Repairable Systems Reliability Analysis

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Performability Engineering Series
Series Editors: Krishna B. Misra (kbmisra@gmail.com)

Scope: A true performance of a product, or system, or service must be judged over the entire life cycle activities connected with design, manufacture, use and disposal in relation to the economics of maximization of dependability, and minimizing its impact on the environment. The concept of performability allows us to take a holistic assessment of performance and provides an aggregate attribute that reflects an entire engineering effort of a product, system, or service designer in achieving dependability and sustainability. Performance should not just be indicative of achieving quality, reliability, maintainability and safety for a product, system, or service, but achieving sustainability as well. The conventional perspective of dependability ignores the environmental impact considerations that accompany the development of products, systems, and services. However, any industrial activity in creating a product, system, or service is always associated with certain environmental impacts that follow at each phase of development. These considerations have become all the more necessary in the 21st century as the world resources continue to become scarce and the cost of materials and energy keep rising. It is not difficult to visualize that by employing the strategy of dematerialization, minimum energy and minimum waste, while maximizing the yield and developing economically viable and safe processes (clean production and clean technologies), we will create minimal adverse effect on the environment during production and disposal at the end of the life. This is basically the goal of performability engineering.

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Publishers at Scrivener
Martin Scrivener (martin@scrivenerpublishing.com)
Phillip Carmical (pcarmical@scrivenerpublishing.com)

Repairable Systems Reliability Analysis

A Comprehensive Framework

Rajiv Nandan Rai

Indian Institute of Technology Kharagpur, Kharagpur, India

Sanjay Kumar Chaturvedi

Indian Institute of Technology Kharagpur, Kharagpur, India

Nomesh Bolia

Indian Institute of Technology Delhi, Delhi, India

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Series Editor Preface

This is the 10th book in the series on performability engineering since the series was launched in 2014. The subject of this book is special as not many books on Repairable System Reliability are available in the literature on reliability engineering. All the three authors of this book come from the reputed academic institutes of technology in India, but have also have had rich experience of working on field projects of practical importance. Incidentally, the two of the authors, namely, Rajiv Nandan Rai and Sanjay Kumar Chaturvedi are the postgraduate and doctorate, respectively, of the first Centre of Reliability Engineering established in India by the series Editor in 1983 at the Indian institute of Technology, at Kharagpur. Rajiv Nandan Rai has also served with the Indian Air Force and has had about 20 years of industrial experience in military aviation, which is reflected in the treatment of the subject.

Actually, the research in repairable systems reliability is limited and very few textbooks are available on the subject. The available textbooks generally provide coverage of non-homogeneous Poisson process (NHPP) where the repair effectiveness index (REI) is considered one. Few more textbooks provide treatment of non-parametric reliability analysis of repairable systems. However, this book aims to provide a comprehensive framework for the analysis of repairable systems considering both the non-parametric and parametric estimation of the failure data. The book also provides discussion of generalized renewal process (GRP) based arithmetic reduction of age (ARA) models along with its applications to repairable systems data from aviation industry.

Repair actions in military aviation may not fall under 'as good as new (AGAN)' and 'as bad as old (ABAO)' assumptions which often find limited uses in practical applications. But actual situation could lie somewhere between the two. A repairable system may end up in one of the five likely states subsequent to a repair: (i) as good as new, (ii) as bad as old, (iii) better than old but worse than new, (iv) better than new, and lastly, (v) worse than old. Existing probabilistic models used in repairable system analysis, such as the perfect renewal process (PRP) and the non-homogeneous Poisson process (NHPP), account for the first two states. In the concept of imperfect repair, the repair actions are unable to bring the system to as good as new state, but can transit to a stage that is somewhere between new and that of one preceding to a failure. Because of the requirement to have more precise analyses and predictions, the GRP can be of great interest to reduce the modelling ambiguity resulting from the above repair assumptions. The authors have discussed to a great extent various possibilities under repairability environment and applied them to physical systems. The book also summarizes the models and approaches available in the literature on the analysis of repairable system reliability.

It is expected the book will be very useful to all those who are designing or maintaining repairable systems.

Krishna B. Misra
Series Editor

Preface

Conventionally, a repair action usually is assumed to renew a system to its "as good as new" condition. This assumption is very unrealistic for probabilistic modeling and leads to major distortions in statistical analysis. Most of the reliability literature is directed toward non-repairable systems, that is, systems that fail are discarded. This book is mainly dedicated toward providing coverage to the reliability modeling and analysis of repairable systems that are repaired and not replaced when they fail.

During his journey in the military organization, the first author realized that most of the industries desire to equip its scientists, engineers, and managers with the knowledge of reliability concepts and applications but have not been able to succeed completely. Repairable systems reliability analysis is an area where the research work is quite limited and very few text books are available. The available text books are also limited in providing a coverage only up to the concepts of non-homogeneous Poisson process (NHPP) where the repair effectiveness index (REI) is considered one. Few more textbooks provide knowledge only on non-parametric reliability analysis on repairable systems.

This book provides a comprehensive framework for the modeling and analysis of repairable systems considering both the non-parametric and parametric estimation of the failure data. The book provides due exposure to the generalized renewal process (GRP)–based arithmetic reduction of age (ARA) models along with its applications to repairable systems data from aviation industry. The book also covers various multi-criteria decision making (MCDM) techniques, integrated with repairable systems reliability analysis models to provide a much better insight into imperfect repair and maintenance data analysis. A complete chapter on an integrated framework for procurement process is added which will of a great assistance to the readers in enhancing the potential of their respective organization. It is intended to be useful for senior undergraduate, graduate, and post-graduate students in engineering schools as also for professional engineers, reliability administrators, and managers.

This book has primarily emerged from the industrial experience and research work of the authors. A number of illustrations have been included to make the subject pellucid and vivid even to the readers who are new to this area. Besides, various examples have been provided to showcase the applicability of presented models and methodologies, besides, to assist the readers in applying the concepts presented in the book.

The concepts of random variable and commonly used discrete and continuous probability distributions can readily be seen in various available texts that deal with reliability analysis of non-reparable systems. The reliability literature is in plenty to cover such aspects in reliability data analysis where the failure times are modeled by appropriate life distributions. Hence, the readers are advised to refer to any such text book on non-repairable systems reliability analysis for a better comprehension of this book.

Chapter 1 presents various terminologies pertaining to repairable systems followed by the description of repair concepts and repair categories.

The mean cumulative function (MCF)–based graphical and non-parametric methods for reparable systems are simple yet powerful option available to analyze the fleet/system events recurrence behavior and their recurrence rate. Chapter 2 is dedicated to MCF-based non-parametric analysis through examples with a case study of remotely operated vehicle (ROV).

The renewal and homogeneous Poisson processes (HPPs) followed by an exhaustive description of NHPP are covered in Chapter 3 along with solved examples. Thereafter, the chapter brings out a detailed description of ARA and ARI models along with their applicability in maintenance. The chapter also derives the maximum likelihood estimators (MLEs) for Kijima virtual age models with the help of GRP. The models are demonstrated with the help of suitable examples.

Chapter 4 provides various goodness-of-fit (GOF) tests for repairable systems and their applications with examples.

Chapter 5 presents various reliability and availability-based maintenance models for repairable systems. This chapter introduces the concept of high failure rate component (HFRC)–based thresholds and provides maintenance models by considering the "Black Box" (BB) approach followed by the failure mode (FM) approach. All the models are well-supported with examples.

This book presents modified failure modes and analysis (FMEA) model in Chapter 6. This model is based on the concept of REI propounded by Kijima and is best applicable to the repairable systems reliability analysis.

Chapter 7 provides an integrated approach for weapon procurement systems for military aviation. The combined applications of MCDM tools like AHP, ANP, and optimization techniques can be seen in this chapter. This model can be used for other industries procurement policy as well.

Chapter 8 is aimed at reducing the overhaul time of a repairable equipment to enhance the availability. Various concepts of throughput analysis have been utilized in this chapter.

The book makes an honest attempt to provide a comprehensive coverage to various models and methodologies that can be used for modeling and analysis of repairable systems reliability analysis. However, there is always a scope for improvement and we are looking forward to receiving critical reviews and/or comments of the book from students, teachers, and practitioners. We hope that the readers will all gain as much knowledge, understanding, and pleasure from reading this book as we have from writing it.

Rajiv Nandan Rai
Sanjay Kumar Chaturvedi
Nomesh Bolia
August 2020

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Introduction to Repairable Systems

1.1 Introduction

A system is a collection of mutually related items, assembled to perform one or more intended functions. Any system majorly consists of (i) items as the operating parts, (ii) attributes as the properties of items, and (iii) the link between items and attributes as interrelationships. A system is not only expected to perform its specified function(s) under its operating conditions and constraints but also expected to meet specified requirements, referred as performance and attributes. The system exhibits certain behavioural pattern that can never ever be exhibited by any of its constituent items or their subsets. The items of a system may themselves be systems, and every system may be part of a larger system in a hierarchy. Each system has a purpose for which items, attributes, and relationships have been organized. Everything else that remains outside the boundaries of system is considered as environment from where a system receives input (in the form of material, energy, and/or information) and makes output to the environment which might be in different form as that of the input it had received. Internally, the items communicate through input and output wherein output(s) of one items(s) becomes the input(s) to others. The inherent ability of an item/system to perform required function(s) with specified performance and attributes when it is utilized as specified is known as *functionability* [1]. This definition differentiates between the terms functionality and functionability where former is purely related to the function performed whereas latter also takes into considerations the level of performance achieved.

Despite the system is functionable at the beginning of its operational life, we are fully aware that even after using the perfect design, best technology available for its production or the materials from which it is made, certain irreversible changes are bound to occur due to the actions of various interacting and superimposing processes, such as corrosion, deformations, distortions, overheating, fatigue, or similar. These interacting processes are the

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main reason behind the change in the output characteristics of the system. The deviation of these characteristics from the specifications constitutes a *failure*. The failure of a system, therefore, can be defined as an event whose occurrence results in either loss of ability to perform required function(s) or loss of ability to satisfy the specified requirements (i.e., performance and/or attributes). Regardless of the reason of occurrence of this change, a failure causes system to transit from a state of functioning to a state of failure or state of unacceptable performance. For many systems, a transition to the unsatisfactory or failure state means retirement. Engineering systems of this type are known as *non-maintained* or *non-reparable* system because it is impossible to restore their functionability within reasonable time, means, and resources. For example, a missile is a non-repairable system once launched. Other examples of non-repairable systems include electric bulbs, batteries, transistors, etc. However, there are a large number of systems whose functionability can be restored by effecting certain specified tasks known as *maintenance tasks*. These tasks can be as complex as necessitating a complete overhaul or as simple as just cleaning, replacement, or adjustment. One can cite several examples of repairable systems one's own day-to-day interactions with such systems that include but not limited to automobiles, computers, aircrafts, industrial machineries, etc. For instance, a laptop, not connected to an electrical power supply, may fail to start if its battery is dead. In this case, replacing the battery—a non-maintained item—with a new one may solve the problem. A television set is another example of a repairable system, which upon failure can be restored to satisfactory condition by simply replacing either the failed resistor or transistor or even a circuit board if that is the cause, or by adjusting the sweep or synchronization settings.

The system, in fact, wavers and stays between satisfactory and unsatisfactory states during its operational life until a decision is taken to dispense with it. The proportion of the time, during which the system is functionable, depends on the interaction between the inherent characteristics of a system from the design and utilization function given by the users' specific requirements and actions. The prominent inherent characteristics could be *reliability*, *maintainability*, and *supportability*. Note that these characteristics are directly related to the frequency of failures, the complexity of a maintenance task, and ease to support that task. The utilization characteristics are driven by the users' operational scenarios and maintenance policy adopted, which are further supported by the logistics functions, which is related to the provisioning of operational and maintenance resources needed. In short, the pattern followed by an engineering system can be termed as funtionability profile whose specific shape is governed by the inherent characteristics of design and system's utilization. The metric

Availability or its variants quantitatively summarize the functionability profile of an item/system. It is an extremely important and useful measure for reparable systems; besides, a technical aid in the cases where user is to make decisions regarding the acquisition of one item among several competing possibilities with differing values of reliability, maintainability, and supportability. Functionability and availability brought together indicates how good a system is. It is referred as *system technical effectiveness* representing the inherent capability of the system. Clearly, the biggest opportunity to make an impact on systems' characteristics is at the design stage to won or lost the battle when changes and modifications are possible almost at negligible efforts. Therefore, the biggest challenge for engineers, scientists, and researchers has been to assess the impact of the design on the maintenance process at the earliest stage of the design through field experiences, analysis, planning and management. And, the repairable system analysis is not just constricted on finding out the reliability metrics.

Most complex systems, such as automobiles, communication systems, aircraft, engine controllers, printers, medical diagnostics systems, helicopters, train locomotives, and so on so forth are repaired once they fail. In fact, when a system enters into utilization process, it is exposed to three different performance influencing factors, *viz.*, operation, maintenance, and logistics, which should be strategically managed in accordance with the business plans of the owners. It is often of considerable interest to determine the reliability and other performance characteristics under these conditions. Areas of interest may include assessing the expected number of failures during the warranty period, maintaining a minimum reliability for an interval, addressing the rate of wear out, determining when to replace or overhaul a system, and minimizing its life cycle costs.

Traditional reliability life or accelerated test data analysis—nonparametric or parametric—is based on a truly random sample drawn from a single population and independent and identically distributed (*i.i.d.*) assumptions on the reliability data obtained from the testing/fielded units. This *i.i.d.* assumption may also be valid, intuitively, on the first failure of several identical units, coming from the same design and manufacturing process, fielded in a specified or assumed to be in an identical environment. Life data of such items usually consists of an item's single failure (or very first failure for reparable items) times with some items may be still surviving-referred as censoring or suspension. The reliability literature is in plenty to cover such aspects in reliability data analysis where the failure times are modeled by appropriate life distributions [2].

However, in repairable system, one generally has times of successive failures of a single system, often violating the i.i.d assumption. Hence, it is not surprising that statistical methods required for repairable system differ from those needed in reliability analysis of non-repairable items. In order to address the reliability characteristics of complex repairable systems, a process rather than a distribution is often used. For a repairable system, time to next failure depends on both the life distribution (the probability distribution of the time to first failure) and the impact of maintenance actions performed after the first occurrence of a failure. The most popular process model is the Power Law Process (PLP). This model is popular for several reasons. For instance, it has a very practical foundation in terms of *minimal repair*—a situation when the repair of a failed system is just enough to get the system operational again by repair or replacement of its constituent item(s). Second, if the time to first failure follows the Weibull distribution, then the Power Law model repair governs each succeeding failure and adequately models the minimal repair phenomenon. In other words, the Weibull distribution addresses the very first failure and the PLP addresses each succeeding failure for a repairable system. From this viewpoint, the PLP can be regarded as an extension of the Weibull distribution and a generalization of Poisson process. Besides, the PLP is generally computationally easy in providing useful and practical solutions, which have been usually comprehended and accepted by the management for many real-world applications.

The usual notion and assumption of overhauling of a system is to bringing it back to "as-good-as-new" (AGAN) condition. This notion may not be true in practice and an overhaul may not achieve the system reliability back to where it was when the system was new. However, there is concurrence among all the stakeholders that an overhaul indeed makes the system more reliable than just before its overhaul. For systems that are not overhauled, there is only one cycle and we are interested in the reliability characteristics of such systems as the system ages during its operational life. For systems that are overhauled several times during their lifetime, our interest would be in the reliability characteristics of the system as it ages during its cycles, i.e., the age of the system starts from the beginning of the cycle and each cycle starts with a new zero time.

1.2 Perfect, Minimal, and Imperfect Repairs

As discussed earlier, a repairable system is a system that is restored to its functionable state after the loss of functionability by the actions other than replacement of the entire system. The quantum of repair depends upon various factors like criticality of the component failed, operational status of the system, risk index, etc. Accordingly, the management takes a decision on how much repair a system has to undergo. The two extremes of the repair are perfect and minimal repairs. A system is said to be perfectly repaired, if the system is restored to AGAN condition (as it is replaced with a new one). Normally, a perfect repair in terms of the replacement is carried out for very critical components, which may compromise operation ability, safety of the system, and/or personnel working with the system. On the other hand, a system is said to be minimally repaired, if its working state is restored to "as-bad-as-old" (ABAO). This type of repair is undertaken when there is heavy demand for the system to work for a finite time or the system will be undergoing preventive maintenance shortly or will be scrapped soon.

Any repair other than perfect and minimal repair comes under imperfect repair. Most of the repairs observed in *day-to-day* systems are imperfect repairs, i.e., a system is neither restored to AGAN conditions nor to ABAO conditions. The three types of repairs are pictorially represented in Figure 1.1.

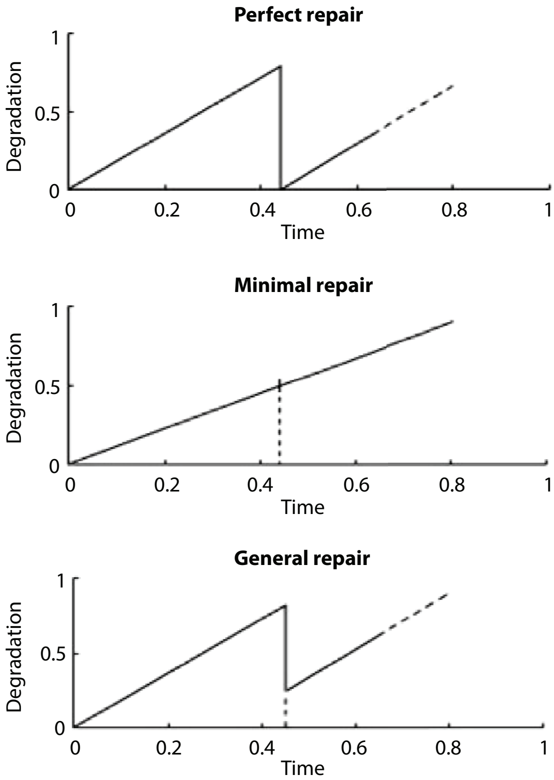

Figure 1.1 Types of repair.

It can be seen from Figure 1.1 that in case of perfect repair, the system is rendered "AGAN" and the life starts at zero in the time scale signifying that the performance degradation is completely restored. In case of minimal repair, after the system is subjected to a repair action, its age remains same as before the repair action and there is no restoration of life below the previous age. So far the general repair is concerned, some of its life is renewed and the system starts functioning after being restored to somewhere between "ABAO" and "AGAN" state.

Figure 1.2 summarizes the techniques in vogue for reliability analysis for both repairable and non-repairable items, respectively.

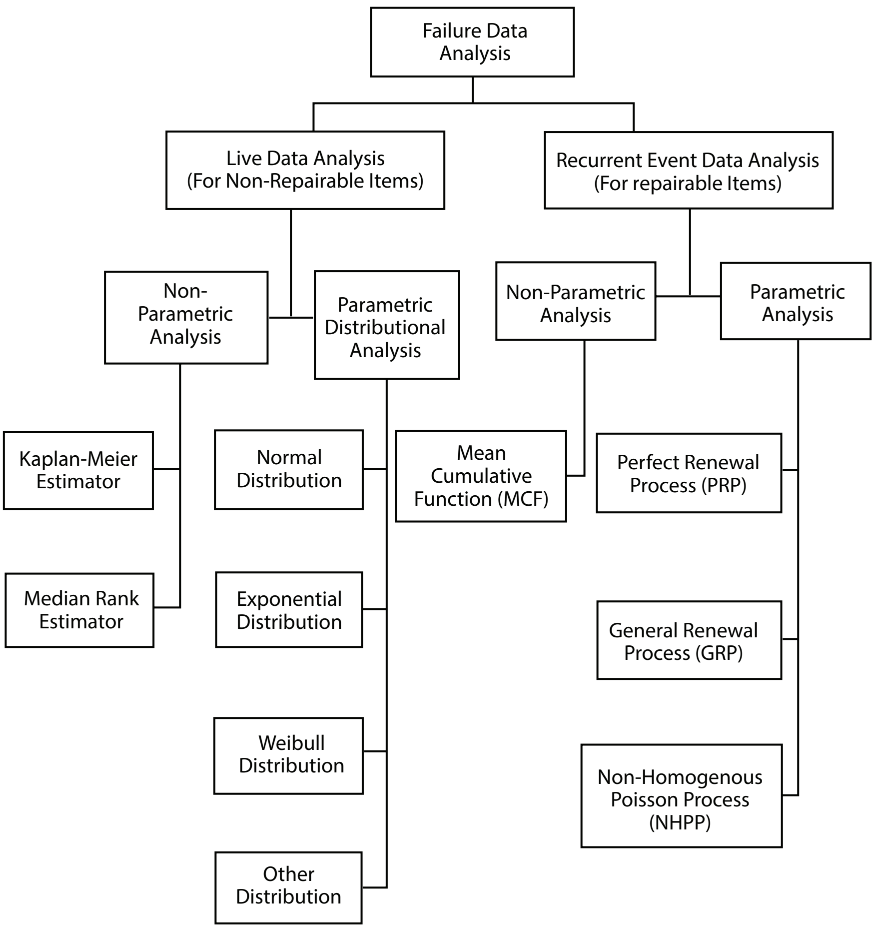

Figure 1.2 Various techniques for reliability analysis.

A *renewal process* (RP) is a counting process where the *inter-occurrence times* are stochastically *i.i.d.* with an arbitrary life distribution. Under the RP, a single distribution can characterize the time between failures (TBF), and the frequency of repair appears constant. The non-renewal behavior occurs if this frequency of repairs increases (deteriorating systems) or decreases (system improving) influencing the corresponding maintenance costs. The *homogeneous Poisson process* (HPP) describes a sequence of statistically *i.i.d.* exponential random variables. Conversely, a non-homogeneous Poisson process (NHPP) [3, 4] describes a sequence of random variables that are neither statistically independent nor identically distributed. The NHPP is often used to model repairable systems that are subject to minimal repair. The generalized renewal process (GRP) allows the goodness of repairs within two extremities, *viz.*, AGAN repair (RP) to the same-as-old repair (NHPP). The GRP is particularly useful in modeling the failure behavior of a specific unit and understanding the effects of repair actions on the age of that system. An example of a system to which the GRP is especially applicable is a system, which is repaired after a failure and whose repair neither brings the system to an AGAN or an ABAO condition, but instead partially rejuvenates the system. Therefore, one should be cautious on the fact that without looking at the actual behavior of the data may lead to underestimation or overestimation of engineering metrics.

The analysis by employing the parametric methods on scenarios of failure-repair requires a certain degree of statistical knowledge, the ability to solve complex equations and verification of distributional assumptions. Further, these equations cannot be solved analytically and require an iterative procedure or special software. Besides, parametric approaches are computationally intensive and not intuitive to a novice or an average person. The analysis of events, irrespective of the nature of the system-reparable or not reparable, should take an analysis path from non-parametric to versatile parametric model with graphical analysis being a common denominator. Undoubtedly, the choice of method depends on the data available and the questions we wish to answer.

1.3 Summary

A repairable system is a system that is restored to its functionable state after the loss of functionability by the actions other than replacement of the entire system. The two extremes of the repair are perfect and minimal repairs. A system is said to be perfectly repaired, if the system is restored to AGAN condition (as it is replaced with a new one). On the other hand, a system is said to be minimally repaired, if its working state is restored to ABAO. Any repair other than perfect and minimal repair comes under imperfect repair. Most of the repairs observed in *day-to-day* systems are imperfect repairs, i.e., a system is neither restored to AGAN conditions nor to ABAO conditions. The RP is used to model AGAN condition. The NHPP is often used to model repairable systems that are subject to minimal repair. The GRP allows the goodness of repairs within two extremities, *viz.*, AGAN repair (RP) to the same-as-old repair (NHPP).

The next chapter describes the mean cumulative function (MCF) based graphical and non-parametric methods for repairable systems.

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2

Repairable Systems Reliability Analysis: Non-Parametric

2.1 Introduction

Many products experience repeated repairs that requires special statistical treatment with respect to formulating methods and models for analysis. The items are usually considered statistically independent, but the times between the occurrences of failure or repair events within a system unit are neither necessarily independent nor identically distributed. The data are usually censored in the sense that system units have different ends of operational histories. A distributional analysis might also be possible for the systems' which undergo a series of failure/repair cycles, on their very first observed failure or if their times-between-failures (TBF) show no trends. However, if a series of multiple failure or repair events, occurring sequentially in time continuum, are to be considered for analysis then order of the failure event's occurrence does matter and ignorance would lead to incorrect analysis and decisions thereof [1, 2]. Since the collection of random variables involved in such systems evolve with time, their behavior is generally modeled through a process rather than a distribution. In reliability engineering, the systems under this category are commonly referred as repairable systems, i.e., systems that are brought to their normal functionable states by means of any minor or major maintenance action(s). Therefore, when assessing and analyzing the system reliability, it is always important to make the distinction between non-repairable components and repairable systems to select an appropriate approach.

For a company or for a competing manufacturer, the common concerns can be [3, 4]:

- The number of repairs, on average, for all system at a specified operational time?
- Expected time to first repair, subsequent repairs, etc.

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- Trends in repair rate or costs whether increasing, decreasing, or substantially constant.
- How to take decisions on burn-in requirements and maintenance or retirement.
- Is burn-in would be beneficial? How long and costs effective it would be?
- How to compare different versions, designs, or performance of systems operating in different environment/ regions, etc.

Each of the above questions and many more can be answered by the method presented in this chapter. This chapter describes mean cumulative function (MCF) based non-parametric graphical approach—a simple yet powerful and informative model—to deal with failure events of systems undergoing a failure and repair cycles wherein the time to effect a repair is assumed to be negligible. This is a reasonable assumption with respect to the operational times of a unit that are usually long than its repair times.

The MCF approach is simple as it is easy to understand, prepare, and present the data. The MCF model is non-parametric in the sense that its estimation involves no assumptions about the form of the mean function or the process generating the system histories. Further, this graphical method allows the monitoring of system recurrences and the maintenance of statistical rigidity without resorting to complex stochastic techniques.

Further, the non-parametric MCF analysis provides similar information as probability plots in a traditional life data analysis. Especially, the plot of the nonparametric estimate of the population MCF yields most of the information sought and the plot is as informative as is the probability plot for life and other univariate data. The sample MCF can be estimated and plot can be constructed for just one single machine or for an entire fleet of machines in a population. Besides, it can also be constructed for all failures events, outages, system failures due to specific failure modes, etc. It can be used to track field recurrences and identify recurrence trends, anomalous systems, unusual behavior, the effect of various parameters (e.g., maintenance policies, environmental, and operating conditions, etc.) on failures, etc. For some situations, this is the only analysis we need to do, and in others, it becomes a precursor to a more advance parametric form of analysis.

2.2 Mean Cumulative Function

A common and popular reliability metric of a repairable system is the cumulative number of failure or repair events, $N(t)$, occurring by time t (also termed as system age). Here, age (or time) means any measure of item's usage, e.g., millage, kilometers, cycles, months, days, and so on. An item's latest observed age is called its suspension or censored age beyond which its history is yet to be observed. A sample item may also not have observed a single failure whereas others have observed one, two, or more failures before its suspension age. Obviously, the number of events occurring by time is random. In non-parametric failure (or repair) events data analysis, every unit of the population can be described by a cumulative history function for the number of failures. The population average of cumulative number of failures (or repairs) at through time t is called Mean Cumulative Function (MCF), $M(t)(= E(N(t)))$. Its derivative, $m(t) = \frac{dM(t)}{dt}$, is assumed to exist and is termed as recurrence rate or intensity function or population instantaneous repair rate at time t. This rate may remain constant, increasing or decreasing in its characteristics and is expressed in terms of occurrences per unit population item per unit time. It is a staircase function with a jump at each event occurrence in time and tracking the accumulated number of events by time t of interest. It is the mean of all staircase functions of every unit in the population.

Figure 2.1 shows an example of the cumulative history function for a single system. The graph on cumulative number of failure events versus

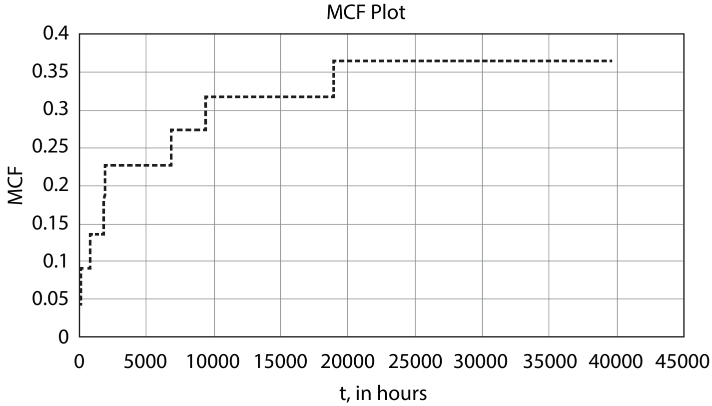

Figure 2.1 A MCF example.

the system age, t. This graph can be viewed as a single observation from a possible curve.

Let us take an example to illustrate the behavior of three units of identically designed systems operating under different environments or maintenance scenarios through MCF.

Example 2.1

Consider of three reparable systems observed until the time of their 12^{th} failure the failure data [5].

System 1: 3, 9, 20, 25, 41, 50, 69, 91, 128, 151, 182, and 227.

System 2: 9, 20, 65, 88, 104, 107, 138, 143, 149, 186, 208, and 227.

System 3: 45, 76, 113, 129, 152, 174, 193, 199, 210, 219, 224, and 227.

Figures 2.2, 2.3, and 2.4, respectively, show the graphs of $N(t_i)$ versus t_i.

It is evident from the plots in Figures 2.2 to 2.4 that the three identical systems are behaving differently wherein the repair rates of these systems show decreasing (an improving system), linear (a stable system), and increasing (a deteriorating system) trends, respectively. In the above, the actual number of events by time t provides an unbiased estimate of the population mean number of failures (or repairs) per system, $M(t)$. This is valid in a situation where only a single system is available at the time of analysis and other systems would appear in future when this single system's design is acceptable and satisfied the intended requirements.

However, in many cases, we are concerned with an overall behavior and performance of several systems manufactured through identical processes

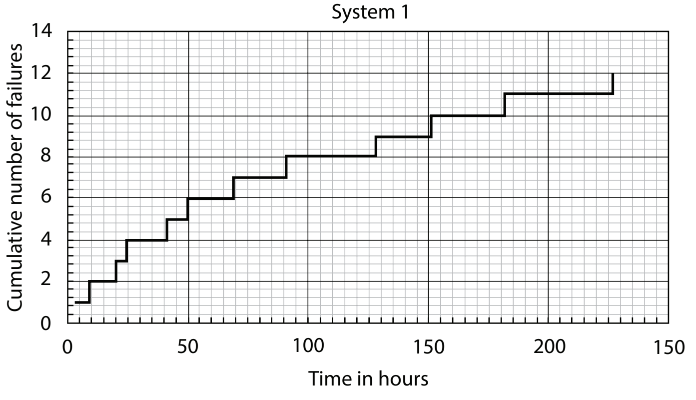

Figure 2.2 An improving system.

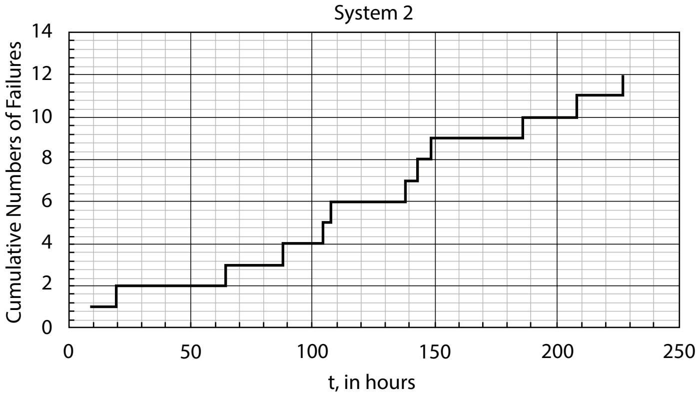

Figure 2.3 A stable system.

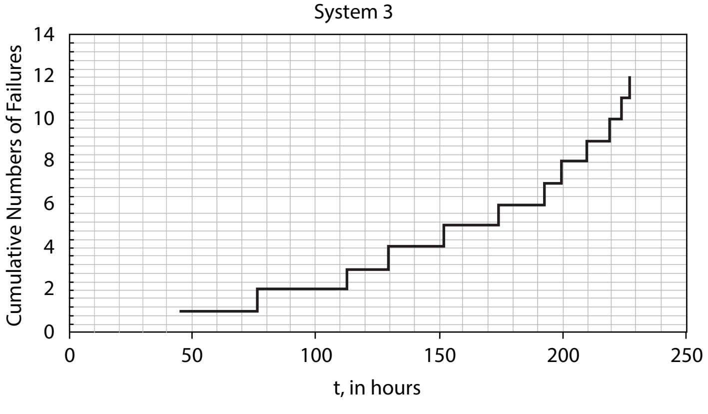

Figure 2.4 A deteriorating system.

and sub items. Even the deployment time of these systems differ and they will have differing failure histories and operating times. Figure 2.5 shows an example of the MCF of a sample of a fleet of randomly selected similar repairable systems from a population.

Note that the intersection of the staircase functions with the vertical line passing through an age t_i creates a distribution for the values of cumulative number of events, wherein a fraction of the population has accumulated no failure, another fraction has accumulated one failure, yet another fraction has accumulated two failures, and so on. This distribution differs at different ages and has a mean $MCF(t_i)$ at age t_i. Thus, the MCF plot is a pointwise average of all population curves passing through the vertical line at each age t_i as can be seen in Figure 2.5.

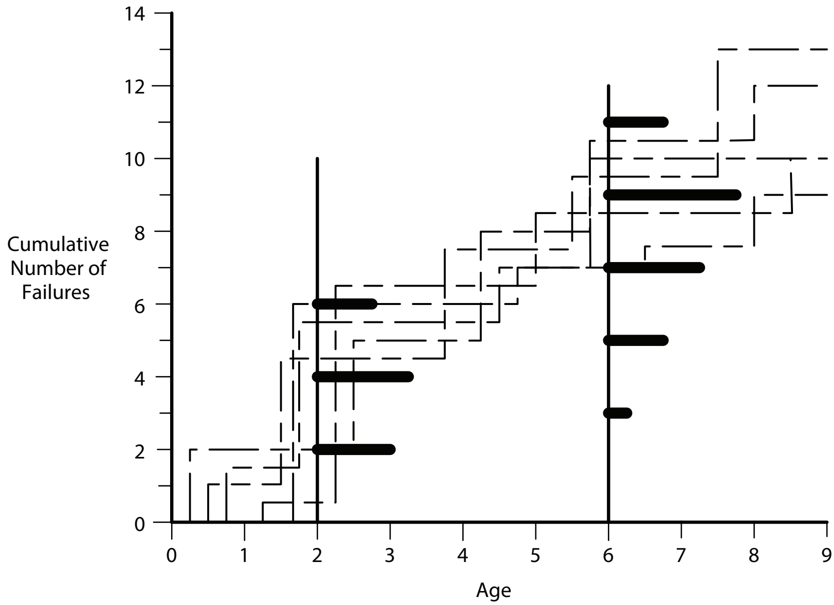

Figure 2.5 History and distribution of failures observed at age t.

Apart from the above type of failure histories, there may exist the data in plenty or in a manner wherein the grouping of the data is viable enough. It is a usually a standard practice of resorting to grouping the data when the sample size is large enough as no significant information is lost in statistical sense if the data is grouped into different intervals—equal or unequal. We will consider both form of data and their subsequent analysis approach later in detail. Let us describe procedural steps of MCF construction [4].

2.3 Construction of MCF Plot and Confidence Bounds: Exact Age Data

2.3.1 MCF Construction: Exact Age Data

The sample MCF can be estimated and its plot can be constructed in a very straightforward manner by following these three simple steps:

1. Rank order all failure and censored times, i.e., sorted from shortest to longest by pooling the failure history of *n* number

of randomly selected systems drawn from a population of such systems. If a failure time for a unit is the same as its censoring time, the failure time is ordered first. In case of multiple units have a common recurrence or censoring age, then follow an arbitrary order.

2. Calculate the number of units, r_i, that have observed life t_i just prior to the occurrence of failure event or number of units after the suspensions occur, i.e.,

$$r_i = \begin{cases} r_{i-1}, & \text{if } t_i \text{ is the age of event recurrence} \\ r_i - 1, & \text{if } t_i \text{ is the age of a suspension,} \\ & \text{with } r_0 = n \text{ at the first observed age} \end{cases}$$

3. The MCF is calculated for each sample recurrence age t_i, as

$$MCF_i = \frac{1}{r_i} + MCF_{i-1} \text{ with } MCF_0 = \frac{1}{r_0}$$

The above procedure is illustrated through following examples.

Example 2.2. [6]
An oil refinery company maintains a rotating machine and wants to make projections about the expected cumulative number of failures for 100 similar rotating machines after 4 years (equal to 35,040 hours) of operation. Failure and suspension data, in hours, for a random sample of 22 machines were collected. Every time a machine fails, a crew repairs the machine to put back again in service. Table 2.1 gives the failure and censoring ages for each machine, where + sign indicates a censored age.

The MCF estimates by following the procedural steps is given in columns 4 to 6 of Table 2.2 and MCF plot is shown in Figure 2.6. The computation of confidence bounds given in the last column #8 of Table 2.2 is explained later.

Looking at MCF plot in Figure 2.6, the company can expect the number of failures per machine after years of operation is, $M(35040) = 0.3658$. Therefore, out of the 100 machines, about 37 failures are expected to occur. It can be observed that if a smooth curve is drawn through the MCF plot, it would have its derivative decreasing in nature, i.e., repair rate decreases as the machine ages and it improves with use. This behavior shown can either be by a typical

Table 2.1 Rotating machine failure data of Example 2.2.

Machine ID#	**t_i (Hours)**
1.	942,435,659+
2.	3,718,412+
3.	64,192,039,779+
4.	70,734,213+
5.	1,898,029,016+
6.	185,128,177+
7.	28,535+
8.	29,168+
9.	679,224,304+
10.	28,921+
11.	27,853+
12.	29,981+
13.	25,761+
14.	28,780+
15.	24,901+
16.	31,360+
17.	23,940+
18.	26,009+
19.	32,236+
20.	30,472+
21.	23,792+
22.	30,183+

Table 2.2 MCF and confidence calculations.

Machine ID# (1)	t_i (2)	State (3)	r_i (4)	$\frac{1}{r_i}$ (5)	$MCF(t_i)$ (6)	$Var(MCF(t_i))$ (7)	$MCF(t_i)$ (8) UB	LB
2	37	F	22	0.0455	0.0455	0.0020	0.1590	0.0130
3	64	F	22	0.0455	0.0909	0.0039	0.2204	0.0375
4	707	F	22	0.0455	0.1364	0.0059	0.2810	0.0662
6	1,851	F	22	0.0455	0.1818	0.0079	0.3400	0.0972
3	1,920	F	22	0.0455	0.2273	0.0099	0.3979	0.1298
9	6,792	F	22	0.0455	0.2727	0.0118	0.4547	0.1636
1	9,424	F	22	0.0455	0.3182	0.0138	0.5107	0.1982
2	18,412	S	21					
5	18,980	F	21	0.0476	0.3658	0.0160	0.5695	0.2350
21	23,792	S	20					
17	23,940	S	19					
9	24,304	S	18					
15	24,901	S	17					
13	25,761	S	16					
18	26,009	S	15					
11	27,853	S	14					
6	28,177	S	13					
7	28,535	S	12					
14	28,780	S	11					
10	28,921	S	10					
5	29,016	S	9					

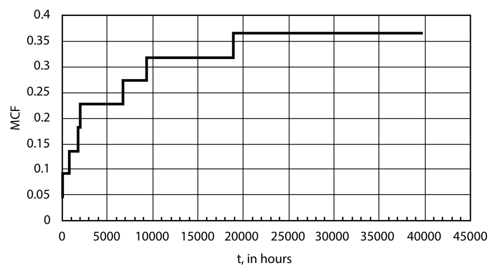

Figure 2.6 MCF plot of Example 2.2.

(*Continued*)

Table 2.2 MCF and confidence calculations. (*Continued*)

Machine ID# (1)	t_i (2)	State (3)	r_i (4)	$\frac{1}{r_i}$ (5)	$MCF(t_i)$ (6)	$Var(MCF(t_i))$ (7)	$MCF(t_i)$ (8) UB	LB
8	29,168	S	8					
12	29,981	S	7					
22	30,183	S	6					
20	30,472	S	5					
16	31,360	S	4					
19	32,236	S	3					
4	34,213	S	2					
1	35,659	S	1					
3	39,779	S	0					

product with manufacturing defects corrected gradually or the effectiveness of the maintenance activity or improved learning curve of the operator.

Example 2.3. [6]

An electronics company subjects its products to a burn-in[1] program to reduce the occurrences for failures and the need for repairs in the field. Every produced unit is run, repaired upon failure and placed back into the test until the population's (instantaneous) recurrence rate decreases to a desired value m_{goal}. What would be the an appropriate burn-in period t_b that would achieve the recurrence rate to a desired value, m_{goal}, equal to 1 failure per 50,000 cycles?

The collected recurrent failures data and the sample *MCF* are shown in Table 2.3. Recall that the derivative, $m(t)=\frac{dM(t)}{dt}$, is termed as recurrence rate or intensity function or population instantaneous repair rate at time t. Therefore, t_b can be estimated from the sample *MCF* by moving a straight line with slope m_{goal}, until it becomes tangent to the smoothed *MCF* plot. The corresponding age at the tangent point determines the appropriate burn-in period. It can be done manually by fitting a smoothed curve and finding the points where its slope becomes 1/50,000.

[1] This type of burn-in program is different from many burn-in programs in which failed units are disregarded and only the units that survive the burn-in are shipped to customers.

Table 2.3 Example 2.3—Burn-in period determination.

Product_ID (1)	**S. No.** (2)	t_i (3)	**State** (4)	r_i (5)	$1/r_i$ (6)	$MCF(t_i)$ (7)
2	1	28	F	17	0.0588	0.0588
3	2	48	F	17	0.0588	0.1176
4	3	530	F	17	0.0588	0.1765
7	4	1,388	F	17	0.0588	0.2353
3	5	14,40	F	17	0.0588	0.2941
7	6	1,455	S			0.2941
1	7	7,068	F	16	0.0625	0.3566
16	8	8,250	F	16	0.0625	0.4191
2	9	13,809	S			0.4191
6	10	14,235	S			0.4191
15	11	14,281	S			0.4191
10	12	19,175	S			0.4191
17	13	19,250	F	12	0.0833	0.5025
8	14	19,403	S			0.5025
11	15	20,425	S			0.5025
9	16	20,997	S			0.5025
13	17	21,144	S			0.5025
14	18	21,237	S			0.5025
5	19	21,762	S			0.5025
17	20	21,888	S			0.5025
16	21	21,974	S			0.5025
12	22	22,149	S			0.5025
4	23	25,660	S			0.5025
1	24	26,744	S			0.5025
3	25	29,834	S			0.5025

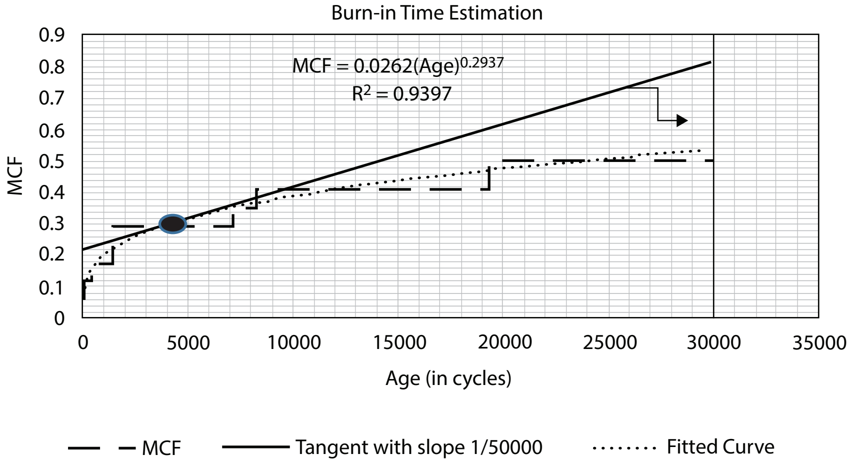

Figure 2.7 Graphical plots of Example 2.3.

Figure 2.7 shows plot of *MCF* (data of column 3rd and 7th in Table 2.3. Example 2.3—Burn-in period determination), power function fitted curve, its equation and index of fit, and the tangent with a slope of 1/50,000. The burn-in period can also be found out analytically as

$$MCF, M(t) = 0.0262t^{0.2937},$$

$$m(t) = \frac{dM(t)}{dt} = 0.0262 \times 0.2937 \times t^{0.2937-1} = \frac{1}{50{,}000}$$

Therefore, t = 4,572.62 cycles with *MCF*(4,572.62) = 0.3114.

2.3.2 Confidence Bounds on MCF: Exact Age Data

Assuming that the values of cumulative number of events at any recurrence age follow a lognormal distribution, then at any recurrence age t_i, the upper and lower confidence bounds of MCF can be computed by [7, 8]

$$MCF(t_i)_L = MCF(t_i)e^{-K_\delta\sqrt{\frac{Var(MCF(t_i))}{MCF(t_i)}}}, \text{ and}$$

$$MCF(t_i)_U = MCF(t_i)e^{K_\delta\sqrt{\frac{Var(MCF(t_i))}{MCF(t_i)}}}, \text{ respectively.}$$

Here, $0.50 < \delta < 1$ is the confidence level, K_δ is the δ standard normal percentile, and $Var(MCF(t_i))$ is the MCF variance at recurrence age t_i. The variance is calculated by

$$Var(MCF(t_i)) = Var(MCF(t_{i-1})) + \frac{1}{r_i^2}\left[\sum_{j \in R_{t_i}} \left(d_{ji} - \frac{1}{r_i}\right)^2\right]$$

Where R_{t_i} is the set of the units that have not been suspended by t_i and d_{ji} is defined as follows:

$d_{ji} = 1$, if j^{th} unit had an event recurrence at age t_i
$d_{ji} = 0$, if j^{th} unit did not have an event recurrence at age t_i

Based on the above equations, the computed variance, lower and upper bounds are provided in the columns #7 and #8, respectively, of Table 2.2. MCF and confidence calculations. Figure 2.8 shows the plots of with confidence bounds on the MCF.

The estimated 90% upper confidence bounds for the cumulative number of failures after four year of operation for 100 units is $0.5695 \times 100 \approx 57$ failures. This information can be used for estimating the repair costs

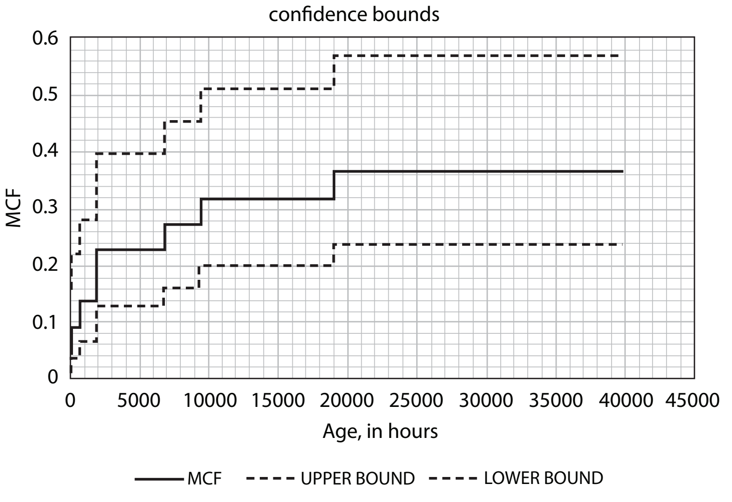

Figure 2.8 MCF with confidence bounds of Example 2.2 data.

or planning for spares. The ensuing section presents the non-parametric estimate of sample *MCF* from the grouped data.

2.3.3 Construction of MCF Plot and Confidence Bounds: Grouped Data

The procedural steps to calculate MCF for grouped data are as follows [4]:

1. If the data is not available in grouped form, determine the numbers of intervals and interval size from the data range. A well-known Sturges's rule can be utilized to determine the desirable number of groups into which a distribution of observations should be classified; the number of groups or classes is 1 + 3.3 *log n*, where *n* is the number of observations.
2. Determine the number of occurrences and suspensions events in each interval and tabulate them.
3. Enter the number of units (N) entering in the successive interval considering suspensions (C_i) with an initial sample size of N_i for $i = 0$. The subsequent samples are computed as, $N_i = N_{i-1} - C_{i-1}$.
4. Compute the MCF, $MCF(t_i) = MCF(t_{i-1}) + m(t_i)$, where the average number of recurrent events per sample unit over an interval i (increment in $MCF(t_i)$) is given by $m(t_i) = \dfrac{R_i}{(N_i - 0.5 \times C_i)}$. The R_i d enotes the number of recurrent event in i^{th} group and denominator provides an average number of units in that group.
5. Plot MCF with respect to the interval ages.

2.3.4 Confidence Bounds on MCF: Grouped Data

As stated in [4] that to calculate approximate confidence bounds for MCF from interval data as computation of correct limits are complicated and cannot be computed using simple procedure. Nelson [4] suggested that the naïve confidence bounds for the exact data can extended to rather than in an easy manner and do not require all the individual item's history. The approximate confidence bounds of MCF are computed by

$$\left(MCF\left(t_i\right)\right)_L = MCF(t_i) - K_\delta \sqrt{var(MCF(t_i))} \text{ and}$$

$$\left(MCF\left(t_i\right)\right)_L = MCF(t_i) - K_\delta\sqrt{var(MCF(t_i))}$$

where

$$var\left(MCF\left(t_i\right)\right) \cong \sum_{j=1}^{i} \frac{m(t_j)}{(N_j - 0.5 \times C_j)}$$

We explain the interval age analysis by applying it on a curtailed problem given in the Chapter 5 of reference [4].

Example 2.4. [4]
Table 2.4 shows the field data (grouped by months in service) on replacements of defrost controls in refrigerators. Calculate the MCF estimate for the cumulative percent replaced. What would be the number of replacements over a 15-year typical life of such refrigerators?

The last column of Table 2.4 shows the $MCF(t_i)$, which is plotted in Figure 2.9. The second-order polynomial is found to be a better fit than a power relation. From this polynomial fit, the number of replacements over a period of 15 years turns out to be replacements per refrigerators.

However, one can observe that there is large number of suspensions occurred in 12th and 24th month, respectively. The investigation revealed that first 12-month data came from refrigerators whose owners had sent in a dated purchase record card and thus has known installation date and this date and date of a control replacement were used to calculate the refrigerator's age at each replacement. The data on month 13 through 24 months came from the refrigerators whose owners' extended the warranty for another year for all parts and labor. The data from 25th through 29th months came from refrigerators whose owners' bought warranty for another year. Thus, the second and third year data representative of subpopulations with high replacement rate, not the population as a whole, consequently, the MCF plot consists of pieces of MCF's of three subpopulations, giving an appearance of increasing replacement rate, whereas the population rate was essentially constant. This can be verified by taking the data of first 12 months' period only.

2.4 Case Study: ROV System

The Remotely Operated Vehicles (ROVs) are unoccupied, highly maneuverable underwater robots that can be used to explore ocean depths while

Table 2.4 Data for Example 2.4.

Month i	**Number of replacements, R_i**	**Number entered, N_i**	**Number suspended, C_i**	$m(t_i)$	$MCF(t_i)$
1.	83	22,914		0.0036	0.0036
2.	35	22,914		0.0015	0.0051
3.	23	22,914		0.0010	0.0062
4.	15	22,914		0.0007	0.0068
5	22	22,914		0.0010	0.0078
6.	16	22,914	3	0.0007	0.0085
7.	13	22,911	36	0.0006	0.0090
8.	12	22,875	24	0.0005	0.0096
9.	15	22,851	29	0.0007	0.0102
10.	15	22,822	37	0.0007	0.0109
11.	24	22,785	40	0.0011	0.0119
12.	12	22,745	**20041**	0.0009	0.0129
13.	7	2,704	14	0.0026	0.0155
14.	11	2,690	17	0.0041	0.0196

(Continued)

Table 2.4 Data for Example 2.4. (*Continued*)

Month *i*	**Number of replacements, R_i**	**Number entered, N_i**	**Number suspended, C_i**	**$m(t_i)$**	**$MCF(t_i)$**
15.	15	2,673	13	0.0056	0.0252
16.	6	2,660	28	0.0023	0.0275
17.	8	2,632	22	0.0031	0.0305
18.	9	2,610	27	0.0035	0.0340
19.	9	2,583	64	0.0035	0.0375
20.	5	2,519	94	0.0020	0.0395
21.	6	2,425	119	0.0025	0.0421
22.	6	2,306	118	0.0027	0.0447
23.	6	2,188	138	0.0028	0.0476
24.	5	2,050	**1188**	0.0034	0.0510
25.	7	862	17	0.0082	0.0592
26.	5	845	28	0.0060	0.0652
27.	5	817	99	0.0065	0.0717
28.	6	718	128	0.0092	0.0809
29.	3	590	590	0.0102	0.0911

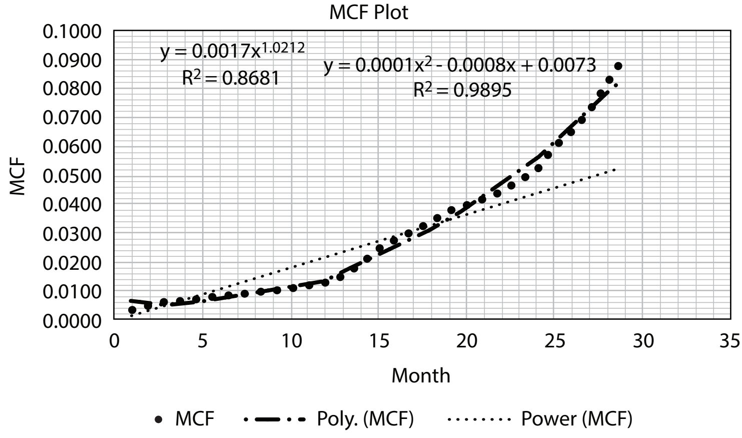

Figure 2.9 MCF plot of Example 2.4.

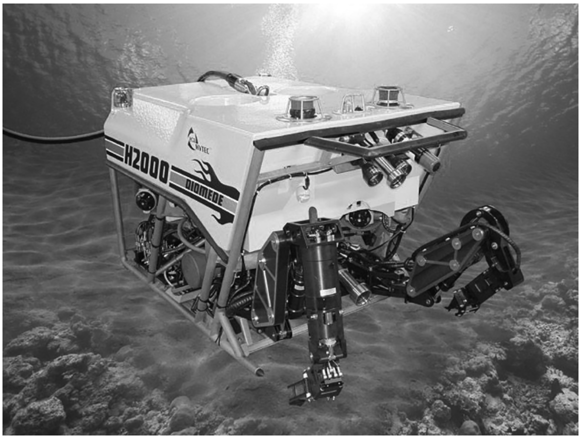

Figure 2.10 A ROV. (Image taken from: https://www.pinterest.com/pin/55267618549 5210644/?nic=1).

being operated by someone at the water surface, i.e., it allows to explore the ocean without actually being inside the ocean (https://oceanexploreer.noaa.gov/facts/rov.html). A typical ROV is shown in Figure 2.10.

These underwater robots are controlled by a person typically on a surface vessel, using a joystick in a similar way that one would play a video game. A group of cables, or tether, connects the ROV to the ship, sending electrical signals back and forth between the operator and the vehicle. Most ROVs are equipped with at least a still camera, video camera, and lights to transmit images and video back to the ship. Additional equipment, such as a manipulator or cutting arm, water samplers, and instruments that measure parameters like water clarity and temperature, may also be added to vehicles to allow for sample collection. Initially developed for industrial applications, such as internal and external inspections of underwater pipelines and the structural testing of offshore platforms, ROVs are now used for many scientific applications. They have proven extremely valuable in ocean exploration and are also used for educational programs at aquaria and to link to scientific expeditions live via the Internet.

ROVs range in size from that of a small computer to as large as a small truck. Larger ROVs are very heavy and need other equipment such as a winch to put them over the side of a ship and into the water. While using ROVs eliminates the "human presence" in the water, in most cases, ROV operations are simpler and safer to conduct than any type of occupied-submersible or diving operation because operators can stay safe (and dry!) on ship decks. ROVs allow to investigate areas that are too deep for humans to safely dive themselves, and ROVs can stay underwater much longer than a human diver, expanding the time available for exploration.

An industry, dealing with remote operation of subsea well equipment, has been observing downtime issues due to a series of failures observed in its remotely operated underwater vehicle system despite following a fixed-time preventive maintenance schedule. The downtime is becoming a cause of concern due to revenue losses and its reputation. Therefore, apart from the other measures, the company decided to conduct a preliminary reliability study to carry out an assessment of its operational fleet by using their recorded failure data. In this study, the data for a fleet of 53 systems with 488-recorded failures during last 5-years were available. A total of 14 failure modes, corresponding to 488 failure entries, were observed and are presented in Table 2.5. Here, we are only presenting the case for the sake of application of the methodology presented in this chapter by applying the approach on the 2-year period data set.

Based on data available and questions, we wish to answer, two approaches had been followed, i.e., (i) interval age analysis and (ii) exact age analysis.

2.5 Interval Age Analysis

2.5.1 MCF With All Types of Failure Modes Combined

Table 2.6 displays failure data of ROV by grouping it in 1-month interval associated to the observed failure modes. There were a total of 267 system

Table 2.5 List of failure categories and counts for all 53 ROVs.

Sr. No.	Failure Mode Category
1	ROV, Manipulator
2	ROV, Tooling
3	Cabling/Connectors
4	Sensors
5	ROV, Hoses/Fittings/Tubing
6	ROV, Hydraulics
7	ROV, Thruster Control
8	ROV, Electrics
9	Electric Motor
10	Operational
11	ROV, Video
12	ROV, Ground fault
13	ROV, Structural
14	ROV, HPU

related failures attributed to 14 failure modes on 54 systems in 2-year duration. Table 2.6 shows the data on a failure mode (denoted with a label). Each row shows failures observed in different modes in a particular month. The number of failures by a specific category in a specific month appears in corresponding column for that category.

This analysis is given in Table 2.7 for the ROV data (Month and Grand Total columns of Table 2.6). The second column contains the total number of failures from all categories observed in each month. The MCF estimated in Table 2.7 is plotted in Figure 2.11 that shows a linear trend with an index of fit close to 1.

It is obvious that this analysis does not brought out any insight in to the systems' behavior, puts some doubt and blindly believing on this analysis, one can conclude that the systems are observing a constant rate of recurrences and erroneous decisions could have followed later!

Table 2.6 Failure Counts in Months Corresponding their Failure Modes.

Month	Censored	A	B	C	D	E	F	G	H	I	J	K	L	M	N	Grand Total	Month
1			3	2	2	2	1	1	1	1	1				1	15	1
2		1	1	1	1	1	3				2					10	2
3		4	2	3	1	1		1	1	2	1	1				17	3
4		1	3	5		2	2	2	1	2	3		1	2		24	4
5		1			1			1							1	4	5
6	1	6	5	1	4	2			2	1						21	6
7	1	2	4		2	2					1		2			13	7
8		2	3					2		1						8	8
9		3										1	1			5	9
10	1	3	2	2	1	2	2	1								13	10
11	1	3		3	1	4		3	1						1	16	11
12		1		6	1		2			2		1				13	12
13	1			1	1						2				1	5	13
14		4	1	1	2			1	1				1			11	14

(*Continued*)

Table 2.6 Failure Counts in Months Corresponding their Failure Modes. (*Continued*)

Month	**Censored**	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**	**N**	**Grand Total**	**Month**
15		1	2	2	2	2										9	15
16	1		2	2		1	3									8	16
17		2	1								1			1		5	17
18	1	5		3			3			1						12	18
19	2	1	2	1		1	2		1	3			1			12	19
20	2	3	2	2	2	1										10	20
21	1	2	1		3	1		2								9	21
22	1	1	2	1	3	1			2		1		1			12	22
23	1	1		1			1			1		1	1			6	23
24		3	1		2	1				1			1			9	24
Month	Censored	A	B	C	D	E	F	G	H	I	J	K	L	M	N	ALL	Month
Grand Total	14	50	37	37	29	24	19	14	10	15	12	4	9	3	4	267	

Table 2.7 MCF calculation by combining all failure modes.

Month i	Number of failure of all modes (R_i)	Suspended item (C_i)	Number enter month(N_i)	Increment m_i	MCF_i
1	15		54	0.2778	0.2778
2	10		54	0.1852	0.4630
3	17		54	0.3148	0.7778
4	24		54	0.4444	1.2222
5	4		54	0.0741	1.2963
6	21	1	53	0.4000	1.6963
7	13	1	52	0.2524	1.9487
8	8		52	0.1538	2.1026
9	5		52	0.0962	2.1987
10	13	1	51	0.2574	2.4561
11	16	1	50	0.3232	2.7794
12	13		50	0.2600	3.0394
13	5	1	49	0.1031	3.1425
14	11		49	0.2245	3.3670

(Continued)

Table 2.7 MCF calculation by combining all failure modes. (*Continued*)

Month i	Number of failure of all modes (R_i)	Suspended item (C_i)	Number enter month(N_i)	Increment m_i	MCF_i
15	9		49	0.1837	3.5506
16	8	1	48	0.1684	3.7191
17	5		48	0.1042	3.8232
18	12	1	47	0.2581	4.0813
19	12	2	45	0.2727	4.3540
20	10	2	43	0.2381	4.5921
21	9	1	42	0.2169	4.8090
22	12	1	41	0.2963	5.1053
23	6	1	40	0.1519	5.2572
24	9		40	0.2250	5.4822

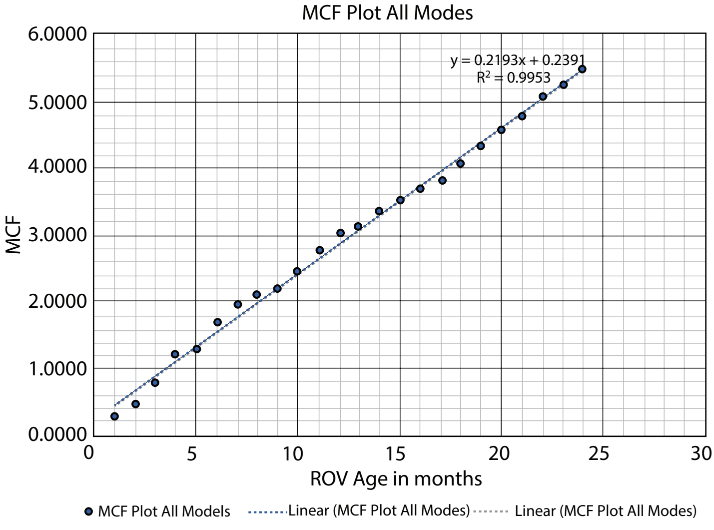

Figure 2.11 MCF by combining all failure modes.

2.5.2 MCF for Individual Failure Modes

The analysis was performed by taking individual failure mode. Repeating process by taking each failure mode in the second column of Table 2.7 and estimated the MCF for each one. Table 2.8 is the result of estimated MCF after 24 months for individual failure modes and arranged according to their respective MCF.

Table 2.8 shows that the MCFs for the failure modes A, B, C, D, and E are very high compared to other failure modes. Note that these five modes are contributing around 70% of the total failures, which became cause of concern.

2.5.3 Exact Age Analysis

Figure 2.12 shows the event plots of the fleet of ROVs. Each horizontal line in this plot shows the history of a ROV unit. Each on a line is the system unit age in days in working when it was repaired. The length of each line tracks each system's length of working.

Figure 2.13 shows the 53 systems' estimated MCF's separately (as one system did not observe any recurrence). It can be observed that the systems show different performance behavior through their respective MCF plots (Recall that the group data analysis combining all failure modes presented earlier showed a constant recurrence rate characteristic, but this analysis reveals another aspect!). Therefore, we decided to divide these systems into three categories according to their MCF shapes as shown in Figure 2.14.

Table 2.8 MCF calculation for individual category at 24 months.

Failure Mode	MCF	Contribution (in %)
A	1.0375	19
C	0.7526	14
B	0.7518	14
D	0.6139	11
E	0.4906	9
F	0.3892	7
I	0.3109	6
G	0.2801	5
J	0.2358	4
H	0.2064	4
L	0.1947	4
K	0.0831	2
N	0.0779	1
M	0.0579	1

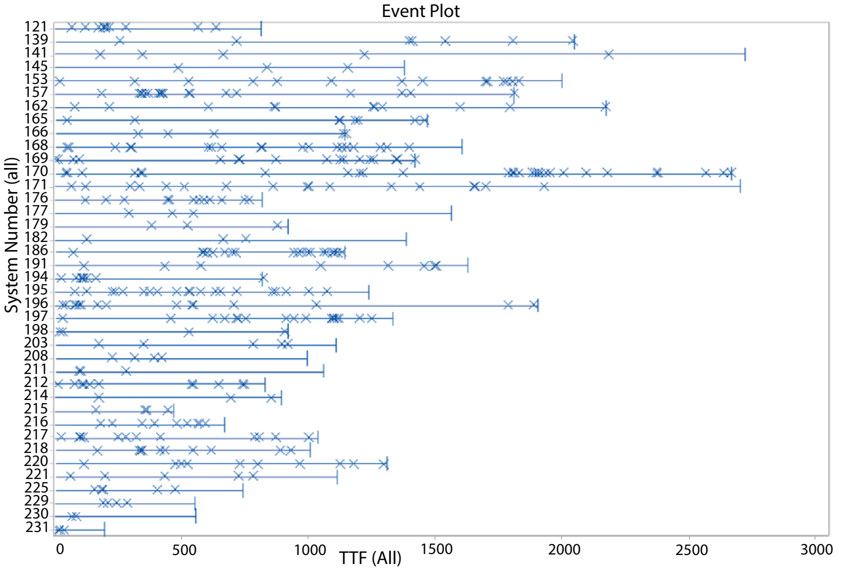

Figure 2.12 Event plot for each system.

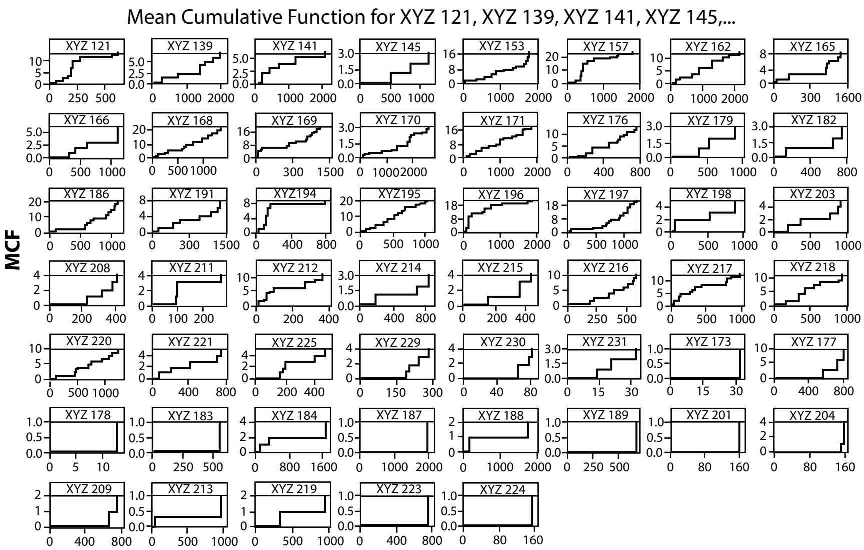

Figure 2.13 MCF plot of each system.

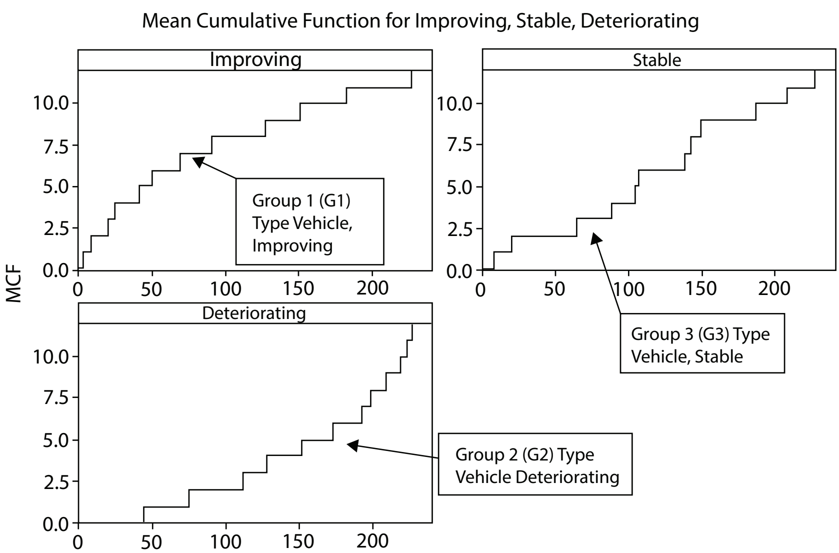

Figure 2.14 Grouping of systems based on performance behavior.

Table 2.9 A sample of collected data.

System Number Group 1	Time to Failure/ Suspended (Days)	System Number Group 2	Time to Failure/ Suspended (Days)	System Number Group 3	Time to Failure/ Suspended (Days)
121	63	139	254	162	77
121	117	139	717	162	215
121	169	139	1,395	162	602
121	190	139	1,406	162	864
121	197	139	1,534	162	869
121	201	139	1,802	162	871
121	210	139	2,040	162	1,254
121	213	139	2,047	162	1,261
121	213	145	483	162	1,287
121	278	145	835	162	1,593
121	280	145	1,151	162	1,788
121	560	145	1,382	162	2,169
121	633	153	19	162	2,169

(*Continued*)

Table 2.9 A sample of collected data. (*Continued*)

System Number Group 1	Time to Failure/ Suspended (Days)	System Number Group 2	Time to Failure/ Suspended (Days)	System Number Group 3	Time to Failure/ Suspended (Days)
121	813	153	312	168	49
141	177	153	524	168	51
141	178	153	777	168	239
141	345	153	872	168	296
141	659	153	876	168	303
141	1,217	153	1,086	168	304
141	2,181	153	1,363	168	602
141	2,720	153	1,447	168	613
157	182	153	1,697	168	654
157	328	153	1,699	168	655
157	329	153	1,704	168	811
157	329	153	1,765	168	813
157	338	153	1,779	168	975

Further, wherever the MCF shape was not discernible due to small set of data, we kept them outside the scope of this study, however, in a thought to include them later in the respective group on receiving some more inputs from the industry. In the process, several systems out of 53 were further left out. The system IDs left out were 173, 178, 183, 184, 187, 188, 189, 201, 204, 209, 213, 219, 223, and 224, respectively.

Table 2.9 below provides snapshots of the data for some selected ROVs with the event plots in Figure 2.15.

Figure 2.16 shows plots of the sample MCFof systems in groups 1, 2, and 3, respectively, with 95% confidence bounds. If we examine these plots, then sample MCF of Group 1 indicates that at the beginning of the operations, systems experienced a higher rate of recurrence than the systems of other groups and got stabilized after a period around 1,000 days. It also shows that the systems of this group showed an improved performance after facing a higher recurrence rate initially. Systems in Group 2 showed a low recurrence rate initially that gradually increased, but after 1,000 days of operation recurrence rate increased drastically causing a heavy deterioration. In Group 3 systems, the plot shows that there was almost a constant recurrence rate until 1,000 days from the beginning of the operations and the aging effects begin to surface out around 1,700 days. Why are systems showing different types of behavior despite being identical in design and configurations? Might be because of active/inactive maintenance activities, working in a different environmental condition compare to other groups system!

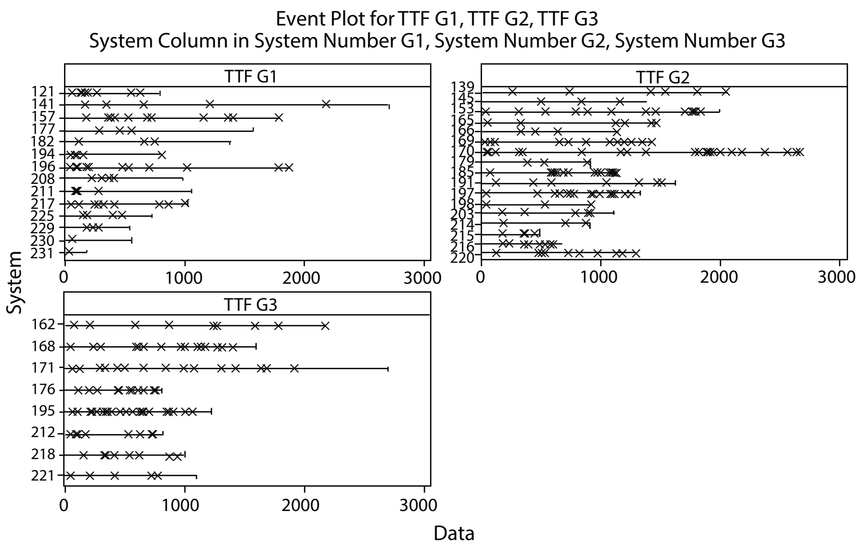

Figure 2.15 Event plot of all three groups.

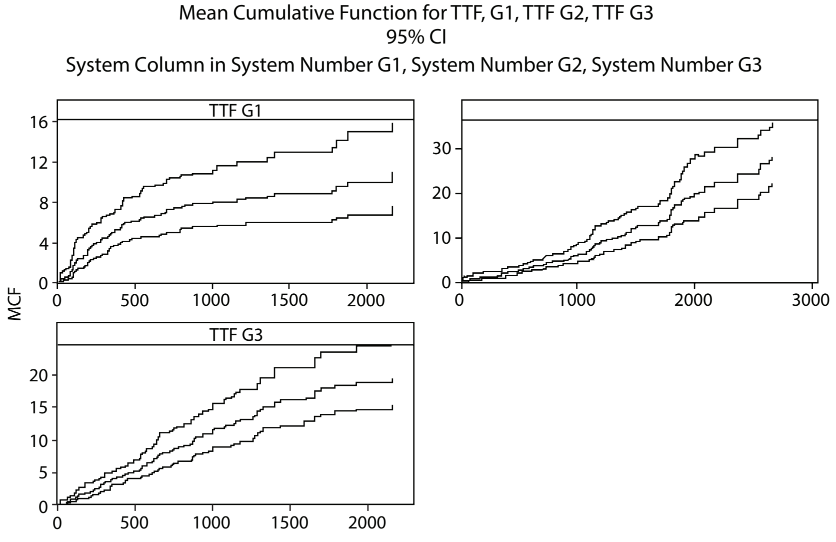

Figure 2.16 MCF plot of all three groups.

Table 2.10 Exercise 1: Failure data of three systems.

System 1	**System 2**	**System 3**
	1.4	0.3
55.6	35.0	32.6
72.7	46.8	33.4
111.9	65.9	241.7
121.9	181.1	396.2
303.6	712.6	480.8
326.9	1,005.7	588.9
1,568.4	1,029.9	1,043.9
1913.5	1675.7	1,136.1
	1787.5	1,288.1
	1867.0	1,408.1
		1,439.4
		1,604.8

2.6 Summary and Conclusion

This chapter has presented sample MCF-based repairable systems analysis of a fleet by considering the exact age and interval age data. Undoubtedly, the choice of method and analysis depend on the data availability and the questions we wish to answer. The presentation has been supported by a case study on a fleet of ROV. The pooled data on the times of successive recurrences of fleet of 54 systems indicated a constant rate of recurrences, whereas individual system behavior showed that the recurrence rates for most of the system were not constant and there existed much difference in their respective MCF and performance behavior. It can be concluded that interval age data analysis sometimes hides the system behavior that entails further analysis, and without looking at the actual behavior of the data may lead to underestimation or overestimation of results. Therefore, based on their MCF shapes of these systems, further categorization and grouping were resorted to. The MCF trend of these groups showed three distinctive rates of recurrences. A further investigation was made to validate the observed outputs by taking the maintenance history and environmental condition data. These results later concluded that many factors can influence the performance of an identical system, for example, completeness and correctness of assigned maintenance task, widely varying environmental and operating conditions and variations in operating levels due to the field environmental condition, correctness of the data records, etc.

Selecting a specific method to assess system reliability is an important aspect; it depends on the kind of data available and questions we wish to answer. Rather than following the common practice of simple parametric distribution analysis for a repairable system, which is not always correct way to do it, it was observed that *MCF*-based non-parametric method is a better choice to analyze the nature of fleet and system recurrence rates through the operational life of a product.

Exercises

(1) The recurrent event data presented in Table 2.10 show the failure data of three systems. The starting time of operation of these three systems is 0 and being observed for 200 hours each. Estimate and draw the MCF plots with 90% confidence bounds [10].

  (i) Analyze the behavior of individual system and comment.

(ii) Combine the data of all three systems and draw the MCF and its 90% confidence interval.

(2) Take the data provided in Table 2.7 and analyze with respect to each failure mode separately.
(3) It is thought that dominant failure modes A, B, and C can be eliminated through redesign. Would the reduction in MCF be fruitful?

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3

Repairable Systems Reliability Analysis: Parametric

3.1 Introduction

Reliability analysis based on different lifetime distributions (exponential, Weibull distribution) are generally preferred for non-repairable systems since the TTF random variables involved are independent and identically distributed (*i.i.d.*). In case of repairable systems, the *i.i.d.* assumption may be violated and processes such as RP and NHPP have been extensively used to model the situation when system is brought to AGAN and ABAO states, respectively. However, a repairable system may end up at other than these two extremities, *viz.*, *better than old but worse than new*, *better than new*, and *worse than old*. The quest to have more accurate analyses and predictions, the GRP can be of great interest to reduce the modeling uncertainty resulting from the aforementioned repair assumptions. This chapter briefly takes up some basic terminologies related to repairable systems followed by modeling and analysis of repairable systems with the help of different processes as mentioned above.

3.2 Basic Terminologies

The following selected definitions and terms pertaining to the repairable systems are useful and are reproduced from [1].

1. Point Process: A point process is a stochastic model (stochastic model possess some inherent randomness) that describes that occurrence of events in time. These occurrences are thought of as point on the time continuum. In general, the times between occurrences may be neither independent nor identically distributed. For our purpose, "occurrences in time" is failure times of a repairable system.

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2. Counting Random Variable: Let N(t) be the counting random variable that denotes number of failures occurred in the interval [0, t]. When N has as its argument an interval, such as N (a, b), the result is the number of failures in that interval. N is called the counting random variable. Number of failures in the interval (a, b) is defined as

$$N(a, b) = N(b) - N(a)$$

Suppose, $N(t_1) = k_1$, $N(t_2) = k_2 \ldots .N(t_n) = k_n$.

Note 3.1.

- A point process has stationary increment if for all k, $P(N(t, t + s) = k)$ is independent of t.
- A point process has independent increment if for all n and for all $a_1 < b_1 \leq a_2 < b_2 \leq \ldots a_n \leq b_n$, the random variables $N(a_1, b_1)$, $N(a_2, b_2) \ldots \ldots, N(a_n, b_n)$, are independent. In other words, $P(N(a_1,b_1) = k_1, \ldots\ldots, N(a_n,b_n) = k_n) = \prod_{i=1}^{n} P\big(N(a_i,b_i)=k_i\big)$.

3. Mean Function of Point Process: It is the expected value of counting random variable N(t) through time, i.e., the expected number of failures through time, $\Lambda(t) = E(N(t))$.
4. Rate of Occurrence of Failures: When $\Lambda(t)$ is differentiable, we define the rate of occurrence of failures ($ROCOF$) as

$$u(t)=\frac{d}{dt}\Lambda(t) \qquad \text{(see Note 3.2)}$$

5. Intensity Function: The intensity function of a point process is the probability of failure in a small interval divided by length of the interval, i.e.,

$$u(t)=\lim_{\Delta t\to 0}\frac{P(N(t,t+\Delta t)\geq 1)}{\Delta t}$$

Note 3.2.

- The intensity function and the ROCOF are measures of the reliability of a repairable system. It turns out that these two functions are equal provided that the simultaneous failures do not occur. (The simultaneous failures can occur only at points at which the mean function Λ (t) is discontinuous.)

- One can observe the similarity between intensity function and hazard rate function. Clearly, the intensity function is the unconditional probability of a failure (not necessarily the first one) in a small interval divided by the length of the interval. Comparing hazard function with intensity function, the hazard function is the limit of a conditional probability; the intensity function is not. The hazard function is the conditional probability that one and only one failure will occur in a small interval, divided by the length of the interval. This may be due to a few of the components having serious defects that will cause this infant mortality. Besides, this probability is conditioned on survival at the beginning of the interval. In other words, for a system that is wearing out, the probability of failure in the interval (t_0, $t_0 + \Delta t_0$) conditioned on survival past time t_0 will be smaller than the probability of failure in a later interval (t_1, $t_1 + \Delta t_1$) conditioned on survival past time t_1.
- Many repairable systems have shape of their ROCOF or intensity function similar to that of bathtub shape of hazard rate curve meant for non-repairable systems. However, their interpretations are entirely different. Figure 3.1 shows the bathtub shaped intensity function. The initial phase in bathtub shape ROCOF or intensity indicates that the system will experience infant mortality failures in its initial life due to design deficiencies and poor manufacturing quality. The intensity function gradually reduces as these infant mortality failures are eliminated. This is followed by a time when the ROCOF is constant and this phase experiences only random failures. Finally, as the system ages, deterioration takes place and the time between failures tend to reduce due to the wear out in the system in this phase.

6. Complete Intensity Function: For some models, it will be more appropriate to consider the conditional probability given the failure history of the process. Let $\mathcal{H}_t$ denote the entire history of the failure process through time t. This

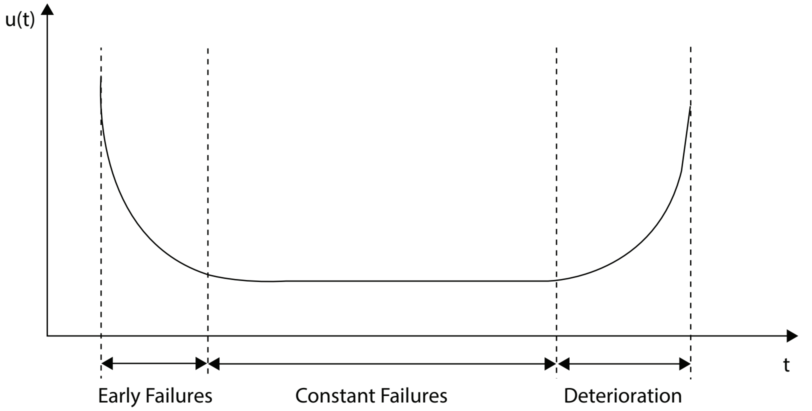

Figure 3.1 Bathtub-shaped intensity function.

history can be represented by the set of failure times $\{t_i: i = 1,2,\ldots,N(t)\}$. Hence, the complete intensity function is

$$u(t) = \lim_{\Delta t \to 0} \frac{P(N(t, t+\Delta t] \geq 1 | \mathcal{H}_t}{\Delta t}$$

3.3 Parametric Analysis Approaches

3.3.1 Renewal Process

A renewal process is an idealized stochastic model for events that occur randomly in time. The basic mathematical assumption in RP is that the times between the successive arrivals of events are i.i.d. In the present context, if a repairable system in service can be repaired to an AGAN condition following each failure such that the *pdf* of the time-between-failures (TBF) does not change from one failure to another, then the failure process is called a renewal process. A special case of this is homogeneous Poisson process (HPP) which is a Poisson process with constant intensity function u. In other words, if TBF, $X_1, X_2\ldots$ are i.i.d. exponential random variables, then $N(t)$ corresponds to a HPP with a constant intensity function, u. The expected number of failures in time interval $[0, t]$, would be

$$\mathrm{E}[\mathrm{N}(t)] = \int_0^t u(t)dt = \mathrm{ut} \tag{3.1}$$

Since the intensity function is constant, the HPP cannot be used to model systems that deteriorates or improve and should be applied with caution. For such situations, a Poisson process with non-constant intensity function can be a viable alternative.

3.3.2 Non-Homogeneous Poisson Process (NHPP)

NHPP is a Poisson process whose intensity function is non-constant. To understand NHPP model, let $N(t)$ be the cumulative number of failures observed in cumulative test time t, and let $u(t)$ be the failure intensity. Under the NHPP model, $u(t)\Delta t$ is the probability of a failure occurring over the interval $[t, t + \Delta t]$ for small Δt. Thus, the expected number of failures experienced over the test interval $[0, t]$ is given by

$$\mathrm{E}[\mathrm{N}(t)] = \int_0^t u(t)dt \tag{3.2}$$

The NHPP model assumes that $u(t)$ may be approximated by the Power Law Model, i.e.,

$$u(t) = a \times b \times t^{b-1};\ a > 0,\ b > 0 \tag{3.3}$$

Where a is called a scale parameter because it depends upon the unit of measurement chosen for t while b is the shape parameter that characterizes the shape of the graph of the intensity function and system behavior. For $b = 1$, $u(\mathrm{t}) = a$, a stable system. For $b > 1$, $u\ (t)$ is increasing. This indicates a deteriorating system, whereas when $b < 1$, $u\ (t)$ is decreasing indicating an improving system.

The power law model has a very practical foundation in terms of minimal repair and is popular for several reasons. Firstly, it models the situation when the repair of a failed system is just enough to get the system operational again. Secondly, if the time to first failure follows the Weibull distribution, then each succeeding failure is governed by the PLP model. From this point of view, the PLP model is an extension of the Weibull distribution. The expected number of failures for this case becomes

$$\mathrm{E}[\mathrm{N}(t)] = \int_0^t u(t)dt = at^b \tag{3.4}$$

This form comes from the assumption that inter-arrival of times between successive failures follow a conditional Weibull probability distribution. It means that the arrival of the i^{th} failure is conditional on the cumulative operating time up to the $(i-1)^{th}$ failure. This conditionality also arises from the fact that the system retains the condition of as bad as old after the $(i-1)^{th}$ repair. Thus, the repair process does not restore any added life to the component or system.

In order to obtain the model parameters, consider the following definition of conditional probability (refer Figure 3.2):

$$\mathrm{P}(T \le t | T > t_1) = \frac{F(t) - F(t_1)}{R(t_1)} = 1 - \frac{R(t)}{R(t_1)} \tag{3.5}$$

Where F (·) and R (·) are the probability of component failure and the reliability at the respective times. Thus,

$$\mathrm{F}(\mathrm{t_i}) = 1 - \exp[\mathrm{a}(\mathrm{t_{i-1}})^{\mathrm{b}} - \mathrm{a}(\mathrm{t_i})^{\mathrm{b}}] \tag{3.6}$$

With the help of Equations (3.3) and (3.6), the probability density function of the i^{th} failure given that $(i-1)^{th}$ failure occurred at time t_{i-1} can be obtained as

$$\mathrm{f}\left(\left(\mathrm{t_i} | \mathrm{t_{i-1}}\right)\right) = \mathrm{ab}\mathrm{t_i^{b-1}} \exp\left[-\mathrm{a}\left\{(\mathrm{t_i})^{\mathrm{b}} - (\mathrm{t_{i-1}})^{\mathrm{b}}\right\}\right] \tag{3.7}$$

The parameters for the NHPP model can be estimated using the maximum likelihood estimation (MLE) method as explained below.

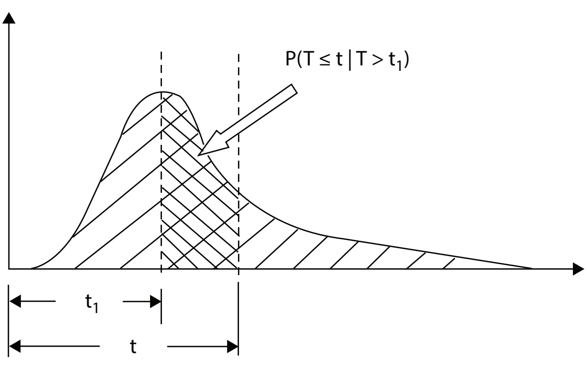

Figure 3.2 Conditional probability of occurrence of failures.

The likelihood function is defined as

$$L = \prod_{i=1}^{n} f\left(t_i \middle| t_{(i-1)}\right); \text{ where } n \text{ is number of failures.}$$

Considering pdf as given in Equation (3.7), the likelihood function is given as follows:

$$L = a^n b^n e^{-at^{*b}} \prod_{i=1}^{n} t_i^{b-1} \tag{3.8}$$

Where

$$t^* = \begin{cases} t_n & \text{if the test is failure terminated} \\ t > t_n & \text{if the test is time terminated} \end{cases}$$

Taking the natural log on both sides of (3.8)

$$lnL = n \ln a + n\, ln\, b - at^{*b} + (b-1)\sum_{i=1}^{n} ln\, t_i \tag{3.9}$$

Differentiating Equation (3.9) with respect to *a* and setting it to zero,

$$\begin{gathered} \frac{\partial(\ln L)}{\partial a} = \frac{n}{a} - t^{*b} = 0 \\ \hat{a} = \frac{n}{t^{*b}} \end{gathered} \tag{3.10}$$

Likewise, taking the first derivative of (3.9) with respect to *b* and setting it to zero give

$$\begin{gathered} \frac{\partial(\ln L)}{\partial b} = \frac{n}{b} - at^{*b} * \ln t^* + \sum_{i=1}^{n} \ln t_i = 0 \\ \hat{b} = \frac{n}{n \ln t^* - \sum_{i=1}^{n} \ln t_i} \end{gathered} \tag{3.11}$$

The above notion can easily be extended to a fleet as well. Let there be a fleet of K identical systems, then

$$L = \prod_{l=1}^{K} \left[a^{n_l} b^{n_l} e^{-at^{*b}} \prod_{i=1}^{n_l} t_i^{b-1} \right]$$

$$lnL = \ln a \sum_{l=1}^{K} n_l + ln\, b \sum_{l=1}^{K} n_l - at^{*b} + (b-1) \sum_{l=1}^{K} \sum_{l=1}^{n_l} ln\, t_i \qquad (3.12)$$

Where $l = 1, 2, \ldots K$.

Differentiating Equation (3.12) with respect to a and b, respectively, and setting it to zero, we get

$$\hat{a} = \frac{\sum_{l=1}^{K} n_l}{K * t^{*b}} \qquad (3.13)$$

$$\hat{b} = \frac{\sum_{l=1}^{K} n_l}{\sum_{l=1}^{K} \sum_{i=1}^{n_l} ln\left(\frac{t^*}{t_i}\right)} \qquad (3.14)$$

Example 3.1.

Aero engines used in military aircraft are empowered with high thrust to enable sudden climb and sustain high "G" loads during manoeuvres. They are also designed to prevent surge and stall due to back pressure resulting from firing of rockets and missiles that create abundance of turbulence in front of the aero engine. These engines are subjected to very high aerodynamic and thermal stresses and hence are subjected to frequent failures. A photograph of such an engine is placed below as Figure 3.3.

The failure times in hours of such an aero engine with time between overhauls of 550 hours are as given below:
203, 477, 318, 536, 494, 213, 303, 525, 345, 299, 154, 230, 132, 321, 123, 351, 188, 49. 02, 267, 548, 380, 61, 160, 375, 550, 174, 176, 257, 102, 81, 541, 518, 533, 547, 299, 208, 326, 451, 349, 152, 509, 249, 325, 261, 328, 48, 19, 142, 200, 426, 90, 522, 446, 338, 55, 549, 84, 342, 162, 250, 368, 96, 431, 14, 207, 324, and 546.

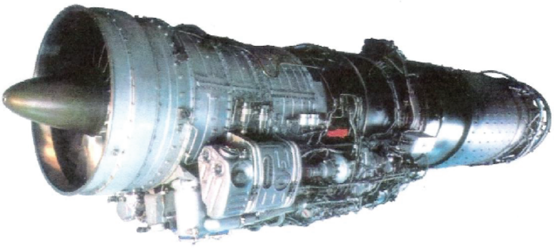

Figure 3.3 Photograph of an aero engine.

It is imperative for the maintenance crew to find out whether the aero engine is deteriorating over a period of 550 hours. This will also assist them to assess the efficacy of present preventive maintenance policy. The crew is also interested in knowing the characteristic life of the aero engine. Since the aero engine is a repairable system, use NHPP model to estimate the scale and shape parameters. Plot the intensity function and suggest remedial measures.

Solution.

Using Equations (3.10) and (3.11), the values obtained are

$$b = \frac{67}{57.6453} = 1.16$$

$$a = \frac{67}{550^{1.16}} = 0.0444$$

Using Equation (3.3), the intensity function becomes

$$u\,(t) = 0.0444 \times 1.16 \times t^{0.16}$$

The intensity function curve is plotted below as Figure 3.4.

The increase in intensity function is due to the wear out in the system. The maintenance crew may think upon giving a re-look into their present maintenance policy and implement the laid down standard maintenance procedures even more efficiently. They may also think upon reviewing the present maintenance policy. Besides, the quality of repair can also be taken into account. These two aspects, however, are dealt in detail in the subsequent sections and chapters.

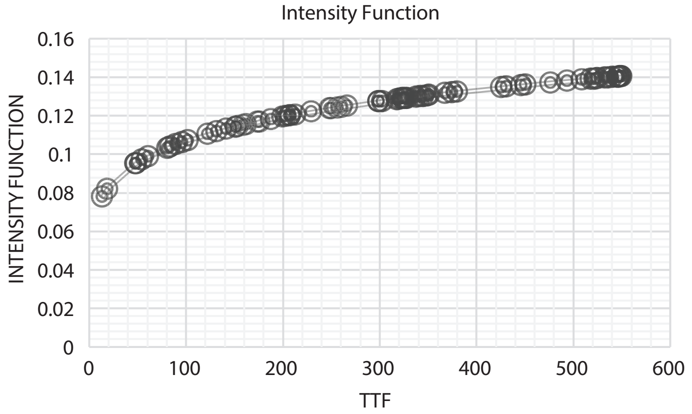

Figure 3.4 Intensity function plot for Example 3.1.

Example 3.2.

The failure times of 18 aero engines with time between overhauls of 550 hours are as given below at Table 3.1.

Estimate the scale and shape parameters using NHPP. Plot the intensity function. Also, estimate instantaneous MTBF at t = 550 hours.

Table 3.1 Time to failure data (hours) of aero engines of Example 3.2.

Engine No.	Times to Failure	Engine No.	Times to Failure
1	324,399,531	10	451
2	342	11	414
3	287,317	12	102
4	531	13	164,176
5	426	14	160,461
6	321,337,495	15	123
7	48,408	16	299
8	325	17	318
9	349	18	203,521

Solution.

Using Equations (3.13) and (3.14), the values obtained are

$$b = \frac{27}{17.2718} = 1.56$$

$$a = \frac{27}{18 \times 550^{1.56}} = 0.000080$$

$$u\ (t) = 0.000080 \times 1.56 \times t^{0.56}$$

$$\text{Instantaneous } MTBF = \frac{1}{u(t)}$$

$$MTBF\ (t = 550\ hours) = 234\ hours$$

The intensity function curve is plotted at Figure 3.5.

The wear out in these aero engines are more compared to that of previous example. The maintenance crew may think upon giving a re-look into their present maintenance policy and implement the laid down standard

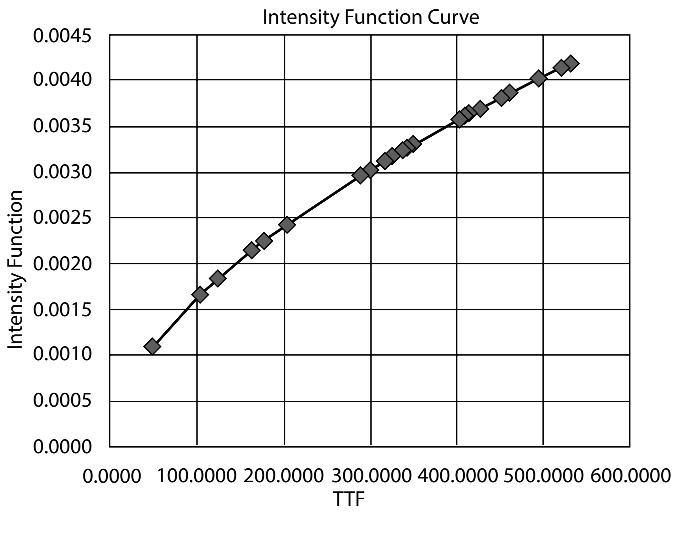

Figure 3.5 Intensity function plot for Example 3.2.

maintenance procedures even more efficiently. They may also think upon reviewing the present maintenance policy. Besides, the quality of repair can also be taken into account. These two aspects, however, are dealt in detail in the subsequent sections and chapters.

3.3.3 Generalized Renewal Process (GRP)

The GRP framework described here is based on two broad approaches for imperfect repair, Arithmetic Reduction of Age (ARA) and Arithmetic Reduction of Intensity (ARI). In ARA models, the effect of repair is expressed by assuming a reduction in actual age of the system and attaining an age termed as virtual age. ARA-based Kijima's virtual age models are most widely cited and effective models in the literature. In ARI approach, the repair effect is considered by the change induced on the failure intensity before and after failure. A brief introduction to ARA and ARI models is covered in the ensuing section.

3.3.3.1 ARA Models

The principle of this class of models is to consider that repair rejuvenates the system. The real age of a system is its functioning time t, and the virtual age is defined as a positive function of its real age, possibly depending on past failures. The idea that repair actions reduce the age of the system is the basis of Kijima's virtual age models [2, 3]. To understand Kijima virtual age models, let's consider a repairable system be observed from time $t_0 = 0$, denoted by t_1, t_2 … the successive failure times. Let the times between the failures be denoted by $X_n = t_n - t_{n-1}$. The concept of virtual age then accounts for the effectiveness of repair in the following way. Let q be the repair effectiveness and V_n be the virtual age of the system after n^{th} repair with $V_0 = 0$. Kijima-I model [2] assume that the n_{th} repair can remove the damage incurred only during the time between $(n - 1)^{th}$ and n^{th} failure, yielding virtual age as

$$V_n = V_{n-1} + qX_n \tag{3.15}$$

$$V_i = V_{i-1} + qX_i \tag{3.16}$$

$$\mathrm{V_i} = \mathrm{q}\sum_{j=1}^{i} \mathrm{X_j} \tag{3.17}$$

Where the distribution of X_n is given by

$$Pr\{X_n < x | V_{n-1} = y\} = \frac{F(x+y) - F(y)}{1 - F(y)} \tag{3.18}$$

However, in practice, the n^{th} repair action may also decrease all damage accumulated up to n^{th} failure, yielding the Kijima-II model for virtual age

$$V_n = q\,(V_{n-1} + X_n) \tag{3.19}$$

$$V_i = q\,(V_{i-1} + X_i) \tag{3.20}$$

$$V_i = \sum_{j=1}^{i} \mathrm{q}^{i-j+1} X_j \tag{3.21}$$

Where the distribution of X_n is given by Equation (3.18).

In Kijima virtual age models, $q = 0$ converges to AGAN condition after the repair and thus can be modeled through RP. Assumption of $q = 1$ signifies that the component is restored to the same condition when it was before the repair, i.e., ABAO condition and can be modeled by NHPP. Therefore, q can be physically interpreted as an index for representing effectiveness and quality of repair. For instance, the values of q that fall in the interval $0 < q < 1$ represent a system state in which the condition of the system is better than old but worse than new. For $q > 1$, the system is in a condition of worse than old. The assumptions for Kijima models are as follows:

- Time to first failure (TTFF) distribution is known and can be estimated from the available data using Weibull distribution MLEs.
- The repair time is assumed to be negligible so that the failures can be viewed as point processes.

The parameters for the GRP model are also estimated using MLE. Currently, the approach of Yanez *et al.* [4] is among the most widely used for GRP parameter estimation. The inter-arrival of failures are assumed to follow the Weibull distribution, and the $f(.)$ and $F(.)$ of the time to i^{th} failure are given by

$$f((t_i|t_{i-1})) = [ab(V_{i-1} + X_i)^{b-1}] \times \exp[a\{(V_{i-1})^b - (V_{i-1} + X_i)^b\}]$$

$$F(t_i) = 1 - \exp[a\{(V_{i-1})^b - (V_{i-1} + X_i)^b\}]$$

In above equations, V_i could be either from Kijima-I or Kijima-II model.

The likelihood, log-likelihood, and MLEs for the failure terminated (single and multiple repairable systems) and time terminated (single and multiple repairable systems) cases for system data are given as follows for both Kijima-I and Kijima-II models.

3.3.3.2 Kijima-I Model

The likelihood, log-likelihood functions, and MLEs for failure terminated single repairable system data set are given as follows:

$$L = \prod_{i=1}^{n}\left[ab\left(q\sum_{j=1}^{i-1}X_j + X_i\right)^{b-1}\right] \times \exp\left[a\left\{\left(q\sum_{j=1}^{i-1}X_j\right)^{b} - (V_{i-1} + X_i)^b\right\}\right] \tag{3.22}$$

By taking log on both sides of Equation (3.22).

$$\ln L = n\log(b) + n\log a + (b-1)\sum_{i=1}^{n}\log\left(q\sum_{j=1}^{i-1}X_j + X_i\right) + \sum_{i=1}^{n}\left[a\left\{\left(q\sum_{j=1}^{i-1}X_j\right)^{b} - \left(q\sum_{j=1}^{i-1}X_j + X_i\right)^{b}\right\}\right] \tag{3.23}$$

To obtain failure terminated MLEs, differentiate the log likelihood function (Equation (3.23)) with respect to each of the three parameters *a*, *b,* and *q* and equate to zero.

$$\frac{\partial \mathrm{Log}(L)}{\partial a} = \frac{n}{a} + \sum_{i=1}^{n}\left[\left(q\sum_{j=1}^{i-1}X_j\right)^{b} - \left(q\sum_{j=1}^{i-1}X_j + X_i\right)^{b}\right] = 0 \tag{3.24}$$

$$\frac{\partial \text{Log}(L)}{\partial b} = \frac{n}{b} + \sum_{i=1}^{n}\left[a\left\{ \left(q\sum_{j=1}^{i-1} X_j \right)^b \log\left(q\sum_{j=1}^{i-1} X_j \right) - \left(q\sum_{j=1}^{i-1} X_j + X_i \right)^b \log\left(q\sum_{j=1}^{i-1} X_j + X_i \right) \right\}\right] + \sum_{i=1}^{n} \log\left(q\sum_{j=1}^{i-1} X_j + X_i \right) = 0 \tag{3.25}$$

$$\frac{\partial \text{Log}(L)}{\partial q} = (b-1)\sum_{i=1}^{n} \frac{\sum_{j=1}^{i-1} X_j}{q\sum_{j=1}^{i-1} X_j + X_i} + a\sum_{i=1}^{n}\left[b\left(q\sum_{j=1}^{i-1} X_j \right)^{b-1} \left(\sum_{j=1}^{i-1} X_j \right) - b\left(q\sum_{j=1}^{i-1} X_j + X_i \right)^{b-1} \left(\sum_{j=1}^{i-1} X_j \right)\right] = 0 \tag{3.26}$$

The likelihood, log-likelihood functions, and MLEs for time terminated single repairable system data set are given as follows:

$$L = \prod_{i=1}^{n}\left[ab\left(q\sum_{j=1}^{i-1} X_j + X_i \right)^{b-1} \right] \times \exp\left[a\left\{ \left(q\sum_{j=1}^{i-1} X_j \right)^b - \left(q\sum_{j=1}^{i-1} X_j + X_i \right)^b \right\}\right] \times \exp\left[a\left\{ \left(q\sum_{j=1}^{n} X_j \right)^b - \left(T - t_n + q\sum_{j=1}^{n} X_j \right)^b \right\}\right] \tag{3.27}$$

Taking log on both sides of Equation (3.27), we obtain

$$\ln L = n\log(b) + n\log a + (b-1)\sum_{i=1}^{n}\log\left(q\sum_{j=1}^{i-1}X_j + X_i\right) + \sum_{i=1}^{n}\left[a\left\{\left(q\sum_{j=1}^{i-1}X_j\right)^b - \left(q\sum_{j=1}^{i-1}X_j + X_i\right)^b\right\}\right] + \left[a\left\{\left(q\sum_{j=1}^{n}X_j\right)^b - \left(T - t_n + q\sum_{j=1}^{n}X_j\right)^b\right\}\right] \quad (3.28)$$

To obtain time terminated MLEs, differentiate the above logarithm (Equation (3.28)) of the likelihood function with respect to each of the three parameters a, b, and q and equate to zero.

$$\frac{\partial \mathrm{Log}(L)}{\partial a} = \frac{n}{a} + \sum_{i=1}^{n}\left[\left(q\sum_{j=1}^{i-1}X_j\right)^b - \left(q\sum_{j=1}^{i-1}X_j + X_i\right)^b\right] + \left[\left(q\sum_{j=1}^{n}X_j\right)^b - \left(T - t_n + q\sum_{j=1}^{n}X_j\right)^b\right] = 0 \quad (3.29)$$

$$\frac{\partial \mathrm{Log}(L)}{\partial b} = \frac{n}{b} + \sum_{i=1}^{n}\left[a\left\{\left(q\sum_{j=1}^{i-1}X_j\right)^b \log\left(q\sum_{j=1}^{i-1}X_j\right) - \left(q\sum_{j=1}^{i-1}X_j + X_i\right)^b \log\left(q\sum_{j=1}^{i-1}X_j + X_i\right)\right\}\right] + \sum_{i=1}^{n}\log\left(q\sum_{j=1}^{i-1}X_j + X_i\right) + \left[a\left\{\left(q\sum_{i=1}^{n}X_j\right)^b \log\left(q\sum_{j=1}^{i-1}X_j\right) - \left(\left(T - t_n + q\sum_{j=1}^{n}X_j\right)\right)^b \log\left(T - t_n + q\sum_{j=1}^{n}X_j\right)\right\}\right] = 0 \quad (3.30)$$

$$\frac{\partial \mathrm{Log}(L)}{\partial q} = (b-1)\sum_{i=1}^{n} \frac{\sum_{j=1}^{i-1} X_j}{q\sum_{j=1}^{i-1} X_j + X_i} + a\sum_{i=1}^{n}\left[b\left(q\sum_{j=1}^{i-1} X_j \right)^{b-1} \left(\sum_{j=1}^{i-1} X_j \right) \right.$$

$$\left. -b\left(q\sum_{j=1}^{i-1} X_j + X_i \right)^{b-1} \left(\sum_{j=1}^{i-1} X_j \right) \right] + a\left[b\left(q\sum_{j=1}^{n} X_j \right)^{b-1} \left(\sum_{j=1}^{n} X_j \right) \right.$$

$$\left. -b\left(T - t_{1,n} + q\sum_{j=1}^{n} X_j \right)^{b-1} \left(\sum_{j=1}^{n} X_j \right) \right] = 0 \tag{3.31}$$

The likelihood, log-likelihood functions, and MLEs for failure terminated multiple repairable system data set are given as follows:

$$L = \prod_{l=1}^{K}\left[\prod_{i=1}^{n_l}\left[ab\left(q\sum_{j=1}^{i-1} X_{l,j} + X_{l,i} \right)^{b-1} \right] \right.$$

$$\left. \times \exp\left[a\left\{ \left(q\sum_{j=1}^{i-1} X_{l,j} \right)^{b} - \left(q\sum_{j=1}^{i-1} X_{l,j} + X_{l,i} \right)^{b} \right\} \right] \right] \tag{3.32}$$

Where K is number of systems (1 = 1, 2,…, K).

Taking log on both sides of Equation (3.32),

$$\ln L = \sum_{l=1}^{K} n_l \log(b) + \sum_{l=1}^{K} n_l \log a + (b-1)\sum_{l=1}^{K}\sum_{i=1}^{n_l} \log\left(q\sum_{j=1}^{i-1} X_{l,j} + X_{l,i} \right)$$

$$+ \sum_{l=1}^{K}\sum_{i=1}^{n_l}\left[a\left\{ \left(q\sum_{j=1}^{i-1} X_{l,j} \right)^{b} - \left(q\sum_{j=1}^{i-1} X_{l,j} + X_{l,i} \right)^{b} \right\} \right] \tag{3.33}$$

To obtain failure terminated MLEs, differentiate the above logarithm of the likelihood function with respect to each of the three parameters *a*, *b*, and *q* and equate to zero.

$$\frac{\partial \mathrm{Log}(L)}{\partial a}=\frac{\sum_{l=1}^{K} n_l}{a}+\sum_{l=1}^{K}\sum_{i=1}^{n_l}\left[\left(q\sum_{j=1}^{i-1}X_{l,j}\right)^{b}-\left(q\sum_{j=1}^{i-1}X_{l,j}+X_{l,i}\right)^{b}\right]=0 \tag{3.34}$$

$$\begin{aligned}\frac{\partial \mathrm{Log}(L)}{\partial b}=&\frac{\sum_{l=1}^{K} n_l}{b}\\&+\sum_{l=1}^{K}\sum_{i=1}^{n_l}\left[a\left\{\left(q\sum_{j=1}^{i-1}X_{l,j}\right)^{b}\log\left(q\sum_{j=1}^{i-1}X_{l,j}\right)\right.\right.\\&\left.\left.-\left(q\sum_{j=1}^{i-1}X_{l,j}+X_{l,i}\right)^{b}\log\left(q\sum_{j=1}^{i-1}X_{l,j}+X_{l,i}\right)\right\}\right]\\&+\sum_{l=1}^{K}\sum_{i=1}^{n_l}\log\left(q\sum_{j=1}^{i-1}X_{l,j}+X_{l,i}\right)=0\end{aligned} \tag{3.35}$$

$$\begin{aligned}\frac{\partial \mathrm{Log}(L)}{\partial q}=&(b-1)\sum_{l=1}^{K}\sum_{i=1}^{n_l}\frac{\sum_{j=1}^{i-1}X_{l,j}}{q\sum_{j=1}^{i-1}X_{l,j}+X_{l,i}}\\&+a\sum_{l=1}^{K}\sum_{i=1}^{n_l}\left[b\left(q\sum_{j=1}^{i-1}X_{l,j}\right)^{b-1}\left(\sum_{j=1}^{i-1}X_{l,j}\right)\right.\\&\left.-b\left(q\sum_{j=1}^{i-1}X_{l,j}+X_{l,i}\right)^{b-1}\left(\sum_{j=1}^{i-1}X_{l,j}\right)\right]=0\end{aligned} \tag{3.36}$$

The likelihood, log-likelihood functions, and MLEs for time terminated multiple repairable system data set are given as follows:

$$
\begin{aligned}
L = \prod_{l=i1}^{K}\Bigg[\prod_{i=1}^{n_l}\Bigg[ab\left(q\sum_{j=1}^{i-1}X_{l,j}+X_{l,i}\right)^{b-1}\Bigg]\times \exp\Bigg[a\Bigg\{\left(q\sum_{j=1}^{i-1}X_{l,j}\right)^{b} \\
-\left(q\sum_{j=1}^{i-1}X_{l,j}+X_{l,i}\right)^{b}\Bigg\}\Bigg]\times \exp\Bigg[a\Bigg\{\left(q\sum_{j=1}^{n_l}X_{l,j}\right)^{b}- \left(T-t_{l,n}+q\sum_{j=1}^{n_l}X_{l,j}\right)^{b}\Bigg\}\Bigg]\Bigg]
\end{aligned}
\tag{3.37}
$$

Taking log on both sides of Equation (3.37), we obtain

$$
\begin{aligned}
\ln L = \sum_{l=1}^{K} n_l \log(b) + \sum_{l=1}^{K} n_l \log a + (b-1)\sum_{l=1}^{K}\sum_{i=1}^{n_l}\log\left(q\sum_{j=1}^{i-1}X_{l,j}+X_{l,i}\right) \\
+\sum_{l=1}^{K}\sum_{i=1}^{n_l}\Bigg[a\Bigg\{\left(q\sum_{j=1}^{i-1}X_{l,j}\right)^{b}-\left(q\sum_{j=1}^{i-1}X_{l,j}+X_{l,i}\right)^{b}\Bigg\}\Bigg] \\
+\sum_{l=1}^{K}\Bigg[a\Bigg\{\left(q\sum_{j=1}^{n_l}X_{l,j}\right)^{b}-\left(T-t_{l,n}+q\sum_{j=1}^{n_l}X_{l,j}\right)^{b}\Bigg\}\Bigg]
\end{aligned}
\tag{3.38}
$$

To obtain time terminated MLEs, differentiate the above logarithm of the likelihood function with respect to each of the three parameters *a, b*, and *q*, and equate to zero.

$$\frac{\partial \text{Log}(L)}{\partial a} = \frac{\sum_{l=1}^{K} n_l}{a} + \sum_{l=1}^{K} \sum_{i=1}^{n_l} \left[\left(q \sum_{j=1}^{i-1} X_{l,j} \right)^b - \left(q \sum_{j=1}^{i=1} X_{l,j} + X_{l,i} \right)^b \right] + \sum_{l=1}^{K} \left[\left(q \sum_{j=1}^{n_l} X_{l,j} \right)^b - \left(T - t_{l,n} + q \sum_{j=1}^{n_l} X_{l,j} \right)^b \right] = 0 \tag{3.39}$$

$$\begin{aligned} \frac{\partial \text{Log}(L)}{\partial b} = {} & \frac{\sum_{l=1}^{K} n_l}{b} \\ & + \sum_{l=1}^{K} \sum_{i=1}^{n_l} \left[a \left\{ \left(q \sum_{j=1}^{i-1} X_{l,j} \right)^b \log\left(q \sum_{j=1}^{i-1} X_{l,j} \right) \right.\right. \\ & \left.\left. - \left(q \sum_{j=1}^{i-1} X_{l,j} + X_{l,i} \right)^b \log\left(q \sum_{j=1}^{i-1} X_{l,j} + X_{l,i} \right) \right\} \right] \\ & + \sum_{l=1}^{K} \left[a \left\{ \left(q \sum_{j=1}^{n_l} X_{l,j} \right)^b \log\left(q \sum_{j=1}^{n_l} X_{l,j} \right) \right.\right. \\ & \left.\left. - \left(T - t_{l,n} + q \sum_{j=1}^{n_l} X_{l,j} \right)^b \log\left(T - t_{l,n} + q \sum_{j=1}^{n_l} X_{l,j} \right) \right\} \right] \\ & + \sum_{l=1}^{K} \sum_{i=1}^{n_l} \log\left(q \sum_{j=1}^{i-1} X_{l,j} + X_{l,i} \right) = 0 \end{aligned} \tag{3.40}$$

$$\frac{\partial \text{Log}(L)}{\partial q} = (b-1)\sum_{l=1}^{K}\sum_{i=1}^{n_l}\frac{\sum_{j=1}^{i-1}X_{l,j}}{q\sum_{j=1}^{i-1}X_{l,j}+X_{l,i}} + a\sum_{l=1}^{K}\sum_{i=1}^{n_l}\left[b\left(q\sum_{j=1}^{i-1}X_{l,j}\right)^{b-1}\right.$$
$$\left.\left(\sum_{j=1}^{i-1}X_{l,j}\right) - b\left(q\sum_{j=1}^{i-1}X_{l,j}+X_{l,i}\right)^{b-1}\left(\sum_{j=1}^{i-1}X_{l,j}\right)\right] + a\sum_{l=1}^{K}$$
$$\left[b\left(q\sum_{j=1}^{n1}X_{l,j}\right)^{b-1}\left(\sum_{j=1}^{n_l}X_{l,j}\right) - b\left(T - t_{l,n} + q\sum_{j=1}^{n_l}X_{l,j}\right)^{b-1}\left(\sum_{j=1}^{n_l}X_{l,j}\right)\right]$$
$$= 0 \tag{3.41}$$

3.3.3.3 Kijima-II Model

The likelihood, log-likelihood functions, and MLEs for failure terminated single repairable system data set are given as follows:

$$L = \prod_{i=1}^{n}\left[ab\left(\sum_{j=1}^{i-1}q^{i-j}X_j + X_i\right)^{b-1}\right]$$
$$\times \exp\left[a\left\{\left(\sum_{j=1}^{i-1}q^{i-j}X_j\right)^{b} - \left(\sum_{j=1}^{i-1}q^{i-j}X_j + X_i\right)^{b}\right\}\right] \tag{3.42}$$

Taking log on both sides of Equation (3.42),

$$\ln L = n\log(b) + n\log a + (b-1)\sum_{i=1}^{n}\log\left(\sum_{j=1}^{i-1}q^{i-j}X_j + X_i\right)$$
$$+\sum_{i=1}^{n}\left[a\left\{\left(\sum_{j=1}^{i-1}q^{i-j}X_j\right)^{b} - \left(\sum_{j=1}^{i-1}q^{i-j}X_j + X_i\right)^{b}\right\}\right] \tag{3.43}$$

To obtain failure terminated MLEs, differentiate the above logarithm of the likelihood function with respect to each of the three parameters *a*, *b*, and *q*, and equate to zero.

$$\frac{\partial \mathrm{Log(L)}}{\partial a} = \frac{n}{a} + \sum_{i=1}^{n}\left[\left(\sum_{j=1}^{i-1} q^{i-j}X_j\right)^{b} - \left(\sum_{j=1}^{i-1} q^{i-j}X_j + X_i\right)^{b}\right] = 0 \tag{3.44}$$

$$\begin{aligned}\frac{\partial \mathrm{Log(L)}}{\partial b} = \frac{n}{b} + \sum_{i=1}^{n}\Bigg[a\Bigg\{ &\left(\sum_{j=1}^{i-1} q^{i-j}X_j\right)^{b} \log\left(\sum_{j=1}^{i-1} q^{i-j}X_j\right) \\ &- \left(\sum_{j=1}^{i-1} q^{i-j}X_j + X_i\right)^{b} \log\left(\sum_{j=1}^{i-1} q^{i-j}X_j + X_i\right)\Bigg\}\Bigg] \\ &+ \sum_{i=1}^{n} \log\left(\sum_{j=1}^{i-1} q^{i-j}X_j + X_i\right) \\ &= 0\end{aligned} \tag{3.45}$$

$$\begin{aligned}\frac{\partial \mathrm{Log(L)}}{\partial q} = (b-1)\sum_{i=1}^{n} &\frac{\sum_{j=1}^{i-1}(i-j)q^{i-j-1}X_j}{\sum_{j=1}^{i-1} q^{i-j}X_j + X_i} \\ &+ a\sum_{i=1}^{n}\Bigg[b\left(\sum_{j=1}^{i-1} q^{i-j}X_j\right)^{b-1}\left(\sum_{j=1}^{i-1}(i-j)q^{i-j-1}X_j\right) \\ &- b\left(\sum_{j=1}^{i-1} q^{i-j}X_j + X_i\right)^{b-1}\left(\sum_{j=1}^{i-1}(i-j)q^{i-j-1}X_j\right)\Bigg] = 0\end{aligned} \tag{3.46}$$

The likelihood, log-likelihood functions, and MLEs for time terminated single repairable system data set are given as follows:

$$L=\prod_{i=1}^{n}\left[ab\left(\sum_{j=1}^{i-1}q^{i-j}X_j+X_i\right)^{b-1}\right]\times\exp\left[a\left\{\left(\sum_{j=1}^{i-1}q^{i-j}X_j\right)^{b}-\left(\sum_{j=1}^{i-1}q^{i-j}X_j+X_i\right)^{b}\right\}\right]\times\exp\left[a\left\{\begin{array}{c}\left(\sum_{j=1}^{n}q^{n-j+1}X_j\right)^{b}-\\\left(T-t_n+\sum_{j=1}^{n}q^{n-j+1}X_j\right)^{b}\end{array}\right\}\right] \tag{3.47}$$

Taking log on both sides of Equation (3.47), we obtain

$$\begin{aligned}\ln L=&\ n\log(b)+n\log a+(b-1)\sum_{i=1}^{n}\log\left(\sum_{j=1}^{i-1}q^{i-j}X_j+X_i\right)\\&+\sum_{i=1}^{n}\left[a\left\{\left(\sum_{j=1}^{i-1}q^{i-j}X_j\right)^{b}-\left(\sum_{j=1}^{i-1}q^{i-j}X_j+X_i\right)^{b}\right\}\right]\\&+\left[a\left\{\left(\sum_{j=1}^{n}q^{n-j+1}X_j\right)^{b}-\left(T-t_n+\sum_{j=1}^{n}q^{n-j+1}X_j\right)^{b}\right\}\right]\end{aligned} \tag{3.48}$$

To obtain time terminated MLEs, differentiate the above logarithm of the likelihood function with respect to each of the three parameters *a*, *b*, and *q*, and equate to zero.

$$\frac{\partial Log(L)}{\partial a} = \frac{n}{a} + \sum_{i=1}^{n}\left[\left(\sum_{j=1}^{i-1} q^{i-j}X_j\right)^b - \left(\sum_{j=1}^{i-1} q^{i-j}X_j + X_i\right)^b\right] + \left[\left(\sum_{j=1}^{n} q^{n-j+1}X_j\right)^b - \left(T - t_n + \sum_{j=1}^{n} q^{n-j+1}X_j\right)^b\right] = 0 \qquad (3.49)$$

$$\frac{\partial Log(L)}{\partial b} = \frac{n}{b} + \sum_{i=1}^{n}\left[a\left\{\left(\sum_{j=1}^{i-1} q^{i-j}X_j\right)^b \log\left(\sum_{j=1}^{i-1} q^{i-j}X_j\right) - \left(\sum_{j=1}^{i-1} q^{i-j}X_j + X_i\right)^b \log\left(\sum_{j=1}^{i-1} q^{i-j}X_j + X_i\right)\right\}\right] + \sum_{i=1}^{n}\log\left(\sum_{j=1}^{i-1} q^{i-j}X_j + X_i\right) + \left[a\left\{\left(\sum_{j=1}^{n} q^{n-j+1}X_j\right)^b \log\left(\sum_{j=1}^{n} q^{n-j+1}X_j\right) = 0 - \left(T - t_n + \sum_{j=1}^{n} q^{n-j+1}X_j\right)^b \log\left(T - t_n + \sum_{j=1}^{n} q^{n-j+1}X_j\right)\right\}\right] = 0 \qquad (3.50)$$

$$\frac{\partial Log(L)}{\partial q}=(b-1)\sum_{i=1}^{n}\frac{\sum_{j=1}^{i-1}(i-j+1)q^{i-j}X_j}{\sum_{j=1}^{i}q^{i-j+1}X_j+X_i}$$
$$+a\sum_{i=1}^{n}\left[b\left(\sum_{j=1}^{i-1}q^{i-j+1}X_j\right)^{b-1}\left(\sum_{j=1}^{i-1}(i-j+1)q^{i-j}X_j\right)\right.$$
$$\left.-b\left(\sum_{j=1}^{i-1}q^{i-j+1}X_j+X_i\right)^{b-1}\left(\sum_{j=1}^{i-1}(i-j+1)q^{i-j}X_j\right)\right]$$
$$+a\left[b\left(\sum_{j=1}^{n}q^{n-j+1}X_j\right)^{b-1}\left(\sum_{j=1}^{n}(n-j+1)q^{n-j}X_j\right)\right.$$
$$\left.-b\left(T-t_n+\sum_{j=1}^{n}q^{n-j+1}X_j\right)^{b-1}\left(\sum_{j=1}^{n_l}(n-j+1)q^{n-j}X_j\right)\right]=0 \tag{3.51}$$

The likelihood, log-likelihood functions, and MLEs for failure terminated multiple repairable system data set are given as follows:

$$L=\prod_{l=1}^{K}\left[\prod_{i=1}^{n_l}\left[ab\left(\sum_{j=1}^{i-1}q^{i-j}X_{l,j}+X_{l,i}\right)^{b-1}\right]\right.$$
$$\left.\times\exp\left[a\left\{\left(\sum_{j=1}^{i-1}q^{i-j}X_{l,j}\right)^{b}-\left(\sum_{j=1}^{i-1}q^{i-j}X_{l,j}+X_{l,i}\right)^{b}\right\}\right]\right] \tag{3.52}$$

Where, K is number of systems (1 = 1, 2,…., K).

Taking log on both sides of Equation (3.52), we obtain

$$\ln L = \sum_{l=1}^{K} n_l \log(b) + \sum_{l=1}^{K} n_l \log a + (b-1)\sum_{l=1}^{K}\sum_{i=1}^{n_l}\log\left(\sum_{j=1}^{i-1} q^{i-j}X_{l,j} + X_{l,i}\right)$$
$$+\sum_{l=1}^{K}\sum_{i=1}^{n_l}\left[a\left\{\left(\sum_{j=1}^{i-1} q^{i-j}X_{l,j}\right)^{b} - \left(\sum_{j=1}^{i-1} q^{i-j}X_{l,j} + X_{l,i}\right)^{b}\right\}\right] \tag{3.53}$$

To obtain failure terminated MLEs, differentiate the above logarithm of the likelihood function with respect to each of the three parameters *a*, *b*, and *q*, and equate to zero.

$$\frac{\partial \mathrm{Log}(L)}{\partial a} = \frac{\sum_{l=1}^{K} n_l}{a} + \sum_{l=1}^{K}\sum_{i=1}^{n_l}\left[\left(\sum_{j=1}^{i-1} q^{i-j}X_{l,j}\right)^{b} - \left(\sum_{j=1}^{i-1} q^{i-j}X_{l,j} + X_{l,i}\right)^{b}\right] = 0 \tag{3.54}$$

$$\frac{\partial \mathrm{Log}(L)}{\partial b} = \frac{\sum_{l=1}^{K} n_l}{b} + \sum_{l=1}^{K}\sum_{i=1}^{n_l}\left[a\left\{\left(\sum_{j=1}^{i-1} q^{i-j}X_{l,j}\right)^{b}\log\left(\sum_{j=1}^{i-1} q^{i-j}X_{l,j}\right) - \left(\sum_{j=1}^{i-1} q^{i-j}X_{l,j} + X_{l,i}\right)^{b}\log\left(\sum_{j=1}^{i-1} q^{i-j}X_{l,j} + X_{l,i}\right)\right\}\right] + \sum_{l=1}^{K}\sum_{i=1}^{n_l}\log\left(\sum_{j=1}^{i-1} q^{i-j}X_{l,j} + X_{l,i}\right) = 0 \tag{3.55}$$

$$\frac{\partial Log(L)}{\partial q} = (b-1)\sum_{l=1}^{K}\sum_{i=1}^{n_l}\frac{\sum_{j=1}^{i-1}(i-j)q^{i-j-1}X_{l,j}}{\sum_{j=1}^{i-1}q^{i-j}X_{l,j} + X_{l,i}}$$

$$+ a\sum_{l=1}^{K}\sum_{j=1}^{n_l}\left[b\left(\sum_{j=1}^{i-1}q^{i-j}X_{l,j}\right)^{b-1}\left(\sum_{j=1}^{i-1}(i-j)q^{i-j-1}X_{l,j}\right)\right.$$

$$\left. - b\left(\sum_{j=1}^{i-1}q^{i-j}X_{l,j} + X_{l,i}\right)^{b-1}\left(\sum_{j=1}^{i-1}(i-j)q^{i-j-1}X_{l,j}\right)\right] = 0$$

(3.56)

The likelihood, log-likelihood functions, and MLEs for time terminated multiple repairable system data set are given as follows:

$$L = \prod_{l=1}^{K}\left[\left[\prod_{l=1}^{n_l}\left[ab\left(\sum_{j=1}^{i-1}q^{i-j}X_{l,j} + X_{l,i}\right)^{b-1}\right]\right.\right.$$

$$\left.\times \exp\left[a\left\{\left(\sum_{j=1}^{i-1}q^{i-j}X_{l,j}\right)^{b} - \left(\sum_{j=1}^{i-1}q^{i-j}X_{l,j} + X_{l,i}\right)^{b}\right\}\right]\right]$$

$$\left.\times \exp\left[a\left\{\begin{array}{c}\left(\sum_{j=1}^{n_i}q^{n-j+1}X_{l,j}\right)^{b} - \\ \left(T - t_{l,n} + \sum_{j=1}^{n_i}q^{n-j+1}X_{l,j}\right)^{b}\end{array}\right\}\right]\right] \quad (3.57)$$

Taking log on both sides of Equation (3.57), we obtain

$$\begin{aligned}\ln L = \sum_{l=1}^{K} n_l \log(b) + \sum_{l=1}^{K} n_l \log a + (b-1) \\ \sum_{l=1}^{K}\sum_{i=1}^{n_l} \log\left(\sum_{j=1}^{i-1} q^{i-j} X_{l,j} + X_{l,i}\right) \\ + \sum_{l=1}^{K}\sum_{i=1}^{n_l}\left[a\left\{\left(\sum_{j=1}^{i-1} q^{i-j} X_{l,j}\right)^{b} - \left(\sum_{j=1}^{i-1} q^{i-j} X_{l,j} + X_{l,i}\right)^{b}\right\}\right] \\ + \sum_{l=1}^{K}\left[a\left\{\left(\sum_{j=1}^{n_i} q^{n-j+1} X_{l,j}\right)^{b} - \left(T - t_{l,n} + \sum_{j=1}^{n_i} q^{n-j+1} X_{l,j}\right)^{b}\right\}\right]\end{aligned} \tag{3.58}$$

To obtain time terminated MLEs, differentiate the above logarithm of the likelihood function with respect to each of the three parameters *a*, *b*, and *q*, and equate to zero.

$$\begin{aligned}\frac{\partial \mathrm{Log}(L)}{\partial a} = \frac{\sum_{l=1}^{K} n_l}{a} + \sum_{l=1}^{K}\sum_{i=1}^{n_l}\left[\left(\sum_{j=1}^{i-1} q^{i-j} X_{l,j}\right)^{b} - \left(\sum_{j=1}^{i-1} q^{i-j} X_{l,j} + X_{l,i}\right)^{b}\right] \\ + \sum_{l=1}^{K}\left[\left(q\sum_{j=1}^{n_l} X_{l,j}\right)^{b} - \left(T - t_{l,n} + q\sum_{j=1}^{n_l} X_{l,j}\right)^{b}\right] = 0\end{aligned} \tag{3.59}$$

$$
\begin{aligned}
\frac{\partial \mathrm{Log}(L)}{\partial b} = {} & \frac{\sum_{l=1}^{K} n_l}{b} \\
& + \sum_{l=1}^{K}\sum_{i=1}^{n_l}\left[a\left\{ \left(\sum_{j=1}^{i-1} q^{i-j} X_{l,j}\right)^{b} \log\left(\sum_{j=1}^{i-1} q^{i-j} X_{l,j}\right) \right.\right. \\
& \left.\left. - \left(\sum_{j=1}^{i-1} q^{i-j} X_{l,j} + X_{l,i}\right)^{b} \log\left(\sum_{j=1}^{i-1} q^{i-j} X_{l,j} + X_{l,i}\right)\right\}\right] \\
& + \sum_{l=1}^{K}\left[a\left\{ \left(\sum_{j=1}^{n_l} q^{n_l-j+1} X_{l,j}\right)^{b} \log\left(\sum_{j=1}^{n_l} q^{n_l-j+1} X_{l,j}\right) \right.\right. \\
& - \left(T - t_{l,n} + \sum_{j=1}^{n_l} q^{n_l-j+1} X_{l,j}\right)^{b} \\
& \left.\left. \log\left(T - t_{l,n} + \sum_{j=1}^{n_l} q^{n_l-j+1} X_{l,j}\right)\right\}\right] \\
& + \sum_{l=1}^{K}\sum_{i=1}^{n_l} \log\left(\sum_{j=1}^{i-1} q^{i-j} X_{l,j} + X_{l,i}\right) = 0 \qquad (3.60)
\end{aligned}
$$

$$\frac{\partial \mathrm{Log(L)}}{\partial q} = (b-1)\sum_{l=1}^{K}\sum_{i=1}^{n_l}\frac{\sum_{j=1}^{i-1}(i-j)q^{i-j-1}X_{l,j}}{\sum_{j=1}^{i-1}q^{i-j}X_{l,j}+X_{l,i}}$$

$$+a\sum_{l=1}^{K}\sum_{i=1}^{n_l}\left[b\left(\sum_{j=1}^{i-1}q^{i-j}X_{l,j}\right)^{b-1}\left(\sum_{j=1}^{i-1}(i-j)q^{i-j-1}X_{l,j}\right)\right.$$

$$\left.-b\left(\sum_{j=1}^{i-1}q^{i-j}X_{l,j}+X_{l,i}\right)^{b-1}\left(\sum_{j=1}^{i-1}(i-j)q^{i-j-1}X_{l,j}\right)\right]$$

$$+a\sum_{l=1}^{K}\left[b\left(\sum_{j=1}^{n_l}q^{n_l-j+1}X_{l,j}\right)^{b-1}\left(\sum_{j=1}^{n_l}(n_l-j+1)q^{n_l-j}X_{l,j}\right)\right.$$

$$\left.-b\left(T-t_{l,n}+\sum_{j=1}^{n_l}q^{n_l-j+1}X_{l,j}\right)^{b-1}\left(\sum_{j=1}^{n_l}(n_l-j+1)q^{n_l-j}X_{l,j}\right)\right]=0 \tag{3.61}$$

Note 3.3.
The MLE equations derived for both Kijima-I and Kijima-II models for various cases are non-linear and complex in nature and cannot be solved easily. These non-linear equations can be solved with the help of software like MATLAB. However, an easier way to obtain the estimators is by maximizing the log likelihood functions. There are various methods to maximize the objective function and any off the shelf softwares can be used for this.

3.3.3.4 Virtual Age–Based Reliability Metrics

The conversion of reliability metrics from real time scale to virtual time scale makes mathematical computation much easier in estimating reliability parameters. The virtual time scale can be reverted into the real time scale later. The Intensity function, MTBF, reliability, availability, and expected number of failure equations [5, 6] based on virtual scale are appended in tabular form as follows in Table 3.2.

Table 3.2 Virtual age–based reliability metrics.

Sl No.	Parameter	Equation	
1	Intensity Function	$u(v_i) = a \times b \times (v_i)^{b-1}$	(3.62)
2	Mean Time Between Failures (MTBF)	$\text{MTBF}(v_i) = \frac{1}{u(V_i)}$	(3.63)
3	Reliability	For first failure, $R(v_1) = \exp\{-a(v_1)^b\}$ (Note: $v_1 = qt_1$)	(3.64)
		For subsequent failures, $R(v_i \mid v_{i-1}) = \frac{R(v_1 + v_{i-1})}{R(v_{i-1})} = \exp a\left[(v_{i-1})^b - (v_i + v_{i-1})^b\right]$	(3.65)
4	Availability	$A(v_i) = \frac{\text{MTBF}(v_i)}{\text{MTBF}(v_i + \text{MTTR})}$ where *MTTR* stands for Mean Time to Repair	(3.66)
5	Expected Number of Failures	$E[N(t)] = \int_0^{Vi} u(V_i)dV$ $i = 1, 2, \ldots n$	(3.67)

Solved Examples

Example 3.3.

Consider the failure times of aero engines of Example 3.1. Estimate the scale, shape parameters and the repair effectiveness index using GRP Kijima-I model. Plot the intensity function curve. If mean time to repair of the aero engines (MTTR = 528 hours). Plot the availability curve. What will be the MTBF and availability of the aero engines at t = 550 hours?

Solution.

Maximizing Equation (3.23) or solving Equations (3.24), (3.25), and (3.26) (see Note 3.3), we obtain the following result:

$$a = 0.00022,\ b = 1.35,\ q = 0.75,\ MTTR = 528 \text{ hours (given)}$$

Using Equations (3.62), (3.63), and (3.66), intensity function, MTBF, and availability can be estimated.

The intensity function equation from the values obtained works out to be

$$u\ (v_i) = 0.00022 \times 1.35 \times (v_i)^{0.35}$$

The intensity function curve is plotted at Figure 3.6.

$$MTBF\ (t = 550\ hours) = 394.5602\ hours$$

The availability curve is plotted at Figure 3.7.

$$Availability\ (t = 550\ hours) = 0.4277$$

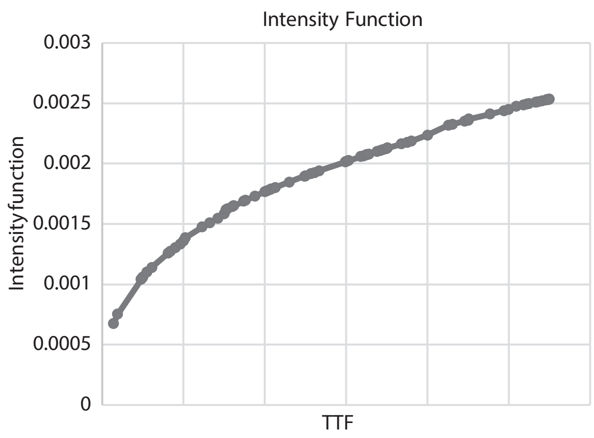

Figure 3.6 Intensity function plot for Example 3.3.

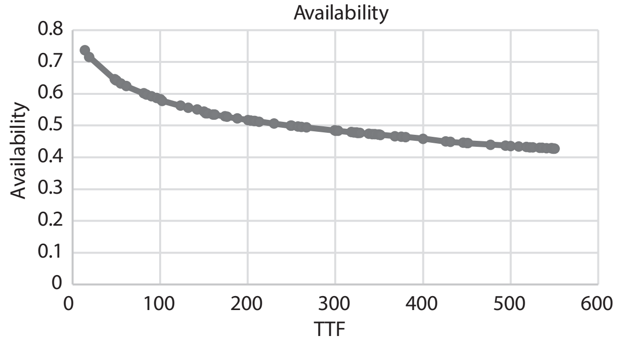

Figure 3.7 Availability plot for Example 3.3.

Example 3.4. [7]
To illustrate the general application of GRP, consider a system tested for T = 395.2 hours with the 56 failure times given in Table 3.3 below. The first failure was recorded at 0.7 hours into the test; the second failure was recorded 3 hours later at 3.7. The last failure occurred at 395.2 hours into the test and the system was removed from the test. This failure data are failure truncated. Find out a, b, and q value for the given data set using Kijima MLEs.

Table 3.3 Time to failure data (hours) of Example 3.4.

0.7	0.63	125	244	315	366
3.7	72	133	249	317	373
1	99	151	250	320	379
1	99	163	260	324	389
1	100	164	263	324	394
2	102	174	273	342	395.2
4	112	177	274	350	
5	112	191	282	355	
5	120	192	285	364	
5	121	213	304	364	

Solution.
Here, $n = 56$, $K = 1$, and $T = 395.2$ hours.

Maximizing Equation (3.23) or solving Equations (3.24), (3.25), and (3.26) (see Note 3.3), we obtain the following results for Kijima-I model:

$$b = 0.9372,\ a = 0.2061,\ q = 1.0$$

Maximizing Equation (3.43) or solving Equations (3.44), (3.45), and (3.46) (see Note 3.3), we obtain the following results for Kijima-II model:

$$b = 0.24725,\ a = 0.89442,\ q = 0.93$$

Example 3.5. [7]
Suppose $K = 6$ systems are observed during $[0, T_i]$ hours, $= 1,\ldots, k$. That is, the data are time truncated with $T_1 = 8760$, $T_2 = 5{,}000$, $T_3 = 6{,}200$, $T_4 = 1{,}300$, $T_5 = 2{,}650$, $T_6 = 500$. Failure data are given in Table 3.4. This failure data set is from multiple repairable systems. Find out a, b, and q values for the given data set using Kijima MLEs.

Solution.
Here, $K = 6$, $T_1 = 8760$, $T_2 = 5{,}000$, $T_3 = 6{,}200$, $T_4 = 1{,}300$, $T_5 = 2{,}650$, $T_6 = 500$.

Maximizing Equation (3.38) or solving Equations (3.39), (3.40), and (3.41) (see Note 3.3), we obtain the following results for Kijima-I model:

$$b = 1.238,\ a = 0.00018,\ q = 0.10$$

Table 3.4 Time to failure data (hours) of Example 3.5.

System 1	System 2	System 3	System 4	System 5	System 6
2,227.08	772.9542	900.9855	411.407	688.897	105.824
2,733.229	1,034.458	1,289.95	1,122.74	915.101	
3,524.214	3,011.114	2,689.878			
5,568.634	3,121.458	3,928.824			
5,886.165	3,624.158	4,328.317			
5,946.301	3,758.296	4,704.24			
6,018.219		5,052.586			
7,202.724		5,473.171			

Maximizing Equation (3.58) or solving Equations (3.59), (3.60), and (3.61) (see Note 3.3), we obtain the following results for Kijima-II model:

$$b = 1.358,\ a = 0.000068,\ q = 0.55$$

3.3.4 Summary

A repairable system may end up at other than the two extremities of AGAN and ABAO conditions after repair, *viz.*, *better than old but worse than new*, *better than new*, and *worse than old*. The quest to have more accurate analyses and predictions, the GRP can be of great interest to reduce the modeling uncertainty resulting from the repair assumptions as mentioned in Section 3.1. This chapter briefly takes up some basic terminologies related to repairable systems followed by modeling and analysis of repairable systems with the help of GRP based ARA concept in form of Kijima-I and Kijima-II virtual age models. The chapter provides likelihood, log-likelihood functions, and MLEs for NHPP, Kijima-I and Kijima-II virtual models for failure and time terminated data for both single and multiple repairable systems. Virtual age based reliability metrics are also provided to reduce the mathematical computing. All the models are demonstrated with the help of solved examples for better comprehension of the readers. Further reading on ARI models have been given for interested readers.

Exercises

(1) A 6-year-old regional transit bus experiences minimal repair upon failure. It was found to have an intensity function given by $u\ (t) = 0.0464\ t^{2.1}$ with t measured in years. Estimate the following:
   a. MTBF (instantaneous)
   b. Expected number of failures over the coming year.
   c. Probability that exactly one failure occurs in the seventh year.

(2) [4] Consider the time between failures of a compressor presented in Table 3.5 below: Estimate the shape and scale parameters. Also, estimate the REI followed by the intensity function and reliability at t = 3,500 hours. Use Kijima-I model for this failure terminated compressor failure time's data.

Table 3.5 Time between failures for a compressor.

No. of Failures	Time Between Failures (h)	No. of failures	Time Between Failures (h)
1	3456	13	360
2	1584	14	998
3	236	15	656
4	516	16	180
5	1820	17	244
6	452	18	1528
7	432	19	44
8	1264	20	3064
9	3072	21	324
10	384	22	1528
11	2448	23	348
12	32	24	336

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Further Reading

Though this chapter is mainly oriented toward providing methods on Kijima-based ARA models but it touches upon the literature in brief about ARI models to provide a broad spectrum of the available models on imperfect repair.

ARI Models

Unlike Kijima virtual age models, in this approach, the repair effect is characterized by the change induced on the failure intensity before and after failure. Repair effectiveness is expressed by a reduction of failure intensity in ARI models.

Gasmi *et al.* [8] worked on GRP virtual age and failure intensity, considering the effect of repair on failure intensities through the virtual age process. Let t_n be the time of completion of n^{th} repair, V_n be the virtual age, $u_n(t)$ be the failure intensity at time t after the n^{th} repair, and ξ_1, ξ_2 are constants that capture the effect of minor and major repairs, respectively. Then, the reduction in intensity is

$$u_{n+1}(t) = u(t - t_n + v_n)$$

$$v_n = \xi_{1,n} \times \xi_{2,n} [v_{n-1} + (t_n - t_{n-1})]$$

A mechanical system with different phases of operating conditions (loaded and unloaded) producing different failure intensities and random failures for both the phases are considered. Minimal, minor, and major repair on failure with change in intensity are modeled. Finally, the optimal maintenance policy on the basis of this model is selected. Kahle [9] uses Kijima models based on failure intensity as a function of virtual age and estimates parameters for Weibull distribution as base line intensity function. She considers Finkelstein's [10–13] time scale transformation to model hazard function and also presents examples for both the Kijima models considering repair actions are discrete time events and repair effectiveness is different at points. Guo *et al.* [14] modeled the system age as expected cumulative number of failure or repair metric. A closed form solution is derived with log linear base line intensity. The author determines the model properties, MLEs for parameter estimation, variance, and covariance metrics, and conducts likelihood ratio test to complete the model. Further results are compared with Kijima models. Doyen and Gaudoin [15] propose two new classes of imperfect repair models wherein the repair effect is described by the changes induced on the failure intensity. The distribution of the failure processes is completely given by the conditional failure intensity defined as

$$u_t = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \Pr(N_{t+\Delta t} - N_t = 1/H_t),$$

where N_t is the number of failures observed up to time t and the probability is conditioned on the information on all failure events H_t.

The time reach of the effect (on failure intensity) of repair action is also been modeled in the literature. Chan and Shaw [16] propose that repair reduces failure intensity by an amount proportional to the current failure intensity. This model is therefore called ARI with infinite memory (ARI_∞) model. The model thus assumes that repair actions impact the overall wear of the system. On the other hand, other researchers have worked to develop models that do away with the assumption of impact of repair action on the global wear of system. Thus, the *ARI* model with memory one called, (ARI_1), considers that repair actions cannot reduce the global wear of the system, but only the relative wear since the last repair. In generalized (ARI_m) model, repair actions reduce the relative wear since the last m repair. Both ARI_1 and ARI_∞ are thus special cases of ARI_m. The authors also compare reduction of age models with the reduction of intensity models and demonstrate that a reduction of age model has a failure intensity which is a function of its virtual age. Hence, properties of failure intensity of age reduction models are same as intensity reduction models. They conclude that we can build reduction of age models by analogy with reduction of intensity models. They arrive at the result that ARI_∞ model is similar to the one introduced by Brown *et al.* [17] and the ARI_1 appears to be same as the Kijima model.

Dijoux [18] proposed a new reliability model in which he combined a bathtub shaped ageing and imperfect maintenance. The repair effect is modeled by reduction of the system virtual age. The most important derivation of the proposed model is that with help of maintenance efficiency the system useful life can be extended.

4

Goodness-of-Fit Tests for Repairable Systems

4.1 Introduction

The Goodness-of-Fit (GOF) of a statistical model describes how well it fits into a set of observations. GOF indices abridge the inconsistency between the observed values and the values expected under a statistical model. It can be said that the concluding step in selection of a theoretical distribution/process is to perform a statistical test for GOF. Such a test, pertinent to the system failure data analysis, compares a null hypothesis H_0 with an alternative hypothesis H_1 having the following form:

H_0: The failure times came from the specified distribution/process.
H_1: The failure times did not come from the specified distribution/process.

A test statistic is estimated based on sample of failure times and is then compared with a critical value (CV) obtained from a table of such values. If the test statistic is found less than the CV, the null hypothesis H_0 is accepted; otherwise, the alternative hypothesis H_1 is acknowledged. The level of significance of the test and the sample size are needed to determine the CV. The probability of erroneously rejecting the null hypothesis and accepting the alternate hypothesis is called the significance level.

Table 4.1 (hypothesis tests) below shows the four probable cases that may occur. Because of the randomness inherent in the sampling process, the test statistic has a probability of exceeding the CV even though H_0 is true. This results in a Type I error having a probability of occurrence equal to the level of significance (α). It is also possible for the test statistic to be less than the CV even though H_1 is true. This results in Type II error. It occurs with a probability that is usually controlled indirectly by the specification of the level of significance and the sample size.

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Table 4.1 Hypothesis tests.

	H_0 **true**	H_1 **true**
Accept H_0	Correct decision	Type II error
Accept H_1	Type I error	Correct decision

It was discussed in the previous chapter that the Power Law model is an extension of the Weibull distribution. In other words, the Weibull distribution addresses the very first failure and the PLP addresses each succeeding failure for a repairable system. Hence, it becomes imperative to check that the first failure times follow Weibull distribution or not. Besides other GOF tests, it is also desirable to test whether some sort of trend exists within the data. Hence, a GOF test to identify such trend becomes essential.

This chapter presents Mann's test for the Weibull distribution, followed by Laplace trend test to check whether some sort of trend exists within the data. The trend tests indicates whether the process is improving or showing increasing intervals between failure arrivals or the process is degrading or showing decreasing intervals between failure arrivals. If a deteriorating trend is observed, then one can decide upon reviewing existing maintenance policies.

The chapter then presents various GOF tests for PLP followed by the Kijima-I virtual age concept-based GOF test model for repairable systems which is a modification of the present Cramer-Von Mises (CVM) GOF test model available for NHPP.

4.2 Mann's Test for the Weibull Distribution

A specific test for the Weibull failure distribution is developed by Mann [1], which is directly applied on the failure data and devoid of any distribution parameter determination a priori. The null and alternative hypotheses for this test are as follows:

H_0: The failure times are Weibull.
H_1: The failure times are not Weibull.

The test statistic is computed by using (4.1)

$$M = \frac{k_1 \sum_{i=k_1+1}^{r-1} [(\ln t_{i+1} - \ln t_i)/M_i]}{k_2 \sum_{i=1}^{k_1} [(\ln t_{i+1} - \ln t_i)/M_i]} \tag{4.1}$$

where $k_1 = \left[\frac{r}{2}\right]$, $k_2 = \left[\frac{r-1}{2}\right]$, k_2 are the number of degrees of freedom for the denominator and numerator, respectively.

Here, r is the number of failures, n is the sample size, and $M_i = Z_{i+1} - Z_i$.

$$Z_i = ln\left[-ln\left(1 - \frac{i-0.5}{n+0.25}\right)\right]$$

and $[r]$ is the integer portion of the number r.

If $M > F_{crit}$, then H_1 is accepted. Values for F_{crit} may be obtained from tables of the F-distribution (Appendix D), if one lets the number of degrees of freedom for the numerator be $2k_2$ and the number of degrees of freedom for the denominator be $2k_1$. This test is for the two-parameter Weibull distribution.

Example 4.1.

The following 76 failure times were recorded for engine fuel pump of an aero engine of a particular make and variant. Conduct Mann's test to check whether the failure times follow Weibull distribution at a significance level of 0.001.

14.07, 14.57, 25.92, 48.45, 55.03, 61.30, 68.18, 76.43, 81.63, 90.30, 102.57, 106.62, 123.70, 152.60, 154.07,160.90, 164.05, 168.22, 174.85, 203.38, 226.77, 230.78, 247.87, 249.25, 254.35, 265.10, 267.73, 277.12, 287.52, 87.55, 299.87, 303.52, 313.17, 318.62, 321.70, 324.93, 325.75, 326.05, 342.95, 349.20, 352.07, 394.58, 414.20, 426.85, 432.10, 451 .42, 531.87, 536.87, 78.92, 585.88, 588.50, 630.13, 631.23, 641.38, 648.23, 661.27, 667.55, 697.52, 701.47, 733.52, 742.67, 750.48, 789.37, 792.67, 818.55, 829.23, 851.67, 865.07, 899.32, 964.87, 966.47, 975.97, 1041.08, 1067.18, 1268.22, 1291.35.

Solution.

H_0: The failure times are Weibull.
H_1: The failure times are not Weibull.

Here, n = 76, $k_1 = 38$, $k_2 = 37$
Test statistic,

$$M = \frac{38 \times 29.17}{37 \times 27.79} = 1.0779 \quad \text{(refer Equation (4.1))}$$

From Appendix D,

$$F_{crit\ 0.001,74,76} = 2.1$$

Since, $M = 1.0779 < F_{crit\ 0.001,74,76}$, H_0 is accepted.

4.3 Laplace Trend Test

The Laplace trend test [2] can determine whether the system is deteriorating, improving, or if there is no trend at all through the hypothesis that a trend does exist within the data. The test statistic, U can be estimated using (4.2)

$$U = \frac{\dfrac{\sum_{i=1}^{N} X_i}{N} - \dfrac{T}{2}}{T \times \sqrt{\dfrac{1}{12N}}} \tag{4.2}$$

where T: total operating time (termination time), X_i: age of the system at the i^{th} successive failure, and N: total number of failures.

The test statistic U is approximately a standard normal random variable. The CV is read from the standard normal tables (*Appendix E*) with a given significance level.

Example 4.2.
Twenty seven failures are recorded for an aero engine for a period of 550 hours. Does the data exhibit any trend? If yes what type of trend exists within the data? Consider a significance level of 0.10. The failure times are as follows:

324.93, 400.47, 531.27, 342.95, 287.52, 317.1, 531.87, 426.85, 321.7, 337, 495.97, 48.45, 408.75, 325.75, 349.2, 451.42, 414.2, 102.57, 164.05, 176.92, 160.9, 461.73, 123.7, 299.87, 318.62, 203.38, 521.57.

Solution.
The Laplace test statistic is estimated to be

$$U = \frac{\left[\left(\frac{8848.68}{27}\right) - \frac{550}{2}\right]}{550 \times \sqrt{\frac{1}{12 \times 27}}} = 1.7257. \quad \text{(refer Equation (4.2))}$$

If $-1.645 < U < 1.645$ at $\alpha = 0.10$ (from standard normal tables, *Appendix E*) then fail to reject the hypothesis of no trend.

Since $U > 1.645$, deteriorating trend exists within system.

4.4 GOF Models for Power Law Process

We have discussed in the previous chapter that the reliability modeling of repairable systems is often represented by a process rather than a distribution. The most popular process model is the PLP model. Note that the Weibull distribution addresses the very first failure and the PLP model addresses each succeeding failure for a repairable system. The ensuing section presents some of GOF tests for assessing the validity of PLP.

4.4.1 Crow/AMSAA Test

The Crow/AMSAA test [3] is based on the assumption that a failure intensity of $u(t) = abt^{b-1}$ is appropriate. When $b = 1$, failure intensity reduces to $u(t) = a$ which means the failure process follows a *HPP*. Then, the test involves whether an estimate of b is significantly different from 1. The hypothesis test is

H_0: $b = 1(HPP)$
H_1: $b \neq 1(NHPP)$

For one system on test, the maximum likelihood estimate (MLE) for b is

$$\hat{b} = \frac{N}{\sum_{i=1}^{N-1} ln\left(\frac{T_N}{T_i}\right)}$$

where N = number of observed failures and T_i = i^{th} *failure arrival time*

Crow [3] argued that the test statistic, $2Nb/\hat{b}$ is chi-squared distributed with $2N$ degree of freedom. So, considering the null hypothesis, the reject criteria is given by

$$\text{Reject } H_0 \quad \text{if } \frac{2N}{\hat{b}} < \chi^2_{2N,1-\alpha/2} \text{ or } \frac{2N}{\hat{b}} > \chi^2_{2N,\alpha/2}$$

4.4.2 Common Beta Hypothesis (CBH) Tests

When conducting an analysis of data consisting of multiple systems, it is expected that each of the systems perform in an identical manner. In particular, it is also expected that the inter-arrival rate of the failures across the systems should be fairly consistent.

Let there be K number of systems. The CBH test [4] is used to compare the intensity functions of the individual systems by comparing the b_q, $q = 1, 2 \ldots K$ values of each system. Each system has an intensity function given by [3]

$$u_q(t) = a_q \times b_q \times t^{b_q - 1} \tag{4.3}$$

Therefore, the *CBH* Test tests the hypothesis, H_0 such that $b_1 = b_2 = \ldots = b_K$.

The conditional MLE of b is given by (4.4)

$$\hat{b} = \frac{\sum_{q=1}^{K} M_q}{\sum_{q=1}^{K} \sum_{i=1}^{Mq} Ln\left(\frac{T_q}{X_{iq}}\right)} \tag{4.4}$$

where M_q: number of failures of the q^{th} system, K: number of systems, T_q: last failure time of the q^{th} system, and X_{iq}: i^{th} failure time of the q^{th} system.

For each system, it is assumed that

$$\chi^2_q = \frac{2M_q b_q}{\hat{b}_q} \tag{4.5}$$

are conditionally distributed as independent chi-squared random variables with $2M_q$ degrees of freedom.

When $K = 2$ the null hypothesis, H_0 can be tested using the following statistic:

$$F = \frac{\dfrac{\chi_1^2}{2M_1}}{\dfrac{\chi_2^2}{2M_2}} \tag{4.6}$$

If H_0 is true, then F equals $\frac{\hat{b}_2}{\hat{b}_1}$ and conditionally has an F-distribution with $(2M_1, 2M_2)$ degrees of freedom. The CV F can then be determined by referring to the chi-squared tables (Appendix F).

Now, if, $K \geq 2$ then the likelihood ratio procedure [4] can be used to test the hypothesis $b_1 = b_2 = \ldots b_K$.

Consider the following statistic:

$$L = \sum_{q=1}^{K} M_q ln(\hat{b}_q) - M ln(b^*) \tag{4.7}$$

where $M = \sum_{q=1}^{K} M_q$

and

$$b^* = \frac{M}{\sum_{q=1}^{K} \dfrac{M_q}{\hat{b}_q}} \tag{4.8}$$

Also, let

$$a = 1 + \frac{1}{6(K-1)}\left[\sum_{q=1}^{K} \frac{1}{M_q} - \frac{1}{M}\right] \tag{4.9}$$

The statistic D is calculated such that

$$D = \frac{2L}{a} \tag{4.10}$$

The statistic D is approximately distributed as a chi-squared random variable with $K - 1$ degrees of freedom. Then, after calculating D, the chi-squared tables with $K - 1$ degrees of freedom can be referred to determine the critical points. H_0 is true if the statistic D falls between the critical points.

Example 4.3.
Consider the failure times of aero engines given in Example 3.2**.** Conduct a Common Beta hypothesis test to check that the inter-arrival rate of the failures across the aero engines is fairly consistent at 5% significance level.

Solution.

$$H_0 = b_1 = b_2 = \cdots.. = b_{18} \qquad \text{(where } \alpha = 0.05\text{)}$$

Here, $M = 27$, $K = 18$

With the help of Equations (4.4) to (4.10), the following values are obtained:

$$b^* = \frac{27}{17.2718} = 1.56$$

$$L = [18.7243 - \{27 \times Ln(1.5632)\}] = 6.6616$$

$$a = 1 + \left(\frac{1}{6 \times 17} \times 13.5\right) = 1.1324$$

The CBH test statistic

$$D = \frac{2 \times 6.6616}{1.1324} = 11.766$$

Since $8.67 < D < 27.6$ with ($K - 1 = 17$) degrees of freedom, from chi-squared table (*Appendix F*), the null hypothesis is accepted at 5% significance level.

4.4.3 CVM Test

As discussed earlier the minimal repair process assumes a NHPP based on the PLP. Like other tests explained in previous sections, to determine whether the NHPP is a more appropriate model than the constant failure rate model (HPP), the hypotheses tested are [1].

H_0: A NHPP with intensity abt^{b-1} models the data.
H_1: A NHPP with intensity abt^{b-1} does not model the data.

An unbiased estimator for the parameter, b for the failure data is obtained from

$$\bar{b} = \frac{n-2}{n}\hat{b} \tag{4.11}$$

where

$$\hat{b} = \frac{n}{(n-1)\ln t^{*} - \sum_{i=1}^{n-1} \ln t_i}$$

and

$$t^{*} = \left(\begin{array}{c} t_n \textit{ if the test is failure terminated} \\ t > t_n \textit{ if the test is time terminated} \end{array} \right)$$

$t_1 < t_2 < \cdots < t_n$ are n successive failure times and t^* is the system time. The CVM test statistic is computed from

$$C_M^2 = \frac{1}{12M} + \sum_{I=1}^{M} \left[\left(\frac{t_i}{t_k} \right)^{b} - \frac{2i}{2M} \right]^2 \tag{4.12}$$

where $t_i = i^{th}$ failure time
b = shape parameter
$M = n$ for time terminated data
$M = n - 1$ for failure terminated data
$t_k = T$ for time-terminated data
$t_k = t_k$ for failure terminated data

Then, for a given level of significance, α, the decision can be made on accepting or rejecting the null hypothesis H_0 by applying the following rule:

$$\text{Reject, if} \quad C_M^2 > C_{cr}^2$$

$$\text{Accept, if} \quad C_M^2 \le C_{cr}^2$$

where C_{cr}^2 is the critical or allowable value for C_M^2 corresponding to the specified significance level α (refer Appendix G)

Example 4.4.
Consider the following 27 failure times of a particular aero engine. Check whether the failure data follows NHPP. Consider $\alpha = 0.10$

48.45, 102.57, 123.70, 160.90, 176.92, 203.38, 28752, 299.87, 317.10, 318.62, 321.70, 324.93, 325.75, 337, 342.95, 349.20, 400.47, 408.75, 414.20, 426.85, 451.42, 461.73, 495.97, 521.57, 531.27, 531.87.

Solution.
Here, $N = 27$, $\alpha = 0.10$

$$\hat{b} = \frac{26}{17.2718} = 1.51 \qquad \text{(refer (4.11))}$$

CVM test statistic

$$C_M^2 = \left(\frac{1}{12 \times 27} + 0.07565 \right) = 0.0787 \qquad \text{(refer (4.12))}$$

CV (Appendix G) for $N = 27$, and $\alpha = 0.10$ is 0.172

Since, $C_M^2 < CV$, the failure times for the repairable systems follow NHPP.

4.5 GOF Model for GRP Based on Kijima-I Model

GOF test for Kijima-I virtual age model [5] is a modified version of CVM GOF test model explained in Section 4.4.3. The intensity function for GRP based on KI model is given by (4.13).

$$u(t_i, a, b, q) = \frac{f(t_i)}{1 - f(t_i)} = a \times b \times \left(t_i + q \sum_{j=1}^{i-1} t_j \right)^{b-1} \qquad (4.13)$$

The hypothesis tested is

H_0: A GRP with intensity function Equation (4.13) describes the failure data.

H_1: The above process does not describe the failure data.

The present CVM GOF statistic used for NHPP is computed as explained in Section 4.4.3. Replacing t_i with V_i as expressed in Equation (3.16) and t_k with V_n as presented at Equation (3.15) in Equation (4.14), the following equation is arrived at

$$MC_{MKI}^2 = \frac{1}{12M} + \sum_{i=1}^{M}\left[\left(\frac{V_i}{V_n}\right)^b - \frac{2i-1}{2M}\right]^2 \tag{4.14}$$

Substituting expressions for V_i and V_n (refer (3.17)) in Equation (4.15), following equation is obtained

$$MC_{MKI}^2 = \frac{1}{12M} + \sum_{i=1}^{M}\left[\left(\frac{q\sum_{j=1}^{i} x_j}{q\sum_{i=1}^{n} x_i}\right)^b - \frac{2i-1}{2M}\right]^2 \tag{4.15}$$

where MC_{MKI}^2: modified CVM GOF statistic for GRP based on Kijima-I ARA model.

After estimation of V_i and V_n and b as explained in detail in the previous chapter, for a given level of significance, α, the decision can be made on rejecting, or not rejecting, hypothesis H_0 by applying the following rule:

$$\text{Reject, if} \quad MC_{MKI}^2 > C_{cr}^2$$

$$\text{Do not reject, if} \quad MC_{MKI}^2 > C_{cr}^2$$

where C_{cr}^2 is the critical or allowable value (Appendix G) for MC_{MKI}^2 corresponding to the specified significance level α.

Thus, it is observed that the concept of virtual age as conceived by Kijima and the effect of q is incorporated in the present form of CVM GOF model. This modified CVM GOF test equation can now be used for GRP based on Kijima-I concept.

Example 4.5.
Consider the failure times of aero engines as given at Example 3.1. Conduct a GOF test to check whether the failure times follow Kijima- I model.

Solution.
Maximizing Equation (3.23) or solving Equations (3.24), (3.25), and (3.26) (see Note 3.3), we obtain the following result:

$$\alpha = 0.00022,\ b = 1.35,\ q = 0.75,$$

Using (4.15),

$$MC_{MKI}^2 = \frac{1}{12 \times 67} + \sum_{i=1}^{67}\left[\left(\frac{0.75\sum_{j=1}^{i} x_j}{0.75\sum_{i=1}^{n} x_i}\right)^{1.35} - \frac{2i-1}{2\times 67}\right]^2 = 0.3249$$

The value of MC_{MKI}^2 obtained is 0.3249 which is below the CV at significance level of 0.01.. Thus, the failure times follow Kijima-I model.

4.6 Summary

This chapter presents Mann's test for the Weibull distribution, followed by Laplace trend test to check whether some sort of trend exists within the data. The chapter then presents various GOF tests for PLP followed by the Kijima-I virtual age concept-based GOF test model for repairable systems which is a modification of the present CVM GOF test model available for NHPP. Solved examples are provided for better comprehension of the subject for the readers.

Exercises

(1) Following failure times are acquired from a test where the test was terminated after 15 failures:
3, 15, 35, 58, 113, 187, 225, 465, 732, 1123, 1587, 2166, 5423, 8423, 12,035.
Check whether the failure times follow NHPP. Take $\alpha = 0.10$.

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5
Maintenance Modeling in Repairable Systems

5.1 Introduction to Maintenance Policies Using Kijima Virtual Age Model

Maintenance actions can be mainly classified into two: 1) corrective maintenance (CM) and 2) preventive maintenance (PM). CM also termed as repair is performed on occurrence of failure. PM, on the other hand, is performed at fixed intervals of time to prevent the system from failure or malfunctions. PM can be both time-based and age-based and is also termed proactive maintenance in literature since it includes the predictive maintenance as well. But, in predictive maintenance, the system is continuously monitored with the help of sensors to predict the failures before it occurs. PM is divided into two categories in terms of activities performed on the system, 1) normal scheduled PM and 2) periodic overhauls. Scheduled PM includes timely inspections, lubrication, small adjustments in the system, filter cleaning, oil filling, or changing, etc., whereas overhaul includes thorough examination, dismantling of whole system, repairs or replacements of the parts if required, functional testing of systems, sub-systems, components, followed by re–assembling of the complete system. Overhaul reviews the system to such extent that it is assumed that the system is restored to as-good-as-new (AGAN) condition.

Basically, any type of intervention or maintenance action in repairable systems, whether it is CM or PM, changes its state to some extent in real life scale as discussed earlier. Furthermore, most of the large repairable systems undergo PM, CM, as well as overhaul. Overhaul is considered essential and important PM for large, complex, and critical systems. The organizations dealing with complex repairable systems state their maintenance policy in terms of specified periods for PM and overhaul.

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A brief literature review on maintenance modeling for repairable systems is presented at Table 5.1 for better appreciation of the subject by the readers.

In this chapter, the maintenance policies for repairable systems based on Kijima-I virtual age model are developed and are explained in context of military aviation (MA) with the help of reliability data of aero engines and combat aircraft. The methodologies described in this chapter for these maintenance policies are equally applicable to any other complex repairable equipment having similar maintenance policies. The chapter considers three variants of the same aero engine as a case study to demonstrate the methodology for determination of high failure rate components (HFRCs) and review of present maintenance policies. The presented case study is on a turbojet with twin spool compressor, an annular type combustion chamber, two stage gas turbine, and of reheated type with a variable area jet nozzle. The aero engines are put back into operation after general repair. The aero engines undergo periodic overhauls; Original Equipment Manufacturer (OEM)–specified TBO being 550 hours. Within an overhaul cycle, the aero engines are subjected to OEM specified periodic maintenance at fixed intervals of 50 hours (i.e. at 50, 100, 150, …, 500 hours). The OEM-specified total technical life (TTL) of the aero engine is 1,800 hours.

For further treatment of declared HFRCs, the chapter also provides a revised maintenance policy by reviewing the time between overhauls (TBOs) for the HFRCs.

5.2 Need for HFRC Threshold

In general, there has been a practice in MA to designate certain frequently failing components as HFRC. This categorization of components is done based on intuition, experience, and the number of failures and are considered critical to the working of the system. Such components are shortlisted for reliability improvement as a part of corrective measures. The components, which fail in the fielded units, are sent to the respective depots or at an appropriate level of maintenance channel for repair. Depots not only handle components arriving for repairs but also induct components being sent for defect investigations and overhaul. Thus, at a given point of time, the repair/overhaul line is heavily loaded with the task of repair/overhaul. Such components are shortlisted as per the existing criteria for reliability improvement measures at depots. The HFRCs pose an extra burden on the repair/overhaul lines which have components already lined up for repair/overhaul. If more components are categorized as HFRCs, then the timely

Table 5.1 Maintenance modeling in repairable systems.

Authors	Brief Work Summary
Kijima *et al.* [1]	Develop a stochastic model for a repairable system which is maintained by a general repair by introducing the virtual age of a system. The model includes the minimal repair case as a special case.
Kijima and Sumita [2]	Develop a model for the long run expected cost per unit time of the system for a general repair which is performed upon a failure. The effect from the general repair is investigated through data and numerical examples to conclude whether minimal repair assumption is acceptable or not.
Makis and Jardine [3]	Develop optimal replacement policy formulation with GRP for slowly deteriorating system with high replacement cost. Authors propose optimal replacement policy based on semi Markov decision process for infinite time horizon as a function of optimum expected average cost.
Jack [4]	Explains implementation of Kijima models for a combined PM and CM cycle deriving likelihood function for parameter estimation and also presents simulation to determine optimal PM policy.
Shirmohammadi *et al.* [5]	Defines optimal maintenance policy for repairable system as preventive replacement cycle time with emergency repair on failure and age based replacement at preventive maintenance time. For this optimal cut off age is a decision parameter which is estimated such that replacement cost can be minimized.
Scarsini and Shaked [6]	Develop a model with concept of benefit rate. Any item generates some benefit during its operating life or age and this benefit rate is modeled as function of virtual age. The authors also derived monotonicity property for the model.
Yevkin and Krivtsov [7]	Demonstrate the efficiency of the approximate and improved Monte Carlo methods in providing the solutions with application to optimal maintenance problems.

induction and production of components are likely to be badly impacted. Note that the threshold for declaring HFRCs is purely based on intuition, experience, and number of failures results into a large number of undesirable components being declared HFRC and omission of eligible components in the list of HFRC cannot be ruled out.

Clearly, the above discussion necessitates a need of a scientific methodology to decide a threshold for HFRC declaration, which will greatly help in tiding over the crisis of more precise shortlisting and hence considering a component for the reliability improvement measures. Moreover, a model for HFRC threshold declaration not only reduces the extra burden on the repair/overhaul lines but will also help in placing the eligible components in this domain leading to avoidance of unnecessary expenditure incurred on reliability improvement measures for non-qualifying components.

5.3 Reliability-Based Methodology for Optimal Maintenance Policies in MA

In this section, we present a reliability based maintenance policy for MA [8] considering the aero engines as a case study. This methodology can be easily extended to any other complex repairable systems that are subjected to imperfect repairs.

This section provides a model for defining threshold for declaring components HFRCs by treating them as repairable systems through the use of GRP models.

5.3.1 Reliability-Based Threshold Model for HFRC

Whenever a failure occurs, the aero engine is exposed to repair at depot and planted back into function. We consider the time to the first overhaul (OH) as t_{1OH}. Prior to presenting the threshold model for HFRCs, our main concern is to estimate *a, b*, and *q* with the assistance of failure data for the first overhaul cycle. We work out the consequent intensity function and availability as discussed earlier.

We firstly need to find the threshold age, t^*, at which the component should be stated HFRC. Let there be *K* number of aircraft in the fleet and the total number of aero engines in the inventory be *E.* Assuming a critical situation where all aircraft are required to deliver and none of them is desired to be unavailable due to requirement of aero engines, the total number of aero engines required for a desired level of flying availability is decided by the organization and is say K + Δ*K*, where Δ*K* is the number of additional aero engines to be made available in the inventory to meet any unserviceability of the aero engines which are part of the flying aircraft at any given point of time. We assume Δ*K* is given to us and decided by the organization based on utilization of engines. Thus, *E* – (*K* + Δ*K*) is the number of aero engines at the overhaul depot either undergoing/waiting for repair/overhaul. The probability of survival (*P*) of aero engines can be defined as the ratio

$$P = \frac{\text{Total number of aeroengines required}}{\text{Total number of aeroengines in the inventory}} \tag{5.1}$$

The least required reliability R^* is estimated as

$$R^* \leq \frac{K + \Delta K}{E}$$

The aero engine is supposed to be affirmed HFRC at the age t^* as soon as the reliability R(t*) goes below R^*. Therefore, the proposed criteria for designating a component HFRC is

$$R(t^*) = \exp\left[a\left(q\sum_{j=1}^{i-1} t_j\right)^b - a\left(t_i + q\sum_{j=1}^{i-1} t_j\right)^b\right] \leq \frac{A + \Delta A}{E}$$

Let us illustrate this through a hypothetical data set. Let the total number of aircraft in the fleet be 75 and the total number of aero engines in the inventory be 190. Let the total number of additional aero engines to be available in the inventory to meet any unserviceability of the aero engines which are part of the flying aircraft at any given point of time be 25. Thus, K = 75, *E* = 190, Δ*K* = 25, *K* + Δ*K* = 100. From Equation (5.1), R(t*) = 100/90 = 0.5263. Subsequently, the age t^*at which $R(t^*) \leq 0.5263$ indicates the age at which the aero engine is supposed to be acknowledged HFRC. The threshold reliability and intensity function can be estimated thereafter. Hence, we can plot reliability $R(t)$ as a function of time by means of the best-fit curve from the data set $[t_i, R(t_i)]$ as shown in Figure 5.1 below.

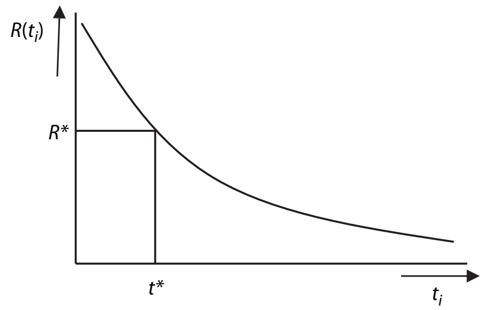

Figure 5.1 Reliability-based HFRC threshold.

Obviously, if the aero engines are declared HFRCs below t^*, then their number visiting the repair depot will be very high. Depots will face an additional task of undertaking HFRCs for reliability improvement besides meeting the commitments of planned overhaul tasks and unscheduled repairs.

5.3.1.1 Review of Present Maintenance Policy for HFRCs

This section reviews the current maintenance policy followed for the HFRCs by reviewing the present TBO of the aero engines. Hence, the objective here is to estimate the time to next overhaul, t_{2OH}. We obtain the GRP MLEs for a, b, q from the failure data of the first overhaul cycle. Subsequently, we find the optimal time for the next overhaul t_{2OH}, of the aero engines, subject to breakdowns. We present a model based on downtime for this analysis.

Reviewed TBO Model

The aim is to find optimal time for the next overhaul. According to the overhaul policy, overhaul will be done at a fixed time and the aero engines that fail are subjected to repair and then put back into the use. We establish the optimal interval to second overhaul to minimize the total downtime per unit time.

Let,

MTTR: Mean downtime necessary to carry out an imperfect repair.

T_{OH}: Mean downtime required to perform an overhaul.

t_{2OH}: Time to second overhaul

$E(t_{2OH})$: Expected number of failures in the interval $(0, t_{2OH})$

The overhaul strategy is to carry out second overhaul at t_{2OH}, irrespective of the age of the equipment, and imperfect repairs occur as many times as required in the interval $(0,t_{2OH})$

The overall downtime per unit time, for overhaul at time t_{2OH}, denoted by $\mathrm{D}(\mathrm{t}_{2\mathrm{OH}})$ is

$$\mathrm{D}(\mathrm{t}_{2\mathrm{OH}}) = \frac{\text{Expected downtime due to repairs} + \text{downtime due to overhaul}}{\text{cycle length}}$$

Expected downtime due to repairs =
Expected number of failures in interval $(0, \mathrm{t}_{2\mathrm{OH}})$
$\times$ Mean time to repair = $\mathrm{E}(\mathrm{t}_{2\mathrm{OH}}) \times \mathrm{MTTR}$.

Therefore,

$$\mathrm{D}(\mathrm{t}_{2\mathrm{OH}}) = \frac{E(t_{2OH}) \times MTTR) + T_{OH}}{t_{2OH} + T_{OH}} \tag{5.2}$$

We achieve $E(t_{2OH})$ as explained in the previous chapter. We also carry out repetitive simulation to estimate the value of t_{2OH} that minimizes $\mathrm{D}(\mathrm{t}_{2\mathrm{OH}})$ specified by Equation (5.2)

Example 5.1.
The aero engine is a repairable system which is placed at the concerned depot for repair and is put back into function after the imperfect repair. OEM-specified TBO for the aero engine is 550 hours and TTL is 1,800 hours. PM is carried out at an interval of 50 hours till the time to overhaul is reached. These aero engines belong to different variants. The aero engines belonging to a particular variant are identical.

The failure times in hours of Variant 1 aero engines are:

203, 477, 318, 536, 494, 213, 303, 525, 345, 299, 154, 230, 132, 321, 123, 351, 188, 49.02, 267, 548, 380, 61, 160, 375, 550, 174, 176, 257, 102, 81, 541, 518, 533, 547, 299, 208, 326, 451, 349, 152, 509, 249, 325, 261, 328, 48, 19, 142, 200, 426, 90, 522, 446, 338, 55, 549, 84, 342, 162, 250, 368, 96, 431, 14, 207, 324, 546.

The time to overhaul T_{OH} (from overhaul manual) and mean time to repair are obtained as: $T_{OH} = 6336$ *hours* and $MTTR = 528$ *hours* respectively.

(1) Estimate scale, shape parameters, repair effectiveness index using Kijima-I model.

(2) Determine the threshold time using reliability-based threshold model, at which the aero engines should be declared HFRC. Determine the time to next overhaul and comment on the results so obtained.
(3) Also estimate reliability, MTBF and availability at the end of both the overhaul cycles and compare them in a tabular form. Comment on the results.

Solution.

(1) Variant 1 underwent 67 failures during the first overhaul cycle. Maximizing Equation (3.23)
or solving Equations (3.24), (3.25), and (3.26) (see Note 3.3), we obtain the following result:

$$a = 0.00022, b = 1.35, q = 0.75,$$

(2) We then evaluate t^* and t_{2OH} as per the methodology discussed in the previous section.

$$t^* = 173\ hours, t_{2OH} = 152\ hours$$

Note that $t_{2OH} < t^*$which means that if the reviewed overhaul policy is followed, the aero engine will never fall in the domain of HFRC.
(3) In Table 5.2, we present relative outcomes of both the overhaul cycles with the help of reliability, MTBF, and availability figures at the end of each overhaul cycle using our recommended overhaul policy. We also present the percent enhancement in these performance indices in the Table 5.2.

Table 5.2 Relative outcome of the two overhaul cycles.

Cycles	**R(t)**	**MTBF(t)**	**A(t)**
1OH ($t= t_{1OH}$)	#	117.28	0.1818
2OH ($t = t_{2OH}$)	0.5960	286.69	0.3519
% Enhancement	#	#	93.56

"#": Not reported

Comments:

- The reliability index obtained at the end of first overhaul cycle is too low, hence not reported. This clearly indicates that the present overhaul policy of performing overhaul at 550 hours needs a review since the performance indices obtained are too low signifying an intense wear out and degradation of the aero engines by the time, they reach their TBO.
- The time to next overhaul has been reviewed to 152 hours which helps the aero engines to avoid the HFRC threshold time of 152 hours. Since the reviewed TBO is reduced from 550 hours to 152 hours, we can infer from Table 5.1 that by means of reviewed overhaul policy, we gain a significant improvement in reliability, MTBF, and availability. The improvement in reliability and availability is too high, hence not reported.

Example 5.2.

Consider the 142 failure times in hours of *Variant* 2 aero engines as placed below:

1, 13, 22, 34, 42, 54, 57, 59, 80, 81, 92, 102, 104, 106, 107, 128, 132, 137, 145, 155, 158, 168, 170, 173, 174, 182, 197, 199, 202.35, 203, 212, 221, 222, 228, 230, 237, 239, 244, 245, 247, 253, 254, 258, 260, 263, 265, 272, 274, 277, 278, 279, 287, 288, 292, 294, 295, 296, 298, 303, 304, 307, 309, 313, 326.73, 329, 332, 333, 339, 345, 346, 349, 352, 353, 354, 360, 367, 368, 368.91, 369, 370, 373, 376, 378, 379, 380, 388, 394, 397, 398.49, 404, 405, 407, 409, 410, 411, 412, 413, 416, 418, 419, 422, 428, 429, 432, 432.15, 434, 446, 451, 452.47, 457, 474, 477.36, 485, 488, 489, 496.79, 497, 201.2, 503, 504, 511, 515, 516.75, 517, 522.74, 528, 529, 530, 534, 536, 538, 541, 542, 543, 544, 545, 546, 547.36, 548, 549.

The values of time to overhaul T_{OH} (from overhaul manual) and Mean time to repair are given as: $T_{OH} = 6{,}336$ *hours* and $MTTR = 2{,}000$ *hours*.

(1) Estimate scale, shape parameters, repair effectiveness index using Kijima-I model.
(2) Also find the threshold time using reliability based threshold model, at which the aero engines should be declared HFRC. Determine the time to next overhaul and comment on the results so obtained.
(3) Estimate reliability, MTBF, and availability at the end of both the overhaul cycles and compare them in a tabular form. Comment on the results.

Solution.

(1) Variant 2 faced 142 failures during the first overhaul cycle. The analysis is similar to Example 5.1 and the results are as follows:

$$a = 1.64 \times 10^{-16},\ b = 4.87,\ q = 4.3$$

(2) We then evaluate $t^*$and t_{2OH} as per the methodology discussed in the previous section

$$t^* = 94.63\ hours,\ t_{2OH} = 50\ hours$$

Note that $t_{2OH} < t^*$ which means that if the reviewed overhaul policy is followed, the aero engines will never fall in the domain of HFRC.

(3) In Table 5.3 below, the relative outcomes of both the overhaul cycles with the help of reliability, MTBF, and availability figures at the end of each overhaul cycle, using our recommended overhaul policy are presented.

Comments

- This is a case of worse repair as evident from the estimated value of $q = 4.3$. This could mainly be due to usage of poor quality of spare parts or re-use of mandatory spare parts like rubber seals and washers. Environmental conditions, skill, and human errors may also be contributory factors. In addition, aero engines are experiencing an extreme wear out, as observed from the value of $b = 4.87$, due to which the quality of repair has suffered.
- The reliability and MTBF obtained at the end of first overhaul cycle is too low, hence not reported. This clearly indicates that the present overhaul policy of performing overhaul

Table 5.3 Relative outcome of the two overhaul cycles.

Cycles	R(t)	MTBF(t)	A(t)
1OH ($t = t_{1OH}$)	#	#	0.1818
2OH ($t = t_{2OH}$)	0.7703	274.65	0.38
% Enhancement	#	#	#

"#": Not reported

at 550 hours needs a review since the performance indices obtained are too low signifying an intense wear out and degradation of the aero engines by the time, they reach their TBO.

- The time to next overhaul has been reviewed to a very low value of 50 hours which helps the aero engines to avoid the HFRC threshold time of 94.63 hours. Since the reviewed TBO is reduced from 550 hours to 50 hours, we can infer from Table 5.3 that by means of reviewed overhaul policy, we gain a significant improvement in reliability, MTBF, and availability. The improvement in reliability and availability is too high, hence not reported.

5.4 Availability-Based HFRC Analysis

In this section, an availability-based methodology is presented by firstly considering the aero engine as a "Black Box" (BB) to determine HFRC [9]. A maintenance policy is then formulated with the results obtained from the BB approach. An extension to the BB approach to determine HFRC based on availability after identifying dominant failure modes (FMs) is presented in the next section. Initially, independent FMs are not considered in the analysis. Motivated by the fact that such a detailed analysis yield better results, an extension to the BB approach to determine HFRC based on availability after identifying dominant FMs is also presented [10]. A methodology to review TBO of the aero engine by considering the dominant FMs of the aero engine is further provided.

5.4.1 Availability-Based Criteria for HFRC (BB Approach)

The aim in this approach is to find t^* at which the component should be declared HFRC. The first step to do that is to define the minimum level of acceptable availability say A^*. The aero engine should be declared HFRC at the first time t^* when the availability $A(t^*)$ goes below A^*. Thus, the proposed criteria for designating a component HFRC is

$$A(t^*) = \frac{\text{MTBF}(t^*)}{\text{MTBF}(t^*) + \text{MTTR}} \le A^*$$

For instance, if the number of flying hours required by a unit flying a type of aircraft, in a month be α. Taking into account 20 days of flying in a month, number of flying hours required per day by the unit is β which works out to be α/20. Considering that 01 aircraft can do at least one turn around, i.e., it will fly at least 2 sorties per day and 1 sortie will last for a duration of 1-hour, number of aircraft required per day is α/40. Let the total number of aircraft in the inventory of unit per day be δ. Then, the required availability of aircraft (hence the required availability of aero engines) to achieve the required flying hours, $A = \alpha/40\delta$. This availability can be taken to be a measure of the minimum required availability A^* implying that for HFRC:

$$A^* = \alpha/40\delta$$

Illustration of the methodology discussed above is provided on a real data set. Let α = 346 *hours*, δ = 18, $A^* = \frac{\alpha}{40\delta} = \frac{346}{40 * 18} = 0.48$, then the first time t^* at which $A(t^*) \leq 0.48$ defines the time at which the aero engine should be declared HFRC. The threshold reliability and intensity function can be calculated after estimation of other parameters like *a, b,* and *q*. This allows plotting availability $A(t)$ as a function of time by using the best fit curve obtained from the data set $[t_i, A(t_i)]$ The best fit curve is plotted in Figure 5.2. In present scenario, declaring HFRC at an age lower than t^* is not efficient.

It is worthwhile mentioning that if the flying task (α) is required to be increased, then A^* has to be increased accordingly. The flying task (α) vs. availability (A) is plotted in Figure 5.3 to highlight the sensitivity of *A* with respect to changes in α.

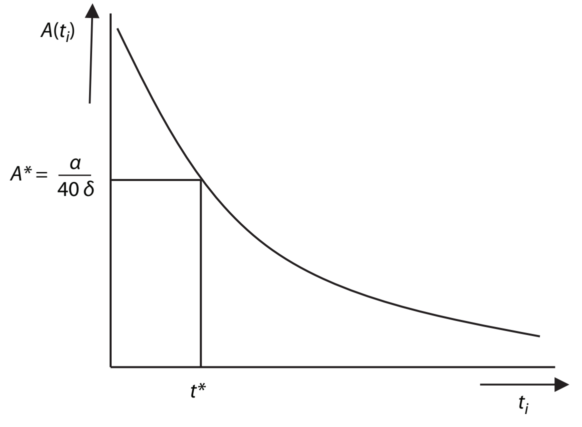

Figure 5.2 Availability-based threshold for HFRC.

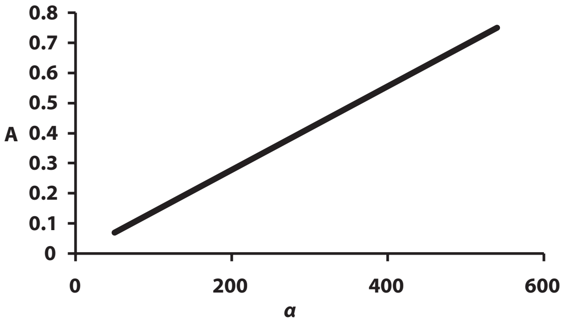

Figure 5.3 Flying task vs. availability.

5.4.1.1 *Review of Overhaul Cycle (BB Approach)*

An optimal interval for overhaul of the case during the second overhaul cycle is presented in this section. According to the overhaul policy, overhaul will be done at a fixed time and the aero engines that fail are subjected to general repair. Aero engines also undergo periodic maintenance at fixed intervals. The optimal interval to second overhaul to minimize the total downtime per unit time is determined by using the following method.

Let T_{PMi} $(i = 1, 2\ldots n)$ the downtime required to carry out concerned PM. The overhaul policy is to perform overhaul at t_{2OH} irrespective of the age of the equipment, and general repairs occur as many times as required in the interval $(0, t_{2OH})$. The total downtime per unit time, for overhaul at time t_{2OH}, denoted by $D(t_{2OH})$ is

$$D(t_{2OH}) = \frac{\textit{Expected downtime due to repairs} + \textit{downtime due to PM} + \textit{downtime due to overhaul}}{\textit{cycle length}}$$

Expected downtime due to repairs =

Expected number of failures in interval $(0, 2_{OH})$ × Mean time to repair = $E(t_{2OH}) \times MTTR$.

Downtime due to PM = TPM $= \sum_{i=1}^{n} T_{PM_i}$; Downtime due to overhaul = T_{OH}. Therefore,

$$D(t_{2OH}) = \frac{[E(t_{2OH}) \times MTTR)] + T_{PM} + T_{OH}}{t_{2OH} + T_{PM} + T_{OH}} \tag{5.3}$$

Repeated simulations are performed to estimate the value of t_{2OH} that minimizes $D(t_{2OH})$ given by Equation (5. 3).

Example 5.3.

Consider the failure data of aero engines provided at Example 5.1.

(1) Estimate scale, shape parameters, repair effectiveness index using Kijima-I model.
(2) Determine the threshold time using availability based threshold model (BB approach), at which the aero engines should be declared HFRC. Determine the time to next overhaul and comment on the results so obtained. Also, determine reliability and intensity function at threshold time.
(3) Estimate reliability, MTBF, and availability at the end of both the overhaul cycles and compare them in a tabular form. Comment on the results. Consider $T_{PM} = 560$ *hours.*

Solution.

(1) Variant 1 underwent 67 failures during the first overhaul cycle. Maximizing Equation (3.23) or solving Equations (3.24), (3.25), and (3.26) (see Note 3.3), we obtain the following result:

$$a = 00022, b = 1.35, q = 0.75,$$

(2) We then evaluate t^* and t_{2OH} as per the methodology discussed in the previous section for BB approach. The results are as follows:

$$t^* = 315 \textit{ hours}, t_{2OH} = 248 \textit{ hours},$$

$$R(t^*) = 0.6165, u(t^*) = 0.0020516$$

It should be noted that $t_{2OH} < t^*$ which means that if this overhaul policy is followed, the aero engine will not fall in domain of HFRC.

(3) In Table 5.4, the values of reliability, MTBF, and Availability at the end of first and second overhaul cycle using the recommended overhaul policy along with the percent improvement obtained in these performance measures are presented.

Table 5.4 Comparative results of both overhaul cycles (Variant 1).

Cycles	R(t)	MTBF(t)	A(t)
First overhaul cycle ($t = t_{1OH}$)	0.3338	394.56 h	0.1818
Second overhaul cycle ($t = t_{2OH}$)	0.7217	541.67 h	0.3519
Percent improvement	#	37.28	93.56

"#": Not reported

As per the new overhaul policy, the TBO is now reduced to 248 hours from 550 hours resulting into a significant improvement in reliability, MTBF, and availability. The improvement in reliability index is too high to be reported.

Example 5.4.
Consider the failure data of aero engines provided at Example 5.2.

(1) Estimate scale, shape parameters, repair effectiveness index using Kijima-I model.
(2) Determine the threshold time using availability-based threshold model (BB approach), at which the aero engines should be declared HFRC. Determine the time to next overhaul and comment on the results so obtained. Also, determine reliability and intensity function at threshold time.
(3) Estimate reliability, MTBF, and availability at the end of both the overhaul cycles and compare them in a tabular form. Comment on the results. Consider. $T_{PM} = 760\ hours$.

Solution.

(1) This variant underwent 142 failures during the first overhaul cycle. A similar analysis to Example 5.2 is performed and the results obtained are as follows:

$$a = 1.64 \times 10^{-16},\ b = 4.87,\ q = 4.3$$

(2) We then evaluate $t^*$and t_{2OH} as per the methodology discussed in the previous section for BB approach. The results are as follows:

$$t^* = 258\ hours,\ T_{PM} = 760\ hours,$$

$$t_{2OH} = 250\ hours,\ R(t^*) = 0.1585,\ u(t^*) = 0.000545$$

It should be noted that $t_{2OH} < t^*$ which means that if this overhaul policy is followed, the aero engine will not fall in domain of HFRC.

(3) In Table 5.5, the values of reliability, MTBF, and availability at the end of first and second overhaul cycle using the recommended overhaul policy along with the percent improvement obtained in these performance measures are presented.

Comments

- The values of reliability and availability obtained during first overhaul cycle are too low to be reported. This is due to the extreme wear out and very poor maintenance during the first overhaul cycle as evident from the values of *b* and *q*, respectively. Other reasons for this are already explained in Example 5.2.
- Due to very low values of reliability, MTBF, and availability obtained during the first overhaul cycle, application of corrective measures in form of reviewing the second overhaul cycle to 250 hours from 550 hours has seen an upsurge in the improvement of all these three parameters. The improvement observed in all the performance measures is seemingly unrealistic and hence not reported.

5.4.2 Availability-Based HFRC Threshold Model Considering FMs

To build up an availability-based threshold model considering FMs [10], all the FMs of the aero engine are firstly identified with the help of the failure data of the first overhaul cycle. Then, a selection for the dominant FMs is made. Parameters *a,b,* and *q* for all FMs from the failure data of the first overhaul cycle are then estimated.

Table 5.5 Comparative results of both overhaul cycles (Variant 2).

Cycles	R(t)	MTBF(t)	A(t)
First overhaul cycle ($t = t_{1OH}$)	#	102.94 h	#
Second overhaul cycle ($t = t_{2OH}$)	0.3470	2858.63 h	0.5884
Percent improvement	#	#	#

“#”: Not reported

The intensity function, MTBF, reliability, and availability for the dominant FMs are computed using equations (5.4), (5.6), (5.1), and (5.12). The same performance parameters for the complete aero engine after combining all the FMs can be estimated from Equations (5.5), (5.7), (5.11), and (5.13).

Intensity function Equation

$$u_j\left(v_i^j\right) = a_j \times b_j \times \left(v_i^j\right)^{b-1} \tag{5.4}$$

$$u_E(v_i) = \sum_{j=1}^{n} u_j\left(v_i^j\right) \tag{5.5}$$

MTBF Equation

$$MTBF_j\left(v_i^j\right) = \frac{1}{u_j\left(V_i^j\right)} \tag{5.6}$$

$$MTBF_E(v_i) = \frac{1}{u_E(v_i)} \tag{5.7}$$

Reliability Equation

For first failure,

$$R_j\left(v_1^j\right) = \exp\left\{-a\left(v_1^j\right)^b\right\} \left(\text{Note: } v_1^j = qt_1^j\right) \tag{5.8}$$

$$R_E(v_1) = \prod_{j=1}^{n} R_j\left(v_1^j\right) \tag{5.9}$$

For subsequent failures,

$$R_j\left(v_i^j \middle| v_{i-1}^j\right) = \frac{R_j\left(v_i^j + v_{i-1}^j\right)}{R\left(v_{i-1}^j\right)} = \exp a\left[\left(v_{i-1}^j\right)^b - \left(v_i^j + v_{i-1}^j\right)^b\right] \tag{5.10}$$

$$R_E(v_i|v_{i-1}) = \prod_{j=1}^{n} R_j\left(v_i^j \middle| v_{i-1}^j\right) \tag{5.11}$$

Availability Equation

$$Aj\left(v_i^j\right) = \frac{MTBF_j\left(v_i^j\right)}{MTBF_j\left(v_i^j\right) + MTTR_j} \tag{5.12}$$

$$A_E(v_i) = \prod_{j=1}^{n} A_j\left(v_i^j\right) \tag{5.13}$$

Note: $i = 2, 3 \ldots, n$ (*for every j*) & $j = 1, 2 \ldots, n$(*FMs*).

We now determine the age, t^* at which the aero engine should be declared HFRC. The methodology discussed below has already been discussed for the BB approach in previous section. Formulating a HFRC threshold model considering FMs is an extension of the methodology discussed earlier. It can be seen from the result arrived for "BB approach" that the required availability of aircraft (hence, the required availability of aero engines) to achieve the required flying hours is $A_E^* = \alpha / 40\delta$.

This availability can be taken to be a measure of the minimum required availability A_E^* implying that for HFRC:

$$A_E^* = \alpha / 40\delta$$

The aero engine should be declared HFRC at the age t^* when the availability $A_E(t^*)$ goes below A_E^*. Thus, the proposed criteria for designating a component HFRC is

$$A_E(t^*) = \frac{MTBF_E(t^*)}{MTBF_E(t^*) + MTTR_E} \leq A_E^*$$

The methodology is now extended by considering dominant FMs of the aero engine. Since the FMs are in series, for n FMs, the proposed criteria for designating a component HFRC, based on availability is

$$A_E(t^*) = \prod_{j=1}^{n} A_j\left(t_j^*\right) \le A_E^*$$

In sync with the earlier discussion, it is observed that designating engines HFRC at an age below t^* will result into the shortlisting of unwanted components for being undertaken for reliability improvement measures. This will impose an extra burden on the repair/overhaul line of the depot and at the same time will inflict extra expenditure to the organization. Correct HFRC threshold designation will also help in reduced queuing at repair/overhaul lines at depots resulting into enhanced throughput. This will lead to increased availability of the aero engines in the fleet.

5.4.2.1 Maintenance Strategy for HFRCs With FM Approach

Once the HFRCs have been recognized it is required to review the current maintenance policy for the HFRCs. In this section, a methodology to review the present TBO of the aero engines that fall under the domain of HFRCs, by considering dominant FMs is presented. The next aim is to estimate the time of next overhaul, t_{2OH}. Before resorting to estimate the time to next overhaul, the GRP MLEs for *a, b,* and *q* from the failure data of the first overhaul cycle are required to be obtained. Other performance parameters are then estimated as discussed in earlier sections.

5.4.2.2 TBO Model Considering FMs

After declaring HFRC and identifying the dominant FMs, it is intended to find the optimal interval for overhaul of the aero engine for the duration of the second overhaul cycle. The objective is to establish the optimal interval to second overhaul to minimize the total downtime per unit time.

The relation between $MTTR_E$ and $MTTR_i$ is given by (5.14).

$$MTTR_E = \frac{\sum_{j=1}^{n} E_j(t_{1OH}) \times MTTR_j}{\sum_{j=1}^{n} E_j(t_{1OH})} \quad n = \text{number of FMs} \tag{5.14}$$

The relation between T_{PM}^{E} and T_{PM}^{k} is given by (5.15)

$$T_{PM}^{E} = \sum_{k=1}^{m} T_{PM}^{k} \quad \text{m = Number of PMs in an overhaul cycle} \tag{5.15}$$

The objective is to overhaul the aero engine at t_{2OH}, whatever be the age of the aero engine at that point of time, and the aero engines are repaired as many times as necessary in the interval (0, t_{2OH}). The overall downtime per unit time, for overhaul at time t_{2OH} denoted by $D(t_{2OH})$ is

$$D(t_{2OH}) = \frac{\textit{Expected downtime due to repairs} + \textit{downtime due to PM} + \textit{downtime due to overhaul}}{\textit{cycle length}}$$

Expected downtime due to repairs =
Expected number of failures in interval (0, t_{2OH}) × Mean time to repair = $(t_{2OH})_E \times MTTR_E$.

Therefore,

$$D(t_{2OH}) = \frac{[(E(t_{2OH})_E \times MTTR_E)] + T_{PM}^{E} + T_{OH}}{t_{2OH} + T_{PM}^{E} + T_{OH}} \tag{5.16}$$

The values of expected number of failures can be estimated from Equation (3.67). $MTTR_i$ is estimated for *jth* FM from the repair data of last 10 years. $MTTR_E$ is then obtained from Equation (5.14). The values of T_{PM}^{k} are acquired from the maintenance manual of the aero engine. T_{PM}^{E} is thereafter calculated from Equation (5.15). The value of T_{OH} is provided in certain available publications like overhaul manual, hence can be accessed directly from it. It is then necessitated to carry out simulation to estimate the value of t_{2OH} that minimizes $D(t_{2OH})$ specified by Equation (5.16).

Example 5.5.
The aero engine represents the most important technology with advanced manufacturing, quality control, design evaluation and extensive testing. This equipment, with its new hardware and systems can accomplish very high standards of reliability. The aero engine failures are dependent on a range of external and internal factors such as (a) component-related factors (design, manufacturing); (b) operational factors (pressure, temperature); (c) environmental factors (ambient conditions, temperature, humidity); (d) maintenance factors (servicing rate, overhaul policy). An aero engine

has various FMs that form the basis for this example. OEM-specified TBO for the aero engine is 550 hours and TTL is 1,800 hours PM is carried out at an interval of 50 hours till the time to overhaul is reached. There are many variants of aero engines in the fleet.

The failure times in hours of aero engines in three dominant FMs were extracted from their operational records during first overhaul cycle (TBO = 550 hours) of a particular variant of aero engines used in a military aircraft. The data is hereunder:

FM1: Corrosion of compressor blades (41 failures)
55, 81, 84, 102, 106, 142, 152, 157, 160, 184, 200, 203, 226, 241, 250, 257, 267, 280, 295, 298, 302, 303, 318, 321, 325, 326, 327, 329, 338, 340, 345, 356, 372, 379, 405, 416, 423, 429, 464, 477, 500.

FM2: Wear out of bearings (17 failures)
61, 249, 250, 265, 287, 299, 317, 321, 351, 361, 379, 394, 408, 426, 438, 442, 461.

FM3: Creep-fatigue fracture of turbine blades (15 failures)
14, 48, 63.28, 123, 130, 154, 172, 174, 208, 230, 313, 349, 451, 462, 482.

Assume MTTR due to three FMs as $MTTR_1 = 492.5\ hours$, $MTTR_2 = 364.81\ hours$, $MTTR_3 = 466.82\ hours$, respectively.

(1) Estimate scale, shape parameters along with REI for each FM separately using Kijima-I model and expected number of failures due to each FM.
(2) Estimate reliability, MTBF, and availability at the end of the first overhaul cycle ($t = t_{1OH}$).
(3) Estimate the HFRC threshold time t^*, availability at the threshold time $A_E(t^*)$, and the time to next overhaul t_{2OH}. Consider $T_{PM}^{E} = 560\ hours, T_{OH} = 6,336\ hours$.
(4) Estimate reliability, MTBF, and availability at the end of second overhaul cycle ($t = t_{2OH}$). Compare the results obtained from both the overhaul cycles and comment.

Solution.

(1&2) The aero engines faced 41 failures due to FM1, 17 failures due to FM2 and 15 failures because of FM3 during the first overhaul cycle. Maximizing Equation (3.23) or solving Equations (3.24), (3.25), and (3.26) (see Note 3.3), we obtain the MLEs for *a, b,* and *q*. The values of $E_j(t_{1OH})$ are obtained with the help of Equation (3.67). Further, the values of $R(t = t_{1OH})$, $MTBF(t = t_{1OH})$, and $A(t = 1_{OH})$ are then estimated as explained in Section 5.4.2. FM wise results for the aero engine for the first overhaul cycle are provided in Table 5.6.

Since the reliability for FM2 is very low, it is not reported This is mainly due to intense wear out and poor quality of the repair process as apparent from the values of shape parameter (b) of 2.14 and repair effectiveness index (q) of 2.7. The reasons for the poor quality of repair have been explained at Example 5.2.

(3) On pooling the data irrespective of the three FMs, the following results for the aero engine are arrived at

$$t^* = 220\ hours,\ t_{2OH} = 220\ hours,\ A_E(t^*) = 0.1738,$$

The values of t^* and t_{2OH} are estimated as per the methodology discussed in previous section. It is observed that availability of the aero engine at all times remain lower than the HFRC threshold availability A_E^* of 0.48 all through the aero engine utilization during the first overhaul cycle. Since $A_E(t)$ is always less then $A^* = 0.48$ and $t_{2OH} = 220$ *hours*, it would be fairly reasonable to exploit the aero engine till 220 hours and pronounce it HFRC at $t^* = 220$ *hours*. With this availability of $A_E(t^*) = 0.1738$, a maximum of , $\propto^* = 125$ hours can be achieved by the unit. If added flying hours are desired, then availability is required to be enhanced.

(4) In Table 5.7, the FM wise results for the second overhaul cycle are presented.

In Table 5.8, the FM wise, percent improvement observed, using our new overhaul policy, are presented.

Table 5.6 Results-failure mode wise for first overhaul cycle.

FMs	*a*	*b*	*q*	$MTTR_j$	$E_j(t_{1OH})$	$R(t = t_{1OH})$	$MTBF(t = t_{1OH})$	$A(t = t_{1OH})$
*FM*1	0.000153	1.46	0.85	492.5	1.55	0.1817	261.96	0.3472
*FM*2	$8.37*10^{-7}$	2.14	2.7	364.81	0.59	#	206.92	0.3619
*FM*3	0.003644	0.92	1.0	466.82	1.21	0.3183	494.16	0.5142

"#": Not reported

Table 5.7 Results-failure mode wise for t_{2OH}.

FMs	*R* $(t = t_{2OH})$	*MTBF* $(t = t_{2OH})$	*A* $(t = t_{2OH})$
*FM*1	0.6807	412.72	0.4559
*FM*2	0.4422	546.39	0.5996
*FM*3	0.6410	455.74	0.4940

Table 5.8 Failure mode wise percent improvement.

	Percent improvement		
Failure modes	**Reliability**	**MTBF**	**Availability**
*FM*1	#	57.55	31.31
*FM*1	#	#	65.68
*FM*3	#	#	#

"#": Not reported

It is observed from Table 5.8 that by using the new overhaul policy, significant improvement is achieved in the performance parameters for all the three FMs. In case of FM2, due to very low value of reliability obtained during the first overhaul cycle, application of corrective measures in form of reviewing the second overhaul cycle to 220 hours has seen an upsurge in the improvement of reliability making it seemingly unrealistic and hence is not reported. In case of *FM*3, the value of *b* is less than 1 thus causing no improvement in MTBF and availability, hence not reported.

In Table 5.9, the values of reliability, MTBF, and availability at the end of first and second overhaul cycle for the aero engine using the recommended overhaul policy, after combining results of all the three FMs are presented.

Table 5.9 Comparative results of both overhaul cycles for the aero engine.

Cycles	**Reliability**	**MTBF(t)**	**Availability**
$(t = t_{1OH})$	#	94	0.0646
$(t = t_{2OH})$	0.1929	155	0.1350
Percent improvement	-	64.89	#

"#": Not reported

Since any FM can cause an engine to fail (series model in reliability) MTBF, reliability, and availability are estimated as explained in Section 5.4.2. The first observation from Table 5.9 is extremely poor reliability and seemingly unrealistic improvement in reliability and, hence, is not reported. This is an outcome of the extremely poor wear out and repair effectiveness index. Further, 64.89% improvement in *MTBF* and significant improvement in availability are achieved.

5.5 Summary

At some point of time of equipment exploitation, one may need to define a threshold for an equipment to consider them under the domain of a HFRC. This chapter has provided a model for deciding a threshold for deeming a component HFRC. Thereafter, a methodology is presented for providing a suitable treatment for the declared HFRCs in form of reviewing the present maintenance policy. The repairs are considered imperfect and Kijima-I based GRP MLEs have been used to estimate all performance indices. The methodology not only provides shape and scale parameters but also renders an index to assess repair efficacy. Investigating these indicators individually and together provides an ample insight into the current maintenance practices. The examples provided reveal that the aero engines have been rendered HFRC earlier than their OEM approved TBO. In view of the fact that the TTL of the aero engines is 1,800 hours, a dire need to review the TBO exists so that the aero engines do not come into the sphere of influence of HFRCs. We also present a methodology to review the TBO for the next overhaul cycle. We detect noteworthy enhancement in all the performance parameters in the revised maintenance policy.

Exercises

1. The failure times of Variant 3 aero engines in hours are as placed below:
   0.5, 12, 13, 15, 17, 31, 38, 39, 42, 47, 52, 67, 98, 101, 125, 133, 144, 166, 167, 177, 179, 189.46, 198, 206, 211, 212, 226, 264, 267, 269, 273, 293, 298, 321, 344.42, 354, 361, 366, 383, 387, 390, 401, 408, 425, 443, 461, 475, 507, 520, 542, 544, 545, 547, 548, 549.

The time to overhaul and mean time to repair are
T_{OH} = 6336 *hours* and *MTTR* = 520 *hours*

(a) Estimate scale, shape parameters, repair effectiveness index using Kijima-I model.
(b) Determine the threshold time using reliability based threshold model, at which the aero engines should be declared HFRC. Determine the time to next overhaul and comment on the results so obtained.
(c) Estimate reliability, MTBF, and availability at the end of both the overhaul cycles and compare them in a tabular form. Comment on the results.

2. Consider the failure times of Exercise 1.
   (a) Estimate scale, shape parameters, repair effectiveness index using Kijima-I model.
   (b) Also find the threshold time using availability-based threshold model (BB approach), at which the aero engines should be declared HFRC. Determine the time to next overhaul and comment on the results so obtained.
   (c) Estimate reliability, MTBF, and availability at the end of both the overhaul cycles and compare them in a tabular form. Comment on the results.
3. The failure times in hours of three FMs worked out from the failure data of the first overhaul cycle (TBO = 550 hours) of Variant 2 aero engines used in a military aircraft are as placed below:

FM1: Compressor blades crack

1, 22, 34, 54, 57, 81, 92, 102, 104, 107, 128, 132, 155, 158, 170, 182,173, 199, 201.2, 202.35, 212,222, 239, 245, 254, 265, 277, 288, 298, 307, 326.73, 333, 349, 353, 354, 368.91, 378, 398.49, 405, 409, 412, 413,416, 418, 419, 422, 428, 429, 432.15, 434, 446, 452.47, 457, 474, 477.36, 485, 488, 496.79, 503, 511, 515, 516.75, 522.74, 528, 530, 534, 538, 541, 543, 546, 547.36, 549.

FM2: Wear out of bearings

13, 59, 80, 137, 168, 197, 221, 228, 230, 244, 253, 258, 260, 263, 274, 278, 279, 287, 294, 296, 304, 309, 313, 329, 339, 346, 360, 367, 368, 370, 376, 388, 397, 407, 411, 451, 504, 529, 542, 548.

FM3: Turbine blades overheat

42, 106, 145, 174, 203, 237, 247, 272, 292, 295, 303, 332, 345, 352, 369, 373, 379,380, 394, 404, 410, 432, 489, 497, 517, 536, 544, 545.

(a) Estimate scale, shape parameters along with REI for each FM using Kijima-I model. Also, estimate expected number of failures due to each FM.

(b) Estimate reliability, MTBF, and availability at the end of the first overhaul cycle ($t = t_{1OH}$). Consider MTTR due to all three FMs as

$MTTR_1 = 492.5\ hours$, $MTTR_2 = 364.81\ hours$, $MTTR_3 = 466.82\ hours$

(c) Estimate the HFRC threshold time t^*, availability at the threshold time $A_E(t^*)$ and the time to next overhaul t_{2OH}. Consider $T_{PM}^{E} = 560\ hours, T_{OH} = 6336\ hours$
Estimate reliability, MTBF, and availability at the end of second overhaul cycle ($t = t_{2OH}$).

(d) Compare the results obtained from both the overhaul cycles and comment.

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6

FMEA for Repairable Systems Based on Repair Effectiveness Index

6.1 Introduction

Every industry following reliability practices and standards invariably uses failure mode and effect analysis/criticality analysis (FMEA/CA) for its product and/or process where risk priority number (RPN)–based approach has taken the front seat than the approach suggested in MIL-STD 1629A—a defunct and disowned standard by its propounder. A traditional RPN is an arithmetic products of three factors, *viz.*, Severity of the effects of a failure mode, Occurrence frequency of its causes and means of Detectability of such mode in the design/process/service, i.e., RPN = S × O × D. These three factors are normally assessed, subjectively and/or objectively, through a customized 10- or 5-point scale for each factor [1] for that particular product or process. These three scales are constructed from experts' opinion and/or the use of available data of that particular product/process. Ideally, FMEA begins during the earliest conceptual stages of design and continues throughout the life of the product or service. Begun in the 1940s by the U.S. military, FMEA is a step-by-step approach for identifying all possible failures in a design, a manufacturing or assembly process, or a product or service. Failure modes means the ways, or modes, in which something might fail. Failures are any errors or defects, especially ones that affect the customer, and can be potential or actual. Effects analysis refers to studying the consequences of those failures. Failures are prioritized according to how serious their consequences are, how frequently they occur, and how easily they can be detected. The purpose of the FMEA is to take actions to eliminate or reduce failures, starting with the highest-priority ones, whose priority can be decided by its RPN and/or its factor(s). FMEA also documents current knowledge and actions about the risks of failures, for use in continuous improvement. FMEA is used during design to prevent

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failures and later it is used for control, before and during ongoing operation of the process. There have been several extensions to the standard FMEA procedure though the basic philosophy of conducting the FMEA remains the same. A brief literature review on such extensions is placed at Table 6.1 for a better appreciation of the readers.

The suggested means to reduce RPN are by governing its multiplicative factors singly or in combinations, e.g., increasing detectability by incorporating various detection techniques through condition monitoring. Some may suggest directing the efforts to reduce RPN by reducing probability of occurrence of failure causes mainly through design modifications. Such techniques indeed require additions and alterations to carry out on the existing system, which not only prove to be expensive but also consume time. Besides, for all these techniques to be incorporated, a complex equipment is required to be sent to OEM further complicating the issue with lots of administrative hassles.

This chapter deliberates and advocates an alternative to reduce RPN through the use of REI. Note that by improving REI for a critical and high priority failure mode directly reduces the probability of occurrence since it is a function of q. This also indirectly enhances the reliability and availability of the equipment. REI is directly related to a number of factors like human error, repair cost, preventive maintenance effectiveness, maintainability, equipment effectiveness, skill, tools testers and ground equipment environmental conditions, spare parts quality, equipment complexity, task complexity, etc. which can be addressed at the level of depot. Hence, it can just be improved by following correct maintenance procedures, creating an appropriate maintenance environment, use of correct tools, testers and ground equipment, using standard spare parts, reducing human error, etc. This only requires a correct mind set of the maintenance manager rather than involving any major design changes or advanced condition monitoring techniques in the existing setups. Since improving q is in hands of the user itself, it will greatly help in improving the performance parameters over a short period of time without intervention of any external agencies and putting any extra burden to the exchequer. Thus, this chapter introduces an algorithm for estimating RPN by introducing (q) supported by sensitivity analysis of (q) on (O) to obtain a final RPN to take appropriate actions.

Table 6.1 Extensions to standard FMEA.

Authors	**Brief Work Summary**
Bowles and Pelaez [2]	Projects a method based on fuzzy logic for prioritization of failures for remedial actions in a Failure Mode Effects and Criticality Analysis (FMECA).
Franceschini and Galetto [3]	Develop an exclusive methodology which can handle situations having diverse significance levels for the three *FM* component indices: severity, occurrence, and detection to decide the risk priority level (RPL) for the FMs.
Sankar and Prabhu [4]	Propose a modified FMEA approach by introducing a new Risk Priority Rank (RPR) technique that utilizes a ranking scale of 1 to 1,000 to characterize the increasing risk of severity (S), probability of occurrence (O), and detection (D) combinations to prioritize failures in a system FMEA for initiating corrective actions.
S.R. Devadasan *et al.* [5]	A modified adaptation of FMEA known as Total Failure Mode Effects Analysis (TFMEA) is developed to bring out failure prevention for achieving continuous quality improvements.
Pillay and Wang [6]	Develop Evidential Reasoning (ER) using fuzzy rules base and gray relation theory to rank the risks of different FMs in order to overcome the drawbacks of the conventional FMEA approach.
Rhee and Ishii [7]	Present the details of life-cost–based FMEA that measures risk in terms of cost over the life cycle.

(*Continued*)

Table 6.1 Extensions to standard FMEA. (*Continued*)

Authors	Brief Work Summary
Hosseini and Safaei [8]	Introduce Decision Making Trial and Evaluation Laboratory (DEMATEL) for reprioritization of FMs based on severity of consequence or weight.
V.P. Arunachalam and C, Jegadheesan [9]	A modified FMEA model with reliability and cost based approach is proposed to overcome the drawbacks of traditional FMEA.
C.Dong [10]	Fuzzy-based utility theory and fuzzy membership functions are utilized to gauge severity, occurrence, and detection. Structure of hierarchy and interdependence of corrective action is evaluated by Interpretive Structural Model (ISM). The weight of a corrective action is then estimated through the analytic network process (ANP).
Chen J. K [11]	Combines the utility of corrective actions to make a decision on improvement priority order of FMEA using Utility Priority Number (UPN).
Wang *et al.* [12]	Fuzzy Risk Priority Numbers (FRPNs) is used to prioritize FMs and used fuzzy geometric means to weigh the fuzzy ratings for *O*, *S*, and *D*.

6.2 A Brief Overview on Performing FMEA

FMEA is performed by firstly identifying the failure modes, finding out their causes and consequences, then estimating the probabilities of occurrence and finally determining corrective actions or preventive measures. It is generally performed as a bottom-up analysis, though it may be functional at any level in which there is adequate description to present the needed data. Distinctive steps in conducting an FMEA include system definition, identification of failure modes, determination of cause, assessment of effect, classification of severity evaluation of probability of occurrence and establishing corrective action. Each of them is briefly explained in subsequent sections by considering aero engine as a case.

6.2.1 System Definition

A functional and physical (hardware) description of the system provides the definition and margins for performing the analysis. Functional and physical descriptions are represented by flow diagrams depicting subassemblies, components, and parts along with their hierarchical interactions. With the help of these two diagrams, a reliability block diagram (RBD) is formed and used as the foundation for carrying out the analysis. A RBD [13] is a diagrammatic method for showing how component reliability contributes to the success or failure of a complex system. RBD is also known as a dependence diagram (DD). An RBD or DD is drawn as a series of blocks connected in parallel or series configuration. Each block represents a component of the system with a failure rate. Parallel paths are redundant: all of the parallel paths must fail for the system to fail. By contrast, any failure along a series path causes the entire series path to fail.

6.2.2 Identification of Failure Modes

Failure modes are known either by physical or functional failure. FMs display an evident behavior in which a component fails. A few examples of FMs for an aero engine include corrosion on compressor blades, oil leak, engine overheat, damaged turbine blades, fuel leak, RPM fluctuation, excessive oil consumption, excessive vibration, etc. Failure may also happen as an outcome of a premature incident, failure to function or become non-operational at a given time, irregular operations, or degraded performance. Failure modes are provided in certain databases like FMD 2016, etc. This product contains field failure mode and mechanism distribution data on a variety of electrical, mechanical, and electromechanical parts and assemblies. This data can be used to assist in the performance of reliability analyses and assessments such as Failure Modes, Effects and Criticality Analysis (FMECA) and Fault Tree Analysis (FTA). The data contained in FMD-2016 represents over 990,000 new records, a 10-times increase over the 98,000 records presented in its predecessor, FMD-2013. The CD-ROM version of FMD-2016 incorporates a user interface with search capabilities that assist in rapid data retrieval. Data was collected for this product from a wide variety of commercial and military sources.

The failure cause-mode-effect relationship shifts in the FMEA as a function of the system level is illustrated in Table 6.2 [14]. For example,

Table 6.2 Failure cause-mode effect relationship.

System	Assembly	Part	Part Manufacturing Process
Effect			
Mode	Effect		
Cause	Mode	Effect	
	Cause	Mode	Effect
		Cause	Mode
			Cause

at the most basic level, the part manufacturing process, the cause of failure may be a process step that is out of control. The ultimate effect of that cause becomes the failure mode at the part level. The failure effect of the part becomes the failure mode at the next level of assembly, and so forth. The failure modes of the system functions/attributes are the effects of failure modes at the subordinate hierarchical level. This tiering continues as the system is broken down to the lowest level that the analysis takes place.

6.2.3 Determination of Cause

A probable cause or causes are bound to exist for each failure mode. Some specific examples of causes in case of aero engines are poor hydraulic discipline, environmental stress, poor maintenance practices, mechanical stress, fuel and oil contamination, fatigue, non-adherence to standard operating procedures, friction, temperature cycling, aging and wear out, substandard or defective parts, operator or maintenance induced error, corrosion, etc.

6.2.4 Assessment of Effect

Each failure has an impact on system operations, which affects the mission reliability and overall availability of the equipment. Effects may vary from total system failure to limited degradation on performance. System performances generally do not get affected immediately on failure of a redundant unit but it influences system reliability. Overall maintenance potential and system safety also gets affected.

Table 6.3 Severity rating.

Severity Rating	Description
8–10	Catastrophic. Significant system failure occurs that can result in injury, loss of life, or major damage.
5–7	Critical. Complete loss of system occurs. Performance is unacceptable.
2–4	Marginal. System is degraded, with partial loss in performance
1	Negligible. Minor failure occurs with no effect on acceptable system performance

6.2.5 Classification of Severity (*S*)

Severity is defined as the rank associated with the most serious effect for a given failure mode [14]. A numerical value of one (1) to ten (10), proportional to this severity, is assigned. High numbers are applicable to effects for which the consequences are severe. For example, if the effect of a particular failure mode is that a critical module will fail catastrophically (i.e., no output), then the assigned severity value will be close to 10. Table 6.3 provides guidelines for assigning this value.

6.2.6 Estimation of Probability of Occurrence *(O)*

The probability of occurrence is based on the expected number of occurrences of each failure mode over a definite time period. This interval may be a mission time, a planned maintenance interval or the system design life. The following CDF can be used to estimate the probability of failure.

For the first failure,

$$F(v_1) = 1 - e^{v_1^b} \tag{6.1}$$

For subsequent failures,

$$F(v_i | v_{i-1}) = 1 - \frac{R(v_i + v_{i-1})}{R(v_{i-1})} \tag{6.2}$$

$$F(v_i | v_{i-1}) = 1 - \exp\left[a \left\{ v_{i-1}^b - (v_i + v_{i-1})^b \right\} \right] \tag{6.3}$$

Table 6.4 Occurrence rating.

Occurrence Rating	Description
$P \geq 0.20$	Frequent. High probability of failure
$0.10 \leq p \leq 0.20$	Probable. Moderate probability of failure
$0.01 \leq p \leq 0.10$	Occasional. Marginal probability of failure
$0.001 \leq p \leq 0.01$	Remote. Unlikely probability of failure

Table 6.4 provides grouping of failure mode frequencies over the operating time interval in our context.

O is rated on a 10-point scale. For example, frequent occurrences can be rated as 10 and remote as1.

6.2.7 Detection

Detection is the assessment of the likelihood that the mechanisms provided to prevent the cause of the failure mode from occurring will detect the cause of the FM or the failure mode itself. The rank associated with the best detection control listed in the design control columns, for the specific failure cause/mechanism under analysis. A numerical value of 1 to 10, inversely proportional to the level of detectability, is assigned. High numbers are applicable to causes/mechanisms that are virtually undetectable. For example, if it is known that a specific failure cause/mechanism will be difficult to detect if it occurs, then the assigned value will be close to 10. Hence, detection is also described on a 10-point scale. Bad detection can be rated as 10 and good detection as 1.0.

6.2.8 Computation of Conventional RPN

This is a quantitative measure known as RPN for each failure mode that combines the probability of the failure modes occurrence with its severity ranking.

$$RPN = O \times S \times D \qquad 1 \leq RPN \leq 1{,}000 \tag{6.4}$$

6.2.9 Determination of Corrective Action

Determination of corrective action depends on the failure modes and their RPN and/or its respective factors. It is needed to provide more attention to the FMs having high RPN and severity classification. The corrective actions are initiated either to reduce *O*, *S* or improve *D*. This can be done by removing the cause of failure, decreasing probability of occurrence and reducing the severity of failure. Decreasing *O* may require design modifications which might be time consuming and costly as well. This will also have a direct impact on the availability of the fleet.

6.3 Estimating RPNs Through the Modified Approach [15]

Recall Kijima-II virtual age equation (derived in Chapter 4):

$$v_i = q\sum_{j=1}^{i} X_j \tag{6.5}$$

Substituting Equation (6.5) in Equation (6.3) to obtain (6.6):

$$F(v_i|v_{i-1}) = 1 - \exp a\left[\left\{q\sum_{j=1}^{i-1} X_j\right\}^b - \left\{\left(q\sum_{j=1}^{i} X_j\right) + \left(q\sum_{j=1}^{i-1} X_j\right)\right\}^b\right] \tag{6.6}$$

From Equation (6.6) it can be seen that the failure probability $F(v_i|v_{i-1})$ and hence the probability of occurrence is directly related to *q*. It is recalled from Equation (6.4) that the RPN is given by $O \times S \times D$. Here, *O* is a measure of the probability of occurrence of the failures. The issue with using *O*, if the failure dynamics are governed by GRP is that it doesn't incorporate the effect of REI. In other words, it is merely a measure of unconditional probability of occurrence, which is meaningful only for distribution where *q* is fixed to be 0 (renewal process) or 1 (minimal repair).

However, the only meaningful probability of occurrence for *GRP* is the conditional probability of occurrence, $F(v_i|v_{i-1})$. Therefore, the modified approach works on replacing *O* with a measure of the conditional probability of occurrence (which we denote by *Q*), thus incorporating the effect of *q*. The values of *Q* can be obtained by using following steps:

(1) A curve between q vs. $F(v_i|v_{i-1})$ is plotted.
(2) From Equation (6.6), the corresponding values of $F(v_i|v_{i-1})$ for the estimated q from the failure data are obtained.
(3) The value of $F(v_i|v_{i-1})$ is rated on a scale of 1 to 10 depending upon the range it falls in as explained in Section 6.2.6.
(4) Q is the value of the rating obtained in step 3.

Hence, the modified algorithm incorporating the effect of conditional probability of occurrence and REI for estimating RPN is placed below as Equation (6.7):

$$\text{RPN} = \text{Q} \times \text{S} \times \text{D} \qquad 1 \leq \text{RPN} \leq 1{,}000 \tag{6.7}$$

Let us illustrate its applicability with the failure modes data of the aero engines in Example 5.5 presented in Chapter 5.

In Example 5.5, three failure modes as observed from the failure data of the first overhaul cycle are given and FM wise MLEs, and other performance parameters are estimated.

Now, for FM1, since $F(t_{1OH}) = 0.8183$ which is greater than 0.20, the computed value of $q = 0.85$ can be rated as ($Q = 10$). The probable reason could be that the compressor blade damage can lead to engine flame out in air and may cause loss of both, the Pilot's life and aircraft, the severity can also be rated as 10. There is no cockpit indication for compressor blade damage but this can be checked only on ground, hence detection can be rated as 8.

In case of FM2, $F(t_{1OH}) = 0.9938$ which is greater than 0.20, the computed value of $q = 2.7$ can be rated as ($Q = 10$). The oil leak can cause seizure of bearings and may lead to failure of engine lubrication system. Hence, the consequences are quite severe, but since both visual and audio indications are provided, the aircraft can be landed under emergent conditions. Thus, the severity (S) can be rated as 8 on a 10-point scale. The detection of oil leak can be done manually on ground and both visual and audio indications are provided in the cockpit, the detection is good both on ground as well as in the air. Hence, detection (D) can be rated as 1 on 10-point scale.

For FM3, $F(t_{1OH}) = 0.6817$ which is again greater than 0.20 but comparatively lesser than the earlier two failure modes and $q = 1$, hence Q can be rated as 8 on a 10-point scale. Note that the consequences of engine overheat could be disastrous and may lead to engine flame out in air leading to aircraft accident, loss of aircraft and loss of Pilot's life. Thus, severity (S) can be rated as 10. Indications of two types—audio and visual—have been

Table 6.5 Failure mode wise initial RPN.

Failure modes	*Q*	*S*	*D*	Initial RPN ($Q * S * D$)
FMI	10	10	8	800
FM2	10	8	1	80
FM3	8	10	2	160

provided in the cockpit, but there is no manual ways of checking engine overheat. Hence, detection (D) can be rated as 2.

In Table 6.5, the summary of failure mode wise of the values of Q, S, and D obtained as discussed above and estimated values of initial RPN is presented.

It can be observed from the table above that FM1 is the most serious failure mode having highest RPN followed by FM2 and FM3.

6.4 Corrective Actions

Various PMs for all the three FMs are presented to appreciate that if they were carried out as per laid down standard maintenance instructions, the failures could have been avoided and the number of failures against each FM would have decreased. This must have resulted in an improved REI (q). In case of compressor blades (FM1) if proper anti corrosive spray of proper specification is carried out during PM, corrosion can be prevented. Similarly, if proper lubrication is carried out on concerned bearings with correct grade of oil, the effect and number of failures due to FM2 can be reduced. If Jet nozzles are properly serviced and electrical systems checked during PM, engine overheat can be avoided and failure of turbine blades (FM3) can be reduced. Various repair processes undergone at repair facilities by the affected components against the three failure modes are also presented to appreciate the effectiveness of REI in improving the RPNs of these FMs. For FM1, the compressor blades have to go through the following processes:

(1) Vibro grinding and Vibro tumbling
(2) Root sand blasting
(3) Root silver coating

For FM2, the only repair process is the replacement of bearings for which a number of components are required to be dissembled for the replacement. An engine test run is then given to ascertain the serviceability.

For FM3, the turbine blades undergo the following repair processes:

(1) Grinding of shroud platform
(2) Welding of blades
(3) Pre-final grinding
(4) Preliminary tool finish along the contour of overlaying and fillets.
(5) Heat treatment
(6) Final grinding
(7) Cleaning and finishing

A sensitivity analysis between q and $F(v_i|v_{i-1})$ is carried out for all the failure modes to arrive at a value of improved q at which $F(v_i|v_{i-1})$ is brought below 0.20. The new RPNs are estimated thereafter and the improvement on all the performance parameters is checked accordingly.

The values of q vs. $F(v_i|v_{i-1})$ for FM1 are plotted in Figure 6.1 to highlight the sensitivity of q with respect to changes in F(.).

From Figure 6.1, it is observed that if the value of q is improved to 0.20 for FM1, then $F(v_i|v_{i-1}) = 0.185$ (hence, less than 0.20), thus $Q = 5$ is assigned.

Similarly, q vs. $F(v_i|v_{i-1})$ is plotted for FM2 in Figure 6.2 to highlight the sensitivity of q with respect to changes in F(.). From Figure 6.2, the improved value of $q = 0.42$ is obtained which gives $F(v_i|v_{i-1}) = 0.19$ (hence, less than 0.20), thus $Q = 5$ is assigned.

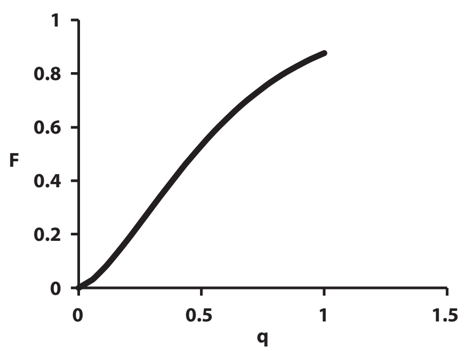

Figure 6.1 q vs. $F(v_i \mid v_{i-1})$ for FM1.

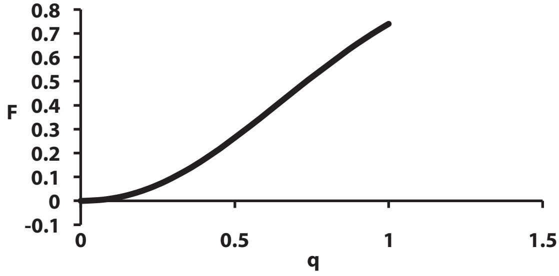

Figure 6.2 q vs. $F(v_i \mid v_{i-1})$ for FM2.

The plot of q vs. $F(v_i|v_{i-1})$ for FM3 is plotted in Figure 6.3 to highlight the sensitivity of q with respect to changes in F.

It is observed from Figure 6.3 that if q is improved to 0.15 for FM3 then $F(v_i|v_{i-1})$ comes down to 0.1821 (hence, less than 0.20) and $Q = 5$ can be assigned.

In Table 6.6, the values of initial q and the improved q obtained through sensitivity analysis along with the rated value of q, i.e., Q, are presented.

In Table 6.7, the values of final RPNs obtained with the value of newly acquired *Qs* are presented.

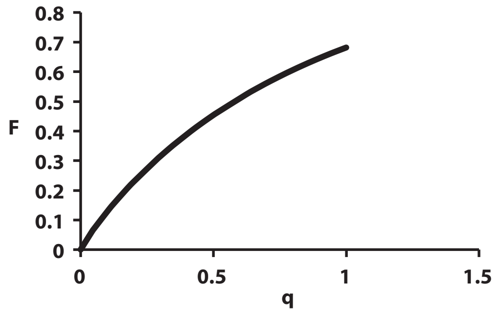

Figure 6.3 q vs. $F(v_i \mid v_{i-1})$ for FM3.

Table 6.6 Failure mode wise values of final Q.

Failure Modes	Estimated *q*	Initial *Q*	Improved *q*	Improved *Q*
FM1	0.85	10	0.20	5
FM2	2.7	10	0.42	5
FM3	1.0	8	0.15	5

Table 6.7 Final RPNs.

Failure Modes	***Q***	***S***	***D***	**Final *RPN***
FM1	5	10	8	400
FM2	5	8	1	40
FM3	5	10	2	100

It is observed from Table 6.7, that RPNs have reduced by almost half on reducing the value of q, i.e., by improving the repair effectiveness without bringing out any changes in mitigating the consequences of failure or improving the detection techniques.

In Tables 6.8, 6.9, and 6.10, the comparative improvement in the performance parameters on applying corrective measures by improving the repair effectiveness q is presented FM wise.

The improvement observed in reliability is seemingly unrealistic and hence not reported. This is due to the extreme wear out and very poor maintenance during the first overhaul cycle as evident from the values of b and q, respectively, for FM1.

The improvement observed in all the performance measures except availability is seemingly unrealistic and hence not reported. This is due to

Table 6.8 FM1.

q	$R(t = t_{1OH})$	$MTBF(t = t_{1OH})$	$A(t = t_{1OH})$
0.85	0.1817	262 h	0.3472
0.20	0.8193	515 h	0.5112
Percent Improvement	#	97	47

"#" Not Reported

Table 6.9 FM2.

q	$R(t = t_{1OH})$	$MTBF(t = t_{1OH})$	$A(t = t_{1OH})$
2.7	#	207 h	0.3619
0.42	0.8102	1128 h	0.7556
Percent Improvement	#	#	109

"#" Not Reported

Table 6.10 FM3.

q	$R(t = t_{1OH})$	$MTBF(t = t_{1OH})$	$A(t = t_{1OH})$
1.0	0.3183	494 h	0.5142
0.15	0.8188	425 h	0.4763
% Improvement	157	#	#

"#" Not Reported

the extreme wear out and very poor maintenance during the first overhaul cycle as evident from very high values of b and q, respectively. Due to very low values of reliability, MTBF, and availability obtained during the first overhaul cycle, application of corrective measures in form of improving q from 2.7 to 0.42 has been seen an upsurge in the improvement of reliability, MTBF, and availability (Table 6.9).

Note that improvement in case of MTBF and availability since value of b in case of FM3 is less than 1 may not be registered.

A significant improvement in all performance parameters is observed by improving the repair effectiveness without actually resorting to adopting any drastic improvement measures.

In Table 6.11, the comparative results of applying the corrective measures by improving the repair effectiveness (q), for the complete aero engine by combining the failure modes are presented. Since the failure modes behave as series model in reliability, the (t), $MTBF$(t), and A(t) are computed as

$$R(t)_{\text{aero engines}} = R(t)_{FM1} \times R(t)_{FM2} \times R(t)_{FM3} \tag{6.8}$$

$$A(t)_{\text{aero engines}} = A(t)_{FM1} \times A(t)_{FM2} \times A(t)_{FM3} \tag{6.9}$$

$$u(t)_{\text{aero engines}} = u(t)_{FM1} + u(t)_{FM2} + u(t)_{FM3} \tag{6.10}$$

Table 6.11 Comparative results.

q	$R(t = t_{1OH})$	$MTBF(t = t_{1OH})$	$A(t = t_{1OH})$
Before correction	#	94 h	0.0646
After correction	0.5435	193 h	0.1840
Percent Improvement	#	105	185

"#" Not Reported

$$MTBF(t)_{aeroengine} = \frac{1}{u(t)_{aero\ engines}} \tag{6.11}$$

It is known from experience that improving the repair quality results into the improvement in the repair effectiveness index (*q*) and leads to a significant improvement in reliability, *MTBF*, and availability of the aero engines. The proposed methodology allows a means to quantify and hence plan the desired improvements. The improvement in case of reliability is seemingly unrealistic, hence not reported for the reasons that the rate of wear out has been quite high and the quality of repair extremely poor during the first overhaul cycle, which is evident from the values of b and q for the first two failure modes. The reasons in detail have already been explained in Example 5.2 of Chapter 5.

6.5 Summary

Conventional FMEA technique estimates RPN without incorporating repair effectiveness and suggests corrective measures to improve RPN by reducing *O, S* and increasing *D*. This can only be done through design changes or incorporating advanced condition monitoring techniques. This chapter presents a methodology for estimating RPN by incorporating repair effectiveness into the conventional RPN algorithm. The chapter presents the repair processes of the identified FMs for better appreciation of REI and suggests ways for improvement of RPN by improving the repair effectiveness index *q* through adoption of measures like following correct maintenance procedures, creating proper maintenance hygiene, using correct tools, testers and ground equipment, using standard spare parts, and so on. REI can also be mathematically modeled as a function of all these factors. A significant improvement in all the performance parameters such as reliability and availability can be seen by improving *q* rather than incorporating measures which involve design changes, sophisticated reliability improvement techniques, and consequent high costs coupled with organizational resistance to big changes.

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7

An Integrated Approach to Weapon Procurement Systems

7.1 Introduction

In this chapter, a framework for an integrated approach to an entire procurement process is developed. An overview of the approach is explained through a flow chart placed at Figure 7.1.

Combat effectiveness and weapon systems selection for combat forces with various qualitative and quantitative methods have been a desired area in military research. To make a proper effectiveness assessment of weapon systems, a number of factors and criteria need to be considered. As shown in Figure 7.1, Air Power (*AP*), Land Power, and Sea Power constitute the three components of combat power. AP is fundamentally important to the security of a country since it is the dominant component of combat power in modern warfare. AP is distinguished by its flexibility, speed, ubiquity, range, and shock effect which, in combination, give it the unique ability to concentrate force and maneuver rapidly over long distances. The objective of AP is to gain maximum military effectiveness from the use of air. Due to crucial importance of AP in modern warfare, it becomes imperative to evaluate its effectiveness and potential. Only if the overall effectiveness of the AP of enemies is known, a country can orient itself to build up appropriate defense. Further, weapon systems are considered essential to the outcome of war, and hence, their procurement becomes an important national decision. If the procurement of weapon systems relies on the intuition of high-level commanders rather than a systematic decision-making process, it can lead to subsequent changes and revisions that waste the defense budget and increase the lead time for procurement.

A brief literature review (Table 7.1) on weapon systems capability evaluation (WSCE) using multi-criteria decision making (MCDM) tools and techniques is presented below for better understanding of the subject by the readers.

Rajiv Nandan Rai, Sanjay Kumar Chaturvedi and Nomesh Bolia. Repairable Systems Reliability Analysis: A Comprehensive Framework, (143–168) © 2020 Scrivener Publishing LLC

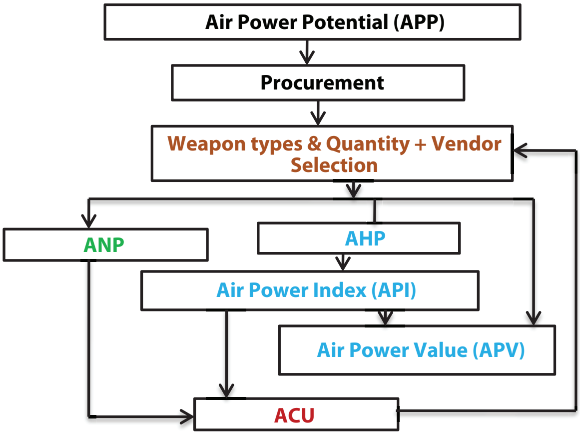

Figure 7.1 Overview of the approach.

The AHP is aimed at integrating different measures into a single overall score for ranking decision alternatives. Its main characteristic is that it is based on pairwise comparison judgment. However, the standard AHP eigenvalue prioritization approach cannot be used, when the decision maker faces a complex and uncertain problem and expresses his/her comparison judgment as uncertain ratios. AHP is thus ineffective when applied to ambiguous problems. Since the real world is highly ambiguous, decision makers usually feel more confident giving interval judgment than fixed value judgment. This is because, usually, they are unable to be explicit about their preferences due to fuzzy nature of the comparison process. For this reason, the authors at Sl No. 3 of Table 7.1 proposed AHP based on fuzzy scales in the field of weapon system capability assessment and selection.

Fuzzy AHP and TOPSIS and other methodologies as evident from the literature review are basically adopted to deal with incompleteness, ignorance, fuzziness, and vagueness of the information. The full AHP–fuzzy AHP solution is only practically usable if the number of criteria and alternatives is sufficiently low so that the number of pairwise comparisons performed by the evaluator must remain below a reasonable threshold. For example, if there are n criteria that have been assigned the importance weights and m alternatives, then to run a full AHP, AHP-fuzzy solution there are $nm(m - 1)/2$ pairwise comparisons remaining to be performed [14]. In this study, $n = 41$, $m = 3$ (described in later section). Hence, number of pairwise comparisons to be carried out is 123 which is substantially large.

Table 7.1 Brief literature review on WSCE using MCDM.

Sl No.	Authors	Brief Work Summary
1	Lee and Ahn [1]	Discuss a hierarchical weapon systems assessment model framework and investigate the applicability of analytic hierarchy process (AHP) [2] in establishing static valuation of combat force potential of a territorial army.
2	T.L. Saaty [2]	Presents fundamentals of AHP.
3	Mon *et al.* [3]; Cheng and Mon [4]; Chen [5]; Cheng [6]; Cheng *et al.* [7]; Cheng and Lin (2002) [8]	Propose AHP based on fuzzy scales in the field of weapon system capability assessment and selection.
4	Chai *et al.* [9]	Provides extensive literature review on decision making techniques in which they have listed a number of application areas based on fuzzy approaches. Besides AHP, fuzzy AHP, fuzzy TOPSIS, and other techniques, various artificial intelligence (AI) techniques like neural network (NN), rough set theory (RST) and gray system theory (GST) are also used to identify approximate solutions for complete optimization problems.
5	Hwang and Yoon [10]	Among numerous MCDM methods developed to solve real-world decision problems, technique for order preference by similarity to ideal solution (TOPSIS) continues to work satisfactorily in diverse application areas. The authors propose TOPSIS to help select the best alternative with a finite number of criteria.

(*Continued*)

Table 7.1 Brief literature review on WSCE using MCDM. (*Continued*)

Sl No.	Authors	Brief Work Summary
6	Behzadian *et al.* [11]	The standard TOPSIS method attempts to choose alternatives that simultaneously have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. The authors in a detailed literature review enumerates a state-of-art survey of TOPSIS applications.
7	Wang and Chang [12]; J. Wang *et al.* [13]; M. Dag deviren *et al.* [14]	The key papers in the field of WSCE using TOPSIS for ranking.
8	Greiner *et al.* [15]	Propose a hybrid approach using AHP and integer programming to screen weapon systems projects.
9	Lee *et al.* [16]	Proposed a hybrid approach of goal programming for WSCE.
10	Jiang *et al.* [17]	Propose an algorithm for WSCE on the basis of belief structure mode and evidential reasoning approach developed to deal with various types of uncertainties such as ignorance and subjectiveness.
11	Meade and Presley [18]; Gencer and Gurpinar [19]; C. Valmohammadi [20]; Chen *et al.* [21]; Kumar and Maiti [22]; Shiue and Lin [23]; Atmaca and Basar [24]; Sadeghi and Manesh [25]; Hasanzadeh *et al.* [26]; Saaty and Vargas [27]	Analytic network process (ANP) has also been paid due importance in its application in various fields for ranking as evident from the literature. ANP, unlike other approaches, also considers interactions of higher level elements from lower level elements and the interactions of elements within the same level.

We involve 15 senior experts who have been working for more than 25 years in this field to decide the criteria. We decide the 41 criteria based on the military aviation doctrine duly deliberated by the experts, considering AP operations, weapon categories, weapon performance categories, and their performance characteristics. We then carry out pairwise comparisons with the help of same experts. Thus, in this case, the inputs obtained from the experts are rigorous and the weapon systems selected to develop various models have been in use for a prolonged period. The computations involved in fuzzy versions, TOPSIS, and AI techniques are too complex to be implemented by combat forces; hence, there was a need to satisfy the desired objectives in a simplified manner without compromising on any required information to build up the model and the methodologies that can be easily implemented by the combat forces. Although ANP has been extensively used for numerous applications, it has been least explored in WSCE problems.

We initially present a model for the selection of alternatives for procurement of AP weapon systems using ANP with help of 41 criteria including reliability, maintainability, and various armament capabilities. The chapter then presents an optimization model for forming a new Air Combat Unit (ACU) in both attack and defense modes, considering several constraints. To achieve this, we present an AHP-based method to compute APP as measured by an index called API and APV on similar lines to that of [1]. The estimation of API and APV is utilized to determine the quantum of enhancement in APP needed against the hostile forces and decide the extent of expansion of APP required for a combat force based on budget constraints, hostile forces capabilities, and other parameters. The indices so estimated are further used or quantifying APP in formulating the objective function for raising an ACU subjected to various constraints.

7.2 Analytic Network Process Model

The first step in developing the ANP model is the analysis of the alternatives selection problem. The main goal of the alternative selection problem is choosing the best alternative that meets the requirement or criteria of the combat unit to build up its APP. Forty criteria is then determined with the help of specialists in this field as mentioned in Section 7.1, under the below mentioned three main criteria clusters:

- Air Power Weapon Category (APWC)
- Air Power Weapon Performance Category (APWPC)
- Air Power Weapon Performance Characteristics (APWPCH)

The criteria are developed during multiple brains storming sessions with the concerned specialists in sync with the doctrine of the combat aviation. All previous weapon systems procurement and future vision of combat aviation were extensively referred. While developing the criteria, all possible roles of the combat force in AP have been considered and decided upon all possible combinations of the weapon systems, which would accomplish the desired role. For every criteria developed, due attention is paid to all aspects of AP and exhaustive deliberations were made accordingly. Further, the interactions between and within clusters and their elements were established.

Figure 7.2 shows alternatives selection network model's control hierarchy according to the determined criteria and also the interactions between the various clusters and their elements. A brief description of the criteria clusters and their elements are now presented to have a better appreciation of the ANP process in deciding the best alternative for weapon selection.

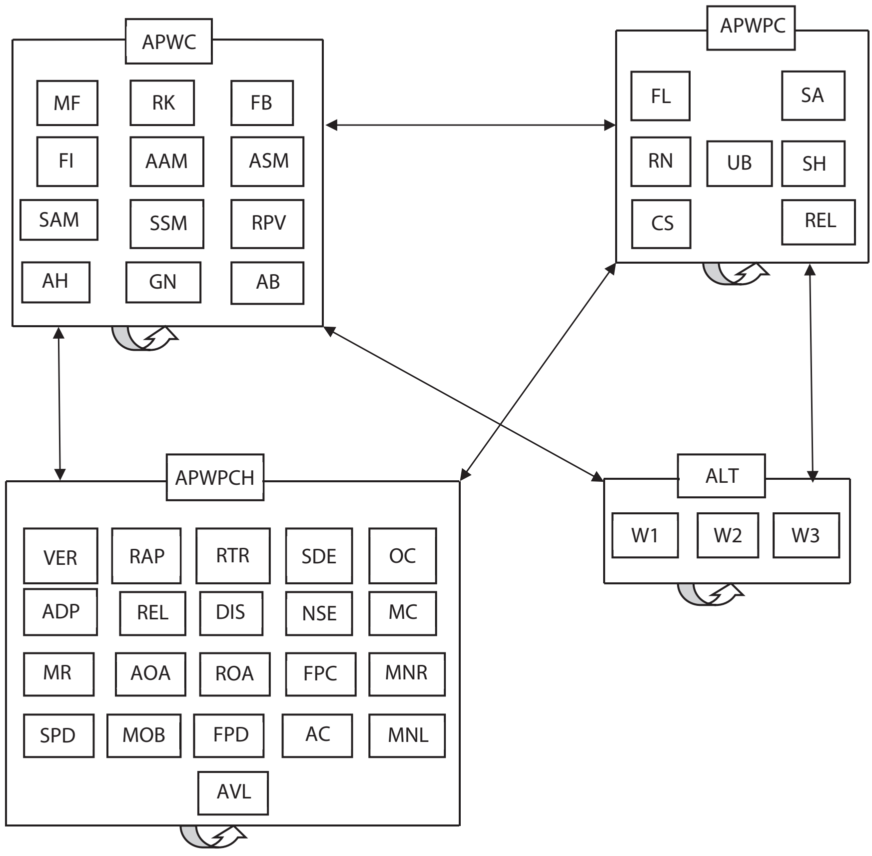

Figure 7.2 Evaluation of alternatives through ANP.

APWC: This cluster houses *AP* weapons of various categories used for defense, attack, or both. Multirole fighters (MFs) can defend the aerospace by intercepting the enemy aircraft and can also be launched into enemy's territory for bombing missions. Fighter Interceptors (FI) are exclusively used for interception missions and can only defend an aerospace. Surface to Air Missiles (SAM) is the second line of defense used for interception of hostile aircraft if the MFs or FIs fail in their missions. Rockets (RK), Air to Air Missiles (AAM), and Guns (GN) are used by MFs and FIs to protect themselves from hostile aircraft in the process of interception. Attack helicopters (AH), fighter bombers (FB), remotely piloted vehicles (RPV), and surface to surface missiles (SSM) are used for attacking enemy's aerospace. Aerial Bombs (AB) and Air to Surface Missiles (ASM) are the payloads of FBs, AHs, and RPVs that are dropped inside enemy's territory to achieve the attack mission.

APWPC: The speed with which AP can be delivered is very different from the speed of sea power applications while the range of AP is a lot greater than that of land power. The magnitude of the differences between AP and other forms of combat power leads to some special qualities which set AP into a different category. This cluster brings out various performance categories pertaining to all weapon categories. The AP has Flexibility (FL) so that the assets can be diverted quickly and effectively from one task to another and from one target to another. AP must possess swiftness of applications (SA) so as to cover distance quickly and to apply force with little delay. Applied to AP, Ubiquity (UB) really means ability to operate almost anywhere within the air and over the surface. The AP can deliver fire power over long distances, i.e., it should have adequate range (RN). AP shall acquire shock (SH) effect, i.e., the AP shall have sudden, disruptive effect of its presence. AP can deliver enormous firepower but it does so at tremendous cost (CS). The resources needed, the effort required and the penalties imposed are the costs of AP. However, all the performance categories explained above can only be tested to their extremes if the AP is able to execute its mission reliably. Hence, reliability (REL) factors in as one of the most important attributes.

APWPCH: All categories of AP weapon performance have certain distinguished characteristics. A related characteristic is versatility (VER) which is evident in the wide range of tasks that it can perform. Increasingly AP has adaptability (ADP) to a wide range of roles, termed multi rolling (MR), whereby it can be engaged in different modes of employment with minimum difficulty. Moreover, there are some assets that are individually capable of performing more than one specialist task during one airborne sortie. Speed (SPD), rapidity (RAP), and responsiveness (RES) are the main attributes of SA. Implicit in the meaning are the ideas of going anywhere and covering long distances. SA is therefore associated with the attributes of UB and RN. Mobility (MOB) implies movement unconstrained by physical barriers. Ability to operate anywhere (AOA) and recover, turnaround, and reload (RTR) are also the important attributes of UB. Range is also used for reach which implies radius of action (ROA) and ranging which combines the concepts of distance (DIS) and fire power delivery at long distances (FPD). SH is a product of the noise (NSE) of aircraft, sudden disruptive effect (SDE), and firepower concentration (FPC) quickly delivered. The psychological result is a fear of disproportionate vulnerability perceived by ground forces or civilians who have been subjected to aerial warfare. Shock is the most effective with the element of surprise. AP is expensive and involves acquisition costs (AC), operations cost (OC), and maintenance cost (MNC). All assets responsible to deliver incessant AP are required to be maintained in an excellent condition and should be such that they recover in a very short span of time after failure. Thus, maintainability (MNL) plays a vital role in strengthening the AP. For instantaneous and effective application of AP in both tactical and strategic modes of warfare availability (AVL) of all the assets related to AP is of utmost importance. These two attributes of MNL and AVL directly affects the mission reliability (MNR).

Alternatives are then determined. Alternatives, i.e., W1, W2, and W3 are selected from the successful ones in their field of activity since all the weapon procurements are mainly resorted to from these alternatives as they are the key players in the field of supplying weapons.

The next step is to construct the super matrix according to the network built in Figure 7.2. The 1–9 scale (Appendix A) developed by Saaty [2] is used and pairwise comparisons are made with the help of judgment from the domain experts. While carrying out pairwise comparisons, the median value of the judgment is taken for estimation of eigenvalues. Also, the consistency of each comparison is checked (Appendix A). The pairwise comparisons for ANP model are provided in Appendix "A" (Tables A1–A65). Each column of a super matrix is a normalized eigenvector. The super matrix is an unweighted one, because each column consists of several eigenvectors that sum to one, and hence, the entire column of the matrix may sum to a number greater than one. The super matrix needs to be stochastic to derive meaningful limiting properties [20].

Therefore, there is a need to obtain the weighted super matrix for which the influence of clusters on each cluster with respect to the control criterion is determined. This gives an eigenvector of influence of the clusters on each cluster. Then, the unweighted super matrix is multiplied by the priority weights from the clusters, which yields the weighted super matrix. Finally, the super matrix is brought to the steady state by multiplying the weighted super matrix by itself until the super matrix's row values converge to the same value for each column of the matrix. This is called the limiting super matrix and this gives the ranking of the alternatives. The unweighted, weighted, and limiting super matrices are given in Appendix "B" (Tables B1–B6). The rankings achieved from the super matrix reveal that W1 emerges as the best choice followed by W2 and W3.

7.3 AP Index and AP Value Estimation

In this section, an AHP-based procedure for estimation of API and APV is presented. An AHP model technique for the parameter estimation process is used on similar lines as that of [1]. The process for enhancing APV based on budget allocation is also demonstrated.

7.3.1 Analytic Hierarchy Process Model

In this chapter, the model adopted to evaluate APE has a detailed multi-level hierarchy and involves a multi-party group judgment. The APE model considers factors relevant to the AP of any combat force. The levels of the hierarchy are as follows: 1) AP operations (e.g., defense, attack); 2) APWC (e.g., fighters, attack helicopters, rockets, missiles, bombs, etc.); 3) APWPC (e.g., flexibility, swiftness of application, ubiquity, range, shock effect, cost, reliability, etc.); 4) APWPCH (e.g., versatility, speed, distance, adaptability, firepower, etc.); 5) Best weapon procurement alternative (e.g., western, russian, indigenized, etc.). The hierarchy of these factors is shown in Figure 7.3.

7.3.2 AP Index Estimation

An effectiveness score of a weapon system relative to other weapon systems within the same category is evaluated via the lower three levels of the hierarchy. The weights of APWC, APWPC, and APWPCH are evaluated through pair wise comparisons, the judgment of which are obtained from various specialists in the field. All the judgments are found at the limit of consistency with consistency ratio, $CR < 0.1$. The pairwise comparisons for AHP model are provided in Appendix "C" (Tables C1–C23). Once we assign the effectiveness scores (APIs) to weapons within a given category, we scale them by the weight of that category relative to other weapon categories to yield the final scaled effectiveness scores. In this case, the weights assigned to the base weapons of respective categories serve as scaling factors. Table 7.2 presents *API* estimation of weapons systems.

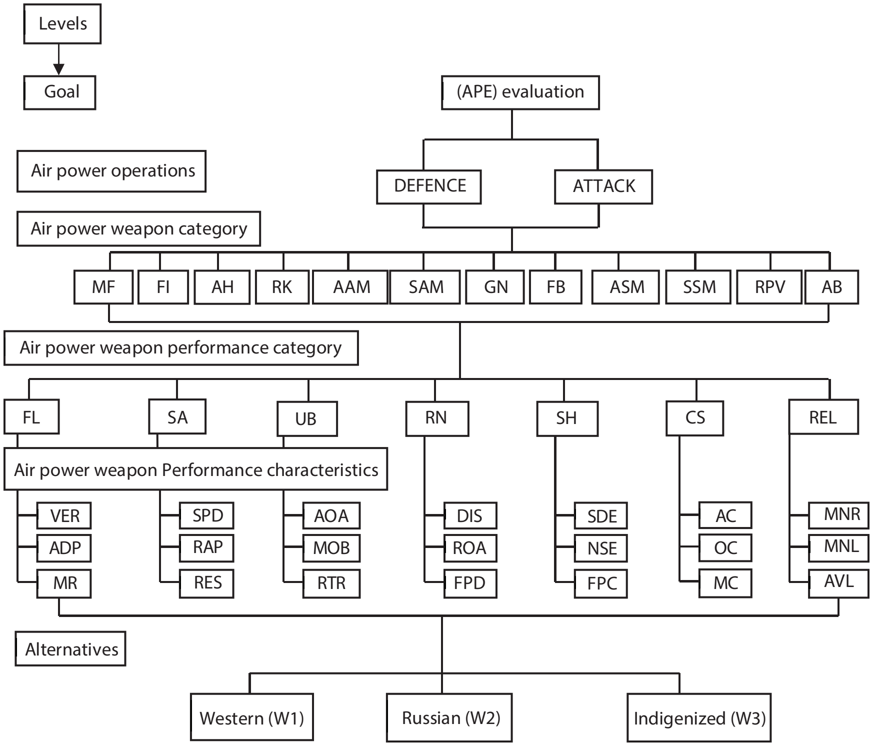

Figure 7.3 Evaluation of APE through AHP.

7.3.3 Sample AP Index Estimation

In this section, the procedure for estimating API is illustrated. The estimation for FI is carried out. Suppose that W1 is the base weapon of this category, the API value of W1, denoted as API_{W1}, is calculated as follows:

$$API_{W1} = (0.11 \times FL) + (0.26 \times SA) + (0.14 \times UB) + (0.16 \times RN) + (0.17 \times SH) + (0.07 \times CS) + (0.12 \times REL) \qquad \text{(Refer) (7.1)}$$

$$= 0.11 \times (0.25 \times VER + 0.16 \times ADP + 0.59 \times MR) + 0.26 \times (0.59 \times SPD + 0.25 \times RAP + 0.16 \times RES) + 0.14 \times (0.53 \times AOA + 0.33 \times MOB + 0.14 \times RTR) + 0.16 \times (0.25 \times DIS + 0.16 \times ROA + 0.59 \times FPD) + 0.17 \times (0.53 \times SDE + 0.14 \times NSE + 0.33 \times FPC) + 0.07 \times (0.16 \times AC + 0.25 \times OC + 0.59 \times MC) + 0.12 \times (0.59 \times MNR + 0.25 \times MNL + 0.16 \times AVL)$$

Table 7.2 Weights for API.

APWPC	APWPCH	Alternatives		
		W1	W2	W3
FL (0.11)	*VER* (0.25) *ADP* (0.16) *MR* (0.59)	0.35 0.36 0.35	0.35 0.34 0.35	0.30 0.30 0.30
SA (0.26)	*SPD* (0.59) *RAP* (0.25) *RES* (0.16)	0.54 0.53 0.54	0.30 0.33 0.30	0.16 0.14 0.16
UB (0.14)	*AOA* (0.53) *MOB* (0.33) *RTR* (0.14)	0.43 0.62 0.53	0.43 0.27 0.33	0.14 0.11 0.14
RN (0.16)	*DIS* (0.25) *ROA* (0.16) *FPD* (0.59)	0.54 0.64 0.56	0.30 0.26 0.35	0.16 0.10 0.09
SH (0.17)	*SDE* (0.53) *NSE* (0.14) *FPC* (0.33)	0.40 0.33 0.54	0.40 0.33 0.30	0.20 0.33 0.16
CS (0.07)	*AC* (0.16) *OC* (0.25) *MC* (0.59)	0.16 0.53 0.64	0.30 0.33 0.26	0.54 0.14 0.10
REL (0.12)	*MNR* (0.59) *MNL* (0.25) *AVL* (0.16)	0.62 0.69 0.64	0.30 0.24 0.26	0.09 0.07 0.10

$= 0.11 \times (0.25 \times 0.35 + 0.16 \times 0.36 + 0.59 \times 0.35) + 0.26 \times (0.59 \times 0.54 + 0.25 \times 0.53 + 0.16 \times 0.54) + 0.14 \times (0.53 \times 0.43 + 0.33 \times 0.62 + 0.14 \times 0.53) + 0.16 \times (0.25 \times 0.54 + 0.16 \times 0.64 + 0.59 \times 0.56) + 0.17 \times (0.53 \times 0.40 + 0.14 \times 0.33 + 0.33 \times 0.54) + 0.07 \times (0.16 \times 0.16 + 0.25 \times 0.53 + 0.59 \times 0.64) + 0.12 \times (0.59 \times 0.62 + 0.25 \times 0.69 + 0.16 \times 0.64)$

$= 0.53$

On similar lines, API_{W2} and API_{W3} for FI are estimated as 0.33 and 0.17. API_{W1}, API_{W2}, and API_{W3} are adjusted so that the API of the base weapon,

Table 7.3 Actual and scaled API values of each weapon category.

Weapons	$APIW_1$		$APIW_2$		$APIW_3$	
	Actual	**Adjusted**	**Actual**	**Adjusted**	**Actual**	**Adjusted**
MF	0.53	1.00	0.33	0.62	0.16	0.31
FI	0.53	1.00	0.33	0.63	0.17	0.32
AH	0.48	1.00	0.33	0.69	0.25	0.52
RK	0.60	1.00	0.48	0.80	0.15	0.25
AAM	0.64	1.00	0.46	0.72	0.13	0.20
SAM	0.58	1.00	0.52	0.90	0.12	0.21
GN	0.50	1.00	0.48	0.96	0.20	0.40
FB	0.56	1.00	0.38	0.68	0.14	0.25
ASM	0.64	1.00	0.46	0.72	0.13	0.20
SSM	0.54	1.00	0.35	0.65	0.50	0.93
RPV	0.64	1.00	0.42	0.66	0.17	0.27
AB	0.54	1.00	0.35	0.65	0.48	0.89

API_{W1}, becomes 1. This adjustment aids in better comparison of alternate systems. Therefore, the adjusted API of W2 is 0.33/0.53 = 0.63 and API of W3 is 0.17/0.53 = 0.32. The estimated actual and adjusted values of API of each weapon category as explained above are given at Table 7.3.

7.3.4 AP Value Estimation

Once the *APE* scores for all weapons by each category and the priority weights of all APWC are derived, the next step is to aggregate them into APV using the actual fleet strength. API scores of weapons by category, number of weapons, and category weights are aggregated to compute APV for the combat unit using the following formula.

$$APV = \sum_{i=1}^{n} (APWCW)_i \left[\sum_{j=1}^{m} x_{ij} (API)_{ij} \right] \tag{7.2}$$

where $(APWCW)_i$ is the category weight of $APWC(i)$; n is the number of APWC; x_{ij} is the number of weapons of type j in category i; and API_{ij} is the API of weapon type j in i^{th} category. The APV scores reflect the APP of a combat unit and can be used to determine the quantum of enhancement in APP needed against the hostile forces. A hypothetical example of deriving APV of combat force "ALPHA" is presented in Table 7.4.

Weapons belonging to $W1$ are weapons of a particular origin and have been considered as base weapons. Weapons belonging to W2 and W3 are from different vendors then W1. The numbers taken for calculating APV are hypothetical. APIs have been obtained from Table 7.3. Weapon unit costs are approximate and hypothetical and have been used merely to demonstrate the procedure. APVs are estimated with the help of Equation (7.2). The present APV of the combat force "ALPHA" has come out to be 6,548.48. The APV so obtained is indicative of the APP with respect to the APE and the strength of weapons of a particular category. APV of the entire combat force is an indicator of the APP of that force. "ALPHA" combat force can use this APV to assess its potential *vis-à-vis* its enemies in respective weapon categories and overall potential as well. Hence, the "ALPHA" combat force can plan its war strategies accordingly.

Percentage contribution of various weapon categories toward overall *APP* can also be calculated from Table 7.4 Any combat force can plan its procurement of weapons based on the budget allocation and the quantum of desired APV increment for that particular category of weapon.

A hypothetical example of enhancing the present overall APV of the combat force "ALPHA" based on the budget allocation is placed below at Table 7.5 Assume that the combat unit "ALPHA" wants to add MF, FI, RK, and AB to its inven tory to supplement its APP.

It can be observed from Table 7.5 that an overall increase of 7.67% in APV of combat unit "ALPHA" can be achieved with a budget allocation of approximately 1,365 million USD. The effect of individual weapons on the AP potential can also be judged, for example, procurement of only 15 MF-W1 increases APV by (2.7/502.02 = 0.54%) and incurs a cost of 493.4 million USD, whereas procurement of 2,000 RK-W1 increases APV by (80/502.02 = 15.93%) and incurs a cost of just 65.8 million USD. Thus, APV proves to be a very important quantitative measure in deciding the extent of expansion of the AP potential of a combat unit based on budget constraints, hostile forces capabilities, and other operational parameters.

Table 7.4 APV estimation of combat force “ALPHA”.

APWC	Category weight	Air Power weapons	Number	API	APV	Weapon unit cost (in million USD)
MF	0.18	MF-W1	54	1	9.72	33
		MF-W2	152	0.62	16.96	26
		MF-W3	0	0.31	0	20
FI	0.11	FI-W1	0	1	0	20
		FI-W2	252	0.63	17.46	13
		FI-W3	0	0.32	0	10.5
AH	0.09	AH-W1	0	1	0	9.2
		AH-W2	20	0.69	1.24	9.2
		AH-W3	20	0.52	0.94	5.2
RK	0.04	RK-W1	5,000	1	200	0.03
		RK-W2	75,000	0.8	2,400	0.03
		RK-W3	0	0.25	0	0.026

(Continued)

Table 7.4 APV estimation of combat force "ALPHA". (*Continued*)

APWC	Category weight	Air Power weapons	Number	API	APV	Weapon unit cost (in million USD)
AAM	0.05	AAM-W1	7,000	1	350	0.07
		AAM-W2	1,000	0.72	36	0.07
		AAM-W3	0	0.20	0	0.40
SAM	0.09	SAM-W1	0	1	0	3.30
		SAM-W2	150	0.9	12.15	2.6
		SAM-W3	0	0.21	0	2.4
GN	0.03	GN-W1	144	1	4.32	0.02
		GN-W2	496	0.96	14.28	0.02
		GN-W3	0	0.40	0	0.016
FB	0.18	FB-W1	90	1	16.2	20
		FB-W2	72	0.68	8.81	13
		FB-W3	0	0.25	0	10.5

(*Continued*)

Table 7.4 APV estimation of combat force "ALPHA". (*Continued*)

APWC	Category weight	Air Power weapons	Number	API	APV	Weapon unit cost (in million USD)
ASM	0.05	ASM-W1	0	1	0	0.07
		ASM-W2	150	0.72	5.4	0.07
		ASM-W3	0	0.20	0	0.05
SSM	0.09	SSM-W1	0	1	0	6.6
		SSM-W2	0	0.65	0	3.9
		SSM-W3	50	0.93	4.18	3.3
RPV	0.04	RPV-W1	20	1	0.8	6.6
		RPV-W2	0	0.66	0	5.2
		RPV-W3	0	0.27	0	3.9
AB	0.06	AB-W1	25,000	1	1,500	0.03
		AB-W2	50,000	0.65	1,950	0.03
		AB-W3	100,000	0.89	5,340	0.026

Table 7.5 APV enhancement.

APWC	Category weight	Air Power weapons	Number	API	APV	Cost incurred (in million USD)
MF	0.18	MF-W1	15	1	2.7	493.40
		MF-W2	0	0.62	0	0
		MF-W3	18	0.31	1.00	355.20
FI	0.11	FI-W1	12	1	1.32	237
		FI-W2	0	0.63	0	0
		FI-W3	0	0.32	0	0
RK	0.04	RK-W1	2,000	1	80	66
		RK-W2	0	0.8	0	0
		RK-W3	0	0.25	0	0
AB	0.06	AB-W1	2,500	1	150	82
		AB-W2	0	0.65	0	0
		AB-W3	5,000	0.89	267	132

7.4 Formation of an ACU

In the previous section, the ANP model for selection of the best alternative for procurement of the weapons is presented and followed by the AHP-based APP computation wherein estimation of two vital parameters, API and APV for APE and APP, is performed. In this section, an optimization model [28] is presented with representative constraints for forming a new ACU. The parameter API estimated in previous section is used as an important tool in development of the optimal decision model. The basic idea is to maximize the APP subject to budgetary and other constraints as explained below. For instance, an organization might have indigenization requirements for better access and cost. In addition, there are constraints which exist due to functional role of an ACU. For example, if the ACU is supposed to play an attack role, it will need certain minimum number *of MFs, FIs AHs, RKs*, and other weapons in its inventory. Further, all weapons necessarily undergo the process of continuous PM at specified intervals. In fact, some defense establishments in the world maintain in house repair shops at significant expenses and efforts for efficient and reliable war readiness. Further, all weapon systems in the inventory of an ACU may not be available at a given point of time. Therefore, ACUs are also required to maintain minimum standards of weapon systems available to strike at almost instant notice. These minimum standards determine what is termed as its critical mission readiness strategy (CMRS), i.e., the minimum number of weapons of each category that should be available, taking into account the (repair) downtime. Optimization models for an ACU in both attack and defense roles that maximize the APP and satisfies the constraints of i) budget, ii) indigenization, iii) functional role strategy, and iv) CMRSs are presented next.

7.4.1 Attack Model

An optimization model for an ACU with a predominantly attack role is firstly presented. The following notations are useful for this as well as the next section. Let x_{ij} be the number of weapons of i^{th} category from j^{th} vendor, API_{ij} be the AP indices as estimated earlier for the i^{th} weapon category from the j^{th} vendor, C_{ij} be the unit cost of i^{th} category from j^{th} vendor, B be the total budget to raise an ACU, p be the minimum proportion of indigenization of weapon systems as per the indigenization policy, A_{ij} be the availability of i^{th} weapon category from j^{th} vendor at a given point of time. Further, let $a = \{a_1, a_2, \ldots, a_{12}\}$ represent the minimum number of weapons of i^{th} category in attack role and $d = \{d_1, d_2, \ldots, d_{12}\}$ minimum number of weapons of i^{th} category in defense role required to raise an ACU in accordance with the functional role strategy of the *ACU*. Lastly, let $S = \{m_1, m_2, \ldots, m_{12}\}$ represent the minimum number of weapons of i^{th} category for critical mission readiness according to the CMRS of the ACU.

The attack model can be formulated as constrained optimization problem as shown below:

$$\text{Maximize:} \quad \sum_i \sum_j API_{ij} x_{ij}$$

Subject to

$$\sum_i \sum_j C_{ij} x_{ij} \leq B \qquad \text{Budget} \qquad (7.3)$$

$$\sum_j x_{ij} \geq a_i \ \forall i \quad \text{and} \ (a_1, a_2, \ldots, a_7) = 0 \qquad \text{Role} \qquad (7.4)$$

$$\sum_j A_{ij} x_{ij} \geq m_i \forall_i \ \text{and} \ (m_1, m_2, \ldots, m_7) = 0 \qquad \text{CMRS} \qquad (7.5)$$

$$\sum_i x_{i3} \geq p \sum_i \sum_{j=1}^{2} x_{ij}, \quad 0 \leq p \leq 1 \qquad \text{Indigenization} \qquad (7.6)$$

7.4.2 Defense Model

Similarly, if the predominant task of the ACU is to defend, its configuration can be determined by solving the following optimization problem:

$$\text{Maximize:} \quad \sum_i \sum_j API_{ij} x_{ij}$$

Subject to

$$\sum_i \sum_j C_{ij} x_{ij} \leq B \qquad \text{Budget} \qquad (7.7)$$

$$\sum_j x_{ij} \geq d_i \ \forall i \quad \text{and} \ (d_8, d_9, \ldots, d_{12}) = 0 \qquad \text{Role} \qquad (7.8)$$

$$\sum_{j} A_{ij} x_{ij} \geq m_i \forall_i \text{ and } (m_8, m_9, \ldots, m_{12}) = 0 \qquad \text{CMRS} \tag{7.9}$$

$$\sum_{i} x_{i3} \geq p \sum_{i} \sum_{j=1}^{2} x_{ij}, \quad 0 \leq p \leq 1 \qquad \text{Indigenization} \tag{7.10}$$

7.4.3 Illustrative Example

An illustrative example for each role to demonstrate that the optimization problem can be solved readily is now presented. These are standard integer programs that can be solved using any of the commonly available solvers. The solution of a hypothetical example as an illustration is presented.

Let the allotted budget B = 3290 *million USD*. The values of API_{ij} for i^{th} weapon category from j^{th} vendor as estimated earlier are shown below at Table 7.6.

Table 7.6 API_{ij} values.

j			
i	1	2	3
1	1	0.62	0.31
2	1	0.63	0.32
3	1	0.69	0.52
4	1	0.80	0.25
5	1	0.72	0.20
6	1	0.90	0.21
7	1	0.96	0.40
8	1	0.68	0.25
9	1	0.72	0.20
10	1	0.65	0.93
11	1	0.66	0.27
12	1	0.65	0.89

Table 7.7 C_{ij} (million USD), a_i, d_i, and m_i.

J				a_i	d_i	m_i
i	**1**	**2**	**3**			
1	33	26	20	0	20	9
2	20	13	11	0	20	9
3	10	9	5	0	2,000	5
4	0.03	0.03	0.026	0	1,000	100
5	0.066	0.066	0.05	0	100	50
6	3.3	2.6	2.4	0	40	25
7	0.02	0.02	0.016	0	0	9
8	20	13	11	20	0	9
9	0.066	0.066	0.05	500	0	50
10	6.6	3.9	3.3	50	0	10
11	6.6	5.3	3.9	20	0	6
12	0.03	0.03	0.026	2,000	0	500

The values of C_{ij} and a_i, d_i, and m_i presented in Table 7.7 are hypothetical and are considered for illustration purpose.

The values of A_{ij} can be estimated from the failure and repair data of the weapons available with the ACU for a period of time with the help of (7.11).

$$A_{ij} = \frac{MTBF_{ij}}{MTBF_{ij} + MTTR_{ij}} \tag{7.11}$$

where A_{ij} is the availability of i^{th} weapon category from the j^{th} vendor, $MTBF_{ij}$ is the mean time between failures of i^{th} weapon category from the j^{th} vendor, and $MTTR_{ij}$ is the mean time to repair of i^{th} weapon category from the j^{th} vendor. In Table 7.8, the hypothetical values of A_{ij} are presented for illustration purpose.

The optimization problem is solved on IBM ILOG CPLEX Optimization Studio V 12.4 and the optimal values so obtained are presented in Table 7.9.

Table 7.8 A_{ij} values.

j			
i	**1**	**2**	**3**
1	0.60	0.50	0.45
2	0.55	0.50	0.45
3	0.65	0.55	0.50
4	0.70	0.65	0.60
5	0.75	0.65	0.55
6	0.60	0.50	0.45
7	0.80	0.75	0.65
8	0.60	0.50	0.45
9	0.75	0.65	0.55
10	0.70	0.60	0.55
11	0.65	0.60	0.55
12	0.80	0.70	0.70

Comparing the achieved optimal values shown in Table 7.9 with the values of a_i, d_i, and m_i, it is observed that with the allotted budget of 3,290 million USD an ACU can be raised and the budget can also successfully meet the CMRS of the *ACU*. However, to arrive at an optimal solution, most of the procurements are to be resorted from the third vendor.

The potential available for a force at any point of time is dependent on the number and operational readiness of the weapon systems. Thus, to enhance APP by increasing APV, either more weapons can be procured or the availability of the weapon systems can be improved.

7.5 Summary

Air power is fundamentally important to security of a country and plays a decisive role in the outcome of hostilities. Any air force in the world is faced with two major problems: first, it wants to know the optimal configuration of weapons subject to various constraints for forming an ACU

Table 7.9 Optimal x_{ij} values.

j			
i	**1**	**2**	**3**
1	0	0	20
2	0	0	20
3	0	0	20
4	2,000	0	0
5	0	0	1,000
6	0	0	100
7	64,190	0	0
8	0	0	20
9	0	0	500
10	0	0	50
11	0	0	20
12	0	0	18,107

and also assess its potential to enter into a war. Second, it seeks to know about the procurement agency which can provide it the best available and latest weapons at a reasonable cost. This chapter initially presents a model for selection of alternatives of the sources supplying weapons by considering ANP which takes into account the interactions between the various elements at the same level and at different levels of hierarchy. The analysis reveals that *W*1 has been ranked first followed by *W*2 and *W*3.

The study then presents an AHP model for evaluating the AP effectiveness of a combat force quantitatively measured by an index named API. Thereafter, this index along with the numbers of weapons of a particular category is used to evaluate the AP potential of a combat unit measured through an index named APV. This index not only helps in determining the AP potential of a combat unit but can also help in planning an expansion of AP potential of the concerned combat force based on the budget allocation. APV also provides weapon wise contribution to the overall AP potential. This paper then develops an optimization model subject to constraints and subsequently demonstrates the model with the help of an illustrative example. The optimal solution is highly biased in favor of indigenization ($W3$) weapon system which can be attributed to high value of p. Note that p is a policy parameter and changing its value can significantly alter procurement decisions.

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8

Throughput Analysis of the Overhaul Line of a Repair Depot

8.1 Introduction

The methodologies provided in Chapter 5 of this book considers the time to overhaul T_{OH} to be provided and then seek to improve availability. This chapter aims to reduce T_{OH} itself. This is achieved by enhancing the present throughput (TH) of the repair and overhaul (RAOH) line by modeling the repair processes of three critical components of the aero engine for identifying the bottlenecks in the repair processes. Remedial measures are then suggested after due analysis to reduce the cycle times (CTs), and work in process (WIP) of critical repair processes resulting in enhancement of the overall throughput of the aero engines in the repair depot which in turn reduces the time to overhaul. The relation of this chapter to the rest of the book is explained through Figure 8.1.

Depots are expected to (a) supply the repaired and overhauled equipment in time and in required numbers, (b) maintain right quality and reliability of the supplied components, and (c) supply the equipment which can be readily deployable for operational role. Presently, effectiveness of a RD is governed by its ability to convert annual tasks into weekly and monthly task, monitoring of progress, and effecting mid-course corrections. During hostility, the RDs are also required to achieve enhanced production rate and sustain it. Thus, RDs are mainly entrusted with the functions of overhaul and repair. Overhaul lines in military aviation (MA) can be broadly classified into following categories: (a) aircraft overhaul line, which deals with the *RAOH* of aircraft and its components; (b) engine overhaul line that deals with the RAOH of aero engines and their aggregates and accessories; (c) system overhaul line dealing with RAOH of electronic systems, e.g., missile equipment, radar, communication, etc. This case study focuses on a typical aero engine overhaul line.

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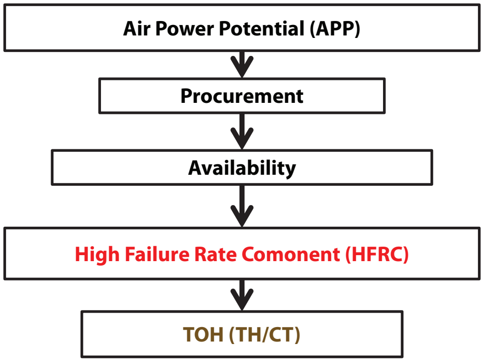

Figure 8.1 Overview of Chapter 8.

Reliability and pre-determined performance during their service life is built into airborne/system equipment, but failure/defects do occur. These may lead to accidents, incidents, or pre-mature withdrawal of components/aero engines causing unserviceability or mission failure. It is hence necessary that these defects are investigated and remedial measures instituted to prevent their recurrence. Presently, the engine repair depot (ERD), which is the focal point of this study, is entrusted with an annual production task of overhaul/repair.

The TH of a plant is a measure of major importance in its assessment. Managers often rely on changes in capacity and process improvements as two major factors that impact TH. The optimal allocation of resources of these two factors is difficult to determine without the support of appropriate mathematical models. The overhaul line is a combination of material handling and processing devices that is common to modern industries. A major task for industrial engineers is to design and operate overhaul lines efficiently and TH is of key interest.

A brief literature survey on TH analysis is carried out for better understanding of the subject by the readers and is placed at Table 8.1.

Three major components, namely, (1) Low Pressure Compressor Rotor Blades (LPCR Blades), (2) Combustion Chamber Outer Casing (CCOC), (3) Low Pressure Turbine Blades (LPTR Blades), are then identified and shortlisted. They are repaired at industrial engineering line after the engines are dismantled at the overhaul line. The repair lines for all these three components are modeled with the help of workstations (WSs) used for concerned repair activities. Concepts of variability and queuing models for carrying out TH analysis are applied to all these three repair lines of the industrial engineering line. The bottlenecks in the repair process of the

Table 8.1 Brief literature review on TH analysis.

Authors	**Brief Work Summary**
Muth [1]	Propose a novel mathematical model that describes a system in terms of the service time, blocking periods, and idle periods of jobs passing through the system. This approach is not restricted by the assumption of exponential service time and gives an approximation of the production rate.
Conway *et al.* [2]	Summarize the impact of buffers on the TH of balanced lines.
Hillier [3]	Extended the results of [2] to find near optimal allocation of finite buffer spaces.
Askin and Iyer [4]	Compare scheduling policies for manufacturing cells.
Lynes and Miltenburg [5]	Describe the relationship between, CT, and work in process (WIP) and cost at work centers in a microelectronics plant. They model plants as open queuing networks and also present effect of variability in both the service time and the arrival process
Britan and Sarkar [6]	Make an attempt to quantify the trade-offs between capacity and process improvements through variance reductions and TH enhancement.
Hayes and Wright [7]; Bitran and Tirupati [8] and Boxma *et al.* [9].	The importance of relationships between performance criteria such as WIP, lead times, TH, manufacturing costs, operation, and capital investments has been emphasized by these authors.
Magazine and Stecks [10]	Consider the problem of improving the output rates from unpaced production lines having a fixed process flow and finite buffers by manipulation of the numbers of workstations, the number of parallel facilities at each workstation, the amount of buffer storage between workstations, and the distribution of workload among the stations.

(*Continued*)

Table 8.1 Brief literature review on TH analysis. (*Continued*)

Authors	Brief Work Summary
Papadopoulos and Heavey [11]	Queuing network modeling of manufacturing systems has been addressed by a large number of researchers. The authors provide a bibliography of material concerned with modeling of production and transfer lines using queuing networks.
Enns [12]	Presents application of queuing decomposition models to study the workflow behavior under various shop routing assumptions. This approach proves valuable in helping to understand variance and covariance effects.
Govil and Fu [13]	Survey the contribution and applications of queuing theory in the field of discrete part manufacturing with concise descriptive summaries rather than detailed mathematical models of various queuing theories results in the manufacturing context.
Angelo *et al.* [14]	Focus on the level of physical system planning and try to define the best shop configuration in terms of process resources layout, considering different variability conditions for demand.
He *et al.* [15]	Address production variability of a production line with M machines and M-1 buffers of finite size. All the processing times by these machines are assumed to be exponential.
Ching *et al.* [16]	Provide methods for TH analysis and bottlenecks identification in assembly systems with non-exponential machines.
Manitz [17]	Studies the production process on multi-stage assembly lines.
Kalir *et al.* [18]	Propose a strategy to reduce inter-departure time variability in production lines.

(*Continued*)

Table 8.1 Brief literature review on TH analysis. (*Continued*)

Authors	Brief Work Summary
Colledani *et al.* [19]	Present a methodology to analyze the production rate variability in unreliable manufacturing systems. The dependency of the variance on the system parameters is investigated.
Barabadi *et al.* [20]	Develop a methodology for TH capacity analysis considering environmental conditions.
Wu *et al.* [21]	Authors inspired by the underlying structure of tandem queues derive an approximate model to characterize the system performance. The model decomposes system queue time and variability into bottleneck and non-bottleneck parts while capturing the dependence upon workstations.
Huang *et al.* [22]	Establish and solve the re-scheduling under a flow-shop mixed-line production planning.

shortlisted three engine components are identified and the remedial measures are discussed in detail. Incorporation of remedial measures would lead to enhancement of the TH of the three selected components, leading to an overall increase in the TH of engines to 26 engines per year.

8.2 Basic Definitions, Parameters, and Relationships

In this section, various basic definitions and relationships pertaining to an industrial engineering line of an overhaul depot are presented [23] for the benefits of the readers and ready reference.

Workstation (WS): A WS is a collection of one or more machines or manual stations that perform identical functions.

Throughput (TH): The average output of a production process (machine, WS, line, plant) per unit time (e.g., parts per hour) is defined as the system's TH or, sometimes, throughput rate.

Capacity: An upper limit on the throughput of a production process is its capacity.

Work in Process (WIP): The inventory between the start and end points of a product routing is called work in process.

Cycle Time (CT): This is the average time from release of a job at the beginning of the routing until it reaches an inventory point at the end of the routing, i.e., the time the part spends as WIP.

Utilization (y): The utilization of a WS is the fraction of time it is not for lack of parts. This includes the fraction of time the WS is working on parts or has parts waiting and is unable to work on them because of a machine failure, setup, or detractor. We can compute utilization as

$$\text{Utilization} = \frac{\text{arrivalrate}}{\text{effective production rate}} \tag{8.1}$$

where the effective production rate is defined as the maximum average rate at which the WS can process parts, considering the effects of failures, setups, and all other detractors that are relevant over the planning period of interest.

Bottleneck rate (r_b): The bottleneck rate of the line is the rate (parts per unit time or jobs per unit time) of the WS having the highest long-term utilization.

Little's law: At every WIP level, WIP is equal to the product of TH and CT.

$$\text{WIP} = \text{TH} \times \text{CT} \tag{8.2}$$

8.3 Variability

Variability exists in all production systems and can have an enormous impact on performance. Physical dimensions, process times, machine failure/repair times, quality measures, temperatures, material hardness, setup times, and so on are examples of characteristics that are prone to non-uniformity.

8.3.1 Measures and Classes of Variability

Variance, commonly denoted by σ^2, is a measure of absolute variability, as is the standard deviation (SD) σ, defined as the square root of the variance. A reasonable relative measure of the variability of a random variable is the SD divided by the mean, called as the coefficient of variation (CV). The coefficient of variation (SCV) can be written as:

$$c^2 = \frac{\sigma^2}{t^2}$$

where t is the mean of the process time.

8.3.2 Causes of Variability

The most prevalent sources of variability in manufacturing environments are:

- Natural variability: This is the variability inherent in natural process time, which excludes random downtimes, setups, or any other external influences. This includes minor fluctuations in process time due to differences in operators, machines, and material. We can express the *CV* of natural process time as

$$c_0 = \frac{\sigma_0}{t_0}$$

where
σ_0: Natural SD of process time
t_0: Mean of natural process time

- Random outages
- Setups
- Operator availability

8.3.3 Variability from Preemptive Outages (Breakdowns)

In this chapter, the main concern is with the variability resulting from the breakdowns of machines; hence, certain terminologies and relations pertaining to variability resulting from breakdowns are defined below.

Availability: It is given by

$$A = \frac{M_F}{M_F + M_R} \tag{8.3}$$

where M_F is mean time to failure (MTTF) and M_R is mean time to repair (MTTR).

Effective Mean Process Time (t_e): The natural process time t_o is adjusted to account for the fraction of time the machine is unavailable results in t_e given by

$$t_e = \frac{t_0}{A}$$

Effective Capacity Rate (r_e): If m is number of machines, then the effective capacity rate is

$$r_e = \frac{m}{t_e}$$

Variance and Squared Coefficient of Variation: Variance $\left(\sigma_e^2\right)$ and squared coefficient of variation $\left(c_e^2\right)$ of the effective process time can be estimated [23] as

$$\sigma_e^2 = \left(\frac{\sigma_0}{A}\right)^2 + \frac{\left(M_R^2 + \sigma_r^2\right)(1-A)t_0}{AM_R} \tag{8.4}$$

$$c_e^2 = \frac{\sigma_e^2}{t_e^2} = c_0^2 + A(1-A)\frac{M_R}{t_0} + c_r^2 A(1-A)\frac{M_R}{t_0} \tag{8.5}$$

8.3.4 Variability in Flows

Figure 8.2. Propagation of Variability between Workstations in Series, the propagation of variability between WSs in series [10], [19] and [23] is characterized.

The following relations are used to estimate various rates and CVs to characterize variability in flows [23]:

Arrival rate: $r_a = \frac{1}{t_a}$ where t_a is the mean time between arrivals.

Arrival CV: $c_a = \frac{\sigma_a}{t_a}$ where σ_a is the SD of time between arrivals.

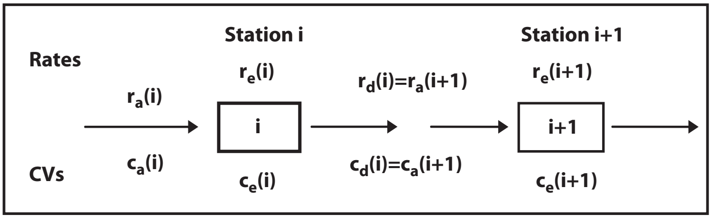

Figure 8.2 Propagation of variability between workstations in series.

Departure rates: $r_d = \frac{1}{t_d}$ where t_d is mean time between departures.

$$t_a(i + 1) = t_d(i)$$

$$c_a(i + 1) = c_d(i)$$

$$c_d^2 = y^2 c_e^2 + (1 - y^2) c_a^2$$

where y= $r_a t_e$ and c_d is the departure CV.

8.3.5 Variability Interactions Queuing

A queuing system [5], [11] and [13] is characterized by a host of specific assumptions, including the type of arrival and process time distributions. A partial classification of single-station, single-job-class queuing systems are given by Kendall's notation [23], which characterizes a queuing station by means of four parameters: D/*B*/*m*/*b*, where *D* describes the distribution of inter arrival times, *B* describes the distribution of process times, *m* is the number of machines at the station, and *b* is the maximum number of jobs that can be in the system.

8.3.5.1 The M/M/1 Queue

This model assumes exponential inter arrival times, a single machine with exponential process times, a first-come first-served protocol, and unlimited space for jobs waiting in queue. The CT in queue CT_q is given by the following relation:

$$CT_q(M/M/1) = \frac{y}{1-y} t_e \tag{8.6}$$

8.3.5.2 The G/G/1 Queue

For this setting, the expression for CT_q can be estimated [23] as

$$CT_q(G/G/1) = \left(\frac{c_a^2 + c_e^2}{2}\right)\left(\frac{y}{1-y}\right) t_e \tag{8.7}$$

Equation (8.7) above separates into three terms: a dimensionless variability term V, a utilization term U, and a time term T. Hence, Equation (8.7) can be written as [23]

$$CT_q = VUT \tag{8.8}$$

Equation (8.8) is referred as Kingman's equation [23] or as the VUT equation.

8.4 Process Batching

In this sub-section, few important relationships of process batching that are used in this case study in problem formulation and estimation of various parameters are presented [23].

Arrival rate: Arrival rate of batch $= \frac{r_a}{k}$ where k = batch size.

Mean time to process a batch: The mean time to process a batch of *k* parts is the sum of the individual process times and is given as

$$t_0(batch) = kt_0$$

Mean effective time to process a batch: The mean effective time to process a batch of *k* parts is the sum of the individual process times and is given as

$$t_e(batch) = kt_e$$

Variance of the time to process the batch: The variance of the time to process the batch is the sum of the individual variances and is given as

$$\sigma_0^2(\text{batch}) = k\sigma_0^2$$

Hence, the CV of the time to process the batch is

$$c_0(\text{batch}) = \frac{\sigma_0(\text{batch})}{t_0(\text{batch})} = \frac{\sqrt{k\sigma_0}}{kt_0} = \frac{c_0}{\sqrt{k}}$$

Similarly, the CV of the effective time to process the batch is $= \frac{c_e}{\sqrt{k}}$.
Utilization rate: The utilization rate of the batch is

$$u = \frac{r_a}{k} \times kt_e = r_a \times t_e$$

8.5 System Flow and Parameters

The work flow/work stations for engine overhaul line are presented in Figure 8.3. In the rest of this case study, the analysis is done for a specific data set and context. The first step in this analysis is to collect data and estimate the means and variances of specific WSs. The data collection involved significant efforts and has been reported in the process requirement tables placed subsequently. Now, the annual required overhaul task of the aero engines is $TH_{req}^{AE} = 26 \; per \; year = 0.013 \; per \; hour$ (1 year = 2,000 working hours).

The annual overhaul task is presently not happening and the actual throughput is 12 engines per year. The aim is to identify the bottlenecks for not achieving the desired TH and suggest remedial measures accordingly. After dismantling, the aero engine components are sent to the repair line for required repairs. After a detailed survey and a thorough study following three components are shortlisted for detailed analysis: (1) LPCR

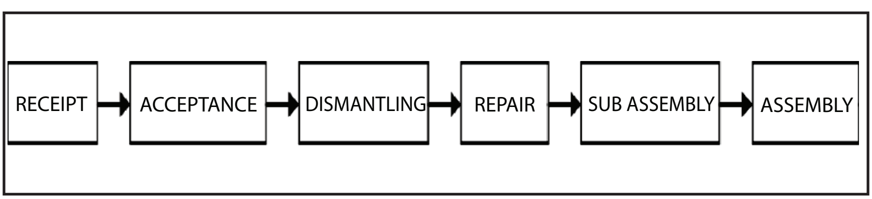

Figure 8.3 Work flow/work stations for engine overhaul line.

Table 8.2 Required TH of the three components.

Sl. Number	Component	Required Annual *TH* (Blades per year)	Required Hourly *TH* (Blades per Hour)
1	LPCR Blades	254 × 26 = 6,604	3.3
2	CCOC	1 × 26 = 26	0.013
3	LPTR Blades	88 × 26 = 2,288	1.14

Blades, (2) CCOC, and (3) LPTR Blades. Table 8.2 provides the details of the required TH of these components to meet the required task of the aero engines as noted earlier.

The work flow/work stations for repair of LPCR blades are presented in Figure 8.4.

In Table 8.3, the process requirements of repair of LPCR blades are provided.

The work flow/work stations for repair of CCOC are now presented in Figure 8.5.

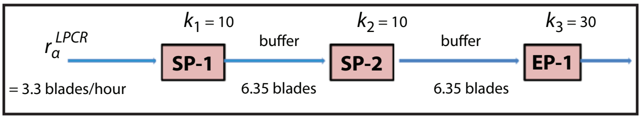

Figure 8.4 Work flow/work stations for LPCR blades.

Table 8.3 Process requirements of LPCR blades repair.

Work Station	Process	Process Time as per *WP*	Actual Process Time	Task
Special process-1 (SP-1)	Vibro grinding and vibro tumbling	$t_o = 8$ *hours*, $\sigma_o = 3$ *hours*	$t_e = 16.6$ *hours* $\sigma_e = 9.4$ *hours*	LPCR (206 blades)
Special process-2 (SP-2)	Root sand blasting	$t_o = 22$ *hours*, $\sigma_o = 7.2$ *hours*	$t_e = 46$ *hours* $\sigma_e = 21.5$ *hours*	LPCR (254 blades)
Electroplating-1 (EP-1)	Root silver coating	$t_o = 48$ *hours*, $\sigma_o = 15$ *hours*	$t_e = 69$ *hours* $\sigma_e = 31.2$ *hours*	LPCR (254 blades)

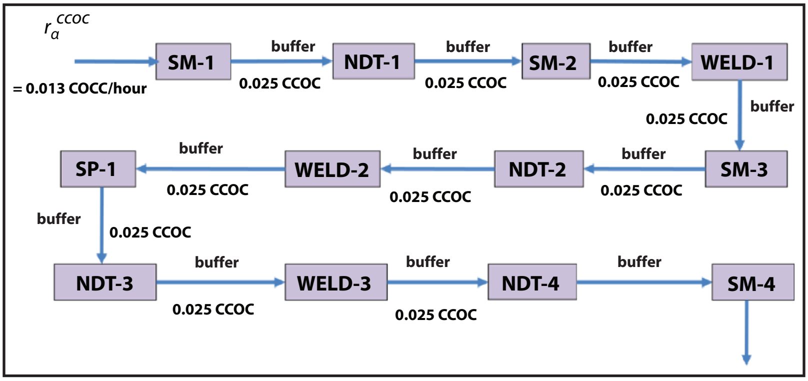

Figure 8.5 Work flow/work stations for CCOC.

In Table 8.4, the process requirements of repair of CCOC are presented.

The work flow/work stations for repair of LPTR blades are presented in Figure 8.6.

In Table 8.5, the process requirements of repair of LPTR blades are presented.

8.6 System Analysis and Discussion

In this section, bottlenecks are identified for all the three selected components, the corresponding results are presented along with a detailed analysis to establish the root cause of the present decline in desired throughput. Bottlenecks are identified as follows: utilization of all WSs is computed using Equation (8.1). The bottleneck WS is the one with the highest utilization. To check whether blocking is an issue CT_q and WIP_q are computed. The results are presented in the following subsections. Possible remedial measures are suggested to clear the bottleneck and their efficacy is checked.

8.6.1 Component 1: LPCR Blades

In Table 8.6, the computational results for LPCR blades are presented, and later, the root cause and remedial measures are discussed.

Capacity calculations presented in Table 8.6 reveal the bottleneck to be WS3, i.e., Root Silver Coating, which requires an average of 69 hours to process the complete job. In addition, WS3 has MTTF of 3.27 hours and a MTTR of 1.43 hours. From past experience, the industrial engineering line knows to be incapable of achieving the target throughput of 3.3 blades per

Table 8.4 Process requirements of CCOC repair.

Work Station	**Process**	**Process Time as per *WP***	**Actual Process Time**	**Task**
Sheet Metal-1 (SM-1)	Cleaning	$t_o = 3$ *hours* $\sigma_o = 1.2$ *hours*	$t_e = 10$ *hours* $\sigma_e = 4.8$ *hours*	1 CCOC
Non-Destructive Testing-1 (NDT-1)	Pre weld X-ray check of Welded joints-(NDT-1)	$t_o = 5$ *hours* $\sigma_o = 1.6$ *hours*	$t_e = 10$ *hours* $\sigma_e = 5.2$ *hours*	1 CCOC
Sheet Metal-2 (SM-2)	Drilling holes if defect found	$t_o = 10$ *hours* $\sigma_o = 2.7$ *hours*	$t_e = 20$ *hours* $\sigma_e = 9.7$ *hours*	1 CCOC
WELD-1	Welding	$t_o = 10$ *hours* $\sigma_o = 2$ *hours*	$t_e = 35$ *hours* $\sigma_e = 19.6$ *hours*	1 CCOC
Sheet Metal-3 (SM-3)	Cleaning and finishing	$t_o = 4$ *hours* $\sigma_o = 1.2$ *hours*	$t_e = 10$ *hours* $\sigma_e = 4.3$ *hours*	1 CCOC
Non-Destructive Testing-2 (NDT-2)	Post weld X-ray checks of welded joints	$t_o = 4$ *hours* $\sigma_o = 1.4$ *hours*	$t_e = 10$ *hours* $\sigma_e = 4.5$ *hours*	1 CCOC
WELD-2	Re welding if defect found	$t_o = 10$ *hours* $\sigma_o = 3.2$ *hours*	$t_e = 20$ *hours* $\sigma_e = 11.3$ *hours*	1 CCOC
Special Process-1 (SP-1)	Heat treatment	$t_o = 12$ *hours* $\sigma_o = 4.3$ *hours*	$t_e = 25$ *hours* $\sigma_e = 12.9$ *hours*	1 CCOC

(*Continued*)

Table 8.4 Process requirements of CCOC repair. (*Continued*)

Work Station	**Process**	**Process Time as per *WP***	**Actual Process Time**	**Task**
Non-Destructive Testing-3 (NDT-3)	Post weld X-ray checks after HT	t_o = 1.5 *hours* σ_o = 0.2 *hours*	t_e = 2 *hours* σ_e = 1.2 *hours*	1 CCOC
WELD-3	Re welding if defect found	t_o = 3 *hours* σ_o = 0.8 *hours*	t_e = 5 *hours* σ_e = 3.2 *hours*	1 CCOC
Non-Destructive Testing-4 (NDT-4)	Re X–ray check	t_o = 5 *hours* σ_o = 1.3 *hours*	t_e = 10 *hours* σ_e = 4.5 *hours*	1 CCOC
Sheet Metal-4 (SM-4)	Inspection of welded joints	t_o = 6 *hours* σ_o = 1.8 *hours*	t_e = 10 *hours* σ_e = 4.1 *hours*	1 CCOC

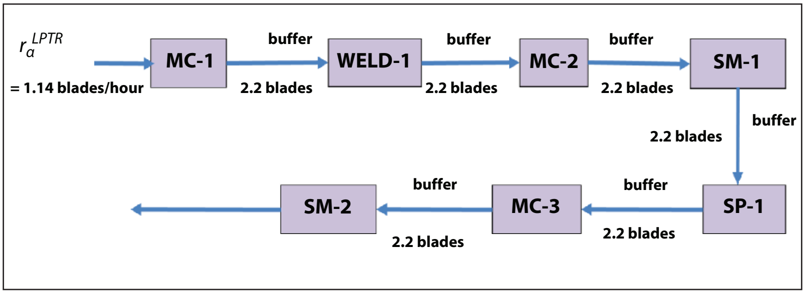

Figure 8.6 Work flow/work stations for LPTR blades repair.

Table 8.5 Process requirements of LPTR blades repair.

Work Station	Process	Process Time as per *WP*	Actual Process Time	Task
Machine Shop-1 (MC-1)	Grinding the shroud platform	t_o = 35 *hours* σ_o = 5 *hours*	t_e = 76 *hours* σ_e = 26.4 *hours*	88 blades
WELD-1	Welding of blades	t_o = 38 *hours* σ_o = 11 *hours*	t_e = 77 *hours* σ_e = 26.4 *hours*	88 blades
Machine Shop-2 (MC-2)	Pre final grinding	t_o = 56 *hours* σ_o = 18 *hours*	t_e = 120 *hours* σ_e = 59.7 *hours*	88 blades
Sheet Metal-1 (SM-1)	Preliminary tool finish along the contour of overlaying and fillets	t_o = 41 *hours* σ_o = 14 *hours*	t_e = 88 *hours* σ_e = 46.2 *hours*	88 blades
Special Process-1 (SP-1)	Heat treatment	t_o = 10 *hours* σ_o = 2.8 *hours*	t_e = 24 *hours* σ_e = 11.9 *hours*	88 blades
Machine Shop-3 (MC-3)	Final grinding	t_o = 36 *hours* σ_o = 12 *hours*	t_e = 80 *hours* σ_e = 41.3 *hours*	88 blades
Sheet Metal-2 (SM-2)	Cleaning and finishing	t_o = 20 *hours* σ_o = 8 *hours*	t_e = 44 *hours* σ_e = 22.6 *hours*	88 blades

Table 8.6 Component 1—LPCR blades.

WS	t_o (batch)	t_e (batch)	A	y	c_o^2	c_e^2	M_R	M_F	c_d^2	c_a^2	V	U	T	CT_q	WIP_q
1	0.39	0.81	0.48	0.27	1.41	3.21	0.14	0.13	1.16	1.00	2.10	0.36	0.08	0.06	0.20
2	0.87	1.81	0.48	0.60	1.07	2.18	0.19	0.18	1.53	1.16	1.67	1.49	0.18	0.45	1.48
3	5.67	8.15	0.70	0.90	2.93	6.13	1.43	3.27	5.23	1.53	3.83	8.66	0.27	9.01	29.74

hour. To remedy this situation, the line is in favor of installing a new electroplating shop. However, along with being expensive, this effort will also result in loss in production. The challenge, therefore, is to find an alternative and better solution.

The two principal tools at disposal are the VUT equation for computing queue time. By computing the effective squared coefficient of variation, the reasons as to why the line is failing to meet its TH target can be analyzed.

The waiting time in queue at WS3 is estimated to be 9.01 hours and WIP in queue WIP_q to be 29.74 against the holding capacity of 6.35 blades per hour, when the arrival rate is 3.3 blades per hour. This reveals why the system cannot make 3.3 blades per hour, even though the utilization of the bottleneck (*WS*3) is only 90%. Since the real system cannot allow WIP in front of WS3 to reach this level of 29.74, WS2 will occasionally become blocked and the resulting lost production at WS2 eventually cause WS3 to become starved (i.e., idled by lack of parts to work on). As a result, neither station can maintain the utilization necessary to produce 3.3 blades per hour. Thus, it is concluded that the problem is rooted in the long queue at WS3.

By Little's law, reducing average queue length is equivalent to reducing average queue time. So, the queue time at WS3 is now considered more closely.

$$CT_q(3) = \left(\frac{c_a^2(3) + c_e^2(3)}{2}\right)\left(\frac{y_3}{1 - y_3}\right) t_e(3)$$

$$CT_q(3) = \left(\frac{1.53 + 6.13}{2}\right)\left(\frac{0.90}{1 - 0.90}\right) \times 0.27$$

$$CT_q(3) = (3.83) \times (8.66) \times (0.27)$$

The third term $t_e(3)$ is the effective process time at WS3, which is simply raw process time divided by availability.

$$t_e(3) = \frac{t_0(3)}{A(3)}$$

$$t_e(3) = \frac{t_0(3)}{M_F(3)/M_F(3) + M_R(3)}$$

$$t_e(3) = \frac{0.19(3.27 + 1.43)}{3.27} = 0.27$$

Since the value of $t_e(3)$ is only slightly larger than the raw process time of 0.19 hours, there is a little room for improvement by increasing availability. The second term in VUT equation is the utilization term. Although at first glance a value of 8.66 may appear large, it corresponds to a utilization of 90%, which is large but not excessive. Although increasing, the capacity of this station would certainly reduce the queue time (and queue size) but as already discussed this is an expensive option.

As an alternate to the first term in VUT equation, the variability inflation factor is considered. Moderate variability in arrivals (i.e., $c_a^2(3) = 1$) and moderate variability in process times (i.e., $c_e^2(3) = 1$) result in a value of one for this term. Therefore, a value of 3.83 is large than usual. To investigate why this occurs it is broken down into its constituent parts, which reveal $c_a^2(3) = 1.53$ and $c_e^2(3) = 6.13$. Obviously, variability due to process time is dominant source of variability. From Equation (8.5), the following equations are arrived at

$$c_e^2(3) = c_0^2(3) + \frac{A_3(1 - A_3)M_R(3)\left[1 + c_r^2(3)\right]}{t_0(3)}$$

$$c_e^2(3) = 2.93 + \frac{2 \times 1.43 \times 0.70 \times 0.30}{0.19}$$

$$c_e^2(3) = 2.93 + 3.16$$

It can be seen that $c_e^2(3)$ is made up of two components, $c_o^2(3)$, which is due to natural variability and the other one is due to random outages. Natural variability is more on manual process than an automated one because natural variability is due to unidentified sources of variability attributed to operators. Hence, there is a requirement for the process of root silver coating to be made more automatic. The major share of $c_e^2(3)$ is a result of random outages. This suggests that an alternative to increasing capacity at *WS*3 is to improve the breakdown situation at WS3.

Various practical options might be available for mitigating the outage problem at WS3. For instance, the line could attempt to reduce MTTR by holding "field ready spares" for parts subject to failures. If such a policy could halve the MTTR, the resulting increase in effective capacity and reduction in process time variability would cause queue time to fall to 1.58 hours which is a significant desirable reduction. The WIP_q at WS3 also goes below from 29.74 to 5.20 which is well within the capacity. With this improvement in place, it turns out be feasible to run at the desired rate of 3.3 blades per hour. Since improving the repair profile of WS3 is likely to be less expensive and disruptive than adding an electroplating shop, this alternative deserves serious consideration.

8.6.2 Component 2: CCOC

In Table 8.7, the results for CCOC are now presented. Root cause for present TH not happening and the remedial measures are then discussed.

From Table 8.7, it is observed that the WS4, i.e., the WS handling the welding process is the bottleneck. Again, considering VUT equation, the following equation is arrived at

$$CT_q(4) = 0.62 \times 0.83 \times 35$$

In this case, the values of first two terms are well within limits; hence, the utilization and variability are well within control. It is needed to focus on the effective process time at WS4 which seems to be at the higher side.

$$t_e(4) = \frac{t_0(4)}{A(4)} = \frac{10}{0.29} = 35 \text{ hours}$$

The effective process time is much larger than the raw process time of 10 hours; hence, there is an ample room for improvement by increasing the availability. In addition, the $WIP_q(4) = 0.24$ which is beyond the holding capacity of 0.025 CCOC per hour. Thus, the system cannot make 0.013 CCOC per hour even though the utilization of the bottleneck is only 46%.

Availability at WS4 is estimated as

$$A(4) = \frac{M_F(4)}{M_F(4) + M_R(4)} = \frac{2.68}{2.68 + 6.70} = 0.29$$

If it is tried to reduce the MTTR to 1.6 hours by holding spares which can be offered readily, by reducing administrative downtimes and also by utilizing skilled technicians for rectification then the improved availability

Table 8.7 Component 2—CCOC.

WS	t_o	t_e	r_e	A	y	c_o^2	c_e^2	M_R	M_F	c_d^2	c_a^2	V	U	T	CT_q	WIP_q
1	3	10	0.10	0.30	0.13	0.16	0.23	0.50	0.22	0.99	1	0.62	0.15	10	0.92	0.01
2	5	10	0.10	0.50	0.13	0.10	0.27	1.68	1.68	0.98	0.99	0.63	0.15	10	0.94	0.01
3	10	20	0.05	0.50	0.26	0.07	0.24	3.25	3.25	0.93	0.98	0.61	0.35	20	4.27	0.06
4	10	35	0.03	0.57	0.46	0.04	0.31	11.17	14.90	0.80	0.93	0.62	0.83	35	18.17	0.24
5	4	10	0.1	0.4	0.13	0.08	0.18	0.89	0.59	0.79	0.80	0.49	0.15	10	0.74	0.01
6	4	10	0.1	0.4	0.13	0.12	0.2	0.67	0.44	0.78	0.79	0.50	0.15	10	0.74	0.01
7	10	20	0.05	0.5	0.26	0.10	0.32	4.34	4.33	0.75	0.78	0.55	0.35	20	3.86	0.05
8	12	25	0.04	0.48	0.33	0.13	0.27	3.31	3.06	0.70	0.75	0.51	0.48	25	6.12	0.08
9	1.5	2	0.5	0.75	0.03	0.02	0.36	1.37	4.1	0.70	0.70	0.53	0.03	2	0.03	0.0004
10	3	5	0.2	0.6	0.07	0.07	0.41	2.12	3.17	0.70	0.70	0.55	0.07	5	0.19	0.003
11	5	10	0.1	0.5	0.13	0.07	0.20	1.35	1.35	0.69	0.70	0.45	0.15	10	0.67	0.009
12	6	10	0.1	06	0.13	0.09	0.17	0.98	1.46	0.68	0.69	0.43	0.15	10	0.64	0.008

works out to be 67%. The effective process time $t_e(4)$ reduces to 15 hours and WIP_q to 0.02. Thus, with this improvement in place it turns out to be feasible to run at the desired rate of 0.013 CCOC per hour. Hence, improving the repair profile of WS4 is a less expensive option rather than enhancing the capacity to achieve the desired throughput.

8.6.3 Component 3: LPTR Blades

In Table 8.8, the results for LPTR blades are presented. Causes for low TH and the corrective measures are then discussed.

From Table 8.8, it is observed that the WS responsible for grinding the shroud platform of the LPTR blades, i.e., WS1 is the bottleneck with utilization of 98%.

Considering VUT equation, the following equation is arrived at

$$\mathrm{CT_q}(1) = 0.56 \times 63.71 \times 0.86$$

It can be seen that the value of U is extremely high due to extensive utilization of grinding machine at WS1. The variability is well within the limits. However, the effective process time is 0.86 hours almost double than that of raw process time. Presently, WIP_q in front of WS1 is 35.14 against the holding capacity of 2.2 blades per hour; hence, the TH of 1.14 blades per hour is not happening.

Since the flow variability value is well within limits, it is needed to focus on the utilization factor U. There may not be any choice but to add capacity in terms of adding one more grinding machine at WS1. Reducing y from 0.98 to 0.78, i.e., by 26%, leads to reduction of effective process time from 0.86 to 0.68 hours. By including an additional capacity, the WIP_q in front of WS1 falls to 1.62 from 35.14 which are well below the holding capacity of 2.2 blades per hour. Hence, with this improvement in place, it turns out to be feasible to run at the desired rate of 1.14 blades per hour.

Thus, to summarize, the following remedial measures [24] can be recommended to get the desired TH.

(1) In case of LPTR blades, the repair profile of WS3 can be improved by reducing MTTR by holding "field ready spares" for parts subjected to failures. If MTTR is halved, the queue time falls from 9.01 to 1.58 hours and WIP_q goes below from 29.74 to 5.20. With this improvement in place it would be feasible to run at the desired rate of 3.3 blades per hour.

Table 8.8 Component 3—LPTR blades.

WS	t_o	t_e	r_e	A	y	c_o^2	c_e^2	M_R	M_F	c_d^2	c_a^2	V	U	T	CT_q	WIP_q
1	0.40	0.86	1.16	0.46	0.98	0.02	0.12	0.08	0.07	0.15	1.00	0.56	63.71	0.86	30.83	35.14
2	0.43	0.84	1.19	0.51	0.96	0.08	0.13	0.04	0.04	0.13	0.15	0.14	23.18	0.84	2.74	3.12
3	0.64	0.85	1.17	0.75	0.97	0.10	0.21	0.18	0.52	0.20	0.13	0.17	34.20	0.85	4.93	5.61
4	0.46	0.82	1.22	0.57	0.93	0.12	0.19	0.07	0.09	0.19	0.20	0.19	13.86	0.82	2.20	2.51
5	0.11	24	0.04	0.42	0.31	6.90	21.63	3.44	2.46	2.26	0.19	10.91	0.45	0.27	1.34	1.53
6	0.41	0.81	1.24	0.51	0.92	0.11	0.21	0.08	0.08	0.52	2.26	1.23	11.46	0.81	11.40	13.00
7	0.23	0.50	2.00	0.45	0.57	0.16	0.26	0.05	0.04	0.44	0.52	0.39	1.33	0.50	0.26	0.30

(2) In case of CCOC, again the repair profile of WS4 needs to be improved. If the MTTR is reduced to 1.6 hours by holding spares which can be offered readily, by reducing administrative downtimes and also by utilizing skilled technicians for rectification, then the improved availability could become 67%. The effective process time $t_e(4)$ reduces to 15 hours and WIP_q to 0.02. With this improvement, it will be feasible to run at the desired rate of 0.013 CCOC per hour.
(3) For LPTR blades WS1 is being over utilized. Thus, adding capacity by adding one more grinding machine at WS1 is the most feasible solution. Reducing y from 0.98 to 0.78, i.e., by 26% leads to reduction of effective process time from 0.86 to 0.68 hours. By including an additional capacity, the WIP_q in front of WS1 falls to 1.62 from 35.14 which is well below the holding capacity of 2.2 blades. Hence, with this improvement in place, the desired rate of 1.14 blades per hour can be achieved.

Thus, it is seen that the recommendations range from adding new capacity, when absolutely needed, to doing simple tasks more efficiently to enhance the repair work. It is noteworthy that such analysis can thus help in meeting the target TH without simplistic recommendations of adding capacity everywhere.

8.7 Summary

In this chapter, the aim was to improve availability of aircraft by reducing time to overhaul of aero engines. This was done by enhancing the TH of the overhaul line by identifying the bottlenecks in the repair line of three most critical components of the aero engine and then suggesting the concerned remedial measures. The three components shortlisted for detailed analysis were LPCR blades, CCOC, and LPTR blades. The repair lines for these components are then modeled with the help of WSs used for concerned repair activities. The concepts of variability and queuing models are used for carrying out TH analysis of all the three repair lines.

In case of LPCR blades, WS3 emerged as the bottleneck. The key insight that emerged from a detailed analysis is that the WS3 is a bottleneck not due to any inherent machine attributes but due to high MTTR. Practical measures to reduce MTTR are suggested and are shown that the desired

TH can be obtained by reducing the MTTR. Hence, a less expensive measure rather than enhancing the capacity is identified.

In case of CCOC, WS4 was the bottleneck. The cause of bottleneck is similar to that of WS3 for LPCR blades and therefore a similar remedial approach is recommended.

In case of LPTR blades, the flow variability value is well within limits, so the focus was on the utilization factor U. High WIP_q causes excessive buffer leading to blocking. The only feasible solution in this case is to add capacity. It is shown that this capacity addition results in meeting TH target. Hence, with this improvement in place, it turns out to be feasible to run at the desired rate of 1.14 blades per hour.

The chapter provides a complete framework for TH analysis of the overhaul line of a RAOH depot. Incorporation of remedial measures lead to enhancement of the TH of the three selected components leading to an overall increase in the TH of engines to 26 engines per year.

Thus, it is seen that the recommendations range from adding new capacity, when absolutely needed, to doing simple tasks more efficiently to enhance the repair work. It is noteworthy that such analysis can thus help in meeting the target TH without simplistic recommendations of adding capacity everywhere. Such recommendations are implementable at the shop floor and can help floor managers in easy implementation.

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Appendix A

The Saaty Rating Scale

Intensity of importance	Definition	Explanation
1	Equal importance	Two factors contribute equally to the objective
3	Somewhat more important	Experience and judgement slightly favour one over the other.
5	Much more important	Experience and judgement strongly favour one over the other.
7	Very much more important	Experience and judgement very strongly favour one over the other. Its importance is demonstrated in practice.
9	Absolutely more important.	The evidence favouring one over the other is of the highest possible validity.
2,4,6,8	Intermediate values	When compromise is needed

The Consistency Index for a matrix = $(\lambda_{max}-n)/(n-1)$

Pairwise Comparisons and Estimation of Weights for ANP

Table A1 Pairwise comparison in cluster APWC WRT MF.

MF	**RK**	**AAM**	**SAM**	**FB**	**ASM**	**SSM**	**AB**	**nth root**	**WTS**
RK	1	3	5	5	3	5	2	3.01	0.33
AAM	0.33	1	5	5	2	5	2	2.07	0.23
SAM	0.2	0.2	1	3	0.33	2	0.33	0.59	0.07
FB	0.2	0.2	0.33	1	0.33	0.33	0.33	0.34	0.04
ASM	0.33	0.5	3	3	1	3	0.5	1.12	0.12
SSM	0.2	0.2	0.5	3	0.33	1	0.33	0.49	0.05
AB	0.5	0.5	3	3	2	3	1	1.45	0.16
								9.07	
CONSISTENCY INDEX (CI) COMPUTATION									
RK	2.49		**λ**s	7.50					
AAM	1.69			7.37					
SAM	0.49			7.47					
FB	0.28			7.63					
ASM	0.90			7.25		CI = 0.07			
SSM	0.40			7.50					
AB	1.16			7.23		CONSISTENCY RATIO			
		λ MEAN=		7.42		CR = 0.05			

Table A2 Pairwise comparison in cluster APWC WRT FI.

FI	MF	GN	RPV	AB	nth root	WTS
MF	1	3	5	5	2.94	0.55
GN	0.33	1	3	3	1.31	0.25
RPV	0.2	0.33	1	3	0.67	0.13
AB	0.2	0.33	0.33	1	0.38	0.07
					5.31	
CONSISTENCY INDEX (CI) COMPUTATION						
MF	2.29		λs	4.12	CI = 0.06	
GN	1.02			4.14		
RPV	0.54			4.26	CONSISTENCY RATIO	
AB	0.31			4.23		
			λ MEAN=	4.19	CR = 0.07	

Table A3 Pairwise comparison in cluster APWC WRT GN.

GN	MF	FI	AH		WTS	
MF	1	1	3	1.44	0.43	
FI	1	1	3	1.44	0.43	
AH	0.33	0.33	1	0.48	0.14	
				3.36		
CONSISTENCY INDEX (CI) COMPUTATION						
MF	1.28		λs	2.99	CI = 0.00	
FI	1.28			2.99		
AH	0.43			2.99	CONSISTENCY RATIO	
			λ MEAN=	2.99	CR = -0.01	

Table A4 Pairwise comparison in cluster APWC WRT AB.

AB	**MF**	**FB**	**RPV**		**WTS**	
MF	1	0.33	3	1.00	0.26	
FB	3	1	5	2.47	0.64	
RPV	0.33	0.2	1	0.40	0.10	
				3.87		
CONSISTENCY INDEX (CI) COMPUTATION						
MF	0.78		**λs**	3.03	CI = 0.01	
FI	1.93			3.03	CONSISTENCY RATIO	
AH	0.32			3.03	CR = 0.03	
			λ MEAN=	3.03		

Table A5 Pairwise comparison in cluster APWPC WRT MF.

MF	**FL**	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**		**WTS**
FL	1	3	3	2	5	5	3	2.80	0.32
SA	0.33	1	3	2	3	5	3	1.90	0.22
UB	0.33	0.33	1	2	3	3	2	1.22	0.14
RN	0.5	0.5	0.5	1	2	3	2	1.06	0.12
SH	0.2	0.33	0.33	0.5	1	3	2	0.68	0.08
CS	0.2	0.2	0.33	0.33	0.33	1	0.33	0.34	0.04
REL	0.33	0.33	0.5	0.5	0.5	3	1	0.63	0.07
								8.62	
CONSISTENCY INDEX (CI) COMPUTATION									
FL	2.46		**λs**	7.58					
SA	1.65			7.47		CI = 0.07			
UB	1.07			7.56					

(*Continued*)

Table A5 Pairwise comparison in cluster APWPC WRT MF. (*Continued*)

MF	**FL**	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**		**WTS**
RN	0.89			7.21		CONSISTENCY RATIO			
SH	0.59			7.48					
CS	0.29			7.34			CR = 0.05		
REL	0.54			7.37					
			λ MEAN=	7.43					

Table A6 Pairwise comparison in cluster APWPC WRT FI.

FI	**FL**	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**		**WTS**
FL	1	0.2	0.2	0.5	2	0.33	0.2	0.43	0.05
SA	5	1	3	3	5	3	3	2.97	0.33
UB	5	0.33	1	2	3	3	3	1.90	0.21
RN	2	0.33	0.5	1	3	3	2	1.29	0.14
SH	0.5	0.2	0.33	0.33	1	0.2	0.2	0.33	0.04
CS	3	0.33	0.33	0.33	5	1	0.33	0.78	0.09
REL	5	0.33	0.33	0.5	5	3	1	1.22	0.14
								8.92	
CONSISTENCY INDEX (CI) COMPUTATION									
FL	0.36		**λ**s	7.50					
SA	2.50			7.53	CI = 0.12				
UB	1.64			7.69					
RN	1.11			7.64	CONSISTENCY RATIO				
SH	0.29			7.83					
CS	0.69			7.88		CR = 0.09			
REL	1.08			7.86					
			λ MEAN=	7.70					

Table A7 Pairwise comparison in cluster APWPC WRT AH.

AH	**FL**	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**		**WTS**
FL	1	3	3	3	5	5	3	2.97	0.34
SA	0.33	1	2	3	3	3	3	1.77	0.21
UB	0.33	0.5	1	2	3	3	2	1.29	0.15
RN	0.33	0.33	0.5	1	2	3	2	0.94	0.11
SH	0.2	0.33	0.33	0.5	1	2	2	0.64	0.07
CS	0.2	0.33	0.33	0.33	0.5	1	0.5	0.41	0.05
REL	0.33	0.33	0.5	0.5	0.5	2	1	0.60	0.07
								8.61	
CONSISTENCY INDEX (CI) COMPUTATION									
FL	2.55		λs	7.40					
SA	1.52			7.41	CI = 0.06				
UB	1.09			7.26					
RN	0.79			7.27	CONSISTENCY RATIO				
SH	0.55			7.38					
CS	0.34			7.24		CR = 0.04			
REL	0.51			7.37					
			λ MEAN=	7.33					

Table A8 Pairwise comparison in cluster APWPC WRT RK.

RK	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**		**WTS**
SA	1	3	0.5	0.5	2	2	1.20	0.17
UB	0.33	1	0.33	0.33	0.2	0.2	0.34	0.05
RN	2	3	1	0.5	3	2	1.62	0.23
SH	2	3	2	1	3	2	2.04	0.29
CS	0.5	5	0.33	0.33	1	0.33	0.67	0.10

(*Continued*)

Table A8 Pairwise comparison in cluster APWPC WRT RK. (*Continued*)

RK	SA	UB	RN	SH	CS	REL		WTS
REL	0.5	5	0.5	0.5	3	1	1.11	0.16
							6.98	
CONSISTENCY INDEX (CI) COMPUTATION								
SA	1.09		λs	6.33				
UB	0.33			6.83	CI = 0.10			
RN	1.47			6.35				
SH	1.85			6.33	CONSISTENCY RATIO			
CS	0.65			6.76				
REL	1.04			6.51		CR = 0.08		
			λ MEAN=	6.52				

Table A9 Pairwise comparison in cluster APWPC WRT AAM.

AAM	SA	UB	RN	SH	CS	REL		WTS
SA	1	3	0.5	0.5	2	2	1.20	0.17
UB	0.33	1	0.33	0.33	0.2	0.2	0.34	0.05
RN	2	3	1	0.5	3	2	1.62	0.23
SH	2	3	2	1	3	2	2.04	0.29
CS	0.5	5	0.33	0.33	1	0.33	0.67	0.10
REL	0.5	5	0.5	0.5	3	1	1.11	0.16
							6.98	
CONSISTENCY INDEX (CI) COMPUTATION								
SA	1.09		λs	6.33				
UB	0.33			6.83	CI = 0.10			
RN	1.47			6.35				

(*Continued*)

Table A9 Pairwise comparison in cluster APWPC WRT AAM. (*Continued*)

AAM	SA	UB	RN	SH	CS	REL		WTS
SH	1.85			6.33	CONSISTENCY RATIO			
CS	0.65			6.76				
REL	1.04			6.51		CR = 0.08		
			λ MEAN=	6.52				

Table A10 Pairwise comparison in cluster APWPC WRT SAM.

SAM	SA	RN	SH	CS	REL		WTS
SA	1	0.5	0.33	3	2	1.00	0.17
RN	2	1	2	5	2	2.09	0.35
SH	3	0.5	1	5	2	1.72	0.29
CS	0.33	0.2	0.2	1	0.5	0.37	0.06
REL	0.5	0.5	0.5	2	1	0.76	0.13
						5.93	
CONSISTENCY INDEX (CI) COMPUTATION							
SA	0.88		λs	5.24			
RN	1.83			5.20	CI = 0.04		
SH	1.53			5.30			
CS	0.31			5.01	CONSISTENCY RATIO		
REL	0.66			5.14			
			λ MEAN=	5.18		CR = 0.04	

Table A11 Pairwise comparison in cluster APWPC WRT FB.

FB	SA	UB	RN	SH	CS	REL		WTS
SA	1	2	0.5	0.33	3	2	1.12	0.15
UB	0.5	1	0.5	0.33	3	2	0.89	0.12
RN	2	2	1	0.5	5	3	1.76	0.24
SH	3	3	2	1	5	3	2.54	0.35
CS	0.33	0.33	0.2	0.2	1	0.5	0.36	0.05
REL	0.5	0.5	0.33	0.33	2	1	0.62	0.08
							7.29	
CONSISTENCY INDEX (CI) COMPUTATION								
SA	0.95		**λ**s	6.19				
UB	0.75			6.16	CI = 0.03			
RN	1.47			6.07				
SH	2.16			6.19	CONSISTENCY RATIO			
CS	0.30			6.09				
REL	0.52			6.11		CR = 0.02		
			λ MEAN=	6.14				

Table A12 Pairwise comparison in cluster APWPC WRT RPV.

RPV	FL	SA	UB	RN	SH	CS	REL		WTS
FL	1	2	2	3	3	3	2	2.16	0.27
SA	0.5	1	2	2	3	3	2	1.67	0.21
UB	0.5	0.5	1	2	3	3	2	1.37	0.17
RN	0.33	0.5	0.5	1	2	3	2	1.00	0.12
SH	0.33	0.33	0.33	0.5	1	3	2	0.73	0.09
CS	0.33	0.33	0.33	0.33	0.33	1	0.5	0.41	0.05

(*Continued*)

Table A12 Pairwise comparison in cluster APWPC WRT RPV. (*Continued*)

RPV	FL	SA	UB	RN	SH	CS	REL		WTS
REL	0.5	0.5	0.5	0.5	0.5	2	1	0.67	0.08
								8.00	
CONSISTENCY INDEX (CI) COMPUTATION									
FL	2.00		λs	7.42					
SA	1.53			7.34	CI = 0.06				
UB	1.25			7.33					
RN	0.91			7.27	CONSISTENCY RATIO				
SH	0.69			7.58					
CS	0.38			7.38		CR = 0.05			
REL	0.62			7.36					
			λ MEAN=	7.38					

Table A13 Pairwise comparison in cluster APWPC WRT AB.

AB	RN	SH	CS	REL		WTS	
RN	1	0.33	0.33	0.33	0.44	0.09	
SH	3	1	3	3	2.28	0.48	
CS	3	0.33	1	0.5	0.84	0.18	
REL	3	0.33	2	1	1.19	0.25	
					4.74		
CONSISTENCY INDEX (CI) COMPUTATION							
RN	0.39		λs	4.26	CI = 0.07		
SH	2.04			4.24			
CS	0.74			4.16	CONSISTENCY RATIO		
REL	1.04			4.15			
			λ MEAN=	4.20		CR = 0.07	

Table A14 Pairwise comparison in cluster APWPCH WRT MF.

MF	VER	SPD	RAP	MOB	FPD	SDE	FPC	AC	MC	MR	MNL	AVL		WTS
VER	1	3	3	3	3	5	3	3	5	2	3	3	2.88	0.19
SPD	0.33	1	2	3	3	5	3	3	4	2	3	3	2.28	0.15
RAP	0.33	0.5	1	3	3	4	2	3	4	2	3	3	1.92	0.13
MOB	0.33	0.33	0.3	1	3	3	3	3	4	2	3	3	1.56	0.10
FPD	0.33	0.33	0.3	0.33	1.00	3	3	4	5	2	3	3	1.36	0.09
SDE	0.2	0.2	0.3	0.33	0.33	1	0.5	3	4	0.5	2	2	0.70	0.05
FPC	0.33	0.33	0.5	0.33	0.33	2	1	3	3	0.5	2	2	0.88	0.06
AC	0.33	0.33	0.3	0.33	0.25	0.33	0.3	1	3	0.2	0.33	0.33	0.41	0.03
MC	0.2	0.25	0.3	0.25	0.2	0.25	0.3	0.33	1	0.2	0.33	0.33	0.29	0.02
MR	0.5	0.5	0.5	0.5	0.5	2	2	5	5	1	5	4	1.41	0.09
MNL	0.33	0.33	0.3	0.33	0.33	0.5	0.5	3	3.00	0.20	1	0.33	0.54	0.04
AVL	0.33	0.33	0.3	0.33	0.33	0.5	0.5	3	3.00	0.25	3	1	0.66	0.04
													14.88	

(Continued)

Table A14 Pairwise comparison in cluster APWPCH WRT MF. (*Continued*)

MF	VER	SPD	RAP	MOB	FPD	SDE	FPC	AC	MC	MR	MNL	AVL		WTS
CONSISTENCY INDEX (CI) COMPUTATION														
	VER	2.65		λs	13.68									
	SPD	2.07			13.51									
	RAP	1.75			13.57									
	MOB	1.44			13.76									
	FPD	1.24			13.60		CI = 0.13							
	SDE	0.61			13.06									
	FPC	0.75			12.62		CONSISTENCY RATIO							
	AC	0.38			13.90									
	MC	0.26			13.24		CR = 0.08							
	MR	1.24			13.02									
	MNL	0.49			13.43									
	AVL	0.59			13.38									
				λ MEAN=	13.40									

Table A15 Pairwise comparison in cluster APWPCH WRT FI.

FI	SPD	RAP	MOB	FPD	SDE	FPC	AC	MC	MR	MNL	AVL		WTS
SPD	1	3	3	3	5	4	3	5	2	5	3	3.09	0.22
RAP	0.33	1	3	3	4	2	3	4	2	3	3	2.17	0.16
MOB	0.33	0.33	1	3	3	3	3	4	2	3	3	1.80	0.13
FPD	0.33	0.33	0.33	1	3	3	4	5	2	3	3	1.54	0.11
SDE	0.2	0.25	0.33	0.33	1.00	0.5	3	4	0.5	2	2	0.78	0.06
FPC	0.25	0.5	0.33	0.33	2	1	3	3	0.5	2	2	0.94	0.07
AC	0.33	0.33	0.33	0.25	0.33	0.33	1	3	0.2	0.33	0.33	0.42	0.03
MC	0.2	0.25	0.25	0.2	0.25	0.33	0.3	1	0.2	0.33	0.33	0.30	0.02
MR	0.5	0.5	0.5	0.5	2	2	5	5	1	5	4	1.55	0.11
MNL	0.2	0.33	0.33	0.33	0.5	0.5	3	3	0.2	1	0.33	0.54	0.04
AVL	0.33	0.33	0.33	0.33	0.5	0.5	3	3	0.25	3.00	1	0.70	0.05
												13.82	

(Continued)

Table A15 Pairwise comparison in cluster APWPCH WRT FI. (*Continued*)

FI	SPD	RAP	MOB	FPD	SDE	FPC	AC	MC	MR	MNL	AVL		WTS
CONSISTENCY INDEX (CI) COMPUTATION													
	SPD	2.74		λs	12.27								
	RAP	1.99			12.62								
	MOB	1.63			12.54								
	FPD	1.37			12.31								
	SDE	0.67			11.78		CI = 0.12						
	FPC	0.78			11.57								
	AC	0.38			12.76		CONSISTENCY RATIO						
	MC	0.26			12.11								
	MR	1.33			11.82		CR = 0.08						
	MNL	0.47			12.12								
	AVL	0.62			12.17								
				λ MEAN=	12.19								

Table A16 Pairwise comparison in cluster APWPCH WRT RK.

RK	**SPD**	**MOB**	**FPD**	**SDE**	**FPC**	**AC**	**MR**		**WTS**
SPD	1	3	3	5	3	5	3	2.97	0.34
MOB	0.33	1	3	3	3	3	2	1.77	0.20
FPD	0.33	0.33	1	3	2	3	2	1.22	0.14
SDE	0.2	0.33	0.33	1	0.2	0.33	0.2	0.31	0.04
FPC	0.33	0.33	0.5	5	1.00	3	0.3	0.83	0.09
AC	0.2	0.33	0.33	3	0.33	1	0.2	0.46	0.05
MR	0.33	0.5	0.5	5	3	5	1	1.30	0.15
								8.85	
CONSISTENCY INDEX (CI) COMPUTATION									
SPD	2.50		λs	7.46					
MOB	1.56			7.81	CI = 0.12				
FPD	1.06			7.69					
SDE	0.28			7.91	CONSISTENCY RATIO				
FPC	0.72			7.68					
AC	0.40			7.63	CR = 0.09				
MR	1.14			7.79					
			λ MEAN=	7.71					

Table A17 Pairwise comparison in cluster APWPCH WRT SAM.

SAM	SPD	MOB	FPD	SDE	FPC	AC	MC	MR	MNL	AVL		WTS
SPD	1	3	3	5	5	3	5	3	5	3	3.30	0.26
MOB	0.33	1	3	3	3	3	3	2	3	3	2.07	0.16
FPD	0.33	0.33	1	3	3	3	5	2	3	3	1.75	0.14
SDE	0.2	0.33	0.33	1	0.33	0.33	2	0.2	3	0.33	0.50	0.04
FPC	0.2	0.33	0.33	3	1.00	0.33	2	0.2	0.5	0.33	0.52	0.04
AC	0.33	0.33	0.33	3	3	1	5	0.2	3	3	1.11	0.09
MC	0.2	0.33	0.2	0.5	0.5	0.2	1	0.2	0.3	0.33	0.32	0.03
MR	0.33	0.5	0.5	5	5	5	5	1	5	3	1.94	0.15
MNL	0.2	0.33	0.33	0.33	2	0.33	3	0.2	1	0.33	0.50	0.04
AVL	0.33	0.33	0.33	3	3	0.33	3	0.33	3	1	0.89	0.07
											12.90	
CONSISTENCY INDEX (CI) COMPUTATION												
	SPD	2.77		λs	10.86							
	MOB	1.85			11.51	CI = 0.14						

(*Continued*)

Table A17 Pairwise comparison in cluster APWPCH WRT SAM. (*Continued*)

SAM	SPD	MOB	FPD	SDE	FPC	AC	MC	MR	MNL	AVL		WTS
	FPD	1.52			11.21							
	SDE	0.45			11.61	CONSISTENCY RATIO						
	FPC	0.46			11.33							
	AC	0.98			11.41	CR = 0.09						
	MC	0.28			11.02							
	MR	1.73			11.51							
	MNL	0.44			11.33							
	AVL	0.76			10.96							
				λ MEAN=	11.27							

Table A18 Pairwise comparison in cluster APWPCH WRT GN.

GN	SPD	FPD	FPC	MR	MNL	AVL		WTS
SPD	1	3	3	3	5	3	2.72	0.35
FPD	0.33	1	3	3	5	3	1.88	0.25
FPC	0.33	0.33	1	3	5	3	1.30	0.17
MR	0.33	0.33	0.33	1	5	3	0.90	0.12
MNL	0.2	0.2	0.2	0.2	1.00	0.33	0.28	0.04
AVL	0.33	0.33	0.33	0.33	3	1	0.57	0.07
							7.67	
CONSISTENCY INDEX (CI) COMPUTATION								
	SPD	2.36		λs	6.66	CI = 0.12		
	FPD	1.64			6.66			
	FPC	1.13			6.65	CONSISTENCY RATIO		
	MR	0.78			6.64			
	MNL	0.24			6.46	CR = 0.09		
	AVL	0.48			6.40			
				λ MEAN=	6.58			

Table A19 Pairwise comparison in cluster APWPCH WRT FB.

FB	SPD	RAP	MOB	FPD	FPC	AC	MC	MR	MNL	AVL		WTS
SPD	1	3	3	3	3	4	5	3	5	3	3.06	0.25
RAP	0.33	1	3	2	3	3	4	2	3	2	1.96	0.16
MOB	0.33	0.33	1	2	3	3	4	2	3	2	1.58	0.13
FPD	0.33	0.5	0.5	1	3	4	4	3	3	2	1.53	0.12
FPC	0.33	0.33	0.33	0.33	1.00	3	4	2	3	2	1.05	0.09
AC	0.25	0.33	0.33	0.25	0.33	1	3	2	3	2	0.78	0.06
MC	0.2	0.2	0.2	0.25	0.25	0.33	1	0.33	0.2	0.2	0.27	0.02
MR	0.33	0.5	0.5	0.33	0.5	0.5	3	1	3	3	0.84	0.07
MNL	0.2	0.33	0.33	0.33	0.33	0.33	5	0.33	1	0.33	0.46	0.04
AVL	0.33	0.5	0.5	0.5	0.5	0.5	5	0.33	3	1	0.74	0.06
											12.29	
CONSISTENCY INDEX (CI) COMPUTATION												
		SPD	2.68		λs	10.76						
		RAP	1.78			11.15						

(*Continued*)

Table A19 Pairwise comparison in cluster APWPCH WRT FB. (*Continued*)

FB	SPD	RAP	MOB	FPD	FPC	AC	MC	MR	MNL	AVL		WTS
		MOB	1.42			11.07						
		FPD	1.39			11.14						
		FPC	0.95			11.10		CI = 0.13				
		AC	0.72			11.33						
		MC	0.25			11.10		CONSISTENCY RATIO				
		MR	0.77			11.22						
		MNL	0.43			11.37		CR = 0.08				
		AVL	0.67			11.07						
					λ MEAN=	11.13						

Table A20 Pairwise comparison in cluster APWPCH WRT RPV.

RPV	VER	SPD	RAP	MOB	FPD	FPC	AC	MC	MR	MNL	AVL		WTS
VER	1	3	3	3	3	3	4	5	3	3	3	2.92	0.21
SPD	0.33	1	3	3	3	3	4	5	2	3	3	2.30	0.17
RAP	0.33	0.33	1	3	3	3	4	4	2	3	3	1.85	0.13
MOB	0.33	0.33	0.33	1	3	3	3	3	2	3	3	1.43	0.10
FPD	0.33	0.33	0.33	0.33	1.00	3	4	4	2	3	3	1.24	0.09
FPC	0.33	0.33	0.33	0.33	0.33	1	3	4	2	3	3	0.98	0.07
AC	0.25	0.25	0.25	0.33	0.25	0.33	1	3	0.3	0.33	0.33	0.40	0.03
MC	0.2	0.2	0.25	0.33	0.25	0.25	0.3	1	0.2	0.33	0.33	0.30	0.02
MR	0.33	0.5	0.5	0.5	0.5	0.5	3	5	1	5	3	1.08	0.08
MNL	0.33	0.33	0.33	0.33	0.33	0.33	3	3	0.2	1	0.33	0.52	0.04
AVL	0.33	0.33	0.33	0.33	0.33	0.33	3	3	0.33	3.00	1	0.67	0.05
												13.69	

(*Continued*)

Table A20 Pairwise comparison in cluster APWPCH WRT RPV. (*Continued*)

RPV	VER	SPD	RAP	MOB	FPD	FPC	AC	MC	MR	MNL	AVL		WTS
CONSISTENCY INDEX (CI) COMPUTATION													
	VER	2.65		λs	12.40								
	SPD	2.09			12.41								
	RAP	1.68			12.49								
	MOB	1.33			12.73								
	FPD	1.13			12.55		CI = 0.14						
	FPC	0.90			12.50								
	AC	0.36			12.18		CONSISTENCY RATIO						
	MC	0.26			12.09								
	MR	0.97			12.26		CR = 0.09						
	MNL	0.48			12.65								
	AVL	0.60			12.33								
				λ MEAN=	12.42								

Table A21 Pairwise comparison in cluster APWPCH WRT AB.

AB	MOB	FPD	FPC	SDE	AC	MR		WTS
MOB	1	3	3	0.33	3	2	1.78	0.24
FPD	0.33	1	3	0.33	2	2	1.05	0.14
FPC	0.33	0.33	1	0.33	2	2	0.68	0.09
SDE	3	3	3	1	3	3	3.00	0.40
AC	0.33	0.5	0.5	0.33	1	0.5	0.42	0.06
MR	0.5	0.5	0.5	0.33	2	1.00	0.61	0.08
							7.54	
CONSISTENCY INDEX (CI) COMPUTATION								
	MOB	1.39		λs	5.88			
	FPD	0.89			6.38	CI = 0.10		
	FPC	0.62			6.88			
	SDE	2.20			5.54	CONSISTENCY RATIO		
	AC	0.42			7.49			
	MR	0.56			6.92	CR = 0.08		
				λ MEAN=	6.51			

Table A22 Pairwise comparison in cluster ALT WRT MF.

MF	**W1**	**W2**	**W3**		**WTS**
W1	1	0.5	0.2	0.46	0.11
W2	2	1	0.2	0.74	0.18
W3	5	5	1	2.92	0.71
				4.12	
CONSISTENCY INDEX (CI) COMPUTATION					
W1	0.34		λs	3.05	CI = 0.02
W2	0.55			3.05	
W3	2.16			3.05	CONSISTENCY RATIO
					CR = 0.04
			λ MEAN=	3.05	

Table A23 Pairwise comparison in cluster ALT WRT FI.

FI	**W1**	**W2**	**W3**		**WTS**
W1	1	5	5	2.92	0.71
W2	0.2	1	0.5	0.46	0.11
W3	0.2	2	1	0.74	0.18
				4.12	
CONSISTENCY INDEX (CI) COMPUTATION					
W1	2.16		λs	3.05	CI = 0.02
W2	0.34			3.05	
W3	0.55			3.05	CONSISTENCY RATIO
					CR = 0.04
			λ MEAN=	3.05	

Table A24 Pairwise comparison in cluster ALT WRT AH.

AH	W1	W2	W3		WTS
W1	1	2	0.33	0.87	0.25
W2	0.5	1	0.33	0.55	0.16
W3	3	3	1	2.08	0.59
				3.50	
CONSISTENCY INDEX (CI) COMPUTATION					
W1	0.76		λs	3.05	CI = 0.02
W2	0.48			3.05	
W3	1.81			3.05	CONSISTENCY RATIO
					CR = 0.04
	λ MEAN=			3.05	

Table A25 Pairwise comparison in cluster ALT WRT FB.

FB	W1	W2	W3		WTS
W1	1	0.2	0.33	0.40	0.10
W2	5	1	3	2.47	0.64
W3	3	0.33	1	1.00	0.26
				3.87	
CONSISTENCY INDEX (CI) COMPUTATION					
W1	0.32		λs	3.03	CI = 0.01
W2	1.93			3.03	
W3	0.78			3.03	CONSISTENCY RATIO
					CR = 0.03
			λ MEAN=	3.03	

Table A26 Pairwise comparison in cluster APWC WRT FL.

FL	**MF**	**AH**	**RPV**		**WTS**
MF	1	3	3	2.08	0.59
AH	0.33	1	2	0.87	0.25
RPV	0.33	0.5	1	0.55	0.16
				3.50	
CONSISTENCY INDEX (CI) COMPUTATION					
MF	1.81		λs	3.05	CI = 0.02
AH	0.76			3.05	CONSISTENCY RATIO
RPV	0.48			3.05	CR = 0.04
			λ MEAN=	3.05	

Table A27 Pairwise comparison in cluster APWC WRT SA.

SA	MF	FI	AH	RK	AAM	SAM	FB	ASM	SSM	RPV		WTS
MF	1	1	5	0.2	0.2	0.2	2	0.2	0.2	5	0.66	0.05
FI	1	1	5	0.2	0.2	0.2	2	0.2	0.2	5	0.66	0.05
AH	0.2	0.2	1	0.14	0.14	0.14	0.2	0.14	0.1	0.33	0.21	0.01
RK	5	5	7	1	0.5	3	5	0.5	3	7	2.59	0.18
AAM	5	5	7	2.00	1.00	3	5	1	3	7	3.19	0.22
SAM	5	5	7	0.33	0.33	1	5	1	3	5	2.07	0.14
FB	0.5	0.5	5	0.2	0.2	0.2	1	0.2	0.2	5	0.54	0.04
ASM	5	5	7	2	1	1	5	1	3	7	2.86	0.20
SSM	5	5	7	0.33	0.33	0.33	5	0.33	1	5	1.48	0.10
RPV	0.2	0.2	3	0.14	0.14	0.2	0.2	0.14	0.2	1	0.28	0.02
											14.54	

(*Continued*)

Table A27 Pairwise comparison in cluster APWC WRT SA. (*Continued*)

SA	MF	FI	AH	RK	AAM	SAM	FB	ASM	SSM	RPV		WTS
CONSISTENCY INDEX (CI) COMPUTATION												
MF	0.50		λs	10.97								
FI	0.50			10.97								
AH	0.16			11.51	CI = 0.13							
RK	1.99			11.17	CONSISTENCY RATIO							
AAM	2.38			10.83	CR = 0.09							
SAM	1.61			11.33								
FB	0.42			11.27								
ASM	2.09			10.64								
SSM	1.18			11.55								
RPV	0.22			11.53								
			λ MEAN=	11.18								

Table A28 Pairwise comparison in cluster APWC WRT UB.

UB	MF	FI	AH	RK	AAM	SAM	GN	FB	ASM	SSM	RPV	AB		WTS
MF	1	1	0.5	5	5	3	5	1	5	3	1	5	2.22	0.14
FI	1	1	0.5	5	5	3	5	1	5	3	1	5	2.22	0.14
AH	2	2	1	5	5	5	5	2	5	3	2	5	3.09	0.19
RK	0.2	0.2	0.2	1	0.5	0.5	1	0.2	0.5	0.33	0.2	1	0.39	0.02
AAM	0.2	0.2	0.2	2.00	1.00	0.5	2	0.2	1	0.33	0.33	2	0.55	0.03
SAM	0.33	0.33	0.2	2	2	1	2	0.33	1	0.33	0.33	2	0.69	0.04
GN	0.2	0.2	0.2	1	0.5	0.5	1	0.2	0.5	0.33	0.2	0.5	0.37	0.02
FB	1	1	0.5	5	5	3	5	1	5	3	1	5	2.22	0.14
ASM	0.2	0.2	0.2	2	1	1	2	0.2	1	0.33	0.2	2	0.55	0.03
SSM	0.33	0.33	0.33	3	3	3	3	0.33	3	1	0.33	3	1.09	0.07
RPV	1	1	0.5	5	3	3	5	1	5	3	1	5	2.12	0.13
AB	0.2	0.2	0.2	1	0.5	0.5	2	0.2	0.5	0.33	0.2	1	0.42	0.03
													15.92	

(*Continued*)

Table A28 Pairwise comparison in cluster APWC WRT UB. (*Continued*)

UB	MF	FI	AH	RK	AAM	SAM	GN	FB	ASM	SSM	RPV	AB		WTS
CONSISTENCY INDEX (CI) COMPUTATION														
MF	1.70		λs	12.21										
FI	1.70			12.21										
AH	2.43			12.56	CI = 0.03									
RK	0.30			12.26	CONSISTENCY RATIO									
AAM	0.43			12.48	CR = 0.02									
SAM	0.54			12.34										
GN	0.29			12.43										
FB	1.70			12.21										
ASM	0.43			12.41										
SSM	0.87			12.76										
RPV	1.63			12.23										
AB	0.33			12.46										
		λ MEAN=		12.38										

Table A29 Pairwise comparison in cluster APWC WRT RN.

RN	MF	FI	AH	RK	AAM	SAM	GN	FB	ASM	SSM	RPV		WTS
MF	1	3	5	5	5	5	5	2	5	2	2	3.21	0.21
FI	0.33	1	3	5	5	3	5	0.33	5	0.33	0.33	1.46	0.10
AH	0.2	0.33	1	5	3	3	5	0.2	3	0.33	0.33	1.00	0.07
RK	0.2	0.2	0.2	1	0.5	0.33	3	0.2	0.5	0.2	0.2	0.37	0.02
AAM	0.2	0.2	0.33	2.00	1.00	0.5	3	0.2	1	0.2	0.2	0.48	0.03
SAM	0.2	0.33	0.33	3	2	1	3	0.2	1	0.2	0.2	0.59	0.04
GN	0.2	0.2	0.2	0.33	0.33	0.33	1	0.2	0.3	0.2	0.2	0.28	0.02
FB	0.5	3	5	5	5	5	5	1	5	2	2	2.83	0.19
ASM	0.2	0.2	0.33	2	1	1	3	0.2	1	0.2	0.2	0.51	0.03
SSM	0.5	3	3	5	5	5	5	0.5	5	1	0.5	2.10	0.14
RPV	0.5	3	3	5	5	5	5	0.5	5	2	1	2.38	0.16
												15.22	

(*Continued*)

Table A29 Pairwise comparison in cluster APWC WRT RN. (*Continued*)

RN	**MF**	**FI**	**AH**	**RK**	**AAM**	**SAM**	**GN**	**FB**	**ASM**	**SSM**	**RPV**		**WTS**
CONSISTENCY INDEX (CI) COMPUTATION													
MF	2.52		λs	11.95									
FI	1.18			12.21									
AH	0.80			12.18	CI = 0.10								
RK	0.29			12.26	CONSISTENCY RATIO								
AAM	0.37			11.61	CR = 0.06								
SAM	0.45			11.68									
GN	0.23			12.67									
FB	2.23			11.98									
ASM	0.39			11.48									
SSM	1.63			11.83									
RPV	1.85			11.81									
			λ MEAN=	11.97									

Table A30 Pairwise comparison in cluster APWC WRT SH.

SH	MF	FI	AH	RK	AAM	SAM	FB	ASM	SSM	AB		WTS
MF	1	1	5	0.5	0.5	0.33	1	0.5	0.2	0.2	0.62	0.05
FI	1	1	5	0.5	0.5	0.33	1	0.5	0.2	0.2	0.62	0.05
AH	0.2	0.2	1	0.5	0.5	0.33	0.2	0.5	0.2	0.2	0.33	0.02
RK	2	2	2	1	1	0.5	0.5	1	0.2	0.2	0.78	0.06
AAM	2	2	2	1.00	1.00	0.5	0.5	1	0.2	0.2	0.78	0.06
SAM	3	3	3	2	2	1	0.5	1	0.2	0.2	1.08	0.08
FB	1	1	5	2	2	2	1	2	0.2	0.2	1.12	0.08
ASM	2	2	2	1	1	1	0.5	1	0.2	0.2	0.83	0.06
SSM	5	5	5	5	5	5	5	5	1	3	4.04	0.30
AB	5	5	5	5	5	5	5	5	0.3	1	3.24	0.24
												13.44

(*Continued*)

Table A30 Pairwise comparison in cluster APWC WRT SH. (*Continued*)

SH	**MF**	**FI**	**AH**	**RK**	**AAM**	**SAM**	**FB**	**ASM**	**SSM**	**AB**		**WTS**
CONSISTENCY INDEX (CI) COMPUTATION												
MF	0.52		λs	11.30								
FI	0.52			11.30								
AH	0.28			11.70	CI = 0.10							
RK	0.60			10.39	CONSISTENCY RATIO							
AAM	0.60			10.39	CR = 0.07							
SAM	0.87			10.86								
FB	0.92			11.02								
ASM	0.64			10.34								
SSM	3.31			11.01								
AB	2.63			10.90								
			λ MEAN=	10.92								

Table A31 Pairwise comparison in cluster APWC WRT MOB.

MOB	MF	FI	AH	RK	AAM	SAM	GN	FB	ASM	SSM	RPV	AB		WTS
MF	1	1	1	5	5	3	5	1	5	3	2	5	2.49	0.15
FI	1	1	1	5	5	3	5	1	5	3	2	5	2.49	0.15
AH	1	1	1	5	3	3	5	1	3	3	1	5	2.16	0.13
RK	0.2	0.2	0.2	1	0.5	0.33	3	0.2	0.5	0.2	0.2	3	0.44	0.03
AAM	0.2	0.2	0.33	2.00	1.00	0.5	3	0.2	1	0.2	0.2	3	0.56	0.03
SAM	0.33	0.33	0.33	3	2	1	3	0.2	1	0.2	0.2	3	0.71	0.04
GN	0.2	0.2	0.2	0.33	0.33	0.33	1	0.2	0.3	0.2	0.2	0.33	0.28	0.02
FB	1	1	1	5	5	5	5	1	5	2	2	5	2.51	0.16
ASM	0.2	0.2	0.33	2	1	1	3	0.2	1	0.2	0.2	3	0.59	0.04
SSM	0.33	0.33	0.33	5	5	5	5	0.5	5	1	0.5	5	1.51	0.09
RPV	0.5	0.5	1	5	5	5	5	0.5	5	2	1	5	1.99	0.12
AB	0.2	0.2	0.2	0.33	0.33	0.33	3	0.2	0.3	0.2	0.2	1	0.34	0.02
													16.06	

(Continued)

Table A31 Pairwise comparison in cluster APWC WRT MOB. (*Continued*)

MOB	MF	FI	AH	RK	AAM	SAM	GN	FB	ASM	SSM	RPV	AB		WTS
CONSISTENCY INDEX (CI) COMPUTATION														
MF	1.95		λs	12.59										
FI	1.95			12.59										
AH	1.68			12.53	CI = 0.08									
RK	0.36			13.15	CONSISTENCY RATIO									
AAM	0.45			12.77	CR = 0.05									
SAM	0.57			12.94										
GN	0.24			13.42										
FB	1.94			12.44										
ASM	0.47			12.65										
SSM	1.29			13.71										
RPV	1.59			12.80										
AB	0.28			13.50										
		λ MEAN=		12.92										

Table A32 Pairwise comparison in cluster APWC WRT DIS.

DIS	**MF**	**FI**	**AH**	**RK**	**AAM**	**SAM**	**GN**	**FB**	**ASM**	**SSM**	**RPV**	**AB**		**WTS**
MF	1	3	5	7	7	5	7	2	7	2	5	7	4.14	0.23
FI	0.33	1	3	7	7	5	7	0.5	7	0.5	2	7	2.43	0.13
AH	0.2	0.33	1	5	5	3	7	0.2	5	0.5	1	7	1.49	0.08
RK	0.14	0.14	0.2	1	0.5	0.33	5	0.14	0.5	0.14	0.14	2	0.38	0.02
AAM	0.14	0.14	0.2	2.00	1.00	0.5	5	0.14	1	0.2	0.2	3	0.51	0.03
SAM	0.2	0.2	0.33	3	2	1	5	0.2	1	0.2	0.2	3	0.68	0.04
GN	0.14	0.14	0.14	0.2	0.2	0.2	1	0.14	0.3	0.14	0.14	0.33	0.21	0.01
FB	0.5	2	5	7	7	5	7	1	7	2	2	7	3.30	0.18
ASM	0.14	0.14	0.2	2	1	1	3	0.14	1	0.2	0.2	3	0.52	0.03
SSM	0.5	2	2	7	5	5	7	0.5	5	1	1	7	2.43	0.13
RPV	0.2	0.5	1	7	5	5	7	0.5	5	1	1	7	1.89	0.10
AB	0.14	0.14	0.14	0.5	0.33	0.33	3	0.14	0.3	0.14	0.14	1	0.29	0.02
													18.28	

(*Continued*)

Table A32 Pairwise comparison in cluster APWC WRT DIS. (*Continued*)

DIS	MF	FI	AH	RK	AAM	SAM	GN	FB	ASM	SSM	RPV	AB		WTS
CONSISTENCY INDEX (CI) COMPUTATION														
MF	3.10		λs	13.68										
FI	1.74			13.07										
AH	1.07			13.07	CI = 0.10									
RK	0.28			13.27	CONSISTENCY RATIO									
AAM	0.36			12.87	CR = 0.07									
SAM	0.47			12.70										
GN	0.16			14.36										
FB	2.36			13.06										
ASM	0.36			12.53										
SSM	1.67			12.58										
RPV	1.33			12.79										
AB	0.21			13.19										
		λ MEAN=		13.10										

Table A33 Pairwise comparison in cluster APWC WRT FPD.

FPD	MF	FI	AH	RK	AAM	SAM	GN	FB	ASM	SSM	RPV	AB		WTS
MF	1	3	5	7	7	7	7	1	7	2	2	7	3.72	0.20
FI	0.33	1	3	5	5	3	5	0.33	5	0.33	0.33	5	1.62	0.09
AH	0.2	0.33	1	3	3	3	5	0.2	3	0.2	0.2	5	1.01	0.05
RK	0.14	0.2	0.33	1	1	0.5	3	0.14	1	0.14	0.14	2	0.45	0.02
AAM	0.14	0.2	0.33	1.00	1.00	0.5	3	0.14	1	0.14	0.14	2	0.45	0.02
SAM	0.14	0.33	0.33	2	2	1	3	0.14	1	0.14	0.14	3	0.58	0.03
GN	0.14	0.2	0.2	0.33	0.33	0.3	1	0.14	0.5	0.14	0.14	0.5	0.27	0.01
FB	1	3	5	7	7	7	7	1	7	2	2	7	3.72	0.20
ASM	0.14	0.2	0.33	1	1	1	2	0.14	1	0.14	0.14	2	0.46	0.03
SSM	0.5	3	5	7	7	7	7	0.5	7	1	1	7	2.95	0.16
RPV	0.5	3	5	7	7	7	7	0.5	7	1	1	7	2.95	0.16
AB	0.14	0.2	0.2	0.5	0.5	0.33	2	0.14	0.5	0.14	0.14	1	0.32	0.02
													18.52	

(*Continued*)

Table A33 Pairwise comparison in cluster APWC WRT FPD. (*Continued*)

FPD	MF	FI	AH	RK	AAM	SAM	GN	FB	ASM	SSM	RPV	AB		WTS
CONSISTENCY INDEX (CI) COMPUTATION														
MF	2.53		λs	12.62										
FI	1.11			12.69										
AH	0.70			12.94	CI = 0.06									
RK	0.30			12.41	CONSISTENCY RATIO									
AAM	0.30			12.41	CR = 0.04									
SAM	0.40			12.66										
GN	0.19			13.26										
FB	2.53			12.62										
ASM	0.31			12.17										
SSM	2.02			12.64										
RPV	2.02			12.64										
AB	0.22			12.80										
		λ MEAN=		12.65										

Table A34 Pairwise comparison in cluster APWC WRT MR.

MR	MF	FI	AH	RK	AAM	SAM	GN	FB	ASM	SSM	RPV	AB		WTS
MF	1	3	3	5	5	2	5	1	5	2	2	5	2.79	0.18
FI	0.33	1	3	3	3	2	3	0.33	3	2	1	3	1.62	0.10
AH	0.33	0.33	1	3	3	2	3	0.33	3	2	1	3	1.34	0.09
RK	0.2	0.33	0.33	1	0.33	0.33	3	0.2	0.3	0.2	0.2	3	0.44	0.03
AAM	0.2	0.33	0.33	3.00	1.00	0.5	3	0.2	1	0.2	0.2	2	0.58	0.04
SAM	0.5	0.5	0.5	3	2	1	5	0.2	1	0.2	0.33	2	0.82	0.05
GN	0.2	0.33	0.33	0.33	0.33	0.2	1	0.2	0.3	0.2	0.2	0.33	0.29	0.02
FB	1	3	3	5	5	5	5	1	5	1	1	5	2.69	0.17
ASM	0.2	0.33	0.33	3	1	1	3	0.2	1	0.2	0.33	2	0.64	0.04
SSM	0.5	0.5	0.5	5	5	5	5	1	5	1	1	5	1.88	0.12
RPV	0.5	1	1	5	5	3	5	1	3	1	1	5	1.94	0.13
AB	0.2	0.33	0.33	0.33	0.5	0.5	3	0.2	0.5	0.2	0.2	1	0.41	0.03
													15.46	

(*Continued*)

Table A34 Pairwise comparison in cluster APWC WRT MR. (*Continued*)

MR	MF	FI	AH	RK	AAM	SAM	GN	FB	ASM	SSM	RPV	AB		WTS
CONSISTENCY INDEX (CI) COMPUTATION														
MF	2.30		λs	12.72										
FI	1.42			13.57										
AH	1.17			13.50	CI = 0.10									
RK	0.39			13.70	CONSISTENCY RATIO									
AAM	0.49			12.85	CR = 0.06									
SAM	0.69			12.95										
GN	0.26			13.55										
FB	2.21			12.72										
ASM	0.53			12.66										
SSM	1.64			13.49										
RPV	1.55			12.34										
AB	0.34			12.97										
		λ MEAN=		13.09										

Table A35 Pairwise comparison in cluster APWC WRT W3.

W3	**MF**	**FI**	**AH**	**RK**	**AAM**	**SAM**	**GN**	**FB**	**ASM**	**SSM**	**RPV**	**AB**		**WTS**
MF	1	7	7	7	7	7	7	7	7	7	7	7	5.95	0.35
FI	0.14	1	3	3	3	3	5	1	3	3	1	3	1.84	0.11
AH	0.14	0.33	1	3	3	3	5	0.33	3	3	1	3	1.40	0.08
RK	0.14	0.33	0.33	1	1	0.33	3	0.2	1	0.33	0.33	1	0.51	0.03
AAM	0.14	0.33	0.33	1.00	1.00	0.33	3	0.2	1	0.33	0.33	1	0.51	0.03
SAM	0.14	0.33	0.33	3	3	1	5	0.33	1	0.33	0.5	1	0.76	0.04
GN	0.14	0.2	0.2	0.33	0.33	0.2	1	0.2	0.3	0.2	0.2	0.33	0.26	0.02
FB	0.14	1	3	5	5	3	5	1	5	3	3	3	2.29	0.13
ASM	0.14	0.33	0.33	1	1	1	3	0.2	1	0.33	0.33	1	0.56	0.03
SSM	0.14	0.33	0.33	3	3	3	5	0.33	3	1	1	3	1.16	0.07
RPV	0.14	1	1	3	3	2	5	0.33	3	1	1	3	1.35	0.08
AB	0.14	0.33	0.33	1	1	1	3	0.33	1	0.33	0.33	1	0.59	0.03
													17.20	

(*Continued*)

Table A35 Pairwise comparison in cluster APWC WRT W3. (*Continued*)

W3	MF	FI	AH	RK	AAM	SAM	GN	FB	ASM	SSM	RPV	AB		WTS
CONSISTENCY INDEX (CI) COMPUTATION														
MF	4.92		λs	14.23										
FI	1.40			13.09										
AH	1.08			13.27	CI = 0.09									
RK	0.37			12.49	CONSISTENCY RATIO									
AAM	0.37			12.49	CR = 0.06									
SAM	0.58			13.17										
GN	0.21			13.63										
FB	1.74			13.07										
ASM	0.40			12.30										
SSM	0.89			13.15										
RPV	0.97			12.35										
AB	0.42			12.30										
		λ MEAN=		12.96										

Table A36 Pairwise comparison in cluster APWC WRT W1.

W1	MF	FI	AH	RK	AAM	SAM	GN		WTS
MF	1	0.33	3	3	3	3	5	2.01	0.21
FI	3	1	7	5	5	3	5	3.60	0.38
AH	0.33	0.14	1	0.33	0.33	0.2	0.3	0.32	0.03
RK	0.33	0.2	3	1	0.33	0.33	3	0.68	0.07
AAM	0.33	0.2	3	3.00	1.00	0.33	3	0.93	0.10
SAM	0.33	0.33	5	3	3	1	3	1.47	0.16
GN	0.2	0.2	3	0.33	0.33	0.33	1	0.46	0.05
								9.46	
CONSISTENCY INDEX (CI) COMPUTATION									
MF	1.66		λs	7.79					
FI	2.81			7.38					
AH	0.26			7.73	CI = 0.11				
RK	0.55			7.67	CONSISTENCY RATIO				
AAM	0.76			7.72	CR = 0.08				
SAM	1.17			7.56					
GN	0.38			7.73					
			λ MEAN=	7.65					

Table A37 Pairwise comparison in cluster APWC WRT W2.

W2	MF	AH	FB	ASM	SSM	RPV	AB		WTS
MF	1	3	0.33	3	3	3	3	1.87	0.21
AH	0.33	1	0.2	3	0.33	0.33	0.3	0.49	0.06
FB	3	5	1	5	3	3	3	2.97	0.34
ASM	0.33	0.33	0.2	1	0.33	0.33	0.3	0.36	0.04

(*Continued*)

Table A37 Pairwise comparison in cluster APWC WRT W2. (*Continued*)

W2	**MF**	**AH**	**FB**	**ASM**	**SSM**	**RPV**	**AB**		**WTS**
SSM	0.33	3	0.33	3.00	1.00	3	3	1.36	0.16
RPV	0.33	3	0.33	3	0.33	1	3	1.00	0.11
AB	0.33	3	0.33	3	0.33	0.33	1	0.73	0.08
								8.78	
CONSISTENCY INDEX (CI) COMPUTATION									
MF	1.67		λs	7.84					
AH	0.43			7.71					
FB	2.52			7.45	CI = 0.12				
ASM	0.31			7.65	CONSISTENCY RATIO				
SSM	1.22			7.83	CR = 0.09				
RPV	0.89			7.82					
AB	0.64			7.79					
			λ MEAN=	7.73					

Table A38 Pairwise comparison in cluster APWC WRT VER.

VER	**MF**	**AH**	**RPV**		**WTS**	
MF	1	3	5	2.47	0.64	
AH	0.33	1	3	1.00	0.26	
RPV	0.2	0.33	1	0.40	0.10	
				3.87		
CONSISTENCY INDEX (CI) COMPUTATION						
MF	1.93		λs	3.03	CI = 0.01	
AH	0.78			3.03	CONSISTENCY RATIO	
RPV	0.32			3.03	CR = 0.03	
			λ MEAN=	3.03		

Table A39 Pairwise comparison in cluster APWC WRT SPD.

SPD	MF	FI	AH	RK	AAM	SAM	FB	ASM	SSM	RPV	AB		WTS
MF	1	3	5	0.33	0.33	0.33	3	0.33	0.33	5	5	1.14	0.08
FI	0.33	1	5	0.33	0.33	0.33	1	0.33	0.33	5	5	0.85	0.06
AH	0.2	0.2	1	0.2	0.2	0.2	0.2	0.2	0.2	1	3	0.34	0.02
RK	3	3	5	1	0.5	0.5	3	0.5	0.33	5	5	1.57	0.11
AAM	3	3	5	2.00	1.00	1	3	1	0.33	5	5	2.02	0.14
SAM	3	3	5	2	1	1	3	1	0.33	5	5	2.02	0.14
FB	0.33	1	5	0.33	0.33	0.33	1	0.33	0.33	5	5	0.85	0.06
ASM	3	3	5	2	1	1	3	1	0.33	5	5	2.02	0.14
SSM	3	3	5	3	3	3	3	3	1	5	5	3.12	0.22
RPV	0.2	0.2	1	0.2	0.2	0.2	0.2	0.2	0.2	1	3	0.34	0.02
AB	0.2	0.2	0.33	0.2	0.2	0.2	0.2	0.2	0.2	0.33	1	0.25	0.02
												14.51	

(*Continued*)

Table A39 Pairwise comparison in cluster APWC WRT SPD. (*Continued*)

SPD	MF	FI	AH	RK	AAM	SAM	FB	ASM	SSM	RPV	AB		WTS
CONSISTENCY INDEX (CI) COMPUTATION													
MF	1.00		λs	12.64									
FI	0.71			12.17									
AH	0.29			12.14	CI = 0.01								
RK	1.30			12.02	CONSISTENCY RATIO								
AAM	1.61			11.62	CR = 0.01								
SAM	1.61			11.62									
FB	0.71			12.17									
ASM	1.61			11.62									
SSM	2.70			12.55									
RPV	0.29			12.14									
AB	0.22			12.60									
			λ MEAN=	12.12									

Table A40 Pairwise comparison in cluster APWC WRT RAP.

RAP	MF	FI	AH	FB	RPV		WTS
MF	1	2	3	1	3	1.78	0.31
FI	0.5	1	3	1	3	1.35	0.24
AH	0.33	0.33	1	0.33	1	0.51	0.09
FB	1	1	3	1	3	1.55	0.27
RPV	0.33	0.33	1	0.33	1.00	0.51	0.09
						5.71	
CONSISTENCY INDEX (CI) COMPUTATION							
MF	1.60		λs	5.12			
FI	1.20			5.09			
AH	0.45			5.01	CI = 0.01		
FB	1.36			5.01	CONSISTENCY RATIO		
RPV	0.45			5.01	CR = 0.01		
			λ MEAN=	5.05			

Table A41 Pairwise comparison in cluster APWC WRT RES.

RES	MF	FI	AH	FB	SSM	RPV		WTS
MF	1	3	5	1	0.33	3	1.57	0.22
FI	0.33	1	3	0.33	0.33	0.33	0.57	0.08
AH	0.2	0.33	1	0.33	0.33	1	0.44	0.06
FB	1	3	3	1	0.33	3	1.44	0.20
SSM	3	3	3	3.00	1.00	3	2.50	0.35
RPV	0.3	3	1	0.33	0.33	1	0.68	0.09
							7.20	

(*Continued*)

Table A41 Pairwise comparison in cluster APWC WRT RES. (*Continued*)

RES	**MF**	**FI**	**AH**	**FB**	**SSM**	**RPV**		**WTS**
CONSISTENCY INDEX (CI) COMPUTATION								
MF	1.36		λs	6.24				
FI	0.55			6.86				
AH	0.41			6.65	CI = 0.11			
FB	1.24			6.19	CONSISTENCY RATIO			
SSM	2.31			6.64	CR = 0.09			
RPV	0.64			6.79				
		λ MEAN=		6.56				

Table A42 Pairwise comparison in cluster APWC WRT RTR.

RTR	**MF**	**FI**	**AH**	**FB**	**RPV**		**WTS**
MF	1	3	3	1	3	1.93	0.33
FI	0.33	1	3	0.33	3	1.00	0.17
AH	0.33	0.33	1	0.33	1	0.51	0.09
FB	1	3	3	1	3	1.93	0.33
RPV	0.33	0.33	1	0.33	1.00	0.51	0.09
						5.89	
CONSISTENCY INDEX (CI) COMPUTATION							
MF	1.69		λs	5.14			
FI	0.91			5.38			
AH	0.45			5.12	CI = 0.04		
FB	1.69			5.14	CONSISTENCY RATIO		
RPV	0.45			5.12	CR = 0.04		
			λ MEAN=	5.18			

Table A43 Pairwise comparison in cluster APWC WRT ROA.

ROA	MF	FI	AH	FB		WTS
MF	1	1	5	3	1.97	0.39
FI	1	1	5	3	1.97	0.39
AH	0.2	0.2	1	0.33	0.34	0.07
FB	0.33	0.33	3	1	0.76	0.15
					5.03	
CONSISTENCY INDEX (CI) COMPUTATION						
MF	1.57		λs	4.01	CI = 0.01	
FI	1.57			4.01	CONSISTENCY RATIO	
AH	0.27			4.06	CR = 0.01	
FB	0.61			4.06		
			λ MEAN=	4.04		

Table A44 Pairwise comparison in cluster ALT WRT FL.

FL	W1	W2	W3		WTS	
W1	1	0.5	0.33	0.55	0.16	
W2	2	1	0.33	0.87	0.25	
W3	3	3	1	2.08	0.59	
				3.50		
CONSISTENCY INDEX (CI) COMPUTATION						
W1	0.48		λs	3.05	CI = 0.02	
W2	0.76			3.05		
W3	1.81			3.05	CONSISTENCY RATIO	
					CR = 0.04	
			λ MEAN=	3.05		

Table A45 Pairwise comparison in cluster ALT WRT SA.

SA	**W1**	**W2**	**W3**		**WTS**
W1	1	0.33	0.33	0.48	0.14
W2	3	1	0.5	1.14	0.33
W3	3	2	1	1.82	0.53
				3.44	
CONSISTENCY INDEX (CI) COMPUTATION					
W1	0.42		λs	3.05	CI = 0.02
W2	1.01			3.05	
W3	1.61			3.05	CONSISTENCY RATIO
					CR = 0.04
			λ MEAN=	3.05	

Table A46 Pairwise comparison in cluster ALT WRT UB.

UB	**W1**	**W2**	**W3**		**WTS**
W1	1	0.33	0.2	0.40	0.10
W2	3	1	0.33	1.00	0.26
W3	5	3	1	2.47	0.64
				3.87	
CONSISTENCY INDEX (CI) COMPUTATION					
W1	0.32		λs	3.03	CI = 0.01
W2	0.78			3.03	
W3	1.93			3.03	CONSISTENCY RATIO
					CR = 0.03
			λ MEAN=	3.03	

Table A47 Pairwise comparison in cluster ALT WRT SH.

SH	W1	W2	W3		WTS
W1	1	0.2	0.33	0.40	0.10
W2	5	1	3	2.47	0.64
W3	3	0.33	1	1.00	0.26
				3.87	
CONSISTENCY INDEX (CI) COMPUTATION					
W1	0.32		**λ**s	3.03	CI = 0.01
W2	1.93			3.03	
W3	0.78			3.03	CONSISTENCY RATIO
					CR = 0.03
			λ MEAN=	3.03	

Table A48 Pairwise comparison in cluster ALT WRT CS.

CS	W1	W2	W3		WTS
W1	1	3	5	2.47	0.64
W2	0.33	1	3	1.00	0.26
W3	0.2	0.33	1	0.40	0.10
				3.87	
CONSISTENCY INDEX (CI) COMPUTATION					
W1	1.93		**λ**s	3.03	CI = 0.01
W2	0.78			3.03	
W3	0.32			3.03	CONSISTENCY RATIO
					CR = 0.03
			λ MEAN=	3.03	

Table A49 Pairwise comparison in cluster ALT WRT REL.

REL	**W1**	**W2**	**W3**		**WTS**	
W1	1	0.33	0.2	0.40	0.10	
W2	3	1	0.33	1.00	0.26	
W3	5	3	1	2.47	0.64	
				3.87		
CONSISTENCY INDEX (CI) COMPUTATION						
W1	0.32		**λ**s	3.03	CI = 0.01	
W2	0.78			3.03		
W3	1.93			3.03	CONSISTENCY RATIO	
					CR = 0.03	
			λ MEAN=	3.03		

Table A50 Pairwise comparison in cluster ALT WRT SPD.

FL	**W1**	**W2**	**W3**		**WTS**
W1	1	0.33	0.2	0.40	0.10
W2	3	1	0.33	1.00	0.26
W3	5	3	1	2.47	0.64
				3.87	
CONSISTENCY INDEX (CI) COMPUTATION					
W1	0.32		**λ**s	3.03	CI = 0.01
W2	0.78			3.03	
W3	1.93			3.03	CONSISTENCY RATIO
					CR = 0.03
			λ MEAN=	3.03	

Table A51 Pairwise comparison in cluster ALT WRT FL.

FL	W1	W2	W3		WTS
W1	1	0.5	0.33	0.55	0.16
W2	2	1	0.33	0.87	0.25
W3	3	3	1	2.08	0.59
				3.50	
CONSISTENCY INDEX (CI) COMPUTATION					
W1	0.48		λs	3.05	CI = 0.02
W2	0.76			3.05	
W3	1.81			3.05	CONSISTENCY RATIO
					CR = 0.04
			λ MEAN=	3.05	

Table A52 Pairwise comparison in cluster ALT WRT FL.

FL	W1	W2	W3		WTS
W1	1	0.5	0.33	0.55	0.16
W2	2	1	0.33	0.87	0.25
W3	3	3	1	2.08	0.59
				3.50	
CONSISTENCY INDEX (CI) COMPUTATION					
W1	0.48		λs	3.05	CI = 0.02
W2	0.76			3.05	
W3	1.81			3.05	CONSISTENCY RATIO
					CR = 0.04
			λ MEAN=	3.05	

Table A53 Pairwise comparison in cluster ALT WRT FL.

FL	**W1**	**W2**	**W3**		**WTS**
W1	1	0.5	0.33	0.55	0.16
W2	2	1	0.33	0.87	0.25
W3	3	3	1	2.08	0.59
				3.50	
CONSISTENCY INDEX (CI) COMPUTATION					
W1	0.48		**λ**s	3.05	CI = 0.02
W2	0.76			3.05	
W3	1.81			3.05	CONSISTENCY RATIO
					CR = 0.04
			λ MEAN=	3.05	

Table A54 Pairwise comparison in cluster ALT WRT FL.

FL	**W1**	**W2**	**W3**		**WTS**
W1	1	0.5	0.33	0.55	0.16
W2	2	1	0.33	0.87	0.25
W3	3	3	1	2.08	0.59
				3.50	
CONSISTENCY INDEX (CI) COMPUTATION					
W1	0.48		**λ**s	3.05	CI = 0.02
W2	0.76			3.05	
W3	1.81			3.05	CONSISTENCY RATIO
					CR = 0.04
			λ MEAN=	3.05	

Table A55 Pairwise comparison in cluster ALT WRT FL.

FL	W1	W2	W3		WTS
W1	1	0.5	0.33	0.55	0.16
W2	2	1	0.33	0.87	0.25
W3	3	3	1	2.08	0.59
				3.50	
CONSISTENCY INDEX (CI) COMPUTATION					
W1	0.48		λs	3.05	CI = 0.02
W2	0.76			3.05	
W3	1.81			3.05	CONSISTENCY RATIO
					CR = 0.04
			λ MEAN=	3.05	

Table A56 Pairwise comparison in cluster ALT WRT FL.

FL	W1	W2	W3		WTS
W1	1	0.5	0.33	0.55	0.16
W2	2	1	0.33	0.87	0.25
W3	3	3	1	2.08	0.59
				3.50	
CONSISTENCY INDEX (CI) COMPUTATION					
W1	0.48		λs	3.05	CI = 0.02
W2	0.76			3.05	
W3	1.81			3.05	CONSISTENCY RATIO
					CR = 0.04
			λ MEAN=	3.05	

Table A57 Pairwise comparison in cluster ALT WRT FL.

FL	**W1**	**W2**	**W3**		**WTS**
W1	1	0.5	0.33	0.55	0.16
W2	2	1	0.33	0.87	0.25
W3	3	3	1	2.08	0.59
				3.50	
CONSISTENCY INDEX (CI) COMPUTATION					
W1	0.48		λs	3.05	CI = 0.02
W2	0.76			3.05	
W3	1.81			3.05	CONSISTENCY RATIO
					CR = 0.04
			λ MEAN=	3.05	

Table A58 Pairwise comparison in cluster ALT WRT FL.

FL	**W1**	**W2**	**W3**		**WTS**
W1	1	0.5	0.33	0.55	0.16
W2	2	1	0.33	0.87	0.25
W3	3	3	1	2.08	0.59
				3.50	
CONSISTENCY INDEX (CI) COMPUTATION					
W1	0.48		λs	3.05	CI = 0.02
W2	0.76			3.05	
W3	1.81			3.05	CONSISTENCY RATIO
					CR = 0.04
			λ MEAN=	3.05	

Table A59 Pairwise comparison in cluster ALT WRT FL.

FL	**W1**	**W2**	**W3**		**WTS**
W1	1	0.5	0.33	0.55	0.16
W2	2	1	0.33	0.87	0.25
W3	3	3	1	2.08	0.59
				3.50	
CONSISTENCY INDEX (CI) COMPUTATION					
W1	0.48		**λs**	3.05	CI = 0.02
W2	0.76			3.05	
W3	1.81			3.05	CONSISTENCY RATIO
					CR = 0.04
			λ MEAN=	3.05	

Table A60 Pairwise comparison in cluster ALT WRT FL.

FL	**W1**	**W2**	**W3**		**WTS**
W1	1	0.5	0.33	0.55	0.16
W2	2	1	0.33	0.87	0.25
W3	3	3	1	2.08	0.59
				3.50	
CONSISTENCY INDEX (CI) COMPUTATION					
W1	0.48		λs	3.05	CI = 0.02
W2	0.76			3.05	
W3	1.81			3.05	CONSISTENCY RATIO
					CR = 0.04
			λ MEAN=	3.05	

Table A61 Pairwise comparison in cluster ALT WRT FL.

FL	W1	W2	W3		WTS
W1	1	0.5	0.33	0.55	0.16
W2	2	1	0.33	0.87	0.25
W3	3	3	1	2.08	0.59
				3.50	
CONSISTENCY INDEX (CI) COMPUTATION					
W1	0.48		λs	3.05	CI = 0.02
W2	0.76			3.05	
W3	1.81			3.05	CONSISTENCY RATIO
					CR = 0.04
			λ MEAN=	3.05	

Table A62 Pairwise comparison in cluster ALT WRT FL.

FL	W1	W2	W3		WTS
W1	1	0.5	0.33	0.55	0.16
W2	2	1	0.33	0.87	0.25
W3	3	3	1	2.08	0.59
				3.50	
CONSISTENCY INDEX (CI) COMPUTATION					
W1	0.48		λs	3.05	CI = 0.02
W2	0.76			3.05	
W3	1.81			3.05	CONSISTENCY RATIO
					CR = 0.04
	λ MEAN=			3.05	

Table A63 Pairwise comparison in cluster ALT WRT FL.

FL	**W1**	**W2**	**W3**		**WTS**
W1	1	0.5	0.33	0.55	0.16
W2	2	1	0.33	0.87	0.25
W3	3	3	1	2.08	0.59
				3.50	
CONSISTENCY INDEX (CI) COMPUTATION					
W1	0.48		λs	3.05	CI= 0.02
W2	0.76			3.05	
W3	1.81			3.05	CONSISTENCY RATIO
					CR = 0.04
			λ MEAN=	3.05	

Table A64 Pairwise comparison in the cluster MATRIX WRTAPWC.

	APWC	**APWPC**	**APWPCH**	**ALT**			**WTS**
APWC	1	3	3	0.33		1.31	0.26
APWPC	0.33	1	3	0.33		0.76	0.15
APWPCH	0.33	0.33	1	0.2		0.38	0.08
ALT	3	3	5	1		2.59	0.51
						5.04	1.00
CONSISTENCY INDEX (CI) COMPUTATION							
APWC	1.11		λs	4.26	CI = 0.06		
APWPC	0.63			4.23			
APWPCH	0.31			4.13	CONSISTENCY RATIO		
ALT	2.13			4.14	CR = 0.07		
			λ MEAN	4.19			

Table A65 Pairwise comparison in the cluster MATRIX WRTAPWPC.

	APWC	**APWPC**	**APWPCH**	**ALT**			**WTS**
APWC	1	2	3	0.33		1.19	0.25
APWPC	0.5	1	3	0.33		0.84	0.18
APWPCH	0.33	0.33	1	0.33		0.44	0.09
ALT	3	3	3	1		2.28	0.48
						4.74	1.00
CONSISTENCY INDEX (CI) COMPUTATION							
APWC	1.04		λs	4.15	CI = 0.07		
APWPC	0.74			4.16			
APWPCH	0.39			4.26	CONSISTENCY RATIO		
ALT	2.04			4.24	CR = 0.07		
			λ MEAN	4.20			

Appendix B

Unweighted Super-Matrix (Part 1)

				APWC									
		MF	**FI**	**AH**	**RK**	**AAM**	**SAM**	**GN**	**FB**	**ASM**	**SSM**	**RPV**	**AB**
APWC	MF	0	0.55	0	0.55	0.6	0.13	0.43	0	1	0	0.13	0.26
	FI	0	0	0	0.45	0.4	0	0.43	0	0	0.6	0.18	0
	AH	0	0	0	0	0	0.06	0.14	0	0	0	0	0
	RK	0.33	0	0.26	0	0	0	0	0	0	0	0	0
	AAM	0.23	0	0.3	0	0	0	0	0	0	0	0	0
	SAM	0.07	0	0	0	0	0	0	0.4	0	0.4	0	0
	GN	0	0.25	0.08	0	0	0	0	0	0	0	0	0
	FB	0.04	0	0	0	0	0.4	0	0	0	0	0.08	0.64
	ASM	0.12	0	0.15	0	0	0	0	0	0	0	0.31	0
	SSM	0.05	0	0	0	0	0.26	0	0	0	0	0.06	0
	RPV	0	0.13	0	0	0	0.15	0	0	0	0	0	0.1
	AB	0.16	0.07	0.21	0	0	0	0	0.6	0	0	0.25	0

(Continued)

(*Continued*)

				APWC									
		MF	**FI**	**AH**	**RK**	**AAM**	**SAM**	**GN**	**FB**	**ASM**	**SSM**	**RPV**	**AB**
APWPC	FL	0.32	0.05	0.34	0	0	0	0	0	0	0	0.27	0
	SA	0.22	0.33	0.21	0.17	0.17	0.17	0	0.15	0.17	0.17	0.21	0
	UB	0.14	0.21	0.15	0.05	0.05	0	0	0.12	0	0	0.17	0
	RN	0.12	0.14	0.11	0.23	0.23	0.35	0.6	0.24	0.35	0.35	0.12	0.09
	SH	0.08	0.04	0.07	0.29	0.29	0.29	0.4	0.35	0.29	0.29	0.09	0.48
	CS	0.04	0.09	0.05	0.1	0.1	0.06	0	0.05	0.06	0.06	0.05	0.18
	REL	0.07	0.14	0.07	0.16	0.16	0.03	0	0.08	0.03	0.03	0.08	0.25
APWPCH	VER	0.19	0	0.19	0	0	0	0	0	0	0	0.21	0
	ADP	0	0	0	0	0	0	0	0	0	0	0	0
	MR	0	0	0	0	0	0	0	0	0	0	0	0
	SPD	0.15	0.22	0.15	0.34	0.34	0.26	0.35	0.25	0.34	0.26	0.17	0

(*Continued*)

(*Continued*)

				APWC									
		MF	**FI**	**AH**	**RK**	**AAM**	**SAM**	**GN**	**FB**	**ASM**	**SSM**	**RPV**	**AB**
	RAP	0.13	0.16	0.13	0	0	0	0	0.16	0	0	0.13	0
	RES	0	0	0	0	0	0	0	0	0	0	0	0
	AOA	0	0	0	0	0	0	0	0	0	0	0	0
	MOB	0.1	0.13	0.1	0.2	0.2	0.16	0	0.03	0.2	0.16	0.1	0.24
	RTR	0	0	0	0	0	0	0	0	0	0	0	0
	DIS	0	0	0	0	0	0	0	0	0	0	0	0
	ROA	0	0	0	0	0	0	0	0	0	0	0	0
	FPD	0.09	0.11	0.09	0.14	0.14	0.14	0.25	0.12	0.14	0.14	0.09	0.14
	SDE	0.05	0.06	0.05	0.04	0.04	0.04	0	0	0.04	0.04	0	0.4
	NSE	0	0	0	0	0	0	0	0	0	0	0	0
	FPC	0.06	0.07	0.06	0.09	0.09	0.04	0.17	0.09	0.09	0.04	0.07	0.09
	AC	0.03	0.03	0.03	0.05	0.05	0.09	0	0.06	0.05	0.09	0.03	0.06

(*Continued*)

(Continued)

				APWC									
		MF	**FI**	**AH**	**RK**	**AAM**	**SAM**	**GN**	**FB**	**ASM**	**SSM**	**RPV**	**AB**
	OC	0	0	0	0	0	0	0	0	0	0	0	0
	MC	0.02	0.02	0.02	0	0	0.03	0	0.02	0	0.03	0.02	0
	MNR	0.09	0.11	0.09	0.15	0.15	0.15	0.12	0.07	0.15	0.15	0.08	0.08
	MNL	0.04	0.04	0.04	0	0	0.04	0.04	0.04	0	0.04	0.04	0
	AVL	0.04	0.05	0.04	0	0	0.07	0.07	0.06	0	0.07	0.05	0
ALT	W1	0.64	0.58	0.53	0.58	0.64	0.53	0.48	0.64	0.53	0.48	0.67	0.5
	W2	0.26	0.31	0.33	0.31	0.26	0.33	0.35	0.26	0.33	0.35	0.24	0.25
	W3	0.1	0.11	0.14	0.11	0.1	0.14	0.17	0.1	0.14	0.17	0.09	0.25

			APWPC					
		FL	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**
APWC	MF	0.59	0.05	0.14	0.21	0.05	0.04	0.14
	FI	0	0.05	0.14	0.1	0.05	0.05	0.14
	AH	0.25	0.01	0.19	0.07	0.02	0.23	0.06
	RK	0	0.18	0.02	0.02	0.06	0	0.04
	AAM	0	0.22	0.03	0.03	0.06	0	0.05
	SAM	0	0.14	0.04	0.04	0.08	0.32	0.09
	GN	0	0	0.02	0.02	0	0	0.03
	FB	0	0.04	0.14	0.19	0.08	0.05	0.15
	ASM	0	0.2	0.03	0.03	0.06	0	0.05
	SSM	0	0.1	0.07	0.14	0.3	0.11	0.11
	RPV	0.16	0.02	0.13	0.16	0	0.2	0.1
	AB	0	0	0.03	0	0.24	0	0.04
APWPC	FL	0.33	0.33	0.33	0.33	0.33	0.33	0.33
	SA	0.24	0.24	0.24	0.24	0.24	0.24	0.24

(*Continued*)

(*Continued*)

			APWPC					
		FL	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**
	UB	0.16	0.16	0.16	0.16	0.16	0.16	0.16
	RN	0.11	0.11	0.11	0.11	0.11	0.11	0.11
	SH	0.05	0.05	0.05	0.05	0.05	0.05	0.05
	CS	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	REL	0.08	0.08	0.08	0.08	0.08	0.08	0.08
APWPCH	VER	0.33	0.33	0.33	0.33	0.33	0.33	0.33
	ADP	0	0	0	0	0	0	0
	MR	0	0	0	0	0	0	0
	SPD	0.24	0.24	0.24	0.24	0.24	0.24	0.24
	RAP	0	0	0	0	0	0	0
	RES	0	0	0	0	0	0	0
	AOA	0	0	0	0	0	0	0
	MOB	0	0	0	0	0	0	0
	RTR	0.16	0.16	0.16	0.16	0.16	0.16	0.16

(*Continued*)

(Continued)

			APWPC					
		FL	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**
	DIS	0.11	0.11	0.11	0.11	0.11	0.11	0.11
	ROA	0	0	0	0	0	0	0
	FPD	0	0	0	0	0	0	0
	SDE	0.05	0.05	0.05	0.05	0.05	0.05	0.05
	NSE	0	0	0	0	0	0	0
	FPC	0	0	0	0	0	0	0
	AC	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	OC	0	0	0	0	0	0	0
	MC	0	0	0	0	0	0	0
	MNR	0.08	0.08	0.08	0.08	0.08	0.08	0.08
	MNL	0	0	0	0	0	0	0
	AVL	0	0	0	0	0	0	0
ALT	W1	0.53	0.48	0.43	0.43	0.33	0.64	0.64
	W2	0.33	0.35	0.43	0.43	0.33	0.26	0.26
	W3	0.14	0.17	0.14	0.14	0.33	0.1	0.1

Unweighted Super-Matrix (Part 2)

		APWPCH												
		VER	**ADP**	**MR**	**SPD**	**RAP**	**RES**	**AOA**	**MOB**	**RTR**	**DIS**	**ROA**	**FPD**	**SDE**
APWC	MF	0.64	0.64	0.64	0.08	0.31	0.22	0.14	0.15	0.33	0.23	0.39	0.2	0.29
	FI	0	0	0	0.06	0.24	0.08	0.14	0.15	0.17	0.13	0.39	0.09	0
	AH	0.26	0.26	0.26	0.02	0.09	0.06	0.19	0.13	0.09	0.08	0.07	0.05	0.06
	RK	0	0	0	0.11	0	0	0.02	0.03	0	0.02	0	0.02	0
	AAM	0	0	0	0.14	0	0	0.03	0.03	0	0.03	0	0.02	0
	SAM	0	0	0	0.14	0	0	0.04	0.04	0	0.04	0	0.03	0
	GN	0	0	0	0	0	0	0.02	0.02	0	0.01	0	0.01	0
	FB	0	0	0	0.06	0.27	0.2	0.14	0.16	0.33	0.18	0.15	0.2	0.29
	ASM	0	0	0	0.14	0	0	0.03	0.04	0	0.03	0	0.03	0.04
	SSM	0	0	0	0.22	0	0.35	0.07	0.09	0	0.13	0	0.16	0.14
	RPV	0.1	0.1	0.1	0.02	0.09	0.09	0.13	0.12	0.09	0.1	0	0.16	0.06

(Continued)

(Continued)

		APWPCH												
		VER	ADP	MR	SPD	RAP	RES	AOA	MOB	RTR	DIS	ROA	FPD	SDE
	AB	0	0	0	0.02	0	0	0.03	0.02	0	0.02	0	0.02	0.12
APWPC	FL	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33
	SA	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24
	UB	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
	RN	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
	SH	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
	CS	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	REL	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
APWPCH	VER	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33
	ADP	0	0	0	0	0	0	0	0	0	0	0	0	0
	MR	0	0	0	0	0	0	0	0	0	0	0	0	0
	SPD	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24

(Continued)

(*Continued*)

		APWPCH												
		VER	**ADP**	**MR**	**SPD**	**RAP**	**RES**	**AOA**	**MOB**	**RTR**	**DIS**	**ROA**	**FPD**	**SDE**
	RAP	0	0	0	0	0	0	0	0	0	0	0	0	0
	RES	0	0	0	0	0	0	0	0	0	0	0	0	0
	AOA	0	0	0	0	0	0	0	0	0	0	0	0	0
	MOB	0	0	0	0	0	0	0	0	0	0	0	0	0
	RTR	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
	DIS	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
	ROA	0	0	0	0	0	0	0	0	0	0	0	0	0
	FPD	0	0	0	0	0	0	0	0	0	0	0	0	0
	SDE	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
	NSE	0	0	0	0	0	0	0	0	0	0	0	0	0
	FPC	0	0	0	0	0	0	0	0	0	0	0	0	0
	AC	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	OC	0	0	0	0	0	0	0	0	0	0	0	0	0

(*Continued*)

(*Continued*)

APWPCH														
		VER	**ADP**	**MR**	**SPD**	**RAP**	**RES**	**AOA**	**MOB**	**RTR**	**DIS**	**ROA**	**FPD**	**SDE**
	MC	0	0	0	0	0	0	0	0	0	0	0	0	0
	MNR	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
	MNL	0	0	0	0	0	0	0	0	0	0	0	0	0
	AVL	0	0	0	0	0	0	0	0	0	0	0	0	0
ALT	W1	0.53	0.53	0.53	0.48	0.48	0.48	0.43	0.64	0.64	0.43	0.48	0.48	0.33
	W2	0.33	0.33	0.33	0.35	0.35	0.35	0.43	0.26	0.26	0.43	0.35	0.35	0.33
	W3	0.14	0.14	0.14	0.17	0.17	0.17	0.14	0.1	0.1	0.14	0.17	0.17	0.33

		APWPCH								ALT		
		NSE	**FPC**	**AC**	**OC**	**MC**	**MNR**	**MNL**	**AVL**	**W1**	**W2**	**W3**
APWC	MF	0.29	0.32	0.04	0.04	0.04	0.18	0.04	0.23	0.4	0.55	0.45
	FI	0	0.19	0.05	0.05	0.05	0.1	0.05	0.13	0	0.3	0.00
	AH	0.06	0.05	0.23	0.23	0.23	0.09	0.23	0.07	0	0.15	0.30
	RK	0	0	0	0	0	0.03	0	0.03	0	0	0.00
	AAM	0	0	0	0	0	0.04	0	0.03	0	0	0.00
	SAM	0	0.08	0.32	0.32	0.32	0.05	0.32	0.08	0	0	0.00
	GN	0	0	0	0	0	0.02	0	0.02	0	0	0.00
	FB	0.29	0.19	0.05	0.05	0.05	0.17	0.05	0.16	0.3	0	0.00
	ASM	0.04	0	0	0	0	0.04	0	0.03	0	0	0.00
	SSM	0.14	0.06	0.11	0.11	0.11	0.12	0.11	0.09	0	0	0.00
	RPV	0.06	0.1	0.2	0.2	0.2	0.13	0.2	0.11	0.3	0	0.00
	AB	0.12	0	0	0	0	0.03	0	0.03	0	0	0.25
APWPC	FL	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.1	0.2	0.20

(*Continued*)

(*Continued*)

		APWPCH								ALT		
		NSE	**FPC**	**AC**	**OC**	**MC**	**MNR**	**MNL**	**AVL**	**W1**	**W2**	**W3**
	SA	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0	0	0.00
	UB	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0	0	0.00
	RN	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0	0	0.00
	SH	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0	0	0.00
	CS	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.4	0.5	0.60
	REL	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.5	0.3	0.20
APWPCH	VER	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.1	0.2	0.30
	ADP	0	0	0	0	0	0	0	0	0	0	0.00
	MR	0	0	0	0	0	0	0	0	0	0	0.00
	SPD	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0	0	0.00
	RAP	0	0	0	0	0	0	0	0	0	0	0.00

(*Continued*)

(Continued)

		APWPCH								ALT		
		NSE	**FPC**	**AC**	**OC**	**MC**	**MNR**	**MNL**	**AVL**	**W1**	**W2**	**W3**
	RES	0	0	0	0	0	0	0	0	0	0.15	0.00
	AOA	0	0	0	0	0	0	0	0	0	0	0.00
	MOB	0	0	0	0	0	0	0	0	0	0	0.00
	RTR	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0	0	0.00
	DIS	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0	0	0.00
	ROA	0	0	0	0	0	0	0	0	0	0	0.00
	FPD	0	0	0	0	0	0	0	0	0	0	0.00
	SDE	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0	0	0.00
	NSE	0	0	0	0	0	0	0	0	0	0.1	0.00
	FPC	0	0	0	0	0	0	0	0	0	0	0.00
	AC	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0	0	0.50
	OC	0	0	0	0	0	0	0	0	0	0	0.00

(Continued)

(Continued)

		APWPCH								ALT		
		NSE	**FPC**	**AC**	**OC**	**MC**	**MNR**	**MNL**	**AVL**	**W1**	**W2**	**W3**
	MC	0	0	0	0	0	0	0	0	0.1	0.05	0.00
	MNR	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.2	0.1	0.00
	MNL	0	0	0	0	0	0	0	0	0.3	0.2	0.00
	AVL	0	0	0	0	0	0	0	0	0.3	0.2	0.20
ALT	W1	0.33	0.53	0.1	0.53	0.64	0.64	0.64	0.64	0.64	0.64	0.64
	W2	0.33	0.33	0.26	0.33	0.26	0.26	0.26	0.26	0.26	0.26	0.26
	W3	0.33	0.14	0.64	0.14	0.1	0.1	0.1	0.1	0.1	0.1	0.10

Weighted Super-Matrix (Part 1)

				APWC									
		MF	**FI**	**AH**	**RK**	**AAM**	**SAM**	**GN**	**FB**	**ASM**	**SSM**	**RPV**	**AB**
APWC	MF	0	0.14	0	0.14	0.16	0.03	0.11	0	0.26	0	0.03	0.07
	FI	0	0	0	0.12	0.1	0	0.11	0	0	0.16	0.05	0
	AH	0	0	0	0	0	0.02	0.04	0	0	0	0	0
	RK	0.09	0	0.07	0	0	0	0	0	0	0	0	0
	AAM	0.06	0	0.08	0	0	0	0	0	0	0	0	0
	SAM	0.02	0	0	0	0	0	0	0.1	0	0.1	0	0
	GN	0	0.07	0.02	0	0	0	0	0	0	0	0	0
	FB	0.01	0	0	0	0	0.1	0	0	0	0	0.02	0.17
	ASM	0.03	0	0.04	0	0	0	0	0	0	0	0.08	0
	SSM	0.01	0	0	0	0	0.07	0	0	0	0	0.02	0
	RPV	0	0.03	0	0	0	0.04	0	0	0	0	0	0.03
	AB	0.04	0.02	0.05	0	0	0	0	0.16	0	0	0.07	0

(*Continued*)

(Continued)

				APWC									
		MF	**FI**	**AH**	**RK**	**AAM**	**SAM**	**GN**	**FB**	**ASM**	**SSM**	**RPV**	**AB**
APWPC	FL	0.05	0.01	0.05	0	0	0	0	0	0	0	0.04	0
	SA	0.03	0.05	0.03	0.03	0.03	0.03	0	0.02	0.03	0.03	0.03	0
	UB	0.02	0.03	0.02	0.01	0.01	0	0	0.02	0	0	0.03	0
	RN	0.02	0.02	0.02	0.03	0.03	0.05	0.09	0.04	0.05	0.05	0.02	0.01
	SH	0.01	0.01	0.01	0.04	0.04	0.04	0.06	0.05	0.04	0.04	0.01	0.07
	CS	0.01	0.01	0.01	0.02	0.02	0.01	0	0.01	0.01	0.01	0.01	0.03
	REL	0.01	0.02	0.01	0.02	0.02	0	0	0.01	0	0	0.01	0.04
APWPCH	VER	0.02	0	0.02	0	0	0	0	0	0	0	0.02	0
	ADP	0	0	0	0	0	0	0	0	0	0	0	0
	MR	0	0	0	0	0	0	0	0	0	0	0	0
	SPD	0.01	0.02	0.01	0.03	0.03	0.02	0.03	0.02	0.03	0.02	0.01	0
	RAP	0.01	0.01	0.01	0	0	0	0	0.01	0	0	0.01	0

(Continued)

(*Continued*)

				APWC									
		MF	**FI**	**AH**	**RK**	**AAM**	**SAM**	**GN**	**FB**	**ASM**	**SSM**	**RPV**	**AB**
	RES	0	0	0	0	0	0	0	0	0	0	0	0
	AOA	0	0	0	0	0	0	0	0	0	0	0	0
	MOB	0.01	0.01	0.01	0.02	0.02	0.01	0	0	0.02	0.01	0.01	0.02
	RTR	0	0	0	0	0	0	0	0	0	0	0	0
	DIS	0	0	0	0	0	0	0	0	0	0	0	0
	ROA	0	0	0	0	0	0	0	0	0	0	0	0
	FPD	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.01	0.01	0.01	0.01
	SDE	0	0	0	0	0	0	0	0	0	0	0	0.03
	NSE	0	0	0	0	0	0	0	0	0	0	0	0
	FPC	0	0.01	0	0.01	0.01	0	0.01	0.01	0.01	0	0.01	0.01
	AC	0	0	0	0	0	0.01	0	0	0	0.01	0	0

(*Continued*)

(Continued)

				APWC									
		MF	**FI**	**AH**	**RK**	**AAM**	**SAM**	**GN**	**FB**	**ASM**	**SSM**	**RPV**	**AB**
	OC	0	0	0	0	0	0	0	0	0	0	0	0
	MC	0	0	0	0	0	0	0	0	0	0	0	0
	MNR	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MNL	0	0	0	0	0	0	0	0	0	0	0	0
	AVL	0	0	0	0	0	0.01	0.01	0	0	0.01	0	0
ALT	W1	0.33	0.3	0.27	0.3	0.33	0.27	0.24	0.33	0.27	0.24	0.34	0.26
	W2	0.13	0.16	0.17	0.16	0.13	0.17	0.18	0.13	0.17	0.18	0.12	0.13
	W3	0.05	0.06	0.07	0.06	0.05	0.07	0.09	0.05	0.07	0.09	0.05	0.13

			APWPC				
		FL	**SA**	**UB**	**RN**	**SH**	**CS**
APWC	MF	0.15	0.01	0.04	0.05	0.01	0.01
	FI	0	0.01	0.04	0.03	0.01	0.01
	AH	0.06	0	0.05	0.02	0.01	0.06
	RK	0	0.05	0.01	0.01	0.02	0
	AAM	0	0.06	0.01	0.01	0.02	0
	SAM	0	0.04	0.01	0.01	0.02	0.08
	GN	0	0	0.01	0.01	0	0
	FB	0	0.01	0.04	0.05	0.02	0.01
	ASM	0	0.05	0.01	0.01	0.02	0
	SSM	0	0.03	0.02	0.04	0.08	0.03
	RPV	0.04	0.01	0.03	0.04	0	0.05
	AB	0	0	0.01	0	0.06	0
APWPC	FL	0.06	0.06	0.06	0.06	0.06	0.06
	SA	0.04	0.04	0.04	0.04	0.04	0.04

(Continued)

(*Continued*)

			APWPC				
		FL	**SA**	**UB**	**RN**	**SH**	**CS**
	UB	0.03	0.03	0.03	0.03	0.03	0.03
	RN	0.02	0.02	0.02	0.02	0.02	0.02
	SH	0.01	0.01	0.01	0.01	0.01	0.01
	CS	0.01	0.01	0.01	0.01	0.01	0.01
	REL	0.01	0.01	0.01	0.01	0.01	0.01
APWPCH	VER	0.03	0.03	0.03	0.03	0.03	0.03
	ADP	0	0	0	0	0	0
	MR	0	0	0	0	0	0
	SPD	0.02	0.02	0.02	0.02	0.02	0.02
	RAP	0	0	0	0	0	0
	RES	0	0	0	0	0	0
	AOA	0	0	0	0	0	0
	MOB	0	0	0	0	0	0
	RTR	0.01	0.01	0.01	0.01	0.01	0.01

(*Continued*)

(*Continued*)

			APWPC				
		FL	**SA**	**UB**	**RN**	**SH**	**CS**
	DIS	0.01	0.01	0.01	0.01	0.01	0.01
	ROA	0	0	0	0	0	0
	FPD	0	0	0	0	0	0
	SDE	0	0	0	0	0	0
	NSE	0	0	0	0	0	0
	FPC	0	0	0	0	0	0
	AC	0	0	0	0	0	0
	OC	0	0	0	0	0	0
	MC	0	0	0	0	0	0
	MNR	0.01	0.01	0.01	0.01	0.01	0.01
	MNL	0	0	0	0	0	0
	AVL	0	0	0	0	0	0
ALT	W1	0.25	0.23	0.21	0.21	0.16	0.31
	W2	0.16	0.17	0.21	0.21	0.16	0.12
	W3	0.07	0.08	0.07	0.07	0.16	0.05

Weighted Super-Matrix (Part 2)

		APWPCH												
		VER	**ADP**	**MR**	**SPD**	**RAP**	**RES**	**AOA**	**MOB**	**RTR**	**DIS**	**ROA**	**FPD**	**SDE**
APWC	MF	0.06	0.06	0.06	0.01	0.03	0.02	0.01	0.01	0.03	0.02	0.04	0.02	0.03
	FI	0	0	0	0.01	0.02	0.01	0.01	0.01	0.02	0.01	0.04	0.01	0
	AH	0.02	0.02	0.02	0	0.01	0.01	0.02	0.01	0.01	0.01	0.01	0	0.01
	RK	0	0	0	0.01	0	0	0	0	0	0	0	0	0
	AAM	0	0	0	0.01	0	0	0	0	0	0	0	0	0
	SAM	0	0	0	0.01	0	0	0	0	0	0	0	0	0
	GN	0	0	0	0	0	0	0	0	0	0	0	0	0
	FB	0	0	0	0.01	0.02	0.02	0.01	0.01	0.03	0.02	0.01	0.02	0.03
	ASM	0	0	0	0.01	0	0	0	0	0	0	0	0	0
	SSM	0	0	0	0.02	0	0.03	0.01	0.01	0	0.01	0	0.01	0.01
	RPV	0.01	0.01	0.01	0	0.01	0.01	0.01	0.01	0.01	0.01	0	0.01	0.01
	AB	0	0	0	0	0	0	0	0	0	0	0	0	0.01

(Continued)

(*Continued*)

		APWPCH												
		VER	**ADP**	**MR**	**SPD**	**RAP**	**RES**	**AOA**	**MOB**	**RTR**	**DIS**	**ROA**	**FPD**	**SDE**
APWPC	FL	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
	SA	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
	UB	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
	RN	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
	SH	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	CS	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	REL	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
APWPCH	VER	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
	ADP	0	0	0	0	0	0	0	0	0	0	0	0	0
	MR	0	0	0	0	0	0	0	0	0	0	0	0	0
	SPD	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07

(*Continued*)

(Continued)

		APWPCH												
		VER	**ADP**	**MR**	**SPD**	**RAP**	**RES**	**AOA**	**MOB**	**RTR**	**DIS**	**ROA**	**FPD**	**SDE**
	RAP	0	0	0	0	0	0	0	0	0	0	0	0	0
	RES	0	0	0	0	0	0	0	0	0	0	0	0	0
	AOA	0	0	0	0	0	0	0	0	0	0	0	0	0
	MOB	0	0	0	0	0	0	0	0	0	0	0	0	0
	RTR	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
	DIS	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	ROA	0	0	0	0	0	0	0	0	0	0	0	0	0
	FPD	0	0	0	0	0	0	0	0	0	0	0	0	0
	SDE	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	NSE	0	0	0	0	0	0	0	0	0	0	0	0	0
	FPC	0	0	0	0	0	0	0	0	0	0	0	0	0
	AC	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01

(Continued)

(*Continued*)

		APWPCH												
		VER	**ADP**	**MR**	**SPD**	**RAP**	**RES**	**AOA**	**MOB**	**RTR**	**DIS**	**ROA**	**FPD**	**SDE**
	OC	0	0	0	0	0	0	0	0	0	0	0	0	0
	MC	0	0	0	0	0	0	0	0	0	0	0	0	0
	MNR	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	MNL	0	0	0	0	0	0	0	0	0	0	0	0	0
	AVL	0	0	0	0	0	0	0	0	0	0	0	0	0
ALT	W1	0.08	0.08	0.08	0.08	0.08	0.08	0.07	0.1	0.1	0.07	0.08	0.08	0.05
	W2	0.05	0.05	0.05	0.06	0.06	0.06	0.07	0.04	0.04	0.07	0.06	0.06	0.05
	W3	0.02	0.02	0.02	0.03	0.03	0.03	0.02	0.02	0.02	0.02	0.03	0.03	0.05

		APWPCH								ALT		
		NSE	**FPC**	**AC**	**OC**	**MC**	**MNR**	**MNL**	**AVL**	**W1**	**W2**	**W3**
APWC	MF	0.03	0.03	0	0	0	0.02	0	0.02	0.1	0.14	0.11
	FI	0	0.02	0	0	0	0.01	0	0.01	0	0.08	0
	AH	0.01	0	0.02	0.02	0.02	0.01	0.02	0.01	0	0.04	0.08
	RK	0	0	0	0	0	0	0	0	0	0	0
	AAM	0	0	0	0	0	0	0	0	0	0	0
	SAM	0	0.01	0.03	0.03	0.03	0	0.03	0.01	0	0	0
	GN	0	0	0	0	0	0	0	0	0	0	0
	FB	0.03	0.02	0	0	0	0.02	0	0.01	0.08	0	0
	ASM	0	0	0	0	0	0	0	0	0	0	0
	SSM	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0	0	0
	RPV	0.01	0.01	0.02	0.02	0.02	0.01	0.02	0.01	0.08	0	0
	AB	0.01	0	0	0	0	0	0	0	0	0	0.06

(Continued)

(*Continued*)

		APWPCH								**ALT**		
		NSE	**FPC**	**AC**	**OC**	**MC**	**MNR**	**MNL**	**AVL**	**W1**	**W2**	**W3**
APWPC	FL	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.01	0.03	0.03
	SA	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0	0	0
	UB	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0	0	0
	RN	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0	0	0
	SH	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0	0	0
	CS	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.05	0.07	0.08
	REL	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.07	0.04	0.03
APWPCH	VER	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.01	0.01	0.02
	ADP	0	0	0	0	0	0	0	0	0	0	0
	MR	0	0	0	0	0	0	0	0	0	0	0

(*Continued*)

(*Continued*)

		APWPCH								ALT		
		NSE	**FPC**	**AC**	**OC**	**MC**	**MNR**	**MNL**	**AVL**	**W1**	**W2**	**W3**
	SPD	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0	0	0
	RAP	0	0	0	0	0	0	0	0	0	0	0
	RES	0	0	0	0	0	0	0	0	0	0.01	0
	AOA	0	0	0	0	0	0	0	0	0	0	0
	MOB	0	0	0	0	0	0	0	0	0	0	0
	RTR	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0	0	0
	DIS	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0	0	0
	ROA	0	0	0	0	0	0	0	0	0	0	0
	FPD	0	0	0	0	0	0	0	0	0	0	0
	SDE	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0	0	0
	NSE	0	0	0	0	0	0	0	0	0	0.01	0
	FPC	0	0	0	0	0	0	0	0	0	0	0

(*Continued*)

(*Continued*)

		APWPCH								ALT		
		NSE	**FPC**	**AC**	**OC**	**MC**	**MNR**	**MNL**	**AVL**	**W1**	**W2**	**W3**
	AC	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0	0	0.04
	OC	0	0	0	0	0	0	0	0	0	0	0
	MC	0	0	0	0	0	0	0	0	0.01	0	0
	MNR	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.01	0
	MNL	0	0	0	0	0	0	0	0	0.02	0.01	0
	AVL	0	0	0	0	0	0	0	0	0.02	0.01	0.01
ALT	W1	0.05	0.08	0.02	0.08	0.1	0.1	0.1	0.1	0.35	0.35	0.35
	W2	0.05	0.05	0.04	0.05	0.04	0.04	0.04	0.04	0.14	0.14	0.14
	W3	0.05	0.02	0.1	0.02	0.02	0.02	0.02	0.02	0.06	0.06	0.06

Limit Super-Matrix (Part 1)

				APWC									
		MF	**FI**	**AH**	**RK**	**AAM**	**SAM**	**GN**	**FB**	**ASM**	**SSM**	**RPV**	**AB**
APWC	MF	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
	FI	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	AH	0.02	0.02	0.02	0.02	0.02	0.01	0.02	0.01	0.01	0.01	0.02	0.02
	RK	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	AAM	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	SAM	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	GN	0	0	0	0	0	0	0	0	0	0	0	0
	FB	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	ASM	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	SSM	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	RPV	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	AB	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01

(Continued)

(*Continued*)

				APWC									
		MF	**FI**	**AH**	**RK**	**AAM**	**SAM**	**GN**	**FB**	**ASM**	**SSM**	**RPV**	**AB**
APWPC	FL	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
	SA	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	UB	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	RN	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	SH	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	CS	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	REL	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
APWPCH	VER	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	ADP	0	0	0	0	0	0	0	0	0	0	0	0
	MR	0	0	0	0	0	0	0	0	0	0	0	0
	SPD	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01

(*Continued*)

(Continued)

				APWC									
		MF	**FI**	**AH**	**RK**	**AAM**	**SAM**	**GN**	**FB**	**ASM**	**SSM**	**RPV**	**AB**
	RAP	0	0	0	0	0	0	0	0	0	0	0	0
	RES	0	0	0	0	0	0	0	0	0	0	0	0
	AOA	0	0	0	0	0	0	0	0	0	0	0	0
	MOB	0	0	0	0	0	0	0	0	0	0	0	0
	RTR	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	DIS	0	0	0	0	0	0	0	0	0	0	0	0
	ROA	0	0	0	0	0	0	0	0	0	0	0	0
	FPD	0	0	0	0	0	0	0	0	0	0	0	0
	SDE	0	0	0	0	0	0	0	0	0	0	0	0
	NSE	0	0	0	0	0	0	0	0	0	0	0	0
	FPC	0	0	0	0	0	0	0	0	0	0	0	0

(Continued)

(*Continued*)

				APWC									
		MF	**FI**	**AH**	**RK**	**AAM**	**SAM**	**GN**	**FB**	**ASM**	**SSM**	**RPV**	**AB**
	AC	0	0	0	0	0	0	0	0	0	0	0	0
	OC	0	0	0	0	0	0	0	0	0	0	0	0
	MC	0	0	0	0	0	0	0	0	0	0	0	0
	MNR	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MNL	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
ALT	AVL	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	W1	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27
	W2	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12
	W3	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05

		APWPC				
		FL	**SA**	**UB**	**RN**	**SH**
APWC	MF	0.07	0.07	0.07	0.07	0.07
	FI	0.02	0.02	0.02	0.02	0.02
	AH	0.02	0.02	0.02	0.02	0.01
	RK	0.01	0.01	0.01	0.01	0.01
	AAM	0.01	0.01	0.01	0.01	0.01
	SAM	0.01	0.01	0.01	0.01	0.01
	GN	0	0	0	0	0
	FB	0.03	0.03s	0.03	0.03	0.03
	ASM	0.01	0.01	0.01	0.01	0.01
	SSM	0.01	0.01	0.01	0.01	0.01
	RPV	0.03	0.03	0.03	0.03	0.03
	AB	0.01	0.01	0.01	0.01	0.01
APWPC	FL	0.04	0.04	0.04	0.04	0.04
	SA	0.02	0.02	0.02	0.02	0.02

(Continued)

(*Continued*)

		APWPC				
		FL	**SA**	**UB**	**RN**	**SH**
	UB	0.01	0.01	0.01	0.01	0.01
	RN	0.01	0.01	0.01	0.01	0.01
	SH	0.01	0.01	0.01	0.01	0.01
	CS	0.03	0.03	0.03	0.03	0.03
	REL	0.03	0.03	0.03	0.03	0.03
APWPCH	VER	0.02	0.02	0.02	0.02	0.02
	ADP	0	0	0	0	0
	MR	0	0	0	0	0
	SPD	0.01	0.01	0.01	0.01	0.01
	RAP	0	0	0	0	0
	RES	0	0	0	0	0
	AOA	0	0	0	0	0
	MOB	0	0	0	0	0
	RTR	0.01	0.01	0.01	0.01	0.01

(*Continued*)

(*Continued*)

		APWPC				
		FL	**SA**	**UB**	**RN**	**SH**
	DIS	0	0	0	0	0
	ROA	0	0	0	0	0
	FPD	0	0	0	0	0
	SDE	0	0	0	0	0
	NSE	0	0	0	0	0
	FPC	0	0	0	0	0
	AC	0	0	0	0	0
	OC	0	0	0	0	0
	MC	0	0	0	0	0
	MNR	0.01	0.01	0.01	0.01	0.01
	MNL	0.01	0.01	0.01	0.01	0.01
ALT	AVL	0.01	0.01	0.01	0.01	0.01
	W1	0.27	0.27	0.27	0.27	0.27
	W2	0.12	0.12	0.12	0.12	0.12
	W3	0.05	0.05	0.05	0.05	0.05

Limit Super-Matrix (Part 2)

		APWPCH													
		VER	**ADP**	**MR**	**SPD**	**RAP**	**RES**	**AOA**	**MOB**	**RTR**	**DIS**	**ROA**	**FPD**	**SDE**	**NSE**
APWC	MF	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
	FI	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	AH	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	RK	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	AAM	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	SAM	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	GN	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	FB	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	ASM	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	SSM	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	RPV	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	AB	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01

(*Continued*)

(*Continued*)

		APWPCH													
		VER	**ADP**	**MR**	**SPD**	**RAP**	**RES**	**AOA**	**MOB**	**RTR**	**DIS**	**ROA**	**FPD**	**SDE**	**NSE**
APWPC	FL	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
	SA	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	UB	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	RN	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	SH	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	CS	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	REL	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
APWPCH	VER	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	ADP	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	MR	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	SPD	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01

(*Continued*)

(Continued)

		APWPCH													
		VER	**ADP**	**MR**	**SPD**	**RAP**	**RES**	**AOA**	**MOB**	**RTR**	**DIS**	**ROA**	**FPD**	**SDE**	**NSE**
	RAP	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	RES	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	AOA	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	MOB	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	RTR	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	DIS	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	ROA	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	FPD	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	SDE	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	NSE	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	FPC	0	0	0	0	0	0	0	0	0	0	0	0	0	0

(Continued)

(*Continued*)

		APWPCH													
		VER	**ADP**	**MR**	**SPD**	**RAP**	**RES**	**AOA**	**MOB**	**RTR**	**DIS**	**ROA**	**FPD**	**SDE**	**NSE**
	AC	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	OC	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	MC	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	MNR	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MNL	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
ALT	AVL	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	W1	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27
	W2	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12
	W3	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05

		APWPCH							ALT		
		FPC	**AC**	**OC**	**MC**	**MNR**	**MNL**	**AVL**	**W1**	**W2**	**W3**
APWC	MF	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
	FI	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	AH	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	RK	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	AAM	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	SAM	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	GN	0	0	0	0	0	0	0	0	0	0
	FB	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	ASM	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	SSM	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	RPV	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	AB	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01

(*Continued*)

(*Continued*)

		APWPCH							ALT		
		FPC	**AC**	**OC**	**MC**	**MNR**	**MNL**	**AVL**	**W1**	**W2**	**W3**
APWPC	FL	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
	SA	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	UB	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	RN	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	SH	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	CS	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	REL	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
APWPCH	VER	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	ADP	0	0	0	0	0	0	0	0	0	0
	MR	0	0	0	0	0	0	0	0	0	0
	SPD	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01

(*Continued*)

(*Continued*)

		APWPCH							ALT		
		FPC	**AC**	**OC**	**MC**	**MNR**	**MNL**	**AVL**	**W1**	**W2**	**W3**
	RAP	0	0	0	0	0	0	0	0	0	0
	RES	0	0	0	0	0	0	0	0	0	0
	AOA	0	0	0	0	0	0	0	0	0	0
	MOB	0	0	0	0	0	0	0	0	0	0
	RTR	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	DIS	0	0	0	0	0	0	0	0	0	0
	ROA	0	0	0	0	0	0	0	0	0	0
	FPD	0	0	0	0	0	0	0	0	0	0
	SDE	0	0	0	0	0	0	0	0	0	0
	NSE	0	0	0	0	0	0	0	0	0	0
	FPC	0	0	0	0	0	0	0	0	0	0

(*Continued*)

(*Continued*)

		APWPCH							ALT		
		FPC	**AC**	**OC**	**MC**	**MNR**	**MNL**	**AVL**	**W1**	**W2**	**W3**
	AC	0	0	0	0	0	0	0	0	0	0
	OC	0	0	0	0	0	0	0	0	0	0
	MC	0	0	0	0	0	0	0	0	0	0
	MNR	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MNL	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
ALT	AVL	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	W1	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27
	W2	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12
	W3	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05

Appendix-C

Pairwise Comparisons and Estimation of Weights for AHP

Table C1 Comparison of APWC WRT DEFENCE.

	MF	FI	AH	RK	AAM	SAM	GN	FB	ASM	SSM	RPV	AB	nth root	WEIGHTS
MF	1	3	3	3	3	3	5	3	5	7	5	3	3.34	0.20
FI	0.33	1	3	3	3	3	5	3	5	7	5	5	2.90	0.18
AH	0.33	0.33	1	3	3	0.33	3	0.33	5	5	5	3	1.49	0.09
RK	0.33	0.33	0.33	1	0.33	0.33	3	0.33	3	5	3	3	0.95	0.06
AAM	0.33	0.33	0.33	3	1	0.33	3	0.33	5	5	3	3	1.19	0.07
SAM	0.33	0.33	3	3	3	1	3	3	7	7	7	7	2.51	0.15
GN	0.20	0.2	0.33	0.33	0.33	0.33	1	0.33	3	3	3	3	0.69	0.04
FB	0.33	0.33	3	3	3	0.33	3	1	5	5	5	3	1.79	0.11
ASM	0.2	0.2	0.2	0.33	0.2	0.14	0.33	0.2	1	3	3	0.33	0.40	0.02
SSM	0.14	0.14	0.2	0.2	0.2	0.14	0.33	0.2	0	1	0.33	0.2	0.24	0.01
RPV	0.2	0.2	0.2	0.33	0.33	0.14	0.33	0.2	0	3	1	0.5	0.35	0.02
AB	0.33	0.2	0.33	0.33	0.33	0.14	0.33	0.33	3	5	2	1	0.57	0.03
													16.41	

(*Continued*)

Table C1 Comparison of APWC WRT DEFENCE. (*Continued*)

	MF	FI	AH	RK	AAM	SAM	GN	FB	ASM	SSM	RPV	AB	nth root	WEIGHTS
CONSISTENCY INDEX (CI) COMPUTATION														
MF	2.83		λs	13.90										
FI	2.41			13.63		$CI = \frac{\lambda_{MEAN} - n}{n-1}$								
AH	1.22			13.48										
RK	0.76			13.22										
AAM	0.98			13.47										
SAM	2.06			13.44		CI = 0.13								
GN	0.56			13.27										
FB	1.48			13.55		CONSISTENCY RATIO								
ASM	0.33			13.65										
SSM	0.19			13.22		CR = 0.09								
RPV	0.28			13.16										
AB	0.47			13.59										
				λ MEAN	13.47									

Table C2 Comparison of APWC WRT ATTACK.

	MF	FI	AH	RK	AAM	SAM	GN	FB	ASM	SSM	RPV	AB	nth root	WEIGHTS
MF	1	3	3	5	5	5	5	0.33	3	3	3	3	2.70	0.16
FI	0.33	1	0.33	3	3	3	3	0.2	0	0.2	0.33	0.33	0.69	0.04
AH	0.33	3	1	3	3	3	3	0.2	3	0.2	3	2	1.40	0.08
RK	0.2	0.33	0.33	1	2	2	3	0.14	0	0.14	0.33	0.2	0.45	0.03
AAM	0.2	0.33	0.33	0.5	1	1	1	0.14	0	0.14	0.2	0.2	0.33	0.02
SAM	0.2	0.33	0.33	0.5	1	1	1	0.14	0	0.14	0.2	0.2	0.33	0.02
GN	0.20	0.33	0.33	0.33	1	1	1	0.14	0	0.14	0.2	0.2	0.32	0.02
FB	3	5	5	7	7	7	7	1	3	3	5	3	4.13	0.24
ASM	0.33	3	0.33	5	5	5	5	0.33	1	0.33	3	0.33	1.29	0.08
SSM	0.33	5	5	7	7	7	7	0.33	3	1	5	3	2.86	0.17
RPV	0.33	3	0.33	3	5	5	5	0.2	0	0.2	1	0.33	0.95	0.06
AB	0.33	3	0.5	5	5	5	5	0.33	3	0.33	3	1	1.61	0.09
													17.06	

(*Continued*)

Table C2 Comparison of APWC WRT ATTACK. (*Continued*)

	MF	FI	AH	RK	AAM	SAM	GN	FB	ASM	SSM	RPV	AB	nth root	WEIGHTS
CONSISTENCY INDEX (CI) COMPUTATION														
MF	2.21		λs	13.93										
FI	0.53			12.96										
AH	1.17			14.27										
RK	0.34			12.98										
AAM	0.25			12.66										
SAM	0.25			12.66		CI = 0.12								
GN	0.24			12.86										
FB	3.21			13.27		CONSISTENCY RATIO								
ASM	1.03			13.58										
SSM	2.29			13.68		CR = 0.08								
RPV	0.76			13.69										
AB	1.26			13.34										
				λ MEAN	13.32									

Table C3 Composite impact of AP operations on APWC.

	0.5	**0.5**	
	DEFENCE	**ATTACK**	**COMPOSITE WEIGHTS**
MF	0.20	0.16	0.18
FI	0.18	0.04	0.11
AH	0.09	0.08	0.09
RK	0.06	0.03	0.04
AAM	0.07	0.02	0.05
SAM	0.15	0.02	0.09
GN	0.04	0.02	0.03
FB	0.11	0.24	0.18
ASM	0.02	0.08	0.05
SSM	0.01	0.17	0.09
RPV	0.02	0.06	0.04
AB	0.03	0.09	0.06
			1.00

Table C4 Comparison of APWPC WRT MF.

	FL	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**	**nth root**	**WEIGHTS**
FL	1	3	1	3	2	5	3	2.23	0.27
SA	0.33	1	3	3	2	5	3	1.90	0.23
UB	1.00	0.33	1	2	2	3	2	1.34	0.16
RN	0.33	0.33	0.5	1	0.33	3	0.33	0.56	0.07
SH	0.5	0.5	0.5	3	1	3	3	1.19	0.14
CS	0.2	0.2	0.33	0.33	0.33	1	0.33	0.34	0.04
REL	0.33	0.33	0.5	3	0.33	3	1	0.77	0.09
								8.33	

(*Continued*)

Table C4 Comparison of APWPC WRT MF. (*Continued*)

	FL	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**	**nth root**	**WEIGHTS**
CONSISTENCY INDEX (CI) COMPUTATION									
FL	2.08		**λ**s	7.78					
SA	1.77			7.75					
UB	1.23			7.62					
RN	0.51			7.55					
SH	1.07			7.50		CI = 0.10			
CS	0.29			7.27					
REL	0.71			7.64		CONSISTENCY RATIO			
							CR = 0.0745		
			λ MEAN	7.59					

Table C5 Comparison of APWPC WRT FI.

	FL	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**	**nth root**	**WEIGHTS**
FL	1	0.33	0.33	0.33	3	0.33	0.33	0.53	0.06
SA	3	1	3	3	5	5	3	2.97	0.34
UB	3.00	0.33	1	3	5	3	2	1.90	0.21
RN	3	0.33	0.33	1	3	2	0.33	0.94	0.11
SH	0.33	0.2	0.2	0.33	1	0.33	0.33	0.34	0.04
CS	3	0.2	0.33	0.5	3	1	0.33	0.72	0.08
REL	3	0.33	0.5	3	3	3	1	1.45	0.16
								8.84	

(*Continued*)

Table C5 Comparison of APWPC WRT FI. (*Continued*)

	FL	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**	**nth root**	**WEIGHTS**
CONSISTENCY INDEX (CI) COMPUTATION									
FL	0.47		**λ**s	7.86					
SA	2.57			7.64					
UB	1.59			7.38					
RN	0.80			7.51					
SH	0.28			7.48		CI = 0.10			
CS	0.62			7.64					
REL	1.24			7.56		CONSISTENCY RATIO			
			λ MEAN	7.58		CR = 0.07			

Table C6 Comparison of APWPC WRT AH.

	FL	SA	UB	RN	SH	CS	REL	nth root	WEIGHTS
FL	1	3	2	2	5	5	3	2.64	0.30
SA	0.33	1	3	3	5	5	3	2.16	0.25
UB	0.50	0.33	1	2	3	3	3	1.37	0.16
RN	0.5	0.33	0.5	1	3	3	0.33	0.82	0.09
SH	0.2	0.2	0.33	0.33	1	0.33	0.33	0.34	0.04
CS	0.2	0.2	0.33	0.33	3	1	0.33	0.46	0.05
REL	0.33	0.33	0.33	3	3	3	1	1.00	0.11
								8.78	

(Continued)

Table C6 Comparison of APWPC WRT AH. (*Continued*)

	FL	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**	**nth root**	**WEIGHTS**
CONSISTENCY INDEX (CI) COMPUTATION									
FL	2.33		**λ**s	7.74					
SA	1.88			7.65					
UB	1.19			7.62					
RN	0.71			7.64					
SH	0.28			7.45		CI = 0.11			
CS	0.40			7.57					
REL	0.90			7.90		CONSISTENCY RATIO			
			λ MEAN	7.65		CR = 0.08			

Table C7 Comparison of APWPC WRT RK.

	FL	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**	**nth root**	**WEIGHTS**
FL	1	0.2	0.33	0.2	0.2	0.33	0.33	0.31	0.03
SA	5	1	3	3	3	5	3	2.97	0.33
UB	3.00	0.33	1	0.33	0.2	0.33	0.33	0.49	0.05
RN	5	0.33	3	1	2	3	2	1.79	0.20
SH	5	0.33	5	0.5	1	5	3	1.80	0.20
CS	3	0.2	3	0.33	0.2	1	0.33	0.63	0.07
REL	3	0.33	3	0.5	0.33	3	1	1.06	0.12
								9.05	

(*Continued*)

Table C7 Comparison of APWPC WRT RK. (*Continued*)

	FL	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**	**nth root**	**WEIGHTS**
CONSISTENCY INDEX (CI) COMPUTATION									
FL	0.26		**λs**	7.51					
SA	2.55			7.79					
UB	0.43			7.94		CI = 0.11			
RN	1.48			7.49					
SH	1.55			7.78		CONSISTENCY RATIO			
CS	0.55			7.85					
REL	0.87			7.41		CR = 0.09			
			λ MEAN	7.68					

Table C8 Comparison of APWPC WRT AAM.

	FL	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**	**nth root**	**WEIGHTS**
FL	1	0.2	0.33	0.2	0.2	0.33	0.33	0.31	0.03
SA	5	1	3	3	3	5	3	2.97	0.33
UB	3.00	0.33	1	0.33	0.2	0.33	0.33	0.49	0.05
RN	5	0.33	3	1	2	3	2	1.79	0.20
SH	5	0.33	5	0.5	1	5	3	1.80	0.20
CS	3	0.2	3	0.33	0.2	1	0.33	0.63	0.07
REL	3	0.33	3	0.5	0.33	3	1	1.06	0.12
								9.05	

(*Continued*)

Table C8 Comparison of APWPC WRT AAM. (*Continued*)

	FL	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**	**nth root**	**WEIGHTS**
CONSISTENCY INDEX (CI) COMPUTATION									
FL	0.26		**λ**s	7.51					
SA	2.55			7.79		CI = 0.11			
UB	0.43			7.94					
RN	1.48			7.49		CONSISTENCY RATIO			
SH	1.55			7.78					
CS	0.55			7.85		CR = 0.09			
REL	0.87			7.41					
			λ MEAN	7.68					

Table C9 Comparison of APWPC WRT SAM.

	FL	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**	**nth root**	**WEIGHTS**
FL	1	0.2	0.2	0.14	0.14	0.2	0.2	0.23	0.03
SA	5	1	5	2	2	3	3	2.64	0.29
UB	5.00	0.2	1	0.33	0.33	3	0.33	0.73	0.08
RN	7	0.5	3	1	2	3	2	2.00	0.22
SH	7	0.5	3	0.5	1	3	3	1.73	0.19
CS	5	0.33	0.33	0.33	0.33	1	0.33	0.57	0.06
REL	5	0.33	3	0.5	0.33	3	1	1.14	0.13
								9.03	

(*Continued*)

Table C9 Comparison of APWPC WRT SAM. (*Continued*)

	FL	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**	**nth root**	**WEIGHTS**
CONSISTENCY INDEX (CI) COMPUTATION									
FL	0.20		**λ**s	7.76					
SA	2.21			7.57		CI = 0.10			
UB	0.63			7.85					
RN	1.61			7.29		CONSISTENCY RATIO			
SH	1.43			7.46					
CS	0.49			7.76		CR = 0.08			
REL	0.95			7.57					
			λ MEAN	7.61					

Table C10 Comparison of APWPC WRT GN.

	FL	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**	**nth root**	**WEIGHTS**
FL	1	0.2	0.2	0.14	0.14	0.2	0.2	0.23	0.03
SA	5	1	2	2	2	3	3	2.32	0.26
UB	5.00	0.5	1	0.33	0.2	0.33	0.33	0.56	0.06
RN	7	0.5	3	1	2	3	3	2.11	0.24
SH	7	0.5	5	0.5	1	3	3	1.87	0.21
CS	5	0.33	3	0.33	0.33	1	0.33	0.78	0.09
REL	5	0.33	3	0.33	0.33	3	1	1.07	0.12
								8.94	

(*Continued*)

Table C10 Comparison of APWPC WRT GN. (*Continued*)

	FL	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**	**nth root**	**WEIGHTS**
CONSISTENCY INDEX (CI) COMPUTATION									
FL	0.19		λs	7.62					
SA	2.02			7.81		CI = 0.12			
UB	0.51			8.06					
RN	1.77			7.49		CONSISTENCY RATIO			
SH	1.57			7.53					
CS	0.68			7.73		CR = 0.09			
REL	0.93			7.77					
			λ MEAN	7.72					

Table C11 Comparison of APWPC WRT FB.

	FL	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**	**nth root**	**WEIGHTS**
FL	1	0.2	0.2	0.3	0.2	0.33	0.33	0.31	0.03
SA	5	1	3	3	3	5	3	2.97	0.33
UB	5.00	0.33	1	2	0.33	3	3	1.39	0.15
RN	3	0.33	0.5	1	2	5	3	1.47	0.16
SH	5	0.33	3	0.5	1	5	3	1.68	0.19
CS	3	0.2	0.33	0.2	0.2	1	0.33	0.43	0.05
REL	3	0.33	0.33	0.33	0.33	3	1	0.73	0.08
								8.96	

(Continued)

Table C11 Comparison of APWPC WRT FB. (*Continued*)

	FL	SA	UB	RN	SH	CS	REL	nth root	WEIGHTS
CONSISTENCY INDEX (CI) COMPUTATION									
FL	0.26		λs	7.59					
SA	2.50			7.56		CI= 0.12			
UB	1.21			7.84					
RN	1.31			7.98		CONSISTENCY RATIO			
SH	1.50			8.00					
CS	0.36			7.65		CR= 0.09			
REL	0.60			7.44					
			λ MEAN	7.72					

Table C12 Comparison of APWPC WRT ASM.

	FL	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**	**nth root**	**WEIGHTS**
FL	1	0.2	0.33	0.2	0.2	0.33	0.33	0.31	0.03
SA	5	1	3	3	3	5	3	2.97	0.33
UB	3.00	0.33	1	0.33	0.2	3	0.33	0.68	0.07
RN	5	0.33	3	1	2	3	2	1.79	0.20
SH	5	0.33	5	0.5	1	5	3	1.80	0.20
CS	3	0.2	0.33	0.33	0.2	1	0.33	0.46	0.05
REL	3	0.33	3	0.5	0.33	3	1	1.06	0.12
								9.07	

(Continued)

Table C12 Comparison of APWPC WRT ASM. (*Continued*)

	FL	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**	**nth root**	**WEIGHTS**
CONSISTENCY INDEX (CI) COMPUTATION									
FL	0.26		λs	7.53					
SA	2.52			7.69		CI = 0.11			
UB	0.58			7.79					
RN	1.48			7.51		CONSISTENCY RATIO			
SH	1.55			7.81					
CS	0.39			7.65		CR = 0.08			
REL	0.87			7.45					
			λ MEAN	7.63					

Table C13 Comparison of APWPC WRT SSM.

	FL	SA	UB	RN	SH	CS	REL	nth root	WEIGHTS
FL	1	0.33	0.33	0.14	0.14	0.33	0.2	0.28	0.03
SA	3	1	3	0.33	0.33	3	0.33	1.00	0.11
UB	3.00	0.33	1	0.33	0.33	3	0.33	0.73	0.08
RN	7	3	3	1	3	3	3	2.89	0.32
SH	7	3	3	0.33	1	5	3	2.27	0.25
CS	3	0.33	0.33	0.33	0.2	1	0.2	0.46	0.05
REL	5	3	3	0.33	0.33	5	1	1.58	0.17
								9.21	

(*Continued*)

Table C13 Comparison of APWPC WRT SSM. (*Continued*)

	FL	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**	**nth root**	**WEIGHTS**
CONSISTENCY INDEX (CI) COMPUTATION									
FL	0.23		**λ**s	7.24					
SA	0.84			7.66		CI = 0.11			
UB	0.61			7.58					
RN	2.53			7.93		CONSISTENCY RATIO			
SH	1.92			7.66					
CS	0.40			7.83		CR = 0.08			
REL	1.34			7.70					
			λ MEAN	7.66					

Table C14 Comparison of APWPC WRT RPV.

	FL	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**	**nth root**	**WEIGHTS**
FL	1	3	0.33	0.33	7	3	3	1.54	0.17
SA	0.33	1	0.33	0.33	5	3	2	1.01	0.11
UB	3.00	3	1	2	5	5	3	2.80	0.30
RN	3	3	0.5	1	7	5	3	2.41	0.26
SH	0.14	0.2	0.2	0.14	1	0.2	0.2	0.23	0.02
CS	0.33	0.33	0.2	0.2	5	1	0.33	0.49	0.05
REL	0.33	0.5	0.33	0.33	5	3	1	0.83	0.09
								9.31	

(*Continued*)

Table C14 Comparison of APWPC WRT RPV. (*Continued*)

	FL	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**	**nth root**	**WEIGHTS**
CONSISTENCY INDEX (CI) COMPUTATION									
FL	1.27		**λs**	7.69					
SA	0.81			7.44					
UB	2.29			7.63		CI = 0.10			
RN	1.93			7.47					
SH	0.19			7.95		CONSISTENCY RATIO			
CS	0.41			7.67					
REL	0.66			7.45		CR = 0.08			
			λ MEAN	7.62					

Table C15 Comparison of APWPC WRT AB.

	FL	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**	**nth root**	**WEIGHTS**
FL	1	2	0.5	1	0.14	0.2	0.2	0.48	0.04
SA	0.5	1	0.5	1	0.14	0.2	0.2	0.39	0.04
UB	2.00	2	1	1	0.14	0.2	0.2	0.58	0.05
RN	1	1	1	1	0.14	0.2	0.2	0.48	0.04
SH	7	7	7	7	1	5	5	4.82	0.45
CS	5	5	5	5	0.2	1	3	2.33	0.22
REL	5	5	5	5	0.2	0.33	1	1.70	0.16
								10.77	

(*Continued*)

Table C15 Comparison of APWPC WRT AB. (*Continued*)

	FL	**SA**	**UB**	**RN**	**SH**	**CS**	**REL**	**nth root**	**WEIGHTS**
CONSISTENCY INDEX (CI) COMPUTATION									
FL	0.33		λs	7.36					
SA	0.27			7.36		CI = 0.09			
UB	0.40			7.35					
RN	0.32			7.15		CONSISTENCY RATIO			
SH	3.57			7.99					
CS	1.67			7.73		CR = 0.07			
REL	1.21			7.68					
			λ MEAN	7.52					

Table C16 Composite impact of APWPC on APWC.

	0.19	0.12	0.09	0.05	0.05	0.1	0.03	0.16	0.04	0.08	0.04	0.06	
	MF	**FI**	**AH**	**RK**	**AAM**	**SAM**	**GN**	**FB**	**ASM**	**SSM**	**RPV**	**AB**	**COMPOSITE WEIGHTS**
FL	0.27	0.06	0.30	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.17	0.04	0.11
SA	0.23	0.34	0.25	0.33	0.33	0.29	0.26	0.33	0.33	0.11	0.11	0.04	0.26
UB	0.16	0.21	0.16	0.05	0.05	0.08	0.06	0.15	0.07	0.08	0.30	0.05	0.14
RN	0.07	0.11	0.09	0.20	0.20	0.22	0.24	0.16	0.20	0.32	0.26	0.04	0.16
SH	0.14	0.04	0.04	0.20	0.20	0.19	0.21	0.19	0.20	0.25	0.02	0.45	0.17
CS	0.04	0.08	0.05	0.07	0.07	0.06	0.09	0.05	0.05	0.05	0.05	0.22	0.07
REL	0.09	0.16	0.11	0.12	0.12	0.13	0.12	0.08	0.12	0.17	0.09	0.16	0.12
													1.01

Table C17 Comparison of APWPCH WRT FL.

	VER	ADP	MR	nth root	WEIGHTS	
VER	1	2	0.33	0.87	0.25	
ADP	0.5	1	0.33	0.55	0.16	
MR	3.00	3	1	2.08	0.59	CI = 0.02
				3.50		
						CONSISTENCY RATIO
CONSISTENCY INDEX (CI) COMPUTATION						
						CR = 0.0431
VER	0.76		λs	3.05		
ADP	0.48			3.05		
MR	1.81			3.05		
			λ MEAN	3.05		

Table C18 Comparison of APWPCH WRT SA.

	SPD	RAP	RES	nth root	WEIGHTS	
SPD	1	3	3	2.08	0.59	
RAP	0.33	1	2	0.87	0.25	CI = 0.02
RES	0.33	0.5	1	0.55	0.16	
				3.50		CONSISTENCY RATIO
						CR = 0.043103
CONSISTENCY INDEX (CI) COMPUTATION						
SPD	1.81		λs	3.05		
RAP	0.76			3.05		
RES	0.48			3.05		
			λ MEAN	3.05		

Table C19 Comparison of APWPCH WRT UB.

	AOA	**MOB**	**RTR**	**nth root**	**WEIGHTS**	
AOA	1	2	3	1.82	0.53	
MOB	0.5	1	3	1.14	0.33	
RTR	0.33	0.33	1	0.48	0.14	CI = 0.02
				3.44		
						CONSISTENCY RATIO
CONSISTENCY INDEX (CI) COMPUTATION						CR = 0.043103
AOA	1.61		**λ**s	3.05		
MOB	1.01			3.05		
RTR	0.42			3.05		
			λ MEAN	3.05		

Table C20 Comparison of APWPCH WRT RN.

	DIS	**ROA**	**FPD**	**nth root**	**WEIGHTS**	
DIS	1	2	0.33	0.87	0.25	
ROA	0.5	1	0.33	0.55	0.16	CI = 0.02
FPD	3.00	3	1	2.08	0.59	
				3.50		CONSISTENCY RATIO
CONSISTENCY INDEX (CI) COMPUTATION						CR = 0.043103
DIS	0.76		**λ**s	3.05		
ROA	0.48			3.05		
FPD	1.81			3.05		
			λ MEAN	3.05		

Table C21 Comparison of APWPCH WRT SH.

	SDE	NSE	FPC	nth root	WEIGHTS	
SDE	1	3	2	1.82	0.53	
NSE	0.33	1	0.33	0.48	0.14	CI = 0.02
FPC	0.50	3	1	1.14	0.33	
				3.44		CONSISTENCY RATIO
CONSISTENCY INDEX (CI) COMPUTATION						CR = 0.043103
SDE	1.61		λs	3.05		
NSE	0.42			3.05		
FPC	1.01			3.05		
			λ MEAN	3.05		

Table C22 Comparison of APWPCH WRT CS.

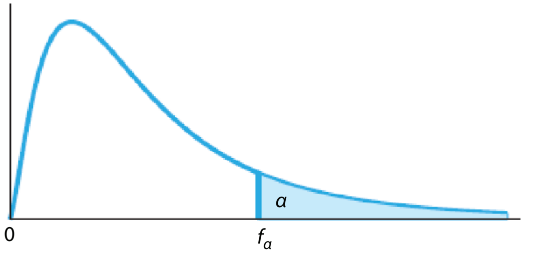
	AC	OC	MC	nth root	WEIGHTS	
AC	1	0.5	0.33	0.55	0.16	
OC	2	1	0.33	0.87	0.25	CI = 0.02
MC	3.00	3	1	2.08	0.59	
				3.50		CONSISTENCY RATIO
						CR = 0.0431
CONSISTENCY INDEX (CI) COMPUTATION						
AC	0.48		λs	3.05		
OC	0.76			3.05		
MC	1.81			3.05		
			λ MEAN	3.05		

Table C23 Comparison of APWPCH WRT REL.

	MNR	**MNL**	**AVL**	**nth root**	**WEIGHTS**	
MNR	1	3	3	2.08	0.59	
MNL	0.33	1	2	0.87	0.25	
AVL	0.33	0.5	1	0.55	0.16	CI = 0.02
				3.50		CONSISTENCY RATIO
						CR = 0.04310345
CONSISTENCY INDEX (CI) COMPUTATION						
MNR	1.81		**λ**s	3.05		
MNL	0.76			3.05		
AVL	0.48			3.05		
			λ MEAN	3.05		

Appendix D

F Distribution Table



F Distribution critical values for P=0.10

	d_1													
d_2	**1**	**2**	**3**	**4**	**5**	7	**10**	**15**	**20**	**30**	**60**	**120**	**500**	**1000**
1	39.864	49.500	53.593	55.833	57.240	58.906	60.195	61.220	61.740	62.265	62.794	63.061	63.264	63.296
2	8.5264	8.9999	9.1618	9.2434	9.2926	9.3491	9.3915	9.4248	9.4413	9.4580	9.4745	9.4829	9.4893	9.4902
3	5.5384	5.4624	5.3907	5.3426	5.3092	5.2661	5.2304	5.2003	5.1845	5.1681	5.1513	5.1425	5.1358	5.1347
4	4.5448	4.3245	4.1909	4.1073	4.0505	3.9790	3.9198	3.8704	3.8443	3.8175	3.7896	3.7753	3.7643	3.7625
5	4.0605	3.7798	3.6194	3.5202	3.4530	3.3679	3.2974	3.2379	3.2067	3.1740	3.1402	3.1228	3.1094	3.1071
7	3.5895	3.2575	3.0740	2.9605	2.8833	2.7850	2.7025	2.6322	2.5947	2.5555	2.5142	2.4927	2.4761	2.4735
10	3.2850	2.9244	2.7277	2.6054	2.5216	2.4139	2.3226	2.2434	2.2007	2.1554	2.1071	2.0818	2.0618	2.0587
15	3.0731	2.6951	2.4898	2.3615	2.2729	2.1582	2.0593	1.9722	1.9243	1.8727	1.8168	1.7867	1.7629	1.7590
20	2.9746	2.5893	2.3801	2.2490	2.1582	2.0397	1.9368	1.8450	1.7939	1.7383	1.6768	1.6432	1.6163	1.6118
30	2.8808	2.4887	2.2761	2.1423	2.0493	1.9269	1.8195	1.7222	1.6674	1.6064	1.5376	1.4990	1.4669	1.4617
60	2.7911	2.3932	2.1774	2.0409	1.9457	1.8194	1.7070	1.6034	1.5435	1.4756	1.3953	1.3476	1.3060	1.2989
120	2.7478	2.3473	2.1300	1.9924	1.8959	1.7675	1.6523	1.5450	1.4821	1.4094	1.3203	1.2646	1.2123	1.2026
500	2.7157	2.3132	2.0947	1.9561	1.8588	1.7288	1.6115	1.5009	1.4354	1.3583	1.2600	1.1937	1.1215	1.1057
1000	2.7106	2.3080	2.0892	1.9505	1.8530	1.7228	1.6051	1.4941	1.4281	1.3501	1.2500	1.1813	1.1031	1.0844

F Distribution critical values for P=0.05

	d_1													
d_2	**1**	**2**	**3**	**4**	**5**	**7**	**10**	**15**	**20**	**30**	**60**	**120**	**500**	**1000**
1	161.45	199.50	215.71	224.58	230.16	236.77	241.88	245.95	248.01	250.10	252.20	253.25	254.06	254.19
2	18.513	19.000	19.164	19.247	19.296	19.353	19.396	19.429	19.446	19.462	19.479	19.487	19.494	19.495
3	10.128	9.5522	9.2766	9.1172	9.0135	8.8867	8.7855	8.7028	8.6602	8.6165	8.5720	8.5493	8.5320	8.5292
4	7.7086	6.9443	6.5915	6.3882	6.2560	6.0942	5.9644	5.8579	5.8026	5.7458	5.6877	5.6580	5.6352	5.6317
5	6.6078	5.7862	5.4095	5.1922	5.0504	4.8759	4.7351	4.6187	4.5582	4.4958	4.4314	4.3985	4.3731	4.3691
7	5.5914	4.7375	4.3469	4.1202	3.9715	3.7871	3.6366	3.5108	3.4445	3.3758	3.3043	3.2675	3.2388	3.2344
10	4.9645	4.1028	3.7082	3.4780	3.3259	3.1354	2.9782	2.8450	2.7741	2.6996	2.6210	2.5801	2.5482	2.5430
15	4.5431	3.6823	3.2874	3.0556	2.9013	2.7066	2.5437	2.4035	2.3275	2.2467	2.1601	2.1141	2.0776	2.0718
20	4.3512	3.4928	3.0983	2.8660	2.7109	2.5140	2.3479	2.2032	2.1241	2.0391	1.9463	1.8962	1.8563	1.8498
30	4.1709	3.3159	2.9223	2.6896	2.5336	2.3343	2.1646	2.0149	1.9317	1.8408	1.7396	1.6835	1.6376	1.6300
60	4.0012	3.1505	2.7581	2.5252	2.3683	2.1666	1.9927	1.8365	1.7480	1.6492	1.5343	1.4672	1.4093	1.3994
120	3.9201	3.0718	2.6802	2.4473	2.2898	2.0868	1.9104	1.7505	1.6587	1.5544	1.4289	1.3519	1.2804	1.2674
500	3.8601	3.0137	2.6227	2.3898	2.2320	2.0278	1.8496	1.6864	1.5917	1.4820	1.3455	1.2552	1.1586	1.1378
1000	3.8508	3.0047	2.6137	2.3808	2.2230	2.0187	1.8402	1.6765	1.5811	1.4705	1.3318	1.2385	1.1342	1.1096

F Distribution critical values for P=0.02

	d_1													
d_2	**1**	**2**	**3**	**4**	**5**	**7**	**10**	**15**	**20**	**30**	**60**	**120**	**500**	**1000**
1	1012.5	1249.5	1350.5	1405.8	1440.6	1481.8	1513.7	1539.1	1551.9	1564.9	1578.0	1584.6	1589.6	1590.4
2	48.505	49.000	49.166	49.249	49.299	49.356	49.398	49.432	49.448	49.465	49.482	49.490	49.496	49.497
3	20.618	18.858	18.110	17.694	17.429	17.110	16.860	16.657	16.553	16.448	16.340	16.286	16.244	16.238
4	14.040	12.142	11.344	10.899	10.616	10.274	10.003	9.7828	9.6696	9.5540	9.4359	9.3760	9.3300	9.3227
5	11.323	9.4544	8.6702	8.2330	7.9530	7.6137	7.3438	7.1234	7.0094	6.8926	6.7728	6.7119	6.6649	6.6573
7	8.9877	7.2026	6.4539	6.0347	5.7647	5.4354	5.1711	4.9531	4.8392	4.7220	4.6007	4.5384	4.4902	4.4825
10	7.6384	5.9336	5.2182	4.8157	4.5549	4.2347	3.9750	3.7580	3.6437	3.5245	3.3999	3.3354	3.2850	3.2770
15	6.7730	5.1355	4.4475	4.0584	3.8052	3.4917	3.2345	3.0168	2.9003	2.7775	2.6468	2.5780	2.5237	2.5151
20	6.3907	4.7875	4.1134	3.7312	3.4817	3.1713	2.9149	2.6955	2.5771	2.4509	2.3148	2.2421	2.1841	2.1747
30	6.0382	4.4695	3.8093	3.4339	3.1877	2.8803	2.6239	2.4020	2.2805	2.1493	2.0047	1.9255	1.8611	1.8505
60	5.7127	4.1785	3.5319	3.1633	2.9207	2.6157	2.3586	2.1326	2.0067	1.8676	1.7085	1.6169	1.5383	1.5251
120	5.5594	4.0423	3.4026	3.0372	2.7963	2.4923	2.2347	2.0059	1.8769	1.7322	1.5613	1.4577	1.3629	1.3458
500	5.4467	3.9428	3.3083	2.9453	2.7057	2.4024	2.1441	1.9128	1.7809	1.6307	1.4468	1.3273	1.2019	1.1750
1000	5.4293	3.9274	3.2937	2.9311	2.6915	2.3884	2.1300	1.8983	1.7659	1.6146	1.4284	1.3053	1.1701	1.1388

F Distribution critical values for P=0.01

	d_1													
d_2	**1**	**2**	**3**	**4**	**5**	**7**	**10**	**15**	**20**	**30**	**60**	**120**	**500**	**1000**
1	4052.2	4999.5	5403.4	5624.6	5763.6	5928.4	6055.8	6157.3	6208.7	6260.6	6313.0	6339.4	6359.5	6362.7
2	98.503	99.000	99.166	99.249	99.299	99.356	99.399	99.433	99.449	99.466	99.482	99.491	99.497	99.498
3	34.116	30.817	29.457	28.710	28.237	27.672	27.229	26.872	26.690	26.504	26.316	26.221	26.148	26.137
4	21.198	18.000	16.694	15.977	15.522	14.976	14.546	14.198	14.020	13.838	13.652	13.558	13.486	13.474
5	16.258	13.274	12.060	11.392	10.967	10.455	10.051	9.7222	9.5526	9.3793	9.2020	9.1118	9.0424	9.0314
7	12.246	9.5467	8.4513	7.8466	7.4605	6.9929	6.6201	6.3143	6.1554	5.9920	5.8236	5.7373	5.6707	5.6601
10	10.044	7.5594	6.5523	5.9944	5.6363	5.2001	4.8492	4.5582	4.4055	4.2469	4.0818	3.9964	3.9303	3.9195
15	8.6831	6.3588	5.4169	4.8932	4.5557	4.1416	3.8049	3.5223	3.3719	3.2141	3.0471	2.9594	2.8906	2.8796
20	8.0960	5.8489	4.9382	4.4306	4.1027	3.6987	3.3682	3.0880	2.9377	2.7785	2.6078	2.5167	2.4446	2.4330
30	7.5624	5.3903	4.5098	4.0179	3.6990	3.3046	2.9791	2.7002	2.5486	2.3859	2.2078	2.1108	2.0321	2.0192
60	7.0771	4.9774	4.1259	3.6491	3.3388	2.9530	2.6318	2.3522	2.1978	2.0284	1.8362	1.7264	1.6328	1.6169
120	6.8509	4.7865	3.9490	3.4795	3.1736	2.7918	2.4720	2.1914	2.0345	1.8600	1.6557	1.5330	1.4215	1.4015
500	6.6858	4.6479	3.8210	3.3569	3.0539	2.6751	2.3564	2.0746	1.9152	1.7353	1.5175	1.3774	1.2317	1.2007
1000	6.6603	4.6264	3.8012	3.3379	3.0356	2.6571	2.3387	2.0564	1.8967	1.7158	1.4953	1.3513	1.1947	1.1586

F Distribution critical values for P=0.005

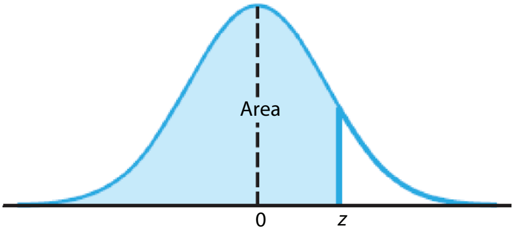
	d_1													
d_2	**1**	**2**	**3**	**4**	**5**	**7**	**10**	**15**	**20**	**30**	**60**	**120**	**500**	**1000**
1	16211	19999	21615	22500	23056	23715	24224	24630	24836	25044	25253	25359	25439	25452
2	198.50	199.00	199.17	199.25	199.30	199.36	199.40	199.43	199.45	199.47	199.48	199.49	199.50	199.50
3	55.552	49.799	47.467	46.195	45.392	44.434	43.686	43.085	42.777	42.466	42.149	41.989	41.867	41.848
4	31.333	26.284	24.259	23.155	22.456	21.622	20.967	20.438	20.167	19.891	19.611	19.468	19.359	19.342
5	22.785	18.314	16.530	15.556	14.940	14.200	13.618	13.146	12.903	12.656	12.402	12.274	12.175	12.159
7	16.235	12.404	10.882	10.050	9.5221	8.8853	8.3803	7.9677	7.7539	7.5345	7.3088	7.1933	7.1044	7.0902
10	12.826	9.4270	8.0807	7.3428	6.8723	6.3025	5.8467	5.4706	5.2740	5.0705	4.8592	4.7501	4.6656	4.6521
15	10.798	7.7007	6.4760	5.8029	5.3721	4.8473	4.4235	4.0697	3.8826	3.6868	3.4802	3.3722	3.2874	3.2739
20	9.9439	6.9865	5.8176	5.1744	4.7616	4.2569	3.8470	3.5020	3.3178	3.1234	2.9159	2.8058	2.7186	2.7046
30	9.1796	6.3547	5.2387	4.6233	4.2275	3.7416	3.3439	3.0058	2.8231	2.6277	2.4151	2.2998	2.2066	2.1914
60	8.4946	5.7950	4.7290	4.1399	3.7599	3.2911	2.9042	2.5705	2.3872	2.1874	1.9621	1.8341	1.7256	1.7073
120	8.1789	5.5393	4.4972	3.9207	3.5482	3.0874	2.7052	2.3728	2.1882	1.9840	1.7468	1.6055	1.4778	1.4550
500	7.9498	5.3548	4.3304	3.7632	3.3963	2.9414	2.5625	2.2303	2.0441	1.8352	1.5844	1.4245	1.2595	1.2247
1000	7.9145	5.3265	4.3049	3.7391	3.3731	2.9191	2.5406	2.2084	2.0219	1.8121	1.5584	1.3945	1.2176	1.1771

F Distribution critical values for P=0.001

	d_1													
d_2	**1**	**2**	**3**	**4**	**5**	**7**	**10**	**15**	**20**	**30**	**60**	**120**	**500**	**1000**
1	405284	499999	540379	562500	576405	592873	605621	615764	620908	626099	631337	633972	635983	636301
2	998.50	999.00	999.17	999.25	999.30	999.36	999.40	999.43	999.45	999.47	999.48	999.49	999.50	999.50
3	167.03	148.50	141.11	137.10	134.58	131.58	129.25	127.37	126.42	125.45	124.47	123.97	123.59	123.53
4	74.137	61.245	56.177	53.436	51.712	49.658	48.053	46.761	46.100	45.429	44.746	44.400	44.135	44.093
5	47.181	37.122	33.202	31.085	29.752	28.163	26.917	25.911	25.395	24.869	24.333	24.061	23.852	23.819
7	29.245	21.689	18.772	17.198	16.206	15.019	14.083	13.324	12.932	12.530	12.119	11.909	11.747	11.722
10	21.040	14.905	12.553	11.283	10.481	9.5174	8.7539	8.1288	7.8038	7.4688	7.1224	6.9443	6.8065	6.7846
15	16.587	11.339	9.3352	8.2526	7.5673	6.7408	6.0808	5.5351	5.2484	4.9502	4.6378	4.4749	4.3478	4.3275
20	14.819	9.9526	8.0984	7.0960	6.4606	5.6920	5.0753	4.5618	4.2900	4.0051	3.7030	3.5439	3.4184	3.3981
30	13.293	8.7734	7.0544	6.1245	5.5339	4.8173	4.2389	3.7528	3.4928	3.2171	2.9197	2.7595	2.6310	2.6100
60	11.973	7.7678	6.1712	5.3067	4.7565	4.0864	3.5415	3.0781	2.8265	2.5549	2.2522	2.0821	1.9390	1.9150
120	11.380	7.3212	5.7814	4.9471	4.4157	3.7669	3.2372	2.7833	2.5345	2.2621	1.9502	1.7668	1.6027	1.5736
500	10.957	7.0041	5.5056	4.6935	4.1757	3.5424	3.0234	2.5759	2.3282	2.0538	1.7292	1.5260	1.3191	1.2759
1000	10.892	6.9556	5.4637	4.6549	4.1392	3.5083	2.9909	2.5443	2.2968	2.0217	1.6946	1.4866	1.2661	1.2162

Appendix E

Normal Distribution Table



Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831

(*Continued*)

(*Continued*)

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
-0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670

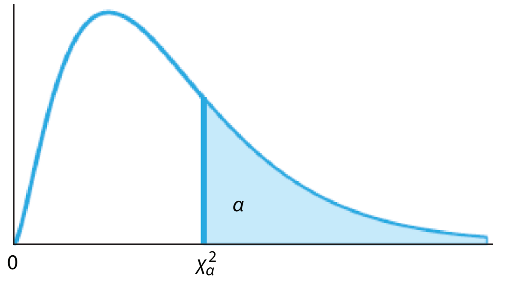
(Continued)

(Continued)

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997

Appendix F

Chi Square Table



DF	0.995	0.975	0.20	0.10	0.05	0.025	0.02	0.01	0.005	0.002	0.001
1	0.0000393	0.000982	1.642	2.706	3.841	5.024	5.412	6.635	7.879	9.550	10.828
2	0.0100	0.0506	3.219	4.605	5.991	7.378	7.824	9.210	10.597	12.429	13.816
3	0.0717	0.216	4.642	6.251	7.815	9.348	9.837	11.345	12.838	14.796	16.266
4	0.207	0.484	5.989	7.779	9.488	11.143	11.668	13.277	14.860	16.924	18.467
5	0.412	0.831	7.289	9.236	11.070	12.833	13.388	15.086	16.750	18.907	20.515
6	0.676	1.237	8.558	10.645	12.592	14.449	15.033	16.812	18.548	20.791	22.458
7	0.989	1.690	9.803	12.017	14.067	16.013	16.622	18.475	20.278	22.601	24.322
8	1.344	2.180	11.030	13.362	15.507	17.535	18.168	20.090	21.955	24.352	26.124
9	1.735	2.700	12.242	14.684	16.919	19.023	19.679	21.666	23.589	26.056	27.877
10	2.156	3.247	13.442	15.987	18.307	20.483	21.161	23.209	25.188	27.722	29.588
11	2.603	3.816	14.631	17.275	19.675	21.920	22.618	24.725	26.757	29.354	31.264
12	3.074	4.404	15.812	18.549	21.026	23.337	24.054	26.217	28.300	30.957	32.909
13	3.565	5.009	16.985	19.812	22.362	24.736	25.472	27.688	29.819	32.535	34.528
14	4.075	5.629	18.151	21.064	23.685	26.119	26.873	29.141	31.319	34.091	36.123
15	4.601	6.262	19.311	22.307	24.996	27.488	28.259	30.578	32.801	35.628	37.697

(Continued)

(*Continued*)

DF	0.995	0.975	0.20	0.10	0.05	0.025	0.02	0.01	0.005	0.002	0.001
16	5.142	6.908	20.465	23.542	26.296	28.845	29.633	32.000	34.267	37.146	39.252
17	5.697	7.564	21.615	24.769	27.587	30.191	30.995	33.409	35.718	38.648	40.790
18	6.265	8.231	22.760	25.989	28.869	31.526	32.346	34.805	37.156	40.136	42.312
19	6.844	8.907	23.900	27.204	30.144	32.852	33.687	36.191	38.582	41.610	43.820
20	7.434	9.591	25.038	28.412	31.410	34.170	35.020	37.566	39.997	43.072	45.315
21	8.034	10.283	26.171	29.615	32.671	35.479	36.343	38.932	41.401	44.522	46.797
22	8.643	10.982	27.301	30.813	33.924	36.781	37.659	40.289	42.796	45.962	48.268
23	9.260	11.689	28.429	32.007	35.172	38.076	38.968	41.638	44.181	47.391	49.728
24	9.886	12.401	29.553	33.196	36.415	39.364	40.270	42.980	45.559	48.812	51.179
25	10.520	13.120	30.675	34.382	37.652	40.646	41.566	44.314	46.928	50.223	52.620
26	11.160	13.844	31.795	35.563	38.885	41.923	42.856	45.642	48.290	51.627	54.052
27	11.808	14.573	32.912	36.741	40.113	43.195	44.140	46.963	49.645	53.023	55.476
28	12.461	15.308	34.027	37.916	41.337	44.461	45.419	48.278	50.993	54.411	56.892
29	13.121	16.047	35.139	39.087	42.557	45.722	46.693	49.588	52.336	55.792	58.301

(*Continued*)

(*Continued*)

DF	0.995	0.975	0.20	0.10	0.05	0.025	0.02	0.01	0.005	0.002	0.001
30	13.787	16.791	36.250	40.256	43.773	46.979	47.962	50.892	53.672	57.167	59.703
31	14.458	17.539	37.359	41.422	44.985	48.232	49.226	52.191	55.003	58.536	61.098
32	15.134	18.291	38.466	42.585	46.194	49.480	50.487	53.486	56.328	59.899	62.487
33	15.815	19.047	39.572	43.745	47.400	50.725	51.743	54.776	57.648	61.256	63.870
34	16.501	19.806	40.676	44.903	48.602	51.966	52.995	56.061	58.964	62.608	65.247
35	17.192	20.569	41.778	46.059	49.802	53.203	54.244	57.342	60.275	63.955	66.619
36	17.887	21.336	42.879	47.212	50.998	54.437	55.489	58.619	61.581	65.296	67.985
37	18.586	22.106	43.978	48.363	52.192	55.668	56.730	59.893	62.883	66.633	69.346
38	19.289	22.878	45.076	49.513	53.384	56.896	57.969	61.162	64.181	67.966	70.703
39	19.996	23.654	46.173	50.660	54.572	58.120	59.204	62.428	65.476	69.294	72.055
40	20.707	24.433	47.269	51.805	55.758	59.342	60.436	63.691	66.766	70.618	73.402
41	21.421	25.215	48.363	52.949	56.942	60.561	61.665	64.950	68.053	71.938	74.745

(*Continued*)

(*Continued*)

DF	0.995	0.975	0.20	0.10	0.05	0.025	0.02	0.01	0.005	0.002	0.001
42	22.138	25.999	49.456	54.090	58.124	61.777	62.892	66.206	69.336	73.254	76.084
43	22.859	26.785	50.548	55.230	59.304	62.990	64.116	67.459	70.616	74.566	77.419
44	23.584	27.575	51.639	56.369	60.481	64.201	65.337	68.710	71.893	75.874	78.750
45	24.311	28.366	52.729	57.505	61.656	65.410	66.555	69.957	73.166	77.179	80.077
46	25.041	29.160	53.818	58.641	62.830	66.617	67.771	71.201	74.437	78.481	81.400
47	25.775	29.956	54.906	59.774	64.001	67.821	68.985	72.443	75.704	79.780	82.720
48	26.511	30.755	55.993	60.907	65.171	69.023	70.197	73.683	76.969	81.075	84.037
49	27.249	31.555	57.079	62.038	66.339	70.222	71.406	74.919	78.231	82.367	85.351

	P										
DF	**0.995**	**0.975**	**0.20**	**0.10**	**0.05**	**0.025**	**0.02**	**0.01**	**0.005**	**0.002**	**0.001**
50	27.991	32.357	58.164	63.167	67.505	71.420	72.613	76.154	79.490	83.657	86.661
51	28.735	33.162	59.248	64.295	68.669	72.616	73.818	77.386	80.747	84.943	87.968
52	29.481	33.968	60.332	65.422	69.832	73.810	75.021	78.616	82.001	86.227	89.272
53	30.230	34.776	61.414	66.548	70.993	75.002	76.223	79.843	83.253	87.507	90.573
54	30.981	35.586	62.496	67.673	72.153	76.192	77.422	81.069	84.502	88.786	91.872
55	31.735	36.398	63.577	68.796	73.311	77.380	78.619	82.292	85.749	90.061	93.168
56	32.490	37.212	64.658	69.919	74.468	78.567	79.815	83.513	86.994	91.335	94.461
57	33.248	38.027	65.737	71.040	75.624	79.752	81.009	84.733	88.236	92.605	95.751
58	34.008	38.844	66.816	72.160	76.778	80.936	82.201	85.950	89.477	93.874	97.039
59	34.770	39.662	67.894	73.279	77.931	82.117	83.391	87.166	90.715	95.140	98.324
60	35.534	40.482	68.972	74.397	79.082	83.298	84.580	88.379	91.952	96.404	99.607
61	36.301	41.303	70.049	75.514	80.232	84.476	85.767	89.591	93.186	97.665	100.888
62	37.068	42.126	71.125	76.630	81.381	85.654	86.953	90.802	94.419	98.925	102.166
63	37.838	42.950	72.201	77.745	82.529	86.830	88.137	92.010	95.649	100.182	103.442

(*Continued*)

(*Continued*)

	P										
DF	**0.995**	**0.975**	**0.20**	**0.10**	**0.05**	**0.025**	**0.02**	**0.01**	**0.005**	**0.002**	**0.001**
64	38.610	43.776	73.276	78.860	83.675	88.004	89.320	93.217	96.878	101.437	104.716
65	39.383	44.603	74.351	79.973	84.821	89.177	90.501	94.422	98.105	102.691	105.988
66	40.158	45.431	75.424	81.085	85.965	90.349	91.681	95.626	99.330	103.942	107.258
67	40.935	46.261	76.498	82.197	87.108	91.519	92.860	96.828	100.554	105.192	108.526
68	41.713	47.092	77.571	83.308	88.250	92.689	94.037	98.028	101.776	106.440	109.791
69	42.494	47.924	78.643	84.418	89.391	93.856	95.213	99.228	102.996	107.685	111.055
70	43.275	48.758	79.715	85.527	90.531	95.023	96.388	100.425	104.215	108.929	112.317
71	44.058	49.592	80.786	86.635	91.670	96.189	97.561	101.621	105.432	110.172	113.577
72	44.843	50.428	81.857	87.743	92.808	97.353	98.733	102.816	106.648	111.412	114.835
73	45.629	51.265	82.927	88.850	93.945	98.516	99.904	104.010	107.862	112.651	116.092
74	46.417	52.103	83.997	89.956	95.081	99.678	101.074	105.202	109.074	113.889	117.346
75	47.206	52.942	85.066	91.061	96.217	100.839	102.243	106.393	110.286	115.125	118.599
76	47.997	53.782	86.135	92.166	97.351	101.999	103.410	107.583	111.495	116.359	119.850

(*Continued*)

(*Continued*)

	P										
DF	**0.995**	**0.975**	**0.20**	**0.10**	**0.05**	**0.025**	**0.02**	**0.01**	**0.005**	**0.002**	**0.001**
77	48.788	54.623	87.203	93.270	98.484	103.158	104.576	108.771	112.704	117.591	121.100
78	49.582	55.466	88.271	94.374	99.617	104.316	105.742	109.958	113.911	118.823	122.348
79	50.376	56.309	89.338	95.476	100.749	105.473	106.906	111.144	115.117	120.052	123.594
80	51.172	57.153	90.405	96.578	101.879	106.629	108.069	112.329	116.321	121.280	124.839
81	51.969	57.998	91.472	97.680	103.010	107.783	109.232	113.512	117.524	122.507	126.083
82	52.767	58.845	92.538	98.780	104.139	108.937	110.393	114.695	118.726	123.733	127.324
83	53.567	59.692	93.604	99.880	105.267	110.090	111.553	115.876	119.927	124.957	128.565
84	54.368	60.540	94.669	100.980	106.395	111.242	112.712	117.057	121.126	126.179	129.804
85	55.170	61.389	95.734	102.079	107.522	112.393	113.871	118.236	122.325	127.401	131.041
86	55.973	62.239	96.799	103.177	108.648	113.544	115.028	119.414	123.522	128.621	132.277
87	56.777	63.089	97.863	104.275	109.773	114.693	116.184	120.591	124.718	129.840	133.512
88	57.582	63.941	98.927	105.372	110.898	115.841	117.340	121.767	125.913	131.057	134.745
89	58.389	64.793	99.991	106.469	112.022	116.989	118.495	122.942	127.106	132.273	135.978

(*Continued*)

(*Continued*)

	P										
DF	**0.995**	**0.975**	**0.20**	**0.10**	**0.05**	**0.025**	**0.02**	**0.01**	**0.005**	**0.002**	**0.001**
90	59.196	65.647	101.054	107.565	113.145	118.136	119.648	124.116	128.299	133.489	137.208
91	60.005	66.501	102.117	108.661	114.268	119.282	120.801	125.289	129.491	134.702	138.438
92	60.815	67.356	103.179	109.756	115.390	120.427	121.954	126.462	130.681	135.915	139.666
93	61.625	68.211	104.241	110.850	116.511	121.571	123.105	127.633	131.871	137.127	140.893
94	62.437	69.068	105.303	111.944	117.632	122.715	124.255	128.803	133.059	138.337	142.119
95	63.250	69.925	106.364	113.038	118.752	123.858	125.405	129.973	134.247	139.546	143.344
96	64.063	70.783	107.425	114.131	119.871	125.000	126.554	131.141	135.433	140.755	144.567
97	64.878	71.642	108.486	115.223	120.990	126.141	127.702	132.309	136.619	141.962	145.789
98	65.694	72.501	109.547	116.315	122.108	127.282	128.849	133.476	137.803	143.168	147.010
99	66.510	73.361	110.607	117.407	123.225	128.422	129.996	134.642	138.987	144.373	148.230

Appendix G

Critical Values for Cramér-von Mises Test

	α				
M	0.20	0.15	0.10	0.05	0.01
2	0.138	0.149	0.162	0.175	0.186
3	0.121	0.135	0.154	0.184	0.23
4	0.121	0.134	0.155	0.191	0.28
5	0.121	0.137	0.160	0.199	0.30
6	0.123	0.139	0.162	0.204	0.31
7	0.124	0.140	0.165	0.208	0.32
8	0.124	0.141	0.165	0.210	0.32
9	0.125	0.142	0.167	0.212	0.32
10	0.125	0.142	0.167	0.212	0.32
11	0.126	0.143	0.169	0.214	0.32
12	0.126	0.144	0.169	0.214	0.32
13	0.126	0.144	0.169	0.214	0.33
14	0.126	0.144	0.169	0.214	0.33
15	0.126	0.144	0.169	0.215	0.33
16	0.127	0.145	0.171	0.216	0.33

(*Continued*)

(*Continued*)

	α				
17	0.127	0.145	0.171	0.217	0.33
18	0.127	0.146	0.171	0.217	0.33
19	0.127	0.146	0.171	0.217	0.33
20	0.128	0.146	0.172	0.217	0.33
30	0.128	0.146	0.172	0.218	0.33
60	0.128	0.147	0.173	0.220	0.33
100	0.129	0.147	0.173	0.220	0.34

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