head and efficiency, both as percentages of design valve. The abscissa is is the nondimensional discharge, ratio of discharge to the design discharge. The full curves are the operating characteristics when there is no cavitation. The dotted lines show the variation when cavitation is induced in the pump. The impairment of efficiency and large loss of head when the pump is cavitating is clearly indicated.

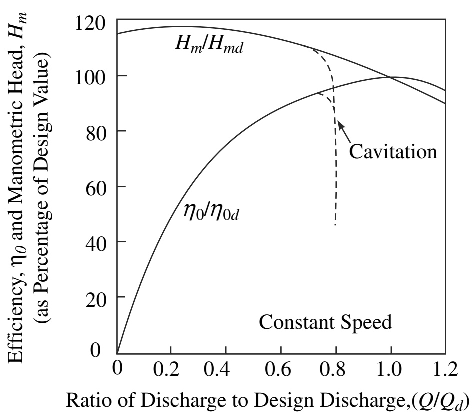


Fig. 5.37 *Effect of cavitation on performance of a pump*

It is seen that at the onset of cavitation there is a sudden drop in the head and in the efficiency. This reduction in the operating head at cavitation is made use of in defining the onset of cavitation. A criterion commonly used is to identify the point at which there is 3% drop in the manometric head as the point corresponding to the incipient cavitation.

5.9.2 Cavitation Parameter, σ

When the local pressures in a flow approaches the vapour pressure of the water at the prevailing temperature, cavitation is imminent. A *cavitation parameter* σ is defined for a centrifugal pump as

$$\sigma = \frac{H_{pi} + \frac{V_s^2}{2g} - H_{vp}}{H_m} \tag{5.59}$$

where

H_{pi} = Pressure head at the pump inlet
H_m = Manometric head of the pump
H_{vp} = Vapour pressure head (absolute)
V_s = Velocity in the suction pipe at the pump inlet

Consider a centrifugal pump with horizontal axis located as shown in Fig. 5.38. The level of the pump axis is taken as the datum. In this

H_{am} = Ambient pressure head (absolute)

H_s = Suction head = Elevation difference from the level of liquid in the supply reservoir (sump) to the datum (i.e. to the axis of the pump)

h_{Ls} = Loss of head due to friction in the suction pipe, including minor losses

Applying Bernoulli's theorem to a point on the liquid surface in the sump and the inlet of the pump:

$$H_{am} - H_s - h_{Ls} = H_{pi} + \frac{V_s^2}{2g}$$

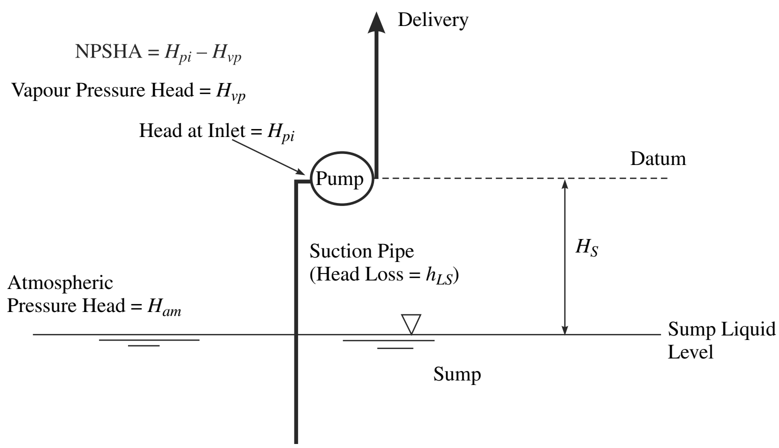


Fig. 5.38 *Definition sketch for σ and NPSHA*

Substituting in Eq. 5.59,

$$\sigma = \frac{H_{am} - H_s - h_{LS} - H_{vp}}{H_m} \tag{5.60}$$

This parameter σ being a function of the head H_m varies with discharge for a given speed of the pump. When a pump is tested at constant speed, fixed discharge and at values of lowered inlet head, it amounts to testing the pump at lowering values of σ. Figure 5.39 represents the result of such a pump test as a plot of efficiency against the value of σ.

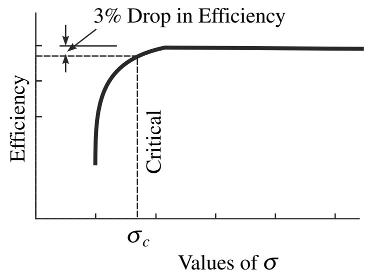


Fig. 5.39 *Definition of Critical Sigma*

Consider the curve from right to left, to study the behaviour of efficiency with decreasing values of σ. With decrease in the value of σ, the efficiency is essentially a constant or gradually decreases until a certain value marked critical in the figure is reached. However, the curve suddenly breaks off into a plunging curve for values of σ less than the critical value (marked as σ_c in the figure). This is due to the onset of cavitation. The smallest value of σ at which incipient cavitation manifests at the pump in question is identified by the criteria of 3% drop in efficiency. This point represents the lowest value of σ that can be allowed in the pump without serious cavitation occurring in the pump. This point is called *critical cavitation parameter* (also known as *critical Thoma Coefficient* or *critical sigma*) and is designated as σ_c. The parameter σ_c is a characteristic of the pump under study. Thus, every pump will have a unique σ_c and one can define the operation space of the pump as follows:

1. if $\sigma \geq \sigma_c$, there will be no cavitation, and
2. if $\sigma < \sigma_c$, the pump installation will suffer cavitation problem.

From Eq. (5.60), at critical cavitation condition, $\sigma_c = \dfrac{H_{atm} - H_v - H_s - h_{Ls}}{H_m}$ and the criteria for cavitation free installation is

$$H_s \leq (H_{atm} - H_v - H_s - h_{Ls} - \sigma_c H_m) \tag{5.61}$$

The maximum suction lift for cavitation free operation of the pump is given by

$(H_s)_{\max} =$ maximum setting height of the pump above the liquid level in the sum

$$= H_{atm} - H_v - H_s - h_{Ls} - \sigma_c H_m \tag{5.61-a}$$

Note that the value given by Eq.(5.61-a), viz. $(H_s)_{\max}$, is the maximum value of H_s that could be adopted and indicates the incipient cavitation conditions at that setting. To allow for unforeseen factors, it is usual to allow a margin (M) of 0.5 to 1.0 m, and the actual H_s adopted will be less by this margin. Examination of Eq. (5.61) reveals that for some high-specific-speed pump installations (that is for low head pumps), the site conditions may necessitate negative settings of the pump. In such cases, the pumps will have to be below the sump liquid level.

NOTE

The 3% drop criteria for defining σ_c has got a variant in practice. Some people adopt 3% drop in manometric head instead of efficiency to define σ_c. In that case, Fig. 5.39 would be a plot of σ *vs.* manometric head.

This critical cavitation parameter being unique to a pump design depends upon the specific speed of the pump. An empirical equation (Shepherd, 1964) governing the critical Thoma coefficient and pump specific speed for a single-suction centrifugal pump is

$$\sigma_c = \frac{N_{sp}^{4/3}}{825} \tag{5.62}$$

This relationship is valid for specific speeds in the range 10 to 240 and thus covers radial, mixed-flow and axial-flow pumps. The value of critical Thoma coefficient for a pump is usually obtained by model tests and its value is furnished by the pump manufacturer, either directly as σ_c or as *NPSHR*. If this information is not available, the empirical equation (Eq.5.62) could be used to estimate the value of σ_c. It is to be noted that the atmospheric pressure decreases with altitude of the place. Roughly, it is 10.35 m of water at sea level and 6.6 m at an altitude of 3000 m above sea level. This should be factored in the major pump installations in mountainous areas. The vapour pressure head H_v is a function of the temperature of liquid; vapour pressure head increases with increase in the temperature of the liquid.

5.9.3 NPSH (Net Positive Suction Head)

1. NPSHA

Even though Thoma coefficient σ is regarded as a fundamental parameter of cavitation, in the context of pumps, the parameter *NPSH* (*Net Positive Suction Head)* is used more often as it is more intuitive and also is directly related to Thoma number σ.

The *actual NPSH* available at a pump installation at the site is known as *NPSHA* (*Net Positive Suction Head Available*). It is defined as the head, in absolute units, measured above the prevailing vapour pressure head that is available to the pump at the inlet.

$$NPSHA = H_{am} - H_s - h_{Ls} - H_{vp} \tag{5.63}$$

It can be regarded as a measure of margin against liquid vaporization and hence cavitation susceptibility. *NPSHA* is dependent on the pump-setting height and pump-operating characteristics. Since *NPSHA* is defined as the head available above the prevailing vapour pressure head at the pump inlet, using Eq. 5.59.

$$NPSHA = H_{pi} - H_{vp} = H_{am} - H_s - h_{Ls} - H_{vp} \tag{5.64}$$

Hence, by using Eq. 5.60, the relation between σ and *NPSHA* can be expressed as

$$\sigma = \frac{NPSHA}{H_m} \tag{5.64-a}$$

2. *NPSHR*

The critical value of σ_c, as determined usually by 3% criteria, corresponds to the critical value of *NPSH*. This critical value of *NPSH* is known variously as *NSPHR*, $NPSH_{min}$, and $NPSHR_{3\%}$. In this book, the term *NPSHR* (an acronym for *Net Positive Suction Head Required*) is used to mean the critical value of *NPSH*. Thus

$$\sigma_c = \frac{NPSHR}{H_m} \tag{5.65}$$

or

$$NPSHR = \sigma_c H_m \tag{5.65-a}$$

As such, *NPSHR* is defined as the head, in absolute units, measured above the prevailing vapour-pressure head, required by the pump to obtain satisfactory pumping head (i.e. no more than 3% reduction in head, or efficiency, at constant flow) and prevent excessive cavitation. *NPSHR* is a pump characteristic. It increases with the discharge due to pump internal losses, friction and other impeller losses. *NPSHR* is determined by the pump manufacturer through tests and the result is depicted as the variation of *NPSHR* against head in the *H-Q* pump performance chart (Fig. 5.34). It should be noted that while *NPSHR* is fixed by the manufacturer, *NPSHA* is dependent on the pump-setting and pump-operating characteristics (i.e. on-site conditions).

3. Margin

In a pump installation, the setting of the pump, relative to the liquid level in the sump, should be such as to achieve the *NPSHR* of the pump with adequate factor of safety, through the provision of a margin (*M*). The difference between *NPSHA* and *NPSHR* is the margin *M* and is expressed as

$$M = NPSHA - NPSHR \tag{5.66}$$

The usual practice is to always provide the margin (*M*) to account for unforeseen conditions; the margin being usually about 10% of *NPSHR* or 0.5 to 1.0 m depending upon the type of liquid and the type of pump.

The pump-setting process can thus be expressed as follows:

- If *NPSHA* > *NPSHR,* the setting is *OK*
- If *NPSHA* < *NPSHR*, setting is not *OK* and cavitation will be a problem.
- Always provide adequate margin *M* = (*NSPHA–NPSHR*) as a factor of safety.
- Thus, *NPSHA* should always be greater than (*NPSHR* + *M*).

Two possible installation types are shown in Fig. 5.40. The centre of the pump is taken as datum. Relative to this datum, the nature of the suction lift in the two cases shown in Fig. 5.40 needs attention. While calculating *NPSHA* by using Eq. (5.64) for cases 1 and 2 shown in the above figure, care must be taken to use appropriate sign for the numerical value of H_s.

The object of the pump setting is to have maximum *NPSHA* at a given location. By doing so, the margin *M* will be large and hence the possibilities of having cavitation conditions will be minimised. The following are some possibilities to maximise *NPSHA*:

- Decrease the static lift H_s
- Reduce friction loss. Towards this, (a) reduce suction pipe length, (b) maximise suction pipe diameter, and (c) minimise the number of bends, tees and valves in the suction pipe.

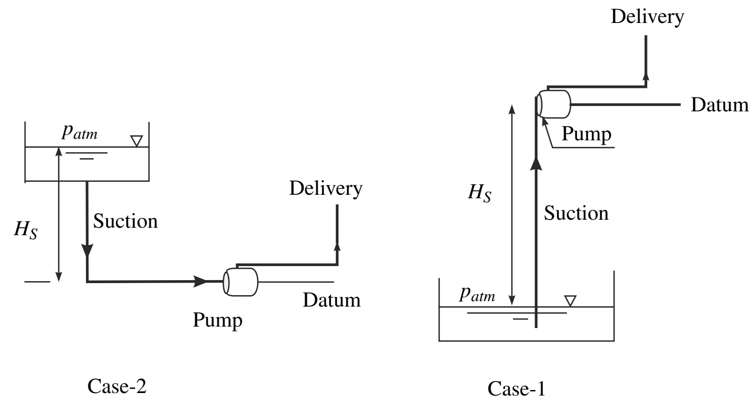


Fig. 5.40 *Two possible pump-installation types*

- Increase ambient pressure (if possible)
- Where applicable, lower the operating temperature of the pumped liquid thereby reducing H_{vp}.

5.10 EFFECT OF VISCOSITY

Centrifugal pumps are used to pump various kinds of liquids; from light oils, chemicals and water to heavy viscous liquids. Invariably, pumps are tested in the manufacturer's laboratory by using water as the testing liquid. Highly developed turbulent flow conditions are the norm in such tests. As such, it can be generalised that for all liquids whose coefficient of viscosity is equal to or less than that of water, there is no change in the nature of flow in the pump and hence the pump characteristics are not affected by the viscosity in that range. However, when liquids thicker (more viscous) than water are used, the nature of the flow would be changed. The viscosity would begin to influence the head-discharge relationship and hence the efficiency characteristic of the pump. Figure 5.41 is a schematic sketch to show the effect of viscosity on H–Q relationship as well as on P–Q relationship.

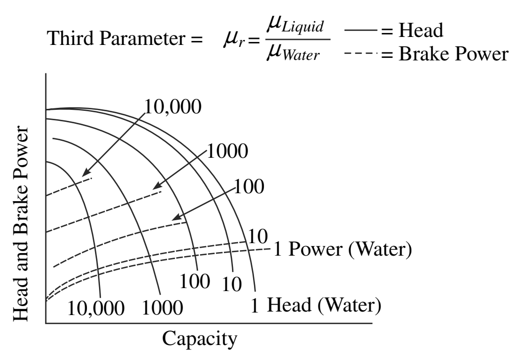


Fig. 5.41 *Effect of viscosity on the performance of a centrifugal pump*

In this figure, the relative dynamic viscosity, defined as $\mu_r = \frac{\mu_{\text{liquid}}}{\mu_{\text{water}}}$, is used as the third parameter.

In this, μ is the dynamic viscosity of the liquid indicated by the suffix. It is seen that for μ_r values up to 10, there is marginal influence of viscosity on the H–Q and

$P-Q$ relationships of the pump. However, for higher values of viscosity, the change is dramatic with considerably large reduction in the value of head for a given discharge and correspondingly large increase in the brake power with discharge. The variation of the typical peak efficiency in this range as reported in various sources is shown in Table 5.6

Table 5.6 *Variation of typical peak of efficiency a pump with μ_r*

Relative viscosity, μ_r	1	10	100	116	396	10000
Peak efficiency	0.85	0.76	0.52	0.47	0.18	0.11

It is obvious that in pumping very viscous liquids, the centrifugal pumps become less efficient. It is generally considered that $\mu_r = 300$ is the upper limit of efficient adaptability of centrifugal pumps and for higher values of μ_r the best choice would be in the domain of positive displacement pumps.

5.11 VARIANTS OF SIMPLE CENTRIFUGAL PUMP

5.11.1 Double-Suction Pump

A single-suction pump, known as *ordinary centrifugal pump*, allows inlet of water at the eye of the impeller on one side only, the other end being completely closed of by the shroud plate. A variant of the ordinary single-suction pump is the double-suction pump, (Fig.5.42). In this, an arrangement is made for the liquid to enter the impeller from both sides. This enables large quantities of liquid to be handled by the pump with relatively low inlet velocity. A *double-suction pump* is similar to two single-suction pumps placed back to back with no solid plate in between them. The end thrust (*axial thrust*) that is always present in single-suction pumps is eliminated in this pump through balancing of the inflow. Double-suction pumps are advantageous in handling large quantity of liquid. However, due to their high costs, double-suction pumps are used only in selected applications.

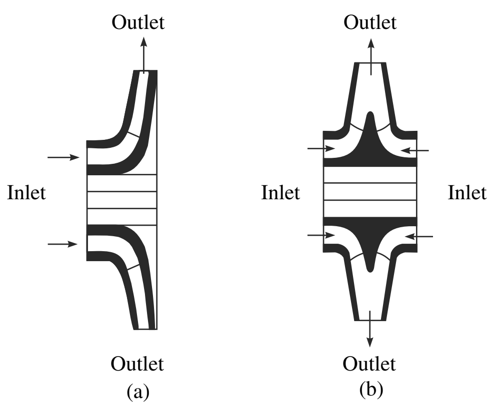


Fig. 5.42 *(a) Single-suction, and (b) double-suction pumps*

5.11.2 Multistage Pumps

When very high heads are involved in pumping, the size of the impeller becomes very large and becomes uneconomical beyond a certain range. A set of pumps in series can produce high heads while the size of each pump in the set can be of smaller dimension. A *multistage pump* uses this principle by joining the essence of the pumps, viz. the impellers, in series under one general compact casing.

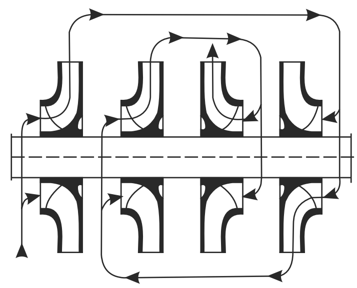


Fig. 5.43 *Multistage pump with four stages*

A multistage centrifugal pump consists of a series of identical impellers mounted on a common shaft. The discharge of one impeller goes into another as inflow through a set of stationary guide vanes. Thus, a multistage pump is a set of identical pumps connected in series. The last impeller discharges to the delivery pipe through a diffuser. The number of stages depends on the impeller design and the desired head. Multistage pumps are available for heads as high as 2000 m. Usually, even numbers of impellers are used. While mounting the impellers on the shaft, one-half of the impellers will have inlet facing one side and the other half will have their inlets in opposite direction (see Fig. 5.42). This arrangement will ensure that the axial thrust of the two groups compensate each other with resultant zero axial thrust on the shaft. Such an arrangement of impellers is known as *opposed mounting*.

Multistage pumps come in both horizontal and vertical mounting. The specific speed of these pumps is calculated based on head per each stage. The analysis of each stage is as for a single-stage pump. Multistage pumps find application where very high heads are required: such as in boiler feed, deep-well pumping and water supply to very high-rise buildings. The normal application areas include: water circulation, booster service, spraying systems and general purpose pumping.

5.12 MIXED-FLOW PUMPS

Common centrifugal pumps are essentially radial-flow pumps that produce the head in the liquid through centrifugal action. The flow is essentially radial in the impeller and the exit flow is in the plane of the impeller that is normal to the axis of the shaft. These pumps are mainly high-head and low-discharge units. As the discharge increases and the head decreases relatively, the pure radial units become very unwieldy and slow. However, by suitably adjusting the configuration of the impeller geometry, relatively higher specific speeds can be accommodated by efficient sized units. The lower half of Fig. 5.35 shows the shape of the vanes at various specific speeds. In the range of $10 < N_{sq} < 100$, the blades are essentially radial. In the range $80 < N_{sq} < 220$, the shapes of the vane undergo a major change; the flow is no more purely radial. It is a mixed flow of liquid transiting from axial at entrance to exit at an angle to the radial direction. The liquid experiences both radial acceleration and lift force. Such pumps are called *mixed-flow pumps.* This range is analogous to a Francis turbine. Mixed-flow pumps are also called variously as, *Francis type pump*, *semiaxial flow pump* and *screw pump*.

Mixed-flow pumps are of use in handling moderate to large quantities of liquid at relatively moderate to small heads. Originally developed for agricultural use, these pumps have been improved manyfold through advances in technology and materials. While typical conventional applications include drainage pumping and lift irrigation, current use includes many types of industrial use of all types, chiefly in condenser circulating service in power plants. These pumps come in single stage as well as in multistage configuration. Also in single stage, there are two types: *Volute type* and *Bowl type*.

- Mixed-flow volute-type pumps find application in condensate cooling in power plants. For very large pumps, the volute is sometimes made of concrete. They are also used in water supply with reservoir intake.
- Mixed-flow bowl-type pumps are usually in vertical or inclined settings and find applications in lift irrigation, land drainage and flood management. Also they are used in water supply from river or reservoir intake. In these pumps, the bowl assembly is submerged in the intake well and as such there is no priming issue.

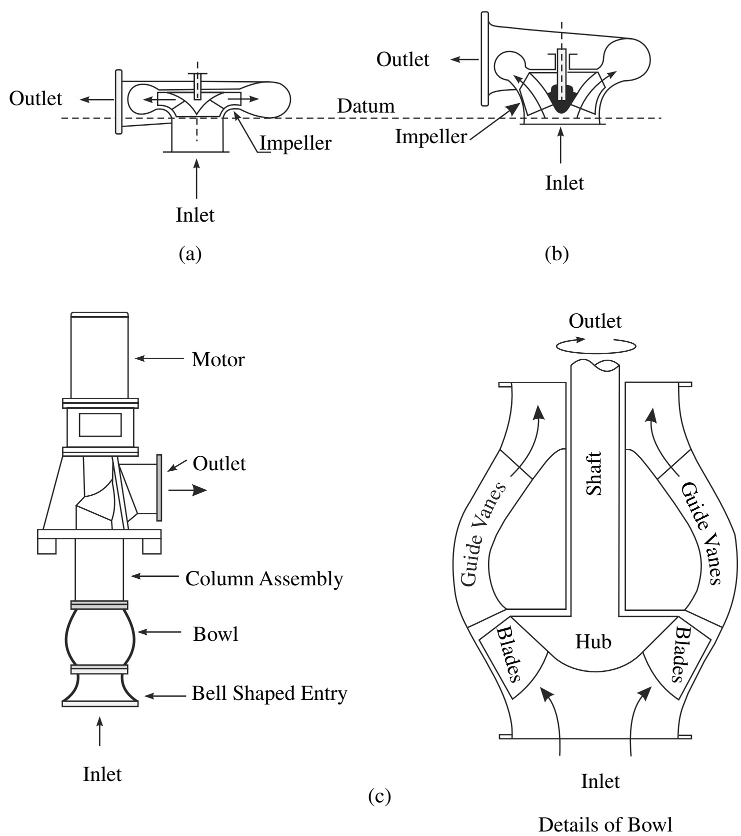


Fig. 5.44 *Comparative impeller configuration of (a) radial flow pump, and (b) mixed-flow volute-type pmps. (c) Schematic sketches of a mixed-flow vertical bowl-type pump*

Mixed-flow pumps come in a wide range of heads and discharges and the following could be considered as a normal average range of variables in single-stage pumps. Instances of specific applications having exceeded this range do exist in plenty.

Ranges of single-stage mixed-flow pump variables:

- Head = 3.0 to 30.0 m
- Discharge = 0.10 m^3/s to 11 m^3/s
- Size = 200 mm to 1500 mm
- Specific speed = 80 to 180
- Efficiency = 80% – 90%

Figure 5.44 (a) and (b) show the comparative characteristic features of the impellers of radial flow and mixed-flow volute-type pumps. Figure 5.44(c) is a schematic sketch showing the details of a vertical (bowl-type) pump. The bulblike feature, called *bowl*, is a characteristic feature of a vertical mixed-flow pump. In a pure axial-flow vertical pump, this bowl will be a cylinder.

Figure 5.45 is a typical performance characteristics of a mixed-flow vertical pump. In the *H–Q* relationship of mixed-flow pumps, the head falls much steeply with the discharge when compared to the similar plot of a centrifugal pump. Notice that the shut-off head is about 180% of normal head. The operating range is limited to 60% to 120% of normal value.

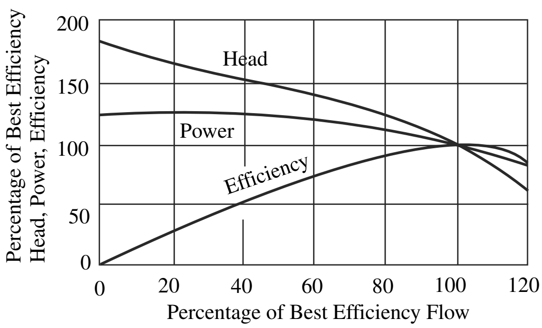


Fig. 5.45 *Typical characteristics of a mixed-flow vertical pump*

5.13 AXIAL-FLOW PUMPS (PROPELLER PUMPS)

5.13.1 Description of the Pump

The progressive change in the shape of the impeller of a rotodynamic pump with the specific speed is illustrated in Fig.5.35. The radial-flow and mixed-flow pumps cater to the specific speed ranges of $10 < N_{sq} < 100$ and $80 < N_{sq} < 220$ respectively. At very high specific speeds ($N_{sq} > 200$), the resulting impeller shape is that of a propeller-shaped pump that receives the flow axially and discharges it axially. The shape resembles that of a propeller turbine. Such high-discharge, low-head rotodynamic pumps with axial flow are called *axial-flow pumps* or *propeller pumps*. They belong to the region of high specific speed, which lie in the range of 180 to 300.

Axial-flow pumps provide thrust to the fluid through creation of lift force due to rotation of specially shaped blades. The blades are aerofoil shaped in cross section and twisted in appearance and are usually 3 to 8 in number. A pressure difference is created between the front and rear of the rotating blades and the fluid moves from the inlet to the outlet of the pump due to this differential pressure. The flow is in the axial direction with no initial and final whirl component. This is assured by providing a set of inlet guide vanes for the flow approaching the impeller and a set of guide vanes at the exit of the vanes to remove any whirl components from the flow and straighten the exit discharge along the axial direction.

The components of the axial-flow pump assembly are

- A bell-shaped entry with inlet guide vanes
- Propeller assembly on a shaft that is connected to the prime mover
- Exit guide vane set to remove whirl component of the flow
- Cylindrical casing that has close clearance with the propeller. Upper part of the casing is connected to a vertical diverging section. The vertical portion of the assembly is called *column assembly*.
- The delivery pipe takes off through an easy elbow/bend from the column assembly. The drive shaft emerges out of the conduit at the bend. The discharge flange can be above or below the floor level. The intake is normally through a well-designed intake structure.

Salient features of a vertical axial flow pump assembly is shown in Fig.5.46 (a) and (b).

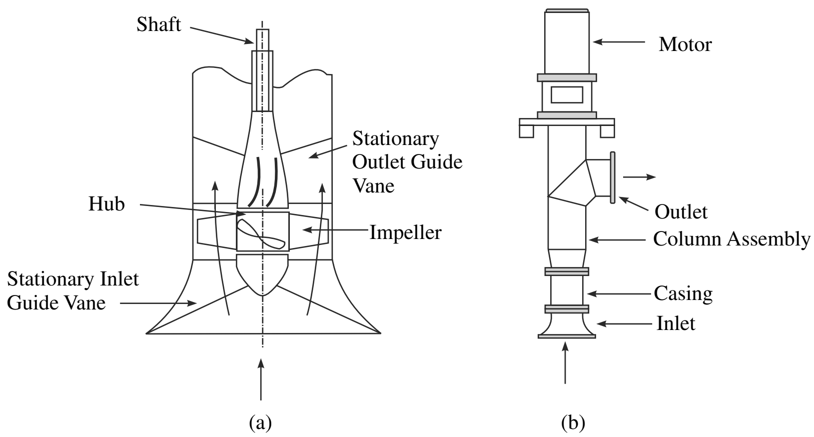


Fig. 5.46 *Details of a vertical axial-flow pump*

5.13.2 Impeller

The impeller of an axial-flow pump is a set of specially designed blades that have the appearance of a propeller. There is no shroud covering the tips of the blades. The blades are connected to the drive shaft at the hub [Fig. 5.46(a)]. The hub itself is a small bulge in the shaft and has the blades attached to it. Generally, the blades are fixed rigidly to the hub. However, as in Kaplan turbine, pumps with adjustable pitch blades are also available for use in specific applications.

The number of blades varies from 3 to 8 and the blades are commonly made of bronze or stainless steel. The shaft is made of structural steel: carbon steel or stainless steel. The column assembly and elbows are usually of mild steel.

5.13.3 Analysis Procedure for Axial-flow Pumps

The analysis of the axial-flow pump is similar to that of a propeller (Kaplan) turbine. In simple one-dimensional approach, two methods are in common use: (a) through use of Euler equation, and (b) through use of aerofoil cascade theory. Advanced analysis techniques on the lines of the design of fans/propellers are in use and are beyond the scope of this book. It would be noticed that the axial flow pump is analogous to the propeller (Kaplan) turbine. A quick but approximate method for preliminary study, on the lines used in the analysis of Kaplan turbine (Sec. 3.3, Chapter 3) is indicated below.

Let D_1 = Outer diameter of the propeller, D_h = Outer diameter of the hub and with suffixes 1 and 2 denoting inlet and outlet respectively.

The flow is purely axial through the blades. As such,

- Area of flow = $A = \dfrac{\pi}{4}(D_1^2 - D_h^2)\,K_1$ where K_1 is the net area factor after deducting for the area occupied by the blades in the cross section. For preliminary studies, it is usual to take $K_1 = 1.0$.

  Hence, $A = \dfrac{\pi}{4}(D_1^2 - D_h^2)$ (5.67)
- $V_{f1} = V_{f2} = V_f$ = Velocity of flow which is taken as constant in the entire inlet – Outlet space, i.e. all along the inlet radius as well as all along the outlet radius.
- Discharge $Q = \dfrac{\pi}{4}(D_1^2 - D_h^2)V_f$ (5.68)
- Blades have aerofoil sections. A set of inlet guide vanes directs the flow properly to the impeller. Further, a set of exit vane blades remove any residual whirl component from the exit flow and direct the flow along the axial direction.
- Let u_1 = Peripheral velocity at inlet at any radius r, and u_2 = peripheral velocity at the outlet at the same radius r. Then

  $$u_1 = u_2 = u = \frac{2\pi rN}{60}$$

  The flow is assumed to enter the pump impeller axially without any whirl component. The guide vane at the inlet guides the flow at a guide angle of 90°. Thus, at inlet $\alpha_1 = 90°$ and $V_1 = V_{f1}$
- The Euler equation is applicable giving the Euler head H_e as

  $$H_e = \frac{V_{u2}u_2 - V_{u1}u_1}{g} = \frac{V_{u2}u_2}{g} \qquad (\because\ V_{u1} = 0) \tag{5.69}$$

  Further, the manometric efficiency $\eta_h = \dfrac{V_{u2}u_2}{gH_m}$ where H_m = Net head.

- Axial pumps are so designed that constant head is developed at all values of the radius of the propeller. Values of α_1, β_1 at inlet and β_2 at outlet change all along the blade length. Angles β_1 and β_2 are both minimum at the blade tip and increases along the radius to a maximum at the hub.

The velocity triangles at the inlet and at the outlet at any radial location on the blade are acute angles and the velocity triangle is constructed in the same way as done in connection with radial-flow pump. Mean radius $r_m = \frac{(D_1 + D_h)}{2}$ is used sometimes as representative location for estimation of values of specific speed, power and overall efficiency of the pump in a quick calculation.

5.13.4 Performance Characteristics

Figure 5.47 shows the performance characteristic of a typical axial-flow pump. The variation of head, brake power and efficiency with discharge is shown in nondimensional form. The Y-axis is the percentage of best efficiency value and the X-axis is the percentage of best efficiency discharge. Note that as per definition, all the three curves show a common value of (100, 100).

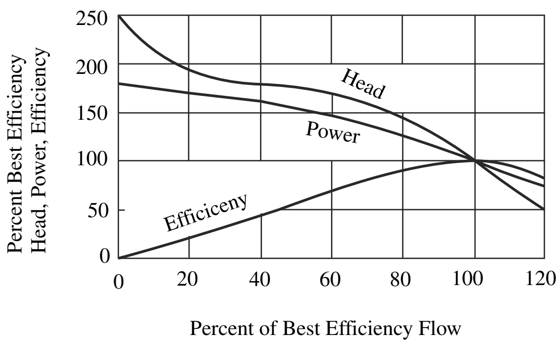


Fig. 5.47 *Performance chart of a typical axial-flow pump*

The head drops off rapidly with the increase in the discharge. The shut-off value of the head is 250% (that is 2.5 times) the normal head. Further, at only 20% increase in normal discharge, the head falls by about 50% of normal value. It has been found that axial-flow pumps with fixed blades are likely to have flow instability at around 50% normal flow.

The efficiency falls off on either side of the best efficiency point and limits the useful application range. The operation range is confined by the head and efficiency curves to 60% to 120% of normal value. Axial-flow pumps with variable pitch (*Kaplan pumps*) exhibit fairly good efficiency over a wide range of flow. Brake power rises rapidly with reduction in flow and reaches to nearly 180% of normal value at the shut-off. If the supply motor is not designed to take this variation, low flow operation should be eliminated. Alternately, axial-flow pumps are not suitable where the load is subjected to wide range of fluctuation. Note that the shut-off condition is completely different from that of the radial-flow pump, and in view of high head and high power requirement at shut-off, it is desirable that axial-flow pumps are started with control valves open.

5.13.5 Applications

Axial-flow pumps are suitable for lifting large quantities of water through relatively small heads and hence find application in diverse fields where such a need arises. The normal range of heads is from 1.0 m to 12 m, impeller sizes from 200 mm to 3.0 m and discharge range is from 0.1 m^3/s to 50 m^3/s.

Typical applications of axial-flow pumps include:

- Lift Irrigation
- Land drainage
- Flood water management
- Cofferdams
- River intake
- Water treatment
- Pollution and effluent control
- Dry docks
- Dewatering
- Industrial service
- Heat-recovery systems
- High volume mixing application
- Nuclear-reactor water circulation
- Ballast control in marine applications

Since the head involved in axial-flow pumps is rather small, the pumping system is sensitive to even small amounts of head loss as they form a substantial percentage of total head. Further, the *NPSHR* of the pump is generally large requiring submergence of the impeller at the intake. The depth of submergence is a function of the diameter of the pump and is usually recommended by the manufacturer. Proper amount of submergence and good intake design is an essential requirement of these pumps. Inadequacy in the submergence at intake results in formation of *intake vortices*, which in turn hampers the performance of the pump. Typically, the problem due to intake vortices is reflected in the reduction of discharge and mechanical vibration and consequent damages of the pump unit. An advantage of axial-flow pump is that it is not easily clogged and can handle some debris.

5.13.6 Relative Performance of Axial- and Radial-flow Pumps

Based on the above description and the study of the respective pump characteristics, some of the major differences in the operating characteristics of axial-flow and radial-flow pumps are summarised in Table 5.7.

Table 5.7 *Comparative performance of axial-flow and radial-flow pumps*

No.	Item	Axial-Flow Pump	Radial-Flow Pump
1	Specific speed	180–300	10–100
2	Mechanical loss in the pump	Negligible	Significant
3	Intake design	Very important	Moderately important
4	*NPSHR*	High	Low
5	Air handling	Good	Poor
6	When pump is reversed	Flow reverses	Flow is in the same direction as before.
7	H–Q curve	Steep	Flat to moderately steep
8	Ratio of safe operating head to normal head	2.0–3.0	1.1 to 1.6
9	Ratio of safe operating power to normal power	1.5–2.5	Less than 1.0

5.14 MATCHING OF SYSTEM CHARACTERISTICS

5.14.1 Basic Features

The performance characteristics of a pump consists of the *H*–*Q* curve, *P*–*Q* curve, η–*Q* curve and *NPSHR*–*Q* curve and are provided by the manufacturer to the client. The application of this information to efficiently perform a pumping job at a site is in the domain of the user. In a general sense, a pump is used to transfer a liquid from a specified source to a specified destination. Besides the pump, it involves piping with necessary fittings, provision of valves and needs dependent measuring instruments. This whole set of elements comprising of source, destination, pumping liquid, piping and other appurtenances constitute a *pumping system*. It is easy to appreciate that this pumping system is site-specific. In pumping of the liquid from the source to the destination, the system offers resistance due to the difference in elevation of the destination from the source, frictional resistance of the piping and form resistance of the fittings, valves, etc. Additionally, if there is a pressure head to be overcome at the receiving end, as for example in pumping into a pressurised container, this pressure head also will have to be accounted for in the system requirement. The cumulative resistances offered to the pumping by the system is called *system head requirement* and forms a system characteristic. The ideal objective in a pump installation is to match the pump characteristics to the system characteristics and achieve the best efficiency in the process.

5.14.2 H–Q Characteristics

1. Pump Characteristic

The head-discharge relationship of a radial flow pump (shown in Figs. 5.32 to 5.34) is typically a drooping curve and could be expressed as a polynomial in *Q* as

$$H = A + BQ + CQ^2 \tag{5.70}$$

where *A, B* and *C* are constants for a given pump. Normally, the pump is operated at a constant speed.

A typical *H*–*Q* relation of a radial flow pump (for a constant value of speed *N*) is shown in Fig. (5.48) with the head *H* plotted in the vertical axis and the discharge *Q* plotted in the horizontal axis. This curve is known as *pump characteristic* and is popularly called *pump H–Q curve*. The plot of efficiency against discharge is known as *efficiency curve* of the pump.

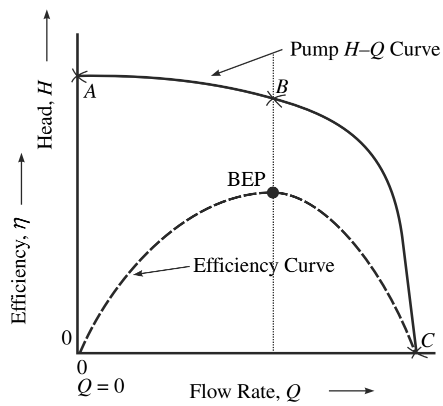


Fig. 5.48 *Characteristics of a radial-flow pump at constant speed*

The efficiency varies parabolically with the discharge, it is zero at the origin, $Q = 0$ and at point marked *C* in the figure where the head $H = 0$. The efficiency reaches a maximum value between these two zero values, and the operating point corresponding to the maximum efficiency is known as *Best Operating Point* (*BEP*).

The head varies from a maximum value at $Q = 0$ and reaches zero at point *C*, where the discharge is maximum. The head corresponding to *BEP*, indicated by the point *B* in Fig. 5.48, lies in between the extreme points *A* and *C*. The maximum discharge which occurs when the net head is zero (indicated by the point *C*) is known as *free delivery*. At the free-delivery condition, the inlet and the outlet are not throttled and there is no restriction of any kind. Thus, there is *no load* on the pump at free delivery. At this free delivery point, $H = 0$, Q is maximum and the efficiency is zero.

Point *A* on the $H–Q$ curve is the shut-off value of head. At this point, $Q = 0$, efficiency = 0 and the head is the maximum.

2. System Characteristic

The system $H–Q$ characteristics consist of two basic components: (i) static head, and (ii) dynamic head loss.

(a) Static Head It is the difference in elevation between the liquid level in the sump and the liquid level in the receiving reservoir. Thus, it is the sum of the suction lift plus delivery lift of a pump. Referring to Fig. 5.49, if Z_a is the elevation of the liquid level in the sump above a datum and Z_b is the liquid level in the receiving reservoir measured above the same datum then

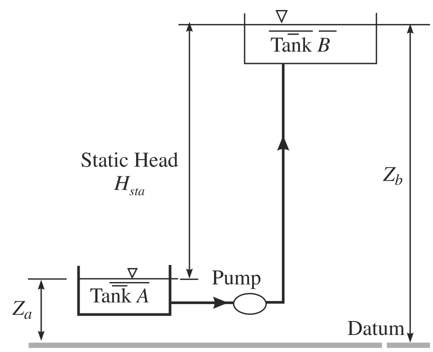


Fig. 5.49 *Definition sketch of static head*

Static head $H_{sta} = Z_b - Z_a$ (5.71)

There are situations in which the supply and receiving reservoirs, either of them or both of them, happen to be pressurised. In such cases, the static head is taken as the difference in the piezometric head of the two reservoirs. Equation (5.71) would read as

$$H_{sta} = \left(\frac{p_b}{\gamma} + Z_b\right) - \left(\frac{p_a}{\gamma} + Z_a\right) \tag{5.72}$$

Notice that Eq. (5.71) is a particular case of Eq. (5.72) when P_a and P_b are both equal to atmospheric pressure. Generally, the static head remains constant and independent of the flow rate.

(b) Dynamic Head Loss, H_{Dy} Dynamic head, also known as *friction head*, represents the energy head loss in the system due to friction and form loss. The losses consist of frictional head loss in the piping and energy loss at the valves and fittings. These could be represented as

Dynamic head loss $H_{Dy} = H_{fL} + H_{Mi}$ (5.73) where H_{fL} = Total pipe frictional loss = $\sum_i \frac{f_i L_i V_i^2}{2gD_i}$

in which for any i^{th} pipe, f_i = Darcy friction factor, L_i = length of the pipe, V_i = Velocity in the pipe,

and d_i = diameter of the pipe and H_{Mi} = Total of minor losses $\sum_i K_i \frac{V_i^2}{2g}$

where for any i^{th} valve or fitting K_i = Loss coefficient.

Since both H_{fL} and H_{Mi} involve the velocity of flow in the pipe, the dynamic head loss $H_{Dy} = f(Q)$ and is unique for each system as it depends on the geometry of the piping configuration. H_{Dy} varies approximately as the square of the discharge and $f(Q)$ can usually be expressed as $H_{Dy} = C + DQ^2$ where C and D are constants for a given system. The system characteristic *H-Q* curve, commonly called *system curve,* is shown in Fig. 5.50.

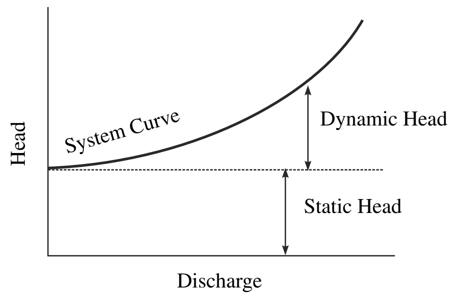


Fig. 5.50 *Definition sketch of system curve*

The total system head $H_{sys} = H_{sta} + H_{Dy}$

$$= \left[\left(\frac{p_b}{\gamma} + Z_b\right) - \left(\frac{p_a}{\gamma} + Z_a\right)\right] + \sum_i \frac{f_i L_i V_i^2}{2gD_i} + \sum_i K_i \frac{V_i^2}{2g} \tag{5.74}$$

Note that if the frictional losses are ignored (or is negligible) the total system head H_{sys} is equal to the static head.

5.14.3 Matching of Pump and System Characteristics

Plot both the pump and system *H–Q* curves to the same scale with head *H* in the vertical scale and *Q* on the horizontal scale, as in Fig. 5.51. This figure graphically represents the situation when a pump installed in a system starts pumping. The discharge from the pump starts with zero value and corresponding shut-off head and gradually increases. The variation of the corresponding head generated could be tracked on the pump curve *mn*: the starting point is point *m* and the pumping state moves along the pump curve from *m* towards *n.* Simultaneously, the system response starts from the point *u* on the system curve, which corresponds to zero discharge. Corresponding to every pumping state on the

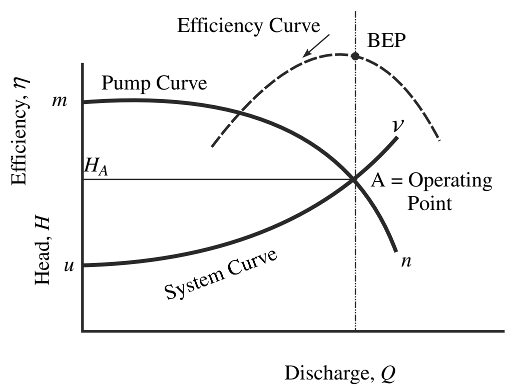


Fig. 5.51 *Pump operating point*

pump curve *mn*, there will be a corresponding point on the system curve representing the resistance head developed by the system. The two curves, pumping curve *mn* and the system curve *uv* intersect at the point *A*. Point *A* is known as *operating point* or *duty point.* At the point *A*, the requirement of the pump is perfectly matched with the requirements of the system curve *uv* and thus, the point *A* represents the operating point of the pump.

Ideally, for a given application, we can make the coordinates of the operating point the design value of the pump, viz. $Q_A = Q_{\text{design}}$ and $H_A = H_{\text{design}}$. Thus, the point *A* being the design point will correspond to *BEP*. If we plot the efficiency curve ($\eta - Q$) on Fig. 5.49, the peak of the efficiency curve representing *BEP* will be on the vertical at the point *A*. That is, if we have a choice to change/tweak the system curve and the pump curve, we would select such a pump that would have *BEP* at the point *A* and achieve the ideal objective in pump-system matching.

In an actual pump installation, the system curve is based on estimated values of various coefficients. Variations in the values of different parameters over time, operational pattern and errors in the estimation of various coefficients may lead to small deviations from the ideal system curve. As such, one would be satisfied if the point *A* were in the neighborhood of *BEP*. Exact coincidence of the point *A* and location of *BEP* is just fortuitous. In reality, one should be looking for *BER* (*Best Operating Range*).

5.14.4 Pumps in Parallel

Generally, radial flow centrifugal pumps have relatively small discharges and relatively high heads. When large flows have to be handled against high heads, a single pump unit would become unwieldy and uneconomical. In such cases, parallel pumping is usually resorted to. If two identical pumps are connected in parallel, and if the flow situation in the system is ideal and frictionless then the head would remain unaltered and the discharge would double. The impact of the real-life system having frictional head loss on the output of two identical pumps connected in parallel is studied in this section.

Consider two identical pumps 1 and 2 connected in parallel. Figure 5.51 shows a pump curve *mn* and this represents the *H–Q* performance of either of the pumps 1 or 2. Curve *mn* is called *single pump curve*. This curve is reproduced in Fig. 5.52. When two identical pumps are connected in parallel, the ideal output would be doubling of

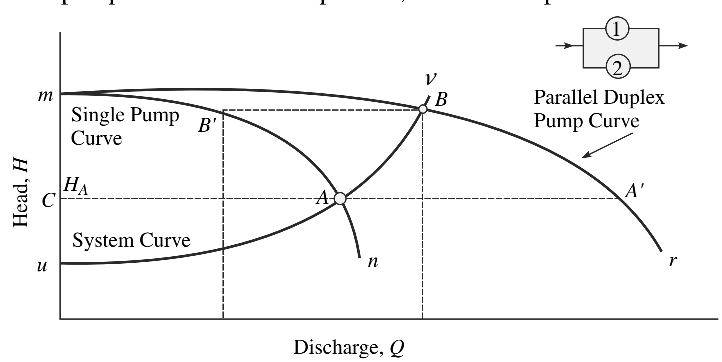


Fig. 5.52 *Pumps in parallel*

the discharge against a given head. The pump curve for this combination of pumps is obtained by doubling the x-coordinate of the pump curve mn. The curves mr shown in Fig.5.52 is obtained by this process and represents the characteristic curve of pumps 1 and 2 connected in parallel. Curve mr is also called *parallel duplex pump curve*. The systems curve representing the variation of the total systems head H_{sys} against the discharge is plotted as curve uv in Fig. 5.52.

The point of intersection of systems curve with the single pump curve mn is marked as the point A. This point A represents the coordinate of the operating point when only one pump (either pump 1 or 2) is working. The coordinates of A are marked as (Q_A, H_A). Further to point A, the systems curve uv intersects the duplex curve mr at the point B. Point B is the operating point of the two pumps 1 and 2 working in parallel. Coordinates of point B are (Q_B, H_B).

- If A is the operating point of a single pump, and if the friction was not present in the system, the operating point of two pumps in parallel would have been at $(2Q_A, H_A)$. This is the coordinate of the point A′ obtained as point of intersection of a horizontal line from A and the duplex line mr. This is shown as line CAA'in Fig. 5.52.
- The actual point of operation of two pumps in parallel is the point B and it is easy to see from Fig.5.52 that $Q_B < Q_{A'}$. Also, the head at B is larger than at A and at the point A', that is $H_B > (H_A = H_{A'})$. This change in the coordinates of the actual operating position B is due to the effect of the dynamic head loss part of the system curve. Thus, we can generalise this situation as follows.

  Two identical pumps in parallel working under a system head consisting of dynamic head loss would produce a net discharge which is less than twice the single pump discharge and also the net head of the parallel duplex pump would be larger than the head of a single pump operation in the same system.
- If a horizontal line is drawn from the point B to intersect the single pump lime mn at the point B' then by the nature of construction of curves mn and mr, the coordinates of B' are $\left(\left(Q_{B'} = \dfrac{Q_B}{2}\right), (H_{B'} = H_B)\right)$. Point B' represents the contribution of discharge and head of each of the pumps 1 and 2. It can thus be concluded that each of the pumps in parallel produce a flow that is less than what each of them would have pumped if they were running singly. Similarly, the head produced by each of the pumps in parallel is more than what each of them would have produced if they were running singly.

Examples 5.39 and 5.42 illustrate the computational aspects of pumps connected in parallel.

5.14.5 Pumps in Series

When it is required to pump a liquid to a head larger than the designed head of an available pump, the pumping could be achieved by combining two or more pumps in series. From elementary fluid mechanics, we know that under ideal frictionless conditions of the system, two identical pumps connected in series pass the same discharge through both the pumps and while doing so, the head is increased two folds. However, when the effect of friction in the system is taken into consideration through

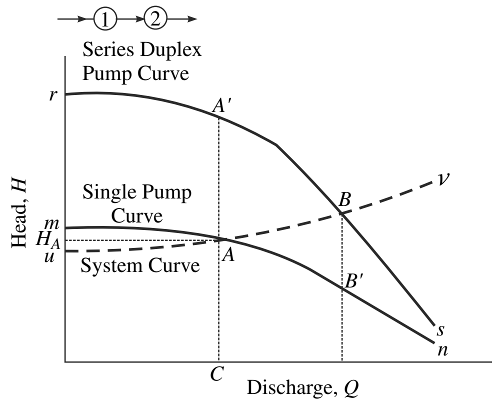


Fig. 5.53 *Two pumps in series*

a system head–discharge curve, the ideal result noted above will undergo changes. The extent of impact of system friction losses on the operation of pumps in series is studied in this section.

Consider two identical pumps 1 and 2 connected in series. The head-discharge curve of the individual pumps being the same, is represented as pump curve *mn* in Fig. 5.53. The characteristics of the combination of pumps 1 and 2 working in series is called *series duplex curve* and is shown in Fig. 5.53 as curve *rs*. To obtain this curve, the ordinates of curve *mn* are first multiplied by a factor of 2 representing the number of pumps in series. Then, the new coordinates are plotted on the same Q–H axes and to the same scale as the *mn* curve. The systems curve is calculated by using Eq. (5.74) and is shown in Fig. 5.53 as the curve *uv*.

Let the system curve intersect the single pump curve *mn* at the point *A*. This intersection point *A* represents the operating point when only one pump (pump 1 or 2) is working. The coordinates of *A* are marked as (Q_A, H_A). Further to the point *A*, the systems curve *uv* intersects the series duplex curve *rs* at the point *B*. Point *B* is the operating point of the two pumps 1 and 2 working in series and the coordinates of the point *B* are (Q_B, H_B).

- If *A* is the operating point of a single pump, and if the friction was not present in the system, the operating point of two pumps in series would have been at (Q_A, $2H_A$). This is the coordinate of the point *A′* obtained as point of intersection of a vertical line from *B* and the series duplex line *rs*. This is shown as the line *CA A′* in Fig. 5.53.
- The actual point of operation of two pumps in series is the point *B* and it is easy to see from Fig. 5.53 that $H_B < H_{A'}$. Also, the discharge at *B* is larger than at *A* and at the point *A′*; that is $Q_B > (Q_A = Q_{A'})$. This change in the coordinates of the actual operating position *B* is due to the effect of the dynamic head loss part of the system curve. Thus, we can generalise this situation as follows:

  Two identical pumps in series working under a system head consisting of dynamic head loss would produce a net head that is less than twice the single pump head and the net discharge of the series duplex pump would be larger than the discharge of a single pump operation in the same system.

- If a vertical line is drawn from the point B to intersect the single pump line mn at the point B' then by the nature of construction of curves mn and rs, the coordinates of B' are $\left((Q_{B'} = Q_B), \left(H_{B'} = \frac{H_B}{2}\right)\right)$. Point B' represents the contribution of discharge and head of each of the pumps 1 and 2. The total head developed is smaller than $2H_A$ and the extra discharge passed by each pump to the system is $(Q_B - Q_A)$. It is interesting to note that each of the pumps in series produce a flow that is more than what each of them would have pumped if they were running singly. Similarly, the head produced by each of the pumps in series is less than what each of them would have produced if they were running singly.
- If the *BEP* of the pump were made to coincide with the point A (i.e. the point corresponding to the single pump operation), when the pumps work as series duplex, the operating point B will be to the right of *BEP* and thus will have less than the optimal efficiency.
- In practice, the operation of pumps in series is very rare. Whenever a situation of very high head and low discharge is encountered, multistage pumps are preferred. For small discharges and high heads, booster pumps are used.
- A special case of pumps in series is the *booster pump*. These pumps are installed at some distance from the primary pump mainly for purposes of increasing the pressure in the downstream. Typically, booster pumps find application in handling the problem areas of a submain in water distribution system.

Examples 5.40 and 5.43 illustrate the computational aspects of pumps connected in series.

5.14.6 Some Aspects Relating to Pump-System Interaction

Some important aspects relating to the use of systems curve and pump curves are listed below. These apply to pumps in single-pump operation as well as in duplex arrangement. The basic idea is applicable to multiplex arrangements also.

- The pump curve supplied by the manufacturer is limited to the recommended operational range of the pump. These curves should not be extended, as one does not know the nature of the behaviour of the pump in the extended region.
- If a systems curve does not intersect a pump curve, it means that that the system flow is not realisable by the pump. The pump is not capable of meeting the system demand for any of the flow in the operating range.
- The pumps used in parallel and in series operations must be identical both geometrically and in the speed also. If the pumps are not identical and are connected in parallel or series, one pump may carry more load and the other may act more of a hindrance than help. Similarly, if the two pumps have different characteristics, say one of them is a radial pump and another a mixed flow pump, a condition of backflow may occur to one of the pumps.
- When two identical pumps are connected in parallel, if the system curve cuts the duplex curve and not the single pump curve, it signifies that the single

pump operation is not feasible. It implies that both the pumps must start simultaneously and shut off simultaneously. Obviously, from the practical point of view, it is a constraint.

- All pumps must operate with *NPSHA* ⩾ *(NPSHR + M).*
- Efforts must be made to select the pumps whose *BEP* is nearer to the operating point of the duplex arrangement.
- In all pump installations, provision for redundancy to allow for maintenance and breakdown of one or more pumps is an important factor.
- When selecting a pump to operate in a system, its shut-off power should be larger than the largest static head between any two points in the flow line in the system. This aspect is illustrated with the following example:

Consider a pumping system as shown in Fig. 5.54. Point *S* in Fig. 5.54 is a local summit whose elevation is above the delivery point. When the pump is running, the piping which lies above the static head H_{st1} will have negative pressures with the point *S* acting as the summit of a siphon. Consider the situation at the starting of the pump. The shut-off head developed by the pump must be larger than the static head of the point *S* (H_{st2}) to push the water past the summit. When once the flow begins, the siphonic action will take over and the effective static head is reduced to H_{st1}.

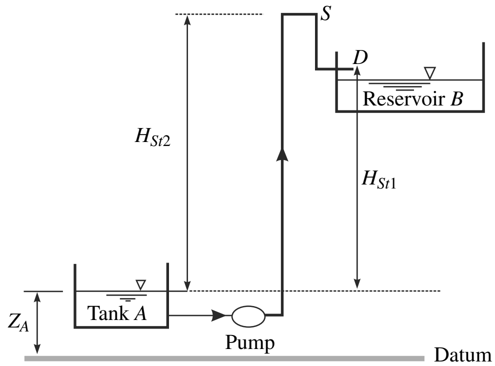


Fig. 5.54 *A pumping system with a local maximum static head*

5.15 PRIMING OF PUMPS

In a centrifugal pump delivering water, the rotation of the impeller creates a head difference between the inlet and the outlet of the pump. When the pump and the suction conduit is completely filled with water, the pressure at the suction side will be lower than the atmospheric pressure. Due to the difference in pressure head between the water surface of the sump and the inlet of the pump, the atmospheric pressure causes the water to be pushed up from the sump into the pump casing. From then onwards, the head created by the pump enables delivery of the flow to the destination. If, however, the pump casing is not filled with water at the start of the pump, the impeller rotates in air or vapour-filled casing. This cannot create the necessary differential head of water between the sump and the inlet as the density of air is much less than that of water. Consequently, the pump does not do its work of pumping of water. Further, dry running of the pump may damage several parts of the pump. It is therefore necessary to assure the pump is always filled with water before start of the pump. If at the start of the pump, the suction pipe or the pump casing is filled with air or vapour pockets, they will have to be replaced by water from an external source to keep the pump full and ready for starting. This practice, viz. the process of filling the casing and suction pipe with the liquid to be pumped from an outside source before starting of the pump, is called *priming*.

A one-way valve, called *foot valve*, is provided at the foot (entrance) of the suction pipe. When a running pump is closed, the foot valve retains water in the suction pipe and in the pump casing, preventing it from draining out to the sump. Thus, when the pump is restarted, it does not need any priming. When a foot valve is leaking or is not functioning properly, the need for the priming arises. Following are some methods adopted for priming of centrifugal pumps:

- In small water pumps, water is poured into the casing through an opening using a funnel. An air vent in the casing is kept open during the filling process. When the casing is filled with water, the pump is started and any residual air is allowed to escape through the air vent. The vent is closed when clear water starts escaping from the vent.
- In larger pumps, sometimes, arrangements are made for evacuation of the air through vacuum-producing devices, such as ejectors and vacuum pumps.
- Priming is not an issue when the pump is located below the sump liquid level.

There are a host of commercially patented devices available that remove air when the pump is started and enable normal functioning of the pump in a very short time. Pumps fitted with such devices are known as *self-priming pumps*. A self-priming pump clears the pump casing of any air and allows normal liquid pumping operation to resume without outside attention. Currently, most of the centrifugal pumps are of the self-priming variety. [Reference **http //www.gouldspumps.com/pag_0029.html** can be consulted for details of a type of self-priming device].

5.16 STANDARDS RELATING TO PUMPS

Some important standards relating to centrifugal pumps are given in Appendix *B*. These cover a variety of topics related to diverse applications including testing. These and such other related and relevant standards have to be consulted in connection with tendering and contract document preparation related to procurement of pumps.

5.17 TESTING OF PUMPS IN LABORATORIES

5.17.1 Purpose of Pump Testing

There is a major difference in the manufacture of pumps and turbines. The number of turbines manufactured in a year in a country is relatively very small in number and most of them are individually manufactured to suit the specific needs of a project. This is particularly true for turbines used in all large hydro projects. Contrary to this, the number of pumps manufactured in a year is a couple of orders of magnitude larger than that of the turbines. Also, a very small fraction of them are of very large size. This is reflected in the testing of pumps.

All major pump manufacturers have their own pump-testing laboratory. In addition, pump-testing laboratories owned by independent organisations, universities and research institutes and by public sector organisations also exist. Pump tests are done, in general, on the prototypes. In the case of large pumps, model testing is resorted to prepare performance characteristics if the laboratory facilities are not adequate to test the prototype. Also, model tests of pumps are conducted to solve specific problems in a class of pumps and in developmental work. In general, pumps are tested for a number of reasons, the important ones being:

1. To produce performance charts of a class of pumps for use in sales promotion and also as guarantee in after sales/ contracts
2. Final inspection of the product by the manufacturer
3. Control systems test
4. As a part of maintenance programme or to sort out a specific operational problem
5. As a part of development or research work
6. To evaluate comparative merits of two or more competing candidate pumps to assist in final selection in a major project

5.17.2 Standard Pump-testing Laboratory

A standard pump-testing laboratory will be similar (in a very general sense) to that of a turbine-testing laboratory but generally of a smaller size. The basic requirements could be listed as follows:

1. A water-recirculating system with adequate pumping capacity
2. A battery of flowmeters (usually magnetic flowmeters are preferred choice) of different ranges for flow measurement
3. Pressure transducers for pressure measurement
4. Dynamometers, torque meters and devices to measure the speed of rotation of shafts. (Handheld tachometers are not preferred)
5. Sophisticated data acquisition system and computer facility with appropriate software (with system design software like Lab*VIEW*)

5.17.3 Measurement of Parameters

All the testing procedures, starting from the test bed and fixtures to the measurements and the accuracies of measurements must all conform to selected relevant standards. The list of relevant standards are given in Appendix *B*. The following two standards are followed extensively and internationally for testing of centrifugal pumps:

- *ISO* 9906: 2012–*Rotodynamic Pumps–Hydraulic Performance Acceptance Test–Grades 1 and 2* and
- *ANSI/HI* 14.6-2011–*Hydraulic Performance Acceptance Tests for Rotodynamic Pumps*

The following basic parameters are measured and acquired digitally in the course of testing of pumps.

1. Discharge, Q

The discharge is measured and recorded continuously through use of magnetic flowmeters of appropriate range and accuracy.

2. Pressure

Accurate measurement of pressure is vital to accurate testing of the pump. Pressures are measured through use of pressure transducers. The location of pressure tapping must be such that the flow is uniform and the velocity at that location does not have any radial component. Bends and valves must be sufficiently far away from the pressure-measuring points. Pressures at the downstream and upstream of the pump are measured at locations situated at a minimum distance of twice the diameters of the pipe. Further, the upstream locations must have a straight stretch of at least four times the diameter of the pipe before the tapping. Similarly, the downstream tapping must be followed by a straight stretch of pipe of at least two diameters in length. The recorded pressures are converted into pressure heads by using the density of water corresponding to the temperature of water as recorded by the data-acquisition system.

3. Speed of Rotation

Optical sensor tachometer is used to acquire the shaft-speed data. Normally, pump tests are conducted at constant speed and the speed sensor will also give inputs to the speed-control governor. To achieve variable speed capability, the motor driving the pump will have a variable frequency drive.

4. Power

The input power is measured in two ways.

(a) By knowing the characteristics of the electric motor driving the pump, the input electrical power is measured by a power meter. In addition the current, voltage, frequency, powerfactor and other electrical power-related parameters are also measured. The input of mechanical energy to the pump is derived from this data as the product of input electrical power and overall efficiency of the drive system.

(b) By knowing the speed of rotation of the pump shaft and the torque transmitted to the pump by the electrical drive, the mechanical power input of the pump is calculated as the product of torque and the angular velocity of the pump shaft. The torque meter connected to the pump shaft measures the torque.

5. Temperature

A thermal sensor located in the suction pipe of the pump continuously senses the temperature of the circulating water.

6. Air Concentration

In tests related to *NPSHR* and cavitation, the air concentration of the water is important and as such, a sensor to measure air concentration continuously is provided in the test set-up.

5.17.4 Tests to Obtain Pump Characteristics

These tests are routine tests for a pump manufacturers' laboratory. The general operation of these tests is through computerised control panel using appropriate software. After the start of the pump in the test circuit, a certain discharge in the operating range of the pump is set. When the flow in the test circuit is stabilised, the readings of the operational parameters of pump, viz. the discharge, head, power input to the pump are recorded at the set point. This constitutes one set of data for one set-point. The discharge is now varied to another set-point and the procedure is repeated.

The usual practice is to start from the highest discharge in the operating range and decrease the discharge to zero value in about 7 to 8 steps. Then the discharge is increased gradually in 7–8 steps up to the maximum value, thus covering about 15 set-points. The flow control valve (usually a gate valve) in the discharge pipe is used to change the discharge through opening or throttling. In each set-point, the flow, pressure, speed, temperature and power input to the pump are recorded. From this data, the performance characteristics of the pump, viz. the variation of head, power and efficiency, with the discharge at the designated speed of rotation is prepared.

Inspection Tests

The object of this test is to inspect the pump for finished quality of manufacturing and to confirm that it follows the predefined operational characteristics. The tests are practically same as the performance tests described above except for the fact that the data is obtained for only two to five selected, predefined, critical set-points. In that way these are short version of the performance tests.

5.17.5 Tests for *NPSHR*

NPSHR is defined (Sec. 5.9.3) as the head, in absolute units, measured above the prevailing vapour-pressure head, required by the pump to obtain satisfactory pumping head (no more than the 3% reduction in head at constant flow) and prevent excessive cavitation. Considering Fig. 5.39, by Eq. (5.65-a)

Defining $NPSH = \sigma H_m$

in which σ is defined as $\sigma = \dfrac{H_{am} - H_s - h_{Ls} - H_{vP}}{H_m}$

At the critical value of $\sigma = \sigma_c$, the $NPSH = NPSHR$. It is also called $NPSHR_{3\%}$ to indicate the criteria by which critical value is determined. For a given pump, running at constant speed, *NPSH* can be reduced by increasing H_s or by decreasing the pressure at the suction end of the pump by throttling the suction pipe.

- The *NPSHR* test consists of selecting a given discharge point in the normal *H–Q* curve and decreasing the *NPSH* as indicated above to obtain a curve of *NPSH* vs. H_m that appears similar to Fig. 5.39. From this curve, $NPSHR_{3\%}$ is determined.
- The experiment is repeated for different discharge points in the normal *H–Q* curve of the pump. For each selected discharge, a value of NPSHR is determined.
- The result is a plot of *NPSHR* on the *y*-axis and discharge *Q* on the *x*-axis that gives the NPSHR curve for the pump at the speed used in the test. The nature of *NPSHR* curve will be as shown in Fig. 5.33 and Fig. 5.34. *NPSHR* increases slowly with *Q*.

There are variants to the above procedure and the details of *NPSHR* tests are beyond the scope of this book. Interested readers may refer to Ref. 5.2 for details of *NPSHR* test of a centrifugal pump.

5.18 ILLUSTRATIVE EXAMPLES—SET 5.2

5.18.1 Affinity Laws and Similitude

*EXAMPLE 5.22

It is required to pump 1.30 m^3/s of water to a total head of 45 m. How many pumps of specific speed 40 and running at 1450 rpm would be needed when connected in parallel, when the dynamic head in the system can be neglected?

Solution

Given: $Q_t = 1.30$ m^3/s, $H_m = 45$ m, $N_{sq} = 40$, $N = 1450$ rpm

If Q_1 = Discharge of one pump,

Specific speed $N_{sq} = \dfrac{N\sqrt{Q_1}}{H_m^{3/4}} = \dfrac{1450\sqrt{Q_1}}{(45)^{3/4}} = 40$

$$\sqrt{Q_1} = \frac{(45)^{3/4} \times 40}{1450} = 0.479$$

$Q_1 = 0.23$ m^3/s

Number of pumps required $\approx \dfrac{1.30}{0.23} = 6$ pumps

*EXAMPLE 5.23

A discharge of 0.4 m^3/s of water is needed to be pumped to a total head of 240 m. How many pumps connected in series and each having a specific speed of 35 and running at a speed of 1500 rpm would be needed for the job? The dynamic head in the system can be neglected.

Solution

Given: $Q = 0.4$ m^3/s, $H_m = 220$ m, $N_{sq} = 35$, $N = 1500$ rpm

If H_{m1} = Head off one pump,

Specific speed $N_{sq} = \dfrac{N\sqrt{Q}}{H_{m1}^{3/4}} = \dfrac{1500\sqrt{0.4}}{(H_{m1})^{3/4}} = 35$

$$(H_{m1})^{3/4} = \frac{1500 \times \sqrt{0.4}}{35} = 27.11$$

$H_{m1} = 81.43$ m

Number of pumps required $\approx \dfrac{220}{81.4} = 3$ pumps

*EXAMPLE 5.24

A centrifugal pump has an impeller of 200 mm and its capacity is 400 L/s at a speed of 1200 rpm against a total head of 12 m. Estimate the speed and head of *a geometrically similar pump with impeller diameter of 300 mm and required to deliver 700 L/s.*

Solution

Given: Q_1 = 0.40 m^3/s, N_1 = 1200 rpm, H_1= 12 m, D_1 = 0.20 m, D_2 = 0.30 m, Q_2 = 0.70 m^3/s

$$\left(\frac{Q}{ND^3}\right)_1 = \left(\frac{Q}{ND^3}\right)_2$$

$$\frac{N_1}{N_2} = \left(\frac{Q_1}{Q_2}\right)\left(\frac{D_2}{D_1}\right)^2 = \left(\frac{0.40}{0.70}\right)\left(\frac{0.30}{0.20}\right)^3 = 1.929$$

$$\text{Speed } N_2 = \frac{N_1}{1.929} = \frac{1200}{1.929} = 622 \text{ rpm}$$

$$\left(\frac{H}{N^2D^2}\right)_1 = \left(\frac{H}{N^2D^2}\right)_2$$

$$\frac{H_1}{H_2} = \left(\frac{N_1}{N_2}\right)^2\left(\frac{D_1}{D_2}\right)^2 = (1.929)^2\left(\frac{0.20}{0.30}\right)^2 = 1.654$$

$$\text{Head } H_2 = \frac{H_1}{1.654} = \frac{12.0}{1.654} = 7.26\,\text{m}$$

*EXAMPLE 5.25

A centrifugal pump is delivering 400 L/s of water against a head of 25 m. If it is desired to pump 600 L/s using the same pump, estimate the percentage increase in speed required. What percentage increase in brake power is required for the new job? What would be the head under the new condition?

Solution

Given: Q_1 = 0.40 m^3/s, Q_2 = 0.60 m^3/s, H_1 = 25 m

Same pump in both the cases. Hence, $D_1 = D_2$

$$\left(\frac{Q}{ND^3}\right)_1 = \left(\frac{Q}{ND^3}\right)_2. \text{ Hence } \frac{Q_2}{Q_1} = \frac{N_2}{N_1}\left(\frac{D_2}{D_1}\right)^3$$

$$\text{Since } \frac{D_2}{D_1} = 1, \frac{Q_2}{Q_1} = \frac{N_2}{N_1} = \frac{0.6}{0.4} = 1.5$$

N_2 is 50% larger than N_1 .

$$\frac{P_2}{P_1} = \left(\frac{N_2}{N_1}\right)^3\left(\frac{D_2}{D_1}\right)^5 = \left(\frac{N_2}{N_1}\right)^3 = (1.5)^3 = 3.375$$

P_2 is 337.5 % more than P_1.

$$\frac{H_2}{H_1} = \left(\frac{N_2}{N_1}\right)^2 \left(\frac{D_2}{D_1}\right)^2 = \left(\frac{N_2}{N_1}\right)^2 = (1.5)^2 = 2.25$$

New head $N_2 = 2.25 \times 25 = 56.25$ m

*EXAMPLE 5.26

A centrifugal pump has a diameter of 325 mm and consumes 15 kW of input power at an efficiency of 0.80. The speed of the pump is 1450 rpm and the total head developed is 21 m. If the impeller size is trimmed to 300 mm, estimate the change in the discharge and head when it is run at the same speed of 1450 rpm.

Solution

Given: $N_1 = 1450$ rpm, $P_1 = 15$ kW, $\eta_1 = 0.80$, $N_2 = 1450$ rpm, $H_1 = 21$ m, $D_1 = 0.325$ m, $D_2 = 0.300$ m

Same speed in both the machines. Hence, $N_2 = N_1$. Given, $\frac{D_1}{D_2} = \frac{0.325}{0.300} = 1.0833$

$$Q_1 = \frac{P_1 \eta}{\gamma H_1} = \frac{15.0 \times 0.8}{9.79 \times 21} = 0.0584 \text{ m}^3/\text{s}$$

Calculations by taking $\left(\frac{N_1}{N_2}\right) = 1$:

Discharge	Head
$\frac{Q_1}{Q_2} = \left(\frac{N_1}{N_2}\right)\left(\frac{D_1}{D_2}\right)^3 = \left(\frac{D_1}{D_2}\right)^3$	$\frac{H_1}{H_2} = \left(\frac{N_1}{N_2}\right)^2 \left(\frac{D_1}{D_2}\right)^2 = \left(\frac{D_1}{D_2}\right)^2$
$Q_2 = \frac{Q_1}{\left(\frac{D_1}{D_2}\right)^3} = \frac{0.0584}{(1.0833)^3} = 0.0459 \text{ m}^3/\text{s}$	$H_2 = \frac{H_1}{\left(\frac{D_1}{D_2}\right)^2} = \frac{21}{(1.0833)^2} = 17.89 \text{ m}$
Change in discharge = (0.0459 – 0.0584) = – 0.0125 m^3/s (Decrease)	Change in head = (17.89 – 21.0) = – 3.105 m (reduction)

**EXAMPLE 5.27

Following are the details of a 1 : 5 scale model of a large pump:
Power supplied = 10 kW Head developed = 6 m
Speed = 1000 rpm Efficiency = 0.85
(a) Estimate the prototype speed, power required, discharge by considering the prototype head as 36 m and assuming the efficiency to remain the same as that of the model. (b) What would be the efficiency of the prototype by the step-up formula of (i) Moody, and (ii) Anderson?

Solution

Given: $N_m = 1000$ rpm, $P_m = 10$ kW, $\eta_m = 0.85$, $D_r = D_m/D_p = 1/5 = 0.20$, $H_p = 36$ m, $H_m = 6$ m

$$H_r = \frac{H_m}{H_p} = \frac{6}{36} = 0.167 \text{ and } \quad D_r = \frac{D_m}{D_p} = \frac{1}{5} = 0.2$$

$$N_r = \frac{H_r^{1/2}}{D_r} = \frac{\sqrt{0.167}}{0.20} = 2.043$$

$$N_p = \frac{N_m}{2.043} = \frac{1000}{2.043} = 490 \text{ rpm}$$

$Q_r = N_r D_r^3 = 2.043 \times (0.2)^3 = 0.01635$

From expression for power, $P_m = \dfrac{\gamma Q_m H_m}{\eta_0}$

$$Q_m = \frac{\eta_0 P_m}{\gamma H_m} = \frac{0.85 \times 10.0}{9.79 \times 6} = 0.1447 \text{ m}^3/\text{s}$$

$$Q_p = \frac{Q_m}{0.01635} = \frac{0.1447}{0.01635} = 8.85 \text{ m}^3/\text{s}$$

$P_r = N_r^3 D_r^5 = (2.043)^3 \times (0.2)^5 = 0.00273$

$$P_p = \frac{P_m}{0.00273} = \frac{10.0}{0.00273} = 3664 \text{ kW}$$

Efficiency: By step-up formula of Moody (Eq. 5.49):

$$\frac{1-\eta_m}{1-\eta_p} = \left(\frac{D_p}{D_m}\right)^{0.2}$$

$$\frac{1-0.85}{1-\eta_p} = \left(\frac{1}{0.2}\right)^{0.2}$$

$1 - \eta_p = 0.1087$ giving $\eta_p = 0.891$

By Anderson's formula (Eq. 5.50):

$$\frac{0.94-\eta_m}{0.94-\eta_p} = \left(\frac{Q_p}{Q_m}\right)^{0.32}$$

$$\frac{0.94-0.85}{0.94-\eta_p} = \left(\frac{1}{0.01635}\right)^{0.32} = 3.71$$

On simplifying, $\eta_p = 0.916$

*EXAMPLE 5.28

A single-stage centrifugal pump has a discharge of 600 L/minute of oil of relative density 0.9 against a head of 20 m. The speed of the pump is 1250 rpm. The efficiency of the pump can be taken as 0.82. Estimate the brake power at this speed and the expected power requirement if the speed is increased to 1500 rpm.

Solution

Given: Q_1 = 0.60 m^3/s, N_1 = 1250 rpm, H_1= 20 m, η_1 = 0.82, N_2 = 1500 rpm, Rel. density = 0.9

Same pump. Hence, $D_1 = D_2$. $\quad \dfrac{N_1}{N_2} = \dfrac{1250}{1500} = 0.8333$

$$\text{Power } P_1 = \frac{\gamma Q_1 H_1}{\eta_1} = \frac{(0.9 \times 9.79) \times 0.60 \times 20}{0.82} = 128.9\,\text{kW}$$

$$\frac{P_2}{P_1} = \left(\frac{N_2}{N_1}\right)^3 \left(\frac{D_2}{D_1}\right)^5 = \left(\frac{N_2}{N_1}\right) \times 1$$

$$\frac{P_2}{P_1} = \left(\frac{N_2}{N_1}\right)^3 = \frac{1}{(0.8333)^3} = \frac{1}{0.579}$$

$$P_2 = \frac{P_1}{0.579} = \frac{128.9}{0.579} = 222.7\,\text{kW}$$

*EXAMPLE 5.29

Two geometrically similar pumps A and B have the same speed of 1500 rpm. Pump A has a diameter of 0.35 m and discharge of 36 L/s against a head of 25 m. Pump B gives a discharge of 18 L/s. Estimate the total head and impeller diameter of Pump B.

Solution

For geometrically similar pumps, the specific speed is the same. Hence,

$$N_{sq} = \frac{N_A \sqrt{Q_A}}{(H_{mA})^{3/4}} = \frac{N_B \sqrt{Q_B}}{(H_{mB})^{3/4}}$$

$$N_{sq} = \frac{N_A \sqrt{Q_A}}{(H_{mA})^{3/4}} = \frac{1500\sqrt{0.0356}}{(25)^{3/4}} = 25.31$$

$$\text{For Pump } B;\; N_{sq} = 25.31 = \frac{1500\sqrt{0.018}}{(H_{mB})^{3/4}} = \frac{201.2}{(H_{mB})^{3/4}}$$

H_{mB} = 15.86 m

$$\text{From affinity laws: } \left(\frac{Q_A}{N_A D_A^3}\right) = \left(\frac{Q_B}{N_B D_B^3}\right)$$

$$D_B^3 = \frac{Q_B}{Q_A} \times \frac{N_A}{N_B} \times D_A^3 = \left(\frac{0.018}{0.036}\right)(1)(0.35)^3 = 0.00214$$

D_B = 0.278 m

**EXAMPLE 5.30

Pump A is designed to pump 0.5 m^3/s of sea water (ρ = 1025 kg/m^3) to a head of 30 m when running at a speed of 1500 rpm. This pump is to be studied in a 1 : 2 *model in the laboratory that uses fresh water (ρ = 998 kg/m³). The model pump has a rotative speed of 1200 rpm.*

(a) Calculate the discharge, total head and the brake power of the model. Assume an efficiency of 0.80 for both model and prototype pumps.

(b) Calculate the specific speed of the model and prototype respectively.

Solution

Given: $\rho_p = 1025$ kg/m^3, $Q_p = 0.5$ m^3/s, $H_p = 30.0$ m, $N_p = 1500$ rpm,
$\rho_m = 998$ kg/m^3, $D_r = ½ = 0.5$, $N_m = 1200$ rpm

$$N_r = \frac{N_m}{N_p} = \frac{1200}{1500} = 0.8; \quad D_r = \frac{D_m}{D_p} = 0.5$$

$Q_r = N_r D_r^3 = (0.8) \times (0.5)^3 = 0.1$

$Q_m = Q_p \times 0.1 = 0.5 \times 0.1 = 0.05$ m^3/s

$H_r = N_r^2 D_r^2 = (0.8)^2 \times (0.5)^2 = 0.16$

$H_m = H_p \times 0.16 = 30 \times 0.16 = 4.8$ m

Prototype power = $P_p = \dfrac{\gamma_p Q_p H_p}{\eta_p} = \dfrac{1}{0.8} \times \left(\dfrac{1025}{1000} \times 9.81\right) \times 0.5 \times 30 = 188.5$ kW

Model power = $P_p = \dfrac{\gamma_m Q_m H_m}{\eta_m} = \dfrac{1}{0.8} \times \left(\dfrac{998}{1000} \times 9.81\right) \times 0.05 \times 4.8 = 2.937$ kW

Specific speed:

Prototype	Model
$N_{sq} = \dfrac{N_p \sqrt{Q_p}}{H_p^{3/4}}$	$N_{sq} = \dfrac{N_m \sqrt{Q_m}}{H_m^{3/4}}$
$N_{sq} = \dfrac{1500\sqrt{0.5}}{(30)^{3/4}} = 82.74$	$N_{sq} = \dfrac{1200\sqrt{0.5}}{(4.8)^{3/4}} = 82.74$

*EXAMPLE 5.31

**In a ¼ model of a pump, the ratio of heads in the model and prototype is 1/3. If the model pump has a brake power of 1.5 kW, estimate the prototype power by assuming the efficiency to be the same for both the model and the prototype. If this model has a speed of 1750 rpm: (a) What is the speed of the prototype? (b) What is the ratio of discharges in the model and prototype?*

Solution

Given: $H_r = 1/3 = 0.333$, $D_r = 1/4 = 0.25$, $P_m = 1.5$ kW, $N_m = 1750$ rpm

$P_r = H_r^{3/2} D_r^2 = (0.333)^{3/2} (0.25)^2 = 0.012$

$$P_p = \frac{p_m}{0.012} = \frac{1.5}{0.012} = 125\,\text{kW}$$

$$N_r = \frac{H_r^{1/2}}{D_r} = \frac{(0.333)^{1/2}}{(0.25)} = 2.308$$

(a) $N_p = \dfrac{N_m}{2.308} = 758\,\text{rpm}$

(b) $Q_r = N_r D_r^3 = 2.308 \times (0.25)^3 = 0.036$

5.18.2 Cavitation

**EXAMPLE 5.32

The NPSHR of a centrifugal pump is given by the manufacturer as 7.5 m. The pump is employed to pump water at 300 L/s from a sump whose water level is 2.05 m below the pump inlet. The atmospheric pressure at the site is read in a pressure gauge as 97.0 kPa (abs) and the vapour pressure at the relevant temperature is 2.35 kPa (abs). Total head loss in the suction pipe is estimated at 0.95 m. A margin of 2.0 m is mandatory. Determine the NPSHA and comment on the suitability of the installation against cavitation problem.

Solution

Given: $H_s = 2.05$ m, $H_{am} = 97$ kPa, $H_{vp} = 2.35$ kPa, $h_{Ls} = 0.95$ m

$NPSHR = 7.5$ m. $M = 2.0$ m

$NPSHA = H_{am} - H_s - h_{Ls} - H_{vp}$

H_{am} = Atmospheric pressure = $\dfrac{p_{atm}}{\gamma} = \dfrac{97.0}{9.79} = 9.91$ m

H_{vp} = Vapour-pressure head = $\dfrac{p_{vp}}{\gamma} = \dfrac{2.35}{9.79} = 0.240$ m

$NPSHA = 9.91 - 2.05 - 0.24 - 0.95 = 6.67$ m

Since $NPSHA < NPSHR$, the pump will have cavitation problems.

*EXAMPLE 5.33

Figure 5.55 shows schematically the installation of a centrifugal pump to pump water from a supply reservoir to a destination. The relevant data of the installation are given:

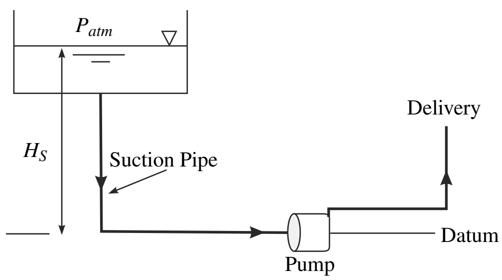


Fig. 5.55 *Example 5.33*

Height of water level in the supply reservoir above the pump = 3.0 m	Atmospheric pressure = 10.10 m
Vapour-pressure head = 0.30 m	*NPSHR* of the pump = 5.0 m
Total head loss in the suction pipe = 0.40 m	

Calculate the *NPSHA* and the safety margin available against cavitation.

Solution

Given: $H_s = 3.0$ m, $H_{am} = 10.1$ m, $H_{vp} = 0.30$, $h_{Ls} = 0.40$ m, NPSHR = 5.0 m

$NPSHA = H_{am} - H_s - h_{Ls} - H_{vp}$

$= 10.10 - 3.0 - 0.4 - 0.3 = 6.4$ m

Margin = $NPSHA - NPSHR = 6.4 - 5.0 = 1.4$ m

**EXAMPLE 5.34

A single-stage centrifugal pump has a specific speed of 30 and pumps water against a total head of 40 m. Calculate the height of the pump above the sump so that the NPSHA has a margin of 2.0 m. The atmospheric pressure and vapour-pressure head can be taken as 10.3 m and 0.33 m respectively. The head loss in the suction pipe can be taken as 10% of the suction lift.

Solution

Given: $N_{sq} = 30$, $H_m = 40$ m, $H_{atm} = 10.3$ m, $H_{vp} = 0.33$, $h_{Ls} = 0.1\ H_s$

$$\sigma_c = \frac{N_{sq}^{4/3}}{825} = \frac{(30)^{4/3}}{825} = 0.013$$

$NPSHR = \sigma_c H_m = 0.113 \times 40 = 4.52$ m

$NPSHA = H_{am} - H_s - h_{Ls} - H_{vp} = (NPSHR + M)$

$10.10 - H_s - 0.33 - 0.1\ H_s = (4.52 + 2.0)$

$1.1\ H_s = 9.97 - 6.52$

$H_s = 3.16$ m

**EXAMPLE 5.35

A centrifugal pump can deliver a discharge of 0.10 m^3/s of water to a head of 30 m. The critical cavitation number σ_c for the pump is found to be 0.12. The pump is to be installed at a location where the barometric pressure is 96.0 kPa (abs) and the vapour pressure is 3.0 kPa (abs). Assuming the intake pipe friction of 0.3 m, determine the value of NPSHR. What would be the maximum allowable elevation above the sump-water surface at which the pump can be installed? Assume a value of 2.0 m for the margin of NPSHR.

Solution

$$\sigma_c = \frac{NPSHR}{H_m}$$

$$0.12 = \frac{NPSHR}{30} \quad \text{giving } NPSHR = 3.6 \text{ m.}$$

Allowing a margin of 2.0 m, $NPSHA = NPSHR + M = 3.6 + 2.0 = 5.6$ m.

$NPSHA = H_{am} - H_s - h_{Ls} - H_{vp}$

$$H_{am} = \text{Atmospheric pressure} = \frac{p_{atm}}{\gamma} = \frac{96.0}{9.79} = 9.806\,\text{m}$$

$$H_{vp} = \text{Vapour-pressure head} = \frac{p_{vp}}{\gamma} = \frac{3.0}{9.79} = 0.306\,\text{m}$$

$NPSHA = 5.6 = (9.806 - H_s - 0.30 - 0.306)$

Maximum allowable suction lift = H_s = 3.60 m

5.18.3 Axial-Flow Pump

**EXAMPLE 5.36

An impeller of an axial flow pump has a hub diameter of 150 mm and tip diameter of 300 mm. At midradius, the inlet blade angle is 30° and the exit blade angle is 38°. The rotation speed of the impeller is 1500 rpm. Calculate the (a) discharge, (b) manometric head, (c) specific speed, and (d) brake power of the pump. Assume manometric and overall efficiencies of the pump as 0.90 and 0.85 respectively.

Solution

Given: $\beta_1 = 30°$, $\beta_2 = 38°$, $D_h = 0.15$ m, $D_t = 0.30$ m, $\eta_{man} = 0.90$ and $\eta_0 = 0.85$, $N = 1500$ rpm

$$D_m = \frac{(D_h + D_t)}{2} = \frac{(0.15 + 0.30)}{2} = 0.225\,\text{m}$$

$$\text{Area} = A = \frac{\pi}{4}((0.30)^2 - (0.15)^2) = 0.053 \text{ m}^2$$

For an axial-flow pump $u_1 = u_2 = u$

At midradius:

$$\text{Peripheral velocity } u = \frac{\pi D_m N}{60} = \frac{\pi \times 0.225 \times 1500}{60} = 17.67 \text{ m/s}$$

From inlet velocity triangle: $V_{f1} = u \tan \beta^1 = 17.67 \times \tan 30° = 10.20$ m/s

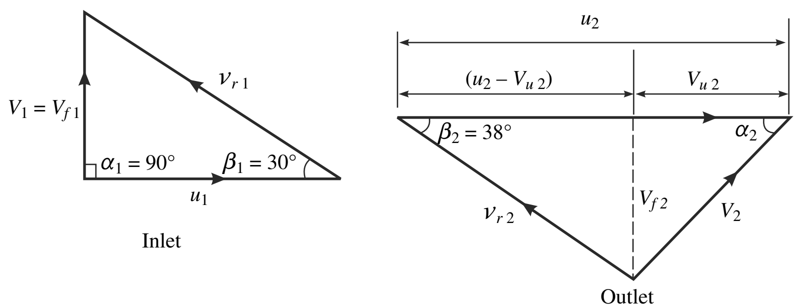


Fig. 5.56 *Velocity triangles for the axial-flow pump, Example 5.36*

(a) Discharge $Q = AV_{f1} = 0.053 \times 10.20 = 0.541$ m^3/s

At the outlet side: $\beta_2 = 38°$. $V_{f1} = V_{f2} = V_f$

$V_{u2} = u - V_f \cot \beta_2$

$$V_{u2} = 17.67 - \left(\frac{10.20}{\tan 38°}\right) = 4.615\,\text{m/s}$$

(b) Manometric head = $H_m = \eta_{ma}\dfrac{u_2 V_{u2}}{g} = \dfrac{0.90 \times 17.67 \times 4.615}{9.81} = 7.48\,\text{m}$

(c) Specific speed = $N_{sq} = \dfrac{N\sqrt{Q}}{H_m^{3/4}} = \dfrac{1500\sqrt{0.541}}{(7.48)^{3/4}} = 244$

(d) Brake power $P_s = \dfrac{\gamma Q H_m}{\eta_0} = \dfrac{9.79 \times 0.541 \times 7.48}{0.85} = 46.6\,\text{kW}$

5.18.4 System Curve and Pump-Curve Interaction

Examples such as No.5.37, No.5.42 and No. 5.43 which use graphical solution procedure are best solved using a spread sheet (such as *MS* Excel).

*EXAMPLE 5.37

A centrifugal pump running at 1200 rpm has the following variation of its prime variables:

Discharge Q (L/m)	0	1200	2400	3600	4200	4800	5500	6000	6600	7200
Head, H_m (m)	30.0	29.0	27.0	24.0	22.5	20.5	18.0	16.0	12.0	8.0
Efficiency (%)	0	35	60	80	84	85	83	78	65	45

Draw the pump (H–Q) curve and the efficiency-head curve on the same H and Q axes. Mark clearly the *BEP* and determine the specific speed of the pump.

Solution

Graphs of head against discharge and efficiency against discharge are plotted on the same x-axis as shown in Fig. 5.57. From this figure , it is seen that the maximum efficiency η_{max} is 85.0 % and occurs at Q = 4800 L/m and H = 20.5 m. The point (Q = 4800 L/m; H = 20.5 m) is thus the *BEP* and is the design point. Coordinates of this *BEP* are used for the calculation of the specific speed.

Discharge Q = 4800 L/m = $\dfrac{4800}{60 \times 1000} = 0.08\,\text{m}^3/\text{s}$

Specific speed $N_{sq} = \dfrac{N\sqrt{Q}}{H^{3/4}} = \dfrac{1200\sqrt{0.08}}{(16.0)^{3/4}} = 42.4$

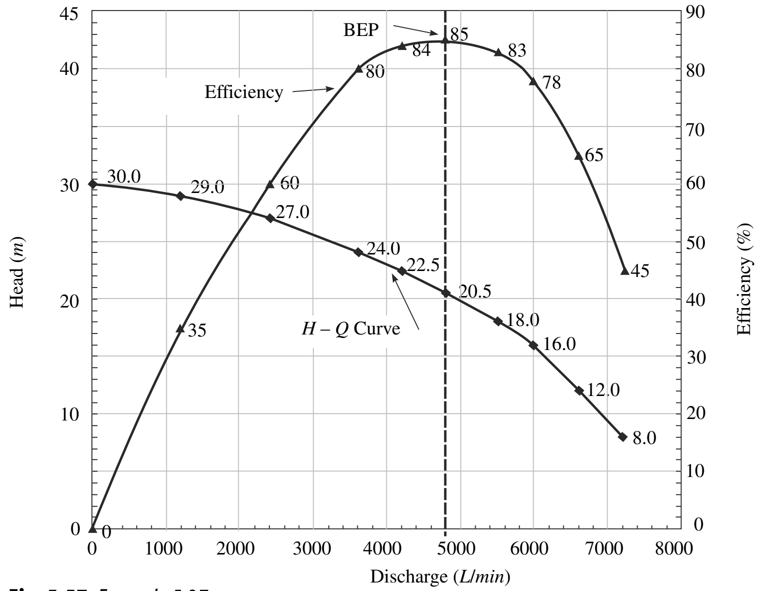


Fig. 5.57 *Example 5.37*

**EXAMPLE 5.38

A pump used to transfer water from a sump to an overhead reservoir has the following H–Q characteristic:

$$H = 23.0 + 11Q - 110Q^2$$

The system curve is given by $H = 25.0 + 90Q^2$. In these equations, H is in metres and Q is in m^3/s. Determine the operating point of the pump. What is the shut-off head of the pump?

Solution

(a) At the operating point

$H = 23.0 + 11Q - 110Q^2 = 15 + 90Q^2$

$200\,Q^2 - 11Q - 8 = 0$

Solving this quadratic equation, the discharge at the operating point $Q_o = 0.2294\ m^3/s$.

Head at the operating point, $H_o = 15 + 90 \times (0.2294)^2 = 19.74$ m

(b) At the shut-off head, $Q = 0$

Putting $Q = 0$ in the pump (H–Q) characteristic equation

$H_{\text{shut-off}} = 23.0$ m

**EXAMPLE 5.39

Following is the head–discharge characteristic of a type of centrifugal pump:

$$H = 40 - 20Q^2$$

where H = Head in metres and Q = Discharge in defined units. The system curve is given by $H = 10 + 10Q^2$

(a) *Calculate the head and discharge at the operating point when two identical pumps of the above type are connected in parallel. Calculate the ratio of* *head and ratio of discharges occurring at parallel duplex operating point and at single pump operating point.*

Solution

Refer to Fig. 5.52.

Single-pump operation curve: $H_1 = 40 - 20\,Q_1^2$

Parallel duplex operation: $Q_d = 2Q_1$

Hence $Q_1 = Q_d/2$ where Q_d = Discharge in parallel duplex operation. The duplex pump curve is $H_d = 40 - 20\left(\frac{Q_d}{2}\right)^2 = 40 - 5Q_d^2$

Systems curve: $H_{sys} = 10 + 10Q^2$

Single-pump operation: At the operating point, $Q = Q_1$ and $H_{sys} = H_1$

$$\therefore\ H_1 = 40 - 20Q_1^2 = 10 + 10Q_1^2$$

$$30Q_1^2 = 30,\ \text{hence } Q_1^2 = 1 \text{ and } Q_1 = 1.0 \text{ unit.}$$

Head $H_1 = 40 - 20 \times (1) = 20$ m

Coordinates of single-pump operating point, A : (Q_A =1.0 unit; H_A = 20 m)

Parallel duplex operation: At the operating point B, discharge $Q = Q_d$ and $H_{sys} = H_d$

$$\therefore\ H_d = 40 - 5Q_d^2 = 10 + 10Q_d^2$$

$$15Q_d^2 = 30,\ \text{hence } Q_d^2 = 2.0 \text{ and } Q_4 = \sqrt{2} = 1.41 \text{ unit.}$$

Head $H_d = 40 - 5 \times (1.41)^2 = 30$ m

Coordinates of parallel duplex pump operating point, B : (Q_B = 1.41 unit; H_B = 30 m)

Ratio of coordinates of point B and A: $\frac{Q_B}{Q_A} = \frac{1.41}{1} = 1.41,\ \frac{H_B}{H_A} = \frac{30.0}{20.0} = 1.5$

**EXAMPLE 5.40

Two pumps each having the same operating characteristics are connected in series. The pump curve is given by $H = 40 - 25Q^2$ and the system curve can be expressed as $H = 30 + 15Q^2$. In the above expressions, H = head in metres and Q = discharge in units of 100 L/s. Calculate the discharge and head at the operating point. If only one pump is working, calculate the ratio of head and ratio of discharges occurring at series duplex operating point and at single pump operating point.

Solution

Refer to Fig.5.53

Single-pump operation curve: $H_1 = 40 - 25\,Q_1^2$

Series duplex operation: $H_d = 2H_1$

Hence $H_1 = H_d/2$ where H_d = Head in series duplex operation. The series duplex pump curve is $\frac{H_d}{2} = 40 - 25Q_d^2$,

$$H_d = 80 - 50\,Q_d^2$$

Systems curve: $H_{sys} = 30 + 15Q^2$

Single-pump operation: At the operating point, $Q = Q_1$ and $H_{sys} = H_1$

$\therefore\ H_1 = 40 - 25Q_1^2 = 30 + 15Q_1^2$

$40Q_1^2 = 10,\ \text{hence, } Q_1^2 = \frac{10}{40} = 0.25$ and $Q_1 = 0.5$ unit.

Head $H_1 = 40 - 25 \times (0.5) = 27.5$ m

Coordinates of single-pump operating point, A : ($Q_A = 0.50$ unit; $H_A = 27.5$ m)

Series duplex operation: At the operating point, $Q = Q_d$ and $H_{sys} = H_d$

$\therefore H_{\text{d}} = 80 - 50\, Q_d^2 = 30 + 15\, Q_d^2$

$$65\, Q_d^2 = 50, \quad \text{hence } Q_d^2 = \frac{50}{65} = 0.769 \text{ and } Q_d = \sqrt{0.769} = 0.877 \text{ units}$$

Head $H_d = 30 + 15 \times (0.877)^2 = 41.54$ m

Coordinates of parallel duplex pump operating point, B: ($Q_B = 0.877$ unit; $H_B = 41.54$ m)

Ratio of coordinates of point B and A:

$$\frac{Q_B}{Q_A} = \frac{0.877}{0.5} = 1.754, \quad \frac{H_B}{H_A} = \frac{41.54}{27.5} = 1.51$$

***EXAMPLE 5.41

A pump is to deliver water from a tank against a total static head of 40 m. The details of suction and delivery pipes are as follows;

Suction pipe	Delivery pipe
Length = 5 m	Length = 1600 m
Diameter = 25 cm	Diameter = 20 cm
Friction factor $f = 0.022$	Friction factor $f = 0.025$

The pump characteristic is given by the equation $H = 100 - 6000\, Q^2$ where H = total pump head in metres and Q = discharge in m^3/s. Assuming an efficiency of 0.80, calculate the operating head, operating discharge and power required by the pump. [Neglect minor losses].

Solution

Static head = 40 m

$$\text{Friction head} = H_L = \frac{f_s L_s V_s^2}{2gd_s} + \frac{f_d L_d V_d^2}{2gd_d}$$

$$H_L = \frac{0.022 \times 5 \times \left(\dfrac{Q}{\frac{\pi}{4} \times (0.25)^2}\right)^2}{2 \times 9.81 \times 0.25} + \frac{0.025 \times 1600 \times \left(\dfrac{Q}{\frac{\pi}{4} \times (0.020)^2}\right)^2}{2 \times 9.81 \times 0.20}$$

$= (8.5 + 10328)Q^2 = 10336.5\, Q^2$

At the operating point, pump head = system head

$100 - 6000\, Q^2 = 40 + 10336.5\, Q^2$

$16336.5\, Q^2 = 60$

$$Q^2 = \frac{60}{16336.5} = 0.003673$$

Discharge = $Q = 0.0606$ m^3/s = 60.6 L/s

Operating head $H = 100 - 6000 \times (0.003673) = 77.96$ m

Power required $P_s = \dfrac{1}{\eta_0}\gamma QH = \dfrac{1}{0.80} \times 9.79 \times 0.0606 \times 77.96 = 57.8$ kW

***EXAMPLE 5.42

Table Ex.5.42–1 below gives the head-discharge-efficiency characteristics of a centrifugal pump. Two pumps of the above type are connected in parallel to a system. The system characteristics are given in Row–4 of the table. (a) Calculate the operating point when (i) only one pump is working, and (ii) when two pumps connected in parallel are working. (b) Calculate the brake power required in both the cases mentioned above.

Table Ex.5.42-1 *Pump Data*

Head, H (m)	22	22	21	19	17	15	12	9	6	3
Discharge, Q (L/s) of one pump	0	20	40	60	70	80	92	100	110	120
Efficiency (%)	0	33	57	76	80	81	79	74	62	30
System (H–Q) characteristics, (H_{sys} in m)	12.0	12.4	13.3	14.6	15.4	16.3	17.5	18.4	19.6	20.8

Solution

To prepare the parallel duplex curve corresponding to two pumps working in parallel, first the head-discharge table is prepared as in Table Ex.5.42-2. In this table, the rows 1, 2, 4 and 5 are the same as given in the data. The values in the third row (discharge) are twice the corresponding values of the row 2, as in parallel operation the discharge of two pumps gets added up. The head-discharge data of a single pump as given in the data along with the discharge-efficiency data of a single pump are plotted in Fig. 5.58 to a common discharge axis. The newly obtained parallel duplex pump data of head vs discharge is now plotted in the same figure and to the same scales. Next, the system curve data is now plotted on the same discharge and head axes and to the same scale on this figure.

Table Ex.5.42-2 *Data for plotting the pump curves and the system curve*

Head (m)	22	22	21	19	17	15	12	9	6	3
Discharge (in L/s) of one pump	0	20	40	60	70	80	92	100	110	120
Discharge (in L/s) of two pumps. [(Row 2) × 2]	0	40	80	120	140	160	184	200	220	240
Efficiency (%)	0	33	57	76	80	81	79	74	62	30
System (H–Q) characteristics, (H_{sys} in m)	12.0	12.4	13.3	14.6	15.4	16.3	17.5	18.4	19.6	20.8

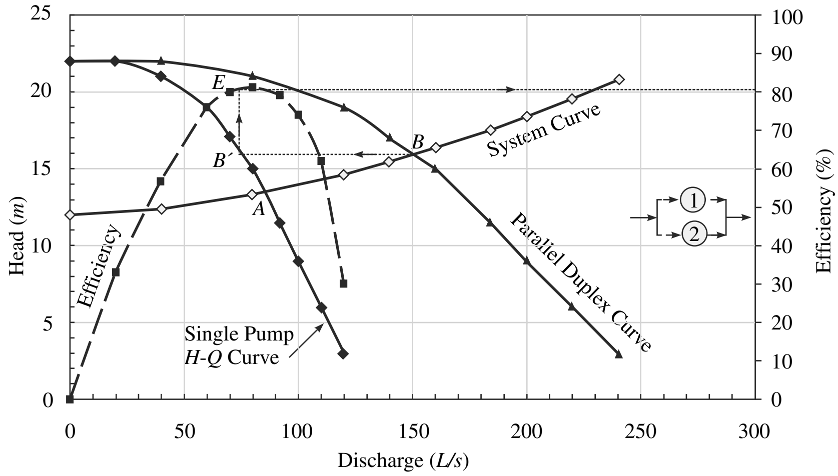


Fig. 5.58 *Two pumps in parallel–Example 5.42*

The intersection of the system curve with the single pump curve, marked as Point *A* in the figure, represents the operating point when only one pump is working. The coordinates of this point *A* are (Q_A = 86 L/s, H_A = 13.2 m, η_A = 80.1%). The intersection of the system curve with the parallel duplex pump curve, marked as Point *B* in the figure, represents the operating point when both the pumps are working in parallel. The coordinates of this point *B* are (Q_B = 151 L/s, H_B = 16.0 m, $\eta_{B'}$ = 80.5%). To obtain the efficiency of each of the two pumps working in parallel, one has to find the discharge corresponding to half of the discharge indicated by Point *B*. Note that the efficiency curve is drawn for the discharge of a single pump. Thus the efficiency of duplex pumps correspond to the discharge of (Q_B/2) = (151/2) = 75.5 L/s for each pump. From the discharge-efficiency relationship of the single pump the efficiency η_B = 80.5%. Graphically, this value can be obtained by drawing a horizontal line from the point *B* to intersect the single pump *H–Q* curve at the point *B′*. The efficiency corresponding to *B′* is obtained by drawing a vertical line through *B′* to intersect the efficiency curve at the point *E*. The procedure is shown by line with arrows in Fig. 5.58. The value of efficiency of the point *E* read on the efficiency axis is 80.5%. In this example, the duplex curve is not intersecting the single pump efficiency curve. The graphical procedure indicated above is to be adopted even if the duplex curve intersects the efficiency curve.

(iii) Brake power: $P_s = \dfrac{1}{\eta_0}\gamma QH$

- When only one pump is working: Operating point is *A*

  Brake power $P_s = \dfrac{1}{0.801} \times 9.79 \times \dfrac{86}{1000} \times 13.2 = 13.87\,\text{kW}$

- When both the pumps are working in parallel: Operating point is *B*

  Brake power $P_s = \dfrac{1}{0.805} \times 9.79 \times \dfrac{151}{1000} \times 16.0 = 29.38\,\text{kW}$

***EXAMPLE 5.43

Table Ex. 5.43-1 below gives the head-discharge-efficiency characteristics of a centrifugal pump. Two pumps of the above type are connected in series to a system. The system characteristics are given in Row 4 of the table. (a) Calculate the operating point when (i) only one pump is working, (ii) when two pumps connected in series are working. (b) Calculate the brake power required in both the cases mentioned above.

Table Ex.5.43-1 *Pump data*

Head, H (m)	22	22	21	19	17	15	12	9	6	3
Discharge (L/s) of one pump	0	20	40	60	70	80	92	100	110	120
Efficiency (%)	0	35	60	80	84	87	83	78	65	45
System (H–Q) Characteristics. H_{sys} in m	10.0	10.3	11.3	12.9	13.9	15.1	16.8	18.0	19.7	21.5

Solution

To prepare the series duplex curve corresponding to the two pumps working in series, the head-discharge table is prepared as in Table Ex. 5.43-2. In this the first four rows are the same as given in the data. The values in the fifth row are twice the corresponding values in the row 1, as in series operation, the head of two pumps gets added up. The head-discharge data of a single pump as given in the data along with the newly obtained series duplex pump data are plotted in Fig. 5.59. The discharge-efficiency data of a single pump are now plotted in Fig. 5.59 to the same discharge scale. The system curve data is plotted next on the same discharge and head axes and to the same scale on this figure.

Table 5.43-2 *Data for plotting the pump curves and the system curve*

Head, H (m)	22	22	21	19	17	15	12	9	6	3
Discharge (L/s) of one pump	0	20	40	60	70	80	92	100	110	120
Efficiency (%)	0	35	60	80	84	87	83	78	65	45
System (H–Q) characteristics. H_{sys} in m	10.0	10.3	11.3	12.9	13.9	15.1	16.8	18.0	19.7	21.5
Head (m) of two pumps in series, [(Row 1) × 2].	44	44	42	38	34	30	24	18	12	6

The intersection of the system curve with the single pump curve, marked as Point A in Fig. 5.59, represents the operating point when only one pump is working. The coordinates of this point A are (Q_A = 80.0 L/s, H_A = 15.1 m, η_A = 87%). The intersection of the system curve with the series duplex pump curve, marked as Point B in the figure, represents the operating point when both the pumps are working in series. The coordinates of this point B are (Q_B = 100.0 L/s, H_B= 16.5 m, η_B = 78.0%). Note that unlike the parallel pump case, the systems curve has the same discharge as the

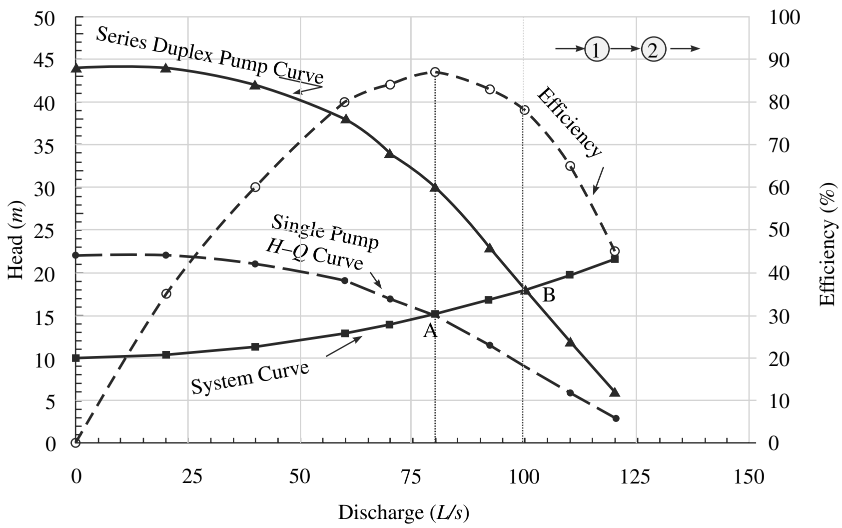


Fig. 5.59 *Example 5.43 (two pumps in series)*

individual pumps and hence there is no difficulty in estimating the efficiency of the duplex pump.

(iii) Brake power: $P_s = \dfrac{1}{\eta_0}\gamma Q H$

- When only one pump is working: Operating point is *A*

  Brake power $P_s = \dfrac{1}{0.87} \times 9.79 \times \dfrac{80}{1000} \times 15.1 = 13.59 \text{ kW}$

- When both the pumps are working in parallel: Operating point is *B*

  Brake power $P_s = \dfrac{1}{0.78} \times 9.79 \times \dfrac{100}{1000} \times 16.5 = 20.71 \text{ kW}$

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Review Questions

5.1 What are manometric head and manometric efficiency of a centrifugal pump?
5.2 With the aid of neat sketches, describe three kinds of casings adopted in centrifugal pumps.
5.3 Explain different types of energy losses encountered in centrifugal pumps.
5.4 Explain different types of efficiencies used in connection with centrifugal pumps.
5.5 Derive Euler's equation for a centrifugal pump and explain the physical significance of each of the terms in the equation.
5.6 How are pumps classified? List the further subclassifications of centrifugal pumps.
5.7 Explain with appropriate sketches the components of a typical centrifugal pump.
5.8 Draw the inlet and outlet velocity diagram of a centrifugal pump with radial inlet velocity. Label the various velocities and angles involved.
5.9 Describe the need for priming of centrifugal pumps. How is it achieved?
5.10 Distinguish between Euler head and manometric head of a centrifugal pump.
5.11 Describe a multistage centrifugal pump.
5.12 Explain why the head-discharge curve of an ideal centrifugal pump is linear and that of an actual pump is non-linear.
5.13 Explain the head-discharge characteristics of a centrifugal pump. Discuss the *slip factor*.
5.14 Discuss the main and operating characteristics of a centrifugal pump. What is the importance of constant efficiency curves?
5.15 Describe the salient features of a mixed-flow pump.
5.16 Explain the chief characteristics of an axial-flow pump.
5.17 Sketch typical performance characteristics curves of an axial flow pump. Use percentages of best efficiency, best efficiency head and best efficiency Power in the y-axis and percentage best efficiency flow in the x-axis.
5.18 Compare the operating characteristics of radial-flow, axial-flow and mixed-flow pumps.
5.19 Explain with a sketch the variation of peak efficiency of a centrifugal pump as a function of specific speed and peak discharge.
5.20 Draw a sketch showing the variation of peak efficiency of large pumps with specific speed. Mark clearly the zones of operation of radial flow, mixed flow and axial flow pumps.
5.21 Distinguish between *NPSHR* and *NPSHA*.
5.22 Explain how the operating point of a pump–pipeline system is determined when
(a) a single pump is working
(b) series duplex pumps are working
(c) parallel duplex pumps are working.
5.23 Write short notes on:
(a) Double suction pump
(b) Booster pump
(c) Priming of pumps
(d) Nondimensional specific speed of centrifugal pumps

5.24 What is the function of the following in a centrifugal pump?
(a) Foot valve
(b) Volute casing
(c) Delivery valve
(d) Priming

5.25 Write short notes on:
(a) Double-suction pump
(b) Multistage pump
(c) Effect of cavitation on the performance of a centrifugal pump

5.26 What types of energy losses occur in an impeller? Explain the nature of losses and sketch their variation with discharge.

5.27 Show in a tabular form the inter dependence of the similitude scale ratios of the following parameters for a centrifugal pump: head, length parameter, speed and discharge.

5.28 Write brief notes on
(a) Circulatory flow in inter-vane passage
(b) Minimum speed of a centrifugal pump
(c) Shutoff head of a centrifugal pump

5.29 Write a brief note on testing of centrifugal pumps in a laboratory.

5.30 Describe briefly the essential components of a standard pump testing laboratory.

Problems

Velocity Triangles and Relationships

P5.1 *Following are the values of some parameters of a centrifugal pump installation:

Static suction head = 4.0 m	Static delivery head = 15.0
Head loss in suction pipe = 2.0 m	Head loss in delivery pipe = 5.0 m
Discharge = 60 litre/s	Relative density of liquid to be pumped = 0.89
Overall efficiency = 0.82	Speed of rotation = 1200 rpm
Outer diameter of the impeller = 30 cm	

Calculate the brake power and the torque applied to the pump shaft.

[***Ans*:** P_s = 15.27 kW, T = 0.1215 kN.m]

P5.2 *A centrifugal pump is to discharge 100 L/s of water at a speed of 1450 rpm against a head of 15 m. The impeller has an outer diameter of 25 cm with a width at the outlet of 6 cm. The manometric efficiency is 0.80. Estimate the blade angle at the outlet. [***Ans*:** β_2 = 12.87°]

P5.3 ** A centrifugal pump impeller has an outer diameter of 30 cm and an inner diameter of 15 cm. The pump runs at 1200 rpm. The impeller vanes are set at a blade angle of 30° at the outlet. If the flow velocity is constant at 2.0 m/s, calculate the (a) velocity and direction of flow at the outlet, (b) head developed, by assuming a manometric efficiency of 0.85, and (c) blade angle at inlet. The flow is radial at the inlet.

[***Ans*:** (a) V_2 = 15.515 m/s, α_2 = 7.4°, (b) H_m = 25.13 m, (c) β_1 = 11.98°]

P5.4 ** A centrifugal pump is required to lift 90 litre/s of water against head of 6.0 m while running at 500 rpm. The flow velocity through the impeller is constant at 2 m/s and the manometric efficiency is 70%. The pump has forward curved blades which make an angle of 150° with the direction of motion. Calculate the (a) diameter, and (b) width of the impeller.

[***Ans*:** (i) D_2 = 0.290 m, b_2 = 0.0494 m]

P5.5 ** A centrifugal pump develops a total head of 25 m. The velocity of flow in the impeler is constant at 2.5 m/s and the blade angl at exit is 35°. The manometric efficiency is 90% and the diameter and breadth of the impeller at the outlet are 40 cm and 8 cm respectively. Calculate the (a) rotational speed of the impeller, and (b) specific speed of the pump in SI units and as Shape factor in revolutions. [***Ans*:** N = 878 rpm, N_{sq} = 39.37, S_q = 0.118]

P5.6 ** A centrifugal pump delivers 50 litre/s of water against a head of 24 m while running at 1500 rpm. The velocity of flow is 2.4 m/s and is constant . The blades are bent backwards at a blade angle of 30° at the outlet. The inner diameter is half the outer diameter. (a) Take manomeric efficiency as 0.8 and find the blade angle at the inlet. (b) Considring the overall efficiency as 0.75, estimate the brake power. [***Ans:*** β_1 = 15.04°, P = 15.66 kW]

P5.7 * A centrifugal pump runs at 800 rpm and delivers 5000 L/min against a head of 7 m. The impeller has an outer diameter of 25 cm and a width of 5 cm at the outlet. If the backward curved vane at the outlet makes an angle of 45° with the tangent to the periphery of the impeller, determine the manometric efficiency. What is the specific speed of the pump?

[***Ans*:** η_{ma} = 0.785, N_{sq} = 53.7]

P5.8 * A centrifugal pump has an impeller of 30 cm diameter whose width at the exit is 6 cm. The velocity of flow through the impeller is constant at 3.0 m/s. The impeller vanes are radial at the outer periphery. If the rotational speed of the impeller is 1000 rpm and the manometric efficiency is 80%, calculate (a) the head produced, and (b) discharge.

[***Ans*:** H_m = 20.12 m, Q = 169.6 L/s]

P5.9 ** A centrifugal pump impeller has an outer diameter which is three times the inner diameter. The pump works at 1000 rpm and develops a net head of 45 m. The velocity of flow is 2.5 m/s and is constant across the impeller. The vanes are curved backwards with an exit angle of 35°. If the outer diameter of the impeller is 45 cm and the width at the outlet is 5 cm, calculate the (a) vane angle at inlet, (b) work done by the impeller on water per second, and (c) manometer efficiency.

[***Ans*:** β_1 = 17.66°, Work done/s/unit weight = 48.0 m, η_{ma} = 0.938]

P5.10 * A centrifugal pump with an impeller of 45 cm outer diameter and 25 cm inner diameter is required to develop a net total head of 20 m. Find the lowest speed to start pumping. [***Ans***: N_m = 1011 rpm]

P5.11 * A centrifugal pump is to run at 1500 rpm and its manometric efficiency is 0.85 . What is the least diameter of the impeller that will just start delivering water at a head of 50 m, if the inside diameter is one third of the outside diameter? [***Ans***: D_{2m} = 0.423 m]

P5.12 * A centrifugal pump has a design discharge of 200 litre/s of water against a head of 10 m at a rotational speed of 600 rpm. The outer and inner diameters of the impeller are 50 cm and 25 cm respectively. Estimate the head loss at the inlet due to shock when the discharge of the pump is reduced by 45% through throttling with the speed remaining the same at 600 rpm.

[***Ans:*** h_q = 0.637 m]

P5.13 * A centrifugal pump delives 0.20 m^3/s of water to a head of 35 m at a speed of 1500 rpm. The outer diameter and width of the impeller at the outlet are 30 cm and 5 cm respectively. (a) If the manometric efficiency is 0.75, calculate the blade angle at the outlet. (b) If the impeller diameter at the inlet is 15 cm, calculate the blade angle at the inlet.

[***Ans***: β_2 = 45.75°, β_1 = 19.8°]

P5.14 * A centrifugal pump impeller rotates at a speed of 1000 rpm. The outer and inner diameters of the impeller are 0.40 cm and 0.20 m respectively. The vanes make an angle of 35° with the peripheral velocity direction at the outlet. If the velocity of flow through the impeller is constant at 1.75 m/s, find the (a) angle of vanes at the inlet, (b) absolute velocity and its direction at the outlet, and (c) manometric head of the pump. Assume manometric efficiency of the pump as 80%.

[***Ans***: (a) β_1 = 9.5°, (b) V_2 = 18.52 m/s, α_2 = 5.42°, (c) H_m = 31.5 m]

P5.15 ** A centrifugal pump running at 1200 rpm delivers at a net head of 10.0 m. At the outlet of the impeller the vane angle is 30° with the peripheral velocity direction. The impeller has an outer diameter of 25 cm and a width of 5 cm at the outlet. (a) Estimate the discharge, and (b) power required to drive the pump. Assume manometric and overall efficiency as 85% and 80% respectively. [***Ans***: Q = 190 L/s, P_s = 23.25 kW]

P5.16 ** A centrifugal pump with an impeller of 50 cm diameter at the outlet and 25 cm at inlet runs at a speed of 1000 rpm. The discharge is 150 litre/s. The width of the impeller is 8 cm at the inlet and 6 cm at the outlet. The vanes are curved back and make an angle of 25° with the peripheral velocity direction at the outlet. Assuming a manometric efficiency of 0.85 and mechanical efficiency of 0.80, calculate (a) the net total head produced by the pump, (b) brake power input to the pump, and (c) specific speed of the pump in *SI* units and as shape factor S_q (revolutions)

[***Ans***: (a) H_m = 51.65m, (b) P_s = 111.5 kW, (c) N_{sq} = 20.1, S_q = 0.0603]

P5.17 * A centrifugal pump has an impeller of outer diameter 35 cm and an inner diameter of 20 cm. The impeller vanes make angle of 35° and 30° with the peripheral velocity direction at the outlet and inlet respectively. The rotational speed of the impeller is 1000 rpm. Assuming the flow velocity to remain constant throughout the impeller, estimate the manometric head developed by the pump. Assume manometric efficiency of the pump as 0.80.

[***Ans***: H_m = 14.45 m]

P5.18 * **Multistage:**
At its normal operating point, a centrifugal pump with one stage delivers 0.3 m^3/s against a head of 30 m at a speed of 1500 rpm. At another site it is required that 0.4 m^3/s be raised over a height of 105 m by using a similar pump operating at the same speed but with multistages (one after the other). How many stages are required? [***Ans***: 3 stages]

P5.19 * A four-stage centrifugal pump has its blades radial at the outlet. The impeller diameter is 40 cm and the speed is 1000 rpm. Calculate the manometer efficiency when the pump is delivering against a total head of 40 m.

[***Ans***: η_{man} = 0.895]

P5.20*** A centrifugal pump installation to pump water has the following characteristics:

Impeller diameter at outlet = 30 cm	Width of impeller at outlet = 6 cm
Blade angle of backward facing blades = 35°	Speed = 1200 rpm
Discharge = 60 L/s	

(a) Calculate the theoretical rise in pressure head across the impeller and the velocity head of the flow leaving the impeller.(b) If a volute casing can convert 60% of the kinetic energy of the exit discharge into pressure energy, calculate the theoretical total pressure head between the outlet and inlet of the pump.

[***Ans***: (a) $\dfrac{\Delta p}{\gamma}$ =18.00 m, (b) $\dfrac{\Delta p_i}{\gamma}$ = 27.17 m.]

P5.21*** Following are some parameters of a centrifugal pump installation.

Manometer efficiency = 0.85	Angle of backward curved blades = 30°
Exit peripheral velocity = 23.0 m/s	Velocity of flow = constant = 1.5 m/s

(a) Calculate the difference in pressure head across the impeller by assuming that there is no loss in the impeller. (b) Estimate the percentage of exit flow kinetic energy recovered in the volute casing.

[***Ans***: (a) $\left(\frac{\Delta p_i}{\gamma}\right)_{theo}$ = 26.62 m, (b) 65.75%]

P5.22*** A diffuser-type centrifugal pump with an impeller diameter of 0.45 m develops a total head of 20 m. The velocity of flow in the impeller passages is constant at 2.0 m/s. The vane angle at the exit of the impeller is 35°. Assume (a) the flow enters the impeller radially, (b) manometric efficiency is 0.85, and (c) loss in the impeller is 10% of the exit velocity head. Estimate (i) the speed of the impeller, (ii) percentage of exit velocity recovered by the diffuser, and (iii) desirable direction of the diffuser vanes.

[***Ans***: N = 708 rpm, 72.48%, α_2 = 8.23°]

Axial-Flow Pump

P5.23 ** An axial-flow pump has an impeller with a hub diameter of 20 cm and tip diameter of 40 cm. At midradius, the inlet blade angle is 15° and the exit blade angle is 35°. The rotation speed of the impeller is 1450 rpm. Calculate the (a) discharge, (b) manometric head, and (c) brake power of the pump. Assume manometric and overall efficiencies of the pump as 0.87and 0.82 respectively. [***Ans***: Q = 0.575 m^3/s, H_m = 28.42 m, P_s = 195.1 kW]

Affinity Laws and Similitude

P5.24 * A centrifugal pump has an impeller of 300 mm and its capacity is 500 L/s at a speed of 1200 rpm against a total head of 15 m. Estimate the speed and head of a geometrically similar pump with impeller diameter of 200 mm and required to deliver 400 L/s. [***Ans***: N_2 = 3240 rpm, H_2 = 48.7 m]

P5.25 ** A centrifugal pump is rated to deliver 120 L/s of water at 1750 rpm under a total head of 80 m. The overall efficiency of the pump is 0.75. If the pump is to be modelled to a scale of 1:2, estimate the discharge, head and power of the model which is to run at a speed of 1250 rpm. Assume the model efficiency to be the same as that of the prototype. Verify your calculations by calculating the respective specific speed of the model and prototype.

[***Ans:*** Q_m = 12.4 L/s, H_m = 13.73 m, P_m = 2.228 kW]

P5.26 * A centrifugal pump running at 1200 rpm delivers 500 L/s of water against a total head of 25 m. Its overall efficiency is 0.80. Find the (a) discharge, (b) head, and (c) power of the same pump when its speed is changed to 1750 rpm? [***Ans***: (a) Q_2 = 0.729 m^3/s, (b) H_2 = 53.17 m, (c) P_2 = 474.5 kW]

P5.27 * It is required to determine the number of stages needed in a multistage pump to lift 3000 L/minute against a total head of 185 m. The operating speed is 1200 rpm and the specific speed of the impeller is limited to 25.

[***Ans***: 8 stages]

P5.28 * A centrifugal pump has an impeller of 35 cm diameter. It is desired to deliver a certain quantity of water to a head of 25 m. Due to a change in the project, the total head is reduced to 20 m and consequently it is proposed to reduce the impeller diameter. If the speed is to remain unchanged as before, estimate the new diameter of the impeller and percentage change in the discharge.

[***Ans***: D_2 = 31.3 cm, Q = —28.45% (reduction in discharge)]

P5.29 * Two homologous pumps *A* and *B* run at the same speed of 600 rpm. Pump *A* has an impeller of 50 cm diameter and discharges 0.4 m^3/s of water under a net head of 50 m. Determine the diameter of impeller of Pump *B* and its net total head if it is to discharge 0.3 m^3/s.

[***Ans***: D_B = 0.454 m, H_{mB} = 41.28 m]

P5.30 * A pump is required to lift 200 L/s from a deep well under a total head of 150 m. A multi-stage pump running at 1200 rpm is proposed. If the specific speed of the impellers is restricted to 20, how many stages are required in the pump? [***Ans***: 2 stages]

P5.31 ** An axial-flow pump has an impeller of 25 cm in diameter and delivers 3 m^3/s at 750 rpm against a head of 8 m. A similar pump is required to deliver

1.75 m^3/s of water when running at a speed of 900 rpm. Calculate the diameter, head and specific speed of this pump.

[***Ans*:** D_2 = 0.1965 m, H_2 = 7.12 m, N_{sq} = 272]

P5.32 ** A 15 cm centrifugal pump running at 1750 rpm delivers 100 L/s against a head of 30 m. The efficiency of the pump is estimated to be 0.83. Determine the discharge, head and power for the pump if the speed is increased to 2000 rpm. [***Ans*:** Q_2 = 0.1145 m^3/s, H_2 = 39.18 m, P_2 = 52.83 kW]

P5.33 ** (a) What is the ratio of power in the model and prototype of a ½ scale pump model, if the ratio of heads in the model and prototype is ¼?

(b) What is the corresponding discharge ratio and speed ratio?

(c) If the model has an efficiency of 0.81, what would be the efficiency of the prototype by the step-up formula of (i) Moody, and (ii) Anderson?

[***Ans*:** (a) P_r = 0.03125, (b) Q_r = 0.125, N_r = 1.0 (c) (i) Moody's η_p = 0.835 (ii) Anderson's η_p = 0.873]

P5.34 * In a ½ model of a pump, the discharge ratio is 1/5. Calculate the head ratio, speed ratio and power ratio. [***Ans*:** H_r = 0.64, N_r = 1.6, P_r = 0.128]

P5.35 ** In the following table, the suffix *r* denotes the ratio of the value of the parameter in the model to the corresponding value of the parameter in the prototype. Fill up the blanks in the table.

Diameter ratio, D_r	Speed ratio, N_r	Discharge ratio, Q_r	Head ratio, H_r	Power ratio, P_r
1/2	–	1/5	–	–
1/2	–	–	1/4	–
1/4	–	–	1/3	–
1/2	–	–	–	1.0

[Answer

Diameter ratio , D_r	Speed ratio, N_r	Discharge ratio, Q_r	Head ratio, H_r	Power ratio, P_r
½	1.6	1/5 = 0.2	0.64	0.128
½	1.0	0.125	1/4	0.03125
¼	2.309	0.1443	1/3	0.01203
½	2	0.25	1.0	1.0

P5.36 *** Pump *X* that has a diameter of 60 cm is designed to pump 0.6 m^3/s of a chemical (ρ = 825 kg/m^3) to a head of 20 m when running at a speed of 1450 rpm. This pump is to be studied in a 1:(1.5) model in the laboratory that uses fresh water (ρ = 998 kg/m^3). The model pump has a rotative speed of 1200 rpm.

(a) Calculate for the model the following parameters:(i) discharge, (ii) total head, (iii) diameter of the impeller, and (iv) the brake power. Assume an efficiency of 0.80 for both model and prototype pumps.

(b) Calculate the specific speed of the model and show that it has the same value as the specific speed of the prototype.

[***Ans*:** (a) Q_m = 0.147 m^3/s, H_m = 6.08 m, D_m = 0.40 m, P_m = 10.94 kW (b) N_s = 118.8]

Cavitation

P5.37 *A water pump has a suction lift of 6.0 m. Given that the atmospheric pressure head is 10.30 m, vapour pressure head is 0.33 m and friction losses in the suction pipe is 0.47 m, estimate the *NPSHA*. If *NPSHR* of the pump is 3.0 m and a margin of 1.0 m is mandatory, is the installation safe against cavitation? [***Ans*****:** *NPSHA* =3.50 m, Not safe]

P5.38 **The *NSPHR* for a centrifugal pump is given by the manufacturer as 6.3 m. The pump is proposed to be installed at a reservoir to lift water at the rate of 0.30 m^3/s when the water level in the reservoir is 1.5 m below the axis of the pump. The atmospheric pressure is 98.00 kPa and the vapour pressure at the site is 2.30 k Pa (abs). Assuming the total head loss in the suction pipe to be 1.70 m, examine whether the proposed installation is safe from cavitation problem. A margin of 1.5 m with respect to *NPSHR* is mandatory.

[***Ans*****:** Not OK]

P5.39 *A pump is installed at an elevation of 1.0 m below the water level in the supply reservoir. Frictional loss in the suction pipe is estimated at 0.6 m. The atmospheric pressure is 10.2 m and the vapour-pressure head is 0.43 m (abs). Estimate the margin if the *NPSHR* of this pump is 5.15 m.

[***Ans*****:** M = 5.02 m]

System Curve and Pump Curve Interaction

P5.40 *A pump lifting water to an overhead reservoir through a pipe system has following $H–Q$ characteristic:

$$H = 18.0 + 10Q - 400\ Q^2$$

The system curve is given by $H = 12.0 + 150\ Q^2$. In these equations H is in meters and Q is in m^3/s. (a) Determine the operating point of the pump. (b) If the efficiency of the pump at the operating point is 78%, calculate the power required to drive the pump.

[***Ans*****:** Q_o= 0.114 m^3/s, H_o = 13.94 m; P_p = 19.95 kW)

P5.41 **Consider the pump whose characteristics are given below. For a system curve given in the table below, determine the operating point of the pump and corresponding brake power.

System Head, H_{sys} (m)	8.0	8.4	9.3	10.9	12.5	14.5	17.0	19.3	21.9	25.6
Discharge, Q(L/minute)	0	1200	2400	3600	4200	4800	5500	6000	6600	7200
Pump Head, H_m (m)	30.0	29.0	27.0	24.0	22.5	20.5	18.0	16.0	12.0	8.0
Efficiency (%)	0	35	60	80	84	85	83	78	65	45

[***Ans*****:** OP: (Q = 5610 L/min, H = 17 m), P_s = 18.98 kW]

P5.42 **Two pumps, each having the same operating characteristics, are connected in parallel. The pump curve is given by $H = 80 - 7000\ Q^2$ and the system curve can be expressed as $H = 50 + 4700\ Q^2$. In the above expressions,

H head in metres and Q = discharge in m^3/s. (a) Calculate the operating point (i) of the two pumps when working in parallel, and (ii) of a single pump when only one pump is working. (b) If the efficiency is 75% at the operating point of the duplex operation, calculate the power input necessary to run the pumps.

[***Ans*:** (a) (Q_B = 68.2 L/s; H_B = 71.86 m), (b) (Q_A = 50.6 L/s; H_A = 62.05 m), P_s = 63.97 kW]

P5.43 * A pump is to deliver water from a sump against a total static head of 12 m. The details of suction and delivery pipes are as follows;

Suction pipe	Delivery pipe
Length = 2.5 m	Length = 400 m
Diameter = 15 cm	Diameter = 20 cm
Friction factor f = 0.018	Friction factor f = 0.020

The pump characteristic is given by the equation $H = 35 - 2200\,Q^2$ where H = total pump head in meters and Q = discharge in m^3/s. Assuming an efficiency of 0.70, calculate the operating head, operating discharge and power required by the pump. [Neglect minor losses].

[***Ans*:** Q = 0.0714 m^3/s, H = 23.78 m, P_s = 23.75 kW]

P.544 ** A centrifugal pump running at 1500 rpm has the following characteristics.

Discharge Q (L/s)	10	15.5	21.5	30.3	36.0	42.5	48.0
Total head H (m)	35.0	34.5	33.3	30.0	26.8	22.8	18.5
Efficiency (%)	60.0	67.5	72.0	73.5	71.0	66.0	60.0

Plot the pump characteristics, find the best efficiency point, and calculate the specific speed of the pump. Estimate the shut-off head of the pump.

[***Ans*:** (Q = 30.3 L/s; H = 30.0 m), η_{max} = 73.5%, N_{sq} = 20.4]

P5.45***Data relating to the head-discharge-efficiency characteristics of a centrifugal pump are given below. Two pumps of the above type are connected in series to a system. The system characteristics are given in Row 4 of the table. (a) Calculate the operating point when (i) only one pump is working, and (ii) when two pumps connected in series are working. (b) Calculate the brake power required in both the cases mentioned above.

Head (m)	20	19.5	18.6	17.4	16.2	15.0	12.5	9.5	7.5	2.5
Discharge (m^3/s) of one pump	0	25	50	75	100	120	150	180	200	250
Efficiency (%)	0	35	60	79	85	86.5	82	74	67	46
System (H–Q) characteristics, (H_{sys} in m)	10	13	17	22	28	34	43	52	58	74

[***Ans*:** (a) (i) (Q_A = 58.0 L/s, H_A = 18 m, η_A = 66%) (ii) (Q_B = 112.0 L/s, H_B = 31 m, $\eta_{B'}$ = 86.0), (b) P_{s1} = 15.49 kW, P_{s2} = 39.52 kW]

P.546***The table given below represents the head-discharge efficiency characteristics of a centrifugal pump. Two pumps of the above type are connected in parallel to a system. The system characteristics are given in Row 4 of the table. (a) Calculate the operating point when (i) only one pump is working, and (ii) when two pumps connected in parallel are working. (b) Calculate the brake power required in both the cases mentioned above.

Head, H (m)	20	19.5	18.6	17.4	16.2	15.0	12.5	9.5	7.5	2.5
Discharge, Q (L/s) of one pump	0	25	50	75	100	120	150	180	200	250
Efficiency (%)	0	34	58	74	80	81	77	69	61	34
System (H–Q) characteristics, (H_{sys} in m)	6.0	6.5	7.2	8.6	10.4	11.9	14.1	16.0	17.2	20.5

[***Ans*:** (a) (i) (Q_A = 142 L/s, H_A = 13.2 m, η_A = 79%) (ii) (Q_B = 190 L/s, H_B = 16.5 m, $\eta_{B'}$ = 79%), (b) P_{s1} = 23.22 kW, P_{s2} = 38.85 kW]

P 5.47 **The shaft power of a centrifugal pump P is expressed in Eq. 5.40 as a function of six variables in the form $P = f(D, N, gH, Q, \rho, \mu)$. In this, the notations for the variables have the usual meaning. Perform dimensional analysis of these seven variables by using Buckingham's Pi theorem method and show that

$$\frac{P}{\rho N^3 D^5} = fn\left(\frac{gH}{N^2 D^2}, \frac{Q}{ND^3}, \frac{\mu}{\rho N D^2}\right)$$

Objective-type Questions

O5.1 *Which one of the following statements is relevant to the specific speed of a centrifugal pump?
(a) Head developed is unity and discharge is unity.
(b) Head developed is unity and power absorbed is unity.
(c) Discharge is unity and power absorbed is unity.
(d) Each of head developed, power absorbed and discharge is equal to unity.

O5.2 **In a centrifugal pump, the inlet angle will be designed to have
(a) relative velocity vector in the radial direction
(b) absolute velocity vector in the radial direction
(c) velocity of flow to be zero
(d) blade angle to be 90°

O5.3 *A fast centrifugal impeller will have
(a) forward facing blades (b) radial blades
(c) backward facing blades (d) propeller-type blades

O5.4 ** In a centrifugal pump with radial entry of liquid, the manometric efficiency η_{ma} is given by

(a) $\eta_{ma} = \dfrac{u_2 V_{u2}}{gH_m}$ (b) $\eta_{ma} = \dfrac{gH_m}{u_2 V_{u2}}$

(c) $\eta_{ma} = \dfrac{u_2 V_{u2}}{2gH_m}$ (d) $\eta_{ma} = \dfrac{2gH_m}{u_2 V_{u2}}$

where H_m = Manometric head, u_2 and V_{u2} are peripheral velocity and swirl velocity at the tip of the impeller.

O5.5 ** Consider the following data for the performance of a centrifugal pump: Speed = 1200 rpm, flow rate = 30 L/s, head = 20 m and power = 5 kW. If the speed is increased to 1500 rpm, the power will be nearly equal to

(a) 6.5 kW (b) 8.7 kW (c) 9.8 kW (d) 10.9 kW

O5.6 * The most commonly used blade configuration of a centrifugal pump has blades that are

(a) bent forward (b) bent backward

(c) straight (d) wave shaped

O5.7 * If N_{sq1} is the specific speed of a centrifugal pump having large discharge at a low head and N_{sq2} is the specific speed of a centrifugal pump having relatively low discharge at a relatively high head, which of the following is true?

(a) $N_{sq1} > N_{sq2}$

(b) $N_{sq1} = N_{sq2}$

(c) $N_{sq1} < N_{sq2}$

(d) All the other three are possible under different conditions

O5.8 ** A pump running at 1000 rpm consumes 1 kW of power and generates head of 10 m of water. When it is operated at 2000 rpm, and if the overall efficiency remains the same, its power consumption and head generated would be about

(a) 4 kW and 50 m of water (b) 6 kW and 20 m of water

(c) 3 kW and 30 m of water (d) 8 kW and 40 m of water

O5.9 *** A pump and its (1/4) scale model are being compared. If the ratio of the heads is 5 : 1 then the ratio of the power consumed by the prototype and the model is

(a) 100 (b) 3.2 (c) 179 (d) 12.8

O5.10 ** A centrifugal pump lifts 0.1 m^3/s of water when operating at 1500 rpm. What is the discharge if the speed of the pump is increased to 3000 rpm?

(a) 0.8 m^3/s (b) 0.05 m^3/s (c) 0.4m^3/s (d) 0.2 m^3/s

O5.11 * A centrifugal pump is started with its delivery valve kept

(a) fully open (b) fully shut (c) partially open (d) 50% open

O5.12 ** In *MLT* system, what is the dimension of the specific speed of a rotodynamic pump?

(a) $LT^{3/2}$ (b) $L^{-3/4} T^{3/2}$ (c) $M^{1/2}L^{1/4}T^{-5/2}$ (d) $L^{3/4}T^{-3/2}$

O5.13 ** In a centrifugal pump of 0.50 m impeller diameter, running at 1500 rpm, the absolute velocity at exit is 8.0 m/s and makes an angle of 45° with the tangential direction. If the manometric efficiency is 80%, the actual total head produced is nearly

(a) 18 m (b) 14 m (c) 9 m (d) 28 m

O5.14 ** A centrifugal pump impeller has outer and inner diameters of 0.30 m and 0.15 m respectively. When running at 1450 rpm, with the discharge pipe completely shut off, the theoretical head produced is about

(a) 53 m (b) 26.5 m (c) 13.3 m (d) 106 m

O5.15 * A pump is required to deliver 150 litres per second of water at a head of 45 m when running at 1750 rpm. The specific speed of the pump in SI units is

(a) 17 (b) 39 (c) 89 (d) 1233

O5.16***The nondimensional specific speed (shape factor) of a pump is given by the relation

(a) $S_q = \dfrac{N\sqrt{Q}}{(gH_m)^{3/4}}$ (b) $S_q = \dfrac{N\sqrt{Q}}{(\rho g H_m)^{3/4}}$

(c) $S_q = \dfrac{N\sqrt{Q}}{(\rho g)^{1/2} H_m^{3/4}}$ (d) $S_q = \dfrac{N\sqrt{\rho Q}}{(gH_m)^{3/4}}$

O5.17***At a rated capacity of 4 m^3/s, a centrifugal pump develops 36 m of head when operating at 600 rpm. What is its *shape factor* (in revolutions)?

(a) 0.082 (b) 0.025 (c) 0.245 (d) 2.45

O5.18***The theoretical shut-off head of a centrifugal pump is given by

(a) $\dfrac{1}{g}(u_2^2 - u_1^2)$ (b) $\dfrac{1}{2g}(u_2^2 - u_1^2)$ (c) $\dfrac{u_1^2}{2g}$ (d) $\dfrac{u_2^2}{g}$

O5.19***A centrifugal pump with an impeller having inlet diameter of 0.30 m and outlet diameter of 0.60 m is to pump water to a head of 25 m. The minimum speed (in rpm) at which this pump would start pumping is

(a) 667 (b) 705 (c) 814 (d) 1410

O5.20 ** In an installation of a centrifugal pump, if the temperature of the liquid in the sump is raised by 20°C, the

(a) *NPSHR* is reduced (b) *NPSHR* is increased

(c) *NPSHA* is increased (d) *NPSHA* is reduced

O5.21***Two identical centrifugal pumps are connected in parallel to a common delivery pipe of a system. The pump curve of each of the pumps is represented by $H = 20 - 60\,Q^2$. The equation of the parallel duplex pump curve is

(a) $H = 20 - 60Q^2$ (b) $H = 40 - 120Q^2$

(c) $H = 10 - 30Q^2$ (d) $H = 20 - 15Q^2$

O5.22***Two identical centrifugal pumps are connected in series to feed a system. The pump curve of each of the pumps is represented by $H = 20 - 60Q^2$. The equation of the series duplex pump curve is

(a) $H = 40 - 240\,Q^2$ (b) $H = 40 - 120Q^2$

(c) $H = 10 - 30\,Q^2$ (d) $H = 20 - 15Q^2$

O5.23 ** Water is pumped through a pipeline to a height of 10 m at the rate of 0.1 m^3/s. The frictional and other losses are 5.0 m. If the efficiency of the pump is 0.75 , the power input to the pump should be

(a) 19.6 kW (b) 13.0 kW (c) 26.7 kW (d) 17.7 kW

O5.24 ***If in a pump the discharge is halved while keeping the speed unchanged, the ratio of new head H_2 to old head H_1 is

(a) $\left(\frac{1}{2}\right)^{1/2}$ (b) $\left(\frac{1}{2}\right)^{3/2}$ (c) $\left(\frac{1}{4}\right)$ (d) $\left(\frac{1}{2}\right)^{1/3}$

O5.25 *A multistage pump is used for obtaining

(a) high flow rate (b) high head

(c) high speed (d) high efficiency

O5.26 *Manometric efficiency of a centrifugal pump

(a) decreases with decrease in the blade angle at the exit

(b) increases with decrease in the blade angle at the exit

(c) is independent of the blade angle at the exit

(d) is maximum at the value of 90° for the blade angle at the exit

O5.27 ** Using of large diameter pipes in a pumping system results in the reduction in

(a) static head (b) frictional head

(c) both static head and frictional heads (d) manometric head

O5.28 ** If the diameter of an impeller is reduced it will result in

(a) flow decreases and pressure increases

(b) both the flow and pressure decrease

(c) both the flow and pressure increase

(d) Flow increases and the pressure decreases

O5.29 *For high-flow requirement , pumps are generally operated in

(a) multistage (b) series

(c) both parallel and series (d) parallel

O5.30 ***Throttling the delivery valve of a centrifugal pump results in

(a) increased head (b) decreased head

(c) increased power (d) increase in both power and head

O5.31 ** The operating point in a pumping system is identified by the point of intersection of

(a) the system curve and the efficiency curve

(b) the system curve and the pump curve

(c) the pump curve and the theoretical power curve

(d) the pump characteristics curve and the *Y*-axis

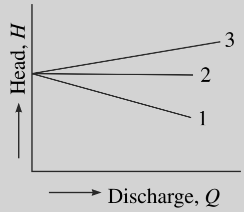
O5.32 ** If the speed of a centrifugal pump is doubled, its power consumption will increase

(a) 2 times (b) 4 times (c) 6 times (d) 8 times

O5.33 ***A centrifugal pump delivers a manometric head of 12 m when pumping a liquid of relative density 0.8. If all other factors remain the same and the liquid has a relative density of 1.2, the new manometric head would be

(a) 8.0 m (b) 10.0 m (c) 12.0 m (d) 18.0 m

O5.34 ** Match List I (outlet vane angle) with List II (Curves labeled 1, 2, and 3 in the given figure) for a pump and select the correct answer using the code given below:

List I (Outlet vane angle)	**List II (Curves labelled 1, 2, and 3 in the given figure)**
A) $\beta_2 < 90°$	
B) $\beta_2 = 90°$	
C) $\beta_2 > 90°$	

Codes:

	A	B	C
(a)	1	2	3
(b)	1	3	2
(c)	2	1	3
(d)	3	2	1

O5.35 * Consider the following statements regarding the volute casing of a centrifugal pump:

(1) Loss of head due to change in the velocity is eliminated

(2) Efficiency of the pump is increased.

(3) Water from the periphery of the impeller is collected and transmitted to the pump at constant velocity

Out of the above, the following statements are correct:

(a) 1, 2 and 3 (b) 1 and 2 (c) 2 and 3 (d) 1 and 3

O5.36 ** Consider the following statement with respect to a centrifugal pump:

If pump *NPSH* requirements are not satisfied, then

(1) sufficient head will not be developed to raise water

(2) efficiency will be lowered

(3) very low discharge will be delivered

(4) cavitation will result

Of these statements,

(a) 1, 2 and 3 are correct (b) 1 and 4 are correct

(c) 2, 3 and 4 are correct (d) All 4 statements are correct

O5.37 ** Consider the following statements pertaining to a centrifugal pump:

1. The manometric head is the head developed by the pump
2. The suction pipe has, generally, a larger diameter as compared to the discharge pipe.
3. The suction pipe is provided with a foot valve and a strainer.
4. The delivery pipe is provided with a foot valve and a strainer

Of these statements,

(a) 1, 2, 3 and 4 are correct (b) 1 and 2 are correct

(c) 2 and 3 are correct (d) 1 and 3 are correct

O5.38 *Consider the following pumps:

(1) Centrifugal pump–single stage (2) Centrifugal pump–Multistage

(3) Reciprocating pump (4) Jet pump

The pump(s) which could be used to lift water through a suction head of 12 m from a well would include

(a) 2 alone (b) 1, 3 and 4 (c) 4 alone (d) 1 and 3

O5.39 A centrifugal pump lifts 100 litres/second of a liquid against a net head of 150 kPa. The brake power required is 18.0 kW when the liquid is water having a specific weight of 9.8 kN/m^3. What would be the brake power if the liquid to be pumped is a solvent, having a relative density of 0.8, instead of water and all other factors remain the same as before?

(a) 14.4 kW (b) 18.0 kW (c) 22.5 kW (d) 17.2 kW

O5.40 Consider the following statements pertaining to a centrifugal pump:

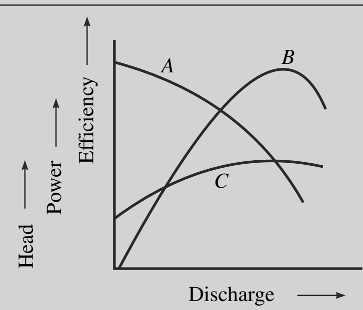
1. The discharge at the free delivery point of a pump is greater than the discharge at its BEP.
2. The efficiency is zero at the shut-off head of the pump.
3. At the BEP, the net head is at its largest value.
4. The efficiency of the pump is zero at its free delivery point.

Out of these, the correct statements are

(a) 1, 2 and 3 (b) 1,2 and 4

(c) 2, 3 and 4 (d) 1, 2, 3 and 4

O5.41 **Match List I with List II for a centrifugal pump and select the correct answer using the code given below.

List I **Title of Pump Operating Characteristic Curve**	**List II** **Curves *A*, *B* and *C***
1) Efficiency-discharge Curve	
2) Head-discharge curve	
3) Power-discharge curve	

Codes:

	A	*B*	*C*
(a)	1	2	3
(b)	1	3	2
(c)	2	1	3
(d)	3	2	1

O5.42 *A centrifugal pump has a normal discharge of 0.4 m^3/s and is working at the design speed of 750 rpm. The outer and inner diameters of the impeller are 0.6 m and 0.3 m respectively. If the discharge is reduced by 40%

by throttling while keeping the speed unchanged, the loss of head due to shock is

(a) 1.13 m (b) 1.81 m (c) 2.55 m (d) 4.52 m

O5.43 ** Figure 5.60 represents the velocity triangle at the outlet of a centrifugal pump.

With reference to this figure, select the correct answer out of the four choices given below:

The outlet blade angle of the pump β_2 has a value of

(a) 166° (b) 90° (c) 76° (d) 14°

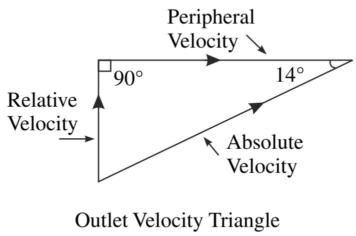


Fig. 5.60 *Objective Question 5.44*

O5.44 ** A centrifugal pump is installed at a height of 5.0 m above the sump water level. Frictional losses in the suction side of the pump amount to 0.6 m. The atmospheric pressure is 10.3 m of water and the vapor pressure is 0.4 m (abs). The *NPSHA* of the pump is

(a) 3.7 m (b) 4.0 m (c) 4.3 m (d) 4.6 m

O5.45 ** A centrifugal pump running at 500 rpm and at its maximum efficiency is delivering a head of 30 m and a flow rate of 60 liters/minute. If the rpm of tahe pump is changed to 1000 rpm then the head H in metres and the flow Q in litres/minute can be estimated to be

(a) (60, 120) (b) (120, 120) (c) (60, 480) (d) (120, 30)

SIX

Reciprocating Pumps

6.1 INTRODUCTION

6.1.1 Basic Principles and Operation

A reciprocating pump is a machine that moves a fluid by means of the reciprocating action of a piston or a plunger inside a cylinder. One-way valves facilitate the movement of the fluid from the sump to a delivery point in the pump system. Reciprocating pumps are positive-displacement pumps as each stroke of the piston displaces a finite quantity of fluid regardless of the resistance against which the pump is operating.

The piston gets its reciprocating motion either by direct connection to a reciprocating prime mover like a steam engine or through a crank and linkages to a rotary prime mover like an *IC* engine or an electric motor. Figure 6.1 (a) shows schematically a single-acting reciprocating pump. In this, the rotation of the crank by a prime mover causes the plunger to move forward or backward inside the cylinder. When the crank angle is 0° to 180°, the backward motion of the plunger, called *suction stroke*, causes a reduction of pressure inside the cylinder and the liquid from

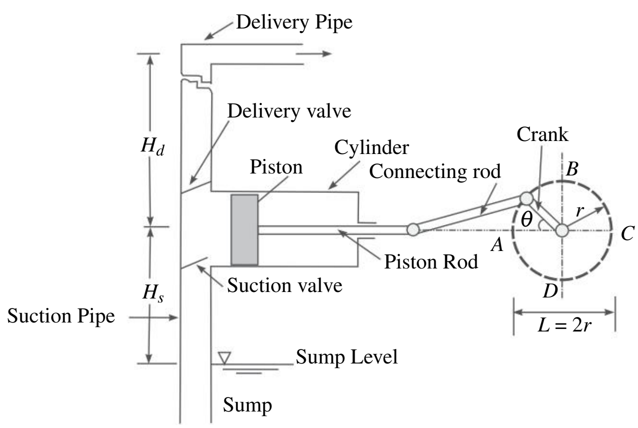


Fig. 6.1 *(a) Schematic sketch of a single-acting reciprocating pump*

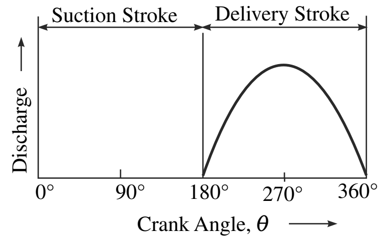


Fig. 6.1 *(b) Variation of discharge with crank angle*

the sump is pushed by atmospheric pressure to occupy the volume vacated by the plunger. The suction valve opens to admit the liquid from the sump to the cylinder while the delivery valve remains closed. In the *delivery stroke* (crank angle 180° to 360°), the suction valve gets closed and the delivery valve opens to allow the liquid displaced by the moving plunger to move up the delivery pipe. Thus, in one set of suction and delivery strokes, that is, in one full rotation of the crank, a volume of liquid equal to the area of cross section of the cylinder multiplied by the stroke length of the plunger gets displaced and the liquid moves up the delivery pipe. The suction and delivery strokes take place alternately and the variation of the discharge in the delivery pipe with the crank angle is shown schematically in Fig. 6.1 (b).

The pump described above, i.e. a pump with a single cylinder and one set of suction and delivery ports, is called a *single-acting pump*. In a *double-acting pump*, the suction and delivery strokes occur simultaneously at both ends of the cylinder due to the presence of two sets of suction and delivery ports. Figure 6.2 (a) and (b) show schematically a layout of a double-acting reciprocating pump and the variation of discharge with crank angle in such a pump. In Fig. 6.2(a), it is seen that while the front part of the cylinder perceives the backward motion of the piston as suction stroke and causes the valve S_1 to open, the rear part of the piston perceives the same motion as delivery stoke and the delivery valve D_2 will be open. The corresponding delivery pattern from the two sets of ports 1 and 2 to the common delivery pipe is shown in Fig. 6.2 (b). Comparing this with Fig. 6.1(a), it is clearly seen that the two-stroke pump acts as equivalent of two simple single-stroke reciprocating pumps with a 180° phase shift.

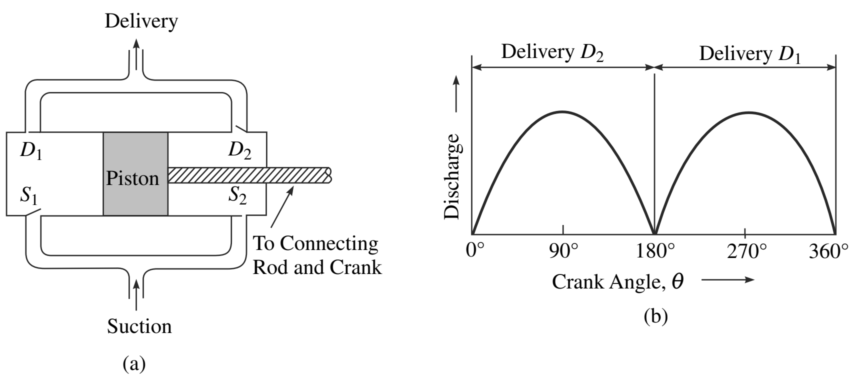


Fig. 6.2 *(a) Double-acting reciprocating pump (b) Variation of discharge with crank angle*

6.1.2 Classification

The following classifications of reciprocating pumps are commonly used in the industry.

1. Direct (Steam Driven) and Indirect Acting (Power Driven)

Some reciprocating pumps are driven directly by a prime mover (such as a steam engine) which itself has a reciprocating motion. In such a case, the piston rod of the steam engine is directly connected to the plunger. Such pump systems are called direct-acting pumps.

As often happens, many prime movers (such as *IC* engines and electric motors) have rotating shafts at the output end and these have to be connected to the reciprocating pump through connecting rods, cross heads and crank shafts. Such pumps are called *indirect-acting* or *power pumps.*

2. Single-Cylinder (Simplex), Double-Cylinder (Duplex) or Multicylinder (Multiplex) Pumps

Depending upon the number of cylinders, the reciprocating pump is classified as single cylinder (simplex), double cylinder (duplex) or multicylinder (multiplex) pump. In a single-cylinder pump, there will be only one cylinder. One-cylinder pumps are called *simple pumps*.

In a double-cylinder (*duplex*) pump, the pistons are mounted on a crankshaft of a common drive with a phase shift of 180°. (See Fig. 6.3). The suction ports of the two cylinders will be connected to a common suction pipe and outlets of the two cylinders will be connected to a common delivery pipe. This type of pump is also known as *two-throw pump*. Note that a duplex single-acting pump is equivalent to a *simple double-acting pump*. A double-cylinder pump can be double-acting also.

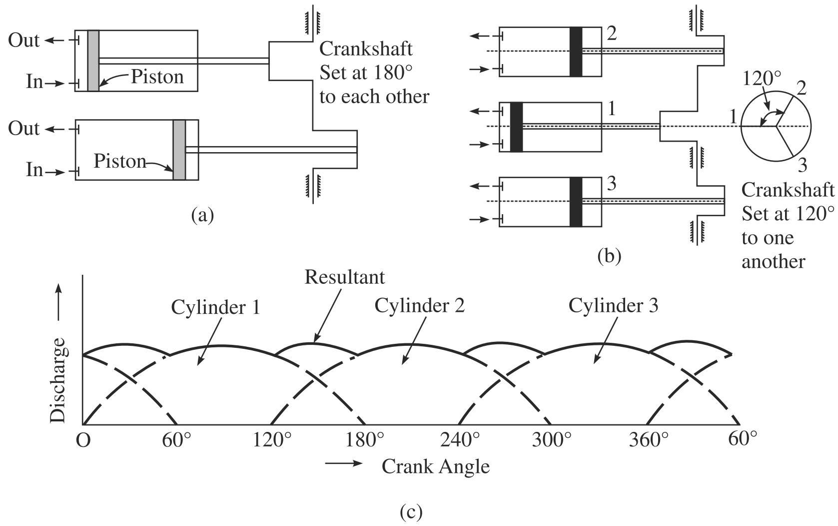


Fig. 6.3 *Multithrow reciprocating pump: (a) Two-throw pump (b) Three-throw pump (c) Variation of discharge in a three-throw pump*

Multicylinder pumping arrangement is employed to get more steady flow in the delivery pipe. In these, the cranks in a common drive provide the necessary phase shift and the outlets from a set of multicylinders are connected to a common delivery pipe. Figure 6.3 (*b* and *c*) show schematically a three-throw (triplex) pump and its discharge performance.

Double or multi cylinder pumps are used with common suction and discharge headers to minimise the fluctuations in discharge. Other than three-throw, multi-cylinder pumps employ 5, 7 and 9 cylinders. Usually the number of cylinders is kept at an odd number to minimise pressure pulsations in the system due to the action of the pistons.

3. Vertical or Horizontal

This classification depends on the mounting of the pump. The reciprocating pump can be mounted either vertically (vertical axis) or horizontally depending upon the space and other operational features.

4. Low Pressure or High Pressure

Reciprocating pumps are employed for low discharge under high pressures; the pressures ranging up to as much as 150 atm. Hence, the terms *low* and *high* are relative terms. Generally, pressures up to 50 atm are treated as low pressure for a reciprocating pump and pressures beyond that as high pressures. It is usual to use *pistons* for low-pressures pumps and *plungers* for high-pressure pumps.

Piston and Plunger In connection with reciprocating pumps, the terms *piston* and *plunger* are used interchangeably. Functionally, both piston and plunger perform the same task of displacing the liquid under the action of a driving mechanism. However, structurally, there is a difference between a piston and a plunger. A plunger is a smooth, cylindrical rod attached to a slider or rod mechanism. A stationary seal is used around the plunger and the plunger moves through the stationary seal ring. Further, because of this construction, plungers are single acting. For a double-acting mechanism with plungers, two cylinders are required. Plungers are used for high discharges, usually greater than 50 atm.

A piston consists of a cylindrical disc provided with a seal at the outer diameter. The piston (disc) is attached to a smooth rod called *piston rod* that imparts reciprocating motion at the liquid end of the piston. The seal in the piston top moves with the piston. Pistons are used for low pressures (up to about 50 atm) and relatively high flows. Further, pistons are used in both single-acting and double-acting pumps.

6.2 ANALYSIS

6.2.1 Discharge

Let L = Length of the stroke, A = Cross-sectional area of the cylinder and N = Speed of the crank in rpm.

1. Single-acting Pump

Consider a single-acting pump. In one stroke, the volume of liquid displaced = AL

$$\text{Discharge of the pump} = Q = \frac{ALN}{60} \qquad (6.1)$$

2. Double-acting Pump

In a double-acting pump, both sides of the piston will be displacing the liquid. In one revolution of the crank, volume of liquid displaced

$$V = AL + (A - A_p)L = (2A - A_p)L$$

where A_p is the area of the piston rod attached to one end of the piston to connect it to the crankshaft. The pump makes N revolutions per minute and hence

$$\text{Discharge of the pump} = Q = \frac{(2A - A_p)LN}{60} \tag{6.2}$$

If the area of the piston rod, A_p, is very small in comparison to area of the cylinder A then Eq. (6.2) can be approximated as

$$Q = \frac{2ALN}{60} \tag{6.2-a}$$

(a) Volumetric Efficiency While Eq. (6.1) and (6.2) are theoretical discharges of an ideal single-acting and double-acting pump respectively; the actual discharge of a pump may be less than the value given by the equations. This may happen for the following reasons:

- There may be leakage of the liquid through seals during the pumping operations.
- The valves may not be in perfect synchronisation with the strokes. This happens to be the major cause for slippage of fluid back past the pump valves before they can close and seal. This will vary between 1 to 5% based upon the pump speed and valve design.

The volumetric efficiency of the pump in percentage is defined as

$$\eta_v = \frac{\text{Actual discharge}}{\text{Theoretical discharge}} \times 100 = \frac{Q_a}{Q_{th}} \times 100 \tag{6.3}$$

where Q_{th} = Theoretical discharge and Q_a = Actual discharge.

(b) Coefficient of Discharge The ratio of actual discharge to theoretical discharge is known as coefficient of discharge, C_d , and thus

$$C_d = \frac{Q_a}{Q_{th}} \tag{6.4}$$

(c) Slip The difference between the theoretical discharge and the actual discharge is called the *slip* of the pump. It is generally expressed in percentage of theoretical discharge and is given by

$$\text{Percentage slip} = \varepsilon_s = \left(\frac{Q_{th} - Q_a}{Q_{th}}\right) \times 100 = (1 - C_d) \times 100 \tag{6.5}$$

It is expected that a pump in good condition will have percentage slip ε_s typically less than 3%. In some cases, the percentage slip at low speed is slightly larger than at high speeds. In such cases, the volumetric efficiency of a reciprocating pump improves as the speed increases.

(d) Negative Slip Generally, the slip of a reciprocating pump is positive and is of the order of 3%. However, in some pump set-ups, there may be some flow going into the delivery pipe without getting acted upon by the piston. In such cases, the actual discharge will be larger than the theoretical discharge and the slip ε_s will be negative. Correspondingly, the coefficient of discharge will be greater than unity. Negative slip can occur when the suction pipe is very long and the delivery pipe is relatively short. In such cases, the inertia of the suction pipe flow may be large enough to open the delivery valve before the beginning of the delivery stroke. This situation is further aggravated if the speed is also large.

6.2.2 Simple Harmonic Motion of Plunger

Following notations are used in the analysis of a reciprocating pump that follows. Refer to Fig. 6.1(a) for the definition of various components of the reciprocating pump.

A = Cross-sectional area
d_p = Diameter of piston/plunger
L = Length of stroke = Twice the crank radius (= 2 r)
r = Crank radius
H_d = Static delivery head
d_d = Diameter of delivery pipe
L_d = Length of delivery pipe
H_s = Static suction head
d_s = Diameter of suction pipe
L_s = Length of suction pipe
Q_t = Theoretical discharge
V = Average velocity in a pipe
P = Power
ω = Angular velocity of the crank
N = Rotational speed in rpm

The motion of the plunger inside the cylinder is treated as a simple harmonic motion. The crank rotates with a constant angular velocity ω radians/second. The connecting rod is considered to be very long compared to the crank radius. Since the delivery stroke and suction stroke are out of phase by 180° and while one of them (say suction) is working, the other (delivery) is shut off. It is common practice to consider the two strokes separately in the analysis. Hence, the displacement angle θ is considered to vary from 0 to 180° only for each stroke. Thus, for any stroke, the angular displacement θ is measured from the instant of commencement of that stroke. In case of suction stroke, the measurement of θ commences (with $\theta = 0$) from *inner dead centre* and ends (with θ = 180°) at the *outer dead centre*. Similarly, for delivery stroke $\theta = 0$, at the outer dead centre and θ = 180° at inner dead centre.

The time is reckoned from the inner dead centre for the suction stroke and from the outer dead centre for the delivery stroke. Suppose the crank has moved through an angle θ in time t. Then $\theta = \omega t$.

1. Velocities and Discharges

The displacement of the piston from the end of the stroke = $x = (r - r\cos\theta) = (r - r\cos\omega t)$

Velocity of piston = $v = \dfrac{dx}{dt} = \omega r \sin\omega t = \omega r \sin\theta$

If A = Area of piston and A_1 = Area of pipe (suction or delivery)

Velocity of water at any instant $v_1 = \dfrac{A}{A_1}\omega r \sin\omega t = \dfrac{A}{A_1}\omega r \sin\theta$ (6.6)

Maximum velocity in the pipe = $v_{1max} = \dfrac{A}{A_1}\omega r$ (6.7)

(a) Single-Acting Pump Time averaged velocity

$$V_1 = \frac{1}{2\pi}\int_0^{\pi}\frac{A}{A_1}\omega r \sin\theta\, d\theta = \frac{v_{1max}}{2\pi}\int_0^{\pi}\sin\theta\, d\theta$$

$$V_1 = \frac{v_{1max}}{2\pi}\left[-\cos\theta\right]_0^{x} = \frac{v_{1max}}{\pi} \qquad (6.8)$$

and thus the time averaged velocity in the pipe $V_1 = \dfrac{A}{A_1}\dfrac{\omega r}{\pi}$ (6.8-a)

Also, from Eq. 6.8, the ratio of maximum discharge (Q_{max}) to mean discharge (Q) in the delivery pipe $\dfrac{Q}{Q_{max}} = \dfrac{V_1}{v_{1max}} = \dfrac{1}{\pi}$. (6.9)

Acceleration of piston = $a = \dfrac{dv}{dt} = \omega^2 r\cos\omega t = \omega^2 r\cos\theta$

Acceleration of liquid in the pipe (suction or delivery) at any instant

$$= a_1 = \frac{A}{A_1}\omega^2 r\cos\theta \qquad (6.10)$$

Maximum acceleration of liquid in the pipe when $\cos\theta = 1.0$ and is given by

$$a_{1max} = \frac{A}{A_1}\omega^2 r \qquad (6.10\text{-a})$$

(b) Double-acting Pump For a double-acting pump, the mean velocity of flow in one complete revolution of the crank is given by

$$V_1 = \frac{2}{2\pi}\int_0^{\pi}\frac{A}{A_1}\omega r \sin\theta\, d\theta = \frac{v_{1max}}{2\pi}\int_0^{\pi}\sin\theta\, d\theta$$

$$V_1 = \frac{2v_{1max}}{2\pi}\left[-\cos\theta\right]_0^{\pi} = \frac{2v_{1max}}{\pi}$$

or

$$\frac{V_1}{v_{1max}} = \frac{2}{\pi} \qquad (6.11)$$

6.2.3 Variation of Discharge in Multithrow Pumps

By careful tracking of discharges in various cylinders at different crank angles, the flow variation in multiplex pumps can be worked out. Figures 6.1, 6.2 and 6.3 show the variation in single-, double- and three-throw pumps. Expressing the variation of discharges as ratios of average discharge, the theoretical flow variation in various commonly used multiplex pumps are represented in Table 6.1.

Table 6.1 *Variation of discharge in multiplex pumps*

Pump Type	$\frac{\text{(Maximum–Average)}}{\text{(Average)}}$	$\frac{\text{(Average–Minimum)}}{\text{(Average)}}$	$\frac{\text{(Maximum–Minimum)}}{\text{(Average)}}$
Simplex–Double-acting	60%	100%	160%
Duplex–Double-acting	24%	22%	46%
Triplex–Single-acting	6%	17%	23%
Quintuplex (5–plungers) – Single-acting	2%	5%	7%
Septuplex (7–plungers) – Single-acting	1.2%	2.6%	3.8%
Nonuplex (9–plungers) – Single-acting	0.64%	1.5%	2.14%

This table clearly shows the advantages of multiplex pumps in obtaining uniform flow in the delivery pipe. An analysis of the flow variation in double-acting multiplex pumps indicate that excepting the simplex pump, all other pumps with an odd number of pistons have the same flow variation for both single- and double-acting piston arrangement.

6.2.4 Acceleration Head

The acceleration of the fluid in the pipe requires a force F given by

$$F = \rho A_1 L_1 \left(\frac{A}{A_1} \omega^2 r \cos \theta \right)$$

The pressure head caused by this force F is

$$H_{a1} = \frac{F}{\rho g A_1} = \frac{L_1}{g} \left(\frac{A}{A_1} \right) \omega^2 r \cos\theta \tag{6.12}$$

The head H_{a1} is known as *acceleration head* or *inertial head*. This is an additional head, over and above the static and friction head that is to be developed by the pump. In the above Eq. (6.12), the suffix 1 was used to denote any pipe. Now, replacing it by the specific symbols, suffix s for suction pipe and suffix d for delivery pipe, we have the following:

1. For Suction Pipe

Acceleration head for suction pipe = $H_{as} = \frac{L_s}{g} \left(\frac{A}{A_s} \right) \omega^2 r \cos\theta$ (6.13)

Maximum acceleration head for suction pipe is at the commencement of the stroke, i.e. at $\theta = 0$, and is given by

$$H_{\text{asm}} = \frac{L_s}{g}\left(\frac{A}{A_s}\right)\omega^2 r \tag{6.14}$$

At the end of the suction stroke when $\theta = 180°$, H_{as} is minimum and

$$H_{\text{as min}} = -\frac{L_s}{g}\left(\frac{A}{A_s}\right)\omega^2 r$$

Also, at the middle of the suction stroke when $\theta = 90°$, $H_{\text{as}} = 0$

2. For Delivery Pipe

Acceleration head for delivery pipe = $H_{\text{ad}} = \dfrac{L_d}{g}\left(\dfrac{A}{A_d}\right)\omega^2 r \cos\theta$ (6.15)

Maximum acceleration head for delivery pipe is at the commencement of the stroke, i.e. is at $\theta = 0$, and is given by

$$H_{\text{adm}} = \frac{L_d}{g}\left(\frac{A}{A_d}\right)\omega^2 r \tag{6.16}$$

At the end of the delivery stroke when $\theta = 180°$, H_{ad} is minimum and

$$H_{\text{ad min}} = -\frac{L_d}{g}\left(\frac{A}{A_d}\right)\omega^2 r$$

Also, at the middle of the delivery stroke when $\theta = 90°$, $H_{ad} = 0$

6.2.5 Correction for Nonsimple Harmonic Motion

When the assumption of simple harmonic motion is not valid due to short length of connecting rod, a correction is applied to pressure head calculated by Eq. 6.13 and Eq. 6.15 by replacing the term (cos θ) by $\left(\cos\ \theta\ (\cos\ \theta + \dfrac{\cos 2\theta}{(L_c / r)}\right)$. In this L_c = Length of the connecting rod. Thus, for suction pipe under nonsimple harmonic motion situation the acceleration head for suction pipe is given by

$$H_{as} = \frac{L_s}{g}\left(\frac{A}{A_s}\right)\omega^2 r\cos\theta\left[\cos\theta + \frac{\cos 2\theta}{L_c / r}\right] \tag{6.13-a}$$

Similarly, for delivery pipe under nonsimple harmonic motion situation the acceleration head for delivery pipe is given by

$$H_{ad} = \frac{L_d}{g}\left(\frac{A}{A_d}\right)\omega^2 r\cos\theta\left[\cos\theta + \frac{\cos 2\theta}{L_c / r}\right] \tag{6.15-a}$$

6.2.6 Friction Head

The flow of liquid in the suction and delivery pipes causes a friction loss and the pump has to provide power to overcome this loss. The friction loss h_f is calculated by using the Darcy–Weisbach equation, in the same form as is used in steady flow in pipes. Thus,

$$h_f = \frac{flV^2}{2gD}$$

where f = Darcy–Weisbach friction factor, V = Average velocity in the pipe and D = Diameter of the pipe. In the present case, the time-averaged velocity in the pipe given by Eq. (6.8) is used and friction factor as given typically in Moody diagram is used.

1. For Suction Pipe

Frictional head loss $h_{fs} = \dfrac{f L_s V_s^2}{2gd_s} = \dfrac{f L_s}{2gd_s}\left(\dfrac{A}{A_s}\omega r \sin\theta\right)^2$ (6.17)

Since h_{fs} is a function of $\sin^2\theta$, the frictional head loss varies parabolically with θ. This gives the average value of head lost in friction as (2/3) times the maximum value of frictional loss of head. Maximum head loss due to friction in suction pipe, h_{fsm}, occurs at $\theta = 90°$ and its value is given by

$$h_{fsm} = \frac{f L_s}{2gd_s}\left(\frac{A}{A_s} r\omega\right)^2 \tag{6.18}$$

Thus, the average frictional head loss in suction pipe is given by

$$\text{Average } h_{fs} = h_{fsav} = \frac{2}{3} h_{fsm} = \frac{1}{3}\frac{f L_s}{gd_s}\left(\frac{A}{A_s}\omega r\right)^2 \tag{6.19}$$

2. For Delivery Pipe

For the delivery pipe, the calculation of frictional head loss is similar to that of suction pipe.

Frictional head loss $h_{fd} = \dfrac{f L_d V_d^2}{2gd_d} = \dfrac{f L_d}{2gd_d}\left(\dfrac{A}{A_d}\omega r \sin\theta\right)^2$ (6.20)

Maximum h_{fd} occurs at = 90° and its value is given by

$$h_{fdm} = \frac{f L_d}{2gd_d}\left(\frac{A}{A_d}\omega r\right)^2 \tag{6.21}$$

and the average frictional head loss in delivery pipe is given by

$$\text{Average } h_{fd} = h_{fdav} = \frac{2}{3} h_{fdm} = \frac{1}{3}\frac{f L_d}{gd_d}\left(\frac{A}{A_d}\omega r\right)^2 \tag{6.22}$$

6.2.7 Combined Effect of Static, Acceleration and Friction Heads

In a simple reciprocating pump, the pump has to produce enough head to overcome static head, friction head and acceleration head at both suction and delivery pipes. Considering the suction pipe first:

The total head in suction pipe measured as absolute pressure head is

$$H_{ts} = H_{atm} - (H_s + H_{as} + h_{fs}) \quad (6.23)$$

where H_{atm} = Atmospheric pressure (absolute),
H_s = Static suction head (see Fig. 6.1),
H_{as} = Acceleration head in suction pipe (Eq. 6.13) and
h_{fs} = Average friction head in suction pipe (Eq. 6.19)

Similarly in delivery pipe:
The total head in delivery pipe measured as absolute pressure head is

$$H_{td} = H_{atm} + (H_d + H_{ad} + h_{fd}) \quad (6.24)$$

where H_{atm} = Atmospheric pressure (absolute),
H_d = Static delivery head (see Fig. 6.1),
H_{ad} = Acceleration head in delivery pipe (Eq. 6.15), and
h_{fd} = Average friction head in delivery pipe (Eq. 6. 22).

It is to be noted that while the static heads H_s and H_d remain constant for a particular installation of the pump, the other two terms, namely the acceleration head and friction head, are functions of time, $\theta = \omega t$, and vary from instant to instant. The salient features of the total suction and total delivery pressure heads at three values of θ are summarized in Table 6.2.

Table 6.2 *Salient features of total suction and delivery pressure heads*

Item	Total Pressure Head in Suction Pipe $H_{ts} = H_{atm} - (H_s + H_{as} + h_{fs})$ (absolute)	Total Pressure Head in Delivery pipe $H_{td} = H_{atm} + (H_d + H_{ad} + h_{fd})$ (absolute)
Beginning of stroke ($\theta = 0$)	$h_{fs} = 0$	$h_{fd} = 0$
Midstroke, ($\theta = 90°$)	$H_{as} = 0$	$H_{ad} = 0$
End of stroke ($\theta = 180°$)	$h_{fs} = 0$	$h_{fd} = 0$

6.3 INDICATOR DIAGRAM

6.3.1 Indicator Diagram for Ideal Pump (No Acceleration and No Friction)

An *indicator diagram* of a reciprocating pump is a plot showing the variation of pressure in the cylinder with the displacement of the piston at various stages of the piston strokes. The area of the indicator diagram represents the work done by the pump per unit weight of liquid in one complete revolution of the crank. Figure 6.4 shows a theoretical indictor diagram of an ideal reciprocating pump; a pump with no acceleration head and no friction losses. The ordinate represents the pressure heads on the plunger and the abscissa the displacement of a plunger of unit length. The horizontal line 0-0 at the bottom of the figure represents the absolute zero pressure head, which is taken as the datum for pressure heads. Line *A*-*B* represents the atmospheric pressure, H_{atm}. Line *jk* represents the suction stroke; point *j* representing $\theta = 0°$ and point *k* representing $\theta = 180°$ of the suction stroke. The suction head developed at the cylinder inlet is H_s at location *j* and remains constant all along the stroke till the end point *k*. The suction head H_s is vacuum pressure and hence is shown below the atmospheric pressure line *AB*.

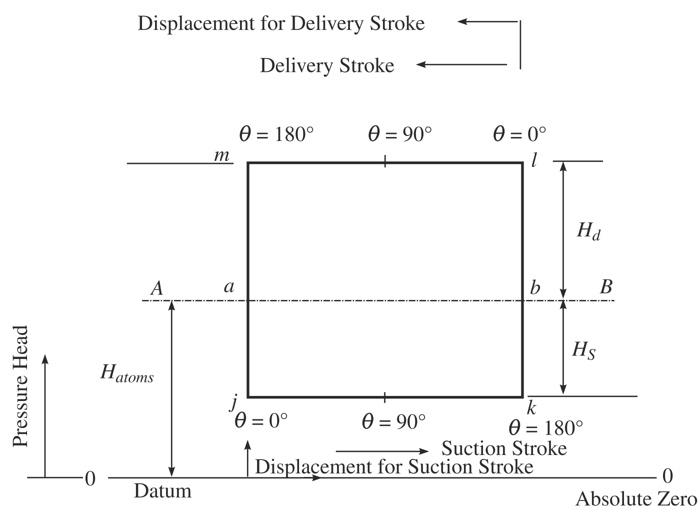


Fig. 6.4 *Theoretical indicator diagram of an ideal pump*

Point *l* marks the beginning of the delivery stroke with $\theta = 0°$. At that point, the pressure head in the cylinder is H_d and the head developed remains constant till the end of the delivery stroke at *m* at which point $\theta = 180°$. The pressure head H_d is above atmospheric pressure and is marked accordingly in the figure.

The stroke length, which is equal to the length of the plunger is taken as of unit dimension.

Area (*jkba*) = H_s = Work done per unit weight of fluid in a suction stroke. Similarly,

Area (*ablm*) = H_d = Work done per unit weight of fluid in delivery stroke.

Hence, total work done per unit weight of liquid = Area (*jkba*) + Area (*ablm*)

$$= \text{Area } (jklm) = (H_s + H_d)$$

If the area of the cylinder is *A* and the stroke length is *L* then

Weight of liquid pumped in one revolution of crank = γAL

Work done per revolution of crank = $\gamma AL\,(H_s + H_d)$

If the speed is *N* = Revolutions per minute,

$$\text{Theoretical work done per second} = P_t = \frac{\gamma ALN}{60}(H_s + H_d) \quad (6.25)$$

For a given pump with a speed N, the quantity $\dfrac{\gamma ALN}{60}$ is constant and hence the area of the indicator diagram is proportional to the work done by the pump per second, that is, the power delivered by the pump to the liquid being pumped.

6.3.2 Indicator Diagram with Acceleration Effect and No Friction

Consider a single-acting simplex pump with acceleration effect but without friction.

1. Suction Stroke

Equation 6.13 gives the acceleration in the suction pipe at any instant as:

Acceleration head for suction pipe = $H_{as} = \dfrac{L_s}{g}\left(\dfrac{A}{A_s}\right)\omega^2\, r\cos\theta$

Maximum acceleration head for suction pipe is at $\theta = 0°$, that is at the beginning of the stroke, and is given by $H_{asm} = \dfrac{L_s}{g}\left(\dfrac{A}{A_s}\right)\omega^2\, r$

The variation of H_{as} with displacement $x = (r - r\cos\theta)$ is linear and has a maximum value of H_{asm} at $\theta = 0°$.

At the end of the stroke when $\theta = 180°$, H_{as} is minimum and represents the deceleration effect of the piston. It is given by $H_{as\,\min} = \dfrac{L_s}{g}\left(\dfrac{A}{A_s}\right)\omega^2\, r$

Also, at the middle of the stroke when $\theta = 90°$, $H_{as} = 0$. The acceleration head is added to the suction head H_s at the beginning of the stroke and at $\theta = 180°$, $H_{as\min}$ being the deceleration effect is deducted from the suction head H_s. The resulting variation of the net suction pressure head, inclusive of acceleration head, is shown by line (pq) in Fig. 6.5. In this line, (jp) is the acceleration head H_{as} deducted from H_s. Line (kq) is the deceleration effect subtracted from H_s. Thus the revised indictor diagram for the suction stroke that includes the acceleration effect is ($apqb$). Note that the triangular portions (jpp_1) and (p_1kq) are identical and hence are of equal area.

Hence, Area ($apqb$) = Area ($ajkb$) = $(H_s) \times 1 = H_s$

= Work done per unit weight in suction stroke

Thus, the acceleration effect does not change the total work done in the suction stroke, but only alters the distribution of pressure heads at various displacements of the suction stroke.

2. Delivery Stroke

The column of water in the delivery pipe will be accelerated at the beginning and in the first half of the delivery stroke. It will be decelerated at the second half of the stroke until the end of the stroke. The acceleration head for delivery pipe is given by Eq. 6.15 as

$$H_{ad} = \frac{L_d}{g}\left(\frac{A}{A_d}\right)\omega^2\, r\cos\theta$$

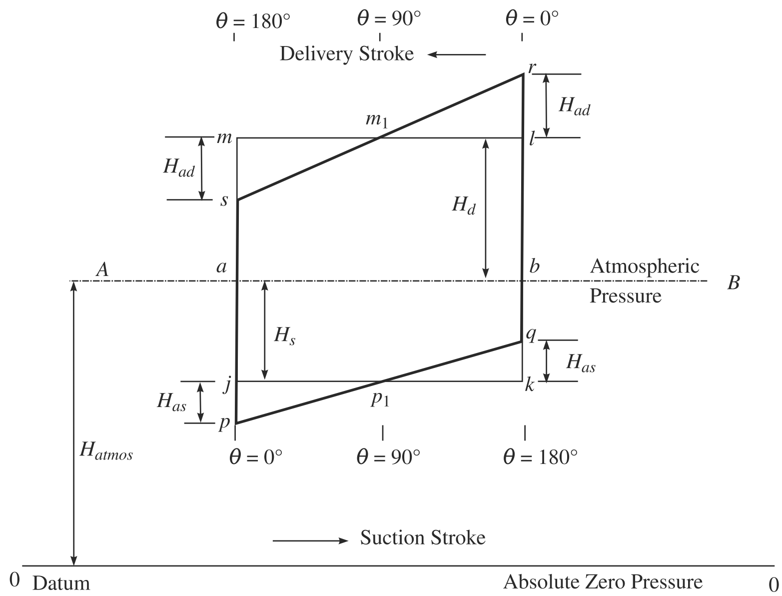


Fig. 6.5 *Indicator diagram showing effect of acceleration head*

H_{ad} is maximum at the beginning of the delivery stroke, i.e. at $\theta = 0$, and the maximum value is given by Eq. (6.16) as

$$H_{adm} = \frac{L_d}{g}\left(\frac{A}{A_d}\right)\omega^2\, r$$

This is indicated by line (*lr*) in the indicator diagram shown in Fig. 6.5.

At the end of the stroke when $\theta = 180°$, H_{ad} is minimum with a value of $H_{ad\,\min}$ $= -\frac{L_d}{g}\left(\frac{A}{A_d}\right)\omega^2\, r$. This is indicated by line (*ms*) in the indicator diagram.

Also, at the middle of the stroke when $\theta = 90°$, $H_{ad} = 0$ and this indicated by the point (m_1). The rectified indicator diagram of delivery stroke accounting for acceleration effect is the area (*abrs*). As in the suction stroke, the two triangular areas, viz. Δ (lrm_1) and Δ (msm_1) are identical and hence are of equal area.

Hence, Area (*abrs*) = Area (*ablm)*, indicating that the acceleration does not change the total work done in the delivery stroke but only alters the distribution of the pressure heads at various displacements of the delivery stroke.

Considering both the suction and delivery strokes together, the total work done per unit weight of the liquid pumped in one complete revolution of the crank is

= Area (*pqblrsa*) = ($H_s + H_d$).

Further, the area (*pqblrsa*) represents the indicator diagram that accounts for the acceleration effect of a single cylinder, single-stroke reciprocating pump without friction.

6.3.3 Indicator Diagram Considering both Acceleration and Friction Effects

1. Suction Stroke

When the friction effects in the suction pipe along with the acceleration effects are considered the additional head loss due to frictional effects has to be superimposed on the indicator diagram of Fig. 6.5. The frictional head loss in the suction pipe is given by Eq. (6.17) as

$$h_{fs} = \frac{f L_s V_s^2}{2gd_s} = \frac{f L_s}{2gd_s}\left(\frac{A}{A_s}\omega r \sin\theta\right)^2$$

Since h_{fs} is a function of $\sin^2\theta$, the frictional head loss varies parabolically with θ. Further, the average value is (2/3) times the maximum value. The maximum h_{fs} occurs at $\theta = 90°$ and its value is given by Eq. (6.18) as:

$$h_{fsm} = \frac{f L_s}{2gd_s}\left(\frac{A}{A_s}\omega r\right)^2$$

The average frictional head loss in suction pipe is given by Eq. (6.19) as

$$\text{Average } h_{fs} = h_{fsav} = \frac{2}{3}h_{fsm} = \frac{1}{3}\frac{f L_s}{gd_s}\left(\frac{A}{A_s}\omega r\right)^2$$

At the beginning of the suction stroke ($\theta = 0°$) and at the end of the suction stroke ($\theta = 180°$), the frictional loss h_{fs} is zero. The maximum ordinate is h_{fsm} and occurs at the midsuction stroke, i.e. at point *e* (of Fig. 6.6) corresponding to $\theta = 90°$. Thus, at the beginning of the suction stroke, the total suction head is ($H_s + H_{as}$) and at the

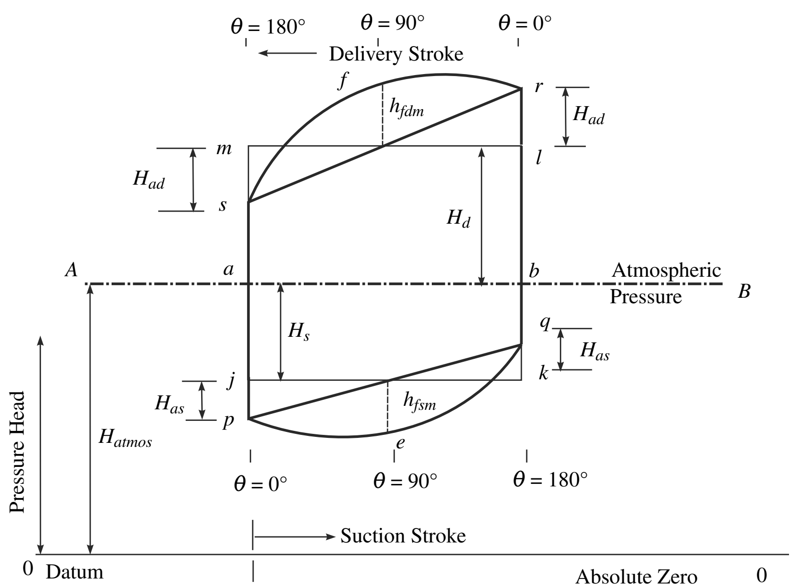


Fig. 6.6 *Indicator diagram showing the effect of acceleration and friction*

end of the suction stroke it is ($H_s - H_{as}$). The equation for h_{fs} being a parabola, if the frictional head is added to the suction portion of the indicator diagram of Fig. 6.5, the indicator diagram for the suction side with friction is obtained as the curve *peq* of Fig. 6.6. Note that the curve *peq* of Fig. 6.6 is also a parabola and its ordinates are calculated by Eq. (6.17) and marked vertically over the inclined line *pq*.

Thus, the frictional loss curve starts from the point *p* of Fig. 6.6 and varies parabolically along the displacement axis to end at the point *q*. The maximum ordinate is h_{fsm} and occurs at the mid- suction stroke, i.e. at point *e* corresponding to $\theta = 90°$. The friction effect is felt as the parabolic curve *peq* of Fig. 6.6, for displacements between the two end points *p* and *q*.

The area of the parabolic curve (*peq*) = $\frac{2}{3}(h_{fsm}) = h_{fsa}$

The total work done in the suction stroke, per unit weight of liquid pumped, is the Area (*apeqb*) = $H_s + \frac{2}{3}(h_{fsm})$

2. Delivery Stroke

The head loss due to friction in the delivery pipe is given by Eq. (6.20) as:

$$h_{fd} = \frac{f L_d V_d^2}{2gd_d} = \frac{f L_d}{2gd_d}\left(\frac{A}{A_d}\omega r \sin\theta\right)^2$$

Since h_{fd} is a function of $\sin^2\theta$, the frictional head loss in delivery stroke varies parabolically with θ. Further, the average value is (2/3) times the maximum value. The maximum h_{fd} occurs at $\theta = 90°$ and its value is given by Eq. (6.21) as

$$h_{fdm} = \frac{f L_d}{2gd_d}\left(\frac{A}{A_d}\omega r\right)^2$$

and the average frictional head loss in delivery pipe is given (Eq. 6.22) as

$$\text{Average } h_{fd} = h_{fdav} = \frac{2}{3}h_{fdm} = \frac{1}{3}\frac{f L_d}{gd_d}\left(\frac{A}{A_d}\omega r\right)^2$$

The indicator diagram for the delivery stroke will have the extra head h_{fd} added to the indicator diagram of Fig. 6.5. The resulting indicator diagram for the delivery pipe is shown in Fig. 6.6. The parabola (*rfs*) reflects the frictional effect. The maximum of this parabola occurs at point *f* corresponding to $\theta = 90°$ of the delivery stroke.

The area of the parabolic curve (*rfs*) = $\frac{2}{3}$ (h_{fdm}) = h_{fdav}

The total work done in the delivery stroke, per unit weight of liquid pumped, is the Area (*apeqb*) = $H_d + \frac{2}{3}$ (h_{fdm})

Theoretical Work Done Per Second The total work done per unit weight of liquid pumped in one complete rotation of the crankshaft is the sum of areas of indicator diagrams of suction and delivery strokes and is given by the area represented by (apeqbrfs) of Fig. 6.6. This is equal to

$$\left[H_s + \frac{2}{3}(h_{fsm}) + H_d + \frac{2}{3}(h_{fdm})\right].$$

For a single-acting, single-cylinder, reciprocating pump having a cylinder of area A and stroke length L,

Theoretical work done in one revolution of the crank is

$$= \gamma AL\left[H_s + H_d + \frac{2}{3}(h_{fsm}) + \frac{2}{3}(h_{fdm})\right]$$

If N = Speed of the pump in revolutions per minute,

Total theoretical work done in one second

$$= P_t = \frac{\gamma ALN}{60}\left[H_s + H_d + \frac{2}{3}(h_{fsm}) + \frac{2}{3}(h_{fdm})\right] \quad (6.26)$$

This represents the theoretical power expended in pumping the liquid through a total lift of $(H_s + H_d)$. The actual power expended is $P = P_t/\eta_0$ where η_0 is the overall efficiency of the pump.

In a double-acting pump, the weight of water pumped per second is = $\frac{2\gamma ALN}{60}$.

This is by considering the diameter of piston rod to be too small in comparison to the diameter of the cylinder. Hence, in a double-acting pump, the total theoretical work done in one second is

$$P_{2t} = \frac{2\gamma ALN}{60}\left[H_s + H_d + \frac{2}{3}(h_{fsm}) + \frac{2}{3}(h_{fdm})\right] \quad (6.27)$$

6.4 ILLUSTRATIVE EXAMPLES—SET 6.1

NOTE

In the following problems, the default liquid involved in the pumping operation is water, with unit weight $\gamma = 9.79$ kN/m^3. Frictional losses are calculated by Darcy–Weisbach formula, $h_f = \frac{flv^2}{2gd}$

*EXAMPLE 6.1

A single-acting reciprocating pump has a 15 cm diameter piston with a crank of 15 cm radius. The delivery pipe is of 10 cm diameter. At a speed of 60 rpm, a discharge of 310 litres/ minute of water is lifted to a total height of 15 m. Find the slip, coefficient of discharge and theoretical power in kW required to drive the pump.

Solution

Cross-sectional area of cylinder = $A = \frac{\pi}{4} \times (0.15)^2 = 0.01767$ m^2

$L = 2\ r = 2 \times 0.15 = 0.30$ m; $N = 60$ rpm

Theoretical discharge $Q_t = \dfrac{ALN}{60} = \dfrac{0.01767 \times 0.30 \times 60}{60} = 0.0053$ m^3/s

$= 0.31806$ m^3/min = 318.06 litres/minute

Actual discharge $Q_a = 310$ litres/min

Slip = $\dfrac{Q_t - Q_a}{Q_t} = \dfrac{318.06 - 310.0}{318.06} = 0.0253 = 2.53\%$

Coefficient of discharge = $C_d = \dfrac{310.0}{318.06} = 0.975$

Total head = $H_t = (H_s + H_d) = 15$ m

Theoretical power = $P_t = \gamma Q_t H_t = 9.79 \times 0.0053 \times 15 = 0.778$ kW

*EXAMPLE 6.2

A three-throw reciprocating pump has a 20 cm diameter pump and a stroke of 40 cm. The pump is required to deliver 5000 litres/minute of water at a static head of 80 m. Friction losses can be taken as 2.0 m in suction pipe and 18 m in delivery pipe. Assume a slip of 2% and pump efficiency of 90%. The velocity head in the delivery can be neglected as too small. Determine the speed of the pump and the power required.

Solution

Cross-sectional area of cylinder = $A = \dfrac{\pi}{4} \times (0.20)^2 = 0.03142$ m^2

Stroke length $L = 0.40$ m

Actual discharge $Q_a = \dfrac{5000}{60 \times 1000} = 0.08333$ m^3/s

Theoretical discharge $Q_t = \dfrac{3ALN}{60} = \dfrac{3 \times 0.03142 \times 0.40 \times N}{60} = 0.0006283$ N

Slip = $0.02 = \dfrac{Q_t - Q_a}{Q_t} = \dfrac{0.0006283\,N - 0.0833}{0.0006283\,N}$

$N = 135.3$ rpm

Total head = $H_t = (H_s + H_d) + (h_{fs} + h_{fd})$

$= 80 + (2.0 + 18.0\ ) = 100.0$ m

Power required = $P = \dfrac{\gamma Q_a H_t}{\eta_0} = \dfrac{9.79 \times 0.08333 \times 100}{0.90} = 90.64$ kW

*EXAMPLE 6.3

A double-acting reciprocating pump has a piston of 30 cm diameter with a piston rod of 6.0 cm. The length of the stroke is 36 cm and the speed of the crank is 30 rpm. The *suction and delivery heads are 5.0 m and 15.0 m respectively. (a) If the percentage slip is 2%, determine the actual discharge from the pump. (b) If the overall efficiency of the pump is 90% and the liquid to be pumped has a specific weight of 8.75 kN/m³, determine the power required to be supplied to the pump.*

Solution

Cross-sectional area of cylinder = $A = \frac{\pi}{4} \times (0.30)^2 = 0.07069\ \text{m}^2$

$$A_p = \frac{\pi}{4}(d_p)^2 = \frac{\pi}{4}(0.06)^2 = 0.00283\ \text{m}^2$$

$L = 0.36$ m; $N = 30$ rpm

Theoretical discharge

$$Q_t = \frac{(2A - A_p)LN}{60} = \frac{[(2\times 0.07069) - 0.00283]\times 0.36 \times 30}{60} = 0.0249\ \text{m}^3/\text{s}$$

If the slip is 2%, actual discharge is $Q_a = Q_1\left(1 - \frac{\varepsilon_s}{100}\right) = 0.0249 \times (1.0 - 0.02) = 0.0244\ \text{m}^3/\text{s}$

Total head = $H_t = (H_s + H_d) = 5 + 15 = 20$ m

Power required = $P = \frac{\gamma Q_a H_t}{\eta_0} = \frac{1}{0.90} \times 8.75 \times 0.0244 \times 20 = 4.74$ kW

**EXAMPLE 6.4

A single-acting reciprocating pump has a 25 cm cylinder with a stroke of 40 cm. The diameters of suction and delivery pipes are 15 cm and 20 cm respectively. If the crank makes 40 revolutions per minute, estimate the maximum velocity and acceleration of water in the suction and delivery pipes.

Solution

Angular velocity $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 40}{60} = 4.189$ rad/s

Radius of crank = $r = 0.40/2 = 0.20$ m

$$\left(\frac{A}{A_s}\right) = \left(\frac{25}{15}\right)^2 = 2.778 \text{ and } \left(\frac{A}{A_d}\right) = \left(\frac{25}{20}\right)^2 = 1.5625$$

Calculations of velocity and acceleration are shown in the table given below:

Item	Suction Pipe	Delivery Pipe
Maximum Velocity	$v_{sm} = \left(\frac{A}{A_s}\right)\omega r = 2.778 \times 4.189$ $\times 0.20 = 2.327$ m/s	$v_{dm} = \left(\frac{A}{A_d}\right)\omega r = 1.5625 \times 4.189$ $\times 0.20 = 1.309$ m/s

Maximum Acceleration	$a_{sm} = \left(\dfrac{A}{A_s}\right)\omega^2 r = 2.778 \times (4.189)^2 \times 0.20 = 9.749\ \text{m/s}^2$	$a_{dm} = \left(\dfrac{A}{A_d}\right)\omega^2 r = 1.5625 \times (4.189)^2 \times 0.20 = 5.484\ \text{m/s}^2$

**EXAMPLE 6.5

In a process industry, glycerine (ρ = 1260 kg/m^3) is pumped by using a single-acting reciprocating pump. The relevant details of the installation are as below:

Diameter of the piston = 20 cm	*Diameter of delivery pipe = 10 cm*
Length of stroke = 40 cm	*Length of delivery pipe = 60 m*
Crank speed = 30 rpm	*Static delivery head = 50 m*
Darcy-Weisbach friction factor f = 0.02	

Estimate the pressure in kPa on the piston at the (a) beginning, (b) middle, and (c) end of the delivery stroke.

Solution

Angular velocity $\omega = \dfrac{2\pi N}{60} = \dfrac{2\pi \times 30}{60} = 3.1416$ rad/s

Radius of crank $= r = 0.40/2 = 0.20$ m.

$$\left(\frac{A}{A_d}\right) = \left(\frac{20}{10}\right)^2 = 4.0$$

Delivery stroke:

Acceleration head: $H_{ad} = \dfrac{L_d}{g}\left(\dfrac{A}{A_d}\right)\omega^2\, r\cos\theta$

$$= \frac{60}{9.81} \times 4 \times (3.1416)^2 \times 0.20 \times \cos\theta = 48.29\cos\theta$$

Friction head: $h_{fd} = \dfrac{f\,L_d}{2gd_d}\left(\dfrac{A}{A_d}\omega r\sin\theta\right)^2$

$$h_{fd} = \frac{0.02 \times 60}{2 \times 9.81 \times 0.10} \times (4 \times 3.1416 \times 0.2 \times \sin\theta)^2 = 3.863\sin^2\theta$$

Pressure head on the piston

$$H_{td} = H_{\text{atmos}} + (H_d + H_{ad} + h_{fd}) = (H_d + H_{ad} + h_{fd}) \text{ (Gauge pressure)}$$

$$= 60.0 + (H_{ad} + h_{fd})$$

$$= 60.0 + (48.29\cos\theta + 3.863\sin^2\theta)$$

Values of H_{td} calculated for various values of θ are shown in the table given below. Note that the pressure head as well as gauge pressures in kPa at the cylinder during delivery stroke are calculated.

Unit weight of liquid = $\gamma = \rho g = 1260 \times 9.81 = 12360.6$ N/m^3 = 12.36 kN/m^3.

Pressure head on the piston during delivery stroke:

Condition	θ	H_{ad} (m)	h_{fd} (m)	Total pressure head H_{td} (gauge) in metres of liquid	Total pressure p_p (gauge) ($p_p = \gamma H_{td}$)
Beginning of stroke	0°	48.29	0	108.29 m	1338.5 kPa
Midstroke	90°	0	3.863	63.863 m	789.3 kPa
End of stroke	180°	–48.29	0	11.71 m	144.7 kPa

***EXAMPLE 6.6

A single-acting reciprocating pump used for lifting water has the following data:

Cylinder diameter = 10 cm	Stroke = 25 cm
Static suction head = 4.0 m	Crank speed = 30 rpm
Diameter of suction pipe = 5.0 cm	Delivery-pipe diameter = 5.0 cm
Length of suction pipe = 6.0 m	Length of delivery pipe = 25 m
Darcy–Weisbach friction factor = f = 0.02	Static delivery head = 16.0 m

Estimate the pressure head on the piston at the (a) beginning, (b) middle, and (c) end of the suction and delivery strokes. Assume atmospheric pressure = 10.0 m of water.

Solution

Angular velocity $\omega = \dfrac{2\pi N}{60} = \dfrac{2\pi \times 30}{60} = 3.1416$ rad/s,

Radius of crank = r = 0.25/2 = 0.125 m. $\left(\dfrac{A}{A_s}\right) = \left(\dfrac{10}{5}\right)^2 = 4.0$ and $\left(\dfrac{A}{A_d}\right) = \left(\dfrac{10}{5}\right)^2 = 4.0$

Suction stroke:

Acceleration head: $H_{as} = \dfrac{L_s}{g}\left(\dfrac{A}{A_s}\right)\omega^2 r \cos\theta$

$$= \frac{6.0}{9.81} \times 4 \times (3.1416)^2 \times 0.125 \times \cos\theta = 3.0182 \cos\theta$$

Friction head: $h_{fs} = \dfrac{f\,L_s}{2gd_s}\left(\dfrac{A}{A_s}\omega\, r \sin\theta\right)^2$

$$h_{fs} = \frac{0.02 \times 6}{2 \times 9.81 \times 0.05} \times (4 \times 3.1416 \times 0.125 \times \sin\theta)^2 = 0.302 \sin^2\theta$$

Pressure head on the piston

$$\begin{aligned} H_{ts} &= H_{\text{atmos}} - (H_s + H_{as} + h_{fs}) \\ &= 10.0 - 4.0 - (H_{as} + h_{fs}) = 6.0 - (H_{as} + h_{fs}) \\ &= 6.0 - (3.0182 \cos\theta + 0.302 \sin^2\theta) \end{aligned}$$

Values of H_{ts} calculated for various values of θ are shown in the table given below.

Pressure head on the piston during suction stroke:

Condition	θ	H_{as} (m)	h_{fs} (m)	Total pressure head H_{ts} (abs)
Beginning of stroke	0°	3.0182	0	2.982 m
Midstroke	90°	0	3.302	5.698 m
End of stroke	180°	–3.0182	0	9.018 m

Delivery Stroke

Acceleration head: $H_{ad} = \frac{L_d}{g}\left(\frac{A}{A_d}\right)\omega^2 r \cos\theta$

$$= \frac{25}{9.81} \times 4 \times (3.1416)^2 \times 0.125 \times \cos\theta = 12.576 \cos\theta$$

Friction head: $h_{fd} = \frac{f L_d}{2gd_d}\left(\frac{A}{A_d}\omega r \sin\theta\right)^2$

$$h_{fd} = \frac{0.02\times 25}{2\times 9.81\times 0.05} \times (4 \times 3.1416 \times 0.125 \times \sin\theta)^2 = 1.2576 \sin^2\theta$$

Pressure head on the piston

$$H_{td} = H_{atmos} + (H_d + H_{ad} + h_{fd})$$

$$= 10.0 + 16.0 + (H_{ad} + h_{fd}) = 26 + (H_{ad} + h_{fd})$$

$$= 26.0 + (12.576 \cos\theta + 1.2576 \sin^2\theta)$$

Values of H_{td} are calculated for various values of θ and are shown in the table given below. Note that the absolute as well as gauge pressures at the cylinder during delivery stroke are calculated.

Pressure head on the piston during delivery stroke:

Condition	θ	H_{as} (m)	h_{fs} (m)	Total pressure head H_{ts} (abs)	Total pressure head H_{ts} (gauge)
Beginning of stroke	0°	12.576	0	38.576 m	28.576 m
Midstroke	90°	0	1.258	27.258 m	17.258 m
End of stroke	180°	– 12.576	0	13.424 m	3.424 m

**EXAMPLE 6.7

A single-acting reciprocal pump lifts a liquid of specific weight 7.50 kN/m³ from a pressurised storage reservoir to an overhead container. The free surface in the supply reservoir is at an elevation of 5.0 m above the centre of the pump. The ambient *pressure over the liquid surface in the supply reservoir is 20 kN/m². The other relevant data relating to the pump are as follows:*

Length of suction pipe = 5.0 m	*Length of stroke = 40 cm*
Crank speed = 60 rpm	*Diameter of cylinder = 20 cm*
Diameter of suction pipe = 10 cm	*Atmospheric pressure = 100 kN/m² (abs)*

Calculate the pressure in the cylinder and force on the piston at the beginning of the suction stroke.

Solution

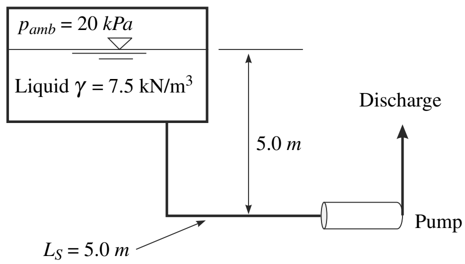


Fig. 6.7 *Example 6.7*

Consider the suction pipe:

Angular velocity

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 60}{60} = 6.283 \text{ rad/s}$$

Radius of crank = r = 0.40/2 = 0.20 m

$$\left(\frac{A}{A_s}\right) = \left(\frac{20}{10}\right)^2 = 4.0$$

Acceleration head: $H_{as} = \frac{L_s}{g}\left(\frac{A}{A_s}\right)\omega^2 r\cos\theta$

At start of the stroke $\theta = 0°$ and cos $\theta = 1.0$

$$H_{as} = \frac{L_s}{g}\left(\frac{A}{A_s}\right)\omega^2 r$$

$$= \frac{5.0}{9.81} \times 4 \times (6.283)^2 \times 0.2 = 16.096 \text{ m}$$

Ambient pressure head in the supply reservoir = $H_{amb} = \frac{p_{amb}}{\gamma} = \frac{20}{7.5}$ = 2.667 m of liquid (gauge)

Atmospheric pressure = 100 kN/m² = $\frac{100}{7.5}$ = 13.333 m of liquid

Ambient pressure (abs) = 2.667 + 13.333 = 16.0 m of liquid (abs)

Pressure head in the cylinder = $H_p = H_{amb} + H_s - H_{as}$ = 16.0 + 5.0 – 16.096

= 4.904 m of liquid (abs)

Pressure in the cylinder at start of suction stroke = p_p = 4.904 × 7.5 = 36.78 kPa (abs)

Since the atmospheric pressure is 100 kPa,

Suction pressure on the piston = (100.0 – 36.78) = 63.22 kPa (vacuum)

Suction force on the piston = 63.22 × $\frac{\pi}{4}$ (0.2)² = 1.986 kN

***EXAMPLE 6.8

A horizontal double-acting single-cylinder reciprocating pump has the following features:

Cylinder diameter = 20 cm	*Length of stroke = 40 cm*
Speed = 30 rpm	*Suction lift = 1.5 m*
Length of suction pipe = 4.0 m	*Diameter of suction pipe = 10 cm*
Delivery lift = 30 m	*Length of delivery pipe = 40 m*
Diameter of delivery pipe = 10 cm	*Darcy–Weisbach friction factor f = 0.02 for both the pipes*

Determine the net force due to water pressure on the piston when the crank has moved by an angle of 45° from the inner dead centre. The size of the piston rod can be neglected as relatively too small.

Solution

Angular velocity $\omega = \dfrac{2\pi N}{60} = \dfrac{2\pi \times 35}{60}$ = 3.665 rad/s, Radius of crank = r = 0.40/2

= 0.20 m $\left(\dfrac{A}{A_d}\right) = \left(\dfrac{20}{10}\right)^2$ = 4.0

L_d = 40.0 m, d_d = 0.10 m, H_d = 30.0 m

In a double-stroke pump when it is 45° for the suction stroke, the corresponding crank angle for the delivery will also be 45°.

Delivery stroke

Acceleration head: $H_{ad} = \dfrac{L_d}{g}\left(\dfrac{A}{A_d}\right)\omega^2\, r\cos\theta$

$$= \frac{40}{9.81} \times 4 \times (3.665)^2 \times 0.20 \times \cos 45° = 30.982 \text{ m}$$

Friction head: $h_{fd} = \dfrac{f\,L_d}{2gd_d}\left(\dfrac{A}{A_d}\omega r\sin\theta\right)^2$

$$h_{fd} = \frac{0.020 \times 40}{2 \times 9.81 \times 0.1} \times (4 \times 3.665 \times 0.20 \times \sin 45°)^2 = 1.753 \text{ m}$$

Total absolute pressure head on the piston face doing delivery stroke:

$$H_{td} = H_{\text{atm}} + H_d + H_{ad} + h_{fd}$$

$$= H_{\text{atm}} + 30.0 + 30.982 + 1.753 = H_{\text{atm}} + 62.735 \text{ m}$$

Suction stroke:

$\left(\dfrac{A}{A_s}\right) = \left(\dfrac{20}{10}\right)^2$ = 40, L_s = 4. 0 m, d_s = 0.10 m, H_s = 4.0 m

Acceleration head: $H_{as} = \frac{L_s}{g}\left(\frac{A}{A_s}\right)\omega^2 r \cos\theta$

$$= \frac{40}{9.81} \times 4 \times (3.665)^2 \times 0.20 \times \cos 45° = 30.98 \text{ m}$$

Friction head: $h_{fs} = \frac{f\,L_s}{2gd_s}\left(\frac{A}{A_s}\omega\, r \sin\theta\right)^2$

$$h_{fs} = \frac{0.02 \times 4}{2 \times 9.81 \times 0.1} \times (4 \times 3.665 \times 0.2 \times \sin 45°)^2 = 0.1753 \text{ m}$$

Total absolute pressure head on the piston face doing suction stroke:

$$H_{ts} = H_{atm} - H_s - H_{as} - h_{fs}$$

$$= H_{atm} - 4.0 - 3.098 - 0.175 = H_{atm} - 7.273 \text{ m}$$

Pressure force on the piston face at the suction side = $F_s = \gamma A H_{ts}$

Pressure force on the piston face at the delivery side = $F_d = \gamma A H_{td}$

Net force on the piston = $F_{Net} = F_{td} - F_{ts} = \gamma A H_{td} - \gamma A H_{ts} = \gamma A(H_{td} - H_{ts})$

$$= \gamma A(62.753 + 7.273)$$

$$= 9.79\left[\frac{\pi}{4} \times (0.2)^2\right] 70.326 = 21.63 \text{ kN}$$

**EXAMPLE 6.9

The data given below refer to a single-acting reciprocating pump:

Cylinder diameter = 35 cm	*Stroke length = 35 cm*
Static suction head = 3.0 m	*Diameter of suction pipe = 20 cm*
Suction pipe length = 6.0 m	*Crank speed = 20 rpm*
Length of delivery pipe = 25.0 m	*Static delivery head = 20.0 m*
Delivery pipe diameter = 20 cm	*Length of delivery pipe = 25 m*
Darcy–Weisbach friction factor f = 0.02	*Overall efficiency of the pump = 0.9*

Estimate the brake power the pump.

Solution

$$A = \frac{\pi}{4} \times (0.35)^2 = 0.09621 \text{ m}^2, \text{ Stroke length } L = 0.35 \text{ m}$$

Angular velocity $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 20}{60} = 2.0944$ rad/s

Radius of crank = $r = 0.35/2 = 0.175$ m

$$H_s = 3.0 \text{ m and } H_d = 20.0 \text{ m}$$

$$\left(\frac{A}{A_s}\right) = \left(\frac{35}{20}\right)^2 = 3.0625 \text{ and } \left(\frac{A}{A_d}\right) = \left(\frac{35}{20}\right)^2 = 3.0625$$

$$h_{fsav} = \frac{2}{3} h_{fsm} = \frac{1}{3} \frac{f L_s}{g d_s} \left(\frac{A}{A_s} \omega r \right)^2$$

$$= \frac{1}{3} \times \frac{0.02 \times 6.0}{9.81 \times 0.20} \times (3.0625 \times 2.0944 \times 0.175)^2 = 0.0257 \text{ m}$$

$$h_{fdav} = \frac{2}{3} h_{fdm} = \frac{1}{3} \frac{f L_d}{g d_d} \left(\frac{A}{A_d} \omega r \right)^2$$

$$= \frac{1}{3} \times \frac{0.02 \times 25}{9.81 \times 0.20} \times (3.0625 \times 2.0944 \times 0.175)^2 = 0.1070 \text{ m}$$

$$Q_t = \frac{ALN}{60} = \frac{0.09621 \times 0.35 \times 20}{60} = 0.0112 \text{ m}^3/\text{s}$$

$$\text{Power required} = P = \frac{\gamma ALN}{60 \times \eta_0} \left[H_s + H_d + \frac{2}{3}(h_{fsm}) + \frac{2}{3}(h_{fdm}) \right]$$

$$P = \frac{9.79 \times 0.0112}{0.90} [3.0 + 20 + 0.0257 + 0.1070] = 2.82 \text{ kW}$$

*EXAMPLE 6.10

Following data pertain to a single-acting reciprocating pump installation:

Cylinder diameter = 15 cm	*Crank speed = 60 rpm*
Stroke length = 20 cm	*Delivery-pipe diameter = 10.0 cm*
Static suction head = 4.0 m	*Length of delivery pipe = 30.0 m*
Diameter of suction pipe = 7.5 cm	*Static delivery head = 25.0 m*
Suction pipe length = 8.0 m	*Darcy–Weisbach friction factor f = 0.025*

The liquid being pumped is a chemical of density 850 kg/m^3. Estimate the power required to drive the pump by assuming an overall efficiency of 80%.

Solution

$$A = \frac{\pi}{4} \times (0.15)^2 = 0.01767 \text{ m}^2$$

Stroke length $L = 0.20$ m

Angular velocity $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 60}{60} = 6.283$ rad/s

Radius of crank = $r = 0.20/2 = 0.10$ m

$$H_s = 4.0 \text{ m. } d_s = 0.075 \text{ m; } H_d = 25.0 \text{ m, } d_d = 0.1 \text{ m}$$

$$\left(\frac{A}{A_s} \right) = \left(\frac{15}{7.5} \right)^2 = 4.0 \text{ and } \left(\frac{A}{A_d} \right) = \left(\frac{15}{10} \right)^2 = 2.25$$

$$h_{fsav} = \frac{2}{3} h_{fsm} = \frac{1}{3} \frac{f L_s}{g d_s} \left(\frac{A}{A_s} \omega r \right)^2 = \frac{1}{3} \times \frac{0.025 \times 8.0}{9.81 \times 0.075} (4 \times 6.283 \times 0.1)^2$$

$$= 0.5723 \text{ m}$$

$$h_{fdav} = \frac{2}{3}h_{fdm} = \frac{1}{3}\frac{f\,L_d}{g\,d_d}\left(\frac{A}{A_d}\,\omega r\right)^2 = \frac{1}{3}\times\frac{0.025\times 30}{9.81\times 0.1}\times(2.25\times 6.283\times 0.10)^2$$

$$= 0.5093 \text{ m}$$

$$Q_t = \frac{ALN}{60} = \frac{0.01767\times 0.20\times 60}{60} = 0.003534 \text{ m}^3/\text{s}$$

$$\gamma = \rho\, g = 850 \times 9.81 = \frac{850\times 9.81}{1000} = 8.339 \text{ kN/m}^3$$

$$\text{Brake power} = P\ \frac{\gamma Q_t}{\eta_0}\left[H_s + H_d + \frac{2}{3}(h_{fsm}) + \frac{2}{3}(h_{fdm})\right]$$

$$= \frac{8.339\times 0.003534}{0.80}\ [4.0 + 25 + 0.5723 + 0.5093] = 1.108 \text{ kW}$$

6.5 CHARACTERISTICS OF A RECIPROCATING PUMP

6.5.1 General Pump Characteristics

1. Efficiency

The efficiency of a reciprocating pump is made up of two components: volumetric efficiency and mechanical efficiency. Volumetric efficiency is essentially due to slippage at valves and seals. Mechanical efficiency arises due to friction at bearings and at speed-reducing devices. Thus, the overall efficiency depends upon the mechanical configuration, the drive system and on the condition of maintenance of the unit. The efficiency of a reciprocating pump is normally very high, being of the order of 85 to 90% in a well-maintained pump.

2. Pressures

Capacity to produce high pressures is a special feature of reciprocating pumps. Pumps with pressures up to 70 bar are routinely used. Special pumps can have pressures as high as 400 bar. The ability to produce very high pressures is so great in reciprocating pumps that a provision of an automatic over-pressure release devise is obligatory in every reciprocating pump installation.

3. Flow Control

In a reciprocating pump, the delivery pipe should never be throttled at the discharge side for purposes of reducing the flow. A working pump means that it is continuously pushing the flow and with throttling the fluid has nowhere else to go; if it is not the safety valve, something else in the system will yield. Similarly, suction throttling also will not work in reducing the flow. The only sensible way to reduce the flow is to reduce the speed (rpm) of the drive, which can be done easily. In a steam drive, the steam valve is to be throttled and in an electric motor drive, the speed of the motor is to be reduced for achieving reduction of speed of the pump.

Shut-off valves are provided at the suction and delivery end of discharge lines to isolate the pump for maintenance and repair activities. They are ***not*** meant for flow control. These valves must be of full open type, such as gate valve.

4. Priming

The reciprocating pump is *self-priming*. However, where possible to reduce wear or the risk of seizure, it should be flooded with liquid before starting.

5. Pressure Pulsation

Pressure pulsation is inherent in the reciprocating mechanism. This is highest in the simplex pumps and is considerably reduced in multiplex pumps. The extent of flow variations in various types of multiplex pumps is shown in Table 6.1. Flow variation is an indication of scope of pressure fluctuations. Pressure fluctuations are a product of acceleration created by the pump. Some applications may require pulsating dampeners to be installed. Also, a pulsating flow may require special design considerations in the piping system to avoid vibration of piping and consequent failure of system components by fatigue.

6. Power

The power rating of the reciprocating pumps vary depending upon the application. It varies from as low as 0.25 kW to high values of the order of 2500 kW.

7. Capacity

A reciprocating pump produces constant flow for a given speed, irrespective of the viscosity of the fluid and downstream pressure conditions. This fact makes it an excellent choice for metering and dosing purposes. The discharge capacity has a wide range, anywhere from a few liters per hour used in metering pumps to the order of 0.5 m^3/s in mass conveyance pumps.

8. Speed

Reciprocating pumps are essentially slow-speed pumps. Usual speeds cover a range from about 100 rpm up to about 700 rpm. Reducing gears are invariably used when the prime mover is an electric motor or an *IC* engine.

9. Industrial Use

(a) General Reciprocating pumps can handle a full range of liquids; from low-viscosity chemicals to high-viscosity materials like heavy oils; from clean water to high particle content slurries including concrete. In view of this wide operating range, reciprocating pumps are often the preferred choice for difficult applications, especially when the discharges are low and pressures are high.

Some typical industrial applications include pumping

- Low viscosity chemicals
- Oils
- Ore slurries
- Drilling mud
- Liquids in reverse osmosis process
- Water in high pressure cleaning (e.g. car wash)
- Salt-water injection
- Hot oil applications
- Blow out preventer
- Subsea applications

(b) Specialized Applications Where constant flow, especially under high pressure, is required reciprocating pumps are preferred. Typical applications in this category include metering and dosing applications, boiler feed and large-scale drip, spray or mist irrigation systems. Following is a partial list of other specialised industrial applications of reciprocating pumps:

- Ammonia service
- Cryogenic service
- High-pressure water
- Hydroforming
- Hydrostatic testing
- Power press
- Slush-ash service
- Steel-mill descaling
- Water-blast service

6.5.2 Operational Characteristics

The operational characteristics of reciprocating pumps are considered in four groupings as indicated below:

1. Variation of Discharge and Power with Speed

The capacity Q (flow rate) of a reciprocating pump varies linearly with the speed. Figure 6.8(a) shows schematically the variation of flow rate with the speed of a reciprocating pump. The variation is linear starting from zero flow rate for zero speed and terminates at maximum discharge for the maximum speed. Two lines, one for zero slip and another for a positive slip (which is usually less than about 3%), are shown in the figure.

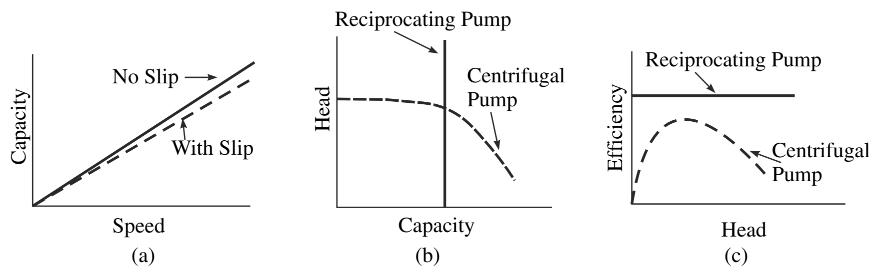


Fig. 6.8 *Operational characteristics of reciprocating pump*

2. Variation of Flow Rate with Pressure Head

Figure 6.8(b) represents schematically the variation of pressure head with capacity at constant speed. The flow rate is constant at all values of head. This constancy of flow rate regardless of the head is a typical characteristic of the reciprocating pump. The flow rate of a reciprocating pump can be varied only by changing the speed or displacement. A pressure relief valve determines the maximum pressure head that can be attained in a pump setting. For comparison purposes, the corresponding variation of flow rate of a centrifugal pump with head is also shown in Fig 6.8(b). The high variation of flow rate with head in a centrifugal pump is to be noted.

3. Variation of Efficiency with Head

Figure 6.8(c) shows schematically the variation of efficiency of a reciprocating pump with head. It is seen that the efficiency is constant over a wide range of heads. Combining the result of this figure with that of Fig. 6.8(a), it can be noted that the brake power *vs* speed curve is also linear, starting from zero–zero of the graph. The corresponding variation of efficiency of a centrifugal pump with head is shown as dotted line in the figure.

4. Effect of Viscosity

Figure 6.9 (a) is a plot of variation of efficiency of a reciprocating pump with viscosity of the liquid. Essentially constant efficiency over a wide range of viscosity of the liquid is a typical characteristic of reciprocating pumps. In some pumps, in a particular range, the efficiency may in fact improve with increase in the viscosity due to reduction of leakage. This is in marked contrast with the corresponding characteristic of a centrifugal pump (shown by dotted line in the figure) that exhibits a marked decrease in the efficiency with increase in viscosity of the liquid.

Figure 6.9(b) represents a plot of head against flow rate, at constant speed, over a wide range of viscosity. As indicated in this figure, the reciprocating pump exhibits constant flow rate over a broad range of pressure head. There is very little effect of viscosity on the flow rate. This characteristic makes reciprocating pumps a better choice when high viscosity liquids are to be pumped.

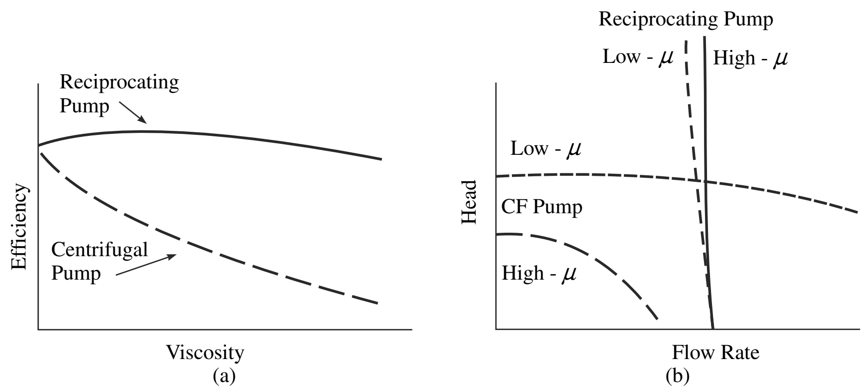


Fig. 6.9 *Effect of viscosity on the performance of a reciprocating pump*

In contrast, the centrifugal pump exhibits a marked impact of viscosity on the head-discharge relationship.

6.5.3 Cavitation, NPSH and Pump Installation

In a reciprocating pump, the suction pipe starts from the suction port of the pump to the foot valve. The pressure at the suction port is of prime importance in deciding on the installation level of the pump. An important aspect of the reciprocating pump is that the liquid is continuous from the foot valve to the delivery end. Since we are concerned here with liquid as the pumping medium, the fluid should be in liquid phase throughout the pumping system. At any point in the suction line, the pressure should be high enough so that the vapour bubbles do not form in the pipeline. If at any point the pressure falls to the neighbourhood of vapour pressure of the liquid, the dissolved gases would first separate from the liquid. This process is called *separation*. The separated gases may form pockets that would lead to flow discontinuities leading to loss of efficiency. Further, pressure pulsation commonly called *Knocking* may appear in the suction pipeline.

If the pressure at any point in the pipeline falls to the level of vapour pressure of the liquid, vapours of the liquid material would be released in the form of bubbles and transported to higher-pressure regions. In regions of higher pressure, these vapour bubbles may collapse leading to cavitation phenomenon and cavitation damage. When cavitation occurs, the performance of the pump is severely degraded. The volume rate of flow drops, pump becomes noisy; erosion of plunger packing area and nonreturn valves takes place. The typical noise due to cavitation is that of gravel being transported along with the liquid. The damages to the pump would be severe if flow under cavitation condition is allowed to continue.

The installation of the pump should be such as to achieve the *NPSHR* of the pump with a margin (*M*). The net positive head available at the site is *NPSHA*. The difference between *NPSHA* and *NPSHR* is the margin *M*.

Thus, $$M = NPSHA - NPSHR \tag{6.28}$$

The usual practice in reciprocating pumps is to provide a margin (*M*) as a percentage of *NPSHR* or 1.5 to 3.5 m depending up on the type of liquid, type of pump and recommendation of the manufacturer.

Pump manufacturers, through tests in their laboratories, determine the suction pressure required to avoid cavitation for each pump design. The result is indicated as *NPSHR* (*Net Positive Suction Pressure Required*) for the pump at designated operating pressure head and discharge. *NPSHR* for a reciprocating pump is defined in a slightly different way than that for a centrifugal pump. For reciprocating pumps, it is defined as the head in absolute units, measured above the prevailing vapour-pressure head, required by the pump to obtain satisfactory volumetric efficiency (usually no more than 3% reduction of capacity) and prevent excessive cavitation. Figure 6.10 shows a definition sketch of *NPSHR* for a reciprocating pump. Interestingly, *NPSHR* is not always expressed in terms of head of liquid. While it is expressed in metres of liquid for a centrifugal pump, for reciprocating pumps, sometimes, it is expressed in terms of Δp = Pressure increment over vapour pressure of liquid. For any plunger size, rotating speed and pumping capacity, there is a specific value of *NPSHR*. Any change in one or more of these variables changes the value of *NPSHR*. Thus, *NPSHR* is a characteristic of the pump and is to be provided by the manufacturer.

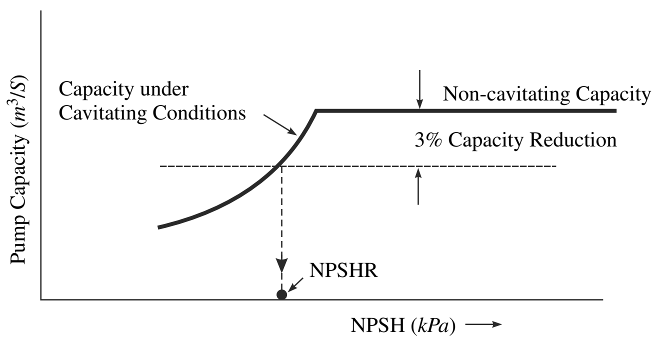


Fig. 6.10 *Definition of NPSHR for a reciprocating pump*

NOTE

NPSHR for a reciprocating pump is defined as *NPSH* corresponding to 3% reduction in capacity. In centrifugal pumps, it is defined as *NPSH* corresponding to 3% reduction in head or efficiency.

While *NPSHR* is a characteristic of the pump, *NPSHA* is dependent on the pump-setting and pump-operating characteristics. It is the head in absolute units, measured above the prevailing vapour pressure head, available to the pump at the pump inlet. With the usual notations, it could be shown that

$$NPSHA = H_{am} - H_s - h_{fs} - H_{as} - H_{vp} \tag{6.29}$$

Consider a reciprocating pump with horizontal axis located as shown in Fig. 6.11(a). The level of the pump axis is taken as the datum. In this

H_{am} = Ambient pressure head (absolute)

H_s = Suction head = Elevation difference from the level of liquid in the supply reservoir (sump) to the datum,(i.e., to the axis of the pump).

H_{vp} = Vapour pressure head (absolute)

h_{fs} = Loss of head due to friction in the suction pipe, including minor losses

H_{as} = Maximum acceleration head at the suction end

H_p = Pressure head at the inlet valve of the pump

Applying Bernoulli's theorem to a point on the liquid surface in the sump and the inlet valve of the cylinder

$$H_{am} - H_s - h_{fs} - H_{as} = H_p \tag{6.30}$$

Since *NPSHA* is defined as the head available above the prevailing vapour pressure head at the pump inlet,

$$H_p - H_{vp} = NPSHA \tag{6.31}$$

Hence, $NPSHA = H_{am} - H_s - h_{fs} - H_{as} - H_{vp}$

Note: For case 2 shown in Fig. 6.11(b), Eq. 6.30 will be

$$H_{am} + H_s - H_{fs} - H_{as} = H_p \tag{6.30a}$$

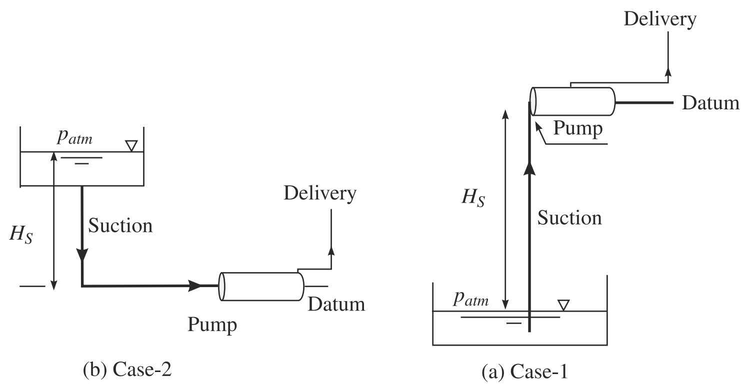


Fig. 6.11 *Two possible pump-installation types*

The object of the pump setting is to have maximum *NPSHA* at a given location. The following are some possibilities to maximise *NPSHA* in the setting of a reciprocating pump:

- Reduce the suction lift H_s.
- Reduce friction loss. Towards this (i) reduce suction pipe length, (ii) maximise suction pipe diameter, and (iii) minimise the number of bends, tees and valves in the suction pipe.
- Increase ambient pressure (if possible)
- Use multiplex pumps
- Use air vessels and such other pulsation reducing devices in the suction pipe.

6.5.4 Minimum Pressure Head at the Pump Cylinder

The minimum safe pressure head in the pump cylinder, H_{pmin}, is based on the cavitation criteria and represents ($NPSHR$ + Margin + H_{vp}). When *NPSHR* values are not known and if water is the working fluid of the pump, it is common practice to take this value of H_{pmin} as about 2.5 m to 3.5 m (abs).

In a single-acting reciprocating pump, the minimum pressure occurring during the start of the suction stroke is important in deciding on safe suction lift from consideration of cavitation aspects. For a known value of H_{pmin} (say, equal to 2.5 m (abs)), the suction lift is related to H_{pmin} from Eq.(6.30), as

$$H_{am} - H_s - h_{fs} - H_{as} = H_{pmin}$$

In this H_{am} is usually the atmospheric pressure = H_{atm} (equal to about 10.3 m of water).

Since at start of the suction stroke $h_{fs} = 0$ and H_{as} is maximum, for design purposes

$$H_{am} - H_s - H_{as} = H_{pmin} = (NPSHR + \mathrm{M} + H_{vp}) \qquad (6.32)$$

where M = Margin in *NPSHR*.

From Eq. (6.32), for known acceleration head H_{as} the value of safe suction lift H_s can be determined.

Also, since $H_{as} = \dfrac{L_s}{g}\left(\dfrac{A}{A_s}\omega^2 r\right)$, and $\omega = \dfrac{2\pi N}{60}$, for a given H_s the value of the safe maximum speed N_{ms} for the pump can be determined. This speed is from the consideration of the suction pipe only.

It should be noted that for water at 20°C, vapour pressure head $H_{vp} = \dfrac{p_v}{\gamma}$. Thus assuming H_{pmin} = 2.5 m (abs) for water is the same as the assumption of ($NPSHR$ + Margin = 2.25 m).

Now, it should be recalled from the description of the indicator diagram that in the delivery stroke the minimum pressure in the cylinder will occur at the end of that stroke. The relationship of the pressure heads at the end of the delivery stroke is

$$H_{am} - H_{ad} + H_d = H_{dp} \qquad (6.33)$$

where H_{am} = Ambient pressure = (usually atmospheric pressure equal to H_{atm})

H_{ad} = Acceleration head in the discharge pipe at the end of the stroke

H_d = Delivery lift

H_{dp} = Pressure head in the pump cylinder at the end of the delivery stroke.

Using this Eq. (6.30), by considering the pressure head in the cylinder to be the minimum safe pressure in the pump = H_{pmin}, the maximum value of H_{ad} for a given delivery head H_d can be determined. Knowing H_{ad}, the maximum value of speed N_{dm} of the pump for the given installation can be estimated. Note that this maximum speed is from the consideration of the delivery pipe only.

Out of the two speed values obtained, one from consideration of the suction pipe (N_{sm}) and another from the consideration of the delivery pipe (N_{dm}), the lowest of the two will be the maximum safe speed which will not cause the pressures in the pump to fall below H_{pmin}. Introduction of air vessels very near to the pump in suction and delivery pipes changes the scene dramatically. With the acceleration head being zero, the friction head corresponding to the average velocity controls the maximum speed. In such cases, the allowable maximum speed increases considerably.

6.6 AIR VESSELS

6.6.1 Flow Dampening

The flow from a single-acting reciprocating pump [Fig. 6.1(b)] is highly fluctuating and the delivery pipe carries the flow in only one-half of the cycle. In the remaining part of the cycle, viz. during the suction stroke, the delivery pipe does not carry any flow at all. The result is a highly variable discharge rate in the pipe. The high variation of flow in each cycle of the crank leads to, not only undesirable flow characteristics, but also uneconomical pipe design as the pipes have to be designed to carry the maximum flow safely. Table 6.1 indicates the variation of flow in three levels, viz. maximum to average, maximum to minimum and the range of maximum to minimum, all the above three as percentages of the average flow for a number of types of multicylinder pumps. It is seen that for a single-acting pump, the variation of the range is 160% of the average and this ratio decreases if more cylinders are used. Thus, for example a nine-throw (nonuplex) pump has a variation of the range as only 2.14% of the mean. While using more cylinders is one way to smooth the flow variation, it is not always an economical proposition. An economical alternative for flow dampening is to use special devices called by the generic name *flow dampeners*. These devices are used as attachments to all types of reciprocating pumps. While there are many types of flow dampeners in use in the industry, one such device used very extensively in reciprocating water pumps is the *air vessel*.

6.6.2 Air Vessels

An air vessel is a large closed chamber fitted to a reciprocating pump to eliminate pulsations of the flow in the pipes connected to the pump. One air vessel is fitted to each of suction and delivery pipes and as close to the pipe as possible. This reduces the flow fluctuations in the rest of the pipe length. Figure 6.12 is a schematic arrangement of two air vessels in a reciprocating pump system.

Typically, an air vessel is a closed chamber made of cast iron or steel and contains compressed air contained in a flexible container called a *bladder* or *bag* placed inside the chamber. The bladder is made of synthetic rubber and its main function is to prevent loss of compressed air. The pressure of compressed air in the bladder is always lower than that of the circuit pressure. There will be a valve on the top of the bladder for purposes of charging with additional air if required. The pumped liquid enters the air vessel from the bottom and occupies the lower portion of the chamber and in the process compresses the air in the bladder. When the pressure in the pipeline connected to the air chamber is lower than that of the bladder, the air pressure in the bladder forces the liquid to move out of the air vessel.

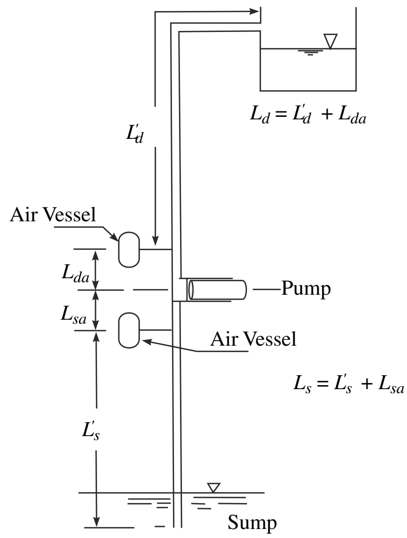


Fig. 6.12 *Schematic arrangement of Air vessels*

Consider the action of a single-acting pump fitted with an air vessel of sufficiently large capacity in the delivery pipe. Let the air vessel be located very near the pump so that for all practical purposes, the entire delivery pipe is considered as being affected by the action of the air vessel. In a delivery stroke, the fluid is being accelerated in the first half of the stroke. The acceleration head is dissipated in the air vessel and *time averaged* means flow will pass down the delivery pipe until the end of the delivery pipe. At any instant, the flow in excess of the average will pass into the air vessel and compresses the air volume there. The pressure of the air in the vessel increases. In the second half of the delivery stroke, the deceleration will also be dissipated at the air vessel and the mean flow passes down the delivery pipe. If the difference between the actual flow pushed by the piston and the average flow is positive, the excess will go into the air vessel. If it is negative, the difference will be made up by the stored water in the air vessel. The built-up pressure in the air vessel pushes the requisite difference into the delivery pipe. In this process, the air pressure in the air vessel will gradually decrease towards its original value.

Consider the delivery pipe of length L_d as in Fig. 6.12. Note that the introduction of air vessel causes the acceleration head of the water column to be confined to the small stretch of pipe (L_{da}) that lies between the pump and the air vessel. The rest of the delivery pipe (of length L'_d) will be having steady, uniform flow with constant mean velocity and hence will be free from any acceleration heads. Further, in this stretch, the frictional losses will also be constant along the time as the flow is no more unsteady.

The process of flow in the suction pipe with an air vessel is similar to that of the delivery pipe past an air vessel, discussed above. In the suction pipe, if an air vessel of sufficiently large capacity is fitted very near to the pump, it will absorb all the acceleration heads created by the simple harmonic action of the piston in the suction stroke. The length of the suction pipe beyond the air vessel towards the foot valve (of length L_s') will be having no acceleration heads at any part of the suction stroke. This will result in steady flow with constant velocity in this stretch of the suction pipe. The acceleration heads would be confined to the small stretch of pipe (of length L_{sa}) that lies between the pump and the air vessel. Further, in the stretch of suction pipe of length L_s' that lies between the air vessel and the foot valve, the frictional losses will also be constant along the time as the flow is steady.

It is necessary that the size of air vessels should be adequate for it to function effectively. For a single-acting pump, two air vessels, each having a volume of six to nine times the volume of the cylinder, is considered adequate.

6.6.3 Indicator Diagram for Pump with Ideal Air Vessels

The indicator diagram for a single-acting reciprocating pump equipped with two ideal air vessels, viz. air vessels of sufficient capacity, one each in suction and delivery pipes, fitted very close to the pump, is shown in Fig. 6.13.

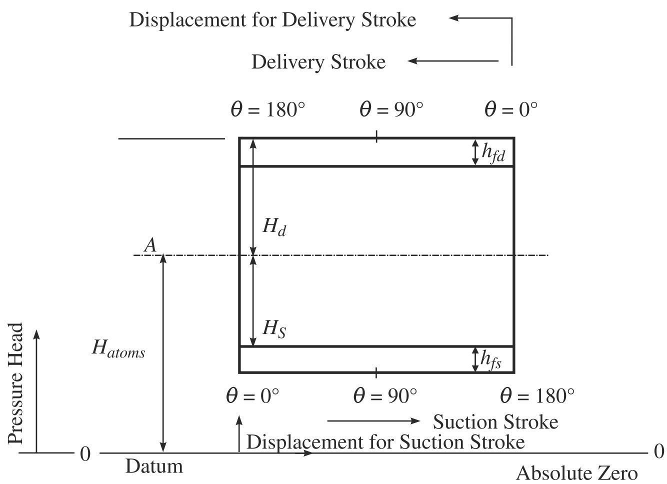


Fig. 6.13 *Indicator diagram with ideal air vessels*

Note the absence of acceleration heads H_{ad} and H_{as} in the delivery and suction pipes respectively. These are negligibly small and hence taken as zero. Because of uniform delivery velocity V_d and uniform suction velocity V_s, the frictional losses are h_{fd} and $h_{fs,}$ in delivery and suction pipes respectively, are constant all along the respective strokes.

The work done per unit weight of liquid pumped is $[(H_s + h_{fs}) + (H_d + h_{fd})]$.

Total work done for a discharge $Q_t = \dfrac{\gamma ALN}{60}$ is

$$P_t = \frac{\gamma ALN}{60}\left[(H_s + h_{fs}) + (H_d + h_{fd})\right] \tag{6.34}$$

6.6.4 Advantages in Using Air Vessels

The major advantages in using air vessels can be listed as follows:

1. Air Vessel in Suction Pipe

(a) For a given speed, there will be reduction in the cavitation susceptibility due to increase in *NPSHA*.

(b) For a given minimum pressure head, the running speed of the pump, hence the discharge, can be increased.

(c) Suction pipe length can be increased.

(d) Power expended in pumping will reduce due to reduction in frictional losses.

2. Air Vessel in Delivery Pipe

(a) Steady discharge in the delivery pipe is assured. If the pipe is to be designed on the basis of maximum discharge, there will be considerable savings as the diameter of the pipe to be used can be reduced.

(b) Reduction in frictional losses, and, hence saving in power, expended. The reduction can be as high as 84%.

(c) For a given minimum pressure head, the running speed of the pump, hence the discharge, can be increased.

Quantifications of some of the chief advantages of using air vessels are indicated in the following sections.

6.6.5 Analysis of Single-Acting Reciprocating Pump with Air Vessels

Refer to Figure 6.15 which is a schematic sketch of a single-acting reciprocating pump with two air vessels, one each in the suction and delivery pipes. In this pump set-up,

L_{sa} = Length of the suction pipe between the cylinder and the air vessel in the suction pipe

L'_s = Length of the suction pipe from the air vessel to the foot valve.

L_{da} = Length of the delivery pipe between the cylinder and the air vessel in the delivery pipe

L'_d = Length of the delivery pipe from the air vessel to the outlet of the delivery pipe.

A = Cross-sectional area of the cylinder

A_d = Cross-sectional area of the delivery pipe

A_s = Cross-sectional area of the suction pipe.

L = Length of stroke

Q_t = Theoretical discharge of the pump.

Total length of delivery pipe is $L_d = L'_d + L_{da}$

Total length of suction pipe is $L_s = L'_s + L_{sa}$

1. Velocity

In the delivery pipe, beyond the air vessels, the flow is steady and the velocity is constant at $V_d = \dfrac{Q_t}{A_d}$ over the length L'_d. Similarly, in the suction pipe, from the foot valve up to the air vessels, the flow is steady and the velocity is constant at $V_s = \dfrac{Q_t}{A_s}$ over the length L'_s. The magnitude of average velocity in delivery and suction pipes can be expressed as follows:

In the delivery pipe: $V_d = \dfrac{Q_t}{A_d} = \left(\dfrac{A}{A_d}\right)\left(\dfrac{LN}{60}\right)$ (6.35)

Also, from Eq.(6.8-a) for *any pipe* (indicated by the suffix 1), time averaged velocity $V_1 = \dfrac{A}{A_1}\dfrac{\omega r}{\pi}$ where r = Radius of the crank.

Thus, for a delivery pipe, the average velocity is also given by

$$V_d = \frac{A}{A_d} = \frac{\omega r}{\pi} \tag{6.36}$$

In the suction pipe: Average velocity $V_s = \dfrac{Q_t}{A_s} = \left(\dfrac{A}{A_s}\right)\left(\dfrac{LN}{60}\right)$ (6.37)

In an analogous manner to that of delivery pipe, the average velocity in the suction pipe is also given by $V_s = \dfrac{A}{A_s} = \dfrac{\omega r}{\pi}$ (6.38)

2. Discharge

Denoting by Q_i, the instantaneous discharge in any pipe (suction or delivery) ,

$$Q_i = A_s v_s = A_s\ \omega\, r \sin\theta \quad \text{for suction pipe, and} \tag{6.39}$$

$$Q_i = A_d v_d = A_d\ \omega\, r \sin\theta \quad \text{for delivery pipe.} \tag{6.40}$$

The average discharge in one cycle = $Q_{av} = A_s v_s = A_d v_d = \dfrac{A\omega r}{\pi}$

3. Discharge in to the Air Vessel

(a) Single-acting Pump Since the air vessel transmits only the time-averaged discharge downstream, the discharge into an air vessel in a single-acting reciprocating pump can be represented as

$$Q_{in} = (Q_i - Q_{av}) = A\ \omega\, r\left(\sin\theta - \frac{1}{\pi}\right) \tag{6.41}$$

If Q_{in} is negative, it indicates that the flow is going *out* of the air vessel.

For zero flow into or out of the air vessel, $Q_{in} = (Q_i - Q_{av}) = A\ \omega\, r\left(\sin\theta - \dfrac{1}{\pi}\right) = 0$

The solution of this equation is $\sin\theta = 1/\pi$ or $\theta = 18.56°$.

(b) Double-acting Pump For a double-acting pump, the average discharge is

$$Q_{av2} = A_s\, V_s = A_d V_d = \frac{2A\omega r}{\pi} \tag{6.42}$$

and hence the discharge going into the air vessel is

$$Q_{in2} = (Q_i - Q_{av}) = A\ \omega\ r\left(\sin\theta - \frac{2}{\pi}\right) \tag{6.43}$$

If Q_{in2} is negative, it indicates that the flow is going *out* of the air vessel.

For zero flow into or out of the air vessel, $Q_{in1} = A\ \omega\ r\left(\sin\theta - \frac{2}{\pi}\right) = 0$

The solution of this equation is $\sin\theta = 2/\pi$

or $\theta = 39.54°$.

4. Pressure Heads

Consider a single-acting reciprocating pump having air vessels in the suction pipe and in the delivery pipe as shown in Fig. 6.14. For the pump, the angular velocity is

$\omega = \frac{2\pi N}{60}$, where N is the speed of rotation of the crank in rpm. With reference to the locations of the air vessels as shown in Fig. 6.14, both suction pipe and the delivery pipe are considered in two reaches each as marked in the figure. The acceleration head and friction loss in the suction and delivery pipes are calculated as below.

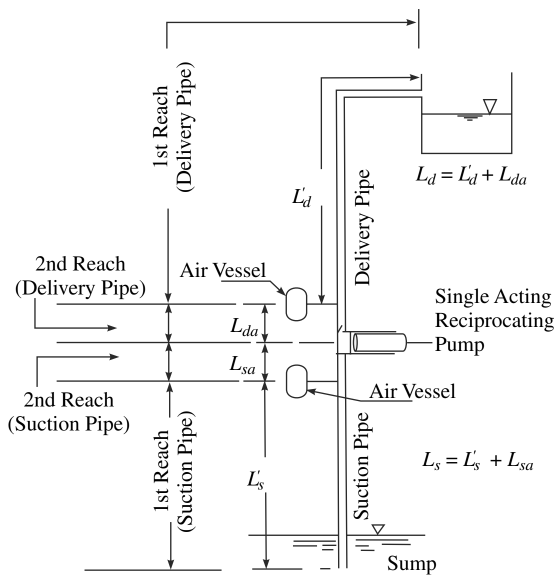


Fig. 6.14 *Definition sketch*

(a) Suction Pipe

(i) Acceleration Head The acceleration of the flow is confined to the reach of length L_{sa} from the pump to the air vessel only and the rest of the suction pipe is free from acceleration effects as steady, uniform flow prevails over there. This reach is called *second reach* of the suction pipe. In this reach, Acceleration head in suction pipe $H_{as} = \frac{L_{sa}}{g}\left(\frac{A}{A_s}\right)(\omega^2 r \cos\theta)$ (6.44)

At $\theta = 0°$, that is at the beginning of the suction stroke, H_{as} will have a maximum value given by

$$H_{asm} = \frac{L_{sa}}{g}\left(\frac{A}{A_s}\right)(\omega^2 r) \tag{6.45}$$

(ii) Frictional Losses Steady uniform flow with an average velocity of V_s [given by Eq. (6.36)] prevails over a length L'_s of the suction pipe. This stretch is called the first reach of the suction pipe. The loss of head due to frictional in this first reach, by Darcy–Weisbach equation is

$$h_{fs1} = \frac{f L'_s V_s^2}{2 g d_s} \tag{6.46}$$

where $$V_s = \frac{A}{A_s} = \frac{\omega r}{\pi}$$

In the suction pipe, in addition to this h_{fs1}, there is another frictional loss due to unsteady flow prevailing in the second reach of length L_{sa} that lies between the air vessel and the cylinder, (see Fig. 6.15). The flow being unsteady, the head loss in this reach varies with the crank angle θ and is given by Eq. (6.17) as

$$h_{fs2} = \frac{f L_{sa} V_s^2}{2 g d_s} = \frac{f L_{sa}}{2 g d_s}\left(\frac{A}{A_s}\omega r \sin\theta\right)^2 \tag{6.47}$$

Note that h_{fs2} is a function of $\sin^2\theta$ and hence the frictional head loss varies parabolically with θ. Further, the average value of frictional head in this pipe is (2/3) times the maximum value. The maximum of h_{fs2} occurs at $\theta = 90°$ and its value is given by

$$h_{fs2m} = \frac{f L_{sa}}{2 g d_s}\left(\frac{A}{A_s}\omega r\right)^2 \tag{6.48}$$

and the average frictional head loss in the second reach of suction pipe is given by

$$\text{Average of } h_{fs2} = h_{fsav2} = \frac{2}{3} h_{fs2m} = \frac{1}{3}\frac{f L_{sa}}{g d_s}\left(\frac{A}{A_s}\omega r\right)^2 \tag{6.49}$$

The total pressure head H_{ts} developed in the cylinder during the suction stroke at any crank angle θ is

$$H_{ts} = H_s + \frac{V_s^2}{2g} + H_{as} + h_{fs1} + h_{fs2} \tag{6.50}$$

In this H_s = Static suction head,

$\frac{V_s^2}{2g}$ = Kinetic energy of the flow in suction pipe. This is usually neglected as too small.

H_{as} = Acceleration head in reach 2 of suction pipe; the flow being steady in the reach 1, there is no acceleration head in that reach.

h_{fs1} = Frictional loss in the reach 1 of suction pipe due to steady flow

h_{fs2} = frictional loss in reach 2 of the suction pipe, due to unsteady flow. Usually the length L_{sa} is very small in relation to the total length L_s of the suction pipe and this head will be relatively very small.

(b) Delivery Pipe

(i) Acceleration Head The acceleration of the flow in the delivery pipe is confined to the reach of length L_{da} from the pump to the air vessel only. This reach is called second reach of the delivery pipe. Rest of the delivery pipe (called first reach) is free from acceleration effects as steady, uniform flow prevails over there. Hence,

Acceleration head in the delivery pipe $$H_{ad} = \frac{L_{da}}{g}\left(\frac{A}{A_d}\right)(\omega^2 r\cos\theta) \tag{6.51}$$

At $\theta = 0°$, that is at the beginning of the delivery stroke, H_{ad} will have a maximum value given by

$$H_{adm} = \frac{L_{da}}{g}\left(\frac{A}{A_d}\right)(\omega^2 r) \tag{6.52}$$

(ii) Frictional Losses Steady uniform flow with an average velocity of V_d [given by Eq. (6.36)] prevails over a length L'_d of the delivery pipe. This stretch will be called the first reach of delivery pipe. The frictional loss of head in this first reach, by Darcy–Weisbach equation is

$$h_{fd1} = \frac{f\,L'_d V_d^2}{2gd_d} \tag{6.53}$$

where $$V_d = \frac{A}{A_d} = \frac{\omega r}{\pi}.$$

In the delivery pipe, in addition to this h_{fd1}, there is another frictional loss due to unsteady flow prevailing in the second reach of length L_{da} that lies between the air vessel and the cylinder (see Fig. 6.15). The flow being unsteady, the head loss in this second reach varies with the crank angle θ and is given by Eq. (6.17) as

$$h_{fd2} = \frac{f\,L_{da}V_s^2}{2gd_d} = \frac{f\,L_{da}}{2gd_d}\left(\frac{A}{A_d}\omega r\sin\theta\right)^2 \tag{6.54}$$

Since h_{fd2} is a function of $\sin^2\theta$, the frictional head loss varies parabolically with θ. Further, the average value is (2/3) times the maximum value. The maximum of h_{fd2} occurs at $\theta = 90°$ and its value is given by

$$h_{fd2m} = \frac{f\,L_{da}}{2gd_d}\left(\frac{A}{A_d}\omega r\right)^2 \tag{6.55}$$

and the average frictional head loss in the second reach of delivery pipe is given by

$$\text{Average } h_{fd2} = h_{fdav2} = \frac{2}{3} h_{fd2m} = \frac{1}{3} \frac{f L_{da}}{g d_d} \left(\frac{A}{A_d} \omega r \right)^2 \tag{6.56}$$

The total pressure head H_{td} developed in the cylinder during the delivery stroke at any crank angle θ is

$$H_{td} = H_d + \frac{V_d^2}{2g} + H_{ad} + h_{fd1} + h_{fd2} \tag{6.57}$$

In this,

H_d = Static delivery head,

$\frac{V_d^2}{2g}$ = Kinetic energy of the flow in delivery pipe; this is usually neglected as too small.

H_{ad} = Acceleration head in the second reach of delivery pipe.

h_{fd1} = Frictional loss in the reach 1 of delivery pipe due to steady flow.

h_{fd2} = Frictional loss in second reach of the delivery pipe, due to unsteady flow.

Usually, the length L_{da} is very small in relation to the total length L_d of the delivery pipe and this head h_{fd2} will be relatively very small.

5. Work Done by Reciprocating Pump with Air Vessels

Theoretical work done per second, per unit weight of the liquid pumped, in a reciprocating pump fitted with air vessels is

$$W_1 = (H_{td} + H_{ts}) - \left[H_d + \frac{V_d^2}{2g} H_{ad} + h_{df1} + h_{fd2} \right] + H_s + \frac{V_s^2}{2g} + H_{as} + h_{fs1} + h_{fs2} \Bigg] \tag{6.58}$$

However, the velocity heads in the suction and delivery pipes being relatively very small, it is usual practice to neglect these two terms.

Theoretical discharge by the pump is $Q_t = \frac{\gamma ALN}{60}$.

Hence, the theoretical work done per second representing the power consumed in lifting the liquid by a total lift of $(H_d + H_s)$ is

$$P_t = \gamma Q_t (H_{td} + H_{ts}) = \gamma Q_t \left(\left[H_d + \frac{V_d^2}{2g} + H_{ad} + h_{fd1} + h_{fd2} \right] + \left[H_s + \frac{V_s^2}{2g} + H_{as} + h_{fs1} + h_{fs2} \right] \right) \tag{6.59}$$

By neglecting the velocity heads, and substituting for Q_t, the simplified expression for power consumed is

$$P_t = \frac{\gamma ALN}{60} \left(\left[h_d + H_{ad} + h_{fd1} + h_{fd2} \right] + \left[H_s + H_{as} + h_{fs1} + h_{fs2} \right] \right) \tag{6.59-a}$$

In actual installations, there will always be mechanical friction and possible slip in the pump leading to mechanical and volumetric efficiencies. Hence, if the overall efficiency of the pump is η_0, the power that is to be supplied to the pump to obtain the theoretical work done per second of P_t is

$$P = \frac{P_t}{\eta_0} \tag{6.60}$$

6.6.6 Indicator Diagram with Air Vessels by Considering all Factors

The expressions for work done per unit weight of liquid as in Eq.(6.58) and the variation of each of the terms can be expressed in the form of the indicator diagram. Figure 6.15 is such a schematic diagram. This figure is somewhat similar to Fig. 6.6 except in the present one there is representation of velocity heads in the suction and delivery tubes and also the frictional head loss due to uniform velocity in length L'_s of suction pipe and L'_d of delivery pipe. As already indicated, the velocity head terms are of relatively very small magnitude and are usually neglected to get the expression for total head as in Eq. (6.59-a). Further, in Fig. 6.16 if the acceleration heads are very small due to positioning the air vessels very close to the cylinder, in the limiting stage. Figure 6.15 will approach the ideal air vessel case shown in Fig. 6.13. The area of the indicator diagram shown in Fig. 6.16 is the expression shown in the right-hand side of Eq. (6.58).

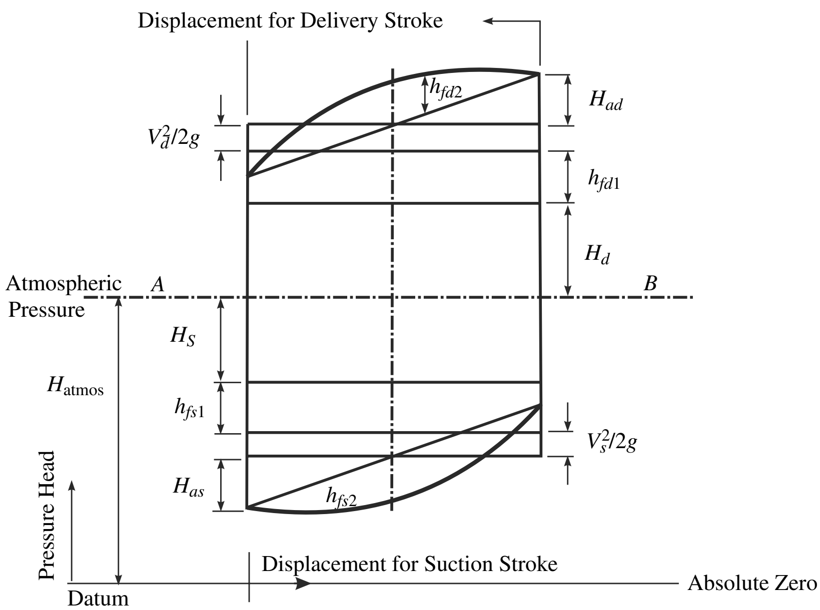


Fig. 6.15 *Indicator diagram for reciprocating pump with air vessels*

6.6.7 Reduction in Frictional Losses due to Fitting of Air Vessel

1. Single-Acting Pump

Consider a single-acting reciprocating pump equipped with air vessels, one each in the suction and delivery pipes. The air vessels are fitted just outside the cylinder of the pump and hence the length of the pipe between the cylinder and the air vessel can be considered zero. The entire pipe (suction or delivery) will have steady flow with constant velocity. Two situations are considered: (i) pipe-lines with air vessels, and (ii) pipelines without air vessels. Suffixes 1 and 2 refer to these two cases respectively.

Case 1: Suction and Delivery Pipelines with Air Vessels This analysis applies to both suction and delivery pipes and as such they are not dealt with separately. Consider a pipe with length L_0 and diameter d_0. The cross-sectional area of this pipe is A_0. The entire pipe is influenced by the air vessel and will have steady flow with an average velocity of V_0. Since the piston of the pump operates in simple harmonic motion, the instantaneous velocity v of flow due to pumping, before being modulated by the air vessel is as below:

Velocity of water at any instant $v = \frac{A}{A_0}\omega r \sin \omega t = \frac{A}{A_0}\omega r \sin \theta$.

The maximum velocity $v_{\max} = \frac{A}{A_1}\omega r$ and the average velocity $V_0 = \frac{A}{A_s}\frac{\omega r}{\pi}$.

Thus $V_0 = \frac{A}{A_s}\frac{\omega r}{\pi} = \frac{v_{\max}}{\pi}$

The head loss due to friction in a pipe of length L_0 is

$$h_{f1} = \frac{f\, L_0 V_0^2}{2\, g d_0} \tag{6.61}$$

Case 2: Same Pipes as Case 1 but without Air Vessel When there is no air vessel in the pipeline, the frictional head loss due to sinusoidal variation of the velocity in the pipeline, as per Eq. (6.19) is

$$h_{f2} = \frac{2}{3}\frac{fL_0}{2\, g d_0}\left(\frac{A}{A_0}\omega r\right)^2 = \frac{2}{3}\frac{f\, L_0}{2\, g d_0}(v_{\max})^2$$

$$= \frac{2}{3}\frac{fL_0(V_0^2\pi^2)}{2\, g d_0} = \frac{2}{3}\pi^2 h_{f1} \tag{6.62}$$

Hence,
$$\frac{h_{f1}}{h_{f2}} = \frac{3}{2\pi^2} \tag{6.63}$$

The theoretical power required to pump a discharge Q over a head H is $P_t = \gamma QH$. The power required being proportional to the head, percentage savings of power required to overcome frictional resistance due to fitting of air vessels is

$$\frac{(P_2 - P_1)}{P_2}\times 100 = \left(1 - \frac{P_1}{P_2}\right)\times 100 = \left(1 - \frac{3}{2\pi^2}\right)\times 100 = 84.8\% \tag{6.63-a}$$

2. Double-Acting Pump

If the pump is double-acting, with the same notations as used in connection with single-acting pump,

The maximum velocity $v_{\max} = \frac{A}{A_t}\omega r$ and the average velocity is $V_0 = \frac{2A}{A_s}\frac{\omega r}{\pi}$.

Thus, $V_0 = \frac{2A}{A_s}\frac{\omega r}{\pi} = \frac{2v_{\max}}{\pi}$

Hence, $h_{f1} = \dfrac{f L_0 V_0^2}{2gd_0}$ and

$$h_{f2} = \frac{2}{3}\frac{f L_0}{2gd_0}(v_{\max})^2 = \frac{2}{3}\frac{f L_0}{2gd_0}\left(\frac{V_0^2\pi^2}{4}\right)$$

$$= \left(\frac{2}{3}\times\frac{\pi^2}{4}\right)h_{f1}$$

Hence, $$\frac{h_{f1}}{h_{f2}} = \left(\frac{6}{\pi^2}\right) \qquad (6.64)$$

Percentage savings of power required to overcome frictional resistance due to fitting of air vessels is

$$\frac{(P_2 - P_1)}{P_2}\times 100 = \left(1 - \frac{P_1}{P_2}\right)\times 100 = \left(1 - \frac{6}{\pi^2}\right)\times 100 = 39.2\% \qquad (6.64\text{-a})$$

6.6.8 Maximum Speed of a Reciprocating Pump Fitted with Air Vessel

As indicated earlier in Sec. 6.5.3, generally the minimum pressure obtained in the suction stroke controls the maximum speed of the pump. Let us consider a single-acting reciprocating pump with an air vessel in the suction side at a distance L_{sa} from the cylinder. L_s is the total length of the suction pipe. Let H_{pmin} be the minimum pressure in the cylinder that is fixed and is mandated not to be exceeded from considerations of cavitation in the suction stroke. Then

$$H_{ts} = H_s + \frac{V_s^2}{2g} + H_{as} + h_{fs1} + h_{fs2} = H_{pmin}$$

$H_{p\min}$

$$= H_s + \frac{\left(\dfrac{A}{A_s}\dfrac{\omega r}{\pi}\right)^2}{2g} + \frac{(L_s - L_{sa})}{g}\left(\frac{A}{A_s}\right)(\omega^2 r) + \frac{f(L_s - L_{sa})\left(\dfrac{A}{A_s}\dfrac{\omega r}{\pi}\right)^2}{2gd_s} + \frac{1}{3}\frac{f L_{sa}}{gd_s}\left(\frac{A}{A_s}\omega r\right)^2 \qquad (6.65)$$

In Eq. (6.65), if the value of H_{pmin} is known, excepting ω and f all the other terms are based on the geometry of the pump system and hence are known quantities. The coefficient f (Darcy–Weisbach friction factor) is assumed or can be determined from known flow properties. As such, the only unknown is the angular velocity ω and it can be obtained from the solution of Eq. (6.62). The speed of the pump N being related to the angular velocity as $\omega = \dfrac{2\pi N}{60}$, could then be determined. This represents the maximum speed of the pump from considerations of cavitation in the suction stroke. Usually, the second term on the right-hand side of Eq. 6.65 representing the velocity head in the suction pipe is neglected as being relatively too small. Example 6.19 illustrates the use of this method to calculate the acceptable maximum speed.

6.7 GENERAL ASPECTS

6.7.1 Comparative Study of Reciprocating Pumps and Centrifugal Pumps

The range of operation of both centrifugal and reciprocating pumps is indeed very large and varied and as such any generalisations can only be in terms of their basic characteristics. Any apparent disadvantage is taken care of by appropriate technical innovation in the application. The following is a brief list of major differences in these two types of pumps.

S1. No	Reciprocating Pumps	Centrifugal pumps
1	Low flow rates and high pressures up to about 700 atmospheres.	High flow rates at moderate pressures of a few atmospheres.
2	At constant speed, constant flow rate at widely different outlet pressures.	At constant speed, variable discharge depending on outlet pressure.
3	Nil or very limited effect of viscosity. Eminently suitable for high-viscosity fluids.	Viscosity greatly affects the performance.
4	Flow-rate control only through control of stroke length or speed.	Flow can be throttled and hence valve-type control is adopted.
5	Efficiency is high at around 90% and remains constant over the entire operating range.	Efficiency around 80% and varies in the operating range.
6	Niche and specialty applications	General and very vast range of applications.
7	Low-speed operation.	High-speed operation.
8	In handling shear sensitive liquids, the pump induced shear is very little.	Considerable shear during pumping—a disadvantage in handling shear sensitive liquids.

6.7.2 Selection of Pump Type

A rough guide to selection of the type of pumps, viz. positive displacement pumps or other types [centrifugal (radial flow), axial flow, mixed flow pumps] for a particular application can be made from the use of Fig. 6.16. Reciprocating pumps occupy a substantial part of the space marked for positive displacement pumps *(PDP)*. Some other types of *PDP*s are discussed in Chapter 7. Further, it is clearly seen from Fig. 6.16 that the *PDPs* have a niche space in the high-head–low-discharge category. The regions of appropriate choice of radial, mixed and axial-flow pumps are also indicted in this figure.

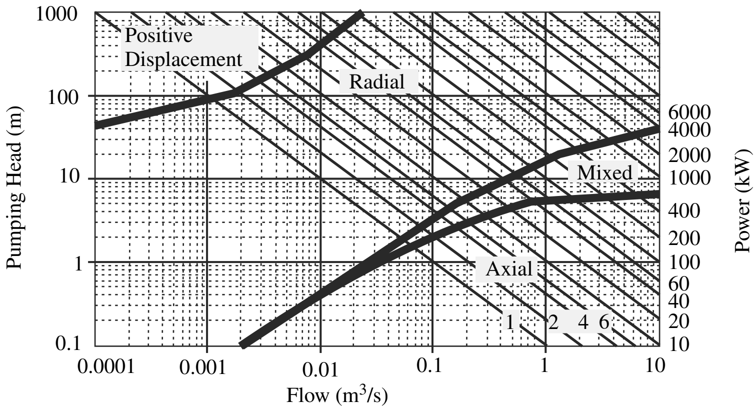


Fig. 6.16 *Chart for selection of pump type [Ref: From PPT on* Hydraulic machines *by Dr. Monroe L. Weber-Shirk, School of Civil & Environmental Engineering, Director of AguaClara, Cornell Univ., USA]*

6.8 ILLUSTRATIVE EXAMPLES—SET 6.2

*EXAMPLE 6.11

A single-acting reciprocating pump has the following data:

Piston diameter = 10 cm	Stroke length = 30 cm
Suction head = 4.0 m	Diameter of suction pipe = 7.5 cm
Length of suction pipe = 4.0 m	Atmospheric pressure = 10.0 m (water) (abs)

If the minimum pressure head in the cylinder from cavitation considerations (NPSHR + Margin) is 2.5 m (abs), determine the maximum speed at which the pump can run without compromising on safety against cavitation. The Darcy–Weisbach friction factor f = 0.03.

Solution

For incipient cavitation in the suction pipe,

$$H_{p\min} = H_{atm} - H_s - H_{as}$$

$$2.50 = 10.0 - 4.0 - H_{as}$$

$$H_{as} = 3.5 \text{ m}$$

At the beginning of suction stroke, the acceleration head = $H_{as} = \frac{L_s}{g}\left(\frac{A}{A_s}\right)\omega^2 r$

Here, $L_s = 4.0$ m, $\left(\frac{A}{A_s}\right) = \left(\frac{10}{7.5}\right)^2 = 1.778$ and $r = 0.30/2 = 0.15$ m.

Substituting in the expression for H_{as},

$$H_{as} = 3.5 = \frac{4}{9.81} \times 1.778 \times 0.15 \times \omega^2$$

$$\omega^2 = 32.185 \text{ giving } \omega = \frac{2\pi N}{60} = 5.673$$

$$N = \frac{60 \times 5.673}{2\pi} = 54.17 \text{ rpm = Maximum speed of the pump}$$

**EXAMPLE 6.12

A plunger is fitted to a vertical pipe filled with water. The lower end of the pipe is submerged in a sump. If the plunger is drawn up with an acceleration of 5.0 m/s², find the maximum height above the water level of the sump at which the plunger will work without cavitation. Assume atmospheric pressure = 10.0 m (abs). Limiting pressure from cavitation consideration = 2.0 m (abs). Take acceleration of gravity as 10 m/s².

Solution

Acceleration $a_p = 5.0$ m/s^2 and acceleration head $H_a = \frac{L_p}{g} a_p$

$$H_a = \frac{L_p}{10} \times 5 = \frac{L_p}{2}$$

Balancing the pressure heads $H_{p\min} = H_{atm} - H_s - H_d$

Here suction head $H_s = L_p$, H_{pmin}= 2.0 (abs) and H_{atm} = 10.0 m (abs)

Hence, $2.0 = 10.0 - L_p - \frac{L_p}{2}$

$$L_p = \frac{8 \times 2}{3} = 5.33 \text{ m}$$

*EXAMPLE 6.13

A single-acting reciprocating pump has a 20 cm diameter piston with stroke of 40 cm. The suction pipe is 10 cm in diameter and 5 m long. The speed of the pump is 30 rpm. Find the maximum suction lift if minimum pressure head in the system is limited to 2.50 m (abs) from cavitation considerations. Assume atmospheric pressure at the site as 10.0 m of water.

Solution

Maximum acceleration in the suction pipe

$$H_{as} = \frac{L_s}{g}\left(\frac{A}{A_s}\right)\omega^2 r$$

Here, N = 30 rpm and hence $\omega = \frac{2\pi \times 30}{60}$ = 3.1416 rad/sec, r = 0.40/2 = 0.20 m

$$L_s = 5.0 \text{ m and } \left(\frac{A}{A_s}\right) = \left(\frac{20}{10}\right)^2 = 4$$

$$H_{as} = \frac{5.0}{9.81} \times 4 \times (3.1416)^2 \times 0.20 = 4.024 \text{ m.}$$

At limiting condition for suction pipe, $H_{p\min} = H_{atm} - H_s - H_{as}$

$$2.5 = 10.0 - H_s - 4.024$$

Maximum suction lift = H_s = 3.476 m

**EXAMPLE 6.14

A double-acting reciprocating pump has a 25 cm cylinder with a stroke of 40 cm. The suction pipe is 5.0 m long and the suction lift is 3.0 m. If the speed of the crank is 25 rpm, determine the minimum diameter of the suction pipe to prevent occurrence of cavitation. The minimum pressure from cavitation consideration (NPSHR + Margin) is limited to 2.5 m (abs). Assume atmospheric pressure as 10.0 m of water (abs).

Solution

Given: L_s = 5.0 m, Diameter of cylinder D = 0.25 m, $r = L/2$ = 0. 40/2 = 0.20 m, N = 25 rpm

Angular velocity $\omega = \dfrac{2\pi N}{60} = \dfrac{2\pi \times 25}{60}$ = 2.618 rad/s

$$\left(\frac{A}{A_s}\right) = \left(\frac{D}{d_s}\right)^2$$

At the beginning of the suction stroke, acceleration head =

$$H_{as} = \frac{L_s}{g}\left(\frac{A}{A_s}\right)\omega^2\, r$$

$$H_{as} = \frac{5}{9.81}\left(\frac{D}{d_s}\right)^2 (2.618)^2 \times 0.20 = 0.699\left(\frac{D}{d_s}\right)^2$$

At limiting condition for the suction pipe,

$$H_{p\min} = H_{atm} - H_s - H_{as}$$

$$2.5 = 10.0 - 3.0 - 0.699\left(\frac{D}{d_s}\right)^2$$

$$0.699\left(\frac{D}{d_s}\right)^2 = 4.5$$

Hence, $\left(\dfrac{D}{d_s}\right)^2$ = 6.438 and $\dfrac{D}{d_s}$ = 2.537

d_s = Diameter of suction pipe = 0.0985 m = 9.85 cm

*EXAMPLE 6.15

A single-acting reciprocating pump has the following data:

Cylinder diameter = 22.5 cm	*Stroke length = 45 cm*
Suction head = 4.5 m	*Diameter of suction pipe = 22.5 cm*
Length of suction pipe = 20 m	*Atmospheric pressure = 10.3 m (water) (abs)*

The NPSHR of the pump is determined by the manufacturer as 1.2 m and a margin of 0.8 m is mandatory. Determine the maximum discharge that can be obtained from the current geometry of the pump without compromising on safety against cavitation. The Darcy–Weisbach friction factor f = 0.03.

Solution

For incipient danger of cavitation in the suction pipe

$$H_{p\min} = NPSHR + M = 1.2 + 0.8 = 2.0 \text{ m}$$

$$H_{p\min} = H_{atm} - H_s - H_{as}$$

$$2.00 = 10.3 - 4.5 - H_{as}$$

$$H_{as} = 3.8 \text{ m}$$

At the beginning of suction stroke, the acceleration head =

$$H_{as} = \frac{L_s}{g}\left(\frac{A}{A_s}\right)\omega^2 r$$

Here, $L_s = 20.0$ m, $\left(\frac{A}{A_s}\right) = \left(\frac{22.5}{22.5}\right)^2 = 1.0$ and $r = 0.45/2 = 0.225$ m

Substituting in the expression for H_{as},

$$H_{as} = 3.8 = \frac{20}{9.81} \times 1.0 \times 0.225 \times \omega^2$$

$$\omega^2 = 8.284 \text{ giving } \omega = \frac{2\pi N}{60} = 2.878$$

$$N = \frac{60 \times 2.878}{2\pi} = 27.5 \text{ rpm} = \text{Maximum speed of the pump}$$

For a given pump geometry, maximum discharge corresponds to the maximum speed. Hence, for this pump, the maximum discharge possible is with speed $N_{max} = 27.5$ rpm.

$$Q_m = \frac{ALN_{\max}}{60} = \frac{\left(\frac{\pi}{4} \times (0.225)^2\right) \times 0.45 \times 27.5}{60} = 0.0082 \text{ m}^3/\text{s}$$

**EXAMPLE 6.16

Determine the maximum permissible speed which will not cause cavitation in the pump system for the single-acting reciprocating pump with following data:

Diameter of piston = 100 mm	*Length of suction pipe = 3.0 m*
Length of stroke = 200 mm	*Length of delivery pipe = 35 m*
Diameter of suction pipe = 35 mm	*Suction lift = 3.0 m*
Diameter of delivery pipe = 25 mm	*Delivery lift = 5.0 m*
Safe minimum pressure head = 2.5 m	*Atmospheric pressure head = 10.1 m of water (abs)*

Solution

(a) For danger of incipient cavitation in the suction pipe

$$H_{p\min} = H_{atm} - H_s - H_{as}$$
$$2.5 = 10.1 - 3.0 - H_{as}$$
$$H_{as} = 4.6 \text{ m}$$

At the beginning of suction stroke, the acceleration head =

$$H_{as} = \frac{L_s}{g}\left(\frac{A}{A_s}\right)\omega^2 r$$

Here , $L_s = 3.0$ m, $\left(\frac{A}{A_s}\right) = \left(\frac{100}{35}\right)^2 = 8.16$ and $r = 0.20/2 = 0.10$ m.

Substituting in the expression for H_{as},

$$H_{as} = 4.6 = \frac{3}{9.81} \times 8.16 \times 0.10 \times \omega^2$$

$$\omega^2 = 18.43 \text{ giving } \omega = \frac{2\pi N}{60} = 4.293$$

$$N = \frac{60 \times 4.293}{2\pi} = 41 \text{ rpm} = N_{smax}$$

= Maximum speed of the pump from consideration of suction pipe

(b) For incipient cavitation in the delivery pipe

$$H_{p\min} = H_{atm} + H_d - H_{ad}$$
$$2.5 = 10.1 + 5.0 - H_{ad}$$
$$H_{ad} = 12.6 \text{ m}$$

At the beginning of suction stroke, the acceleration head =

$$H_{ad} = \frac{L_d}{g}\left(\frac{A}{A_d}\right)\omega^2 r$$

Here $L_d = 35.0$ m, $\left(\frac{A}{A_d}\right) = \left(\frac{100}{25}\right)^2 = 16$ and $r = 0.20/2 = 0.10$ m.

Substituting in the expression for H_{ad},

$$H_{ad} = 12.6 = \frac{35}{9.81} \times 16 \times 0.10 \times \omega^2$$

$$\omega^2 = 5.71 \text{ giving } \omega = \frac{2\pi N}{60} = 2.39$$

$$N = \frac{60 \times 2.39}{2\pi} = 22.8 \text{ rpm} = N_{d\max}$$

= Maximum speed of the pump from consideration of delivery pipe

The lower of the two values of speed obtained as above, viz. ($N_{d\max} = 22.8$ rpm) and ($N_{s\max} = 41.0$), is to be selected as the maximum speed of the pump. Hence for the given pump, the maximum speed is $N_{d\max} = 22.8$ rpm.

***EXAMPLE 6.17

A single-acting reciprocal pump lifts a liquid of specific weight 9.0 kN/m^3 from a pressurised storage reservoir to an overhead container. The free surface in the supply reservoir is at an elevation of 3.5 m above the centre of the pump. The ambient pressure over the liquid surface in the supply reservoir is 27 kN/m^2 (Vacuum). The other relevant data relating to the pump are as follows:

Length of suction pipe = 6.0 m	*Length of stroke = 50 cm*
Diameter of suction pipe = 10 cm	*Diameter of cylinder = 20 cm*
Minimum pressure anywhere in the system from cavitation considerations = 27 kPa (abs)	*Atmospheric pressure = 99 kN/m^2 (abs)*

Calculate the maximum speed admissible.

Solution

Refer to Fig. 6.17.

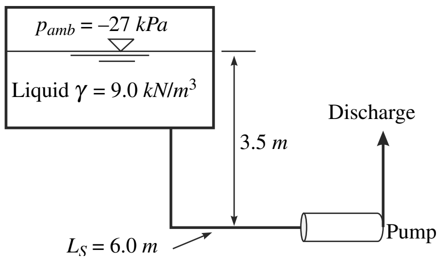


Fig. 6.17 *Example 6.17*

Radius of crank = r = 0.50/2 = 0.25 m

$$\left(\frac{A}{A_s}\right) = \left(\frac{20}{10}\right)^2 = 4.0$$

Acceleration head:

$$H_s = \frac{L_s}{g}\left(\frac{A}{A_s}\right)\omega^2 r\cos\theta$$

At start of the stroke $\theta = 0°$ and $\cos\theta = 1.0$

$$H_{as} = \frac{L_s}{g}\left(\frac{A}{A_s}\right)\omega^2 r$$

Refer to Fig.6.18 that gives salient geometry of the system.

$$H_{as} = \frac{6.0}{9.81} \times 4 \times \omega^2 \times 0.25 = 0.611\ \omega^2$$

Ambient pressure head in the supply reservoir = H_{amb}

$$= \frac{p_{amb}}{\gamma} = \frac{-27}{9} = -3 \text{ m of liquid (gauge)}$$

Atmospheric pressure = 99 kN/m^2 = $\frac{99}{9}$ = 11.0 m of liquid

Ambient pressure (abs) = – 3.0 + 11.0 = 8.0 m of liquid (abs)

Minimum pressure head in the cylinder = $H_{p\min} = H_{amb} - H_s - H_{as} = \frac{27}{9}$ m (abs)

$$3.0 = 8.0 + 3.5 - 0.6116\ \omega^2$$

$$0.6116\ \omega^2 = 8.5$$

$$\omega^2 = 13.9 \text{ leading to } \omega = \frac{2\pi N}{60} = 3.728$$

$$N = \frac{3.728 \times 60}{2\pi} = 35.6 \text{ rpm} = \text{Maximum speed admissible in the system}$$

*EXAMPLE 6.18

A reciprocating pump which is used to pump water has a bore of 120 mm and stroke of 220 mm. It runs at a speed of 40 rpm. The delivery pipe is 80 mm in diameter and 30 m in length. It is proposed to add an air vessel at a distance of 3 m from the cylinder. Determine the acceleration head with and without the air vessel.

Solution

(i) Without air vessel: $H_{ad} = \dfrac{L_d}{g}\left(\dfrac{A}{A_d}\right)\omega^2 r$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 40}{60} = 4.189 \text{ rad/s}, r = 0.22/2 = 0.11 \text{ m}$$

$$H_{ad} = \frac{30}{9.81} \times \left(\frac{120}{80}\right)^2 \times (4.189)^2 \times 0.11 = 13.28 \text{ m}$$

(ii) With air vessel, the length of pipe that undergoes acceleration reduces from 30 m to 3 m.

Hence, acceleration with air vessel $H_{ad1} = \dfrac{3}{30} \times 13.28 = 1.328$ m

*EXAMPLE 6.19

A single-acting reciprocating pump has a stroke length of 45 cm and a cylinder diameter of 30 cm. The suction pipe is 6 m long and has a diameter of 15 cm. The water level in the sump is 3.0 m below the pump axis.
(a) Calculate the maximum speed of the pump if the safe pressure head against cavitation is 2.5 m of water (abs).
(b) If an air vessel is fitted on the suction side at a length of 2.0 m from the cylinder, calculate the admissible maximum speed and the percentage change in the discharge. Take Darcy–Weisbach friction coefficient f = 0.02. Assume atmospheric pressure head = 10.0 m (abs).

Solution

(i) *Without Air Vessel*: $H_{as1} = \dfrac{L_s}{g}\left(\dfrac{A}{A_s}\right)\omega^2 r$

Here, $L_s = 6.0$ m, $\left(\dfrac{A}{A_s}\right) = \left(\dfrac{30}{15}\right)^2 = 4$ and $r = 0.45/2 = 0.225$ m

$$H_{as1} = \frac{6}{9.81} \times 4 \times 0.225 \times \omega^2 = 0.5504\,\omega^2$$

At limiting condition of safe pressure head: $H_{atm} = H_{p\min} + H_s + H_{as1}$

$$10.0 = 2.5 + 3.0 + 0.5504\,\omega^2$$

$$\omega^2 = \frac{10 - 5.5}{0.5504} = 8.176 \text{ giving } \omega = \frac{2\pi N}{60} = 2.859$$

$$N_1 = \frac{60 \times 2.859}{2\pi} = 27.3 \text{ rpm} = \text{Maximum speed of the pump without air vessel}$$

(ii) *With Air Vessel*: Acceleration pressure is confined to a 2.0 m length of pipe next to the cylinder. Friction loss in the remaining 4.0 m part of the suction pipe is constant over time as the flow is steady over there.

$$H_{as2} = \frac{L_{s2}}{g}\left(\frac{A}{A_s}\right)\omega^2 r = \frac{2.0}{9.81} \times 4 \times 0.225\ \omega^2$$

$$H_{as2} = 0.1835\ \omega^2$$

$$h_{fs1} = \frac{f\ L_{s1}\ V_s^2}{2gd_s} = \frac{0.02 \times 4.0}{2 \times 9.81 \times 0.15} V_s^2 = 0.0272\ V_s^2$$

However, V_s = Average steady velocity in the suction pipe =

$$\left(\frac{A}{A_s}\right)\frac{\omega\ r}{\pi} = \frac{4 \times 0.225}{\pi}\omega = 0.2865\ \omega$$

Hence, $h_{fs1} = 0.0272 \times (0.2865\ \omega)^2 = 0.002232\ \omega^2$

At limiting condition of safe pressure head: $H_{atm} = H_{p\,\min} + H_s + H_{as} + h_{fs1}$

$$10.0 = 2.5 + 3.0 + 0.1835\ \omega^2 + 0.002232\ \omega^2$$

$$0.185732\ \omega^2 = 4.5$$

$$\omega = \frac{2\pi N}{60} = 4.922$$

$$N_2 = \frac{60 \times 4.922}{2\pi} = 47.0 \text{ rpm} = \text{Maximum speed of the pump with air vessel}$$

Discharge $Q_t = \dfrac{ALN}{60}$

Ratio of discharges = $\dfrac{Q_2}{Q_1} = \dfrac{N_2}{N_1} = \dfrac{47.0}{27.3} = 1.722$

Percentage change in discharge due to fitting of air vessel = 72.2% increase.

***EXAMPLE 6.20

A single-acting reciprocating pump has a cylinder of 15 cm diameter and a stroke of 30 cm. The delivery pipe is 20 m long and has a diameter of 7.5 cm. The speed of the pump is 40 rpm and the discharges water to a height of 15 m. If an air vessel is fitted to the delivery side at a length of 1.0 m from the cylinder, calculate the pressure head at the beginning and middle of the delivery stroke. Take Darcy–Weisbach friction factor f = 0.02.

Solution

$$A = \frac{\pi}{4}(0.15)^2 = 0.0177 \text{ m}^2,\ L = 0.3 \text{ m}$$

For a single-acting pump, discharge

$$Q_t = \frac{ALN}{60} = \frac{0.0177 \times 0.3 \times 40}{60} = 0.00354 \text{ m}^3/\text{s}$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 40}{60} = 4.189 \text{ rad/s}, \quad r = \frac{0.30}{2} = 0.15 \text{ m}$$

$$\left(\frac{A}{A_d}\right) = \left(\frac{15}{7.5}\right)^2 = 4, \quad A_d = \frac{0.01777}{4} = 0.004425 \text{ m/s}$$

Steady velocity after the installation of air vessel = $V_d = \dfrac{Q_t}{A_d} = \dfrac{0.00354}{0.004425} = 0.8$ m/s

L_{da} = Length of the delivery pipe between the cylinder and the air vessel = 1.0 m

L'_d = Length of pipe with steady flow = 20 – 1 = 19.0 m

At the beginning of the delivery stroke:

Acceleration head = $H_{ad} = \dfrac{L_d}{g}\left(\dfrac{A}{A_d}\right)\omega^2 r$

Here, L_d = 1.0 m, $\left(\dfrac{A}{A_d}\right)$ = 4 and r = 0.15 m. Substituting in the expression for H_{ad},

$$H_{ad} = \frac{1}{9.81} \times 4 \times (4.189)^2 \times 0.40 = 1.073 \text{ m}$$

Friction head due to steady flow =

$$h_{f1} = \frac{f\, L'_d}{2gd} V_d^2 = \frac{0.02 \times 19}{2 \times 9.81 \times 0.075}(0.8)^2 = 0.165 \text{ m}$$

Pressure head on the piston: $H_{td} = H_d + H_{ad} + h_{fd1}$

$= 15.0 + 1.073 + 0.165 = 16.238$ m (gauge)

At the middle of the delivery stroke: Acceleration head = 0

Friction head $h_{fdt} = h_{fd1} + h_{fd2}$

h_{fd1} = Remains same as at the beginning of the stroke = 0.165 m

h_{fd2} = Friction head due to unsteady flow in pipe of length of L_{da}

$$h_{fd2} = \frac{f\, L_{da}}{2gd_d}\left(\frac{A}{A_d}\omega r\right)^2 = \frac{0.02 \times 1.0}{2 \times 9.81 \times 0.075}(4 \times 4.189 \times 0.15)^2 = 0.086 \text{ m}$$

Pressure head on the piston: $H_{td} = H_d + h_{fd1} + h_{fd2}$

$= 15.0 + 0.165 + 0.086 = 15.251$ m (gauge)

**EXAMPLE 6.21

A single-acting reciprocating pump has the following data:

Diameer of the cylinder = 25 cm	Stroke length = 40 cm
diameter of the delivery pipe = 15 cm	Crank speed = 40 rpm
Length of delivery pipe = 20.0 m	

A large-diameter air vessel is fitted to the delivery pipe. Determine the quantity of water going in or coming out of the air vessel when the crank angle is (a) 15°, (b) 90°, and (c) 120°. Also, determine the crank angle at which there is no flow in to or out of the air vessel.

Solution

$$A = \frac{\pi}{4} \times (0.25)^2 = 0.0491 \text{ m}^2, L = 0.4 \text{ m}$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 40}{60} = 4.1888 \text{ rad/s}, r = \frac{0.40}{2} \text{ m} = 0.20 \text{ m}$$

$$\left(\frac{A}{A_d}\right) = \left(\frac{25}{15}\right)^2 = 2.778 \qquad A_d = \frac{0.0491}{2.778} = 0.01768 \text{ m}^2$$

Since the pump is single-acting,

$$\text{Time-averaged discharge } Q_t = \frac{ALN}{60} = \frac{0.0491 \times 0.4 \times 40}{60} = 0.01309 \text{ m}^3/\text{s}$$

v_i = Instantaneous velocity of unsteady flow

$$= \frac{A}{A_d} \omega r \sin\theta = 2.778 \times 4.1888 \times 0.20 \sin\theta = 2.3273 \sin\theta$$

Discharge in this part of the pipe at any time

$$Q_i = v_i A_d = 2.3273 \sin\theta \times 0.01768 = 0.04115 \sin\theta$$

Discharge into/out of air vessel: $Q_{in} = (Q_t - Q_i) = (0.01309 - 0.04115 \sin\theta)$ valid for θ in the range 0° to 90°. For $\theta > 90°$, $Q_i = 0$

If $Q_t < Q_i$, then flow goes out of the air vessel.

If $Q_t > Q_i$, then flow goes in to the air vessel.

Values of $(Q_t - Q_i)$ area calculated for the given two values of θ are as shown below:

Crank Angle θ	Q_t m³/s	Q_i m³/s	$Q_{in} = (Q_t - Q_i)$	Remark
15°	0.01309	0.01065	0.00244	Flow goes out of air vessel
90°	0.01309	0.04115	−0.0281	Flow goes into the air vessel

When there is zero flow into or out of the air vessel

$$Q_{in} = (Q_t - Q_i) = (0.01309 - 0.04115 \sin\theta) = 0$$

$$\sin\theta = \frac{0.01309}{0.04115} = 0.3181$$

θ = 18.55° of the delivery stroke

*EXAMPLE 6.22

A single-acting reciprocating pump has an air vessel in the delivery side fitted very close to the cylinder. The cylinder has a diameter of 30 cm and a stroke length of

45 cm. The delivery pipe is 40 m long and has a diameter of 20 cm. the speed of the pump is 60 rpm. Determine the power saved by the air vessel in overcoming friction in the delivery pipe. Take Darcy–Weisbach friction factor f = 0.03.

Solution

(a) *Without Air Vessel*: $A = \frac{\pi}{4} \times (0.30)^2 = 0.07069 \text{ m}^2$, $L = 0.45$ m

For a single-acting pump, discharge

$$Q_t = \frac{ALN}{60} = \frac{0.07069 \times 0.45 \times 60}{60} = 0.0318 \text{ m}^3/\text{s}$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 60}{60} = 6.283 \text{ rad/s}, \quad r = \frac{0.45}{2} = 0.225 \text{ m}$$

$$\left(\frac{A}{A_d}\right) = \left(\frac{30}{20}\right)^2 = 2.25, \quad A_d = \frac{A}{2.25} = \frac{0.07069}{2.25} = 0.03142 \text{ m}^2$$

Maximum friction head =

$$h_{fdm} = \frac{f L_d}{2gd_d}\left(\frac{A}{A_d}\omega r\right)^2 = \frac{0.03 \times 40}{2 \times 9.81 \times 0.20}(2.25 \times 6.283 \times 0.225)^2 = 3.094 \text{ m}$$

Time-averaged friction head $h_{fd2} = \frac{2}{3} h_{fdm} = \frac{2}{3} \times 3.094 = 2.063$ m

(b) *With Air Vessel*: The velocity is steady in the delivery pipe with an average value of

$$V_d = \frac{Q_t}{A_d} = \frac{0.0318}{0.03142} = 1.012 \text{ m/s}$$

Frictional head loss in delivery pipe,

$$h_{fd1} = \frac{f L_d}{2gd_d} V_d^2 = \frac{0.30 \times 40}{2 \times 9.81 \times 0.20} \times (1.012)^2 = 0.3132 \text{ m}$$

Frictional power saved due to air vessel in the delivery pipe

$$P_{save} = \gamma Q_t (h_{fd2} - h_{fd1}) = 9.79 \times 0.0318 \times (2.068 - 0.3132) = 0.545 \text{ kW}$$

*EXAMPLE 6.23

Following data pertain to a horizontal single acting reciprocating pump installation pumping water:

Cylinder diameter = 15 cm	*Crank speed = 60 rpm*
Stroke length = 20 cm	*Delivery pipe diameter = 7.5 cm*
Static suction head = 4.0 m	*Length of delivery pipe = 30.0 m*
Diameter of suction pipe = 7.5 cm	*Static delivery head = 25.0 m*
Suction pipe length = 8.0 m	*Darcy–Weisbach friction factor f = 0.025*

Air vessels are fitted very near to the cylinder on both the suction and the delivery sides. Estimate the power required to drive the pump by assuming the pump efficiency as 80%.

Solution

$$A = \frac{\pi}{4} \times (0.15)^2 = 0.01767\ \text{m}^2,\ L = 0.2\ \text{m}$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 60}{60} = 6.283\ \text{rad/s}, \qquad r = \frac{0.20}{2} = 0.10\ \text{m}$$

$$\left(\frac{A}{A_s}\right) = \left(\frac{15}{7.5}\right)^2 = 4 \qquad A_d = A_s = \frac{0.01767}{4} = 0.004179\ \text{m}^2$$

Since the pump is single acting,

$$\text{Discharge } Q_t = \frac{ALN}{60} = \frac{0.01767 \times 0.2 \times 60}{60} = 0.003534\ \text{m}^3/\text{s}$$

Since the air vessels are very near to the cylinder, it can be safely assumed that the entire suction pipe and the delivery pipe will have steady flow.

$V_s = V_d$ = Velocity in suction and delivery pipes =

$$\frac{Q_t}{A_s} = \frac{Q_t}{A_p} = \frac{0.003534}{0.004179} = 0.8457\ \text{m/s}$$

$$\frac{V_s^2}{2g} = \frac{V_d^2}{2g} = \frac{(0.8457)^2}{2 \times 9.81} = 0.0365\ \text{m}$$

Hence, $$h_{fd} = \frac{f L_d}{2gd_d} V_d^2 = \frac{0.025 \times 30}{2 \times 9.81 \times 0.075} \times (0.8457)^2 = 0.364\ \text{m}$$

$$h_{fs} = \frac{f L_s V_s^2}{2gd_s} = \frac{0.025 \times 8.0}{2 \times 9.81 \times 0.075} \times (0.8457)^2 = 0.097\ \text{m}$$

Power required, by using Eq. (6.59),

$$P = \frac{\gamma Q_t}{\eta_0}\left([H_d + \frac{V_d^2}{2g} + H_{ad} + h_{fd}] + [H_s + \frac{V_s^2}{2g} + H_{as} + h_{fs}]\right)$$

Noting that $H_{ad} = H_{as} = 0$,

$$P = \frac{9.79 \times 0.003534}{0.8}\ ([25 + 0.0365 + 0.364] + [4.0 + 0.03654 + 0.097])$$

$$= 1.277\ \text{kW}$$

[Note the small value of velocity heads in delivery and suction pipes. They can safely be neglected and Eq. (6.59-a) be used to calculate the power required. The resulting power requirement comes out as 1.274 kW.]

*EXAMPLE 6.24

Show that when a large air vessel is fitted in a pipe (suction/delivery)at a distance x from the cylinder of a single-acting reciprocating pump, the savings in energy in that pipe due to reduction in frictional resistance head is $\left(0.848\left(1-\dfrac{x}{L_0}\right)\right)$ *where L_0 is the total length of the pipe.*

Solution

Consider a suction pipe of a reciprocal pump as shown in Fig. 6.18. The length of the pipe is L_0 and its diameter is d. An air vessel of sufficient capacity is fitted at a distance x from the cylinder of the pump. Stretch BC of the pipe has unsteady flow and the length of this reach is x. The stretch AB, in Fig. 6.18 has steady flow and the length of this reach is $(L_0 - x)$.

Let V = Average velocity in the pipe. In reach BC fluctuation of velocity is sinusoidal with a maximum velocity v_m related to time averaged velocity V as

$$V = \frac{v_m}{\pi}$$

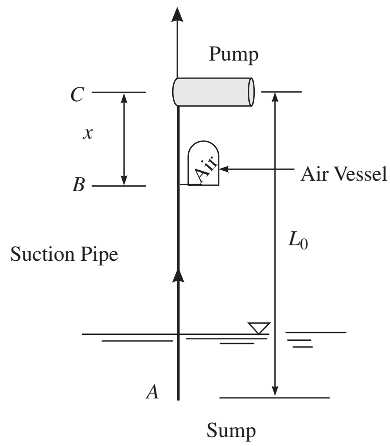


Fig. 6.18 *Example 6.24*

Flow with Air Vessel:

Total frictional head loss

$H_{ft} = h_{f1}$ in reach AB + h_{f2} in reach BC

$$h_{f1} = \frac{f(L_0 - x)}{2gd}V^2 = \frac{f(L_0 - x)}{2gd}\frac{v_m^2}{\pi^2} = \frac{(L_0 - x)}{\pi^2}K$$

where $K = \dfrac{fv_m^2}{2gd}$

$$h_{f2} = \frac{2}{3}\frac{fx}{2gd}v_m^2 = \frac{2}{3}xK$$

Total frictional head loss $H_{ft} = h_{f1} + h_{f2} = \left[\dfrac{(L_0 - x)}{\pi^2} + \dfrac{2}{3}x\right]K$

Without Air Vessel: The frictional loss is due to unsteady flow.

Total frictional head loss $h_{f0} = \dfrac{2}{3}\dfrac{L_0 f v_m^2}{2gd} = \dfrac{2}{3}L_0K$

∴ Reduction in frictional head due to installation of air vessel (expressed as a fraction of initial head loss)

$$\varepsilon = \frac{(h_{f0} - (h_{f1} + h_{f2}))}{h_{f0}} = \frac{\dfrac{2}{3}L_0\, K - \left[\dfrac{(L_0 - x)}{\pi^2} + \dfrac{2}{3}x\right]K}{\dfrac{2}{3}L_0K}$$

$$= \frac{3}{2}\left[\frac{2}{3} - \left(\frac{\left(1-\frac{x}{L_0}\right)}{\pi^2} + \frac{2}{3}\left(\frac{x}{L_0}\right)\right)\right]$$

$$= 1 - \frac{3}{2\pi^2} + \left(\frac{3}{2\pi^2}\right)\left(\frac{x}{L_0}\right) - \left(\frac{x}{L_0}\right)$$

$$= \left(1 - \frac{3}{2\pi^2}\right)\left(1 - \frac{x}{L_0}\right)$$

Percentage of savings in energy loss due to frictional resistance

$$= \varepsilon\,\% = 0.848\left(1 - \frac{x}{L_0}\right)\%$$

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6.3 Tackett, H H, *Positive Displacement Reciprocating Pump Fundamentals–Power and Direct Acting Types,* Proc. 24th Intl. Pump Users Symposium, USA, 2008.
• ***www.turbolab.tamu.edu/uploads/files/papers/p24/P24Tut01.pdf***

Review Questions

6.1 Define *slip* in a reciprocating pump. Explain the phenomenon of negative slip.

6.2 Explain with the help of appropriate sketches the difference among (a) a double-acting simplex pump, (b) a single-acting duplex pump, and (c) a double-acting duplex pump.

6.3 Draw sketches of two-throw and three-throw reciprocating pumps along with their discharge delivery diagrams.

6.4 Differentiate between
(a) Single-acting and double-acting reciprocating pumps,
(b) Single-cylinder and double-cylinder reciprocating pump.

6.5 Draw an indicator diagram of a single-acting reciprocating pump considering acceleration and friction. What would be the change in the indictor diagram if an air vessel is attached (a) to suction side only, (b) to the delivery side only, and (c) to both the suction and delivery sides.

6.6 Draw indicator diagram for the following cases of a reciprocating pump:
(a) When no air vessel is installed
(b) When air vessel is installed on suction side close to the pump
(c) When air vessel is installed on delivery side close to the pump
(d) When air vessels are installed on both sides of the pump and both are close to the pump.

6.7 Define separation in reciprocating pumps and explain its importance.
6.8 What are the factors limiting the suction lift of a reciprocating pump?
6.9 List the factors affecting the maximum speed of a reciprocating pump.
6.10 For a given pump, list the factors that control the suction lift of a pump.
6.11 Why is the speed of a reciprocating pump lower than that of a centrifugal pump? On what factors does the speed of the reciprocating pump depend?
6.12 Explain how the separation of flow is caused in reciprocating pumps. What preventive measures are usually taken to reduce the same appreciably?
6.13 Write a brief note on the salient operational characteristics of a Reciprocating pump.
6.14 How can flow rate of a reciprocating pump be changed?
6.15 Why is it necessary to have a relief valve in a system with a positive displacement pump?
6.16 Write a note on air vessels and list clearly the advantages of using an air vessel in (a) suction side, and (b) delivery side of a pump.
6.17 Derive expressions for the instantaneous velocity and acceleration in the suction and discharge pipes of a single acting reciprocating pump. Show that the instantaneous velocity in the pipe is equal to the time averaged velocity in the pipes when the crank angle $\theta = 18.56^{o}$. What is the corresponding angle for a double-acting pump?
6.18 List the salient differences between centrifugal pumps and reciprocating pumps.
6.19 List at least five major applications where reciprocating pumps are the ideal choice for handling the liquids involved.
6.20 Write a brief note on cavitation in reciprocating pumps.
6.21 List a few specialty uses of reciprocating pumps.

Problems

P6.1 ** A single-acting reciprocating pump used to lift kerosene has the following characteristics:

Piston diameter = 30 cm	Stroke length = 50 cm
Speed = 40 rpm	Total lift = 25 m

If the discharge of kerosene (ρ = 680 kg/m^3) delivered by the pump at the outlet is 1380 litres/minute, calculate the slip, coefficient of discharge and theoretical power required to drive the pump.

[***Ans:*** ε_s = 2.4%, C_d = 0.976, P_t = 3.93 kW]

P6.2 * A double-acting reciprocating pump, running at 40 rpm is discharging 1.0 m^3/minute. The pump has a stroke of 40 cm and the diameter of the piston is 20 cm. The piston rod has a diameter of 5 cm. The delivery and suction heads are 20 m and 5 m respectively. The overall efficiency of the pump is known to be 89%. Find the slip of the pump and the power required to drive the pump. Also, estimate the mechanical efficiency of the pump.

[***Ans:*** ε_s = — 2.7%, P = 4.58 kW, η_m = 86.7%]

P6.3 * A single-acting reciprocating pump has a 10 cm diameter piston with a stroke length of 30 cm. At a speed of 75 rpm, a discharge of 181 litres/minute of liquid was measured. Find the coefficient of discharge and slip of the pump. [***Ans:*** C_d = 1.024, ε_s = –2.553 (negative slip)]

P6.4 *A single-acting reciprocating pump running at 30 rpm delivers 700 liters /minute of a liquid of density 917 kg/m^3. The diameter of the piston is 25 cm and stroke length is 50 cm. Determine the (a) theoretical discharge of the pump, (b) co-efficient of discharge, and (c) volumetric efficiency of the pump. If the total lift of pumping is 25 m and friction and other losses can be considered to be 10% of the total lift, estimate the power required to drive the pump. Assume a mechanical efficiency of 92% for the pump. [***Ans:*** Q_t = 736.3 litres/min, η_v = 95%, P = 3.473 kW]

P6.5 **A double-acting reciprocating pump has a cylinder of 20 cm diameter and stroke of 30 cm. The speed of the pump is 30 rpm. Estimate the maximum velocity and acceleration in the suction pipe of 20 cm diameter and delivery pipe of 25 cm diameter.
[***Ans:*** v_{sm} = 0.471 m/s, a_{sm} = 1.480 m/s^2, v_{dm} = 0.3016 m/s, a_{dm} = 0.9475 m/s^2]

P6.6 **The following data refers to a single-acting horizontal reciprocating pump lifting water from a storage tank.

Length of stroke = 15 cm	Diameter of piston = 15 cm
Speed of the crank = 60 rpm	Length of suction pipe = 6.0 m
Diameter of suction pipe = 7.5 cm	

Calculate the acceleration head at (a) the beginning, (b) middle, and (c) end of the suction stroke. If the water level in the storage tank is 3.5 m above the axis of the cylinder, calculate the magnitude of the maximum suction force on the cylinder. Take atmospheric pressure as 10.3 m of water.
[***Ans:*** (a) 7.243 m (b) 0 (c) —7.243 m, F_p = 0.648 kN]

P6.7 **A single-acting reciprocating pump has a piston of 10 cm diameter and a stroke of 30 cm. The length and diameter of the suction pipe are respectively 10 m and 7.5 cm. The speed of the pump is 30 rpm. Find the absolute pressure in the cylinder at the beginning of the suction stroke (a) when the water level in the supply tank is 3.5 m below the centre of the pump, and (b) when the water level in the supply tank is 3.5 m above the centreline of the pump. Take the atmospheric pressure as 100 kPa.
[***Ans:*** H_{ts} = 60.532 kN (abs), H_{ts} = 108.0 kN (abs)]

P6.8 **The length and diameter of the delivery pipe of a single-acting reciprocating pump are 40 m and 10 cm respectively. The pump has a plunger of 15 cm diameter and a stroke of 35 cm. The delivery head is 30 m above the centre of the sump. The atmospheric pressure head is 10.3 m of water and the speed of the pump is 35 rpm. Determine the acceleration head and total pressure head on the piston at the (a) beginning, (b) middle, and (c) end of the delivery stroke. Take Darcy–Weisbach friction factor f = 0.02. [***Ans:***]

θ	H_{as} (m)	H_{ts} (gauge)	θ	H_{as} (m)	H_{ts} (gauge)	θ	H_{as} (m)	H_{ts} (gauge)
0°	21.565	51.565 m	90°	0	30.85 m	180°	— 21.57 m	8.435 m

P6.9 ** A single-acting reciprocating pump has a stroke length of 15 cm. The suction pipe is 7 m long. The water level in the sump is 2.5 m below the cylinder. The diameters of the suction pipe and the plunger are 7.5 cm and 10.0 cm. If the speed of the pump is 75 rpm, determine the pressure head on the piston at the (a) beginning, (b) middle, and (c) end of the suction stroke. Take Darcy–Weisbach friction factor $f = 0.02$.

[***Ans:*** H_{ts} (gauge): – 8.37 m, – 2.604 m, + 3.370 m]

P6.10 ** Following data pertain to a double-acting reciprocating pump installation:

Cylinder diameter = 15 cm	Crank speed = 45 rpm
Stroke length = 30 cm	Delivery-pipe diameter = 7.5 cm
Static suction head = 4.0 m	Length of delivery pipe = 30.0 m
Diameter of suction pipe = 7.5 cm	Static delivery head = 25.0 m
Suction-pipe length = 8.0 m	Darcy–Weisbach friction factor $f = 0.025$

Calculate the acceleration head and friction head in the suction and delivery pipes at the instant when the crank angle measured from the inner dead centre is 60°.

[***Ans:*** H_{ad} = 20.37 m, h_{fd} = 3.056 m, H_{as} = 5.433 m, h_{fs} = 0.815 m]

P6.11 ** Following data pertain to a single-acting reciprocating pump installation:

Cylinder diameter = 15 cm	Crank speed = 60 rpm
Stroke length = 20 cm	Delivery-pipe diameter = 7.5 cm
Static suction head = 4.0 m	Length of delivery pipe = 30.0 m
Diameter of suction pipe = 7.5 cm	Static delivery head = 25.0 m
Suction-pipe length = 8.0 m	Darcy–Weisbach friction factor $f = 0.025$

Estimate the power required to drive the pump by assuming an overall efficiency of 80%. The liquid being pumped is water.

[***Ans:*** P = 1.372 kW]

P6.12 * A double-acting reciprocating pump has a piston of 30 cm diameter and the diameter of the piston rod is 6 cm. The length of the stroke is 35 cm and the crank moves at a speed of 75 rpm. The suction and delivery heads are 5.0 and 35.0 m respectively. Estimate the discharge capacity of this pump. If the liquid to be pumped has a specific weight of 8.0 kN/m^3, estimate the power required to pump the liquid. Assume the pump efficiency as 85% and allow 5% of total lift to account for frictional losses.

[***Ans:*** Q = 0.0606 m^3/s, P = 23.96 kW]

P6.13 ** A single-acting, single-cylinder reciprocating pump used to pump water has the following details:

Stroke length = 400 mm	Piston diameter = 400 mm
Speed = 20 rpm	Suction head = 4 m
Delivery head = 20 m	Diameter of delivery pipe = 200 mm
Diameter of suction pipe = 200 mm	Length of delivery pipe = 30 m
Length of suction pipe = 6 m	Darcy–Weisbach friction factor $f = 0.02$
Pump efficiency = 0.85	

Estimate the power required to drive the pump.

[***Ans:*** P = 4.696 kW]

P6.14 *A single-acting reciprocating pump has the following characteristics:

Diameter of the cylinder = 15 cm	Length of stroke = 30 cm
Suction head = 2.5 m	Diameter of suction pipe = 5.0 cm
Length of suction pipe = 5.0 m	

The minimum pressure head in the system is limited to 2.0 m (abs) from cavitation considerations. Assuming the atmospheric pressure as 10.0 m water, determine the maximum speed at which the pump can be run.

[***Ans:*** N = 27.0 rpm]

P6.15 *A single-acting reciprocating pump has a 20 cm diameter piston with a crank of 40 cm radius. The delivery pipe is 10 cm in diameter and 45 m long. Water is lifted to a height of 40 m above the axis of the horizontal cylinder. Find the maximum speed at which the pump can run without cavitation. Assume (a) atmospheric pressure at the site as 9.75 m of water, and (b) the minimum pressure head in the system is limited to 2.75 m (abs).

[***Ans:*** N = 24.16 rpm]

P6.16 *The stroke of a double-acting single cylinder reciprocating pump is 37.5 cm and the speed is 40 rpm. The diameter of the piston is 30 cm. The pump is horizontal and the axis is 3.5 m above the water level in the sump. The suction pipe is 7.5 m long. Find the diameter of the suction pipe such that the minimum stipulated pressure head from cavitation considerations is not violated. The minimum pressure head is 2.5 m (abs). Take atmospheric pressure is 10.0 m of water (abs) [***Ans:*** d_s = 23.78 cm]

P6.17 **A double-acting reciprocating pump has a 20 cm cylinder with a stroke of 20 cm. The suction pipe is 15.0 long and has a diameter of 20 cm. If the speed of the pump is 60 rpm, determine the discharge and the maximum suction lift if *NPSHR* is 2.0 m (abs). A margin of 1.0 m above NPSHR is mandatory. Assume atmospheric pressure as 10.1 m of water (abs).

[***Ans:*** Q_t = 0.00628 m^3/s, H_s = 1.064 m.]

P6.18 **Following are the details of a single-acting, single-cylinder reciprocating pump.

Length of stroke = 50 cm	Suction lift = 3.25 m
Diameter of cylinder = 12.5 cm	Delivery lift = 12.0 m
Length of suction pipe = 5.2 m	Atmospheric pressure head = 10.3 m of water (abs)
Diameter of suction pipe = 10.0 cm	Darcy–Weisbach friction factor f = 0.02

Assuming the safe minimum pressure head as 2.5 m,

(a) Calculate the maximum permissible speed of the pump.

(b) If an air vessel is fixed very close to the cylinder in the delivery pipe, calculate the power required to pump water. Assume the frictional head in the delivery pipe to be 0.15 m and the velocity heads in the pipes can be neglected. Efficiency of the pump can be assumed as 90%.

[***Ans:*** N = 44.8 rpm, P = 1.0 kW]

P6.19 ** A single-acting reciprocating pump has a stroke length of 37.5 cm and a cylinder diameter of 22.5 cm. The suction pipe is 12 m long and has a diameter of 15 cm. The water level in the sump is 3.0 m below the centre of the pump. If the speed of the pump is 20 rpm, determine pressure head on the piston at the beginning of the stroke (a) when there is no air vessel in the suction pipe, and (b) when an air vessel is fitted to the suction pipe at the level of the cylinder and at a distance of 1.5 m from the cylinder. Take Darcy–Weisbach friction factor f = 0.02.

[***Ans:*** H_{p0} = – 5.628 m, – 3.2884 m]

P6.20 ** A single-acting reciprocating pump has a cylinder of 15 cm diameter and a stroke length of 30 cm. The delivery pipe is 30 m long and has a diameter of 7.5 cm. The speed of the pump is 60 rpm. Determine the power saved in overcoming friction in the delivery side by fitting an air vessel very near the cylinder on the delivery pipe. Take Darcy–Weisbach friction factor f = 0.02. [***Ans:*** P_{sav} = 0.17 kW]

P6.21***A double-acting reciprocating pump has the following data.

Diameter of cylinder = 20 cm	Length of stroke = 40 cm
Diameter of suction pipe =10.0 cm	Speed of the crank = 90 rpm

If an air vessel is fitted on the suction side, calculate the flow in to or out of the air vessel when the crank angle is (a) 45°, and (b) 150°.

[***Ans:*** Q_{in} = — 0.00419 m^3/s, + 0.00807 m^3/s]

P6.22 ** Following data pertain to a single-acting reciprocating pump installation pumping water:

Cylinder diameter = 10 cm	Crank speed = 60 rpm
Stroke length = 20 cm	Delivery-pipe diameter = 10.0 cm
Static suction head = 4.0 m	Length of delivery pipe = 30.0 m
Diameter of suction pipe = 7.5 cm	Static delivery head = 25.0 m
Suction pipe length = 8.0 m	Darcy–Weisbach friction factor f = 0.025

Air vessels are fitted very near to the cylinder on the suction side and at a distance of 3 m from the cylinder on the delivery side. Estimate the power required to drive the pump by assuming the pump efficiency as 90%.

[***Ans:*** P = 0.517 kW]

P6.23 ** Show that in a suction pipe of a single-acting reciprocating pump, the maximum acceleration head H_{as} is related to the theoretical discharge Q_t and angular velocity ω as $H_{as} = \frac{L_s}{g}\frac{\pi Q_t \omega}{A_s}$ where L_s and A_s are the length and area of the suction pipe respectively.

Objective-type Questions

O6.1 *In positive-displacement pumps, the slip can sometimes be negative when the actual discharge is greater than the theoretical discharge. This happens in

(a) small suction pipes coupled with low delivery head
(b) small suction pipes coupled with medium delivery head
(c) long suction pipes coupled with low delivery head
(d) long suction pipes coupled with medium delivery head

O6.2 *In a reciprocating pump without air vessels, the friction head in the delivery pipe is maximum at crank angle θ =

(a) 0° (b) 90° (c) 60° (d) 180°

O6.3 *In a reciprocating pump without air vessels, the acceleration head in the suction pipe is maximum at the crank angle value of θ =

(a) 0° (b) 90° (c) 60° (d) 180°

O6.4 **In a single-acting reciprocating pump, without air vessels, the average velocity in delivery pipe is given by V_d =

(a) $\left(\frac{A}{A_d}\right)\omega r$ (b) $\left(\frac{A}{A_d}\right)\frac{\omega r}{\pi}$ (c) $\left(\frac{A}{A_d}\right)\frac{\pi\omega r}{2}$ (d) $\left(\frac{A}{A_d}\right)\frac{\omega r}{2\pi}$

where ω = Angular velocity of the crank, r = Radius of crank, A = Area of cylinder, A_d = Area of delivery pipe.

O6.5 **In a single reciprocating pump without air vessels, the ratio of the time-averaged friction head to the instantaneous maximum friction head in delivery pipe is given by $\frac{h_{fdav}}{h_{fdm}}$ =

(a) 1/2 (b) 1/3 (c) 2/3 (d) 4/3

O6.6 *The indicator diagram of a reciprocating pump is a plot of

(a) work done *vs* stroke length
(b) acceleration head *vs* stroke length
(c) pressure head *vs* stroke length
(d) crank speed *vs* power developed

O6.7 **A double-stoke reciprocating pump has its stroke length reduced by half and the speed is doubled. This would cause the discharge to

(a) decrease by 25% (b) remain unaltered
(c) decrease by 50% (d) increase by 100%

O6.8 **Figure 6.19 shows an indicator diagram of a single-acting reciprocating pump. This indictor diagram belongs to

(a) a reciprocating pump with two air vessels, one each in suction pipe and delivery pipe at some distance from the cylinder
(b) a reciprocating pump with one air vessel
(c) a reciprocating pump with two ideal air vessels
(d) a reciprocating pump with no air vessels

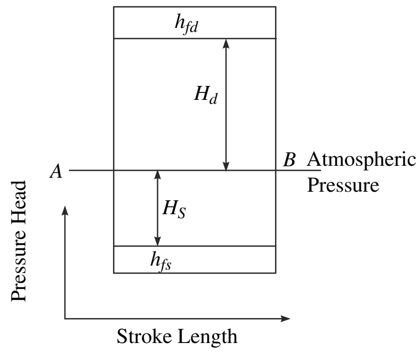


Fig. 6.19 *Objective Question O6.8*

O6.9 ** As compared to single-acting pump, a double-acting reciprocating pump has nearly
(a) double head (b) double efficiency
(c) double flow (d) double speed

O6.10***If pump *NPSHR* is not satisfied,
(a) efficiency will be low
(b) cavitation will take place
(c) the pump will not develop necessary head
(d) the pump will consume excessive power

O6.11 ** A reciprocating water pump delivers 100 litres of water per second against a suction head of 5 m and a delivery head of 15 m. If the overall efficiency of the pump is 80%, the required power to drive the pump is near about
(a) 13 kW (b) 20 kW (c) 25 kW (d) 30 kW

O6.12 ** Saving of work done in overcoming frictional resistance in a pipe (suction/delivery) by fitting an air vessel to a double-acting reciprocating pump is
(a) 49.2% (b) 84.8% (c) 39.2% (d) 66.6%

O6.13 ** In a double-acting, single-cylinder reciprocating pump, the ratio of time-averaged velocity to the instantaneous maximum velocity in the delivery pipe is given by $\frac{V_d}{v_{dm}} =$
(a) $\frac{1}{2\pi}$ (b) $\frac{2}{\pi}$ (c) $\frac{1}{\pi}$ (d) $\frac{2}{3\pi}$

O6.14 ** In a double-acting, single-cylinder reciprocating pump, the theoretical discharge is given by
(a) $\frac{ALN}{60}$ (b) $\frac{2ALN}{60}$ (c) $\left(\frac{ALN}{60}\right)^2$ (d) $\frac{2AN}{60}$

O6.15 ** In a single-acting, single-cylinder, reciprocating pump the theoretical discharge is 50 litres/minute. The actual discharge is measured to be 49 litres/minute. The slip of this pump is
(a) 98% (b) –2% (c) 2% (d) 0.98

O6.16 ** A double-acting duplex reciprocating pump has
(a) 4 pistons (b) 4 cylinders (c) 2 pistons (d) 1 piston

O6.17***In a triplex single-acting reciprocating pump without air vessels, the ratio
$\left[\frac{(\text{Maximum} - \text{Minimum})}{\text{Average}}\right]$ discharge in the delivery pipe is about
(a) 60% (b) 46% (c) 23% (d) 7%

O6.18 ** Consider the following statements:
Before starting a reciprocating pump, the following actions have to be performed:
(1) Completely close the delivery pipe valve
(2) Prime the pump
(3) Reduce the speed of the drive to the lowest possible value
(4) Completely open the delivery valve
The correct statements are
(a) 1 and 2 (b) 2 and 4 (c) 1,2 and 3 (d) 2, 3 and 4

O6.19 ** Consider the following statements:
To maximise the *NPSHA* (Net Positive Suction Head Available) of a reciprocating pump,
(1) reduce suction-pipe friction loss
(2) increase ambience pressure at supply reservoir
(3) increase the pump speed
(4) increase static suction head
The correct statements are:
(a) 1 and 2 (b) 3 and 4 (c) 1,2 and 3 (d) 1, 2, 3 and 4

O6.20 ** In a single-acting reciprocating pump, the acceleration head at the beginning of the suction stroke is 4.0 m. If the pump is 2.0 m below the water level in the supply reservoir the pressure head at the cylinder by considering the atmospheric pressure as 10 m is
(a) 4.0 m (abs) (b) 12. 0 m (abs)
(c) 8.0 m (abs) (d) 6.0 m (abs)

O6.21 * Which of the following statements pertaining to single-acting reciprocating pump without air vessels is correct?
(a) Reciprocating pumps are less efficient than the centrifugal pumps.
(b) Delivery from the reciprocating pump will be pulsating.
(c) Reciprocating pumps are suitable for large discharges under small heads.
(d) For negative slip to occur, a reciprocating pump must have a negative suction head.

O6.22 * An air vessel in the delivery side of a reciprocating pump
(a) maintains steady discharge output
(b) prevents cavitation in the pump
(c) enables the pump to run at higher speeds
(d) enables the suction head to be increase.

O6.23 * The chief function of an air vessel in the delivery side of a reciprocating pump is to obtain
(a) reduction in suction head

(b) rise in delivery head
(c) continuous supply of discharge at uniform rate
(d) increase in the discharge magnitude

O6.24 *Consider the following statements:

Air vessels are fitted to the suction side of a reciprocating pump
(1) to achieve higher speed without separation
(2) reduce work done in overcoming frictional resistance
(3) have uniform discharge
(4) avoid excessive pressure fluctuation and vibration

The correct statements are
(a) 1, 2 and 4
(b) 1 and 2 only
(c) 3 and 4 only
(d) 2, 3 and 4

O6.25 *Chief objective of using air vessels in the suction side of reciprocating pumps is to
(a) increase the delivery head
(b) reduce suction head
(c) minimise delivery head fluctuation
(d) reduce accelerating head

SEVEN

Miscellaneous Hydraulic Machinery and Devices

A: HYDRAULIC DEVICES

7.1 INTRODUCTION

While the turbines and pumps described in the previous chapters form important hydraulic machines, there are a host of other hydraulic machines and devices that are widely used as components of a fluid system. A few of these devices and a few interesting types of pumps are briefly described in this chapter. The list of devices described in this chapter include

- Hydraulic press
- Hydraulic accumulator
- Hydraulic intensifier
- Hydraulic winch
- Jigger
- Hydraulic crane
- Hydraulic lift
- Fluid coupling
- Fluid torque converter
- Hydraulic actuator

The pumps described in this chapter are

- Gear pump
- Lobe pump
- Sliding-vane pump
- Screw pump
- Piston pump
- Diaphragm pump
- Hydraulic ram pump
- Jet pump
- Airlift pump
- Vertical-turbine pump
- Submersible pump
- Regenerative pump

7.2 HYDRAULIC SYSTEMS

7.2.1 Types of Hydraulic Systems

A hydraulic system is a circuit in which power and forces are transmitted through a liquid. These can be classified into two categories as (1) hydrostatic systems, and (2) hydrodynamic systems

1. Hydrostatic Systems

In these systems the power and force are transmitted primarily by the fluid static pressure. The motivating force in such systems is a change in pressure whereas the velocity of the fluid usually remains constant. There is no energy transfer as kinetic energy to and from the fluid. The hydraulic press, hydraulic accumulator and hydraulic intensifier are examples of hydrostatic systems.

2. Hydrodynamic System

In this form of power transmission, energy is transferred by a change in velocity or kinetic energy. The change in the pressure of the working fluid is generally of no consequence. Fluid coupling and torque converter are typical examples of this type of system.

7.2.2 Basic Principles

The basic principles of hydraulic systems of hydrostatic type are indeed very simple to comprehend. The fluid to be used should be incompressible and hence oil is the most common medium. The hydraulic system consists of a closed unit and the liquid is confined in a set of interconnected containers and connecting tubing. The following four basic principles are applicable to this system:

1. Force applied at any one point is transmitted to another point.
2. The applied force is multiplied (or divided) depending on the geometry of the system.

   The above two principles can be comprehended by the following example. Consider two interconnected piston-cylinder units as in Fig. 7.1(a).

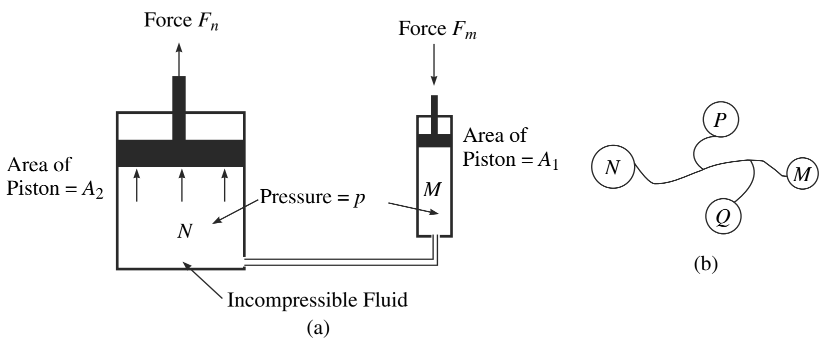


Fig. 7.1 *Basics of hydrostatic systems*

The cylinders M and N are interconnected and contain an incompressible fluid. Force F_m applied on the piston at M transforms into pressure $p = F_m/A_1$ where A_1 is the area of cylinder M. By *Pascal's law*, the pressure is common to all points in the fluid and hence it will also be acting on the bottom of the piston/ram of the cylinder N. The force applied to the bottom of the piston of the larger cylinder N having an area A_2 is then

$$F_n = pA_2 = \frac{F_m A_2}{A_1} \tag{7.1}$$

If the volume of the fluid is held constant, the displacement of the larger piston, relative to the smaller piston, will be proportionately smaller.

From Eq.(7.1), it is seen that the force F_n is related to the force F_m through a multiplication factor (A_2/A_1). Thus, if the diameter ratio of cylinders *N* and *M* is 3, the force applied at *M* is multiplied by a factor of 9 at the bottom of the cylinder *N*.

3. The connecting pipe can be of any shape, size, length and planar layout.

   Thus, the result of Eq. (7.1) will be the same for any arbitrary configuration of the connecting pipe.
4. The pipe can have any branching and the force will be outputted at all branches with multiplying factors depending on the individual geometries.

Thus for example, consider two additional cylinders with pistons, *P* and *Q*, connected to the pipe connecting *N* and *M* as shown schematically in Fig. 7.1(b). For a given force F_m applied to the piston in the cylinder *M*, the pistons in all the other three cylinders *P* , *Q* and *N* will experience forces F_p, F_q and F_n respectively. The forces F_p and F_q will have multiplication factors depending on the areas of pistons at *P* and *Q* respectively. Note that the force F_n is not affected and remain unaltered due to the introduction of additional branches in the tubing connecting *M* and *N*. This leads to an important concept of a *master cylinder* driving more than one slave cylinder. Automobile braking system is a typical example of the use of this concept.

PASCAL'S LAW

Pressure is transmitted undiminished in an enclosed static fluid.

7.3 HYDRAULIC PRESS

A hydraulic press is a device where a smaller force is used to provide a much higher force for purposes of providing compressive or lifting force. Equation (7.1) represents the relationship between applied force and output force. Based on the basic principles of hydrostatic systems stated in Sec. 7.2, working devices have been developed and the hydraulic presses have been used as a standard industry device for providing large compressive forces. Joseph Bramah (1802) is credited as the inventor of the hydraulic press. A schematic hydraulic press is shown in Fig.7.2 (a). The ram (also sometimes called *plunger*) is activated by a pump providing the hydraulic pressure that causes a displacement of a movable platform. The materials placed between the stationary platform and movable platform undergo high compressive forces. Common industrial use of hydraulic presses include, compression forming, blanking, forging and punching.

In the hydraulic press shown in Fig. 7.2(a), the movable platform acts directly against the object to be pressed and the stationary platform takes the reaction load and transmits it to the foundation. This type of press is called *direct-type hydraulic press*. Similar to the principle of a hydraulic press, the arrangement of a ram working in a cylinder under hydraulic pressure created by a pump finds use in many industrial applications. These include *hydraulic jack* and simple *hydraulic lift*. If the stationary platform in the press shown in Fig.7.2(a) is omitted, the device acts like a simple hydraulic jack or a simple hydraulic lift.

Figure 7.2(b) shows an *inverted type of hydraulic press*. In this, the plunger, under hydraulic pressure created by a pump, acts vertically downwards in its pressing action and the reaction force is directly transmitted to the foundation. After the pressing job is over, the return weights assist in quick return of the movable plate to the initial position.

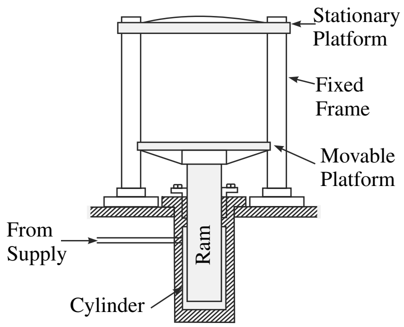


Fig. 7.2(a) *Direct-type hydraulic press*

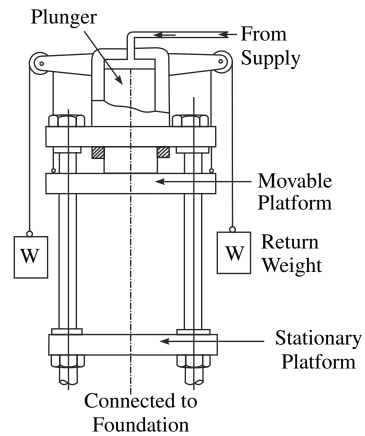


Fig. 7.2(b) *Inverted-type hydraulic press*

7.4 HYDRAULIC ACCUMULATOR

A hydraulic accumulator is a device akin to that of a storage battery, to store hydraulic liquid under pressure when not required by the system. Normally pumps in a hydraulic system work continuously. When they are idling due to no load, hydraulic accumulators serve the purpose of storing the energy of the fluid under pressure to be released as and when required.

7.4.1 Simple Accumulator

A simple (or common) hydraulic accumulator consists principally of a high-pressure cylinder in which a ram can move. Figure 7.3 is a schematic representation of a simple hydraulic accumulator. When the load on the pump in a hydraulic system is reduced or rejected, the high-pressure liquid flows into the accumulator and causes the ram to move. A resistance load on the movement of the ram is provided either through a dead weight or in the form of a compression spring. This arrangement

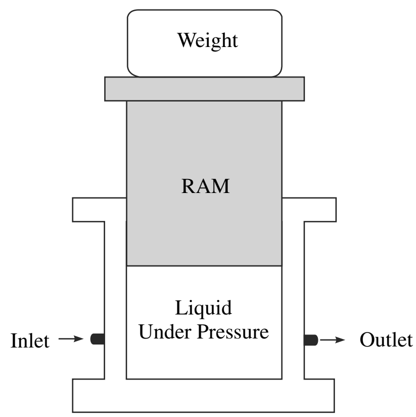


Fig. 7.3 *Simple hydraulic accumulator*

keeps the liquid in the accumulator under desired high pressure. When needed, this pressure liquid from the accumulator is released through the outlet to the chosen hydraulic machine/device. The release of the pressure liquid causes the ram of the accumulator to descend towards its original position.

The commonly used types of accumulators can be classified as:

1. Raised weight type,
2. Spring type
3. Metal bellows type, and
4. Compressed gas type.

Figure 7.3 is of the raised weight type of accumulator. In this the pressure p of the liquid in the accumulator is related to the weight W on the ram of diameter D by the simple relation

$$W = \frac{\pi D^2}{4} p \tag{7.2}$$

If L = Stroke of the ram = Length of the ram movement, the power P supplied by the accumulator when the ram falls uniformly through a distance of L in time t, is given by

$$P = \frac{WL}{t} \tag{7.3}$$

7.4.2 Differential Accumulator

A *differential accumulator* is a variant of the common type of accumulator described in the previous section and has the specific advantage that a relatively high pressure can be obtained by a comparatively small load on the ram. Figure 7.4 shows a differential accumulator. The device consists of a fixed ram in which the introduction of a brass bush makes the lower portion larger than the upper portion. A loaded cylinder slides on the fixed ram. Liquid under pressure is admitted through the fixed ram and fills the annular space between the fixed ram and the cylinder available above the top of the bush. The liquid can enter and leave through valve controlled openings at the bottom of the fixed ram.

During the loading, the liquid from the pump enters from the bottom of the fixed ram and lifts the loaded cylinder upwards. The accumulator is fully loaded when the cylinder is at the end of its stroke. The liquid will be under the pressure due to load W. During the unloaing of the accumulator, the liquid passes out of the outlet at the bottom of the fixed cylinder and the sliding cylinder descends.

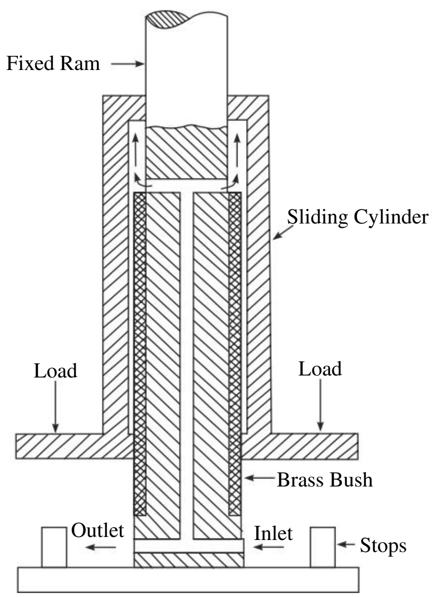


Fig. 7.4 *Differential accumulator*

If D = External diameter of the bush and d = External diameter of the fixed ram, the annular area $A = \frac{\pi}{4}\left(D^2 - d^2\right)$

If W = Load carried by the cylinder, pressure of the liquid in the accumulator

$$p = \frac{W}{A} = \frac{W}{\frac{\pi}{4}\left(D^2 - d^2\right)} \quad (7.4)$$

Capacity of the accumulator = $C = WL = pAL = p \times$ (Annular space between the bush and the cylinder).

It is seen that by making the cross-sectional area of the bush (annular gap) small, for a given load W, the pressure p can be increased. The differential accumulator is thus able to have a smaller footprint to obtain a given pressure p.

Uses of Accumulators

An accumulator is an important component in the present-day hydraulic systems and serves many purposes. Its major uses can listed as below:

1. Stores pressure and hydraulic fluid
2. Acts as a backup power source
3. Acts as a leakage compensator and
4. Acts as a shock absorber of the system

7.5 HYDRAULIC INTENSIFIER

A *hydraulic intensifier* is a component that converts the low pressure from a cylinder into high pressure in a smaller cylinder. Intensifiers consist essentially of two different-sized cylinders with pistons connected by a common piston rod. Intensifiers operate on the ratio-of-areas principle in interconnected cylinders. There are quite a few devices, many being patented ones, which serve as intensifiers. Two commonly used simple varieties are described below:

7.5.1 Simple Hydraulic Intensifier

A common piston rod connects the pistons of two cylinders of different bore (Fig. 7.5). Lower-pressure fluid, acting on the larger piston, exerts a force that is transferred mechanically by the rod to the smaller piston. The smaller piston generates a higher pressure in the fluid in its bore: the pressure ratio is inversely proportioned to the areas ratio.

Let suffixes 1 and 2 denote the larger and smaller cylinders respectively. For small cylinder velocities, that is when there is no acceleration head,

$$p_1 \frac{\pi}{4} D_1^2 = p_2 \frac{\pi}{4} D_2^2$$

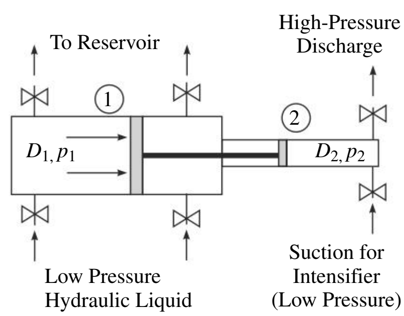


Fig. 7.5 *Hydraulic intensifier*

$$p_2 = p_1 \left(\frac{D_1}{D_2} \right)^2 \quad (7.5)$$

Thus, the input pressure is increased by a factor which is equal to the square of the ratio of the inlet to outlet cylinder diameters. The ratio p_2/p_1 is called *intensification ratio*. In hydraulic circuits, intensifiers are often called *boosters*.

7.5.2 Co-axial-Type Intensifier

Another form of intensifier is the co-axial type [Fig.7.6(a)]. The arrangement consists of a fixed cylinder, a movable hollow cylindrical ram and a fixed ram. Initially, the low-pressure liquid enters through the fixed ram and enters into the hollow of the movable ram. This pushes the sliding ram outwards till it reaches its full stroke length. Now the low-pressure inlet valve is closed and the low-pressure liquid supply is let into the fixed cylinder that pushes the sliding ram (downwards in the figure) causing the liquid in it to escape through the fixed ram to the outlet. The action is similar to that of a syringe. This action causes the outflowing liquid to have a higher pressure. The ratio of the area of outer fixed cylinder to the area of fixed ram is the intensification ratio of this device. Note the co-axial intensifier described above is single-acting.

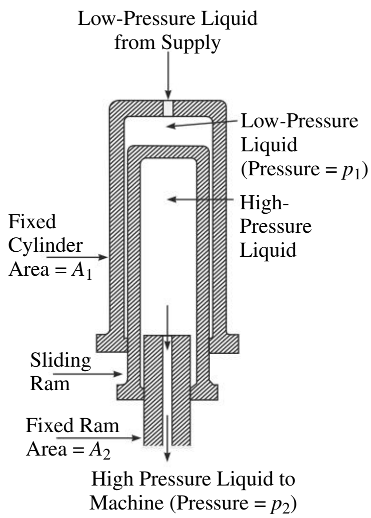


Fig. 7.6 *(a) Coaxial-type hydraulic intensifier*

If p_1 = Pressure of low pressure supply

A_1 = Inside cross sectional area of fixed (larger) cylinder of diameter D_1

A_2 = Inside cross sectional area of fixed ram of inside diameter D_2

Delivery pressure at the outlet $p_2 = p_1 \left(\frac{A_1}{A_2} \right) = p_1 \left(\frac{D_1}{D_2} \right)^2$

The above relationship assumes that there is no frictional loss in the movement of the ram. If, however, there is frictional effect amounting to ε % at each of the packing of the ram, from equilibrium considerations of the moving ram at any position,

$$p_1 A_1 \left(1 - \frac{\varepsilon}{100} \right) = \frac{p_2 A_2}{\left(1 - \frac{\varepsilon}{100} \right)} \quad (7.6)$$

$$p_2 = \frac{p_1 A_1}{A_2} \left(1 - \frac{\varepsilon}{100} \right)^2 \quad (7.7)$$

Hydraulic intensifiers are simple, rugged, and reliable fluid-power components.

7.5.3 Location of Accumulator and Intensifier

In a hydraulic system operating a machine, the intensifiers are located after the accumulator and before the machine. A schematic layout of the basic components of a hydraulic device is shown in Fig.7.6 (b). The hydraulically driven machine (such as a press, crane, winch) is supplied with pressure liquid from the supply reservoir through the action of a pump. The pump is driven by an electric motor or an *IC* engine. Accumulators and intensifiers are provided on the main line in the sequence as shown in the figure. Normally, the pump runs continuously and the working of the accumulator and intensifier is based on the load on the machine.

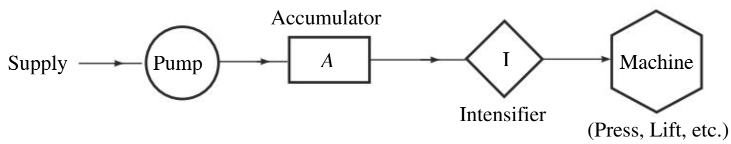


Fig. 7.6 *(b) Schematic layout of a hydraulic device*

7.6 HYDRAULIC WINCH

A *winch* is a device used to pull-in or let-out a rope attached to a load that needs pulling or lifting. It is used in pulling in a heavy load through the action of a pulley. A hydraulic winch consists essentially of a motor-driven drum (spool) on which a wire rope is wound up. Necessary gear train, clutch arrangement and braking and safety devices constitute the adjunct mechanisms of a winch. The driving of the drum is through a hydraulic motor (see Sec.7.22.2) and a drive train. The reduction gear system enables the motor power to be converted into a large pulling force. The direction of rotation can be reversed by control switches. The energy for the hydraulic system of the motor is usually through pumps driven by *IC* engines for mobile units and by electric power for stationary units.

Hydraulic winches find application in hydraulic cranes, ships and marine operations and in industries where heavy objects have to be pulled or lifted as part of the routine operations.

7.7 HYDRAULIC JIGGER

A *hydraulic jigger* is a device to magnify, through a system of pulleys, a small movement of a ram into a large movement of a load. The jigger consists principally of a movable ram inside a fixed high-pressure cylinder and connected to multiple pulley sheaves. Figure 7.7 is a schematic sketch of a hydraulic jigger. A continuous wire rope passes through the pulley system. One end of the rope is anchored by fixing to a strong lug cast into the cylinder and the other end passes over various pulleys, located farther away, to a load. The cylinder of the jigger is connected to a hydraulic circuit of pump–accumulator combination type.

A small displacement of the ram due to injection or withdrawal of hydraulic fluid from the cylinder causes the load at the free end of the rope to experience a large movement due to magnification effect of the pulleys. Application of jiggers includes crane and lift where large controlled movements are needed. The jigger can be placed horizontally or in a vertical position. The jigger was extensively used in water hydraulic power systems and does not find application in the present-day oil hydraulic power systems.

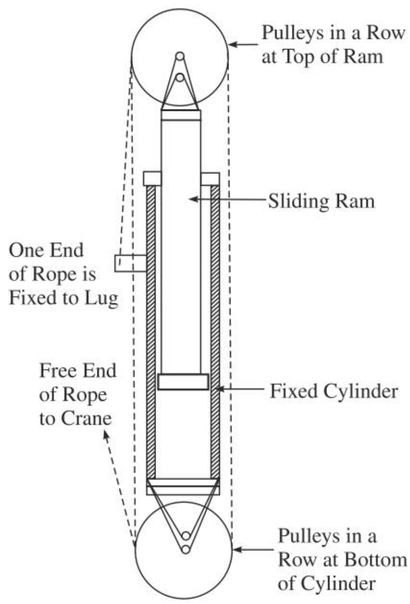


Fig. 7.7 *Hydraulic jigger*

Early 19th century saw many installations of water hydraulic power systems for use in cranes, lifts and hoists which had the jigger as a main component. With the advent of electricity-based motive power applications, the water hydraulic power systems, especially their use in cranes, lifts and hoists, have been completely replaced by advanced technology. Currently, the water hydraulic power systems are of historical interest only. A good record of historical uses of water hydraulic power systems is available in **http://www.ipenz.org.nz/heritage/conference/papers/Gibson_J.pdf** (3rd Australasian Engineering Heritage Conference 2009, *Remnants of Early Hydraulic Power Systems,* by J W Gibson & M C Pierce).

A brief description of early versions of two hydraulic devices (hydraulic crane and hydraulic lift) that were based on the use of jiggers is given below. The modern versions of these two devices are presented in the next section (Sec. 7.8).

1. Hydraulic Crane with Jigger

A hydraulic crane consists essentially of a crane truss that is an assembly of a jib (boom), a tie, a tower and pulleys. The tower is securely anchored to a foundation or is mounted on a rotatable base secured to the foundation. A jigger is suitably located either vertically or horizontally (as in Fig.7.8) and a wire rope extends from the pulleys of the jigger to the pulleys of the crane and finally to the hook of the crane. A heavy weight in the form of a ball above the hook keeps the rope taut. The jigger is operated by a water hydraulic system. The displacement of the ram in the jigger is magnified by the pulley system of the jigger and the result is a magnified movement of the hook of the

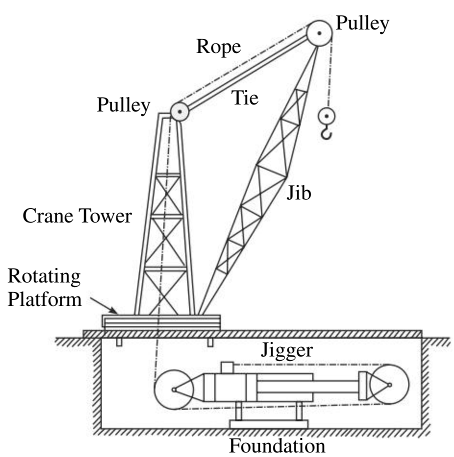


Fig. 7.8 *Swivel-type hydraulic crane with jigger*

crane. Sir William Armstrong (1846) is credited with the invention of the hydraulic crane.

2. Hydraulic Lift with Jigger

A hydraulic lift uses the jigger in the same way as a crane to lift a carriage vertically. The jigger is fixed with the ram working downwards so that its weight will be supported by the cable suspension system. The carriage moves in the guides and is suspended from the top. Sliding balance weights are used to balance the weight of the lift and its load.

7.8 PRESENT-DAY HYDRAULIC CRANE AND LIFT

Since the use of water-hydraulic-power-driven jigger and its use in hydraulic crane and hydraulic lift described in the previous section is obsolete, a brief description of the modern versions of hydraulic crane and lift is given in this section.

7.8.1 Modern Hydraulic Crane

Modern hydraulic cranes use the oil hydraulic system. In this, the hydraulic devices such as winches and rams perform the work and modern electronic devices are adopted for the control of the devices. Most hydraulic cranes use double-gear pumps (Sec. 7.11). Generally, two or more pumps are used; a main pump to raise and lower the main boom and also to operate the telescopic sections of the boom. The other auxiliary pumps are employed for steering, counterweight and out-rigger operations.

- **Main boom:** Large arm, sometimes of telescopic nature, mainly responsible for lifting.
- **Pumps:** Hydraulic pump system usually double-gear or variable displacement pump.
- **Counterweight:** Very heavy weights placed at the back of the cab to prevent the crane from tilting during lifting operations.
- **Rotex gear:** A gear system below the cab that allows the boom to move sideways.
- **Out-rigger:** Support device provided to the wheeled base to keep the crane balanced.
- **Winch:** Hydraulically operated device to pull in or let out the wire rope attached to the hook.

Figure 7.9 shows a wheel-mounted telescopic boom hydraulic crane. The hydraulic cylinder units are the main devices for lifting the boom with the load and also for the telescopic action of the boom. The turntable and the hook position are activated by a hydraulic winch.

Crane operations like lowering and rising of the boom, rotation of the cab, winding and unwinding of the winch are performed inside the cab through control system involving joysticks and foot pedals. The motive power for the hydraulic circuit is usually through a diesel motor for mobile crane units and electrical motors for fixed cranes.

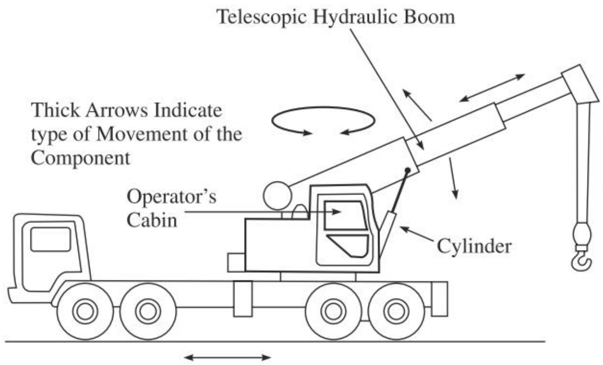


Fig. 7.9 *Schematic sketch of a modern wheel-mounted telescopic boom hydraulic crane*

7.8.2 Modern Hydraulic Lift

The popular modern hydraulic lift (elevator) is a hydraulic piston-type elevator system. In this, the car moves at a relatively low speed of about 45 m/minute. The elevator system is limited to low-rise buildings, generally about six serviced levels. Unlike the electric-motor-driven elevator system, the hydraulic elevator does not have cables, counterweights, brake drums, or costly control equipment and needs fewer safety devices than the cable-type elevator system. Hence, it is a relatively low-capital- cost and low-maintenance-cost unit. Figure 7.10 is a schematic sketch of a modern day hydraulic elevator.

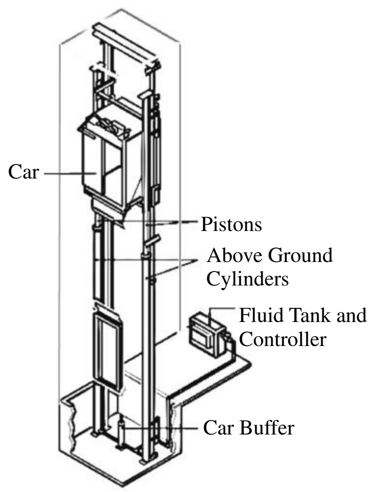


Fig. 7.10 *Schematic sketch of a modern-day hydraulic lift (Ref. 7.6)*

In a hydraulic lift, the car is moved by a hydraulic hoist or piston attached to the car-floor structure. The lift is activated through an oil-pumping system. Oil is pumped from a reservoir into the base of the piston in a cylinder. An electric motor pumps oil into the cylinder through a gear pump to move the piston, which in turn smoothly lifts the elevator car. Removing oil from the piston base with the help of the same pump system lowers the car. Electrically operated valves control the release of the oil for a very smooth operation.

The shaft has guide rails for car movement and has a buffer spring at the bottom. Further, adequate limit switches help in proper operation. If there is an oil leak, the car will slowly descend. Compared to other types of lift systems, the hydraulic elevator system ranks highest from the safety aspect.

These elevators are frequently used as service elevators in low-rise buildings. Since there is no counterweight as in electric traction lifts, the hydraulic lift system carries full load on the up-run and no load on the down-run. Thus for a given load, the horsepower required is greater for hydraulic than for electric traction but the power consumption is approximately equal for both types of equipment.

7.9 FLUID COUPLING

The *hydraulic coupling* (generally called *fluid coupling*) is the simplest means of transmitting torque hydraulically. It can be defined as a device in which a fluid, usually oil, transmits torque from one shaft to another, producing an equal torque in the other shaft. A fluid coupling is widely used to transfer rotating power from a prime mover, such as an internal combustion engine or electric motor, to a rotating driven load.

The fluid coupling consists basically of two elements: a centrifugal pump impeller connected to the driving shaft and a turbine wheel runner on the output (driven) shaft. There is no mechanical connection between both the shafts. Figure 7.11 represents a schematic view of a fluid coupling. The device consists of two split toroidal grooved discs, each facing the other with a small clearance between them. Radial blades are provided across the grooves to divide them into curved cells.

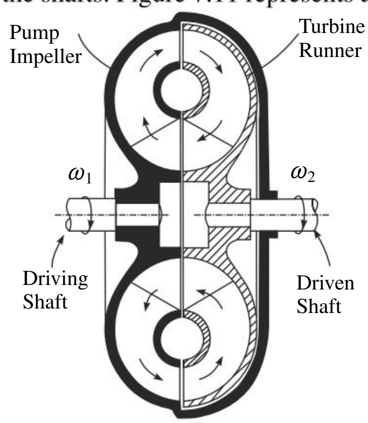


Fig. 7.11 *Fluid coupling*

The hollow space in the impeller and the runner is filled with oil. When the driving member (impeller) begins to rotate (as the engine is started and runs), the oil is set into motion. The vanes in the driving member start to carry the oil round with them. As the oil is spun round, it is thrown outward or away from the shaft, by centrifugal force. However, since the oil is being carried round with the rotating driving member, it is thrown into the driven member. The oil thus strikes the vanes of the driven member at an angle, thereby imparting torque to the driven member due to transfer of kinetic energy.

The inlet angles of the runners are set such that the flow from the impeller enters without any shock. The oil in the turbine moves towards the shaft of the wheel from where it flows to the pump to complete a closed fluid circuit. Since the rate of flow and change in the velocity vector are numerically same in both the impeller and the runner, the torque of the input and output shafts of a fluid coupling are identical at all speeds, if friction and other losses are neglected.

If the speeds of both the impeller and the runner are same, there would be no flow. However, due to fluid friction and turbulence effects, the angular velocity of the output (driven) shaft ω_2 will be a little less than the angular velocity of the driving shaft ω_1. The ratio $S = \left(\dfrac{\omega_1 - \omega_2}{\omega_1}\right)$ is known as *slip* and the slip is generally very small being about 3% at peak speed. The necessary reduction of the speed of the driven shaft thus maintains continuous flow of oil from the impeller to the runner. Thus, the power loss is very small and the torque ratio between the output and input shaft is very near unity. The condition at which the speeds of the driving and driven shafts are the same is known as *stall*. At the stall value, the efficiency drops down to zero value very rapidly.

Efficiency

The power input to the coupling is

$$P_1 = T_1 \omega_1$$

where T_1 is the input torque and ω_1 is the angular velocity of the input shaft.
Similarly for the output shaft let

$$P_2 = T_2 \omega_2$$

whereT_2 is the output torque and ω_2 is the angular velocity of the output shaft.
The efficiency of the coupling is the ratio of the power output to the power input and is given by

$$\eta_c = \frac{P_2}{P_1} = \frac{T_2\,\omega_2}{T_1\,\omega_1} \tag{7.8}$$

If the frictional resistance at the surfaces of the impeller and rotor are neglected as small, the torques of the input shaft and the output shaft are essentially equal and hence the efficiency of the coupling is principally governed by the ratio of angular velocities, $\frac{\omega_2}{\omega_1}$. Further, since the slip S is defined as $S = \left(\frac{\omega_1 - \omega_2}{\omega_1}\right)$, the efficiency is related to slip as

$$S = \frac{\omega_1 - \omega_2}{\omega_1} = 1 - \frac{\omega_2}{\omega_1} = 1 - \eta_c \tag{7.9}$$

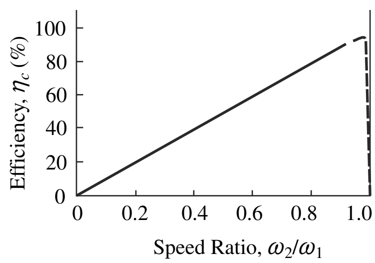


Fig. 7.12 *Efficiency of hydraulic coupling*

The variation of the efficiency with the angular velocity ratio $\frac{\omega_2}{\omega_1}$ (which is the same as speed ratio (N_2/N_1) is shown schematically in Fig.7.12. The efficiency is zero when the speed ratio is zero and increases essentially linearly till about a value of $\frac{\omega_2}{\omega_1} \approx$ 0.95 and then onwards the efficiency drops suddenly to zero value at a speed ratio of unity.

Through dimensional analysis of the pertinent variables controlling the hydraulic coupling it can be shown that

$$\frac{T}{\rho N_1^2 D^5} = Fn\,(S) \quad \text{....(called } \textit{torque factor}\text{)} \tag{7.10}$$

$$\frac{P}{\rho N_1^3 D^5} = Fn\,(S) \quad \text{...(called } \textit{power factor}\text{)} \tag{7.11}$$

where T = Torque input, P = Power input, N_1= Rotational speed of the impeller, D = Diameter of the impeller and $S = 1 - \frac{\omega_2}{\omega_1} = 1 - \frac{N_2}{N_1}$ = Slip.

These two nondimensional factors (viz. power factor and torque factor) would enable the scaling up of parameters for homologous fluid couplings. Further, it is seen from Eq. (7.10) and Eq. (7.11) that for a given value of slip,

1. Torque (T) is directly proportional to the square of the rotational speed N_1 of the impeller,
2. Power (P) is directly proportional to the cube of the rotational speed N_1 of the impeller, and
3. Both the power (P) and torque (T) are directly proportional to the density of the liquid used in the fluid coupling.

In fluid coupling, low-viscosity fluids are generally preferred and the physical properties, like density and viscosity of the oil determine the operation characteristics of the coupling. For example, as noted above, increasing the density of the oil increases the torque that can be transmitted at a given speed. Another interesting characteristic is that the fluid couplings are bi-directional. They can be used in any direction of rotation.

Fluid couplings are very widely used and their application includes diesel locomotives, automobiles, aviation and marine machinery and agricultural machinery.

7.10 TORQUE CONVERTER

A torque converter is a modified form of fluid coupling. Like a fluid coupling, the torque converter normally takes the place of a mechanical clutch, allowing the load to be separated from the power source. Unlike a fluid coupling, however, a torque converter is able to multiply torque when there is a substantial difference between input and output rotational speed, thus providing the equivalent of a reduction gear.

A torque converter consists of three main components: (a) a pump impeller connected mechanically to the driving shaft, (b) a turbine runner connected to the driven shaft, and (c) a stator, usually known as a reaction member, positioned in the middle of the flow from the impeller and the runner. The stator has guide vanes that change the direction of the liquid impinging on the runner and thus causes the torque delivered to the driven shaft to be higher than that of the driving shaft. Figure 7.13 is a schematic sketch of a simple fluid torque converter.

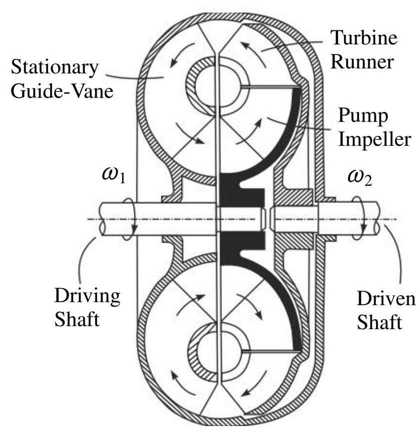


Fig. 7.13 *Torque converter*

In a fluid coupling, under conditions of high slippage, the fluid flow returning from the turbine to the pump opposes the direction of pump rotation. This leads to a significant loss of energy. Under the same condition in a torque converter, the returning fluid will be redirected by the stator so that it aids the rotation of the pump, instead of impeding it. This leads to recovery of much of the energy in the returning fluid and addition to the energy being supplied by the pump itself. This action causes a substantial increase in the mass of fluid being directed to the turbine, producing an increase in the output torque.

Unlike the radially straight blades used in a fluid coupling, the turbine and stator of a torque converter use angled and curved blades. The shape of the blades is important as even minor variations can result in significant changes in the performance of the device.

Efficiency

The torque relationship between the input and output shafts can be expressed as

$$T_2 = T_1 + T_v \tag{7.12}$$

where T_1 = Torque of the input shaft, T_2 = Torque of the output shaft and T_v = Torque of the guide vanes.

Power input = $P_1 = T_1 \omega_1$

Useful power output = $P_2 = T_2 \omega_2$

Efficiency of the torque converted $\eta_T = \dfrac{P_2}{P_1} = \dfrac{T_2\,\omega_2}{T_1\,\omega_1}$ (7.13)

Depending on the design of the guide vanes, T_v may be positive leading to $T_2 > T_1$; in which case it is a *torque multiplier*. If T_v is negative, the converter will be a *torque divider*. Normally, the torque converters are employed for torque multiplication. Figure 7.14 shows schematically the variation of torque ratio and efficiency with the speed ratio in a torque converter.

Figure 7.14 indicates that the converter attains its maximum efficiency at a speed ratio, of about 0.5 and at higher speed ratios, the efficiency drops. The efficiency is zero at zero speed ratio and once again it is zero at a speed ratio of about 0.9. The torque ratio is highest at zero speed ratio and drops continuously, in a nonlinear way to attain zero value at a speed ratio of around 0.9.

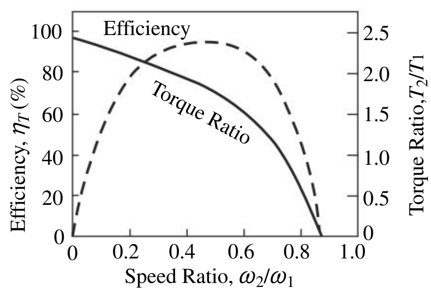


Fig. 7.14 *Characteristics of a torque converter*

The slip, in both the coupling and in converter, represents the inefficiency of the system and hence the energy lost as heat. Dissipation of heat produced in these units is an important factor in the construction of these units. For large units, external cooling system may be required.

There have been many advances over the basic features given above and it is possible to have a variable speed system with several stages of runner and guide vanes. Torque converters are used extensively in (a) automatic transmissions on automobiles, such as cars, buses and light trucks, (b) marine propulsion systems, and (c) industrial power transmission.

B: Rotary Pumps

Rotary pumps form a class of positive-displacement pumps in which the pumping elements are connected to a rotor inside a fixed casing. In this, the motion of pumping elements fitted to a rotor causes the pumping fluid that is trapped in a moving volume to be carried around the casing to be discharged at the outlet. Generally, these pumps are specialty pumps and have relative low speed. They are self-priming and have good suction lift. Five types of rotary pumps, viz. gear pump, lobe pump, sliding-vane pump, screw pump and piston pump are described in the subsequent sections. In the descriptions that follow, note the common features of these pumps like absence of inlet and outlet valves, and ability to generate continuous, nonpulsating flow.

7.11 GEAR PUMP

7.11.1 External Gear Pump

Gear pumps are positive displacement pumps. Here, a pair of identical spur gears meshed inside a closed casing, rotate in opposite directions as shown in Fig.7.15. This type of pump is called *external gear pump* as both the gears mesh on their outer perimeter. Out of the two gears, one is a driver and receives power input through the shaft and the other is an idler. The casing has an inlet and outlet for liquid to be pumped. The teeth of the gear have a perfect meshing that prevents any leakage and aids in pushing the fluid forward. The rotation of the gears causes the liquid in the casing to be bodily pushed continuously. There is no dynamic action of imparting pressure or kinetic energy as in other rotodynamic pumps. The flow is continuous, uniform and very high pressures can be achieved. Note that the liquid flows on the outside of the gear, that is between the casing and the outer portion of the gears and there is no flow between the gears. In addition, there is no inlet and outlet check valves as in a reciprocating pump. If the direction of the drive gear is reversed, the flow will be in opposite direction; hence its bidirectional capability.

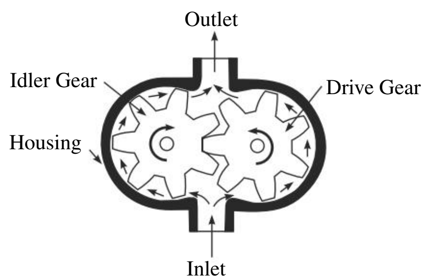


Fig. 7.15 *External gear pump*

Other important characteristic features of the gear pumps are

- No pulsations in the flow
- Self priming
- Bidirectional
- High-pressure and high-temperature capability
- Low *NPSHR*
- Precise and accurate metering capability

These pumps are high-speed, high-pressure pumps and are made of a wide variety of materials depending upon the need of the application. The speeds are in the range of 1750 to 3500 rpm for small pumps, and large pumps have smaller speeds of the order of 650 rpm. While external gear pumps resemble the internal gear pumps in its characteristics, the structural arrangements of the components allow these pumps to be made to closer tolerances and hence they can accommodate larger pressures. For low-viscosity liquids there can be some slip especially at large pressure heads. The efficiency of the pump is around 90%.

The actual discharge from the pump is usually estimated by the following empirical formula:

$$Q = 0.95\ \eta\ \pi\ c\ (D - c)LN / 60 \text{ m}^3/\text{s} \tag{7.14}$$

where D = Outside diameter of the gears

c = Centre to centre distance between the axis of the gears

L = Axial-length of gear teeth

N = Revolutions per minute

η = Volumetric efficiency of the pump

External gear pumps find wide applications in (a) lubrication of internal combustion engines, (b) handling viscous liquids such as lube oils, (c) chemical mixing and blending, and (d) handling of acids and caustics.

7.11.2 Internal Gear Pump

Internal gear pump consists essentially of a rotor with internally cut teeth that meshes with the external teeth of an idler gear wheel. The idler is smaller than the rotor gear and rotates on a stationary pin. The operation of the idler is completely inside the rotor gear. A crescent-shaped partition, which is an integral part of the pump, forms a seal between suction and discharge portions of the pump (see Fig. 7.16). As the rotor rotates, the liquid from the inlet is first trapped between the teeth of the rotor, the idler and the crescent. It is further carried round by the idler and is ejected at the outlet. Note that the rotor teeth and the idler teeth mesh each other completely in the portion midway between the inlet and the outlet ports. Internal gear pumps are exceptionally versatile in handling a very vast range of viscous liquids; from thin solvents to thick asphalt. The range of viscosity ranges from 1 centipoise to 10^6 centipoises. Further, this pump has a very wide range of operational temperature that ranges from room temperature to as high as 400°C. The speeds of these pumps are relatively low, the normal speed being around 1200 rpm. Other, desirable properties of the internal gear pumps are

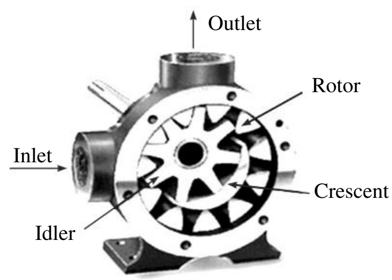


Fig. 7.16 *Internal gear pump*

- Nonpulsating, self-priming, bidirectional capability pump
- Constant discharge irrespective of pressure conditions
- Reliable, easy to handle and maintain

Applications of internal gear pumps include, and are not limited to, the following broad areas of pumping:

- All varieties of lube and fuel oils
- Food products ranging from syrups to peanut butter
- Alcohol and solvents
- Asphalt and bitumen
- Glycol
- Paints, pigments and inks

7.12 LOBE PUMP

Lobe pump, also sometimes called *rotary lobe pump*, is similar in action to that of an external gear pump. In this, a set of lobes (rounded projecting parts), each independently driven, are set in a chamber in a manner similar to that of a set of gears. The positioning of the lobes is so adjusted that they are very close to each other but do not touch each other. A timing gear device is set externally at the drive to achieve desired relative positions and speed of the lobes. When in motion, the lobes trap the liquid at the inlet, displace it around the casing and finally discharge it at the outlet. The motion is continuous without any pulsations. Figure 7.17 shows a schematic sketch of a three-lobe pump.

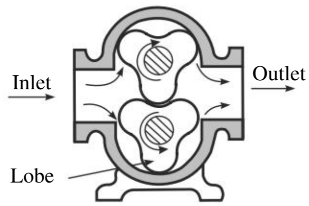


Fig. 7.17 *Lobe pump*

A variety of lobe arrangements are in use: single-lobe, double-lobe, tri-lobe and multilobe arrangements are in use. As in any rotary positive-displacement pump, there are no valves either at the inlet or at the outlet. The suction capacity is low when compared to that of a gear pump. Further, since the lobes do not touch each other, the slip is appreciable in low viscosity flows. However, they have excellent ability to handle high-viscosity flows but with reduced speed. They offer low shear and gentle handling of the pumping commodity. The chief characteristics of the lobe pump can be listed as follows:

- Gentle pumping action
- Flow is independent of pressure conditions
- Constant and nonpulsating rate of flow
- Long-time dry run will not harm the pump
- Can pass medium-sized solid mater without harming the solids
- High reliability and simple operation

Lobe pumps are extensively used in a variety of industries dealing with (a) food and beverage, (b) Pharmaceuticals, and (c) chemicals. In the food industry, the ability to handle solids without damage is used advantageously in moving cherries, olives, apricots, vegetables, etc. Since excellent sanitary conditions are possible in this pump, they are used in pumping of practically all kinds of food products which include among others baby foods, ice cream, yoghurt, honey, syrups and starches.

Typically, a commonly used lobe pump will be made of stainless steel and have a capacity up to 150 m^3/h; handle pressures up to 12 bars and viscosity in the range 1 to 10^5 centistokes. Also, the ability to handle a wide range of temperature from about –25°C to 150°C is normal.

7.13 SLIDING-VANE PUMP

A sliding vane pump is a positive-displacement rotary pump. In this, a rotor with radial slots is fixed eccentrically in a cylindrical housing of short length. Each of the slots has a close-fitting vane inside it. The vanes are capable of sliding easily in the slots. When the rotor is in motion, gaps of varying width form between the rotor and the housing and the vanes slide easily, due to centrifugal force, to fill the gap. Figure 7.18, which is a schematic sketch of a vane pump, shows eight vanes at a given instant having slid out, at various degrees, to fill the gap between the rotor and the housing. At the point *A*, which is the midpoint between the inlet and the outlet, the gap between the rotor and the housing is zero. Beyond *A*, in the clockwise direction, the gap increases in size and reaches a maximum at the point *B* which is diametrically opposite *A*. Note that the point *B* is midway between the inlet and the outlets.

A rotor with radial slots is positioned eccentrically in a housing of circular cross section. The rotor has several evenly placed slots. Vanes made of nonmetallic composite material fit loosely in these slots. The vanes are free to move. As the rotor turns, due to centrifugal force the vanes slide outwards and fit the rim of the casing snugly. While in most designs, the vanes are held against the wall of the housing by centrifugal force, in some designs light springs assist in the snug fit of the vanes against the wall. Figure 7.18 is a schematic sketch of a vane pump. Observe the eccentric opening and the vanes in different levels of extension. There are eight vanes in this pump and two consecutive vanes and the rim of the housing make cavities. The size of these cavities varies depending upon the position of the vanes. Due to eccentric positioning of the rotor in a circular housing, the free space between the rotor and the housing is crescent shaped. With respect to the rotation of the rotor, the inlet is provided at the expanding portion of the crescent and the outlet is at the diminishing part of the crescent. The inlet and outlet are 180° apart.

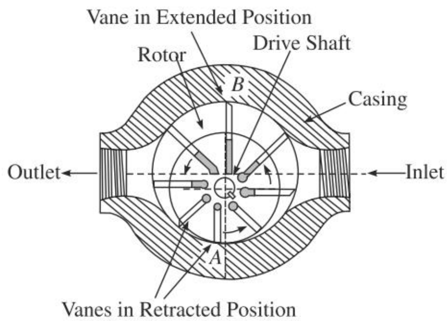


Fig. 7.18 *Schematic sketch of a sliding vane pump*

The flow from the inlet moves into the pocket formed between two adjacent vanes and the gap between the rotor and the housing. As the vane moves in counter-clockwise direction, the volume of the pockets decreases beyond the point *B*. At any instant, for a given pocket, as the vane moves from *B* towards *A,* the amount of decrease in volume represents the flow that has been squeezed out to the outflow pipe. This happens to all the pockets and continuously with the revolution of the rotor. As the rotor rotates with a speed *N*, there will be continuous operation of squeezing out of the flow to the outlet.

Note that there are no inlet and outlet valves. Obviously, the rate of flow is proportion to the speed of rotation of the rotor. In some models, the eccentricity is adjustable thus enabling changing of flow rate for a given speed. The vanes are of nonmetallic composite material and the housing is of stainless steel or cast steel. A removable liner is provided in the housing to enhance the useful life of the pump. For the vane pump shown in Fig. 7.18, since all the pumping action takes place at one side of the housing, the shaft experiences a lateral load. Pumps of this kind are called *unbalanced vane pumps*. Many variants of this simple type, including methods of balancing the lateral thrust, are available.

Vane pumps are good for low-viscosity liquids and are not efficient with highly viscous liquids. It cannot handle solids and is not good with abrasive liquids. As such, they are preferred for thin liquids like solvents. Generally, a sliding vane pump has medium capacity, medium pressure range and medium speed. Typically, the speed range is 900–1500 rpm; Pressure range is 100–1100 kPa, and temperature range of –30°C to 250°C. The pump can run dry for short periods without damage.

Uses

Sliding-vane pumps are generally preferred to convey the following liquids:

- Fuel oil
- Solvents
- Aviation fuel
- *LPG*
- Transformer oil
- Distillates
- Kerosene
- Ammonia
- Lube oils
- Petrol
- Edible oils
- Refrigeration coolants

The services and industries using the vane pump are chiefly

- Transport tanker service
- Power stations
- Aviation industry
- Chemical and pharmaceutical industry
- Petroleum and fuel industry
- Paint industry
- Edible-oil industry
- Automobile industry

7.14 SCREW PUMP

Screw pumps are classified as positive, constant-displacement rotary pumps. They come in three types: one-screw, two-screw and three-screw pumps. The one-screw pump is called *progressive cavity pump*. By far, the largest applications of screw pumps are of the three-screw kind. Figure 7.19 shows schematically a *single-suction three-screw pump*. This pump consists essentially of three intermeshing helical thread screws housed inside a chamber with an inlet and outlet. The middle screw is the drive screw (called *power rotor*) and is directly connected to the power shaft. The other two screws are idlers and are called *idle rotors*. The screw threads of the power rotor sit between the threads of the idlers. Also, note that the threads of the idlers are opposed in direction to that of the driver. There are no valves, gears or vanes in the system.

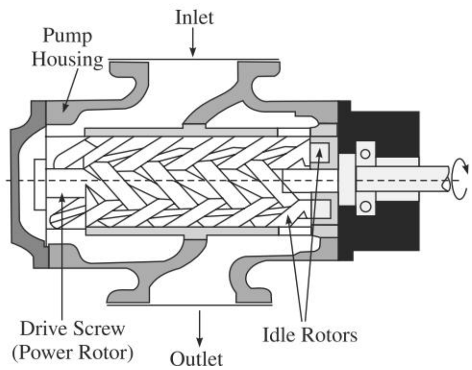


Fig. 7.19 *Screw pump*

The pumping liquid from the inlet is trapped between the teeth of the idler screws. Though continuous, the flow can be considered as a series of packets lying between a set of two adjacent threads of the idler. The rotation of the driver causes each packet of trapped liquid to move progressively forward in the axial direction and ultimately to the outlet. For each turn of the driver, the displacement is a constant quantity. Continuous rotation causes a continuous stream of liquid to be pushed out of the exit as a constant stream of liquid. There is no rotation of the fluid; only linear translation is present. The flow rate is a function of the speed of rotation and also depends as the cube of the diameter of the power rotor.

The screw pumps are classified as high-pressure, high-flow pumps. The speeds are relatively low. They have a reasonably good efficiency of about 80–85%, which depend upon the viscosity of the liquid and the pressure head of the pump. A typical midrange three-screw pump will have a flow range of 10 to 1500 litres/minute; Pressure head of 80 bars; can handle viscous flow in the range 5 to 400 centistokes; speed of 750 to 3500 rpm and a working range of –20°C to 150°C. Two types of suction arrangements are in use: single-suction and double-suction. Single-suction arrangement is used for low to medium flow rates and low to high pressures. The double-suction pumps, being essentially two pumps in parallel, are used for high flow rates.

The output from a screw pump is continuous without pulsations and the pump is self-priming. Screw pumps have high suction capability. They run very quietly and are reliable and robust.

Applications

Screw pumps find considerable applications in machinery lubrication, fuel-oil transport, powering of hydraulic machinery, conveyance of high temperature and viscous fluid like asphalt and residual oils in refinery. They find extensive use in ships, commercial vessels and barges to handle lubricating oils and hydraulic fluids.

7.15 PISTON PUMPS

7.15.1 Introduction

The reciprocating pump consisting of a piston or a plunger is driven either directly or through a crankshaft to convert the rotary motion of a prime mover to reciprocating motion. Even though they use a piston, it is convention to call these pumps reciprocating pumps and these were discussed in Chapter 6. There is a class of pumps used in hydraulic power systems that use the reciprocating action of a set of pistons moving in machined bores of a cylindrical block, the power for the movement of the pistons coming from the rotation of a drive shaft and appropriate mechanisms. In these pumps, called *piston pumps*, the motion of the pistons is primarily activated by the rotation of a drive shaft and hence these pumps are classified as rotary positive-displacement pumps. There are a few variants in the basic mechanisms mentioned above, and depending upon the activating mechanism of the piston and cylinders, piston pumps are classified into several types as shown in Fig. 7.20.

7.15.2 Axial Piston Pump

A set of odd number of pistons is arranged axially, parallel to each other, near around the outer periphery of a cylindrical block. Each piston is housed inside a

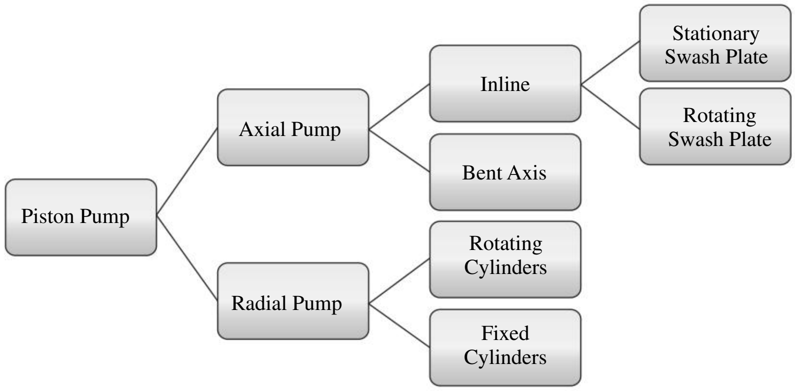


Fig. 7.20 *Classification of piston pumps*

well-machined cylindrical bore and is capable of moving inside it. The pistons bear against an inclined plate (called *swash plate*) at one end and the other end moves inside the bore. While the driving shaft rotates the cylindrical block, the pistons experience an axial thrust that makes them execute a simple harmonic motion in their respective barrels; the return of the pistons being facilitated by springs. In the cylinder, the bores are connected to port plates or check valves and these direct the liquid from the intake to the pump and from the pump to the discharge. The flow rate of the pump is the sum of the flows from individual piston-bore set and is a function of the speed of rotation.

Figure 7.21 and 7.22 show schematically two arrangements of swash plate and cylinder. Note that in Fig. 7.21, the swash plate is fixed and the cylinder rotates and in Fig. 7.22, the swash plate is rotating and the cylinder is fixed in position.

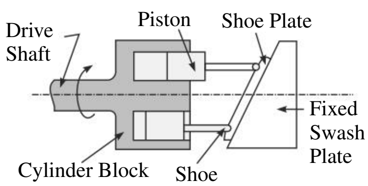


Fig. 7.21 *Fixed swash plate and rotating cylinder block*

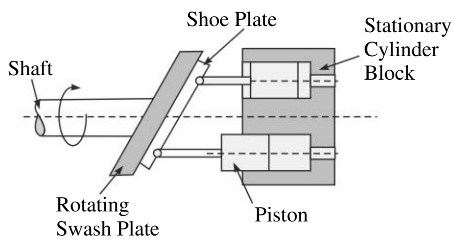


Fig. 7.22 *Fixed-cylinder block and rotating swash plate*

Theoretical delivery from the piston pump having n pistons is

$$Q = nNL\left(\frac{\pi d^2}{4}\right) \tag{7.15}$$

in which n = Number of pistons, N = Speed of rotation, L = Length of the stroke, and d = Diameter of the bore.

Bent-Axis Piston Pump

In this type, the swash plate is absent. The axis has a discontinuity at the pump and the cylinder block is joined to the drive shaft by a universal coupling or a gear arrangement to have a bent axis (Fig. 7.23). The cylinder block is stationary and its

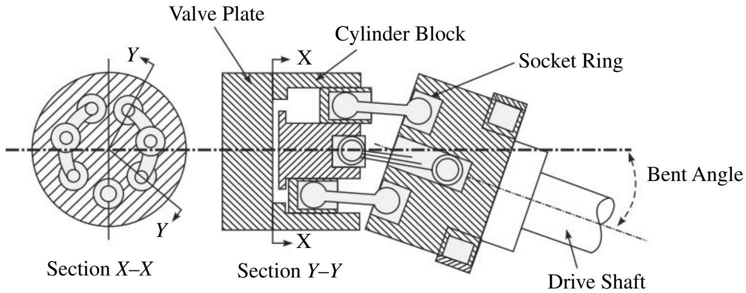


Fig. 7.23 *Bent-axis piston pump*

angle with the drive shaft makes it act as a swash plate in providing lateral (axial) force to the pistons. By adjusting the angle between the shaft and the cylinder block, the discharge rate can be varied. The normal bent angles are 15°, 20°, 25°, and 30°. Ports at the end of the cylinder block and a valve plate provide the necessary porting for inlet and outflow.

Characteristics

Axial pumps are bidirectional and are best suited for high-pressure–low-volume applications. They have very high efficiency, with a range of 90 to 95%, and come in capacity range of 20 LPM to 100 GPM, pressure range of 1.5 MPa to 25 MPa at a speed 1750 rpm. Axial piston pumps are sophisticated, precision-produced equipment and as such are relatively costly.

General applications of these pumps include: mobile equipment, construction equipment, machine tools and in machine forming and stampings.

7.15.3 Radial Piston Pumps

A radial piston pump is a rotary positive-displacement pump. Radial piston pumps come in two kinds: rotating cylindrical block type and fixed cylindrical block type.

1. Rotating Cylindrical Block Type

It consists of a set of pistons arranged radially in a cylindrical block mounted eccentrically inside a cylindrical casing as shown in Fig. 7.24. As the rotor starts rotating, the pistons reciprocate in and out of their respective cylinders. Owing to the centrifugal force, the outer ends of the cylinders bear against a reaction ring. The ring is a part of the rotor, which in turn rotates on a slide block. The cylinder block rotates on a *pintle* (stub shaft), which incorporates a cylindrical sleeve valve with intake and discharge passageways. The

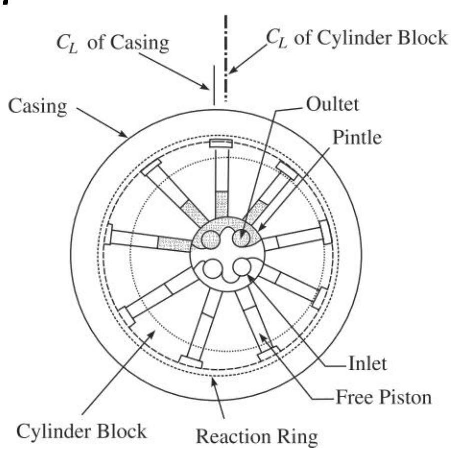


Fig. 7.24 *Radial piston pump (rotating cylindrical block type)*

pistons draw in the liquid during one half of a revolution and discharge during the other half revolution.

The stroke of the piston is equal to twice the eccentricity (2ε) and the theoretical delivery from the pump is given by

$$Q = 2\varepsilon\left(\frac{\pi d^2}{4}\right)nN = \frac{1}{2}\varepsilon nN\left(\pi d^2\right) \tag{7.16}$$

in which n = Number of pistons, N = Speed of rotation, ε = Eccentricity of the rotor in the casing and d = Diameter of the bore.

2. Fixed cylindrical Block

A variant of the above type of radial pump is the one in which the cylindrical block and hence the cylinders are stationary. They do not rotate round the drive shaft. The pistons are actuated by a cam arrangement in the drive shaft. Appropriate inlet and outlet arrangements are made for each of the fixed cylinders. Such radial pumps are sometimes called *plunger pumps*. An interesting characteristic of these pumps is the same direction of delivery for both clockwise and counter-clockwise rotation of the drive shaft. This aspect finds specific application in the automobile industry.

(a) Characteristics Radial piston pumps have smooth and quiet operation. They exhibit high efficiency and low-pressure ripple. They are bidirectional in operation. These pumps are quite robust, and work at high rpm. The normal pressure ratings are 200 to 450 bars, flow rate is in the range of 20 to 80 cm^3 per rotation. The efficiency of these pumps is generally very high.

(b) Applications Typical applications of radial piston pumps include machine tools, automotive sector (in automatic transmission and hydraulic suspension controls) and in plastic and powder injection moulding. Recently, proprietary axial piston pumps have been developed, as a replacement to the legacy turbo-centrifugal pump, for pumping fuel and oxidiser in a rocket engine. Radial piston pumps are suitable for use in potentially hazardous atmospheres and thus find applications in gas and steam turbines, and in the mining industry.

7.16 COMPARATIVE CHARACTERISTICS OF ROTARY PUMPS

The rotary pumps described in the previous sections are compared for five attributes in the following table to help appreciate their specific strong points.

Pump		Abrasives	Thin Liquids	Viscous liquids	Solids	Pressure Head
Gear Pump	Internal gear	Good	Good	Excellent	Poor	Good
	External gear	Poor	Good	Good	Poor	Excellent
Lobe Pump		Good	Average	Excellent	Excellent	Good
Sliding-Vane Pump		Poor	Excellent	Average	Average	Average
Screw Pump		Poor	Excellent	Good	Poor	Good
Piston Pump		Poor	Excellent	Excellent	Poor	Excellent

Rotary pumps belong to the class of positive-displacement pumps and hence it is but natural to know their strong points relative to reciprocating pumps. Some of the main advantages of rotary pumps over the conventional reciprocating pumps can be listed as follows:

- Simple construction
- Direct coupling to electrical motor/*IC* engine
- Continuous, pulsation free, supply of liquid
- Satisfactory power to weight ratio leading to compact units
- Bidirectional
- Can function as hydraulic motors also

C: Miscellaneous Hydraulic Pumps

This section deals with a set of eight popular pumps that differ somewhat from the other types of pumps discussed earlier. The pumps discussed under this section are diaphragm pump, hydraulic ram, jet pump, ejector pump, airlift pump, vertical turbine pump, submersible pump and regenerative pumps.

7.17 DIAPHRAGM PUMP

Diaphragm pumps belong to the category of positive displacement pumps (*PDP*) and are the most widely used pump of *PDP* category. The pump consists essentially of a chamber with a flexible diaphragm at one end. Inlet to and outlet from the chamber is controlled by one-way valves (see Fig.7.25). A reciprocating force is applied to the diaphragm on the side that lies outside the chamber so that the diaphragm flexes into or out of the chamber repeatedly. The reciprocating force can be either a cam linkage to a rotating shaft or a pulsating compressed fluid (air or hydraulic fluid).

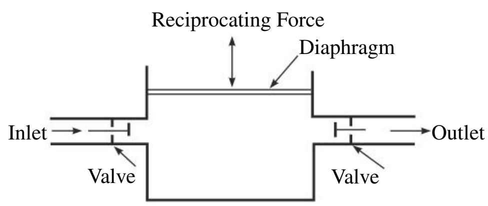


Fig. 7.25 *Schematic sketch of a diaphragm pump*

The action of relaxing and flexing of the diaphragm, due to action of pulsating force on one side, causes the pressure in the chamber to alternate from suction to positive pressures. This action is similar to the action of a piston in the cylinder of a reciprocating pump. Consider a case when the diaphragm is in flexed position into the chamber. When it is relaxed due to withdrawal of external force on it, a suction pressure is created and the inlet valve opens. The liquid in the inlet pipe enters into the chamber. Next, due to the reciprocating action of the force, when the diaphragm is flexed in the next cycle, the action is similar to the delivery stroke of a reciprocating pump. The liquid in the chamber is pushed bodily; the inlet valve is closed and the outlet valve is opened. The liquid flows out of the chamber to the outlet pipe. The action is repeated to cause a positively displaced pulsating delivery into the outlet pipe.

Observe that there is no contact between the pumping liquid and the mechanism that pulsates the diaphragm. There is a possibility of infinitely wide flow control by controlling the frequency and/or magnitude of the pulsating force on the diaphragm. If desired, the pulsation of the flow can be reduced by using flow dampers through adoption of duplex or multiplex units.

The diaphragm is made of *PTFE* (Teflon), synthetic rubber or metal. The pumps are of rugged construction and are good in handling debris-laden liquid and slurries. In fact, the specification for the pump includes the maximum sizes of solids that can be handled. In view of its ability to handle high solid content, the diaphragm pump is sometimes called *mud-sucker pump*. A partial list of materials that can be handled by a diaphragm pump is as follows:

- Low to high viscous liquids
- Solid-laden liquids
- Shear sensitive liquids
- Abrasive liquids
- Flammable liquids
- Hygienic materials like pharmaceutical compounds and food items

Diaphragm pumps come in a wide variety of ranges of sizes and performance parameters. Typically pressures up to 1200 bars, flow rates up to 180 m^3/h and temperature range of –80°C to 200°C are not unusual. Further, the diaphragm pumps are suitable for metering as well as for conveyance. The chief advantages of diaphragm pumps can be listed as below:

- Can run dry without damage for long intervals
- Self-priming
- Infinitely wide flow control
- Can handle debris laden liquids easily
- Use no electricity and hence can be used in explosion proof environment

Uses

Virtually, all major industries use diaphragm pumps in a wide variety of applications. A partial list of industries in which these pumps are used are automobile and aircraft industries, biomedical industries, paper and pulp industry, distilleries and food industry. Major applications include

- Chemicals transfer
- Clay slurry handling
- Residual sludge handling
- Waste products from surface-conditioning plants
- Paint transfer
- Vegetable-oil transfer
- Hygienic applications
- Handling glue, bleaching products and sodium silicate in paper and pulp industry

7.18 HYDRAULIC RAM: (HYDRAULIC RAM PUMP)

The hydraulic ram is a pumping device that utilises the principle of *water hammer* to lift small quantities of water to a higher level through use of large quantities of water at a lower head. Figure 7.26 shows a schematic layout of the hydraulic ram pump set-up. The source is a tank or a stream. A pipeline connects the source to the ram located at a lower level. The pumping device consists essentially of a chamber C, set of valves (V_1 and V_2) and an air vessel. This device, which does not use any other form of energy source than that of the flow of water, is in use since more than two centuries. John Whitehurst (1772) is credited as the inventor of the hydraulic ram pump.

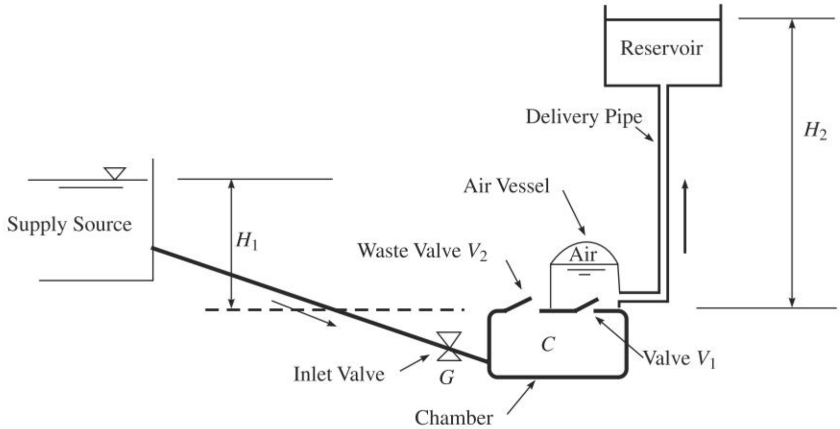


Fig. 7.26 *Hydraulic ram*

Operation

To start with, the gate valve G in the pipe is opened. Water rushes down the pipe into the chamber and some water passes out through the waste valve V_2 which is open. The build-up of the dynamic pressure within the chamber C at the seat of the valve V_2 causes the valve to shut down suddenly. This sudden closure and consequent sudden change of momentum causes a pressure build-up in the chamber. The excess pressure causes the valve V_1 to open and the water rushes to the air chamber. The compression of the air in the air chamber causes the water to pass out through the delivery pipe to the reservoir at its outlet.

The dissipation of the pressure wave caused by the momentum change causes the valve V_1 connecting the air chamber to close and also the waste valve V_2 to open. The process repeats. The valves V_1 and V_2 are one-way valves and act due to self-weight or due to spring loading.

It is usual to provide an air bleed valve, called *snifting* valve, to prevent formation of negative pressures in the chamber due to air entrainment action of the flows in the chamber.

Let h_{fs} and h_{fd} be the frictional head losses in the supply and delivery pipes respectively. Further,

H_1 = Height of water surface in the source above the chamber of the ram

H_2 = Height of delivery reservoir above the chamber

Q_d = Discharge delivered by the ram

Q_w = Discharge wasted by the ram

Q_s = Discharge supplied to the ram

$= (Q_d + Q_w)$

Work done per second = $\gamma Q_d (H_2 + h_{fd})$

Work done per second in supplying the flow to the ram

$$= \gamma Q_s (H_1 - h_{fs}) = \gamma (Q_d + Q_w)(H_1 - h_{fs})$$

$$\text{Efficiency of the ram} = \eta = \frac{Q_d(H_2 + h_{fd})}{Q_s(H_1 - h_{fs})} \tag{7.17}$$

where Q_s = Discharge supplied to the ram = ($Q_d + Q_w$).

If frictional losses are neglected $$\eta = \frac{Q_d H_2}{Q_s H_1} \quad (7.18)$$

A hydraulic ram pump is useful where the water source flows constantly and the usable fall from the water source to the pump location is at least 1.0 m

7.19 JET PUMPS

Jet pump is a generic name for pumps that use, to some extent, the suction created by a jet at a venturi expansion to lift the pumping liquid. Two types of jet pumps in common use are described here. Use of specific type name along with the suffix jet pump to designate a particular kind avoids possible confusion:

7.19.1 Centrifugal-jet Pump

This type of jet pump is a combination of a normal centrifugal pump and a jet device (called the *injector*) at the suction end. When the pump is started, a part of the water from delivery side of the pump is diverted into a nozzle. Water under high pressure is forced through this nozzle into the throat of a venturimeter-shaped pipe located in the suction side of the pump assembly. The negative pressure caused by the jet flow causes the water to be sucked up from the sump and delivers it to the pump. This additional suction enhances the total suction head of the pump assembly. Figure 7.27 is a schematic sketch of a jet-pump assembly. This device can obtain as much as 5 to 6 m of suction lift. Jet pumps are usually recognized in three types: shallow-well jet pumps, deep-well jet pumps and convertible jet pumps.

1. Shallow-Well Jet Pumps

The jet pump shown in Fig.7.27 is a shallow-well jet pump. Here, the jet assembly is integral with the main centrifugal pump. Only one suction pipe leads from the sump/ well into the pump. These pumps have a limitation in terms of the suction head of about 5.5 m and hence the installation has to be within the elevation range of about 5 m from the level of water in the sump.

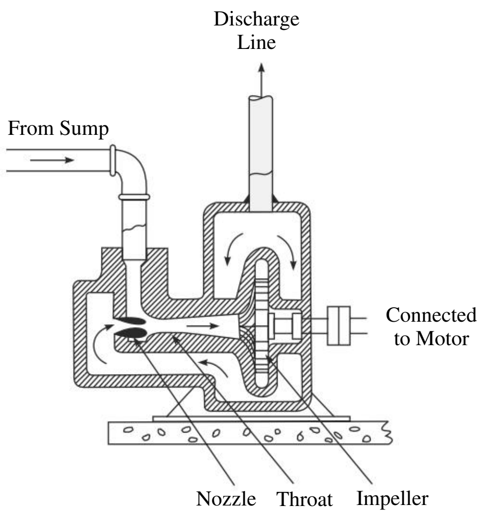


Fig. 7.27 *Jet-pump (for shallow well/sump)*

2. Deep-well Jet Pumps

Figure 7.28 is a schematic representation of a deep-well jet-pump assembly. Here, the injector (i.e. the jet assembly) is remote from the pump and is installed in the well/sump near the foot valve. As such, in a typical deep-well jet-pump installation, two pipes lead from the pump to the sump/well. One of them is the feed from the pump to the jet and the other is the delivery pipe. The injector assembly is attached to the suction pipe with a foot valve at the extreme end. The height of the jet assembly from the water surface of the sump is usually restricted to about 5.0 m from cavitation considerations. The main pump is a centrifugal pump of requisite capacity.

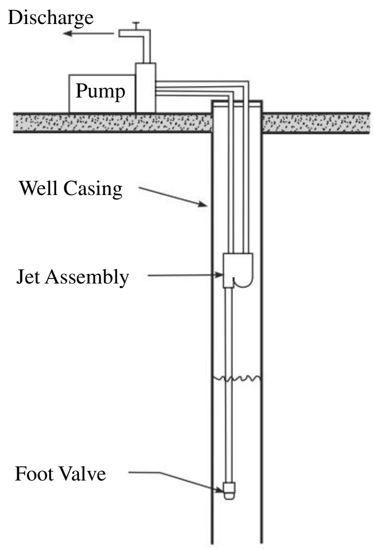


Fig. 7.28 *Deep-well jet-pump assembly*

3. Convertible Jet Pumps

In this type of pumps, the jet assembly is removable and can be fitted in either the pump block or remote from it. Thus, the advantages of both the shallow-well and deep-well pumps are available in this type.

Jet pumps are self-priming and typically provide low rates of flow at high pressure. Since the motors are above the water, jet pumps are easier to service than submersibles. Centrifugal jet pumps find major applications in domestic uses for supply of fresh water from wells as source.

7.19.2 Ejector Pumps

Ejector pumps (also known as *eductor* pumps) are another type of jet pumps in common use. These pumps work on the principle of jet action in a venture expansion. Figure 7.29 shows a schematic sketch of an ejector pump. A jet of water under high pressure is injected at the mouth of an expanding pipe. The reduction in the pressure (vacuum) causes a suction at the inlet for the pumping fluid. The pumped liquid as well as the jet liquid meet and merge and the combined mass is ejected out of the pump at the outlet.

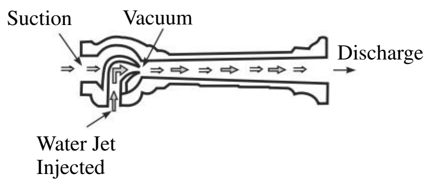


Fig. 7.29 *Ejector-type jet pump*

The efficiency of ejector-type jet pump is defined as

$$\eta = \frac{Q_s(H_s + H_d)}{Q_n(H_n - H_d)} \qquad (7.19)$$

where Q_n = Discharge through the nozzle, Q_s = Discharge through the suction pipe, H_n = Pressure head applied at the nozzle, H_s = Suction head, H_d = Delivery head. Generally, the efficiency of the ejector-type jet pump is low, being less than 50%.

These ejector pumps do not have any moving parts, are robust and provide reliable service. They find diverse specialised applications such as in

- Maritime activities for pumping bilge water, changing of ballast water and pumping of oil residue from ships

- Firefighting equipment
- Oil and gas industry
- Handling of wastewater in domestic and industrial environment

7.20 AIRLIFT PUMP

Airlift pump is a device which works through the buoyancy effect of the air-entrained water. A continuous jet of air is introduced at the bottom of the submerged end of a tube located in a sump. The tube, usually vertical, is called *eductor* and forms the delivery pipe also. there are no valves and no moving parts in this pump system. Figure 7.30 shows a schematic layout of an airlift pump. The submerged portion of the pipe of length S is the suction pipe and the remaining part of the pipe (eductor) that is above the static level of the sump is the delivery pipe. The lift of the pump L is the vertical distance between the centreline of the delivery pipe at its exit and the static water level in the sump. Note that the air supply is external to the pump; it can be a blower or a supply of compressed air from another source.

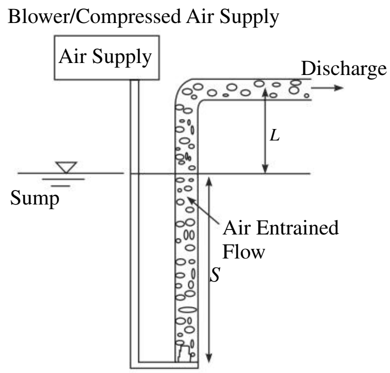
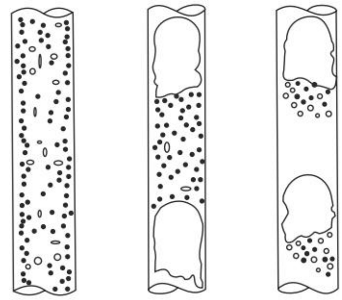


Fig. 7.30 *Schematic sketch of airlift pump*

The injected air enters the tube as a stream of bubbles and very soon, the entire water column in the eductor pipe becomes air entrained. The density of the air-water mixture being considerably less than that of water in the sump, a net buoyant force acts on the column of air-entrained water in the eductor pipe. This causes the air-entrained water to rise in the tube and exit at the outlet.

The rate of discharge is a function of the rate of air volume injected, the diameter of the pipe and the relative values of the submerged pipe length S and lift L. In a general sense, the discharge increases with rate of air volume supply till an optimum value for a given set-up and beyond that value the incremental increase in the water discharge deceases with further increase in the air discharge. It is found that smaller diameter tubes are capable of higher lifts for a given submerged length S. The ratio L/S has an important bearing on the efficiency of the pump. It is found that for S of the order of 15 m, $L/S \approx 0.5$ gives optimum efficiency. As the value of S increases, the ratio L/S for optimum efficiency also increases reaching a value of $L/S \approx 2$ for $S \approx 150$ m. The efficiency of the pump, defined as the ratio of water energy output to the energy expended in pumping in the air, is relatively low being of the order of 25% to 45%. The normally used air speed is 10 m/s. The diameter of the eductor pipe varies from 7.5 cm to 75 cm.

The analytical analysis of the mechanism of pumping in this pump is quite complicated and is usually attempted by considering the flow system as two-phase flow of air and water. Three modes of air-entrained flow are usually recognised (Fig. 7. 31). These are



(a) Bubble Flow (b) Bubbly-Slug Flow (c) Slug Flow

Fig. 7.31 *Modes of two-phase flow in air-lift pump*

1. For low air-injection rates, the distribution of air in water will be in the form of bubbles of different sizes and distributed fairly evenly. This mode is called *bubble flow*.
2. For very high rates of air injection, the air distribution in the water column will be in the form of several air pockets alternating with water pockets. These pockets of air and water are called slugs and this mode of air-entrained flow is called *slug flow*.
3. In between the bubble flow and the slug flow modes, there exists a region in which the slugs of air alternate with slugs of water having air bubbles. This mode is called *bubbly slug* flow.

These three modes are shown schematically in Fig.7.31.

It is found by experience that the bubble flow gives more lift than the slug flow.

The specific advantages of airlift pumps can be listed as

- No moving parts and hence easy installation and maintenance
- Lower initial cost
- Water pumped through the airlift pumps is highly aerated and hence has uniform oxygen level. This feature makes this device especially attractive in waste-water treatment facilities.

Uses

Airlift pumps finds use in a variety of special-purpose situations such as in

- Water circulation and aeration in aquaculture—a type of aquaculture system known as *Recirculating Aquaculture systems* (*RAS*) uses airlift pumps almost like a standard pumping device
- Pumping of slurries and other gritty solutions from ponds and sumps
- Waste-water treatment facilities; especially in handling (a) return activated sludge, (b) waste activated sludge, and (c) Aerobic digester supernatant return.

7.21 DEEP WELL TURBINE PUMP AND SUBMERSIBLE PUMP

These are vertical, multistage, centrifugal pumps specially adopted for pumping from deep tubewells/borewells. In these pumps, bowl assemblies replace the volute of conventional centrifugal pump. The bowls house impellers and guide vanes. All the impellers are connected to a common shaft. The entire assembly is always located beneath the water surface.

7.21.1 Deep-well Turbine Pump (Vertical Turbine Pump)

This is a mixed-flow type pump comprising of multiple stages. The term *turbine* refers to the fixed guide vanes that replace the conventional volute chamber to act as diffusers to the flow from the impeller. The pump consists essentially of a pipe of rather limited size extending from the bottom of the well up to the top. A foot valve with a strainer is fixed at the bottom and above it an assembly of bowls containing impellers and fixed guide vanes are assembled. All the impellers are connected to a common vertical shaft that extends up to the top of the well. At the top, above the ground surface, the shaft is connected to a source of power, viz. electric motor/ IC engine. The diameter of the pump is limited by the diameter of the well. Figure 7.32(a-1) shows a schematic view of a vertical turbine pump. Details of the bowl assembly of a vertical turbine pump are shown in Fig. 7.32(a-2). This figure shows three stages. Roughly, each stage in a pump contributes about 15 m of head to the pump. Vertical alignment of the well is very important in these pumps.

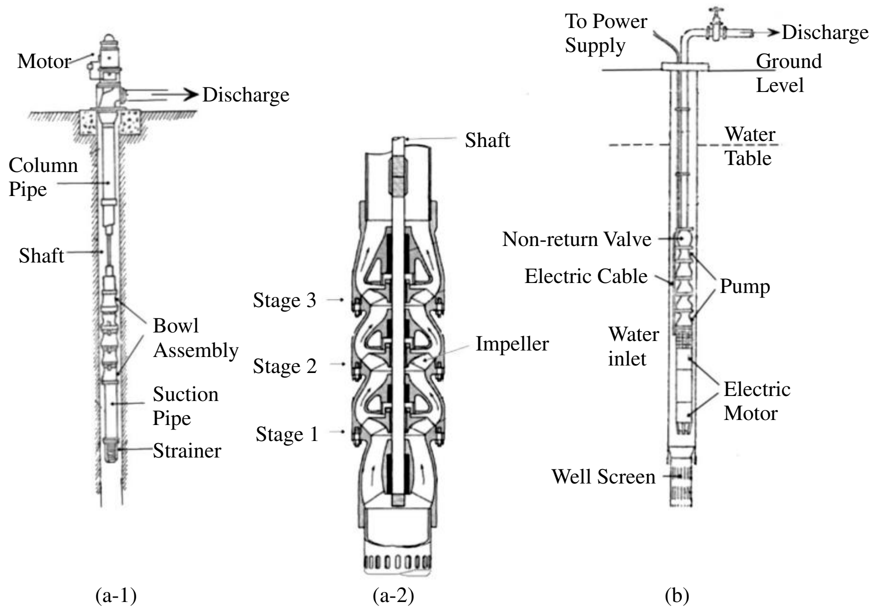


Fig. 7.32 *(a-1) Vertical turbine-pump assembly (a-2) Details of bowl assembly (b) submersible pump*

Since the drive is above the ground level, these pump installations require a pump house at the top. The pumps are sturdy and are eminently suited to the purpose of pumping from deep tubewells. However, the initial cost is high. Deep-well turbine pumps are widely used for large tubewells where the discharges are fairly large and the heads are high. The efficiency of these pumps is good and lies in the range of 70%.

7.21.2 Submersible Pumps

This is essentially a vertical turbine pump with a submersible electric motor close-coupled to it at the bottom. A specially designed submersible motor of cylindrical shape is fixed directly below the intake of the pump. The pump and the motor assembly is lowered into the well and remain submerged. A highly insulated electrical cable from the motor traverses up the well and is connected to the power source at the top [Fig.7.27 (b)]. Elimination of the long vertical shaft and column assembly, as in the case of the vertical turbine pump, is one of the main advantages of this pump. This feature enables the submersible pump to be used in very deep wells where the use of vertical shaft would not be feasible. Further, the elimination of a long shaft reduces bearing friction and provides an unobstructed pipe for delivery of water to the surface. The well need not be truly vertical and in fact, the alignment of the well is not at all a consideration. Further, there is no need of a pump house. Submersible pumps are the preferred choice for small-to medium-sized tubewell/borewells in the domestic water-supply sector.

7.22 REGENERATIVE FLOW PUMP (RFP)

7.22.1 Description

Regenerative Flow Pumps (*RFP*) are rotodynamic pumps capable of high heads at low flow rates. This pump is unique in design and operations as it combines the main characteristics of centrifugal pumps and also some features of a rotary displacement pump. *RFP* is known by several aliases, major ones being *regenerative turbine pump, peripheral pump, friction pump, side channel pump, traction pump* and *vortex pump.* A similar turbomachine used for compressing air is called *regenerative flow compressor* (*RFC*). The basic principles of construction and working and characteristics are common to *RFP* and *RFC*. Even though, Wahle of *USA* patented the regenerative pump in 1915, it is the recent advances in design, materials and manufacturing technology that has made this machine a viable pump in the niche market of low specific speeds.

An *RFP* consists essentially of a disc-shaped impeller with large number of small blades cut in a small band in the rim. In a standard design, the vanes occupy less than about 20 % outer radial space of the impeller. The number of blades (also called vanes) may be as many as 120. The vanes are cut on both sides of the impeller disc through machining. The vanes come in many arrangement patterns and the nature of vane spacing and arrangement on either side of the impeller may be same or different. Web pieces of curved profile span the vanes. Figures 7.33 and 7.34 show schematically the impeller assembly and some details of vane, web and surrounding channel.

A casing to form a toroidal shaped conduit with inlet and outlet ports covers the vane portion of the impeller. The conduit, called channel (or *side channel*), extends from the inlet port to outlet port covering an angular spread of 300° at the centre of the impeller. Figure 7.34 shows the details of the channel formed by the casing, web and vanes. Three types of vane configurations in common use are shown in this figure. Notice that the channel, in which the flow of the liquid takes place during pumping action, consists of two side channels on either side of the web separating the vanes and passage on the top of the web connecting the two side channels.

1. Stripper

The portion of the channel between the inlet and the outlet forming a sector of 60° angular width is blanked off with solid obstruction to prevent the cross-flow from

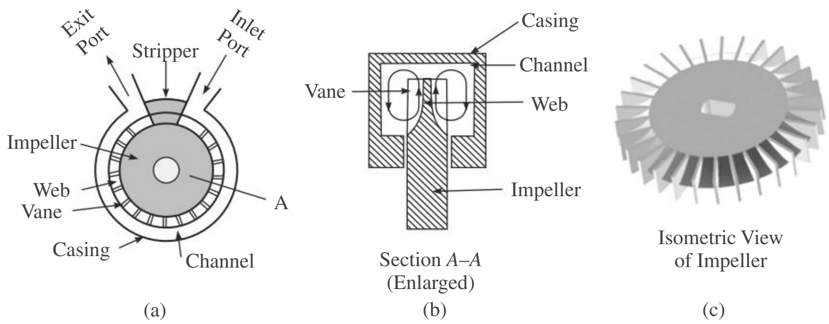


Fig. 7.33 *Details of a regenerative pump*

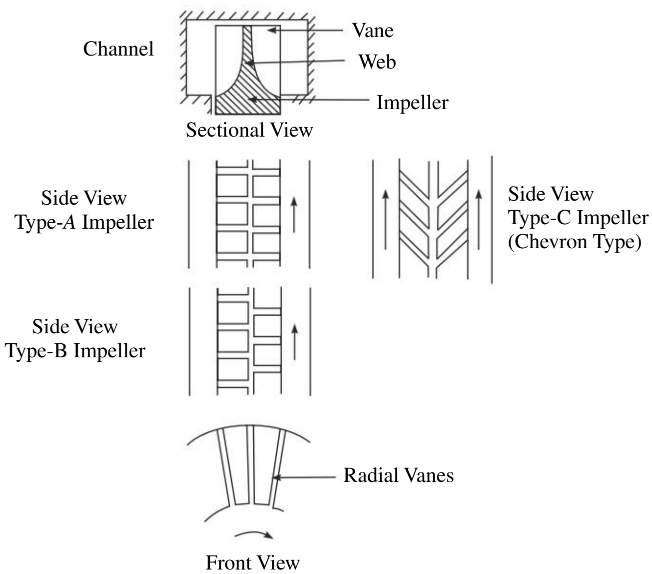


Fig. 7.34 *Details of vane arrangement in an RFP*

high-pressure outlet to the low-pressure inlet. This blanked portion is called *stripper* (or *septum*). To allow for the rotation of the impeller, very small clearances, of the order of a hundredth of a millimeter, is provided in the stripper. The stripper acts as a dead end to the side channel from the inlet and is hydraulically responsible for unique helical flow pattern in the flow channel.

2. Flow in the Channel

When the impeller is rotating, the vanes push the flow between two adjacent vanes in the direction of rotation. Due to the shear of the web portion of this cavity, a lateral flow is also formed in the side channel. The unique shape of the flow channel, with its two interconnected side channels generates, a vortex flow in the two cells, on each side of the web, Fig. 7.35. The interaction of the tangential flow due to vanes and the vortex due to shear action causes a helical (corkscrew type) flow in the channel. Due to this flow pattern, a liquid element gets into the space between the vanes, many times in its passage from inlet to the outlet. In each interaction with the impeller, some energy is transferred to the fluid element from the impeller. Each passage through the vane can be considered as equivalent to a *stage* in the conventional centrifugal pump. Thus, the fluid element in an *RFP* is multistaged in its helicoidal path and this enables the pump to generate high heads at relatively low discharges. High heads, low discharges and moderate speeds of *RFP* lead to low specific speed of these machines.

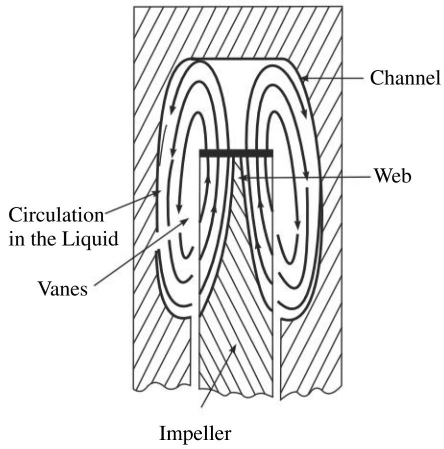


Fig. 7.35 *Circulatory flow in the flow channel*

7.22.2 Characteristics

The regenerative pumps are low-specific-speed pumps. The specific speed N_{sq} is in the range of 5 to 7.0. The speed is moderate and typically is in the range of 1750 rpm to 3500 rpm. The discharges are very low being of the order of magnitude of 0.1 to 1.0 litres/s. The heads are high and normally in the range of 10.0 m to 150 m. The important characteristics of the regenerative pump are the variation of head and efficiency with the capacity at constant speed. Figure 7.36 shows a typical variation of head and efficiency with discharge. The important feature to note is the linear decrease of head with discharge.

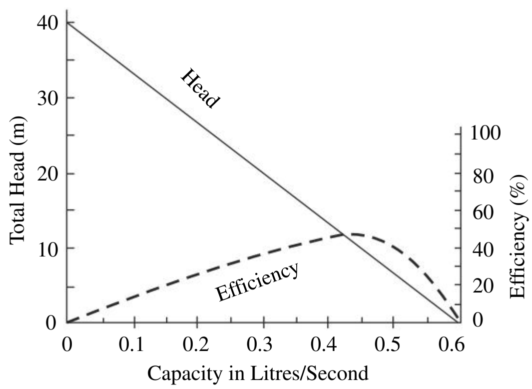


Fig. 7.36 *Typical variation of head and efficiency with capacity in a RFC at constant speed*

This feature enables fine control of the discharge through throttling by means of a valve. The efficiency is very low and in the operating range; it will be in the order of 20 to 45%.The brake power requirement of an *RFP* decreases with increase in the discharge. This feature is contrary to the behaviour of a centrifugal pump where the brake power required generally increases with increase in the discharge.

Other salient characteristics of *RFP* are as follows:

- Low *NPSHR* which enables it to handle vapours and air-entrained flow up to 20% concentration.
- The running tolerances in the pump are very close. This feature restricts the use of the pump only to clear and thin liquids. The viscosities of the liquids for which *RFP* can be used should be less than 250 *SSU*. If the liquid contains

solids in suspension, the pump is likely to be choked or there will be wearing out of parts leading to leaks and loss of efficiency.

- Most of the *RFP* pump models are self-priming. Further, they are insensitive to direction of rotation.
- The pumps are available in a wide range of material from stainless steel to polypropylene and *PVDF*. This enables its applications to cover a wide variety of chemical applications and a wide range of temperatures.
- More compact than the multistage centrifugal pump for the same head and discharge. In addition, the operation is quieter and the pump less expensive than an equivalent centrifugal pump.
- The design is very rugged and the *RFC* pump is normally designed for continuous (24 × 7) duty without damage to the parts.

7.22.3 Applications

*RFC*s find a wide range of applications. Their use includes areas such as

- Automotive and aerospace fuel pumping
- Booster systems
- Domestic water supply
- Agricultural industries
- Chemical and foodstuff industries

7.23 HYDRAULIC ACTUATOR

7.23.1 General

A fluid-power system uses fluid as a means of transferring power. A prime mover such as an electric motor/*IC* engine drives a pump that develops a flow of fluid with requisite pressure. The fluid flow is conveyed to a destination in pipes/tubes. The energy of the fluid at the other end is used in an actuator to develop mechanical motion. The fluid used in the system can be a liquid, in which case the system is called a hydraulic power system. If the fluid is air, it is called a pneumatic system. Valves of different kinds aid in controlling the behaviour of the system.

A hydraulic actuator receives pressure energy and converts it to a mechanical force and motion. An actuator that gives linear motion as the output is called a *cylinder* or a *ram*. It is also known as *reciprocating motor* or *linear motor*. A *rotary actuator* produces torque and rotating motion and is popularly known as *hydraulic motor*.

7.23.2 Hydraulic Motor

A *hydraulic motor*, also known as rotary actuator, is a device that converts hydraulic pressure and flow into torque and rotation of a shaft. Conceptually, it is the inverse of a hydraulic pump. Hydraulic motors find application in hydraulic drive systems that are used in heavy duty systems such as excavators, mixers drives, winches, crane drives, roll mills and four-wheel drives of military vehicles.

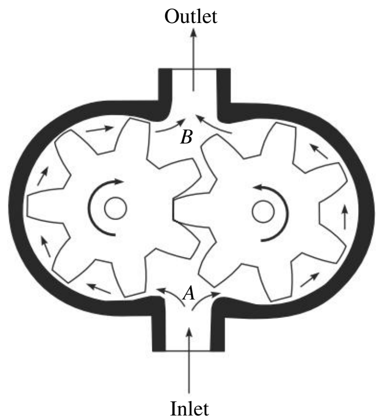


Fig. 7.37 *External gear type hydraulic motor*

The general construction of a hydraulic motor is similar to that of a corresponding hydraulic pump. However, some necessary modifications are done to make the system efficient. Because of the simplicity of operation, all hydraulic motors are of positive-displacement type. Most of the positive-displacement pumps, like gear pump, vane pump and piston pump have their counterpart of a hydraulic motor.

An external-gear-type hydraulic motor is shown in Fig. 7.37. This motor consists essentially of two meshing gears. Both gears are driven gears, but only one is connected to the output shaft. A hydraulic fluid under pressure is fed to one of the ports. Operation is essentially the reverse of that of a gear pump. Flow from the pump enters the chamber *A* and flows in either direction around the inside surface of the casing. The reaction of the fluid on the meshing gears causes the gears to rotate and at the same time the fluid is moved to be ultimately ejected out at the outlet port in the chamber *B*. Continuous supply of fluid under pressure, delivers continuous torque to the shaft of the main gear. The rotary motion of the shaft is now available for work. Note that there are no valves involved in the operation. The inlet and outlet ports are interchangeable to cause change in the direction of rotation. The speed of rotation can be altered by changing the pressure or flow rate of the hydraulic fluid at the inlet port.

Hydraulic motors are rated according to their (a) displacement, (b) torque capacity, and (c) maximum pressure limitation. They come in many types and some of the common types are

- Gear-type motor (both external and internal gear type (called *Gerotor*) motor)
- Vane-type (both unbalanced and balanced type) motors
- Piston-type motors (both axial and radial types)
- Screw motor

Hydraulic motors form the essential component of many heavy-duty machinery used in agricultural, transportation, forestry, recycling and marine operations.

7.24 ILLUSTRATIVE EXAMPLES

7.24.1 Hydrostatic Systems

*EXAMPLE 7.1

An accumulator has a ram of 250 mm in diameter. The effective stroke of the ram is 3.0 m and the total load on the ram is 450 kN. Find the total power delivered by the accumulator when the ram falls through its full stroke in 90 seconds. The frictional effects in the accumulator can be taken as 2% of the total load on the ram. What is the displacement of the accumulator?

Solution

Effective load = 450 – (0.02 × 450) = 441 kN

Power supplied by the accumulator = $\boldsymbol{P} = \dfrac{WL}{t} = \dfrac{441 \times 3.0}{90}$

= 14.7 kW

The displacement of the accumulator is the volume displaced by the ram in one full stroke.

Displacement = $\dfrac{\pi \times (0.250)^2}{4} \times 3.0 = 0.1473 \text{ m}^3 = 147.3$ litres

*EXAMPLE 7.2

An accumulator has a ram of 200 mm in diameter. The effective stroke of the ram is 2.5 m and the total load on the ram is 350 kN. Find the total power delivered by the accumulator to the main when the ram falls through its full stroke in 60 seconds and if at the same time the pumps are delivering 10 litres/second of oil to the main. The frictional effects in the accumulator can be neglected.

Solution

Intensity of pressure = $\dfrac{350}{\left(\dfrac{\pi}{4}\right) \times (0.2)^2} = 11141$ kPa

Equivalent pressure head = $h = \dfrac{p}{\gamma} = \dfrac{11141}{\gamma}$ m of oil

Work done per second by the pump = $P_1 = \gamma Q h$

$= P_1 = \gamma \times \dfrac{10}{1000} \times \dfrac{11141}{\gamma} = 111.4$ kW

Work done by the accumulator due to load coming down = $\boldsymbol{P} = \dfrac{WL}{t}$

$= \dfrac{350 \times 2.5}{60} = 14.58$ kW

Total power delivered to the main = $P_t = P_1 + P_2 = 111.4 + 14.58 = 126$ kW

*EXAMPLE 7.3

An accumulator of a hydraulic system has a ram of 30 cm diameter and stroke of 5.0 m. It is loaded with 800 kN and the friction is assumed to consume 2% of this load. If the full stroke of the ram falls steadily in 2 minutes, while the pumps are delivering 15 litre/s to the machinery, what power is delivered to the machinery by the hydraulic system?

Solution

Effective load = 800 – (0.02 × 800) = 784 kN

Power supplied by the accumulator = $\boldsymbol{P} = \dfrac{WL}{t} = \dfrac{784 \times 5.0}{120}$

= 32.67 kW

Pressure at the accumulator $= p = \dfrac{784}{\left(\dfrac{\pi}{4}\right) \times (0.30)^2} = 11091.3$ kN/m^2

Discharge $Q = 15$ litre/s

Power supplied by the pump $= \gamma Qh$

$$= \gamma Q \frac{p}{\gamma} = pQ = 11091.3 \times \left(\frac{15}{1000}\right) = 166.37 \text{ kW}$$

Total power supplied by the hydraulic system = 32.67 + 166.37 = 199.04 kW

*EXAMPLE 7.4

The diameters of a differential accumulator are as follows: (a) without the bush it is 120 mm, and (b) with the bush it is 150 mm. The length of the stroke is 1.50 m. If a pressure of 8000 kPa is desired in the sliding cylinder, estimate the load on the cylinder. What is the energy capacity of the accumulator?

Solution

$W = p \times$ Annular area $= 8000 \times \dfrac{\pi}{4}\left((0.15)^2 - (0.12)^2\right) = 50.9$ kN

Energy capacity $= C = W \times L = 50.9 \times 1.50 = 76.35$ kN.m

*EXAMPLE 7.5

The sliding ram of a hydraulic accumulator carries a load W. The diameter of the ram is 20 cm and the pressure inside the accumulator is to be maintained at 8000 kPa. If the friction due to packing can be assumed to be 5% of the weight W, estimate the value of the weight W when
(a) the ram descends with uniform velocity, and
(b) it ascends with uniform velocity.

Solution

(a) When the ram descends, the friction force acts upwards.

Hence, $(W - 0.05\,W) = pA$

$$0.95\,W = 8000\,\frac{\pi}{4}(0.2)^2$$

$$W = \frac{251.33}{0.95} = 264.6 \text{ kN}$$

(b) When the ram ascends, the friction force acts downwards.

Hence, $(W - 0.05\,W) = pA$

$$1.05\,W = 8000\,\frac{\pi}{4}(0.2)^2$$

$$W = \frac{251.33}{1.05} = 239.36 \text{ kN}$$

*EXAMPLE 7.6

A hydraulic intensifier has a fixed ram of 10 cm diameter and a sliding ram of 50 cm diameter. Calculate the pressure at the outlet of the intensifier, if the supply pressure is 2 bar. Assume that the loss due to friction at each of the packing of the intensifier is 2% of total force on respective packings.

Solution

From equilibrium considerations of the moving ram at any position,

$$p_1 A_1 (1 - \varepsilon) = \frac{p_2 A_2}{(1-\varepsilon)}$$

$$p_2 = \frac{p_1 A_1}{A_2}(1-\varepsilon)^2$$

$$p_2 = 200 \times \left(\frac{50}{10}\right)^2 (1-0.02)^2 = 4802 \text{ kPa}$$

*EXAMPLE 7.7

A hydraulic intensifier is to increase the pressure of a fluid from 400 kPa to 1800 kPa. The stroke of the intensifier is 0.9 m and the capacity is 20 litres. Determine the inside diameters of the high-pressure fixed ram and the sliding ram.

Solution

Refer to Fig. 7.6.

Displacement volume = 20 litres = 0.02 m^3

Area of low-pressure fixed cylinder = $A_1 = \dfrac{0.02}{0.90} = 0.0222 \text{ m}^2$

Diameter of low-pressure cylinder = $d_1 = \sqrt{\dfrac{4A_1}{\pi}} = \sqrt{\dfrac{4 \times 0.02222}{\pi}} = 0.168 \text{ m}$

$= 16.8$ cm

Since $p_1 A_1 = p_2 A_2$,

Area of high-pressure fixed ram = $A_2 = \dfrac{p_1 A_1}{p_2} = \dfrac{400 \times 0.02222}{1800} = 0.00493 \text{ m}^2$

Diameter of high-pressure fixed ram = $d_2 = \sqrt{\dfrac{4A_2}{\pi}} = \sqrt{\dfrac{4 \times 0.00493}{\pi}}$

$= 0.0793$ m $= 7.93$ cm

*EXAMPLE 7.8

In a hydraulic press a compressive force of 500 kN is required at a job. The frictional resistance of the 30 cm ram operating the loading platform can be assumed to be 25 kN. Calculate the force to be applied on a plunger operating in a 10 cm diameter cylinder that provides the pressure fluid to the press. The frictional resistance at the plunger assembly can be taken as 1% of the external load on the plunger.

Solution

At the press, force to be provided by the fluid pressure = 500 + 25 = 525 kN

Fluid pressure at the base of the ram = $\dfrac{525}{\left(\dfrac{\pi}{4}\right)\times(0.30)^2}$ = 7427 kN/m^2

Area of the plunger = A_p = $\left(\dfrac{\pi}{4}\right)\times(0.10)^2$ = 0.007854 m^2

Force required on the plunger (without frictional losses)

F_1 = 7427 × 0.007854 = 58.33 kN

Friction at plunger assembly = 0.01 × 58.33 = 0.5833 kN

Total force required at the plunger = F_p = (F_1 + 1% of F_1) = (58.33 + 0.583)

= 58.91 kN

*EXAMPLE 7.9

A hydraulic crane that is fed with water at a pressure of 6000 kN/m^2 is required to lift a load of 30 kN through a total height of 10 m. The jigger system gives a velocity ratio of six. (a) Estimate the stroke length and diameter of the ram in the jigger, by assuming an efficiency of 70%. (b) If the pressure of supply is raised to 7000 kN/m^2, what maximum weight can be lifted through the same height of 10 m at the same speed?

Solution

(a) Load on ram × Velocity of ram × η = Load lifted × Velocity of lifting of load

Load on the ram = $\dfrac{W\times\text{velocity ratio}}{\eta}=\dfrac{30\times 6}{0.7}=257.1$ kN

Area of ram A = $\dfrac{\text{Load on ram}}{\text{pressure}}=\dfrac{257.1}{6000}=0.04286$ m^2

Diameter of ram = d = $\sqrt{\dfrac{4A}{\pi}}=\sqrt{\dfrac{4\times 0.04286}{\pi}}=0.2336$ m = 23.36 cm

Stroke of ram = $\dfrac{\text{Distance of Lift}}{\text{Velocity ratio}}=\dfrac{10}{6}=1.667$ m

(b) Load on the ram = p_1A = 7000 × 0.04286 = 300.0 kN

Maximum weight W = $\dfrac{\text{Load on the ram }\times\ \eta}{\text{Velocity ratio}}=\dfrac{300.0\times 0.7}{6}=35.0$ kN

**EXAMPLE 7.10

An accumulator maintains a pressure of 8000 kPa and supplies water under pressure to a hydraulic crane. The frictional losses in the transmission can be taken as 10%. The crane has a jigger with 20 cm ram. The velocity ratio of the jigger pulleys is 8:1.

(a) Assuming frictional losses of 400 kPa in the jigger due to mechanical resistance of the pulleys, etc. calculate the load that can be raised at a speed of 0.5 m/s to a height of 9 m. (b) What is the required stroke of the jigger ram?

Solution

Effective pressure on jigger = 8000 × (1 – 0.1) – 400 = 6800 kPa

$$\text{Load on the jigger ram} = 6800 \times \left(\frac{\pi}{4}\times(0.2)^2\right) = 213.6 \text{ kN}$$

$$\text{Load lifted by crane} = W = \frac{\text{Load on the ram} \times \eta}{\text{Velocity ratio}} = \frac{213.6\times1}{8} = 26.7 \text{ kN}$$

(Efficiency $\eta = 1$ is assumed).

$$\text{Stroke of ram } L = \frac{\text{Distance of lift}}{\text{Velocity ratio}} = \frac{9}{8} = 1.125 \text{ m}$$

**EXAMPLE 7.11

A hydraulic accumulator has a ram of 40 cm diameter and its stroke is 5 m. It takes 2 minutes for the ram to move down one full stoke length with a total load of 500 kN on it. While the ram is moving down, a pump supplies oil to the machine at the rate of 80 litres/s. The friction at the packing can be assumed to be 5% of the total load on the ram. Determine (a) pressure intensity within the accumulator, (b) power delivered to the machine by the accumulator, and (c) power required to drive the pump by assuming an efficiency of 70%.

Solution

(a) $$\text{Pressure intensity } p = \frac{W(1-\varepsilon)}{A_r} = \frac{500\times(1-0.05)}{\frac{\pi}{4}(0.4)^2} = 3780 \text{ kPa}$$

(b) $$\text{Power delivered by the accumulator} = P_a = \frac{W(1-\varepsilon)L}{t} = \frac{500\times0.95\times5}{2\times60} = 19.79 \text{ kPa}$$

$$\text{Power delivered by the pump} = pQ = 3780 \times \frac{80}{1000} = 302.4 \text{ kW}$$

(c) $$\text{Power required to drive the pump} = P_{sp} = \frac{302.4}{0.70} = 432 \text{ kW}$$

*EXAMPLE 7.12

In a material-testing machine, the force is applied to the specimen by the ram of a hydraulic press. The diameter of the ram is 30 cm and the maximum force to be applied on the specimen is 1200 kN. The frictional resistance can be assumed to be 6% of the applied load. The oil in the press is pressurised by loading a plunger in a vertical cylinder of 15 cm diameter. If the frictional effects at the plunger amounts to 50 kN, estimate the necessary load on the plunger needed for the task.

Solution

At the ram:

Pressure required on the ram $p = \dfrac{W_r(1+\varepsilon)}{A_r} = \dfrac{1200 \times (1+0.06)}{\frac{\pi}{4}(0.3)^2} = 17995 \text{ kPa}$

At the plunger: Pressure on the plunger = 17995 = $\dfrac{(W_p - 50)}{\frac{\pi}{4}(0.15)^2}$

$$W_p - 50 = 17995 \times 001767 = 318$$

Weight on the plunger = W_p= 368 kN

***EXAMPLE 7.13

A hydraulic lift raises 200 kN through a height of 20 m once every 3 minutes. The speed of lift is 0.8 m/s. It is worked from an accumulator, which is being continuously charged by a pump. The pressure of water is 3500 kPa. The efficiency of the lift is 0.79 and that of the pump is 0.85. Find the power required to drive the pump and the minimum capacity of the accumulator. Neglect all other losses.

Solution

Work done per second in lifting the load = $\dfrac{W \times u}{\eta} = \dfrac{200 \times 0.8}{0.79} = 202.5 \text{ kW}$

Time taken to lift the weight to 20 m height = $\dfrac{20}{0.8}$ = 25 s

Energy spent when the lift moves the weight by 20 m = 202.5 × 25 = 5063 kW.s

The pump supplies this energy in 180 seconds.

Continuous power transferred to the fluid by the pump = $\dfrac{5063}{180}$ = 28.13 kW

P_p = Power required to drive the pump = $\dfrac{28.13}{0.85}$ = 33.1 kW

Energy supplied to recharge the accumulator = E_s= 28.13 × (180 – 25) = 4360 kW.s

If C is the capacity of the accumulator and p is its pressure then $pC = E_s$

$$C = \frac{E_s}{p} = \frac{4360}{3500} = 1.246 \text{ m}^3$$

The minimum capacity of the accumulator = 1.246 m^3

Check for energy balance: Energy supplied by the accumulator = 4360 kW.s

Energy supplied by the pump during the operation of the lift = 28.13 × 25

= 703 kW.s

Total energy supplied = 4360 + 703 = 5063 kW.s = Energy spent in doing work.

7.24.2 Fluid Coupling

**EXAMPLE 7.14

A fluid coupling has a 4% slip when the input shaft is rotating at 1500 rpm. If the input shaft has a torque of 20 N.m, estimate the output torque and the efficiency of the coupling.

Solution

Given: $N_1 = 1500$ rpm, $T_1 = 20$ kN.m, Slip $S = 4\% = 0.04$

Efficiency $\eta_T = 1 - S = 1 - 0.04 = 0.96$

$$\text{Slip } S = \frac{(\omega_1 - \omega_2)}{\omega_1} = \frac{(N_1 - N_2)}{N_1} = 0.04$$

$$N_2 = 0.96\, N_1 = 0.96 \times 1500 = 1440 \text{ rpm}$$

$$\omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi \times 1500}{60} = 157.08 \text{ rad/s}$$

$$\omega_2 = \frac{2\pi N_2}{60} = \frac{2\pi \times 1440}{60} = 150.8 \text{ rad/s}$$

$$P_1 = T_1 \omega_1 = 20 \times 157.08 = 3141.6 \text{ W}$$

$$P_2 = \eta_T P_1 = 0.96 \times 3141.6 = 3016 \text{ W}$$

$$T_2 = \text{Output torque} = \frac{P_2}{\omega_2} = \frac{3016}{150.8} = 20 \text{ N.m}$$

7.24.3 Torque Converter

**EXAMPLE 7.15

A torque converter has rotational speeds of 2400 rpm at the input shaft and 900 rpm at the output shaft. The input and output torques are 50 N.m and 110 N.m respectively. Determine the corresponding powers of the input and output shafts and the efficiency of the torque converter.

Solution

Given: $N_1 = 2400$ rpm, $T_1 = 50$ N.m, $N_2 = 900$ rpm, $T_2 = 110$ kN.m

$$\omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi \times 2400}{60} = 251.33 \text{ rad/s}$$

$$\omega_2 = \frac{2\pi N_2}{60} = \frac{2\pi \times 900}{60} = 94.25 \text{ rad/s}$$

Power input = $P_1 = T_1 \omega_1 = 50 \times 251.33 = 12566.5$ W

Power output = $P_2 = T_2 \omega_2 = 110.0 \times 94.25 = 10367.5$ W

$$\text{Efficiency of torque converter} = \eta_T = \frac{P_2}{P_1} = \frac{10367.5}{12566.5} = 0.825 = 82.5\%$$

7.24.4 Hydraulic Ram Pump

*EXAMPLE 7.16

A hydraulic ram pump lifts 5 litres/s of water to a height of 18 m through a 100 m long delivery pipe of 10 cm diameter. The supply source is 4.0 m above the ram. It is estimated that the water wasted at the ram is 75 litres/s. Assume the frictional loss at the supply pipe to be negligible and the friction factor of f = 0.02 is appropriate for the delivery pipe. Calculate the efficiency of the hydraulic ram.

Solution

Given:

$H_1 = 4.0$ m

$H_2 = 18.0$, $Q_d = 5.0$ litres/s = 0.005 m^3/s

$Q_w = 75.0$ litres/s = 0.075 m^3/s

Q_s = Discharge supplied to the ram =(0.005 + 0.075) = 0.08 m^3/s

A_d = Area of delivery pipe = $\frac{\pi}{4}(0.1)^2 = 0.007854$ m^2

V_d = Velocity in the delivery pipe = $\frac{0.005}{0.007854} = 0.6367$ m/s

$$h_{fd} = \frac{fLV^2}{2gD_d} = \frac{0.02\times 100\times(0.6367)^2}{2\times 9.81\times 0.10} = 0.413 \text{ m}$$

Efficiency of the ram = $\eta = \frac{Q_d(H_2 + h_{fd})}{Q_s(H_1 - h_{fs})}$

$$\eta = \frac{0.005\times(18+0.413)}{0.08\times 4} = 0.288 = 28.8\%$$

**EXAMPLE 7.17

A hydraulic ram pump receives 80 litre/s of water from a source under a head of 5.0 m and delivers 8.0 litre/s to a reservoir 15 m above the ram. The delivery pipe is 75 m long and has a diameter of 100 mm. The supply pipe is 12 m long and is 200 mm in diameter. (a) Assuming a friction factor f = 0.025 for both the pipes, estimate the efficiency of the ram. (b) What would be the efficiency if the friction in the pipes are neglected?

Solution

Given: $H_1 = 5.0$ m

$H_2 = 15.0$, $Q_d = 8.0$ litres/s = 0.008 m^3/s

Q_s = Discharge supplied to the ram = 80 liter/s = 0.08 m^3/s

(a) A_d = Area of delivery pipe = $\frac{\pi}{4}(0.1)^2 = 0.007854$ m^2

V_d = Velocity in the delivery pipe = $\frac{0.008}{0.007854} = 1.0186$ m/s

$$h_{fd} = \frac{fLV_d^2}{2gD_d} = \frac{0.025 \times 75 \times (1.0186)^2}{2 \times 9.81 \times 0.10} = 0.992 \text{ m}$$

A_s = Area of supply pipe = $\frac{\pi}{4}(0.2)^2 = 0.0314 \text{ m}^2$

V_s = Velocity in the supply pipe = $\frac{0.08}{0.0314} = 2.548$ m/s

$$h_{fs} = \frac{fLV_s^2}{2gD_s} = \frac{0.025 \times 12 \times (2.548)^2}{2 \times 9.81 \times 0.20} = 0.496 \text{ m}$$

Efficiency of the ram = $\eta = \frac{Q_d(H_2 + h_{fd})}{Q_s(H_1 - h_{fs})}$

$$\eta = \frac{0.008 \times (15 + 0.992)}{0.08 \times (5 - 0.496)} = 0.355 = \mathbf{35.5\%}$$

(b) If the friction in the pipes is neglected:

Efficiency of the ram = $\eta = \frac{Q_d H_2}{Q_s H_1}$

$$\eta = \frac{0.008 \times 15}{0.08 \times 5} = 0.30 = \mathbf{30\%}$$

*EXAMPLE 7.18

A source of water can supply 90 litres/s under a head of 5.0 m to a hydraulic ram pump. Estimate the discharge that can be delivered to a tank situated 24 m above the pump. Assume an efficiency of 40% for the pump. The friction head in the supply pipe and delivery pipes can be taken as 0.2 m and 0.70 m respectively.

Solution

Given: $H_1 = 5.0$ m, $H_2 = 24.0$, $h_{fd} = 0.70$ m, $h_{fs} = 0.2$ m

Q_s = Discharge supplied to the ram = 0.090 m^3/s, $\eta = 0.40$

Efficiency of the ram = $\eta = \frac{Q_d(H_2 + h_{fd})}{Q_s(H_1 - h_{fs})}$

$$0.40 = \frac{Q_1 \times (24 + 0.7)}{0.090 \times (5 - 0.2)}$$

$$Q_d = \frac{0.4 \times 0.432}{24.7} = 0.007 \text{ m}^3\text{/s} = 7 \text{ litres/s}$$

7.24.5 Ejector Pump

*EXAMPLE 7.19

An ejector-type jet pump is located at 3.0 m above the sump water level and has a head of 13 m applied at the nozzle. The delivery head is 11.0 m when the discharge *through the pump is 15 l/s and the jet discharge is 2.5 l/s. Estimate the efficiency of the ejector jet pump.*

Solution

Efficiency of the jet pump $\eta = \dfrac{Q_s(H_s + H_d)}{Q_n(H_n - H_d)}$

Here, H_n = 13.0 m, H_d = 11.0 m, H_s = 3.0 m,

$(H_d + H_s) = (11.0 + 3.0) = 14.0$ m, $(H_n - H_d) = 13.0 - 11.0 = 2.0$ m

$Q_n = 2.5$ *l*/s, $Q_s = (15.0 - 2.5) = 12.5$ *l*/s,

$$\eta = \frac{12.5 \times 14.0}{2.5 \times 2.0} = 35.0\%$$

7.24.6 Submersible Pump

*EXAMPLE 7.20

A submersible pump in a well is set at 180 m below the ground level. At maximum drawdown, the water surface in the well is at 160 m below the ground level. The discharge pipe from the pump to the surface is 25 cm in diameter. How many stages with specific speed N_{sq}= 80 are needed if the discharge from the well at maximum drawdown is 0.1 m^3/s? The speed of the pump is 1450 rpm and the Darcy−Weisbach friction factor f for the pipe is 0.02.

Solution

Specific speed $N_{sq} = \dfrac{N\sqrt{Q}}{H_m^{3/4}}$

$$80 = \frac{1450 \times \sqrt{0.1}}{H_m^{3/4}}$$

$H_m^{3/4} = 5.73$ leading to $H_m = 10.25$ m

Velocity in the delivery pipe $V = \dfrac{Q}{A} = \dfrac{0.1}{(\pi/4)\times(0.25)^2} = 2.04$ m/s

Head loss in the delivery pipe = $\dfrac{fLV^2}{2gD} = \dfrac{0.02 \times 180 \times (2.04)^2}{2 \times 9.81 \times 0.25} = 3.05$ m

Total head = 160 + 3.05 = 163.05 m

Number of stages = $\dfrac{\text{Total head}}{H_m} = \dfrac{163.05}{10.25}$ = 16 stages

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Review Questions

7.1 Explain the working principle of
(a) Hydraulic accumulator of ordinary type
(b) Differential-type accumulator

7.2 Write short notes on
(a) hydraulic winch, (b) hydraulic jack, and (c) jigger.

7.3 Sketch a schematic figure of a modern hydraulic crane and label its salient parts. Briefly describe its working.

7.4 Describe briefly the working of a present day hydraulic lift.

7.5 What is a hydraulic system? Distinguish between hydrostatic and hydrodynamic systems with suitable examples.

7.6 What is a rotary pump? What are its chief characteristics?

7.7 Describe briefly the salient features of
(a) gear pump, (b) vane pump, and (c) piston pump.

7.8 Explain the working of (a) fluid coupling, and (b) fluid torque converter. What are their main applications?

7.9 With the help of neat sketches describe the working of a diaphragm pump.

7.10 Distinguish between the following:
(a) Hydraulic accumulator and hydraulic intensifier
(b) Hydraulic coupling and hydraulic torque converter
(c) Hydraulic intensifiers of ordinary type and differential type
(d) Internal gear pump and external gear pump
(e) Centrifugal jet pump and ejector type jet pump
(f) Axial piston pump and radial piston pump

7.11 Explain the working of a hydraulic ram pump.

7.12 What is the principle of operation of an airlift pump?

7.13 Describe the salient features of (a) vertical-turbine pump, and (b) submersible pump.

7.14 Describe the salient features of a regenerative flow pump.

7.15 Compare and contrast the important features of a regenerative pump and centrifugal Pump.

7.16 What is an actuator? Describe a gear-type rotary actuator.

7.17 List the essential components of a hydraulic power system.

Problems

P7.1 *An accumulator is loaded with 800 kN weight. The ram has a diameter of 305 mm and stroke length of 1.2 m. The friction can be assumed to be 3% of the total load on the ram. (a) Find the power supplied by the accumulator when the ram descends through the full stroke length in 100 seconds. (b) If a pump is supplying 6 litres/second of liquid to the machine while the accumulator is descending, calculate the power supplied by the pump.

[***Ans:*** P_1 = 9.312 kW, P_2 = 63.73 kW]

P7.2 *The diameters of a differential accumulator without the bush and with the bush are 125 mm and 150 mm, respectively. Find the load on the ram to have a pressure intensity of 100 bars. What is the length of stroke required to have an energy capacity of 0.03 kWh? [***Ans:*** L = 2.0 m]

P7.3 *An accumulator with 25 cm diameter ram and 30 cm length of stroke is loaded with total weight of 500 kN. The frictional losses can be assumed to be 5% of load on the accumulator. The ram slides down its full length of stroke, with a uniform velocity, in 100 seconds. While the ram is descending, a pump connected to the accumulator system delivers 300 litres/minute of oil to the machine. Calculate the total power being delivered to the machine.

[***Ans:*** P_t = 62.61 kW]

P7.4 *A hydraulic intensifier has a fixed ram of 10 cm diameter and a sliding cylinder of 40 cm diameter. The loss due to friction at each packing of the unit is 4% of the total force on each of the packing. Estimate the pressure of liquid on the low-pressure side if the pressure on the high-pressure side is 30,000 kPa. [***Ans:*** P_1 = 2034.5 kW]

P7.5 *The ram of a hydraulic press is 20 cm in diameter and is worked from an intensifier. The high-pressure intensifier ram is 8.0 cm in diameter and the low-pressure piston is 90 cm in diameter. If the speed of advance of the high-pressure ram in the intensifier is 10 cm/s, calculate the speed of advance of the press ram in cm/minute when exerting a force of 500 kN.

[***Ans:*** V = 96 cm/minute]

P7.6 *A hydraulic press has a ram of 25 cm diameter and a plunger of 25 mm diameter. The plunger has a stroke of 25 cm and makes 30 strokes per minute. If a weight of 40 kN is to be lifted by the press, find the force on the plunger and rate of rise of the weight.

[***Ans:*** F_p= 0.40 kN, V_{ram} = 7.5 cm/minute]

P7.7 *In a hydraulic press, the ram and the plunger have diameters of 25 cm and 2.5 cm respectively. The press is to lift a weight of 60 kN through a height of 1.2 m in 90 seconds. The stroke of the plunger is 25 cm. Calculate the (a) force required to lift the weight, (b) power required to drive the plunger, (c) volume of oil to be pumped, and (d) the number of strokes per minute of the plunger.

[***Ans:*** F_p= 0.60 kN, P = 0.8 kW, $\forall$ = 58.9 litres, N = 320 strokes/min]

P7.8 **In a hydraulic press, the diameter of the ram is 15 cm and the plunger is of 2 cm diameter. Estimate the force on the plunger required to lift a load of 40 kN. (a) How many strokes of the plunger are required to lift the load through 1.5 m, if the stroke of the plunger is 37.5 cm? (b) Determine the power of the motor required to drive the plunger, if the time taken to lift the weight is 2 minutes. Assume overall efficiency of the motor to be 0.70. [***Ans:*** (a) N = 225 strokes, P_m = 0.714 kW]

P7.9 ** The ram in the jigger of a hydraulic crane has a diameter of 25 cm and the jigger has a velocity ratio of 8:1. Water is supplied to the jigger at a pressure of 110 bars and the crane mechanism has an efficiency of 60%. (a) Estimate the load lifted by the crane, and (b) quantity of water used when the load is lifted through a height of 10 m. [***Ans:*** W = 40.5 kN, $\forall$ = 6.136 Litres]

P7.10 ** A hydraulic lift raises 100 kN through a height of 28 m once every 3 minutes. The speed of the lift is 0.75 m/s. It is worked from an accumulator, which is being continuously charged by a pump. The pressure of liquid is 3000 kPa. The efficiency of the lift is 0.80 and that of the pump is 0.75. Find the power required to drive the pump and the minimum capacity of the accumulator. Neglect all other losses.

[***Ans:*** P_p= 25.92 kW, C = 0.925 m^3]

P7.11 * A fluid coupling has a 3% slip when the output shaft is rotating at 1250 rpm. If the input shaft has a torque of 30 N.m, estimate the output torque and the efficiency of the coupling. [***Ans:*** T_2 = 30 N.m, η_T = 97%]

P7.12 ** A torque converter has rotational speeds of 2000 rpm at the input shaft and 1000 rpm at the output shaft. The input and output torques are 80 N.m and 150 N.m respectively. Determine the corresponding powers of the input and output shafts and the efficiency of the torque converter.

[***Ans:*** P_1 = 16. 752 kW, P_2 = 15.705 kW, η_T = 93.75%]

P7.13 * A hydraulic ram pump working under a head of 4.0 m pumps 2.0 litres/s water to a delivery tank situated 15.0 m above the ram. The amount of water wasted is estimated as 15 litres/s. Assuming the loss of head of 1.5 m in the delivery head and the loss in the supply pipe to be negligible, estimate the efficiency of the ram. [***Ans:*** η = 48.5%]

P7.14 ** A hydraulic ram pump receives 100 litre/s of water from a source under a head of 5.0 m and delivers 10.0 litre/s to a reservoir 20 m above the ram. The delivery pipe is 50 m long and has a diameter of 100 mm. The supply pipe is 15 m long and is 200 mm in diameter. Assuming a friction factor f = 0.02 for both the pipes, estimate the efficiency of the ram.

[***Ans:*** η = 49.3%]

P7.15 * A hydraulic ram pump lifts 5 litres/s of water to a height of 20 m. The water wasted at the pump is estimated to be 70 litres/s. The source is 4.0 m above the pump. Assuming an energy loss of 0.4 m in the supply line and 0.6 m in the delivery line, estimate the efficiency of the pump.

[***Ans:*** η = 38.1%]

P7.16 * A hydraulic ram pump utilises a supply under 3.0 m head and delivers to an effective head of 21 m above the pump. The ratio of water wasted to water delivered is 15. Calculate the efficiency of the pump. Neglect frictional losses in the pipes. [***Ans:*** η = 43.8%]

P7.17 *A submersible pump in a well is set at 200 m below the ground level. At maximum drawdown, the water surface in the well is at 170 m below the ground level. The discharge pipe from the pump to the surface is 20 cm in diameter. How many stages with specific speed N_{sq}= 70 are needed if the discharge from the well at maximum drawdown is 0.08 m^3/s? The speed of the pump is 1450 rpm and the Darcy–Weisbach friction factor *f* for the pipe is 0.025.

[***Ans:*** 17 stages]

Objective-type Questions

O7.1 *Which one of the following set represents a power transmission system?

(a) Pump, hydraulic accumulator, hydraulic intensifier and hydraulic coupling
(b) Pump, turbine, hydraulic accumulator and hydraulic coupling
(c) Turbine, accumulator, intensifier and hydraulic coupling
(d) Accumulator, intensifier, hydraulic coupling and torque converter

O7.2 *A hydraulic press has a ram of 20 cm diameter and a plunger of 5 cm diameter. The force required at the plunger to lift a weight of 16×10^4 N shall be

(a) 256×10^4 N (b) 64×10^4 N
(c) 4×10^4 N (d) 1×10^4 N

O7.3 *A hydraulic accumulator is a device to store

(a) sufficient quantity of water to compensate for the change in discharge
(b) sufficient energy to drive the machine when normal source does not function
(c) sufficient energy in case of machines which work intermittently to supplement the energy from the normal source
(d) liquid which would otherwise have gone waste

O7.4 **Fluid flow machines use the principle of either (i) supplying energy to the fluid, or (ii) extracting energy from the fluid. Some fluid-flow machines are a combination of both (i) and (ii) listed above. They are classified as

(a) compressors (b) hydraulic turbines
(c) torque converters (d) windmills

O7.5 **A simple hydraulic intensifier has piston diameters of 4 cm and 8 cm. The pressure to which the liquid is raised when the supply pressure is 750 kPa is

(a) 1500 kPa (b) 2250 kPa
(c) 3000 kPa (d) 6000 kPa

O7.6 **A hydraulic crane utilises 60 litres/s of water at 6000 kPa pressure to lift 20 kN of weight through a height of 12 m. The efficiency of the system is

(a) 87.7% (b) 66.7% (c) 50.3% (d) 45.5%

O7.7 *A hydraulic accumulator has a ram of 300 cm^2 cross-sectional area and a stroke of 2 m. The capacity of the accumulator in kN.m when supplied with water at 500 kPa is

(a) 500 (b) 0.06 (c) 15 (d) 30

O7.8 **In a hydraulic accumulator, a total load of 50 kN produces a pressure of 375 kPa. If 5% of the load is accounted to overcome frictional resistance at packing, what additional load will produce a pressure of 750 kPa in the cylinder?

(a) 47.5kN (b) 50 kN (c) 52.5 kN (d) 55 kN

O7.9 **If ω_s and ω_p represent the angular velocities of driven and driving members of a fluid coupling respectively then the slip is equal to

(a) $1-\dfrac{\omega_s}{\omega_p}$ (b) $\dfrac{\omega_s}{\omega_p}$ (c) $\dfrac{\omega_p}{\omega_s}$ (d) $1-\dfrac{\omega_p}{\omega_s}$

O7.10***Consider the following statements regarding the fluid coupling:

1. Efficiency increases with increase in speed ratio.
2. Neglecting friction, the output torque is equal to input torque.
3. At the same input speed, higher slip requires higher input torque.

Which of the above statements are correct?

(a) 1, 2 and 3 (b) 1 and 2 (c) 2 and 3 (d) 1 and 3

O7.11 *A hydraulic coupling

(a) connects two shafts rotating at about the same speed
(b) connects two shafts running at different speeds
(c) is used to segment the torque to the driven shaft
(d) is used to connect the centrifugal pump to an electric motor for efficient operation

O7.12 **In a fluid coupling, the torque transmitted is 50 kNm when the speeds of driving and driven shafts are 900 rpm and 720 rpm respectively. The efficiency of this fluid coupling is

(a) 20% (b) 25% (c) 80% (d) 90%

O7.13***A hydraulic coupling transmits 1 kW of power at an input speed of 200 rpm with a slip of 2%. If the input speed is changed to 400 rpm, the power transmitted with the same slip is about

(a) 2 kW (b) ½ kW (c) 4 kW (d) 8 kW

O7.14 **Consider the following statements regarding a torque converter:

1. Its maximum efficiency is less than that of the fluid coupling.
2. It has two runners and a set of stationary vanes intercepted between them.
3. It has only one runner and a stationary wheel.
4. The ratio of secondary to primary torque is zero for the zero value of angular velocity of the secondary.

Which of the following is correct?

(a) 1 and 2 (b) 3 and 4 (c) 1 and 4 (d) 2 and 4

O7.15***Which one of the following graphs represents the characteristics of a torque converter? In the figure, the suffix *r* stands for the turbine runner and *p* for the pump impeller.

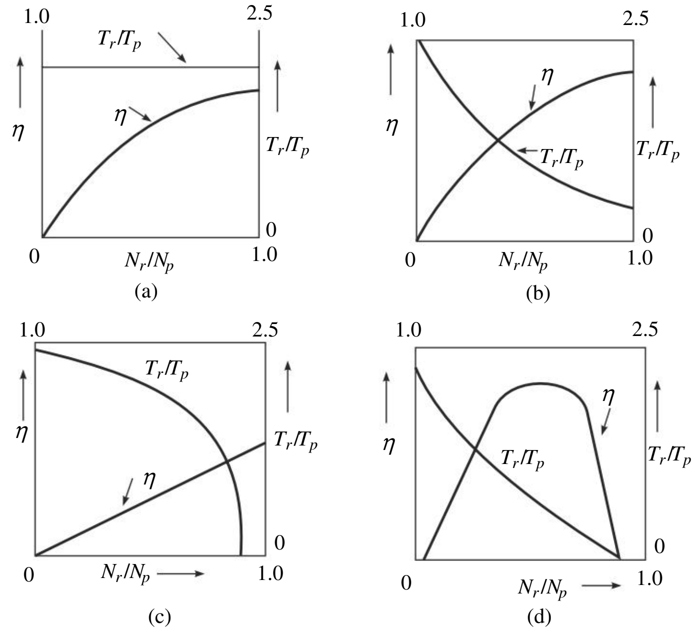

Fig. 7.38 *Objective Question O 7.15*

O7.16 ** Which of the following is pumps is NOT a positive displacement pump?

(a) Reciprocating pump (b) Jet pump
(c) Sliding-vane pump (d) Lobe pump

O7.17 ** Which of the following pumps is NOT a positive displacement pump?

(a) Screw pump (b) Air pump
(c) Diaphragm pump (d) Lobe pump

O7.18 ** The following is not a rotary pump:

(a) Diaphragm pump (b) Screw pump
(c) Lobe pump (d) Sliding vane pump

O7.19 *** Consider the following statements:

(1) A rotary pump does not have inlet and outlet valves.
(2) An airlift pump does not have any moving parts.
(3) A lobe pump gives a continuous flow without pulsations.
(4) A diaphragm pump does not have inlet and outlet valves.

Out of the above, the correct statements are

(a) 1 and 3 only (b) 2 and 4 0nly(c) 1,2 and 3 only (d) 1, 2 , 3 and 4

O7.20 ** In a food-processing industry, food-grade thick syrup is to pumped in a process. The pump of choice would be

(a) lobe pump (b) jet pump
(c) screw pump (d) reciprocating pump

O7.21 ** Hydraulic ram is a pump which works on the principle of

(a) water hammer (b) centrifugal action
(c) reciprocating action (c) hydraulic press

O7.22 ** Pumps which raise oil or water by the buoyancy of an aerated column of oil or water in a submerged tube are called

(a) airlift pumps (b) vacuum pumps
(c) positive displacement pumps (d) peristaltic pumps

O7.23 *** Consider the following statements in regard to the hydraulic ram.

1. Hydraulic ram does not need external power.
2. It works on the fundamental principle of a water hammer.
3. The overall efficiency of the hydraulic ram is only of the order of 8 to 19%.
4. It can be termed as a low, intermittent discharge low-head pumping station.

The correct answer is

(a) 2 and 3 (b) 1, 2, 3 and 4
(c) 1 and 2 (d) 1 and 3

O7.24 ** A high-efficiency pump is required for low discharge, high head and low maintenance cost. Delivery of water need not be continuous. The pump need not run at high speed. Which one of the following is the correct choice?

(a) Centrifugal pump (b) Reciprocating pump
(c) Air lift pump (d) Hydraulic ram

O7.25 * A centrifugal jet pump is classified as

(a) positive displacement pump (b) rotary pump
(c) hydrostatic pump (d) none of these

O7.26 ** Consider the following pumps:

1. Screw pump 2. Diaphragm pump
3. Sliding-vane pump 4. Lobe pump

A pump that would be an ideal selection out these four for pumping debris-laden flow would be

(a) 1 (b) 3 (c) 4 (d) 2

O7.27 ** In a jet pump,

(a) kinetic energy of the high velocity stream is converted into potential energy
(b) energy of high-pressure stream is converted into kinetic energy of low-pressure stream
(c) high-velocity stream aided by the geometry creates a suction pressure that results in increased suction head
(d) potential energy of the fluid is converted into kinetic energy thereby increasing the delivery head

O7.28 ** Water is supplied from a height of 2.8 m at the rate of 35 lps to a hydraulic ram that delivers 2 lps to a height of 28 m above the ram. The efficiency of the ram is

(a) 57% (b) 55% (c) 52% (d) 44%

O7.29 ** By which one of the following devices can a small stream of water can be lifted to a great height?

(a) Hydraulic ram (b) Hydraulic crane
(c) Hydraulic lift (d) Hydraulic coupling

EIGHT

Elements of Hydroelectric Power Engineering

8.1 INTRODUCTION

Hydroelectric power (commonly called *hydropower*) *engineering* is the technology of developing electric power by harnessing the potential energy of flowing water. Hydropower can be captured efficiently whenever a stream of water falls from a higher elevation to a lower elevation. This change in the elevation of water, known as *head,* is an essential requirement of hydropower development. In a hydropower plant, the potential energy of water in a river or stream is converted into rotating energy of a shaft at a hydraulic turbine. The prime movers of the generators in these plants are the hydroturbines. After having discussed in detail the characteristics of different types of hydroturbines, this chapter presents a brief description of the end use of turbines, viz. salient features of hydroelectric plants, to compliment the earlier study of turbines discussed in chapters 1 through 4.

Currently, all the electric power generated in the world can be broadly classified into three categories as thermal, hydro and other non-conventional sources. The thermal power includes the electrical energy produced from all fossil-fuel sources (coal, oil and gas) as well as nuclear energy. Even the power produced from geothermal energy sources can be included under this category. On a global basis, the hydropower production comes next only to that from fossil-fuel sources. The relative electricity generation worldwide from various sources is shown in Table 8.1. It is seen that on a worldwide basis, as of 2009, hydro occupies roughly 16% of the

Table 8.1 *Worldwide electricity generation by Type (IEA data, 2009)*

Total Production in 2009: 20,055 TWh			
Conventional Sources		Nonconventional Sources	
Fossil-fuel	67.1%	Wind	1.7%
Hydropower	16.2%	Solar	0.2%
Nuclear	13.4%	Geothermal	0.3%
		Biomass	1.1%

total electrical power produced. In the Indian scene, currently the ratio of hydro to other sources is about 23:77.

Hydropower has certain unique features that make it highly desirable. The chief among them is that hydropower is clean; with no or very little pollution footprints and is renewable. It is still the largest renewable energy system in use in practically all parts of the world. Some comparative features of hydro and thermal power plants are listed in Table 8.2.

Table 8.2 *Comparative features of hydro and thermal power plants*

Sl. No	*Hydropower*	*Thermal Power*
1	Renewable. No (or negligible) fuel cost.	Non-renewable source. High fuel cost.
2	Clean and safe energy source. No air pollution. No thermal pollution of water.	Heavy pollution and not always safe. Very high safety measures are needed in nuclear thermal plants. Air pollution in fossil-fuel plants. Thermal pollution of water bodies.
3	Ideally suited for peak power. Plant can be run up and synchronised in a few minutes.	Ideally suited for base power. Starting up does take large time.
4	High efficiency, about 90–95% overall energy conversion efficiency.	Efficiency of energy conversion is low with an average of about 35% and a maximum of about 60%
5	High capital cost and very low operating cost. Sometimes, a part of the capital cost of the dam can be shared by making the dam a multipurpose project.	Moderate capital cost and very high operating cost.
6	The locations are geographically fixed leading to large transmission costs.	Can be located in the load centres and transmission costs can be reduced
7	Each hydro plant is site-specific. Standardised modular designs are not possible.	Standardised modular thermal plant units are generally adopted. Leads to considerable savings in time and cost.
8	Moderate environmental damage at the reservoir, canal and other components.	Heavy environmental damage at the source of the fuel as well as at the power plant and its neighbourhood. Large pollution footprints.
9	Problems related to relocation of project-affected people at the reservoir site may be high.	Smaller problems of relocation of project-affected people.
10	The components are rugged and have long life. A hydro plant having an economical life of 50 years or more is common. However, sedimentation and related progressive loss of storage capacity is a concern.	High depreciation of the components due to the high temperatures and nature of the plant: needs frequent replacements of major components. Economic life of the plant is much smaller than that of a hydro plant.
11	Takes very long time for survey, design, mandatory approvals and construction.	Reasonably quick processing and construction time.

From the point of view of current thinking, the following are the *USP* of Hydro plants, especially small hydro plants:
1. Hydropower helps fight climate change.
2. Hydropower can reduce pollution.
3. Hydropower makes a significant contribution to development.
4. Hydropower means clean, affordable power.
5. Hydropower is a key tool for sustainable development.

8.2 LIST OF COMPONENTS OF A HYDRO PLANT

A general list of major components of a typical conventional hydroelectric power plant is as follows:
1. Storage or diversion structure
2. Conveyance devices and appurtenances (power canal, settling basin and sediment-removal devices, tunnel, penstock)
3. Forebay or surge tank
4. Intake (with trash rack and flow control device)
5. Flow-control devices and intake (valves, gates, trash rack, emergency stop logs, etc.)
6. Powerhouse (turbine and generator assembly, transformers, switch gear and control room)
7. Tailrace channel

Practically, all hydropower plants are site specific. In a given hydro project, not all the above components may be there and also there may be some other additional site-specific components that are included. A brief description of the above listed components is given in Sec. 8.5.

8.3 CLASSIFICATION

1. Based on Station Capacity

On the basis of total of *nameplate capacity* of all the generators in the plant (station capacity), the plants are classified as mega, large and small hydro as given below:

Mega Hydro Project	Large Hydro Project	Small Hydropower Project (*SHP*)		
		Small Hydro	Mini Hydro	Micro Hydro
> 500 MW	25 MW to 500 MW	2 MW to 25 MW	101 kW to 2.0 MW	<100kW

NOTE

Water power plants up to 5 kW are sometimes called *pico hydro* plants

2. Based on Head

Depending on the magnitude of the gross head, the hydro projects are classified as

- *High-head plant:* Head > 250 m
- *Medium-head plant:* Head in the range 30 m to 250 m
- *Low-head plant:* Head in the range 2.0 m to 30 m

This is not a rigid classification and is meant only for the purpose of categorising the sites.

3. Basis of Storage

In terms of nature and operation of storage, hydro plants are classified as (a) *run-of-river,* (b) storage, and (c) *pumped-storage* plants. The details of these three kinds are discussed in the subsequent sections.

4. Relative Locations of Storage and Powerhouse

On the basis of relative locations of storage (dam or diversion structure) and powerhouse, hydro plants are classified into two main classes as (a) concentrated fall, and (b) divided-fall.

(a) Concentrated-Fall Plants In this grouping, the powerhouse is built at the toe of the dam or very near it. The head created by the dam is essentially the head of the power plant. The main river itself acts as the tailrace. Figure 8.1(a) shows a schematic sketch of a concentrated fall development. Notice that many of the components listed in Sec. 8.2 are missing in this. Most concentrated fall plants at the foot of a dam are medium-head plants.

Figure 8.1(b) shows a hydro plant that uses the relative small head at a weir. In this type of development, known as *run-of-river (ROR)* plant, the natural drop due to the steep slope of the terrain and the head created at the weir is made use of. There is essentially no storage available and the power generation is dependent purely on the available water in the river. As such, such plants are ideally suited to locations where the discharge in the river is large and the flow is normally perennial.

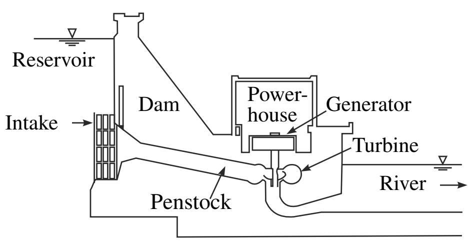

Fig. 8.1(a) *Concentrated fall: powerhouse at the toe of the dam*

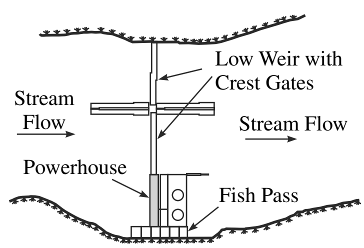

Fig. 8.1(b) *Run-of-river hydro plant*

(b) Divided-Fall Development In this category, the head available in the terrain is made use of in addition to the head created at the dam/diversion structure. High-head hydro plants are mostly divided-fall plants. There are many variations in this type and three typical kinds of divided-fall developments are shown schematically in Fig. 8.2 (a, b and c).

In Figure 8.2(a), a power canal takes off from the dam and ends up at the forebay. Sediment-removal devices are provided at the entrance region of the canal. The forebay also acts as a sedimentation basin. A long and large penstock and of sufficient size takes off from the forebay and near the powerhouse, it is divided into a short length of multiple penstocks to feed individual turbines. The tailrace is relatively short and leads into a the main river. This type can be called Type-A divided-fall development

Figure 8.2(b), is a divided-fall development that is a variant of the type described in Fig. 8.2(a). In this, the penstocks are relatively of small length and each one of them takes off from the forebay and feeds the turbine directly. This plant can be called Type-B divided-fall development.

Figure 8.2(c) shows a development that is normally used in high-head development. In this, the penstock or tunnel takes off from the main dam and leads to a surge tank. From the surge tank, shorter and smaller penstocks take off to feed individual turbines in the powerhouse. The tailrace is very short and leads the water from the turbines to the river. This could be categorised as Type-C divided-fall development.

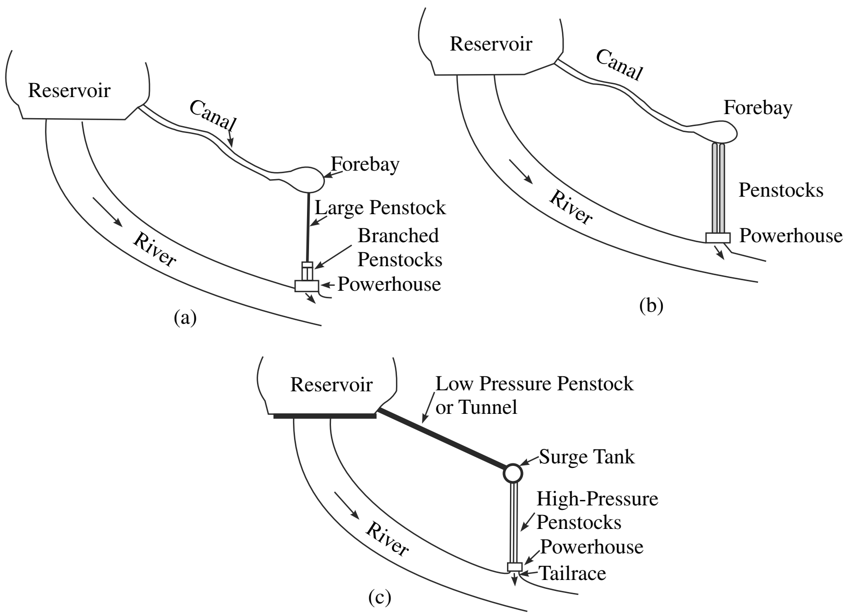

Fig. 8.2 *Typical types of divided-fall hydro developments*

8.4 DEFINITIONS AND EXPLANATION OF TERMS USED IN HYDRO POWER PLANTS

1. Units

The unit of electric power is kilowatt (kW) and the unit of *electrical energy* is kilowatt-hour (kWh) which is the energy of 1 kW of power available continuously for 1 hour.

2. Installed Capacity

It is the sum of capacities of all the generating units installed in the powerhouse.

3. Load-Duration Curve

Load-duration curve is a plot of load on the plant against the percentage of times the load is equalled or exceeded. The hourly load on a plant varies significantly in a day; daily load varies significantly in week; weekly load in a season; and monthly load in a year. Depending on the purpose, different sampling intervals are chosen to plot the load-duration curve. Thus, an annual load curve represents the percentage of times a given load is equalled or exceeded in a year. The *annual load factor* is the ratio of the area bounded by the annual load curve to the area of the rectangle corresponding to the maximum demand occurring during the course of the year.

4. Flow-Duration Curve

One of the popular methods of studying the stream flow variability is through a *flow-duration* curve. A Flow-Duration Curve (*FDC*) of a stream is a plot of discharge against the percent of time the flow was equalled or exceeded. The discharge data of the stream is arranged in the form of a list with descending order of discharges. Class intervals are used if the number of individual values is very large. If N data points used in this listing, the plotting position of any discharge (or class value) Q is

$$P_p = \frac{m}{(N+1)} \times 100 \tag{8.1}$$

where m = Order number of the discharge (class value) in the list. P_p = Percentage probability of the flow magnitude being equalled or exceeded. The plot of the discharge Q (in the y-axis) against P_p (in the horizontal axis) is the flow-duration curve, Fig. 8.3. The plot can be in arithmetic scale, or semi-log plot or log-log plot depending upon the range of the data and the intended use of the plot. The flow-duration curve (*FDC*) represents the cumulative frequency distribution and can be considered to represent the stream-flow variation in an average year.

Figure 8.3 shows an *FDC* of a stream under study for hydropower development. This curve indicates the flow magnitude (Q_p) available at any desired percentage of time, (p_p). Thus in Fig. 8.5, Q_{50} = 83 m^3/s is the flow available for 50% time in a year. Similarly, Q_{90} = 45 m^3/s is the flow that is available 90% of times. Another terminology to denote Q_p is as *p% dependable flow*. Thus, Q_{50} = 50% dependable flow and Q_{95} = 95% dependable flow. In Fig. 8.3, 95% dependable flow is 35 m^3/s.

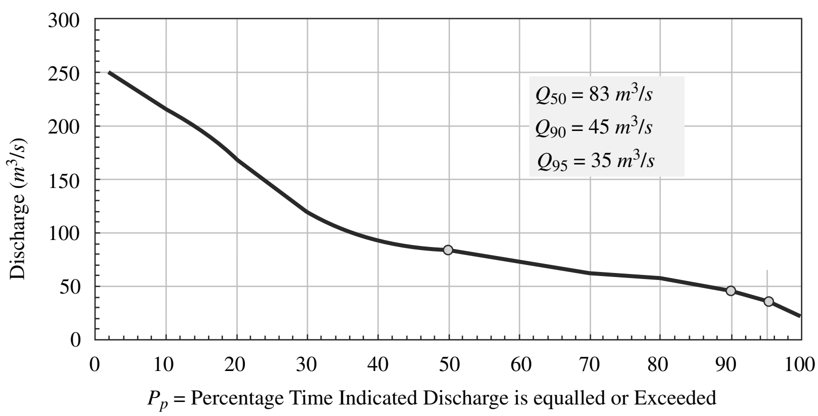

Fig. 8.3 *Flow-duration curve of a stream*

5. Power-Duration Curve

Power-duration curve is a plot of power that could be produced in a plant represented by the vertical axis and the per cent of times a given power will be equalled or exceeded represented on the horizontal axis. If the head is constant during the period under study, the power-duration curve is the same as flow-duration curve with appropriate scaling factor. Power-duration curve is useful in deciding on *base (firm) power* and *secondary power.*

6. Firm Flow

The firm flow to a hydro plant is defined as the flow being available $p\%$ of the time, where p is a percentage specified by the user and usually equal to 95%.

7. Firm Power

The *firm power,* or *primary power,* is the power that is assured to the customer, usually equal to 95% of the time. Such a power would correspond to the firm flow available at the intake. If the flow is unregulated flow, as in the case of run-of-river plant, firm power corresponds to the minimum flow (usually 95% dependable flow) in the stream. With regulation, the primary power will be larger than that due to the unregulated flow in the river. Figure 8.4 is a plot of power-duration curve for a hydro plant on a stream with a constant head. It also represents the flow-duration curve with appropriate scale. The flow with regulation is shown by a dotted line. Notice that the effect of flow regulation by storage is to reduce the peak flows and to increase the low flows. This leads to higher primary flow with storage when compared to primary power that could be produced without storage.

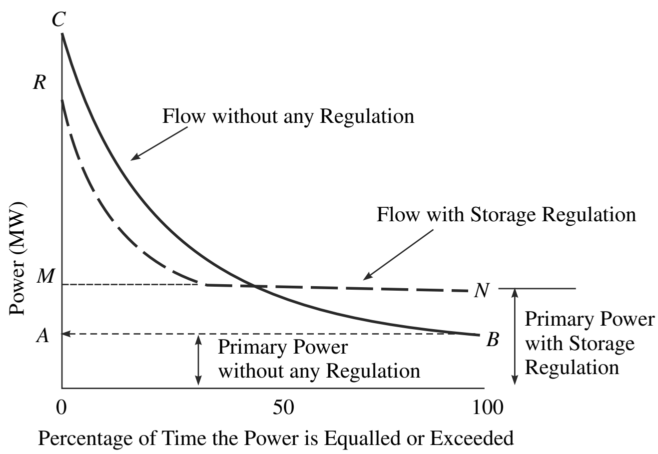

Fig. 8.4 *Power-duration curve and definition of primary power*

Thus, a run-of-river plant has low firm power and a storage plant on the same river can have a considerably enhanced firm power.

In a grid-type power system consisting of several types of power generation, the firm power capacity of a singular small size plant may not be very important. However, if a hydro plant is supplying power to a dedicated load in an isolated area, firm power supply from the plant is very important.

8. Secondary Power

Secondary power (also known as *surplus power*) is the power available in excess of the primary power. It is available, normally, in only certain parts of the year. In Fig. 8.4, the area marked *ABC* is the secondary power without storage. Similarly, the area marked *MNR* is the secondary power with flow regulation through storage.

9. Load Factor

Load factor is the ratio of average load of a plant to the peak-load in a specified time interval. Thus, we have daily load factor, weekly load factor and monthly load factor.

10. Plant-use Factor (Capacity Factor)

The ratio of average load developed during a certain period to the installed capacity of the plant is known as *plant-use factor* or *capacity factor*. It represents the power actually produced by the plant in a given period of time to what it is capable of producing under design conditions during the same period of time at continuous full power operation. If a plant with a capacity of 10 MW produces an average 4.5 MW in a certain period (say a week) then its plant-use factor is $\frac{4.5}{10} = 0.45$. Typically, the hydro projects have a capacity factor of 0.25 to 0.80. The reasons for reduced value of capacity factor are many, but it can mainly be due to (a) the plant working at reduced output or the plant being out of service during the period under study, (b) nonavailability of requisite amount of water, and (c) lack of demand. If peak-load is equal to the plant capacity, the capacity factor is equal to the load factor.

11. Utilisation Factor

The ratio of maximum demand of a system or part of a system in a specified period to the rated capacity of the power plant is known as *utilisation factor*. For a hydro plant, the annual utilisation factor is in the range 0.4 to 0.9 and depends upon the plant installed capacity, load factor and storage.

12. Base-Load Plant

Power plants capable of substantially meeting the stipulated load at 95% of times are known as base-load plants. By design, the thermal plants are capable of working as base-load plants. However, storage-type hydro plants with considerable carryover storage can work as base-load plants.

13. Peak-Load Plants

A peak-load plant works in conjunction with a base-load plant and takes care of the peak-load of the power system. A storage-type hydro plant is ideally suited for this purpose, as it can be started at a very short start-up time which can vary from a few seconds to the order of 3 to 4 minutes depending upon the length of conduit to the nearest storage spot. Pumped-storage hydro plant is an example of using the excess power of the base-load to meet the needs of the peak-load.

14. Storage

Storage is the volume of reservoir available for storing water to balance the seasonal fluctuations of inflow. In a water year, the river experiences a peak flow (wet) season and lean flow (dry) season. By providing adequate storage, the excess flow in the wet season could be stored to be made available in the dry season through regulation of outflow. This would enhance the firm amount of water that could be drawn from the reservoir. The amount of storage volume to be provided depends upon the annual hydrograph of the river as well as the proposed firm withdrawal of water from the reservoir and the regulation procedure. The storage volume of a reservoir is limited by the inflow hydrograph and the size of the reservoir behind the dam Normally, seasonal fluctuations are dampened or eliminated in the storage reservoirs through regulation of outflow. Thus, the term *storage* is used in the context to mean the volume of water required in the reservoir to meet the seasonal fluctuations in the hydrograph of the river. Some very big reservoirs may have *carryover storage* which enable the fluctuations of more than one water year to be evened out.

15. Pondage

In any hydro plant, there would be fluctuations in the load depending upon the time of the day and day of the week. At each of these fluctuation episodes, the governing mechanism of the turbine adjusts the discharge entering into the turbine. If the load is reduced, there will be a rejection of water entering into the turbine. This would mean less flow would get into the penstock at the forebay and the water level in the forebay pond would rise. Conversely, if the load on the turbine increases, more flow is admitted into the turbine and the penstock draws more water. This situation leads to a depletion of the storage from the forebay pond.

Consider a short time period, such as a week in the operation of a hydro plant. In a typical week, the hydro plant being essentially a peak-load station, may work for a few hours in a day and may be idle during a day or two in a week. If the forebay is supplied with regulated water from a storage reservoir, the inflow into the forebay will be essentially constant over a short time such as a week. However, in run-of-the river plants, where a small weir is provided for flow diversion, the pool behind the weir acts as pondage. In this case, the flow in the river is unregulated and its flow rate may change over a week. As such, a storage is to be provided in the pond created by the weir to accommodate the possible change in the inflow volume over the week so that a fixed quantity of water is supplied to the turbines over the week.

The live storage capacity of the forebay pond or any other balancing pond in the system, to essentially accommodate short-term fluctuations in the load and inflow of water is termed *pondage*. Typically, a week (or ten-day unit) is taken as the unit of period to estimate pondage. Thus, in a general sense, the purpose of the pondage can be summed up as follows:

(a) To balance the fluctuations in the load,
(b) To store the idle day's inflow,
(c) To compensate for any loss due to leakage or spillage,
(d) To compensate for short-term fluctuations in the stream flow, and
(e) To enable the use of stored water during hours of peak-load at the plant.

Thus, while the pondage represents the volume of water required to even out the short-term (generally over a week) fluctuations in the load, the storage represents the large volumes of water required to meet the seasonal fluctuations (over a couple of months) in the flow hydrograph of the river.

16. Pondage Factor

Pondage factor is defined as the ratio of total stream-flow hours per week to the number of hours the plant will be operational in that week. For example, if a river has a constant inflow for all the seven days of the week and the plant operates for six days, at eight hours per day in that week, the pondage factor is $\left(\dfrac{7 \times 24}{6 \times 8}\right) = 3.5$. The pondage factor is a measure of the pondage volume needed.

17. Run-of-River Hydro Plant (ROR)

Run-of-River hydro plant (*ROR*) is a type of hydroelectric-generation plant where no storage or very little storage (called pondage) is used. It could be described as a plant where water is used at a rate no greater than that with which it runs down the river. Ideally, it makes use of the relatively large quantities of discharge in the perennial rivers or regulated flow of streams/canals downstream of a very large reservoir and relatively small heads available due to topographical features of the river. The Bureau of Indian Standards Code IS: 4410 defines a run-of-river power station as *"a power station utilising the run of the river flows for generation of power with sufficient pondage for supplying water for meeting diurnal or weekly fluctuations of demand. In such stations, the normal course of the river is not materially altered"*. However, in practice there are many deviations from the above in the ROR developments around the world. Thus, the ROR hydro plant that comes under the above criteria can be called an *ideal ROR plant*.

ROR plants are thus essentially peak-load stations. A majority of them are concentrated-fall developments. Figure 8.1(b) shows a *ROR* where a small weir across a river is used to divert the flow into the powerhouse with sufficient head. When there is pondage available, the minimum pondage capacity is designed to take care of daily and weekly fluctuations in the load. Sometimes weekly fluctuations in the flow are also accommodated in the design of pondage volume.

8.5 BRIEF DESCRIPTION OF COMPONENTS OF A HYDRO PLANT

1. Dam

Depending upon the nature of the valley in which a reservoir across a river is proposed, the following types of dams are in use: gravity dam, arch dam, rockfill dam, earthen dam and composite dam. In a project, depending upon local conditions, one type of dam or a combination of types is used. Provision of adequate capacity spillway with suitable energy dissipater in the dam is an important safety aspect of reservoirs. Other major components of a dam are (a) the intake of proper geometry, (b) trash rack, (c) sluice gate, and (d) spillways. Dams are used in both concentrated-fall type and divided-fall type of hydro plants.

Many of the reservoirs in the middle and lower reaches of a river used in hydropower development will have multipurpose objectives in addition to hydropower generation. Dams for hydro-only projects are usually found in the upper reaches of a river in mountainous areas. The storage created in the reservoir helps even out the seasonal fluctuations in the river over a water year. Some very large reservoirs are provided with facilities to carry over the storage from one water year to the next.

The storage in the reservoir acts as a large settling basin and hence, sediment load in the intake of the conveyance unit (canal, penstock or tunnel) is not, normally, an issue. However, loss of storage due to reservoir silting, as a result of continuous deposition of sediments in the reservoir, is a major concern.

2. Diversion Structure

These can be weirs or barrages and consists of the following components: (a) Trash rack, (b) intake sluice, and (c) sedimentation basin and related sediment-removal devices. Sediment-load in the intake water is a major issue in large alluvial rivers and in streams and rivers in geologically fragile areas such as the Himalayan region of India and Nepal.
Two types of intakes are in use: (a) side intake, and (b) bottom intake.

(a) Side Intake Side intake is the most common type of intake and could be called the conventional type of intake. It consists of a wall set at nearly right angles to the main weir wall and along the side of the river in which the intake sluices are provided. It is simple in construction and consists of a sill over which the water flows from the river into the canal. The offtake flow is controlled through gates. The weir structure used in this arrangement can be partly or completely submerged during operation. Silt excluders are an important component of weirs and barrages on alluvial channels. These conventional intakes are easy to operate and have very good control over the intake of discharge. However, these weir and intake structure units are expensive. In addition, the control in diversion of very small flow is rather difficult.

(b) Bottom Intake It consists of a small weir structure and a bottom intake with grills through which the river flow enters into a tunnel-like structure to be diverted to the canal. These find useful application in small hydro developments in mountainous streams where conventional weir and side intakes are very expensive. Further, bottom intakes are very good in handling highly fluctuating flows and even very low flows can be diverted. These are specialty structures, require good design to prevent clogging of intake grills and are site-specific.

3. Settling Basin

Alluvial rivers and streams, in geologically fragile areas such as the Himalayan region, carry considerable amount of sediment load. The water diverted from these streams and rivers for power-development purposes carries substantial sediment in suspension in spite of silt-excluding arrangements at the intake. If allowed to pass through a turbine, the quartz particles in the suspended sediment cause erosion and related problems in the turbine and impede the overall efficiency power generation. Hence, removal of sediment load is an important task in the design of a water-conveyance system. Silt ejectors and settling tanks (basins) of appropriate design are an integral part of the water-conductor system.

4. Forebay

The forebay is an enlarged portion at the end of a power channel as shown in Fig. 8.2 (a and b). It is essentially a small pond (storage tank) and serves the purpose of steady and continuous supply of water to the turbines. Penstock pipes take off from the forebay to lead the water to the turbines. The storage volume in the forebay is designed to be adequate to take care of small fluctuations in the supply of water to the turbines due to load rejection and acceptance by the turbines. In addition, the forebay acts as the last settling basin to the sediment particles. Components of a forebay are (a) trash rack, (b) sluice gate control to the penstock, and (c) a spillway.

5. Penstock

Penstock is a pipe to carry the water to the turbines. It is of sufficient strength to withstand the static pressure and dynamic pressure due to any water-hammer pressure in the flow. While steel is the most common material for penstocks, some small hydropower units use other pipe material such as concrete. Steel penstocks are made of welded joints and are laid in above-ground or buried configuration, In above-ground layout, the pipes are supported at regular intervals by saddles or piers. These supports are meant to transfer the dead weight of the pipe and contained water to the ground at safe bearing pressure. Provison is made in these supports to allow lateral movement of pipe due to thermal expansion. Expansion joints are provided in the penstocks at appropriate intervals. At every change in direction or grade, concrete anchor blocks are provided to resist thrusts due to change in momentum and other forces that act on the pipe. Air inlet valves and pressure relief valves are provided to maintain safe working and flow conditions in the pipe.

Long penstocks have a surge tank at a convenient location to reduce the length of pipe subjected to water-hammer pressures. Smaller stretches of penstock pipes (Fig. 8.2) will be designed with adequate thickness to withstand the static pressure and water-hammer pressures.

6. Tunnels

In some sites, the terrain may be rugged and site conditions may preclude the use of penstocks. In such cases, tunnels will be the only means of conveying water through the terrain. Tunnels, being the most expensive water-conveying system, are used to the minimum length necessary. Figure 8.2 (c) shows a common arrangement using tunnels. Here, the tunnels would end up in a surge tank and penstocks would take off from the surge tank to feed the turbines. The shape of the tunnel is dependent on the nature of the rock through which it is driven. Invariably, all tunnels are lined with concrete and effort is made to keep the alignment straight. Adequate shafts are provided in the tunnel for inspection and repair. A sand trap is normally provided at the end of the tunnel.

7. Surge Tanks

In many hydropower projects, large penstocks carry considerable quantity of water from a reservoir to the turbines. When there is a sudden drop in the load at the generator, the flow into the turbines is reduced suddenly leading to water-hammer situation. Similarly, when there is sudden increase in the discharge requirement at the turbines, a sudden opening of the control valves may cause negative pressures. To overcome these problems, it is a common practice to install surge tanks in such systems. A simple surge tank (also sometimes called *stand pipe*) is essentially a large cylindrical vertical tank of sufficient height connected to the penstock. Figure 8.5 is a definition sketch of a simple surge-tank installation.

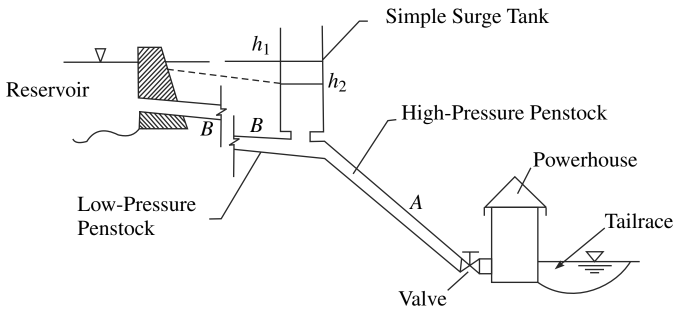

Fig. 8.5 *Simple surge tank*

In this figure, a set of large pipes made up of two parts B and A, connect a reservoir to a turbine and generator set. Pipeline B of length L_b is the low-pressure penstock taking off from the reservoir and the pipe A of length L_a is the high-pressure penstock connected to the turbine. A surge tank is provided as near the turbine valve as is feasible. Generally, the penstock A will be considerably shorter than the penstock B. Let h_1 represent the static level corresponding to no flow in the penstocks. When there is uniform flow in the pipe B, due to friction, the hydraulic grade line will be as shown by a dotted line in the figure and the piezometric head at the surge tank is h_2.

When there is sudden closure of the valve at the turbine, water-hammer pressures are created in the penstock A. The free surface of the surge tank acts as a reservoir type end condition for the water-hammer process in the pipeline and the effect of the water-hammer is confined to the penstock A only. Water from the penstock A rushes into the surge tank and the water surface in the surge tank rises. After the momentum change has dissipated, the water surface moves down and thus a mass oscillation of water is set up in the surge tank. Due to friction this oscillation will be dampened and die down eventually. If there were no surge tank, the entire pipe length $(L_a + L_b)$ would have been affected by the water-hammer effect. Installation of a surge tank confines the water-hammer pressures to the pipe A of length L_a only.

A similar, but converse, situation exists at the sudden load acceptance and opening of the valve at the turbine. In this case, the water flows out quickly out of the surge tank and the water surface in the surge tank will be eventually restored due to flow from the reservoir. Thus, the surge tank in a hydropower system as above,

- Helps reduce the length of the penstock affected by water-hammer effects due to valve operations,
- Provides a source for storage of water rejected by the turbine due to sudden valve,
- Provides a source of water to meet sudden demands created by the turbine due to sudden valve opening.

Simple surge tanks, as described above, are usually open at the top and are of sufficient height so that they do not overflow.

While simple surge tanks are adequate for small and medium heads, high-head hydro plants require some additional devices in the surge tanks to cause additional energy loss and to dampen the surges more quickly. This is to restrict the sizes of the surge tanks which otherwise would be very large and uneconomical. Many designs which are the variants of the basic simple surge tank to achieve additional damping of surge are available. Some of the important varieties are (a) restricted-orifice surge tank, and (b) differential surge tank.

(a) Restricted-Orifice Surge Tanks In this type of surge tank [Fig. 8.6(a)], an orifice at the entrance from the penstock causes energy loss in the surge entering the surge tank. This reduces the height of the initial surge and aids in quick damping.

(b) Differential Surge Tank In this type [Fig. 8.6(b)], a riser of about the same diameter as the penstock is provided inside the surge tank. The riser is provided with a set of ports at the lower level. The surge first enters into the riser and while rising, pours out through the ports into the surrounding space of the surge tank. The flow from the ports causes some energy loss. Further, the surge rises more rapidly in the riser than in the surge chamber and very soon the two columns of water will be out of phase. That is, when the water column is rising in the surge chamber, the water column in the riser may start falling. Since the two columns are connected through the ports, this causes additional energy loss leading to fast dampening of the mass fluctuation in the surge tank. The height of the surge is much smaller than what it would have been in a simple surge tank, thus leading to economy in the cost of the surge tank.

8. Intake

Intake is the structure that leads the water from a storage unit (reservoir or forebay) into the water-conductor unit. At the dam, weir or barrage, the intake enables required quantity of water to be admitted to the canal or tunnel or penstock as the case is. The essential requirements of an intake are as follows:

- The geometry of the entrance and the passages must be streamlined to keep the hydraulic losses to the minimum.
- A flow-control device should be present to enable control over the flow admitted into the conduit. Usually, the device would be a sluice gate or a valve depending upon the conduit.
- A trash rack should be provided to prevent entry of debris and trash into the conduit.

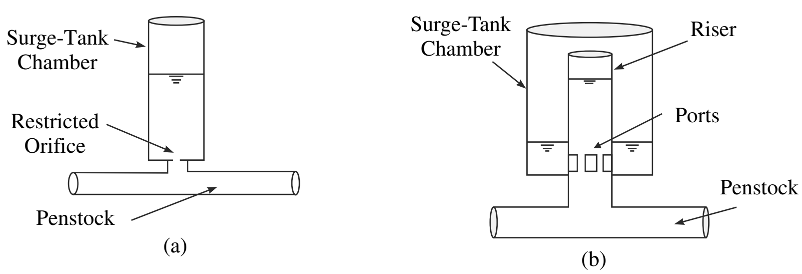

Fig. 8.6 *(a) Restricted-orifice surge tank (b) Differential surge tank*

- Provision for bulkheads are to be made at the entrance of the intake to undertake repair and maintenance of the sluice gates

In canal intake from a weir or a barrage on an alluvial river, a sediment excluder is provided. In some cases it will be followed by a silt ejector also.

9. Powerhouse

Powerhouse is the structure which houses the turbines, generators and other machinery. The heart of the hydropower plant is the generator-turbine assembly. Basically, the power house is equivalent to the main production shop of a factory, and as such it would be provided with machine hall, control room, operations room, ancillary machinery and workshop with facilities like travelling cranes for removing and installation of heavy machinery. While the switchyard will be housed outside the powerhouse, the transformers can be inside or outside the powerhouse. The size of the powerhouse is site-specific. A powerhouse of a large sized (≈400 MW) hydro plant is a very big structure, and may have a size of the order of 150 m × 40 m × 60 m. While the length of the powerhouse depends on the number of machines, its width and height depend upon the dimensions of the turbine and generator units and the site conditions.

In certain special cases, where the gorge is of solid rock, powerhouses are provided underground. While these underground powerhouses are very expensive, they have the advantages of high security and safety from physical surface damages such as avalanches, landslides and even earthquakes.

10. Tailrace

In the powerhouse, the discharge from out of the turbines is collected and lead through an open channel or conduit back into the river. This conveyance channel is called tailrace. In reaction turbine units, the level of the tailrace channel affects the effective head on the turbine. The tailrace channel is usually lined and maintained in good condition till it reaches the main river.

8.6 PUMPED STORAGE PLANTS

A *pumped storage* hydro plant is a system which stores the excess energy of a base-load in the form of potential energy of water at higher elevation and uses this to convert it back to electrical energy during peak power demand. In a sense, it is the equivalent of a hydraulic accumulator on an enormous scale. It is a very large size load-balancing technique and could be called *grid-energy storage system*.

Pumped-storage (*P-S*) schemes can be broadly classified as pure pumped-storage plant, and mixed (*pump-back*) pumped storage plant.

1. Pure P-S Plant

In a pure pumped-storage scheme, shown schematically in Fig. 8.7, there are two storage reservoirs and one pumping station. Reservoir *A* is at high elevation and is called high-level reservoir. Reservoir *B* is at a low-level and is called low-level reservoir. The powerhouse *P* is above *B* but much below *A* in elevation. The reservoirs can be natural or man-made. During a base-load period, if a drop in demand occurs in the grid, the excess electrical energy available is used to pump water from the low-level reservoir *B* to the high-level reservoir *A*. In this way, the excess electrical energy is converted into potential energy of stored water in the reservoir *A*. When there is more demand than the base-load, that is at the peak-load, the water is drawn from the reservoir and electrical energy is produced at the powerhouse. The water after passing through the turbines goes to the low-level reservoir *B*. Thus, it is a closed-loop system in which the total quantity of water is conserved. The loss of water from this closed system is due to evaporation and other seepage and leakage losses and they have to be made good by topping up the loss of water at frequent intervals. The powerhouse is an integral part of the grid and the energy for pumping comes from the grid and the energy produced by the generator is fed back to the grid. Reversible pump turbines of Francis type or Deriaz turbine pumps are employed for turbine and pumping action.

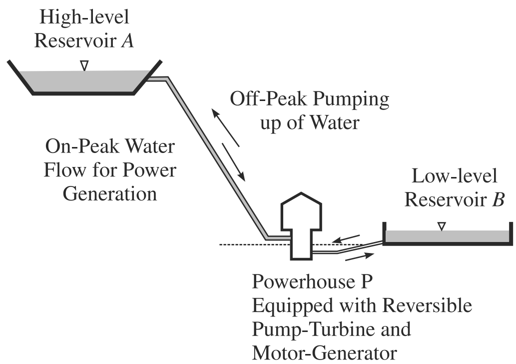

Fig. 8.7 *Schematic sketch of pure pumped storage plant*

It will be noted that there is a net energy loss in each cycle of pumping and power generation by using the same water at the turbine. However, the pumping operation uses cheap, low-cost surplus base power and the turbines help generate high-unit-cost peak power. The arbitrage available in the unit rates of power at pumping and at generation makes this an economically viable operation. Observe that in a pure *P-S* plant, in a certain long period, the total water pumped is equal to the amount of water run through the turbine.

2. Mixed (Pump-back) Pumped Storage Plant

In this type of *P-S* plants, the reservoir *A* is built on a river and the reservoir *B* is allowed to discharge to the river in a controlled way. The power plant operates like a conventional hydro plant during peak-load conditions but is capable of pumping back some amount water from the downstream reservoir to the high-level reservoir *A* during slack demand period. It is thus a combination of pure *P-S* plant and a conventional peak power hydro plant. In this type, over a certain long period, the quantity of water pumped will be less than the quantity of water that has passed through the turbine. There are many variants of this pump-back plant.

3. Present Status of P-S Plants

Presently pumped storage schemes are one of the most cost-effective means of storing huge amounts of electrical energy and considerable interest in its development is shown all around the world. Especially in connection with green sources of energy such as solar power and wind-energy development, which by their very nature are intermittent, there is renewed interest in the use of pure *P-S* schemes to act as mega size power storage unit in large-scale-power harnessing systems. There are about 50 operational *P-S* schemes in the world with a capacity of more than 1000 MW. An estimate of 2009 indicated that there was at least 127,000 MW of pumped hydro storage projects globally and it is expected to reach 203,000 MW by 2014. Further, pumped hydro makes up nearly 99% of the world's energy-storage technologies. Table 8.3 lists five largest operational P-S plants in the world.

Table 8.3 *Five largest pumped-storage plants in the world*

Station	Country	Capacity (MW)	Year
Bath Country Pumped Storage Station	USA	2,772	1985
Guangdong Pumped Storage Power Station	China	2,400	1996–2000
Ludington Pumped Storage Power Plant	USA	1,872	1969–1973
Okutataragi Hydroelectric Power Sation	Japan	1,932	
Tianhuangping Pumped Storage Power Station	China	1,836	2001

Presently, there are quite a few P-S schemes in operation/under construction in India. In terms of potential, 56 P-S schemes having a potential installed capacity of 94,000 MW have been identified in India. Some of the major operational P-S schemes in India are:

- Koyna (Stage-IV), Maharashtra, $4 \times 250 = 1000$ MW
- Sardar Sarovar, Gujarat, $6 \times 200 = 1200$ MW
- Sri Sailam, A P, $6 \times 150 = 900$ MW
- Purulia, W B, $4 \times 225 = 900$ MW

8.7 ESTIMATION OF POWER POTENTIAL OF A STREAM FOR AN SHP

The topographical survey of the area and collection of essential hydrological data relevant to the site, such as flow data spanning several years, are the first phase of the task. Long-term, historical flow data, covering at least 30 years is desirable.

Analysis of Flow Data

Based upon a long length of stream flow data, the *FDC* of the stream is prepared. In preparing *FDC,* it should be noted that only integer number of water year data be used. Stream flow data covering fraction of a water year is not admissible. From this, by considering an unregulated flow, various dependable flow magnitudes are extracted. It is usual practice to take about 95% dependable flow (Q_{95}%) as a characteristic minimum flow available in the stream.

Environmental Flow (Compensation Flow, Reserved Flow, Prescribed Flow) In every natural stream, a certain minimum flow is to be left to flow unhindered into the downstream of an impoundment structure for ecological, environmental and aesthetic considerations. Such a flow is known as *environmental flow*. It is also known as *reserved flow* or *prescribed flow* or *compensation flow.* This is a relatively new concept and a host of methods is proposed to estimate this flow. However, there is no generally accepted methodology to be adopted to estimate the quantum of environmental flow. The amount of environmental flow is site-specific

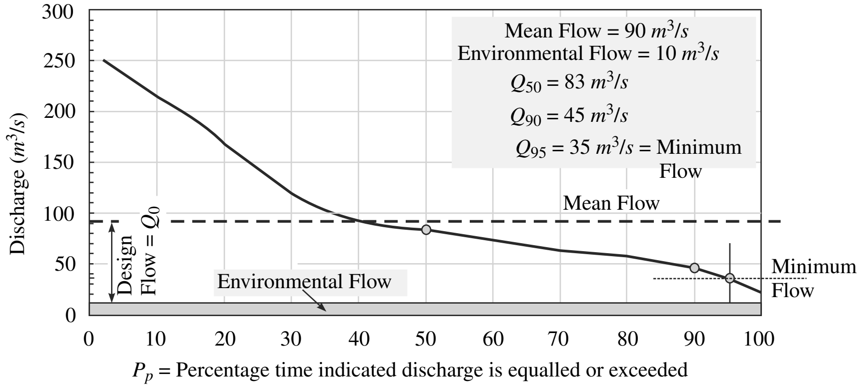

Fig. 8.8 *FDC of a river indicating available flow for a hydro plant.*

and should be carefully selected on consideration of a variety of concerns. A too low value would not serve the intended purpose and may cause permanent damage and pointless extra provision would affect power production and cause economic loss. As a first estimate, a value in the neighborhood of Q_{95} could possibly be taken as a reasonable value for *environmental* flow of a stream.

Figure 8.8 shows the *FDC* of Fig. 8.3 on which the mean flow and the minimum flow (taken as Q_{95}) are marked. It is seen that the *FDC* belongs to a perennial river. The mean flow for this river is 90 m^3/s. Let us assume that the environmental flow for this river is provided at a value of 10 m^3/s from local environmental considerations. This is marked in the figure as a horizontal line at the bottom (shaded portion in Fig. 8.8).

The operating discharge available is the space in the chart that lies between the *FDC* and the environmental flow line marked at the bottom. The available flow for power production consists of

(i) Minimum flow of (35.0–10.0) = 25 m^3/s.
(ii) The available mean flow is (90–10) = 80 m^3/s.
(iii) If the design flow is taken as available mean flow then the design flow = 80 m^3/s.

These items are shown, appropriately marked, in Fig. 8.8. If Power-duration curve (*PDC*) is to be calculated, it will have to be based on the available discharge only by duly discounting the environmental discharge.

8.8 STORAGE IN A RESERVOIR

8.8.1 Hydrograph

The flow in a river, in general, varies daily, weekly, monthly and seasonally. Usually, the discharge is high in rainy (wet) season and least in dry months. The plot of the discharge in a river against time in chronological order is known as a hydrograph. An *annual hydrograph* represents the variation of the flow in the stream over a year. The time interval can be a day, week or a month. The annual hydrograph shows the

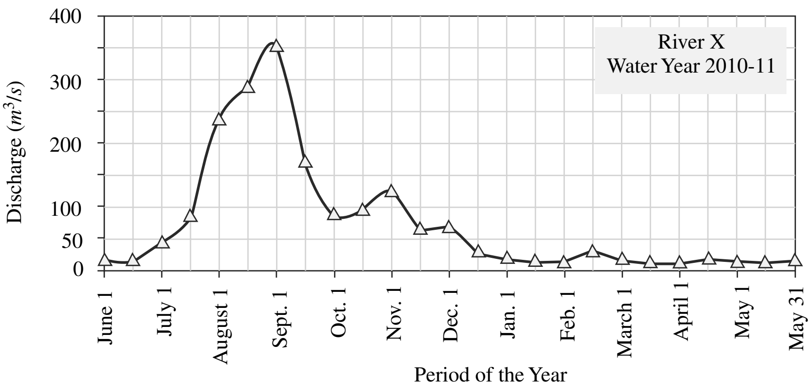

Fig. 8.9 *Schematic representation of a typical annual hydrograph*

seasonal variation of the discharge over a year and forms the basic data for further analysis through *FDC* and storage analysis. Figure 8.9 shows schematically a typical annual hydrograph of a river. Observe the flow depicted in this figure starts from a relatively dry period and ends in a relatively dry period, at the end of a calendar year. Each country has fixed a definite 12-month period to begin and end in a relatively dry season as the *water year*. In India, the water year starts from June 1st and ends on May 31st of the following calendar year. In USA, the water year is from October 1st to September 30th of the following year. All stream-flow records are maintained on water-year basis. In a water year, a complete cycle of climatic changes is expected and hence the annual data on the water year basis is a representative statistic of the annual stream flow. In all stream-flow analysis, such as *FDC* and mass curve, the period of data should be integer multiples of water-years and no fractional year data is permissible.

A hydrograph that has an essentially constant flow all through a water-year is a rarity. A storage reservoir helps through regulation of the flow to even out the fluctuations of discharge with time; it can store excess flow over the demand during wet periods and meet the excess of demand over the stream-flow into the reservoir in dry months.

For a large reservoir project, considerably long length of river flow data is needed to properly estimate the storage needed and to fix the operational policy of the reservoir. Normally, a minimum of 30 years of data is expected. The storage analysis and preparation of working tables of a reservoir is a fairly complicated task and belongs to the realm of advanced hydrological analysis. This section attempts to give a very brief introduction to the topic of fixing up of the storage capacity of a reservoir to meet the specific demands of downstream user such as a hydro plant.

8.8.2 Flow-Mass Curve

The earliest systematic analysis of a reservoir inflow-demand-storage relationship was by Rippl in 1882, who introduced the concept of flow-mass curve. This is a graphical procedure in which the cumulative inflow in the reservoir is plotted against the time in chronological order. The resulting plot is known as *flow-mass curve*. The abscissa, (time scale) can be in weeks or months and the ordinate is in units of volume (such as million m^3, cusec.day, ha.m). Figure 8.10 shows a typical flow-mass curve of a stream. The mass curve is a continuously increasing curve with peaks and valleys. Some of the important aspects of the mass curve (as in Fig. 8.10) are as follows:

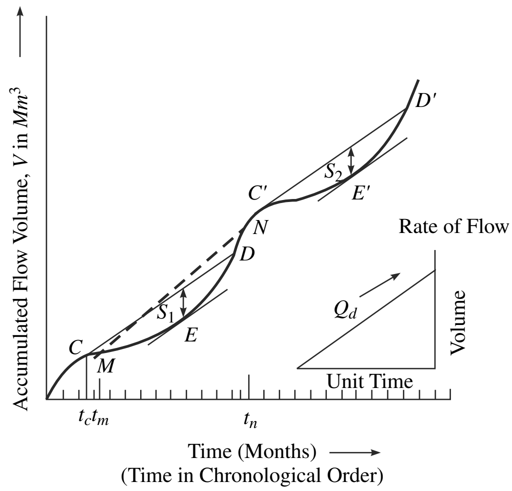

Fig. 8.10 *Definition sketch of a flow-mass curve*

- The slope of the mass curve at any point in the graph represents the rate of flow at that instant.
- If two points *M* and *N* are joined by a straight line, the slope of the line *MN* represents the average rate of flow that can be maintained between the times t_m and t_n if a reservoir of adequate storage capacity is available.
- A peak of the mass curve represents the end of a local wet period and beginning of a dry period.
- A valley of the mass curve represents the end of a local dry period and beginning of a wet period.
- Consider a line *CD* of the slope Q_d drawn tangential to the mass curve at a ridge. This line represents a constant rate of withdrawal Q_d from the reservoir and is called a *demand line*. For a pure hydro project, the demand volume in a certain time interval consists of the need of the turbines plus the evaporation loss from the reservoir.
- The maximum difference in ordinate between the demand line and the mass curve represents the storage S_1 needed to sustain the demand Q_d during the time interval between points *C* and *D*.
- If the reservoir is full at the point *C*, the point of maximum difference in ordinates between the demand line and the mass curve, the valley (point *E*) represents the time instant at which maximum depletion of the reservoir takes place. The reservoir starts filling up after the point *E* and the reservoir will be full again at the point *D*.
- If the length of the record is very long, there will be more such ridges and valleys (roughly one set per water year) and each set represents a storage value. The maximum of such local storage values (S_1. S_2….) represents the storage to be provided to tide over all the dry periods represented in the sample data. This once again highlights the need for long length of data.

The converse problem of finding the maximum demand that can be met by a known reservoir storage can also be solved by the use of mass curve. The mass curve is very versatile and can include a wide variety of situations, like variable demand, etc. When the number of years of record is very large, the mass curve becomes very unwieldy and cumbersome to use. Further, being a graphical procedure, it is obsolete. An equivalent tabular form of calculation of storage required to meet a demand is shown in Example 8.4. Current practice is to use a variant of the mass curve technique in the form of a numerical procedure called *sequent peak algorithm.*

8.8.3 Sequent Peak Algorithm

Sequent peak algorithm is a numerical procedure due to Thomas and Fiering that is used very extensively for analysis of storage in a reservoir through use of computers. A brief description of this procedure is given in this section. The theoretical reasoning of the procedure are beyond the scope of this book. However, the procedure given below is sufficient to solve simple problems associated with reservoir storage.

Consider the stream-flow data of N consecutive years being available for analysis. The data length is doubled by adding another set of N years of same data at the end of the first N years. This makes the length of data to be operated upon as $2N$ years. The data is considered in suitable time interval like week, fortnight or a month. Starting from the beginning, for any i^{th} time interval, let x_i = Inflow volume and D_i = Demand volume (inclusive of evaporation and any other loss from the reservoir). The surplus or storage in that period is the *net-flow volume* given by

$$\text{Net-flow volume} = \text{Inflow volume–Outflow volume}$$
$$= x_i - D_i$$

In the sequent peak algorithm, a mass curve of net-flow volume plotted against chronological time is considered. This curve known as *residual mass curve,* is shown schematically as definition sketch in Fig. 8.11. This curve will have a set of peaks (local maximums) and troughs (local minimums) that appear like the curve shown in Fig. 8.10. For any peak P, the next following peak of magnitude greater than P, is called a *sequent peak*. Figure 8.11 shows the first peak P_1, sequent peaks P_2 and P_3. With this definition of sequent peak, the following numerical procedure is adopted to identify the appropriate storage value:

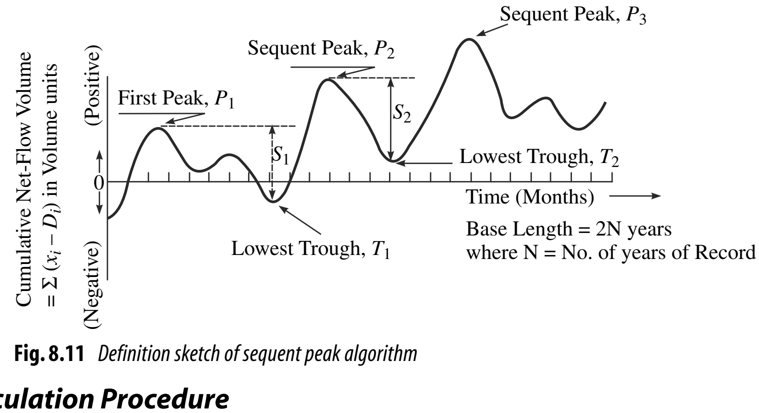

Fig. 8.11 *Definition sketch of sequent peak algorithm*

Calculation Procedure

1. Calculate the cumulative net-flow volumes, viz.,

   $\sum_t (x_i - D_i)$ for t = 1,2,3........($2N$).

2. Locate the first peak P_1 and the sequent peak P_2 (that is next peak of magnitude greater than P_1).

3. Find the *lowest trough* T_1 between P_1 and P_2 and calculate $(P_1–T_1)$.
4. Now move to the point P_2. Starting with P_2, find the next sequent peak P_3 and the lowest trough T_2 (between P_2 and P_3) and calculate $(P_2–T_2)$.
5. Repeat the procedure for all the sequent peaks available in the $2N$ periods of the record considered for analysis. Let there be a total of M lowest troughs and $(M + 1)$ sequent peaks,
6. The required storage capacity is given by

$$S = \text{Maximum of } (P_j - T_j) \text{ in the set } j = 1 \text{ to } M$$

NOTE

Fig. 8.11 being a definition sketch is only for explanation and does not form the solution procedure of sequent peak algorithm. The sequent peak algorithm is a numerical procedure that is very simple and is best solved through use of a spreadsheet such as MS Excel.

For further information relating to *FDC* and storage calculations under various demand and supply scenarios in a stream, refer to a book on Engineering Hydrology, such as

Engineering Hydrology by K. Subramanya
4th Ed., Tata McGraw-Hill Education Pvt. Ltd.,
New Delhi

8.9 OTHER PARAMETERS

8.9.1 Head

If the project is a high-head one, the topographic maps may be used to get the gross head at the project. However, in case of small head projects, the simultaneous stage readings at the upstream and downstream sections of the plant are needed for different ranges of discharges in the stream. This may have to be obtained by field surveying. Normally, a constant head applicable to all discharges (nominal head) at the plant is assumed for preliminary studies.

If a gate-controlled storage is used in a project, the variation of the head is relatively small when compared to projects having uncontrolled diversion structures only.

8.9.2 Design Discharge, Q_0

Depending upon the nature of *FDC*, a design discharge is selected. This discharge will be equalled to or smaller than the mean discharge (Q_{mean}) in the stream. A main consideration is the capacity factor of the plant. The capacity factor (*CF*) of the plant is defined as

$$\text{Capacity Factor } (CF) = \frac{\text{Energy generated per year (kWh/year)}}{\text{Installed capacity} \times 8760 \text{ hours/year}}$$

Smaller the ratio of design flow to mean flow (Q_o/Q_{mean}), larger would be the time the plant would be working in a year. It would have larger capacity factor. A quick estimate of the variation of the capacity factor with design flow is given in Table 8.4.

Table 8.4 *Relationship between capacity factor and design flow*

Design Flow, Q_0	Capacity Factor, *CF*
Q_{mean}	40%
0.75 Q_{mean}	50%
0.50 Q_{mean}	60%
0.33 Q_{mean}	70%

Higher capacity factor means smaller design discharge for a given head. This results in smaller turbines that work more number of hours. Conversely, smaller capacity factor means larger design discharge and hence larger turbines that work less number of hours. There is a trade-off here and economics of the project dictates the capacity factor and hence the design discharge to be adopted. Generally, for mini and small hydro plants, the capacity factor is between 50% and 70%.

Technical Minimum Flow of Turbines

Every turbine has a minimum discharge below that it cannot operate or operate with drastic reduction of efficiency. This flow is known as *technical minimum flow* of the turbine and its value for a given turbine design is supplied by the manufacturer. Table 8.5 gives the normal ranges of the technical minimum flow for various types of turbines.

Thus, the type of turbine to be adopted in a project, especially in an *SHP*, is determined not only by its specific speed and related parameters but also by the operational range of discharge as reflected by the *FDC* of the stream.

Table 8.5 *Technical minimum flow of turbines*

Type of Turbine	Q_{min} (as % of Design flow Q_0)
Francis	50%
Semi Kaplan	30%
Kaplan	15%
Pelton	10%
Turgo	20%
Propeller	75%

8.9.3 Rated Power

For the selected design flow, the head is estimated from the available data and the rated power is etimated as

$$P = \eta\, \gamma Q_0 H \tag{8.2}$$

where η is the overall efficiency of the power generation, and Q_0 is the design discharge.

For preliminary estimates, the efficiency is chosen as 0.715 to get the peak power *P* as

$$P = \eta\,\gamma\,Q_0\,H = 0.715 \times 9.79 \times Q_0\,H = 7Q_0\,H \tag{8.3}$$

In this, Q_0 is in m^3/s and H is in metres and P is the peak power in kW.

8.9.4 Annual Energy Output

For the chosen design discharge, the capacity factor is selected from Table 8.5. The annual energy from the project is then calculated as

$$\text{Energy (kWh/year)} = E = 8760 \times P \times CF \tag{8.4}$$

(In Eq. 8.4, the number 8760 is the number of hours in a year).

8.10 ILLUSTRATIVE EXAMPLES

*EXAMPLE 8.1

A hydroelectric station has two units of 20 MW and one unit of 30 MW. If during a week, the load varies from 40 MW to 65 MW, calculate the (a) weekly load factor, (b) weekly plant-use factor, and (c) weekly utilisation factor.

Solution

Total installed capacity = (2 × 20) + 30 = 70 MW

$$\text{Average power in the week} = \frac{(40+65)}{2} = 52.5\ \text{MW}$$

(a) $$\text{Weekly load factor} = \frac{\text{Average load in the week}}{\text{Peak load in the week}} = \frac{52.5}{65} = 0.808 = 80.8\%$$

(b) $$\text{Weekly plant-use factor} = \frac{\text{Energy actually produced in the week}}{\text{Max. energy producing capability}}$$

$$= \frac{52.5 \times 7\ \text{days}}{70.0 \times 7\ \text{days}} = 0.75 = 75.0\%$$

(c) Weekly utilisation factor

$$= \frac{\text{Maximum power produced in the week}}{\text{Installed capacity}} = \frac{65}{70} = 0.929 = 92.9\%$$

**EXAMPLE 8.2

A run-of-river plant is supplied with a regulated flow at a constant rate of 40 m^3/s. The effective head is 6.0 m and the installed capacity is 7.5 MW. If the plant is to operate as a peak station at 10 hours per day for 5 days a week, estimate the (a) pondage required at the forebay. If this pondage is provided, what is the (i) weekly load factor, (ii) weekly plant-use factor, and (iii) utilisation factor? Assume overall efficiency of the turbine-generator set as 0.8.

Solution

Actual working hours per week = (5 ×10) = 50 hours

Pondage = S = (2 × 24) + 14 = 62 hours of flow

$= 62 \times 3600 \times 40 = 8.928\ \text{Mm}^3$

Total inflow volume in a week = $40 \times 7 \times 24 \times 3600$

$=$ Outflow volume to the turbine = $Q \times 5 \times 10 \times 3600$

$$Q = \text{Discharge available during running} = \frac{7\times 24\times 3600\times 40}{5\times 10\times 3600} = 134.4\ \text{m}^3/\text{s}$$

Power produced = $P = \eta\gamma QH = 0.8 \times 9.79 \times 134.4 \times 6 = 6316$ kW

= 6.316 MW = Peak load

(a) Weekly average power = $\dfrac{6.316\times 50}{7\times 24}$ = 1.88 MW

$$\text{Weekly load factor} = \frac{\text{Average load in the week}}{\text{Peak load in the week}} = \frac{1.88}{6.316} = 0.298 = 29.8\%$$

(b) $\text{Plant-use factor} = \dfrac{\text{Energy actually produced}}{\text{Max. energy producing capability}}$

$$= \frac{6.316\times 50\times 3600}{7\times 24\times 3600\times 7.5} = 0.251 = 25.1\%$$

(c) $\text{Utilisation factor} = \dfrac{\text{Maximum power produced}}{\text{Installed capacity}} = \dfrac{6.316}{7.5} = 0.842 = 84.2\%$

**EXAMPLE 8.3

A run-of-river hydro plant has an average daily flow of 20 m³/s in the first week of November. To even out the daily fluctuations of discharge, it is estimated that 15% of daily mean flow is needed as storage. The effective head of the plant is 8.0 m. The plant operates 20 hours/day, five days a week. The daily load factor is 60%. The pondage requirement is calculated as 20% of the mean flow to the turbine. If the maximum allowable fluctuation in the pond is limited to 0.8 m, estimate (a) the surface area of the pond, and (b) the weekly energy output. Assume the efficiency of the turbine-generator set as 0.72.

Solution

Total inflow volume in a week = $20 \times 7 \times 24 \times 3600$

$=$ Outflow volume to the turbine = $Q \times 5 \times 20 \times 3600$

$$Q = \text{Discharge available during running} = \frac{7\times 24\times 3600\times 20}{5\times 20\times 3600} = 33.6\ \text{m}^3/\text{s}$$

S_1 = Storage required to even out river flow fluctuation = 15% of daily flow

$= 0.15 \times 24 \times 3600 \times 20 = 0.2592\ \text{Mm}^3$

S_2 = Pondage required for variation of load = 20% of the mean flow to the turbine

$= 0.2 \times 33.6 \times 24 \times 3600 = 0.5806 \text{ Mm}^3$

Total pondage required = $S_1 + S_2 = 0.2592 + 0.5806 = 0.8398 \text{ Mm}^3$

(a) Maximum allowed water-level fluctuation in the pond = 0.8 m

$$\text{Area of pond} = \frac{\text{Volume of pond}}{\text{Permitted change in water level}} = \frac{0.8398}{0.8} = 1.0498 \text{ Mm}^2$$

$$\text{Average head during the week} = 8.0 + \frac{0.80}{2} = 8.40 \text{ M}$$

(b) Peak power generated = $P = \eta \gamma QH = 0.72 \times 9.79 \times 33.6 \times 8.40 = 1990$ kW
$= 1.99$ MW

Weekly energy output = $1.99 \times 5 \times 20 \times 0.60 = 119.4$ MWh

***EXAMPLE 8.4

A run-of-river plant supplies power to a variable load as indicated below:

Time	0–4 hours	4–8 hours	8–12 hours	12–16 hours	16–20 hours	20–24 hours
Load in MW	2.1	3.2	7.4	8.2	6.2	4.1

(a) Draw the load curve and determine (i) average load, and (ii) daily load factor.
(b) If the net head is 8.0 m and overall efficiency is 82%, determine the average flow required and the pondage required to meet daily load fluctuation.

Solution

$$\text{Average load} = (2.1 + 3.2 + 7.4 + 8.2 + 6.2 + 4.1) = \frac{31.2}{6} = 5.2 \text{ MW}$$

$$\text{Daily load factor} = \frac{\text{Average load in a day}}{\text{Peak load in a day}} = \frac{5.2}{8.2} = 0.634 = 63.4\%$$

The load curve is plotted from given data and is shown in Fig. 8.12.

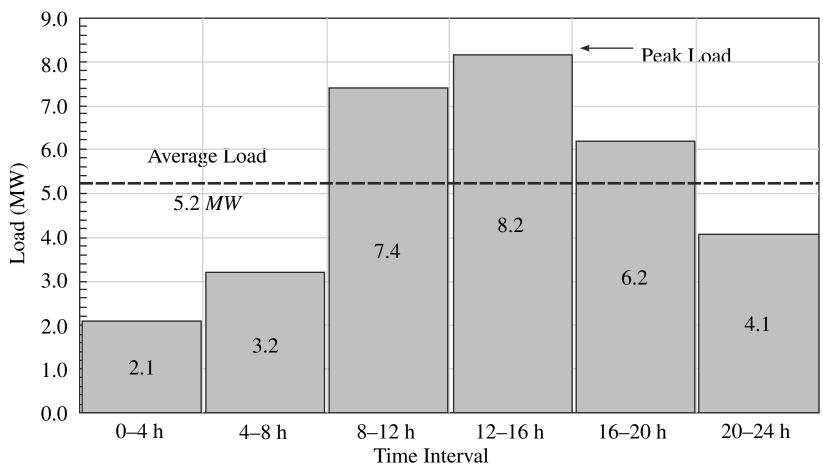

Fig. 8.12 *Load curve*

(ii) Pondage:

$$P \text{ (in MW)} = \frac{\eta\gamma QH}{1000} = \frac{0.82 \times 9.79 \times Q \times 8.00}{1000} = 0.06422\,Q$$

$$\text{Discharge for average load} = Q_a = \frac{5.2}{0.06422} = 80.97 \text{ m}^3\text{/s}$$

Mean inflow in 4-hour interval = 80.97 × 4 = 323.89 (m^3/s).hour

Pondage calculations are done in a tabular manner as shown in Table 8.6.

Table 8.6 *Calculation of pondage*

1	2	3	4	5	6	7	8
Time interval	**Mean flow volume in units of (m^3/s.h)**	**Load in MW**	**Required flow rate (m^3/s)**	**Demand volume (m^3/s .h)**	**Departure from inflow volume (m^3/s.h)**	**Cumulative excess Demand (m^3/s.h)**	**Cumulative excess inflow (m^3/s.h)**
0–4 h	323.882	2.1	32.70	130.799	193.084	193.084	
4–8 h	323.882	3.2	49.83	199.312	124.570	317.654	
8–12 h	323.882	7.4	115.23	460.909	–137.027		–137.027
12–16 h	323.882	8.2	127.69	510.737	–186.855		–323.882
16–20 h	323.882	6.2	96.54	386.167	–62.285		–386.167
20-24 h	323.882	4.1	63.84	255.369	68.514	386.168	
Required pondage ((m^3/s).h)						**386.17**	

Minimum pondage required = 386.17 ((m^3/s).h) = 386.17 × 3600 = 1390212 = 1.39 Mm^3

NOTE

While the solution of Example 8.4 is correct, the method has certain limitations. A foolproof method for calculating storage (pondage) is to use the sequent peak algorithm as in Example 8.7.

**EXAMPLE 8.5

Discharges in a river are considered in 10 class intervals. Three consecutive water years of data of discharge in the river are given below.

Discharge Range in m^3/s	≥ 350	250 to 349	150 to 249	90 to 149	50 to 89	30 to 49	15 to 29	10 to 14	6 to 9	3 to 5
Number of Occurrences	6	30	60	121	137	169	232	183	122	35

Calculate and plot the flow-duration curve of the river and determine:

(a) 50% and 75% dependable flow.

(b) If the net available head at a hydropower plant site in this river is 20 m, calculate the 90% dependable power at the site by assuming the overall efficiency as 72%.

Solution

The data are arranged in descending order of class value and is shown in Table 8.7. In this, Col.2 represents the number of occurrences of the corresponding event in Col.1. Col.3 is the cumulative total of Col.2 and represents the number of days the lowest flow in the class is equalled or exceeded. This gives the value of order number (rank) *m*. The percentage probability P_p = percentage of flow in the class being equalled or exceeded is calculated by Eq. (8.1) as

$$P_p = \frac{m}{(N+1)} \times 100$$

Here, N = Total number of occurrences = Sum of Col.2 = 1095.

P_p is recorded in Col. 4. Col.5 lists the smallest value of discharge (= Q) in each class interval.

The discharge Q is now plotted in the Y-axis against P_p of Col.4 that is plotted in the X-axis as shown in Fig. 8.13. Since the range of variables is very large, log-log plot is adopted. The computations involved are shown in tabular form in Table 8.7.

Table 8.7 *Computation of FDC—Example 8.5*

1	2	3	4	5
Discharge Range (m^3/s)	Number of Occurrences	m = Order Number (Rank) = Cumulative Total of Col. 2	$P_p = \frac{m}{(N+1)} \times 100$	Q = Lowest Value of Discharge in the Range of col. 1
≥ 350	6	6	0.55	350
250 to 349	30	36	3.28	250
150 to 249	60	96	8.76	150
90 to 149	121	217	19.80	90
50 to 89	137	354	32.30	50
30 to 49	169	523	47.72	30
15 to 29	232	755	68.89	15
10 to 14	183	938	85.58	10
6 to 9	122	1060	96.72	6
3 to 5	35	1095	99.91	3
Total Occurrences = N = 1095				

The required dependable flows are read from the graph of Fig. 8.13. They can also be obtained through linear interpolations of two nearest data points on either side of the required value.

Thus: 50% dependable flow = Q_{50} = 28.38 m^3/s.

75% dependable flow = Q_{75} = 13.17 m^3/s.

90% dependable flow = Q_{90} = 8.02 m^3/s .

P_{90} = 90% dependable power = $\eta_0 \gamma Q_{90} H$

$= 0.72 \times 9.79 \times 8.02 \times 20 = 1131$ kW

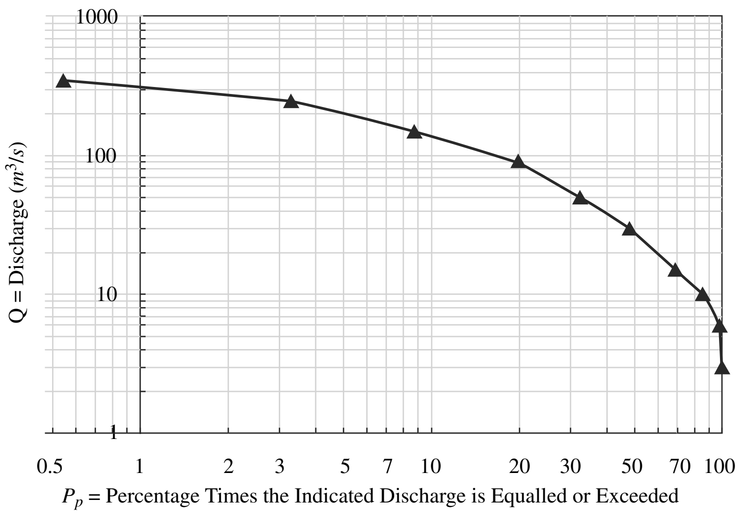

Fig. 8.13 *Flow-duration curve—Example 8.5*

**EXAMPLE 8.6

Stream-flow data of a river pertaining to two consecutive water years are given below.
(a) Calculate the mean flow in the stream.
(b) If the design flow of a hydro plant on this river is taken as 75% of mean flow, calculate the rated power under a net head of 13.5 m and overall efficiency of 80%.

Stream-flow data

Discharge Class in m^3/s		No. of Days the Class has Occurred
From	To	
	$\geqslant$ 16	82
16	14	26
14	13	14
13	11	38
11	10	20
10	9	20
9	6	84
6	4	82
4	3	64
3	2	90
2	0.7	210

Solution

A frequency table of the occurrence of the event, discharge (Q), is constructed as in Table 8.8. The midvalue of the discharge class (Q) is noted in Col.2. The number of occurrences of each Q is recorded in Col.3. The sum of Col.3 = Total number of data points = N = 730. The frequency of occurrence is calculated as ($f = n/N$) and is noted in Col.4. Col.5 is the product of Col.4 and Col.3. The mean flow is calculated as

$$Q_m = \sum_{1}^{N} f_i Q_i$$

where $f = \dfrac{n}{N} = \dfrac{\text{Number of occurrences of each class of discharges}}{\text{Total of all occurrences across all the classes}}$

and is found to be equal to 6.2459 m^3/s.

Table 8.8 *Calculation of mean discharge—Example 8.6*

1		2	3	4	5
Discharge Class in m^3/s		Midvalue of Discharge Class, Q m^3/s	No. of Days the Event (Q) has Occurred (n)	Frequency of Occurrence of the Event ($f = n/N$)	f_iQ_i
From	To				
	≥16	16.0	82	0.112	1.7973
16	14	15.0	26	0.036	0.5342
14	13	13.5	14	0.019	0.2589
13	11	12.0	38	0.052	0.6247
11	10	11.5	20	0.027	0.3151
10	9	10.5	20	0.027	0.2877
9	6	7.5	84	0.115	0.8630
6	4	5.0	82	0.112	0.5616
4	3	3.5	64	0.088	0.3068
3	2	2.5	90	0.123	0.3082
2	0.7	1.4	210	0.288	0.3884
			Total=N = **730**	Total = 1.000	$Q_m = \sum_{1}^{N} f_iQ_i$ = 6.2459

Q_0 = Design flow = 75% of $Q_m = 0.75 \times 6.2459 = 4.68$ m^3/s

Rated power = $P = \eta\gamma Q_0 H = 0.80 \times 9.79 \times 4.68 \times 13.5 = 494.8$ kW

**EXAMPLE 8.7

Following is the stream flow record of two consecutive critical years of a stream which is being considered for hydropower development. Estimate the minimum size of a reservoir on this stream to provide a constant downstream withdrawal of 7.0 m^3/s. Use the sequent peak algorithm.

Month (1st Year)	Monthly Flow Volume (M.m^3)	Month (2nd Year)	Monthly Flow Volume (M.m^3)
June	36.20	June	10.20
July	57.40	July	30.80
August	65.50	August	43.10

(Contd.)

(Contd.)

September	28.60	September	53.10
October	32.80	October	39.00
November	36.90	November	28.90
December	24.60	December	16.50
January	10.20	January	16.40
February	2.10	February	12.30
March	2.10	March	10.30
April	2.10	April	8.30
May	4.10	May	4.90

Solution

The given data is for two years. And as such, the sequent peak algorithm calculations are performed for $2 \times 2 = 4$ years. Calculations are shown in tables: Table 8.9 and 8.10.

- Col.3 is the given data.
- Demand volume (Col. 4) = (Constant withdrawal rate in m^3/s) $\times$ $(60 \times 60 \times 24)$ $\times$ (Number of days in the month) $\times$ $(1/10^6)$ Mm^3 = $0.6048 \times$ Col.2
- Col.4 = Col.3 – Col.4
- Col. 5 = Cumulative values of Col.4

Table 8.9 *Computation sheet of Example 8.7*

1	2	3	4	5	6	7
Month	No. of Days in the Month	Inflow Volume ($M.m^3$)	Demand Volume ($M.m^3$)	Net-Inflow Volume ($M.m^3$)	Cumulative Net-Inflow Volume ($M.m^3$)	Rem-arks
June	30	36.20	18.14	18.06	18.06	
July	31	57.40	18.75	38.65	56.71	
August	31	65.50	18.75	46.75	103.46	
September	30	28.60	18.14	10.46	113.91	
October	31	32.80	18.75	14.05	127.97	
November	30	36.90	18.14	18.76	146.72	
December	31	24.60	18.75	5.85	152.57	Peak-1
January	31	10.20	18.75	–8.55	144.02	
February	28	2.10	16.93	–14.83	129.19	
March	31	2.10	18.75	–16.65	112.54	
April	30	2.10	18.14	–16.04	96.50	
May	31	4.10	18.75	–14.65	81.85	
June	30	10.20	18.14	–7.94	73.90	Trough-1
July	31	30.80	18.75	12.05	85.96	
August	31	43.10	18.75	24.35	110.31	

Contd.

Table 8.9 *(Contd.)*

1	2	3	4	5	6	7
Month	**No. of Days in the Month**	**Inflow Volume ($M.m^3$)**	**Demand Volume ($M.m^3$)**	**Net-Inflow Volume ($M.m^3$)**	**Cumulative Net-Inflow Volume ($M.m^3$)**	**Rem-arks**
September	30	53.10	18.14	34.96	145.26	
October	31	39.00	18.75	20.25	165.51	
November	30	28.90	18.14	10.76	176.27	Peak-2
December	31	16.50	18.75	–2.25	174.02	
January	31	16.40	18.75	–2.35	171.67	
February	28	12.30	16.93	–4.63	167.04	
March	31	10.30	18.75	–8.45	158.59	
April	30	8.30	18.14	–9.84	148.74	
May	31	4.90	18.75	–13.85	134.90	Trough-2
June	30	36.20	18.14	18.06	152.95	
July	31	57.40	18.75	38.65	191.60	
August	31	65.50	18.75	46.75	238.35	
September	30	28.60	18.14	10.46	248.81	
October	31	32.80	18.75	14.05	262.86	
November	30	36.90	18.14	18.76	281.62	
December	31	24.60	18.75	5.85	287.47	Peak-3
January	31	10.20	18.75	–8.55	278.92	
February	28	2.10	16.93	–14.83	264.09	
March	31	2.10	18.75	–16.65	247.44	
April	30	2.10	18.14	–16.04	231.39	
May	31	4.10	18.75	–14.65	216.74	
June	30	10.20	18.14	–7.94	208.80	Trough-3
July	31	30.80	18.75	12.05	220.85	
August	31	43.10	18.75	24.35	245.20	
September	30	53.10	18.14	34.96	280.16	
October	31	39.00	18.75	20.25	300.41	
November	30	28.90	18.14	10.76	311.17	Peak-4
December	31	16.50	18.75	–2.25	308.92	
January	31	16.40	18.75	–2.35	306.57	
February	28	12.30	16.93	–4.63	301.93	
March	31	10.30	18.75	–8.45	293.48	
April	30	8.30	18.14	–9.84	283.64	
May	31	4.90	18.75	–13.85	269.79	

Table 8.10 *Results of computation—Example 8.7*

No	Peak	Trough	Storage $S_j = (P_j - T_j)$
1	152.57	73.90	78.669
2	176.27	134.90	41.37
3	287.47	208.80	78.669
4	311.17		

Required minimum storage = Maximum of $S_j = (P_j - T_j)$ values

$$= 78.669 \text{ Mm}^3$$

8.11 HYDROPOWER POTENTIAL OF INDIA

Development of hydropower in the country could be considered in two categories:

1. Large hydropower projects (the conventional hydro-development strategy), and
2. Small hydropower projects (alternative renewable energy development).

1. Large Hydropower Projects

India is endowed with rich hydroelectric power potential that is estimated at 84,000 MW at 60% load factor corresponding to an installed capacity of about 150,000 MW. India ranks fifth in the world in terms of exploitable hydropower potential. The first hydropower power plant in India is a run-of-river plant in Darjeeling district in West Bengal with an installed capacity of 130 KW and was commissioned in the year 1897. The first major hydro plant in India was the Shivanasamudram project on the Cauveri in Karnataka. It was a 4.5 MW plant commissioned in 1902. Since then there has been considerable progress and the current (2012) installed capacity is 39,527 MW which is nearly 26% of the assessed potential. In the beginning of year 2012, the total installed capacity of electrical energy from all sources was 1,87,550 MW. The share of different types of fuel/energy sources for this generation is shown in Table 8.11.

Table 8.11 *Installed capacity of electrical power by type in India (beginning of 2012)*

Total Installed Capacity: 187550 MW			
Conventional Sources		**Nonconventional sources**	
Coal	55.9%	Wind	7.6%
Gas	9.5%	Solar	0.1%
Oil	0.6%	Co-generation (Bagasse)	0.7%
Nuclear	2.5%	Biomass	0.6%
Hydropower (including *SHP*)	22.5%		

It is seen that the installed hydro capacity is around 23% of all types electrical power development in the country.

The distribution of the hydropower development in different parts of the country is shown in Fig. 8.12. Note that the northern region and southern region account for most of the hydropower in the country. Out of the total hydropower development, about 1.8% is the share of small hydro and the rest (98.2%) is from large hydropower development. A list of some major hydroelectric plants in India is given in Table 8.13. Table 8.14 gives a list of top five largest hydro plants in the world. Note the relatively small sizes of Indian top hydro projects.

All aspects relating to large hydropower development, in the country is vested with the Ministry of Power, Government of India.

2. Small Hydropower Projects (SHP)

Since the beginning of the 21st century, there is renewed interest in the development of small hydropower projects (*SHP*), primarily from its environmental benign aspects and ability to produce power in remote areas. *SHP*s have been identified as an important tool for the economic development of isolated and remote mountainous areas. Further, *SHP*s are not only economically viable but also have the advantage of short gestation period. Currently (2012), *SHP*s in India have substantive private investment. About 2800 MW of installed capacity exists in the *SHP* sector spread over about 760 projects. Facilities for the manufacture of practically all the components of all the types *SHP*s are available in the country.

3. R & D Infrastructure

Many high-quality turbine and pump testing laboratories do exist in both public and private sectors. International standard turbine laboratories exist at the Central Water Power Research Station, Pune, and at some educational and research institutes in the country, for use in verification of designs, generation of design data and validation of designs through high-level *CFD* analysis. Facilities and capabilities are available in the country for *CFD* analysis of various components of both large and small hydro projects.

Table. 8.12 *Regionwise hydropower installed capacity at the beginning of 2012*

Region	Hydro-power Installed Capacity (MW)
Northern	15670
Western	7460
Southern	11356
Eastern	3883
North–Eastern	1158
Total	**39,527**

Table 8.13 *List of some major hydroelectric plants in India*

Sl. No	Hydro Project	State	Generator units (MW)	Capacity (MW)
1	Koyna	Maharashtra	4×65; 4×80; 4×75 2×20; 4×250	1920
2	Sri Sailam	Andhra Pradesh	6×150; 7×110	1670
3	Nathpa Jhakri	Himachal Pradesh	6×250	1500

Contd.

Contd.

4	Sharavati	Karnataka	10 × 103.5; 2 × 27.5; 4 × 80;	1469
5	Sardar Sarovar (on Narmada)	Gujarat	6 × 200; 5 × 140	1450
6	Bhakra dam	Punjab	5 × 108; 5 × 107	1325
7	Kalinadi	Karnataka	5×50; 3×40; 1×135; 5×150	1240
8	Indira Sagar (on Narmada)	Madhya Pradesh	8 × 125	1000

Table 8.14 *Top five largest hydro plants in the world*

Sl.No.	Hydro Project	Country	Installed Capacity (MW)	Year
1	Three Gorges Dam	China	22,500	2011
2	Itaipu Dam	Brazil/Paraguay	14,000	2008
3	Guri	Venezuela	10.200	1986
4	Tucrui	Brazil	8370	1984
5	Grand Coulee	USA	6809	1980

- For official data and information relating to hydropower in India, the website ***http://www.cea.nic.in*** of Central Electricity Authority (*CEA*), Ministry of Power, Government of India, may be consulted.
- Information relating to *SHP* development in India can be obtained from the website of the Ministry of New and Renewable Energy, Government of India: ***http://www.mnre.gov.in/schemes/grid-connected/small-hydro***
- The development of small hydropower (*SHP*) in the country is vested with the Ministry of New and Renewable Energy *(MNRE)* Government of India.

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Review Questions

8.1. Describe various typical types of hydropower plants in use.

8.2. Compare and contrast the salient features of thermal and hydropower plants.

8.3. Draw a neat sketch of a typical high-head, divided-fall hydropower plant and label the salient components.

8.4. Describe with sketches a pure pumped-storage power plant.

8.5. Describe with appropriate sketches an ideal run-of-river plant.

8.6. Explain:
- (a) Load factor
- (b) Capacity factor
- (c) Flow-duration curve
- (d) Power-duration curve

8.7. Distinguish between:
- (a) Base-load and peak-load
- (b) Pondage and storage
- (c) Run-of-river plant and concentrated fall storage-type plant

8.8. Write brief notes on
- (a) Reserved flow
- (b) Effect of storage regulation on *FDC*
- (c) Surge tank
- (d) Forebay

8.9. Following are some of the claims of hydropower from the point of view of environmental concern:
- (a) Hydropower helps fight climate change
- (b) Hydropower can reduce pollution
- (c) Hydropower means clean, affordable power

Elaborate on these views.

8.10. Explain briefly:
- (a) Water year
- (b) Annual hydrograph
- (c) Flow-mass curve
- (d) Environmental flow

8.11. Describe briefly the sequent peak algorithm method of estimation of reservoir capacity to meet a known downstream flow demand.

8.12. Write a brief note on the importance of small hydro projects *(SHP)* in the current Indian energy scene.

Problems

P8.1 * A hydro plant gets a regulated supply of 20 m^3/s. The effective head of the plant is 15 m and the overall efficiency of the turbine–generator set is 0.75. If the plant is to work for eight hours per day and six days a week as a peaking plant, calculate the (a) required pondage and (b) pondage factor. What would be the firm capacity of the plant with pondage?

[***Ans:*** 2.88 Mm^3, Pondage factor = 3.5, P = 7.71 MW]

P8.2** A run-of-river plant on a river works 6 days a week, 24 hours a day. The effective head is 16 m and the overall plant efficiency is 0.82. In a typical week, the flow in the river is as follows.

Day	Sun	Mon	Tue	Wed	Thurs	Fri	Sat
Discharge, m^3/s	31	40	45	55	50	45	35

Estimate (a) the total pondage required to even out the flow fluctuation and to get the maximum output from the plant, and (b) weekly energy output from the plant. [***Ans:*** (a) 2.909 Mm^3 (b) E = 41.36 MWh]

P8.3** A run-of-river hydroelectric plant with a net head of 10 m meets a peak demand of 15 MW. The plant operates under a weekly load factor of 30% when serving as a peak-load plant. What minimum discharge in the stream is needed to make the plant serve as a base-load plant? If the minimum discharge in the stream is 45 m^3/s, what would be the maximum weekly load factor? Assume the plant efficiency as 80%. [***Ans*****:** Q = 57.46 m^3/s, 23.5%]

P8.4** A run-of-river plant is located on a stream that has a regulated supply of 25 m^3/s. The net head available is 15 m.

(a) If the plant is operated 8 hours a day to supply peak-load and if no pondage is provided, calculate the firm capacity of the plant. Assume overall plant efficiency = 0.79.

(b) What would be the firm capacity if a pondage volume of 3 Mm^3 is provided?

(c) What would be the minimum pondage volume necessary to achieve maximum firm capacity at the plant?

(d) What would be the change in the firm capacity if the actual pondage volume provided is larger than that calculated in part (c) of the question?

[***Ans:*** (a) 2.90 MW (b) 4.91 MW (c) 3.6 Mm^3
(d) No change in firm capacity]

P8.5 * A hydroelectric plant has turbine-generator sets as follows: 3 of capacity 5 MW and 4 of 7 MW capacity. During a certain week, the load at the plant varied from 28 MW to 40 MW. Calculate (a) weekly load factor, (b) weekly plant factor, and (c) weekly utilisation factor.

[***Ans:*** Load factor = 80.0%, Plant factor = 70.83, Utilisation factor = 83.3%]

P8.6** Four consecutive water years of data of discharge in the river are given below in Table 8.15. In this, the stream discharges are considered in 12 class intervals. The available net head at a proposed hydro plant site in this river is given in Col.3 of the table.

(a) Compute and plot the (a) flow-duration curve, and (b) power-duration curve for the hydro plant site. Overall efficiency of the plant can be taken as 0.75.

(b) Estimate (a) the 75% dependable flow in this stream, and (b) 90% dependable power at the hydro-plant site.

Table 8.15 *Stream-flow data at the hydro-plant site*

Discharge Range in m^3/s	Number of Occurrences	Available Net Head (m)
≥ 450	19	18.0
449 to 350	38	18.2
349 to 275	90	18.4
274 to 225	87	20.0
224 to 175	212	20.1
174 to 135	267	19.5
134 to 100	230	19.0
99 to 70	156	18.5
69 to 45	143	18.0
44 to 20	86	17.0
19 to 10	77	17.9
9 to 3	56	17.8

[***Ans:*** Q_{75} = 70.48 m^3/s, P_{90}= 2.983 MW]

P8.7.** Stream-flow data of two consecutive years in a perennial stream are given below.

(a) Prepare a flow-duration curve of the stream. (Plot the discharge on logarithmic scale and the percentage probability on arithmetic scale.)
(b) Calculate 75% and 90% dependable flows of the stream.
(c) What is the dependability of a discharge of 5.0 m^3/s in this stream?

Discharge Class in m^3/s		No. of Days the Class has Occurred
From	To	
	⩾16	82
16	14	26
14	13	14
13	11	38
11	10	20
10	9	20
9	6	84
6	4	82
4	3	64
3	2	90
2	0.7	210

[***Ans:*** Q_{90} = 1.0 m^3/s, Q_{75} = 3.32 m^3/s, P_p = 44.46%]

P8.8** Volume of water flow in a stream at a proposed reservoir site in a critical year is tabulated below:

Month	Flow Volume (Mm^3)
June	9.83
July	18.4
August	22.64
September	22.64
October	19.81
November	8.49
December	7.1
January	5.66
February	5.66
March	2.83
April	4.25
May	5.8

Determine the minimum capacity of the reservoir needed to feed the turbines of a hydropower plant at a maximum uniform rate throughout the year. What is the value of this uniform discharge that could be obtained throughout the year if adequate capacity of reservoir is available. Ignore evaporation and other losses. [***Ans:*** S = 38.63 Mm^3, Q_{av} = 4.221 m^3/s]

P8.9** Solve the pondage requirement of Example 8.4 by sequent peak algorithm. [***Ans:*** Pondage = 1.39 Mm^3]

Objective-type Questions

O8.1 * The load factor of a hydroelectric plant is the ratio of
(a) average load to maximum load during a specified time interval
(b) energy consumed in a time to the maximum demand in that time
(c) maximum demand to connected load
(d) average output of the plant in a given period of time to plant capacity

O8.2 * The capacity factor, in a certain period of time, is the ratio of
(a) maximum load to plant capacity
(b) actual capacity to the rated capacity
(d) average load to plant capacity
(d) actual energy output to the installed capacity.

O8.3 * Which one of the following statements is NOT correct?
(a) Storage and pondage can be obtained from the study of the flow-duration curve.
(b) Primary or firm power corresponds to maximum stream-flow condition.
(c) Secondary power is occasionally called surplus power.
(d) Often, flash boards are kept on the weir crest to augment the pondage at low flow-durations.

O8.4 * Pondage in a hydropower unit is defined as

(a) impounding of considerable amount of excess water during seasons of surplus flow
(b) impounding of considerable amount of excess water during seasons of surplus power demand
(c) a regulating body of water in the form of relatively small amount of run-off to regulate flow variations in daily or weekly power.
(d) storage unit to hold excess run-off for a few hours only

O8.5** Consider the following statements:

Weekly pondage requirement of a hydro-power plant includes the need to

1. balance the varying demand within a week
2. compensate the wastage, spillage and evaporation in a week
3. balance short time fluctuations in the inflow
4. store water to meet unforeseen long-term demand

The correct statements are

(a) 1 and 3 (b) 1,2, and 4 (c) 1, 2, and 3 (d) 1, 2, 3 and 4

O8.6*** The daily average flow in a river is 12 m^3/s. For a run-of-river plant with a net head of 10 m and overall efficiency of 80% in this river, the pondage required to use it as a 6 hour peaking station, in Mm^3, is

(a) 0.622 (b) 0.259 (c) 0.973 (d) 0.778

O8.7 * During a certain week, a hydropower plant produces 8,400,000 kWh of energy and the peak-load during that week is 100,000 kW. What is the load factor during the week?

(a) 40% (b) 45% (c) 50% (d) 60%

O8.8 * Pumped storage plants

(a) use thermal energy to pump water and generate hydropower to meet peak demand
(b) allow thermal power to take up peak-load while the hydropower can take up the baseload
(c) convert low value off-peak energy into high-value on-peak capacity and energy
(d) convert low value thermal power into high-value hydropower

O8.9** Match List 1 with List 2 and select the correct answer using the codes given below the lists:

List 1		List 2	
A	Storage	1	Small storage
B	Pondage	2	Ratio of total inflow to total number of days the plant operates in a week
C	Pondage factor	3	Large reservoir
D	Daily load factor	4	Product of average plant load and peak-load
		5	Ratio of average load to peak-load in a day

Codes:

	A	*B*	*C*	*D*
(a)	5	1	2	4
(b)	3	1	2	5
(c)	3	2	1	5
(d)	5	2	1	4

O8.10**Match List 1 with List 2 and select the correct answer using the codes given below the lists:

List 1		List 2	
A	Load factor	1	Ratio of total inflow to total number of days the plant operates in a week
B	Capacity factor	2	Ratio of maximum demand to station capacity
C	Pondage factor	3	Ratio of average load to maximum load
D	Utilisation factor	4	Ratio of energy produced to product of installed capacity and time in hours

Codes:

	A	*B*	*C*	*D*
(a)	1	2	3	4
(b)	3	4	1	2
(c)	1	4	3	2
(d)	3	2	1	4

O8.11* A pumped storage plant is always a

(a) high-head plant (b) run-of-river plant
(c) peak-load plant (d) base-load plant

O8.12* Pondage of a hydro plant is defined as

(a) impounding of considerable amount of excess water flowing in wet season
(b) a body of water to regulate flow variations due to daily or weekly fluctuations in power
(c) excess run-off in a month
(d) flow required to maintain base-load

O8.13**Consider the following statements:

(1) Run-of-river plants can be located in any river.
(2) Runaway speed of Pelton turbine is about 180% of its normal speed.
(3) Underground power stations are advantageous in areas susceptible to landslides.
(4) Higher the specific speed of a turbine, larger would be the head and discharge.

Out of these, the correct statements are

(a) 1 and 2 (b) 2 and 3 (c) 1 and 4 (d) 2 and 4

O8.14* Which one of the following statements are correct?

A forebay in a hydel system is provided at the junction of

(a) the power channel and the tailrace channel
(b) the tail race channel and the penstock
(c) the penstock and the turbine
(d) the power channel and the penstock

O8.15*** A small hydro project gets a continuous, steady, uniform, regulated supply of Ꝟ m^3/hour on all the 7 days of a week from a reservoir to its forebay. The hydro plant works 5 days a week for 8 hours per day. The pondage required at the forebay is

(a) 96 Ꝟ m^3 (b) 56 Ꝟ m^3 (c) 48 Ꝟ m^3 (d) 64 Ꝟ m^3

O8.16**Match List 1 with List 2 and select the correct answer using the codes given below the lists:

List 1		List 2	
A	Pumped storage plant	1	Optimal use of natural fall in the terrain
B	Run-of-river plant	2	Environmentally least harmful
C	Storage-type hydro	3	Has high-capacity factor
D	Divided-fall type hydro	4	Economical storage of energy

Codes:

	A	*B*	*C*	*D*
(a)	1	2	3	4
(b)	4	2	3	1
(c)	4	1	2	3
(d)	3	2	1	4

O8.17* Consider the following statements:

A surge tank provided on the penstock connected to a water turbine

(1) helps reduce the water-hammer effect in a section of the penstock
(2) stores extra water when needed
(3) provides increased demand of water when needed

Which of these statements are correct?

(a) 1, 2 and 3 (b) 2 and 3 (c) 1 and 2 (d) 1 and 3

O8.18* The function of a surge tank is to

(a) avoid reversal of flow
(b) reduce the water-hammer effect in a major section of the pipeline
(c) prevent occurrence of mass oscillation of water
(d) smoothen the flow

O8.19* A surge tank is provided in a hydropower plant to

(a) to strengthen the penstocks
(b) reduce length of pipe affected by water-hammer pressure
(c) reduce frictional losses in the penstocks
(d) increase the net head

APPENDIX A

Dimensional Analysis and Similitude

A1.1 DIMENSIONAL ANALYSIS

A1.1.1 Introduction

Dimensional analysis is a type of compacting technique to reduce the number of variables to be studied in an experimental investigation of a physical phenomenon. All physical phenomena can be expressed in terms of a set of basic or fundamental dimensions. In fluid mechanics, these are usually the mass *M*, length *L*, time *T* and temperature θ in what is known as the *MLT* θ system. Sometimes the force F is used in place of the mass *M* to get *F, L, T* and θ as the basic dimensions of what is known as the *FLT*θ system or *engineering system.* The commonly occurring variables in the subject of fluid mechanics together with their common notations and dimensions are given in Table A1-1.

Table A1-1 *Common variables in fluid flow phenomenon*

Quantity	Notation	Dimensions	
		(MLT θ system)	(FLT θ system)
Length	L	L	L
Area	A	L^2	L^2
Volume	$\forall$	L^3	L^3
Angular velocity	ω	T^{-1}	T^{-1}
Frequency	f	T^{-1}	T^{-1}
Velocity	V	$L T^{-1}$	$L T^{-1}$
Discharge	Q	$L^3 T^{-1}$	$L^3 T^{-1}$
Mass density	ρ	$M L^{-3}$	$F L^{-4} T^{-2}$
Dynamic viscosity	μ	$M L^{-1} T^{-1}$	$F L^{-2} T$
Kinematic viscosity	ν	$L^2 T^{-1}$	$L^2 T^{-1}$
Surface tension	σ	$M T^{-2}$	$F L^{-1}$

(Contd.)

(Contd.)

Quantity	Notation	Dimensions	
		(MLT θ system)	(FLT θ system)
Bulk (volume) modulus of elasticity	K	$M L^{-1} T^{-2}$	FL^{-2}
Specific weight	γ	$ML^{-2}T^{-2}$	FL^{-3}
Relative density	S (or RD)	$M^0L^0T^0$	$F^0L^0T^0$
Force	F	$M L T^{-2}$	F
Torque	T	$M L^2T^{-2}$	FL
Momentum	M	$M L T^{-1}$	FT
Work, energy	W, E	$M L^2T^{-2}$	FL
Power	P	$M L^2T^{-3}$	FLT^{-1}
Rotational speed	N	T^{-1}	T^{-1}
Stress, pressure	p	$M L^{-1}T^{-2}$	$F T^{-2}$
Temperature	T	θ	θ
Specific heat	c_p, c_v	$L^2 T^{-2} \theta^{-1}$	$L^2 T^{-2} \theta^{-1}$
Thermal conductivity	k	$MLT^{-3} \theta^{-1}$	$FT^{-3} \theta^{-1}$

[**Note:** Generally, the temperature θ is important in problems associated with compressible fluids. While dealing with liquids, it is a relatively insignificant parameter and as such the temperature θ is not included in further sections of this essay.]

A1.2 DIMENSIONAL ANALYSIS

In the dimensional analysis of a physical phenomenon, the relationship between the dependent and independent variables is studied in terms of their basic dimensions to obtain information about the functional relationship between the dimensionless parameters that control the phenomenon. There are several methods of reducing the number of dimensional variables into a number of dimensionless parameters. Two of the commonly used methods are (i) Raleigh's method, and (ii) Buckingham Pi theorem method.

A1.2.1 Raleigh's Method

If A_1 is a dependent variable and A_2, A_3,……A_n are independent variables in a phenomenon, the variable A_1 is expressed as

$$A_1 = kA_2^aA_3^bA_4^c......A_n^{ij} \tag{A1-1}$$

where k is a dimensionless constant. The dimensions of each of the quantities A_1, A_2, A_3,……A_n are written and the sum of the exponents of each of the fundamental dimensions (M, L and T or F, L and T whichever set is preferred) on both sides are equated. The solutions of the resulting set of linear equations, on simplification, yields dimensionless groups controlling the phenomenon. Examples A1-1 and A1-2 illustrate the method. While this method is simple for a small number of variables, it becomes rather cumbersome when a large number of variables are involved.

A1.2.2 Buckingham Pi Theorem Method

The *Buckingham Pi theorem* states that if there are m primary dimensions involved in the n variables controlling a physical phenomenon then the phenomenon can be described by $(n - m)$ independent dimensionless groups (known as π s). The word (or symbol) *Pi* here refers to a product of variables and the Greek letter π is used to indicate these products, by designating them as π_1, π_2, π_3, etc.

In the application of this method, m number of repeating variables is first selected and dimensionless groups obtained by considering each one of the remaining $(n - m)$ variables, one at a time. A procedure similar to Raleigh's method is used in this part of the operation. In view of this procedure of having repeating variables, the method is also known as the *method of repeating variables.* Care is needed in selecting repeating variables and the following three guidelines are worth noting:

1. The repeating variables must have, amongst themselves, all the basic dimensions involved in the problem.
2. The dependent variable ***must not*** be chosen as a repeating variable.
3. Normally, a length parameter (such as a characteristic diameter), a characteristic velocity and the fluid density are convenient sets of repeating variables.

Procedure for Obtaining PI Terms

- Let $A_1, A_2, A_3, \ldots\ldots A_n$ be the n variables involved in the physical problem. Since all the variables affecting the phenomenon are included, a functional relationship must exist between them. This could be expressed as

$$F\,(A_1, A_2, A_3, \ldots\ldots A_n) = 0 \qquad \text{(A1-2)}$$

- If m dimensions are involved, let the dimensionless groupings of the n variables be $\pi_1, \pi_2, \pi_3, \ldots\ldots \pi_{n-m}$. These could be expressed as

$$F(\pi_1, \pi_2, \pi_3, \ldots\ldots \pi_{n-m}) = 0 \qquad \text{(A1-3)}$$

- To determine the π terms, select m number of repeating variables. Let they be A_1, A_2, A_3. These are selected as per the guidelines indicated earlier. They contain amongst themselves all the basic dimensions (say M, L and T); not necessarily in each one of them but collectively.
- Using them, the π terms are expressed as follows:

$$\text{First } \pi \text{ term, } \pi_1 = A_1^{x_1}\, A_2^{y_1}\, A_3^{z_1}\, A_4 \qquad \text{(A1- 4)}$$

  In this x_1, y_1 and z_1 are unknown exponents. Note that the variable A_4 is not assigned any unknown exponent; it has an exponent of unity.
- Combining the rest of the variables $\{A\}$s, one A at a time, the other π terms are formed as below:

$$\text{Second } \pi \text{ term, } \pi_2 = A_1^{x_2}\, A_2^{y_2}\, A_3^{z_2}\, A_5 \qquad \text{(A1-5)}$$

$$\text{Third } \pi \text{ term, } \pi_3 = A_1^{x_3}\, A_2^{y_3}\, A_3^{z_3}\, A_6 \qquad \text{(A1-6)}$$

and so on, till the last π term, $\pi_{n-m} = A_1^{x_{n-m}} A_2^{y_{n-m}} A_3^{z_{n-m}} A_n$ (A1- 7)

- Now substitute the dimensions of $\{A\}$ terms in each of the equation (A1-4) through equations (A1-7). Collect the exponents of the dimensions *M, L* and *T* from each equation.

- For each π term, a maximum of three linear equations in three unknowns are obtained. Solving them will yield the π term. All the $(n-m)$ Pi terms are obtained in this manner. The results are expressed as

$$F(\pi_1, \pi_2, \pi_3, \ldots\ldots \pi_{n-m}) = 0 \tag{A1-8}$$

Buckingham's Pi theorem method is relatively simple, straightforward and intuitive. As such, this is the preferred method, over Raleigh's method, for dimensional analysis. The procedure is best learned through a few examples. Examples A1-3 through A1-5 illustrate the use of Buckingham's Pi theorem method to some problems connected with hydraulic machines.

A1.3 SIMILITUDE

A1.3.1 Types of Similitude

In hydraulic engineering and in aeronautical engineering, model studies play a very important role in validating the design as well as in obtaining valuable data for design. In the field of hydraulic machines, model studies are probably the only means of proving the compliance of contract-performance specifications in the installed product such as turbines and large-size pumps. In model studies, experiments are conducted in a scale model of the original (prototype) and valuable results are obtained at relatively small costs and with greater control on the flow parameters. The laws that are used to scale up the model results to prototype behaviour are known as *similarity laws*. The relation between the model and prototype is known as *similitude* and is classified into three kinds.

1. Geometric Similarity

If the ratios of corresponding lengths in a model and prototype are the same, the model is said to be a geometrically similar model of the prototype. In such models,

If $$\frac{L_{\text{model}}}{L_{\text{prototype}}} = L_r$$

then $$\frac{(\text{Area})_{\text{model}}}{(\text{Area})_{\text{prototype}}} = \frac{A_m}{A_p} = L_r^2 \tag{A1-9}$$

and $$\frac{(\text{Volume})_{\text{model}}}{(\text{Volume})_{\text{prototype}}} = \frac{\mathcal{V}_m}{\mathcal{V}_p} = L_r^3 \tag{A1-10}$$

2. Kinematic Similarity

Kinematic similarity means geometric similarity and in addition, the ratio of velocities at all corresponding points in the flow is the same.

If $$\frac{L_{\text{model}}}{L_{\text{prototype}}} = L_r$$

and if $$\frac{(\text{Velocity})_{\text{model}}}{(\text{Velocity})_{\text{prototype}}} = \frac{V_m}{V_p} = V_r \text{, then}$$

(a) Time ratio = $\dfrac{T_{\text{model}}}{T_{\text{prototype}}} = T_r = \dfrac{L_r}{V_r}$ (A1-11)

(b) Acceleration ratio = $\dfrac{a_{\text{model}}}{a_{\text{prototype}}} = a_r = \dfrac{V_r^2}{V_r} = \dfrac{L_r}{T_r^2}$ (A1-12)

(c) Discharge ratio = $\dfrac{Q_{\text{model}}}{Q_{\text{prototype}}} = \dfrac{Q_m}{Q_p} = Q_r = \dfrac{L_r^3}{T_r}$ (A1- 13)

3. Dynamic Similarity

Two systems are dynamically similar if geometric and kinematic similarities exist and further the ratios of all corresponding forces in the two systems are the same. Let the forces due to gravity = F_g, viscosity = F_v, elasticity = F_e, surface tension = F_{st} and inertia = F_i. Using additional suffixes *m* and *p* to stand for model and prototype respectively, strict dynamic similarity means

$$\frac{F_{gm}}{F_{gp}} = \frac{F_{vm}}{F_{vp}} = \frac{F_{em}}{F_{ep}} = \frac{F_{stm}}{F_{stp}} = \frac{F_{im}}{F_{ip}} = \text{Constant} \tag{A1-14}$$

From this, the following relationships can be derived:

$$\frac{(\text{Interia force})_m}{(\text{Viscous force})_m} = \frac{(\text{Interia force})_p}{(\text{Viscous force})_p} = \text{Constant 1}$$

$$\frac{(\text{Interia force})_m}{(\text{Gravity force})_m} = \frac{(\text{Interia force})_p}{(\text{Gravity force})_p} = \text{Constant 2}$$

$$\frac{(\text{Interia force})_m}{(\text{Elasticity force})_m} = \frac{(\text{Interia force})_p}{(\text{Elasticity force})_p} = \text{Constant 3}$$

$$\frac{(\text{Interia force})_m}{(\text{Surface-tension force})_m} = \frac{(\text{Interia force})_p}{(\text{Surface-tension force})_p} = \text{Constant 4}$$

A1.3.2 Important Dimensionless Parameters

The various constants 1 through 4 that are necessary for dynamic similarity are represented in a convenient way as ratios per unit volume and these form the basic ratios for analysis of fluid flow.

1. The ratio of inertial force to viscous force is called *Reynolds number* and is expressed as

$$\text{Reynold Number} = R_e = \frac{\text{Inertial force}}{\text{Viscous force}} = \frac{\rho VL}{\mu} = \frac{VL}{\nu} \tag{A1-15}$$

For dynamic similarity of viscous forces between a model and a prototype

$$R_{em} = \left(\frac{VL}{\nu}\right)_m = R_{ep} = \left(\frac{VL}{\nu}\right)_p \tag{A1-16}$$

2. The ratio of inertial force to gravity forces is called *Froude number* and is expressed as

$$\text{Froude number} = F_r = \frac{(\text{Inertia force})^{1/2}}{(\text{Gravity force})^{1/2}} = \frac{V}{\sqrt{gL}} \tag{A1-17}$$

For dynamic similarity where gravity forces are predominant between a model and a prototype,

$$\left(\frac{V}{\sqrt{gL}}\right)_m = F_{rm} = F_{rp} = \left(\frac{V}{\sqrt{gL}}\right)_p \tag{A1-18}$$

3. The ratio of inertial force to compressibility forces is called *Mach number* and is expressed as

$$\text{Mach number} = \text{M} = \frac{(\text{Inertia force})^{1/2}}{(\text{Compressibility force})^{1/2}} = \frac{V}{\sqrt{K/\rho}} = \frac{V}{C} \tag{A1-19}$$

where C = Velocity of sound in the medium. For dynamic similarity where compressibility effects are predominant between a model and a prototype,

$$\left(\frac{V}{C}\right)_m = M_m = M_p = \left(\frac{V}{C}\right)_p \tag{A1-20}$$

4. The ratio of inertial force to surface tension forces is called *Weber number* and is expressed as

$$\text{Weber number} = W = \frac{\text{Inertia force}}{\text{Surface-tension force}} = \frac{\rho V^2 L}{\sigma} \tag{A1-21}$$

For dynamic similarity where surface-tension effects are predominant between a model and a prototype,

$$\left(\frac{\rho V^2 L}{\sigma}\right)_m = W_m = W_p = \left(\frac{\rho V^2 L}{\sigma}\right)_p \tag{A1-22}$$

A1.3.3 Model Scales

Strict fulfillment of all the requirements of a dynamic similarity is practically impossible to achieve. In view of this, fluid-flow models are usually designed to account for one most dominant force and occasionally for two dominant forces. The effect of other forces, if any, is treated as scale effect and corrected through empirical adjustments.

- If the dominant force is gravity force, Froude law of similarity is adopted. In such models, the Froude number in the model and the prototype are made the same and the corresponding scales of length, velocity, discharge and forces are worked out on the basis of Froude law. These are used in scaling up the model data to prototype data. Typical flow situations that are modeled by Froude law include all free surface-flow phenomenon such as flow over spillways, energy dissipators, river models and also ship models.

- Similarly, where viscous forces are predominant, Reynolds number similarity is adopted. In these models, the Reynolds number in the model is kept the same as in the prototype. Typical flow situations that are modeled by Reynolds law include all closed conduit flows and flow past submerged bodies. These include studies of flow-metering devices, flow in pipes and all subsonic wind-tunnel testing of airplane body shapes.
- Where compressibility effects are predominant, Mach number similarity is adopted in the model studies. Typically, supersonic wind tunnels that are used to test body shapes of missiles, rockets, and supersonic crafts adopt Mach number similarity.
- Weber number similarity is important in problems related to study of capillary waves and droplet formation.

Distorted Models

A large number of problems related to local phenomenon in open-channel flow, such as flow past hydraulic structures, are analysed through strict Froude law similarity. However, modelling of some open-channel flow situations like rivers, harbours, estuaries, etc. that have small longitudinal slope and large areal spread pose many practical problems such as of space constraints, if strict Froude law similitude is attempted. To overcome these problems, such flow situations are modelled by using distorted scales. The vertical flow dimension (viz. depth) is used to simulate Froude law while the other two linear dimensions (viz. length and breadth) are scaled to suit available space. The similitude scales are then worked out separately for each flow parameter. Such models that have different scales for depth and for length are known as *distorted models*.

A1.3.4 Model Studies of Hydraulic Machinery

While conducting model studies on hydraulic machinery, the rotating parts of a hydraulic machine require an extra consideration. This will be in addition to other criteria of similitude discussed earlier. In a hydraulic machine, it should be assured that the streamline patterns due to rotation of the parts are similar in model and prototype. This parameter must relate the discharge to the speed of the rotating parts. For geometrically similar machines, if the vector diagrams of the velocity entering and leaving the moving parts (that is the velocity diagrams at the inlet and outlet of a runner/impeller) are similar, the units are *homologous.* This means that for all practical purposes, the dynamic similarity exists if velocity triangles are similar.

In the flow through hydraulic machines, the Froude number and Weber number are unimportant. Also, the role of the Reynolds number is reflected in terms of scale effect due to relative differences in the viscous effects in the model and the prototype. This scale effect is of the order of 2–3% decrease in the model efficiency of a turbine/pump. It is usually corrected through empirical correction formulae. The Mach number is important only where compressibility is involved as in the case of compressors and gas turbines. As such, in water turbines and pumps handling liquids, the Mach number is unimportant.

In view of the above, the basic nondimensional parameters used in the modelling of hydroturbines and centrifugal pumps are a set of dimensionless parameters, common to both the turbines and the pumps, represented as follows:

$$\frac{P}{\rho N^3 D^5} = fn\left(\frac{gH}{N^2 D^2}, \frac{Q}{ND^3}\right) \tag{A1-23}$$

In this P = power, H = relevant head, N = rpm of the runner/impeller, D = diameter of the runner/impeller, Q = discharge and g = acceleration due to gravity and ρ = density of fluid. The particular application of this equation to turbines is discussed in Chapter 1 and 2 and its application to centrifugal pumps is discussed in Chapter 5.

NOTE

While dealing with similitude in hydraulic machines, it is normal practice to treat the terms g and H as one parameter, instead of two variables. The term (gH) being a very important parameter in the study of turbines and pumps, is given a separate name as *specific energy* and a separate symbol E in *IEC* 60193(Ed.2). The basic unit of specific energy is energy per unit mass (joule/kg = Nm/kg). The dimensions of (gH) are $[L^2T^{-2}]$.

Minimum Size of Turbine Models

If the models are too small, the viscous effects may be highly skewed in the model due to larger laminar boundary layers. This may affect the overall similitude also. In order to achieve good similarity between the model and the prototype, the minimum Reynolds number and sizes of certain parameters have been stipulated in *IEC* 60193(Ed.2). These minimum values applicable to models of various types of turbines are shown in Table A1-2.

Table A1-2 *Minimum reynolds number and size of turbine models*

Parameter	Minimum Values for Model Size and Test Parameters			
	Radial (Francis)	Diagonal (Deriaz)	Axial (Kaplan, Bulb)	Impulse (Pelton)
Reynolds Number	4×10^6	4×10^6	4×10^6	2×10^6
Head, H(m)	10 m	5 m	3 m for $D < 0.4$ m and 2 m for $D \geq 0.4$ m	50 m
Reference diameter, D	0.25 m	0.30 m	0.30 m	—
Bucket width, B	—	—	—	0.08 m
Definition of Reynolds number v = Kinematic viscosity	$R_e = \frac{D\sqrt{2gH}}{v}$	$R_e = \frac{D\sqrt{2gH}}{v}$	$R_e = \frac{D\sqrt{2gH}}{v}$	$R_e = \frac{D\sqrt{2gH}}{v}$

A1.4 ILLUSTRATIVE EXAMPLES

A1.4.1 Raleigh's Method

**EXAMPLE A1.1

In a fluid machine, the torque T of the impeller is known to depend on the diameter D and speed N of the impeller and the density ρ and dynamic viscosity of the liquid. Show by using Raleigh's method of dimensional analysis that the torque could be expressed as

$$\frac{T}{\rho N^2 D^5} = Fn\left[\frac{\mu}{\rho D^2 N}\right]$$

Solution

The given parameters are first expressed in the following functional form:

$T = Fn\ (D, N, \rho, \mu)$

$T = k\ D^a N^b\ \rho^c\ \mu^d$ in which k is a dimensionless constant.

Expressing the above in terms of the basic dimensions of each parameter,

$[ML^2T^{-2}] = [M^0L^0T^0]\ [L]^a\ [T^{-1}]^b\ [ML^{-3}]^c\ [ML^{-1}T^{-1}]^d$

Equating the powers of the basic dimensions:

(i) M: $1 = c + d$

(ii) L: $2 = a - 3\,c - d$

(iii) T: $-2 = -b - d$

There are four variables and three equations. Values of three unknowns (a, b and c) are expressed in terms of the other variable, d.

Expressing the three equations (i), (ii) and (iii) in terms of d:

$c = 1 - d$

$a = 2 + 3\,c + d = 2 + 3 - 3d + d = 5 - 2d$

$b = 2 - d$

Substituting the above in the functional expression for T,

$$T = kD^{(5-2d)}N^{(2-d)}\rho^{(1-d)}\ \mu^d$$

$$T = k(\rho D^5 N^2)\left(\frac{\mu}{\rho D^2 N}\right)^d$$

$$T = \rho D^5 N^2\ Fn\left(\frac{\mu}{\rho D^2 N}\right)$$

Hence, $\dfrac{T}{\rho N^2 D^5} = Fn\left[\dfrac{\mu}{\rho D^2 N}\right]$

***EXAMPLE A1.2

The power required to drive the propeller of a boat is known to depend on the diameter D and angular velocity ω of the propeller, the density ρ and dynamic viscosity μ of water and the free-stream velocity V. Determine the functional relationship P of the power of the propeller as

$$\frac{P}{\rho V^3 D^2} = Fn\left[\left(\frac{V}{D\omega}\right), \left(\frac{\rho VD}{\mu}\right)\right]$$

Solution

The given parameters are first expressed in the following functional form:

$P = Fn\ (D, \omega, \rho, \mu, V)$

$P = kD^a\ \omega^b, \rho^c, \mu^d, V^e$ in which k is a dimensionless constant.

Expressing the above in terms of the basic dimensions of each parameter,

$$[ML^2T^{-3}] = [M^0L^0T^0]\ [L]^a\ [T^{-1}]^b\ [ML^{-3}]^c\ [ML^{-1}T^{-1}]^d\ [LT^{-1}]^e$$

Equating the powers of the basic dimensions:

(i) M: $1 = c + d$

(ii) L: $2 = a - 3\,c - d + e$

(iii) T: $-3 = -b - d - e$

There are five variables and three equations. Values of three unknowns (*a, b* and *c*) are expressed in terms of the other two variables (viz. *d* and *e*). [Note that the selection of the two unknowns *d* and *e* is arbitrary].

Expressing the three equations (i), (ii) and (iii) in terms of *d* and *e.*

$c = 1 - d$

$a = 2 + 3\,c + d - e = 2 + 3 - 3d + d - e = 5 - 2d - e$

$b = 3 - d - e$

Substituting the above in the functional expression for P,

$P = kD^{(5-2d-e)}\ \omega^{(3-d-e)}\ \rho^{(1-d)}\ \mu^d V^e$

$$P = k(\rho D^5\omega^3)\left(\frac{V}{D\omega}\right)^e\left(\frac{\mu}{\rho\omega D^2}\right)^d$$

$$\frac{P}{\rho D^5\omega^3} = Fn\left[\left(\frac{V}{D\omega}\right), \left(\frac{\mu}{\rho\omega D^2}\right)\right]$$

The nondimensional terms could be reconfigured as shown below to obtain the desired form of result:

$$\frac{P}{\rho D^5\omega^3} = \frac{P}{\rho D^5\omega^3} \times \frac{1}{\left(\frac{V}{D\omega}\right)^3} = \frac{P}{\rho V^3 D^2}$$

$$\frac{\mu}{\rho\omega D^2} = \frac{\mu}{\rho\omega D^2} \times \frac{\omega D}{V} = \frac{\mu}{\rho VD}$$

Substituting these, $\dfrac{P}{\rho V^3 D^2} = Fn\left[\left(\dfrac{V}{D\omega}\right), \left(\dfrac{\rho VD}{\mu}\right)\right]$

Application of Buckingham pi theorem method

***EXAMPLE A1.3

The efficiency η of a pump is known to depend upon the kinematic viscosity of the liquid v, the angular velocity ω, diameter D of the impeller, discharge Q and the energy per unit mass (gH) transferred to the liquid by it. Express η in terms of dimensionless parameters using Buckingham Pi theorem method as

$$\eta = F\left[\frac{\omega D^2}{v}, \frac{Q}{\omega D^3}, \frac{gH}{\omega^2 D^2}\right]$$

In the parameter (gH) the term g = acceleration due to gravity and H= manometric head. Take (gH) as one parameter.

Solution

$\eta = fn\,(v, \omega, D, Q, gH)$

Listing the dimensions of each variable:

η	v	ω	D	Q	gH
$M^\circ L^\circ T^\circ$	L^2T^{-1}	T^{-1}	L	L^3T^{-1}	L^2T^{-2}

There are six variables $n = 6$

and two basic dimensions $m = 2$

Hence, number of dimensionless terms = $n - m = 4$

For dimensional analysis, select ω and D as repeating variables.

I *term* π_1: Since η is dimensionless, $\pi_1 = \eta$

II *term* π_2: Consider v and the selected repeating variables ω and D.

$$\pi_2 = \omega^{a_2} D^{b_2} v$$

$$[M^oL^oT^o] = [T^{-1}]^{a_2}\,[L]^{b_2}\,[L^2T^{-1}]$$

By equating the powers of M, L and T,

$b_2 + 2 = 0$ Thus, $b_2 = -2$

$-a_2 - 1 = 0$ Thus, $a_2 = -1$

Hence, $\pi_2 = \left[\frac{v}{\omega D^2}\right]$

III term: $\pi_3 = \omega^{a_3} D^{b_3} Q$

π_3: $[M^oL^oT^o] = [T^{-1}]^{a_3}\,[L]^{b_3}\,[L^3T^{-1}]$

By equating the powers of M, L and T,

$b_3 + 3 = 0$ Thus, $b_3 = -3$

$-a_3 - 1 = 0$ Thus, $a_3 = -1$

Hence, $\pi_3 = \left[\frac{Q}{\omega D^3}\right]$

IV term: $\pi^4 = \omega^{a_4} D^{b_4}\,(gH)$

$[M^oL^oT^o] = [T^{-1}]^{a_4}\,[L]^{b_4}\,[L^2T^{-2}]$

By equating the powers of M, L and T,

$b_4 + 2 = 0$ Thus, $b_4 = -2$

$-a_4 - 2 = 0$ Thus, $a_4 = -2$

Hence, $\pi_4 = \dfrac{gH}{\omega^2 D^2}$

The functional relationship for the efficiency is $\eta = F\left[\dfrac{V}{\omega D^2}, \dfrac{Q}{\omega D^3}, \dfrac{gH}{\omega^2 D^2}\right]$

which can also be written as $\eta = fn\left[\dfrac{\omega D^2}{v}, \dfrac{Q}{\omega D^3}, \dfrac{gH}{\omega^2 D^2}\right]$

**EXAMPLE A1.4

A disc of diameter D is rotating at an angular velocity ω in a fluid of dynamic viscosity μ and density ρ. Obtain an expression for the torque T, which is known to depend upon the above four parameters and also on the clearance ε of the disc in its casing, as $T = (\rho\omega^2 D^5) fn\left(\left[\dfrac{\mu}{\rho D^2 \omega}\right], \left[\dfrac{\varepsilon}{D}\right]\right)$

Solution

$T = fn\,(D, N, \mu, \rho, \varepsilon)$

Listing the dimensions of each variable:

T	D	ω	μ	ρ	gH
ML^2T^{-2}	L	T^{-1}	$ML^{-1}T^{-1}$	ML^{-3}	L

There are six variables $n = 6$

Three basic dimensions $m = 3$

Hence, number of dimensionless terms = $n - m = 3$.

For dimensional analysis, select D, ω and ρ as repeating variables.

I *term* π_1: Consider T and the selected repeating variables D, ω and ρ.

$\pi_1 = D^{a_1}\omega^{b_1}\rho^{c_1}T$

$[M^oL^oT^o] = [L]^{a_1}\,[T^{-1}]^{b_1}\,[ML^{-3}]^{c_1}\,[ML^2T^{-2}]$

By equating the powers of M, L and T,

$c_1 + 1 = 0,$ Thus, $c_1 = -1$

$a_1 - 3c_1 + 2 = 0,$ Thus, $a_1 = -5$

$-b_1 - 2 = 0$ Thus, $b_1 = -2$

Hence $\pi_1 = \left[\dfrac{T}{\rho\omega^2 D^5}\right]$

II Term: Consider μ and the selected repeating variables D, ω and ρ.

$\pi_2 = D^{a_2}\omega^{b_2}\rho^{c_2}\mu$

$[M^oL^oT^o] = [L]^{a_2}\,[T^{-1}]^{b_2}\,[ML^{-3}]^{c_2}\,[ML^{-1}T^{-1}]$

By equating the powers of M, L and T

$c_2 + 1 = 0$ Thus, $c_2 = -1$

$a_2 - 3c_2 - 1 = 0$ Thus, $a_2 = 3\, c_2 + 1 = -2$

$-b_2 - 1 = 0$ Thus, $b_2 = -1$

Hence $\pi_2 = \left[\dfrac{\mu}{\rho D^2 \omega}\right]$

III Term: Consider ε and the selected repeating variables D, ω and ρ

$\pi_3 = D^{a_3} \omega^{b_3} \rho^{c_3} \varepsilon$

$[M^o L^o T^o] = [L]^{a_3} [T^{-1}]^{b_3} [ML^{-3}]^{c_3} [L]$

By equating the powers of M, L and T,

$c_3 = 0$

$a_3 - 3c_3 + 1 = 0$ Thus, $a_3 = 3, c_2 - 1 = -1$

$-b_3 = 0$

Hence, $\pi_3 = \left[\dfrac{\varepsilon}{D}\right]$

Thus, $\left[\dfrac{T}{\rho \omega^2 D^5}\right] = fn\left(\left[\dfrac{\mu}{\rho D^2 \omega}\right], \left[\dfrac{\varepsilon}{D}\right]\right)$

or $T = (\rho \omega^2 D^5)\, fn\left(\left[\dfrac{\mu}{\rho D^2 \omega}\right], \left[\dfrac{\varepsilon}{D}\right]\right)$

**EXAMPLE A1.5

In a hydraulic turbine, the power developed P, under a set of controlled conditions, is found to be a function of (i) the diameter of the runner D,(ii) the width of the runner B, (iii) the rotational speed N, (iv) density of water ρ, (v) coefficient of dynamic viscosity μ, and (vi) energy per unit mass of fluid, (gH) where g = acceleration due to gravity and H = net head over the turbine. Taking (gH) as one parameter, show that the functional relationship for power can be expressed as

$$\frac{P}{\rho N^3 D^5} = F\left[\frac{D}{B}, \frac{\rho D^2 N}{\mu}, \frac{ND}{\sqrt{gH}}\right]$$

Solution

$P = fn\,(D, B, N, \rho, \mu, (gH))$

Listing the dimensions of each variable:

P	D	B	N	ρ	μ	gH
ML^2T^{-3}	L	L	T^{-1}	ML^{-3}	$ML^{-1}T^{-1}$	L^2T^2

There are five variables. $n = 7$

Two basic dimensions $m = 3$

Hence, number of dimensionless terms = $n - m = 4$.

For purposes of dimensional analysis, select ρ, D and N as repeating variables.

I *term* π_1: Consider P and the selected repeating variables ρ, D and N.

$\pi_1 = \rho^{a1} D^{b1} N^{c1} P$

$[M^o L^o T^o] = [ML^{-3}]^{a1} [L]^{b1} [T^{-1}]^{c1} [ML^{-2}T^{-3}]$

By equating the powers of M, L and T,

$a_1 + 1 = 0$ Thus, $a_1 = -1$

$-3a_1 + b_1 + 2 = 0$ Thus, $b_1 = -5$

$-c_1 - 3 = 0$ Thus, $c_1 = -3$

Hence, $\pi_1 = \left[\dfrac{P}{\rho N^3 D^5}\right]$

2[nd] *term:* π_2: Consider B and the selected repeating variables ρ, D and N.

$\pi_2 = \rho^{a2} D^{b2} N^{c2} B$

$[M^o L^o T^o] = [ML^{-3}]^{a2} [L]^{b2} [T^{-1}]^{c2} [L]$

By equating the powers of M, L and T,

$a_2 = 0$

$b_2 + 1 = 0$ Thus, $b_2 = -1$

$-c_2 = 0$

Hence, $\pi_2 = \left[\dfrac{B}{D}\right]$

3[rd] *term* ρ_3: Consider μ and the selected repeating variables ρ, D and N.

$\pi_3 = \rho^{a3} D^{b3} N^{c3} \mu$

$[M^o L^o T^o] = [ML^{-3}]^{a3} [L]^{b3} [T^{-1}]^{c3} [ML^{-1}T^{-1}]$

By equating the powers of M, L and T,

$a_3 + 1 = 0$ Thus, $a_3 = -1$

$-3a_3 + b_3 - 1 = 0$ Thus, $b_3 = -1$

$-c_3 - 1 = 0$ Thus, $c_3 = -1$

Hence, $\pi_3 = \left[\dfrac{\mu}{\rho D^2 N}\right]$

4[th] *term* π_4: Consider (gH) and the selected repeating variables ρ, D and N.

$\pi_4 = \rho^{a4} D^{b4} N^{c4} (gH)$

$[M^o L^o T^o] = [ML^{-3}]^{a4} [L]^{b4} [T^{-1}]^{c4} [ML^{-1}T^{-1}]$

By equating the powers of M, L and T,

$a_4 = 0$

$-3a_4 + b_4 + 2 = 0$ Thus, $b_4 = -2$

$-c_4 - 2 = 0$ Thus, $c_4 = -2$

Hence, $\pi_4 = \left[\dfrac{gH}{D^2 N^2}\right]$

$$F\left\{\left[\frac{P}{\rho N^3 D^5}\right],\left[\frac{B}{D}\right],\left[\frac{\mu}{\rho D^2 N}\right],\left[\frac{gH}{D^2 N^2}\right]\right\}=0$$

Taking the inverse of 2^{nd}, 3^{rd} and fourth terms and further taking the square root of the last term after taking the inverse, we get

$$\frac{P}{\rho N^3 D^5}=F\left[\frac{D}{B},\frac{\rho D^2 N}{\mu},\frac{ND}{\sqrt{gH}}\right]$$

A.2 ABBREVIATIONS

ANSI	American National Standards Institute
ASME	American Society of Mechanical Engineers
BEP	Best Efficiency Point
BER	Best Operating Range
CF	Capacity Factor
CFD	Computational Fluid Dynamics
CV	Control Volume
D	Derivative term (*D*)
ESHA	European Small Hydro Association
FDC	Flow Duration Curve
HI	Hydraulic Institute (*USA*)
I	Integral Stage (I)
IC Engine	Internal Combustion Engine
IEA	International Energy Agency
IEC	International Electrotechnical Commission
IEEE	Institute of Electrical and Electronics Engineers
IS	Indian Standard of Bureau of Indian Standards
ISO	International Organization for Standardization
M	Margin (above *NPSHR*)
MIV	Main Inlet Valve
NPSHA	Net Positive Suction Head Available
NPSHR	Net Positive Suction Head Required
P-S	Pumped Storage
PDP	Positive displacement pump
PI	Proportional Integral function controller

PID	Proportional Integral Derivative (*PID*) controllers
PTFE	Polytetrafluroethylene (DuPont brand name: Teflon)
PVDF	Polyvinylidene Fluoride
RAS	Recirculating Aquaculture Systems
RFC	Regenerative Flow Compressor
RFP	Regenerative Flow Pump
ROR	Run-of-River Plant
SHP	Small Hydro Plant
SSU	Saybolt Seconds Universal (a unit of viscosity)
USBR	*US* Bureau of Reclamation

A3 ADDITIONAL REFERENCES

This list contains books that could be used advantageously as additional reference to the text. This list can be referred to for additional information and for advanced material related to the topics covered in various chapters of the text. These references would normally be available at most engineering college libraries.

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Appendix B

B1.1 SOME IMPORTANT STANDARDS RELATED TO HYDRO TURBINES

Indian Standards	
IS 12800:Part 3:1991	Guidelines for Selection of Hydraulic Turbine, Preliminary Dimensioning and Layout of Surface Hydroelectric Powerhouses—Part 3:Small, Mini and Micro Hydroelectric Powerhouses
IS 12837:1989	Hydraulic Turbines for Medium and Large Powerhouses—Guidelines for Selection
IS 14197:1994	Code for Model Acceptance Tests of Hydraulic turbines
International Standards	
IEC 60041 Ed. 3.0 b:1991	**Field acceptance tests to determine the hydraulic performance of hydraulic turbines, storage pumps and pump turbines** "Specifies methods for any size and type of impulse or reaction turbine, storage pump or pump turbine. Determines whether the contract guarantees have been fulfilled and deals with the rules governing these tests as well as the methods of computing the results and the content and style of the final report."
IEC 60193 Ed. 2.0 b:1999	**Hydraulic turbines, storage pumps and pump turbines—Model acceptance tests**

(Contd.)

(Contd.)

	"This standard covers the arrangements for model acceptance tests to be performed on hydraulic turbines, storage pumps and pump turbines to determine if the main hydraulic performance contract guarantees have been satisfied. Further, it contains rules governing conduct of tests and prescribes measures to be taken if any phase of the tests is disputed."
IEC 60308 Ed. 2.0 b:2005	**Hydraulic turbines—Testing of control systems** "Deals with the definition and the characteristics of control systems. It is not limited to the actual controller tasks but also includes other tasks which may be assigned to a control system, such as sequence control tasks, safety and provision for the actuating energy. The following systems are included: speed, power, opening, water level and flow control for all turbine types; electronic, electrical and fluid power devices; safety devices as well as start-up and shutdown devices."
IEC 60545 Ed. 1.0 b:1976	**Guide for commissioning, operation and maintenance of hydraulic turbines** "Establishes suitable procedures for commissioning, operating and maintaining hydraulic turbines and associated equipment. Applies to impulse and reaction turbines of all types, and especially to large turbines directly coupled to electric generators. Also applies to pump turbines when operating as turbines, and water conduits, gates, valves, drainage pumps, cooling-water equipment, generators, etc., where they cannot be separated from the turbine and its equipment."
IEC 60609-1 Ed. 1.0 b:2004	**Hydraulic turbines, storage pumps and pump turbines—Cavitation pitting evaluation—** **Part 1: Evaluation in reaction turbines, storage pumps and pump turbines** "Provides a basis for the formulation of guarantees applied to cavitation pitting for reaction hydraulic turbines, storage pumps and pump turbines. It addresses the measurement and evaluation of the amount of cavitation pitting on certain specified

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	machine components for given conditions. The cavitation-pitting evaluation is based on the loss of material during a given time and under accurately defined operating conditions. All wetted surfaces are considered.”
IEC 60609-2 Ed. 1.0 b:1997	**Cavitation pitting evaluation in hydraulic turbines, storage pumps and pump turbines Part 2: Evaluation in Pelton turbines** This standard serves as a basis for the formulation of guarantees on cavitation pitting on Pelton turbine runners. It also provides a basis for the measurement and evaluation of the amount of cavitation pitting on Pelton turbine runners. Guarantees which restrict the extent of cavitation pitting and drop erosion on Pelton turbines at the end of an operating period specified in the contract are necessary when the pitting is expected in all or in some operating ranges.”
IEC 61362 Ed. 1.0 b:1998	Guide to specification of hydraulic turbine control systems.
IEC/TR 61366-1 Ed. 1.0 en:1998	Hydraulic turbines, storage pumps and pump-turbines—Tendering Documents - Part 1: General and annexes
IEC/TR 61366-2 Ed. 1.0 en:1998	Hydraulic turbines, storage pumps and pump-turbines—Tendering Documents—Part 2: Guidelines for technical specifications for Francis turbines
IEC/TR 61366-3 Ed. 1.0 en:1998	Hydraulic turbines, storage pumps and pump turbines—Tendering documents—Part 3: Guidelines for technical specifications for Pelton turbines
IEC/TR 61366-4 Ed. 1.0 en:1998	Hydraulic turbines, storage pumps and pump turbines—Tendering documents—Part 4: Guidelines for technical specifications for Kaplan and propeller turbines
IEC/TR 61366-5 Ed. 1.0 en:1998	Hydraulic turbines, storage pumps and pump turbines—Tendering documents—Part 5: Guidelines for technical specifications for tubular turbines
IEC/TR 61366-6 Ed. 1.0 en:1998	Hydraulic turbines, storage pumps and pump-turbines—Tendering documents—Part 6: Guidelines for technical specifications for pump-turbines

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B1.2 SOME IMPORTANT INDIAN STANDARDS RELATED TO PUMPS

IS 1520:1972	Horizontal Centrifugal Pumps for Clear, Cold, Fresh Water
IS1710:1973	*VT* Pumps for Clear, Cold, Fresh Water
IS 11346:2002	Tests for Agricultural and Water Supply Pumps—Code of Acceptance
IS 5600:2002	Pumps—Sewage and Drainage—Specification
IS 5639:1970	Specification for Pumps Handling Chemicals and Corrosive Liquids
IS 5659:1970	Specification for Pumps for Process Water
IS 6070:1983	Code of Practice for Selection, Operation and Maintenance of Trailer-fire Pumps, Portable Pumps, Water Tenders and Motor-fire Engines
IS 6595:Part 1:2002	Horizontal Centrifugal Pumps for Clear, Cold Water—Specification—Part 1: Agricultural and Rural Water Supply Purposes
IS 6595:Part 2:1993	Horizontal Centrifugal Pumps for Clear, Cold Water—Part 2: General Purposes other than Agricultural and Rural Water Supply—Specification
IS 8034:2002	Submersible Pumpsets—Specification
IS 8418:1999	Specification for Horizontal Centrifugal Self-Priming Pumps
IS 8472:1998	Pumps—Regenerative or Clear, Cold Water—Specification
IS 9079:2002	Electric Monoset Pumps for Clear, Cold Water for Agricultural and Water Supply Purposes—Specification
IS 9137:1978	Code for Acceptance Test for Centrifugal, Mixed Flow and Axial Pumps—Class C

B1.3 SOME IMPORTANT ASME/ANSI/HI STANDARDS RELATING TO PUMPS

ANSI/HI 14.6-2011	**Hydraulic Performance Acceptance Tests for Rotodynamic Pumps** This new standard is considered the global reference for testing centrifugal and vertical pumps. Six acceptance grades and tolerance bands have been included. Further, it contains updated test arrangement and information about measuring instruments. This

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	standard supersedes the earlier ANSI/HI 1.6-2000 and ANSI/HI 2.6-2000.
ASME B73.1M-2001 Horizontal End Suction Centrifugal Pumps for Chemical Process, Specifications for	Establishes requirements for centrifugal pumps of horizontal, end suction single stage, centreline discharge design.
ASME B73.2M-2003 Vertical In-Line Centrifugal Pumps for Chemical Process, Specifications for	Covers motor-driven centrifugal pumps of vertical shaft, single-stage design with suction and discharge nozzles in line. It includes dimensional interchangeability requirements and certain design features to facilitate installation and maintenance. It is the intent of this standard that pumps of the same standard dimension designation, from all sources of supply, shall be interchangeable with respect to mounting dimensions and size and location of suction and discharge nozzles

B1.4 SOME IMPORTANT ISO STANDARDS RELATING TO PUMPS

ISO 9906: 2012 Rotodynamic Pumps—Hydraulic Performance Acceptance Test—Grades 1 and 2	Deals with hydraulic test and contains instructions on data treatment and setting up of test equipment. Includes earlier ISO 2548 and ISO 3555.
ISO 5198: 1998 Pumps—Centrifugal, Mixed Flow and Axial Pumps—Code for Performance Tests. Precision Class.	General recommendation, measurements of rates of flow, head, speed of rotation, power input and efficiency. Also, cavitation tests, estimating and analysis of uncertainties.

APPENDIX C

Answers to Objective Questions

Chapter No.	Question No.	0	1	2	3	4	5	6	7	8	9
1	O 1.00	–	d	c	d	a	a	b	d	d	a
	O 1.10	a	a	b	b	d	d	a	c	d	d
	O 1.20	c	b	a	b	b	d	b	b	a	a
	O 1.30	b	a	b	a						
2	O 2.00	–	a	d	d	a	d	c	d	b	b
	O 2.10	b	b	d	b	a	a	c	b	c	d
	O 2.20	a	b	b	c	a	d				
3	O 3.00	–	b	a	d	b	b	d	b	a	c
	O 3.10	d	a	c	a	d	c	a	d	b	d
	O 3.20	b	a	a							
4	O 4.00	–	c	b	b	b	c	a	a	b	c
	O 4.10	d	a	d	a	d	a	a	b	b	b
	O 4.20	c	d	c	d						
5	O 5.00	–	a	b	c	b	c	b	a	d	c
	O 5.10	d	b	d	a	b	b	a	c	d	c
	O 5.20	d	d	b	b	c	b	b	b	b	d
	O 5.30	d	b	d	c	a	a	b	c	c	b
	O 5.40	b	c	a	b	c	b				

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Chapter No.	Question No.	0	1	2	3	4	5	6	7	8	9
6	O 6.00	–	c	b	a	b	c	c	b	c	c
	O 6.10	b	c	c	b	b	c	c	c	b	a
	O 6.20	c	b	a	c	a	d				
7	O 7.00	–	a	d	c	c	c	b	d	b	a
	O 7.10	a	a	c	d	a	d	b	b	a	d
	O 7.20	a	a	a	c	b	d	d	c	a	a
8	O 8.00	–	a	d	b	c	c	d	c	c	b
	O 8.10	b	c	b	b	d	d	b	a	b	b

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