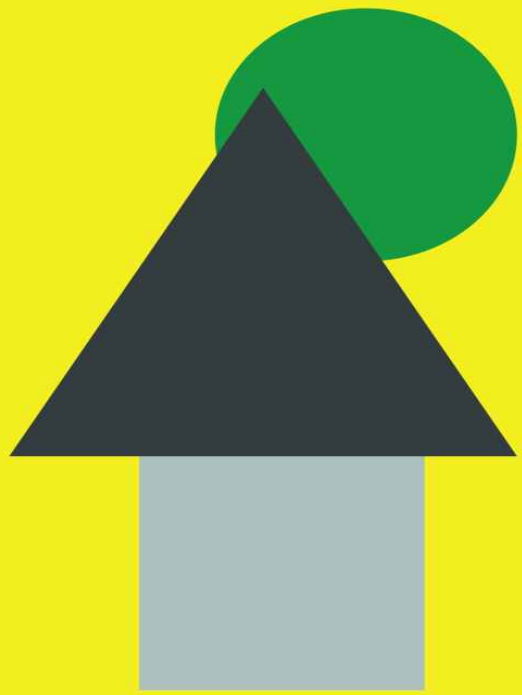
TAMING GEOMETRY



A must for students preparing for competitive examinations

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Taming Geometry

Play with some of the best geometry problems.

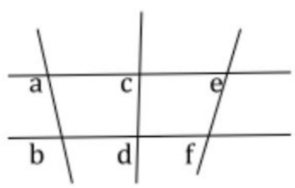
PREFACE :

"I am a Mathematics enthusiast. After completing my Engineering in Electronics and Telecommunications , I started teaching Maths. The love for maths was so much that I started appearing for competitive examinations which had Maths or Quantitative Aptitude just a part of it. Exams like AFCAT, LIC ADO (Mumbai Topper) , Bank PO, Management Trainee , etc were cleared by me , mostly because i was exceptionally well at Quant. Having said that, and years of experience made me write a fun book for all the math lovers and Students preparing for competitive exams known as 111 Math Problems which was accepted as one of the best math puzzle books available in the market.

Continuing with my puzzle series, I am here proudly announcing puzzle book of my favourite subject Geometry. The love and curiosity for the subject, will help a student in 'Taming Geometry'. All the Best "

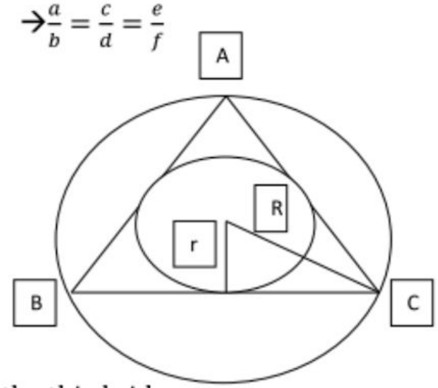
Before we start, let me give you all the important formulae in Geometry to help you crack some of the toughest Problems in this book.

Formulae & Tricks



$\rightarrow \frac{a}{b} = \frac{c}{d} = \frac{e}{f}$

=>Triangle Properties



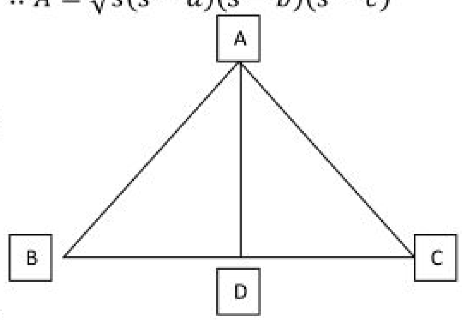
- Sum of two sides always greater than the third side
- Side opposite to biggest angle is largest,while smallest for smallest angle.
- Medians intersect at the <u>Centroid.</u>
- All altitudes {heights from vertices} intersect at orthocenter.
- All angle bisectors intersect at incenter.
- Centroid & incenter always inside triangle
- For an acute angled triangle circumcenter & orthocenter will be insides
- For obtuse angled triangle, circumcenter & orthocenter will be outsides.
- For right angled triangle, circumcenter & orthocenter i.e. lies at the vertex, where angle is 90°.
- Orthocenter, Centroid & Circumcenter always lies on the same line known as 'Euler line'.

Formulae & Tricks

- Orthocenter is twice as for from the centroid of circumcenter is.
- For an isosceles triangle incenter lies on the same line.
- For equilateral triangle, all four are some point.

Area of a triangle, $A = \frac{1}{2} \times base \times height, A = \frac{1}{2}.ab.\sin(c)$

Semiperimeter =S = $\frac{a+b+c}{2}$, $\therefore A = \sqrt{s(s-a)(s-b)(s-c)}$



(Inradius)='r' $\therefore A = r.s$

Circumcenter = R, $\therefore A = \frac{abc}{4R}$

Apollonius Theorem

AD=Median

$AB^2 + AC^2 = 2(AD^2 + BD^2)$

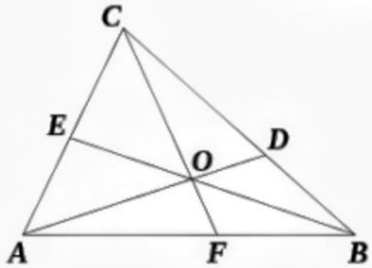
If AD= Angle bisector, then , $\frac{AB}{AC} = \frac{BD}{DC}$

In equilateral triangle → $Area = \frac{\sqrt{3}}{4} \times side^2, Height = \frac{\sqrt{3}}{2} \times side$

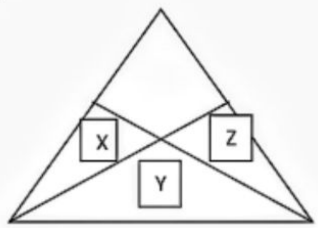
In Isosceles triangle, Area =$\frac{c}{4}\sqrt{4a^2 - c^2}$

Maximum side of a square that can be inscribed in a triangle is

Formulae & Tricks



For a triangle we have,



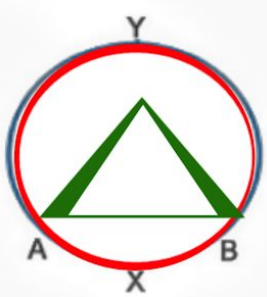
Here X,Y,Z and $\triangle$

Are areas

→ $\frac{1}{\Delta}+\frac{1}{y}=\frac{1}{x+y}+\frac{1}{y+z}$ Thus, Area of ADFE can be found

$\triangle$ = Area of Complete Triangle.

Circles

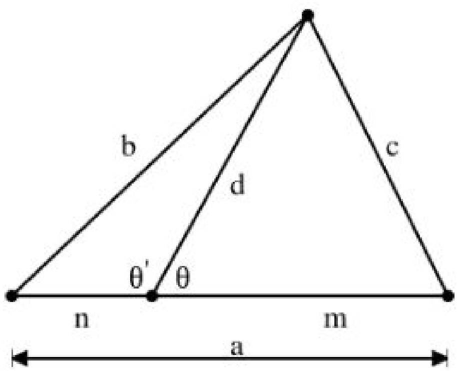


Length of Arc AB $=S=r\theta=r.\frac{\pi\theta}{180}$ {if θ *in degree*}

→ PA =PB

Formulae & Tricks

$\boldsymbol{x = \frac{ch}{c+h}}$**.......{$\boldsymbol{c = side\ of\ triangle\ where\ one\ side\ os\ square\ is\ placed,}$
h=height of triangle on side c}**



Cevian Length, d = cevian

Stewarts theorem

$b^2m + c^2n = a(d^2 + mn)$

If d = ~~angle~~ ~~then~~ its length can be determined by

$\frac{AO}{AD} + \frac{BO}{BE} + \frac{CO}{CF} = 2$

$(b + c)^2 = a\text{^}2\left(\frac{d^2}{mn} + 1\right)$

CEVA'S Theorem:

$\frac{BD}{DC} \times \frac{CE}{AE} \times \frac{AF}{BF} = 1$

$\frac{AO}{OD} = \frac{AF}{BF} + \frac{AE}{EC}$

Formulae & Tricks

→ PB× $PA = PD \times PC$

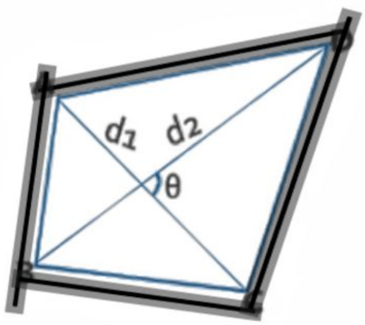
$\theta = \frac{1}{2}\{m(arc\ AC) + m(arc\ BD)\}$

→$PC^2 = PA.PB, \theta = 1/2\{m(arc(AC) - m(arc(BC)$

→$\angle BAQ = \ \angle BCA$......{Alternate Segement Theorem)

→ Length of common direct tangent = $\sqrt{d^2 - (R_1 - R_2)^2}$

→and for common transverse tangent=$\sqrt{d^2 - (R_1 + R_2)^2}$ (Where d=Distance between center of circles)



Polygon	Sum of interior Angles
N = total sides	$=(n-2)180°$
Number of diagonals	Sum of exterior angles
$= \frac{n(n-3)}{2}$	=360°
Area of a regular polygon	Area=$\frac{1}{2} d_1 d_2 sin\theta\ also$

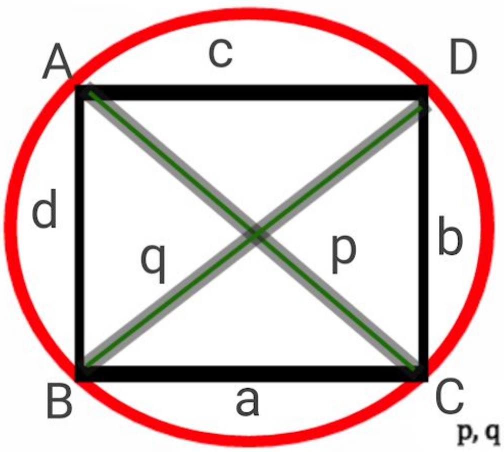
Formulae & Tricks

$=\left(\frac{1}{2}\times perimeter \times in\ radius\right)$ A=$\frac{1}{2}AB.BC.Sin(angle)$

→Cyclic Quadrilateral

Area $=\sqrt{(s-a)(s-b)(s-c)(s-d)}$

$$S=\frac{a+b+c+d}{2}$$



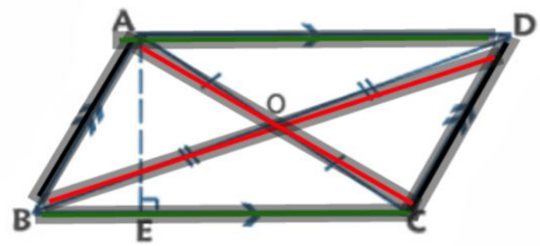
p, q are diagonals

Ptolemy's theorem

$pq = ac + bd$

Parallelogram –

Formulae & Tricks



Area=Base× $Height$

A parallelogram inscribed in a circle is always rectangle

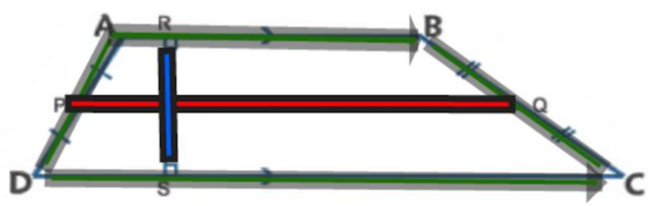
And if it is circumscribed about a circle it is a Rhombus

$$AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2$$

→ A rectangle is formed by four angle bisectors of a//

→Rhombus: Area=$\frac{1}{2}d_1d_2 = a^2\sin(any\ angle)$

Also Area =d× $\sqrt{a^2 - \left(\frac{d^2}{4}\right)}$



→Trapezium Area of trapezium=$\frac{1}{2} \times (sum\ of\ parallel$ sides)× $height$

$$AC^2 + BD^2 = AD^2 + BC^2 + 2.AB \times DC$$

Formulae & Tricks

→Isosceles Trapezium

If a trapezium is inscribed in a circle it has to be an isosceles triangle.

If a circle can be inscribed in a trapezium then

Sum of parallel sides =sum of lateral sides

- ➔ Hexagon: $Area = 6.\frac{\sqrt{3}}{4}.a^2 = \frac{3\sqrt{3}}{2}.a^2, perimeter = 6a$
- ➔ Length of diagonals =2a$(3\ big)$
- ➔ Length of diagonals = $\sqrt{3}a(6\ Small)$ Area of pentagon = 1.72a^2

Figure	Volume	Total Surface Area	Lateral/Curved Surface Area
Cube	$(side)3$	$6 \times side^2$	$4 \times side^2$
Cuboids	$L.B.H$	$2(LB + BH + LH)$	$2(LH + BH)$
Cylinder	$\pi r^2 h$	$2\pi r(r + h)$	$2\pi rh$
Cone	$\frac{\pi r^2 h}{3}$	$\pi r(r + l)$	$\pi rl, \dots\dots \{l = \sqrt{h^2 + r^2}\}$
Sphere	$\frac{4}{3}\pi r^3$	$4\pi r^2$	$4\pi r^2$
Hemisphere	$\frac{2}{3}\pi r^3$	$3\pi r^2$	$2\pi r^2$
Frustum/Furcated cone	$\frac{1}{3}\pi h(R^2 + r^2 + Rr)$	$\pi(R + r)l + \pi(R^2 + r^2)$	$\pi(R + r)L$
Prism	$Area of base \times height$	$(perimeter + height) + 2(Area\ of Base$	$perimeter \times Height$
Pyramid	$\frac{Area\ of\ Base\ \times\ Height}{3}$	$\frac{1}{2} \times perimeter \times slant\ height + Area\ of\ Base$	$\frac{1}{2} \times perimeter \times slant\ height$

Formulae & Tricks

A Sphere circumscribed about a cube of side ‘a’, will have a radius of $\frac{\sqrt{3}a}{2}$

Largest possible sphere inscribed in a ‘Cylinder’ of radius ‘a’

And height ‘h’ will have its radius,

$r = \frac{h}{2}$ ....$\{\text{if } 2a>h\}$

OR r=a ....$\{\text{if } 2a<h\}$

Largest possible sphere inscribed in a cone of radius ‘r’ and (slant height)=2r, then radius of sphere $= \frac{r}{\sqrt{3}}$.

If a cube is inscribed in a hemisphere of radius ‘r’, then side of cube $= r\sqrt{\frac{2}{3}}$

The largest cube that can be chiseled out from a cone of height ‘h’ and radius ‘r’ c.m. then cube edge $= x = \frac{\sqrt{2}rh}{h+\sqrt{2}\,r}$

Maximum volume of cylinder that can be curved out of a cone of radius ‘r’ & height ‘h;’ is $V = \frac{4}{27}\pi r^2 h$

Maximum sphere inside a cone with radius r & height ‘h’ will be
$radius\ of\ sphere = x = \frac{rh}{r+l} \ldots\ldots \{l = \sqrt{r^2+h^2}\}$

Maximum volume of Cone in a sphere of radius r will be

$$V = \frac{32}{81}\pi r^3$$

Formulae & Tricks

Distance between two points (x_1, y_1) & (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

If (x, y) divides a line Q $\{P = (x_1, y_1)$& $Q = (x_2, y_2)\}$ internally in the ratio of m:n , then co-ordinates are $x = \frac{mx_2+nx_1}{m+n}$, $y = \frac{my_2+ny_1}{m+n}$

External division, $x = \frac{mx_2-nx_1}{m-n}$, $y = \frac{my_2-ny_1}{m-n}$

Slope=tan$(\theta) = m \ldots \{\theta = angle\ made\ by\ x\ axis\ in\ positive\ direction\}$

$ax + by + c = 0 \ldots\ldots\ldots\ldots\ldots. m = -\frac{a}{b}$, for$p(x_1, y_1)$& $Q(x_2, y_2)$, $m = \frac{y_2-y_1}{x_2-x_1}$

Equation of a line with slope 'm' & one point (x_1, y_1) is $y - y_1 = m(x - x_1)$

Two intercept form → $\{x\ intercept = a\ \&\ y\ intercept = b\}\ \frac{x}{a} + \frac{y}{b} = 1$

Acute angle between two lines with slope m_1 & m_2 is given by

$$tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Distance of point $P(x_1, y_1)$ from a line $ax + by + c = 0, is\ D = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

Three points of a parallelogram $= A(x_1, y_1), B(x_2, y_2), C(x_3, y_3) then$
Fourth point is given by $\{(x_1 + x_3 - x_2), (y_1 + y_3 - y_2)\}$

Formulae & Tricks

The vertices of a triangle are $P(x_1, y_1), Q(x_2, y_2), R(x_3, y_3)$

In center $= \left\{\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right\}$ $\{a, b, c\ are\ lengths\}$

Centroid $= \left\{\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right\}$

Area $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Circle → General form $=> x^2 + y^2 + 2gx + 2fy + c = 0$ Here

$Center = (-g, -f)$ $and\ radius = \sqrt{g^2 + f^2 - c}$ for circle with center (h,k) & radius=r Equation=>$(x - h)^2 + (y - k)^2 = r^2$ {if h,k = (0,0) origin then $x^2 + y^2 = r^2$}

$2sinA.cosB = sin(A + B) + sin(A - B),$

$2cosAsinB = sin(A + B) - sin(A - B)$

$2cosAcosB = cos(A + B) + cos(A - B)$

$2sinAsinB = cos(A - B) - cos(A + B)$

$$sinC + sinD = 2\sin\left(\frac{C + D}{2}\right).cos\left(\frac{C - D}{2}\right)$$

$$sinC - sinD = 2\cos\left(\frac{C + D}{2}\right).sin\left(\frac{C - D}{2}\right)$$

$$cosC + cosD = 2\cos\left(\frac{C + D}{2}\right).cos\left(\frac{C - D}{2}\right)$$

$$cosC - cosD = 2\sin\left(\frac{C + D}{2}\right).sin\left(\frac{D - C}{2}\right)$$

$sin3x = 3sinx - 4\sin^3 x$

$cos3x = 4\cos^3 x - 3cosx$

$$tan3x = \frac{3tanx - \tan^3 x}{1 - 3\tan^2 x}$$

Now, for the sake of brushing up your Skills, try out these simple Problems and then we'll head to the main game !

Easy Questions (The Foundation)

Q1. In a Δ PQR, D is the midpoint of line QR and E is the mid point of PD. Then find the ratio of area of Δ QEP and ΔPQR ?

Q2. In a Δ PQR, QD and RE are two medians which intersects each other at 'O'. PO intersect the line ED at M. Find the ratio of PM : MO ?

Q3. I is the incentre of a triangle PQR. If angle PRQ = 55°, angle PQR = 65° then the value of angle QIR is ?

Q4. In Δ PQR, angle Q = 60° and angle R = 40°. If PD and PE be respectively the internal bisector of angle P and perpendicular on QR, then the measure of angle DPE is ?

Q5. The angle between the external bisectors of two angles of a triangle is 60°. Then the third angle of the triangle is ?

Q6. PD is the median of a triangle PQR and O is the centroid such that PO = 10 cm. The length of OD (in cm) is ? Q7. In a Δ PQR, QD & RE are the two medians which intersect each other at right angle. PQ = 22, PR = 19, find QR = ?

Q8. In Δ PQR, PD, QE and RF are the altitudes in the ratio 1 : 2 : 3 respectively, then the ratio of PQ : QR : RP is ?

Q9. In Δ PQR, draw QE ⊥PR and RF⊥PQ and the perpendicular QE and RF intersect at the point O. If angle QPR = 70°, then the value of angle QOR is ?

Q10. Let PQR be an equilateral triangle and PX, QY, RZ be the altitudes. Then the right statement out of the four given responses is ? (a) PX = QY = RZ (b) PX ≠ QY ≠ RZ (c) PX = QY ≠ RZ (d) None of these.

Q11. AD is the Internal bisector of∠A, If BD = 4 cm, cm and AB = 6 cm, DC = 12 cm, find AC.

Q12. Two tangents are drawn from point P to a circle at points M and N. O is the centre of circle. Then if angle MOP = 60°, then angle MPN =

?

Q13. PQ is the diameter of a circle with centre O. RS is a chord which is equal to radius in length. PR and QS are produced to meet at M. Then angle PMQ will be ? Q14. If G be the centroid of ΔPQR and if PG = QR then what will be the measure of angle QGR ?

Q15. Two circles each of radii r cut each other and passes through others centre. Then length of common chord is ?

Q16. Tangents drawn at two points A and B lying on circumference of a circle cut each other at M if angle AMB = 68°, then angle MAB is ?

Q17. In ΔPQR internal angle bisector of angle R cuts PQ at point D. In this PQ ≠ PR and E is a point on RD such that PE = PD. If angle PQR = 50° then angle RPE is equal to ?

Q18. PQR is a triangle and A is any point on PQ such that angle PRA = angle PQR, if PR = 9 cm, RA = 12 cm and QR = 15 cm, then PA is equal to ?

Q19. The lengths of perpendiculars drawn from any point in the interior of an equilateral triangle to the respective sides are 6cm, 8cm, and 10cm. The length of each side of the triangle is ?

Q20. In a right angled ΔPQR , angle PQR = 90°; QN⊥PR, PQ = 6 cm, PR = 10 cm. Then PN : NR is ?
Q21. In a ΔPQR a line DE is drawn parallel to QR. If DE/QR =⅖ then find the ratio of area of ΔPDE & Area of DERQ ?
Q22. If G is the centroid and PD is a median of a triangle PQR, if PD is 12 cm then PG is ?

Q23. ΔPQR is a right angled triangle. PD is a perpendicular to hypotenuse QR. If, PQ = √5, PR = 2√5 , then value of QD is equal to ?

Q24. In a right angled triangle PQR, PR = 2.5 cm, cos Q = 0.5, of side PQ in cm is equals to ?

Q25. Two circles of radii 4 cm and 9 cm respectively touch externally at a point and a common tangent touches them at points A and B respectively. Then what will be the area of the square with side AB ?

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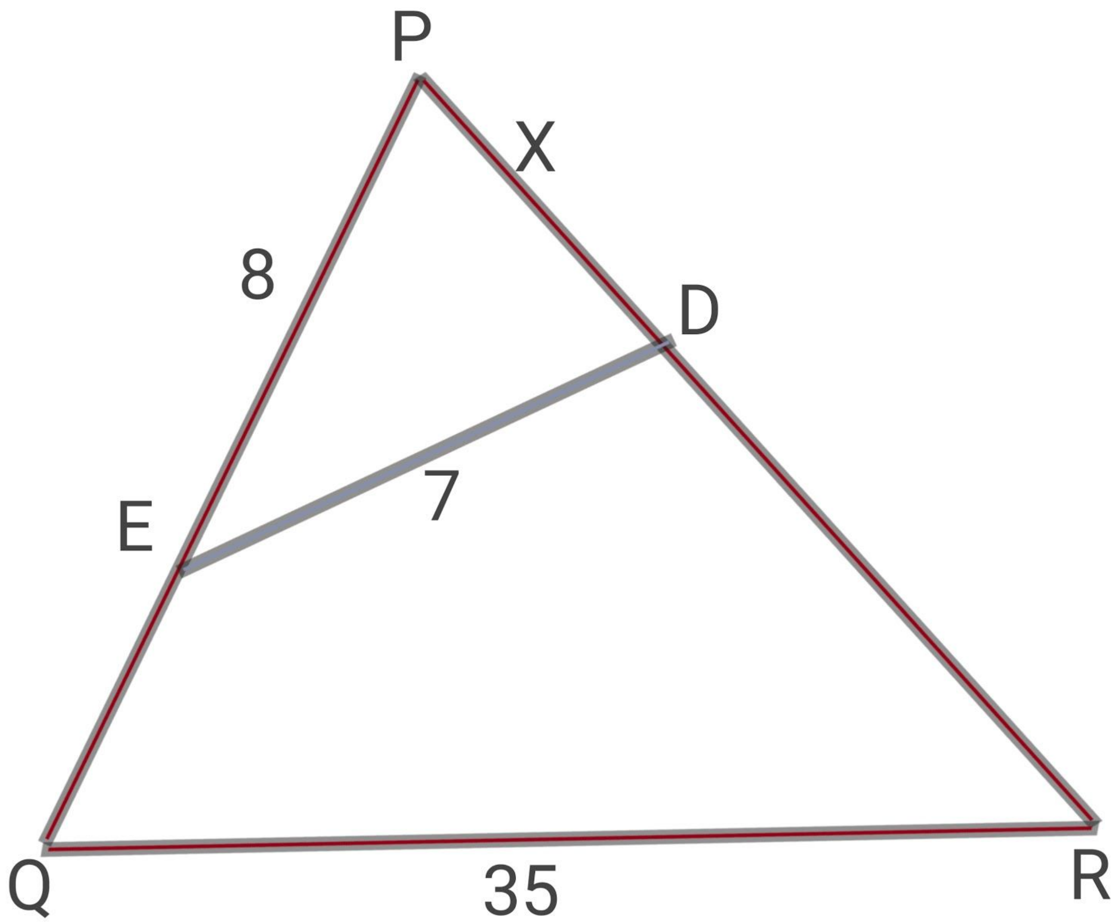
Now, comes the main game. It will start from moderate questions and gradually get to some of the toughest questions.

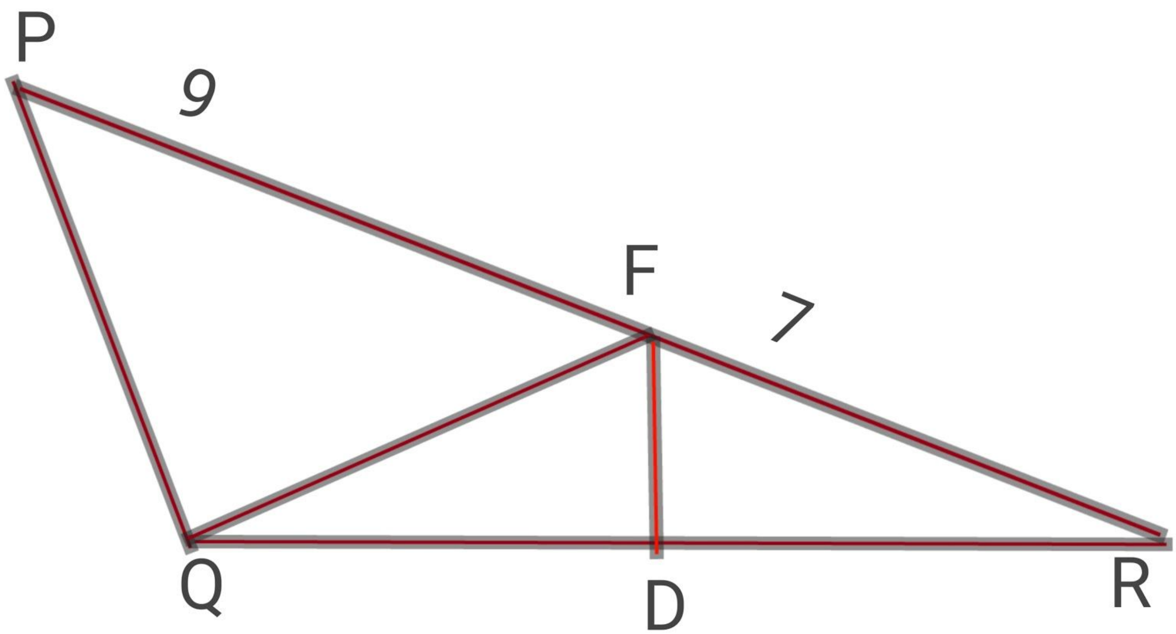
Tough Questions (The TAMING)

Q1• In the given figure Below, EP = 8 , ED = 7 , QR = 35, PD = X, angle PED = angle PRQ. PD = QE, Find the value of 'X'.

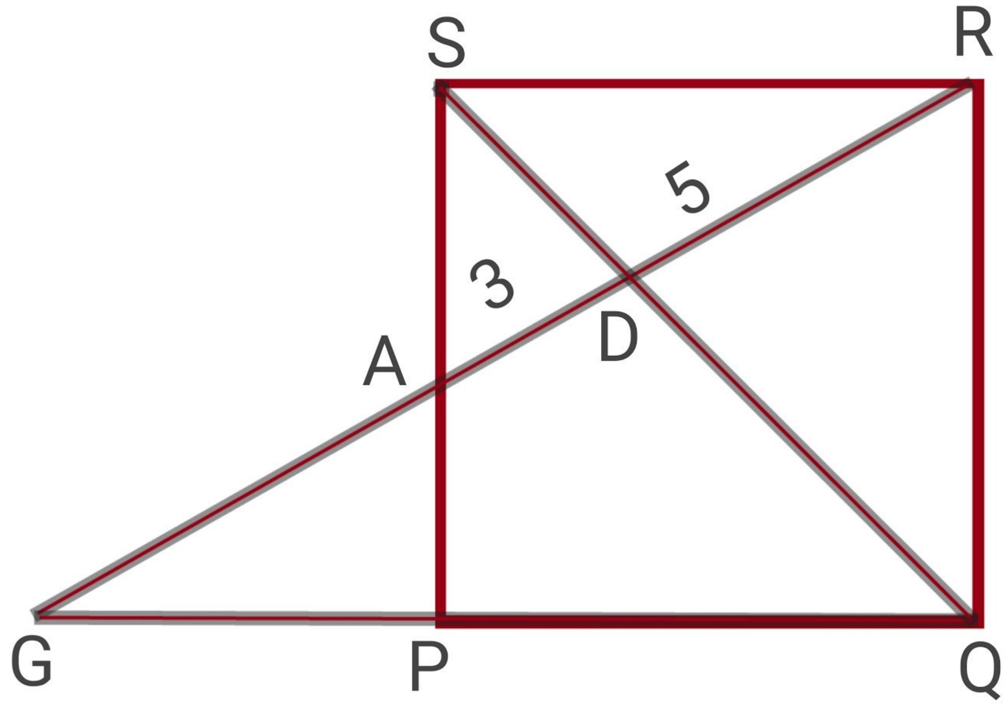


Q2• The tip of the lotus plant , which has a rigid stem, was 1 cm above the lake but forced by the wind it gradually advances and gets submerged into the lake at a distance of 3 cm. Find the depth of the lake.

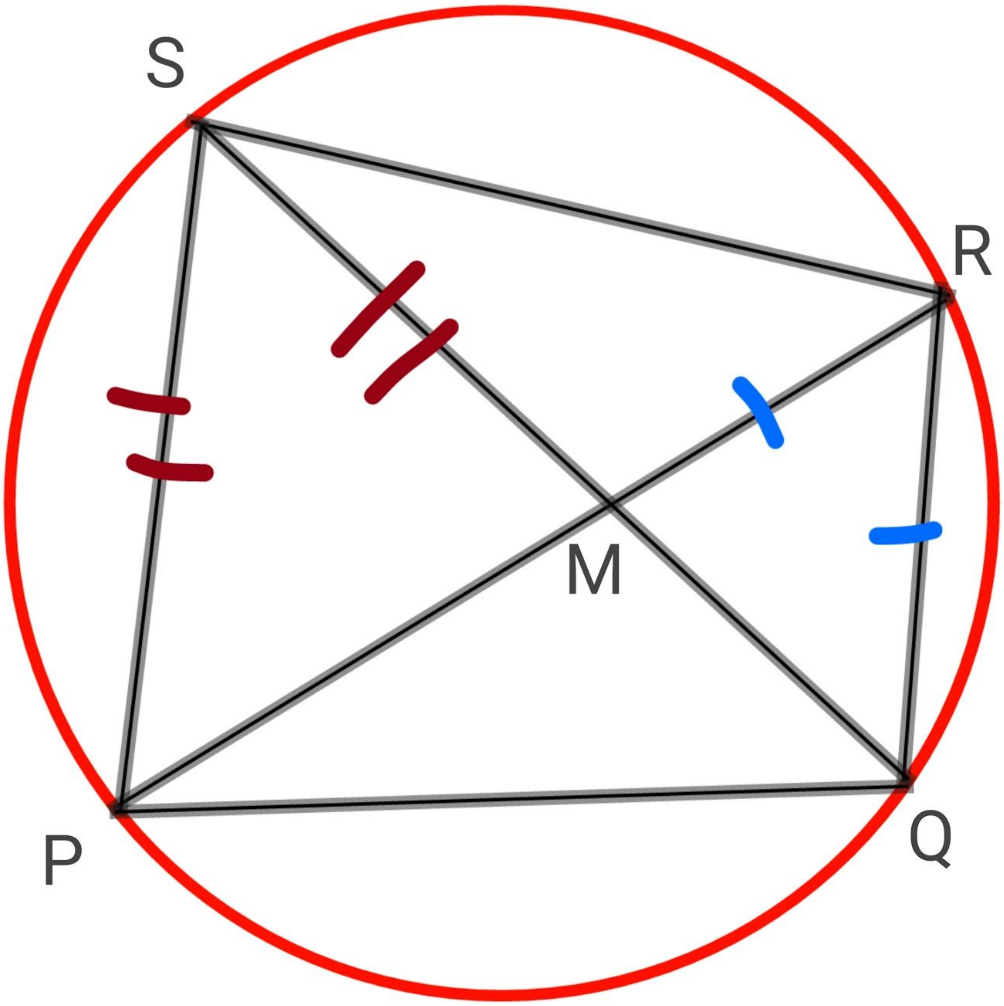
Q3• In Triangle PQR, FD is Perpendicular Bisector of side QR, and QF bisects angle PQR. Given, PF = 9 and FR = 7, find area of Triangle PQF.



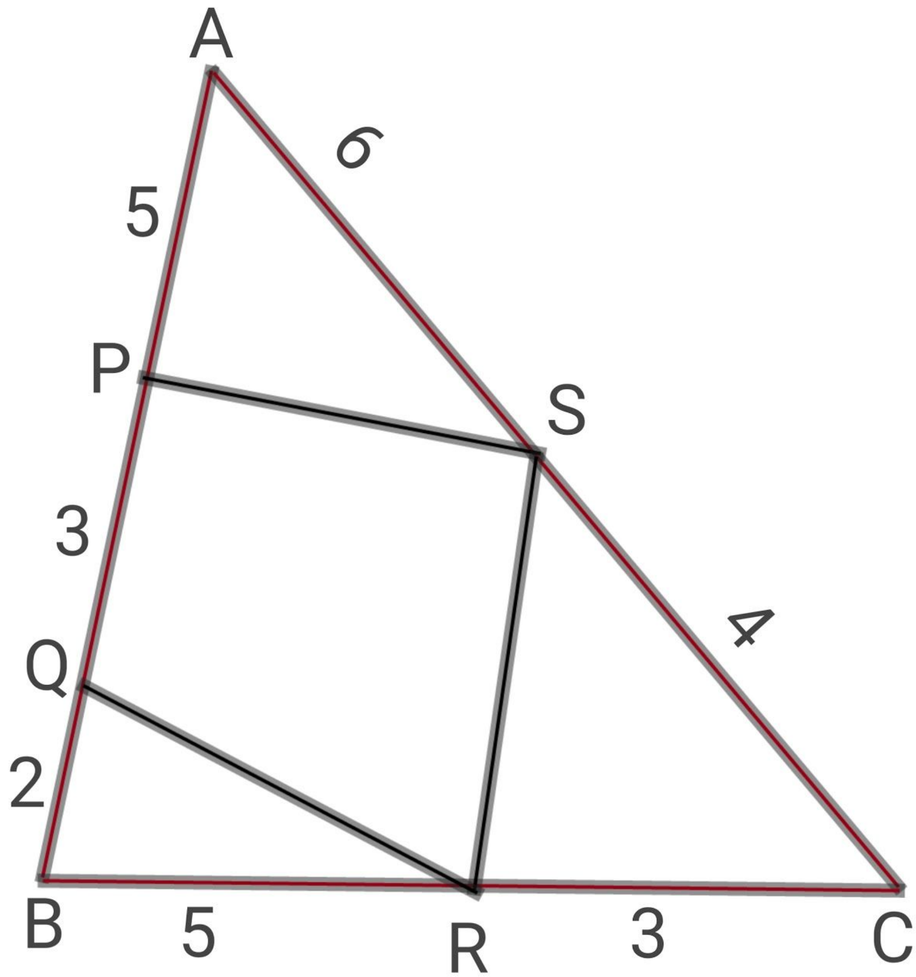
Q4• In the figure below, PQRS is a Square and AD = 3 , DR = 5 , Find the value of AG.



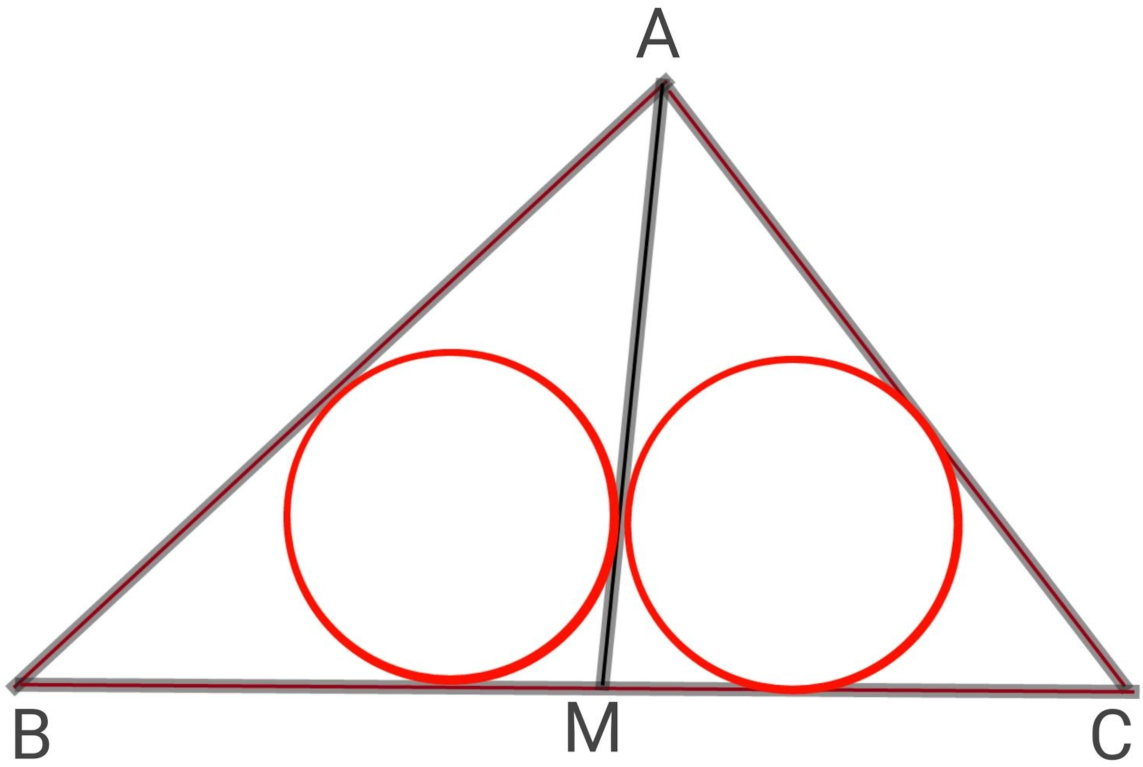
Q5• Following is a Cyclic quadrilateral where PR = diameter = 125 cm. QS = 117 cm. PS = SM , QR = MR. Find the value of PQ.



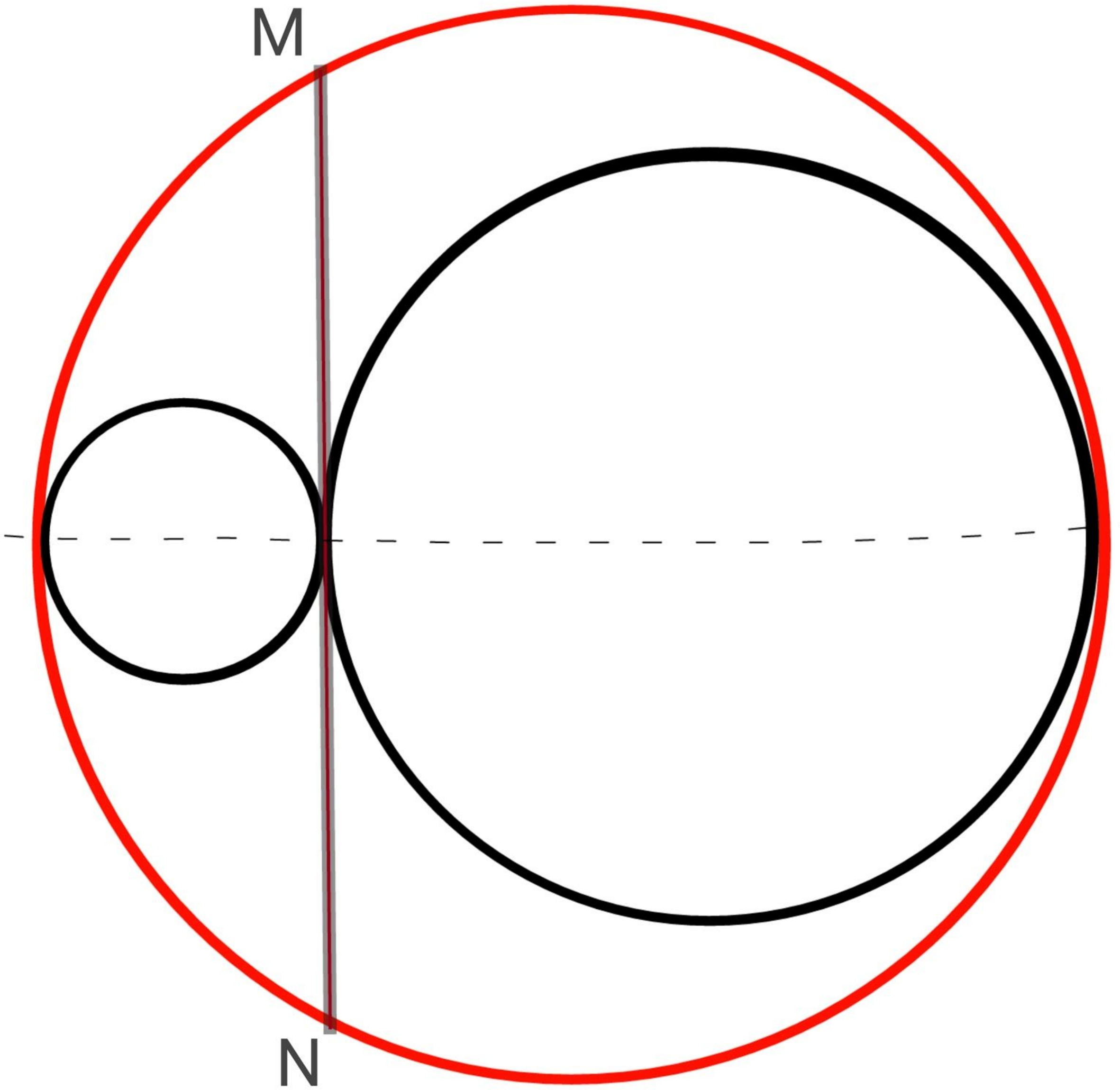
Q6• Following is a Triangle with the respective side lengths given. You need to find the area of the Quadrilateral PQRS.



Q7• In the given triangle below, AB = 12, BC = 18 , AC = 10. Find the length of BM.

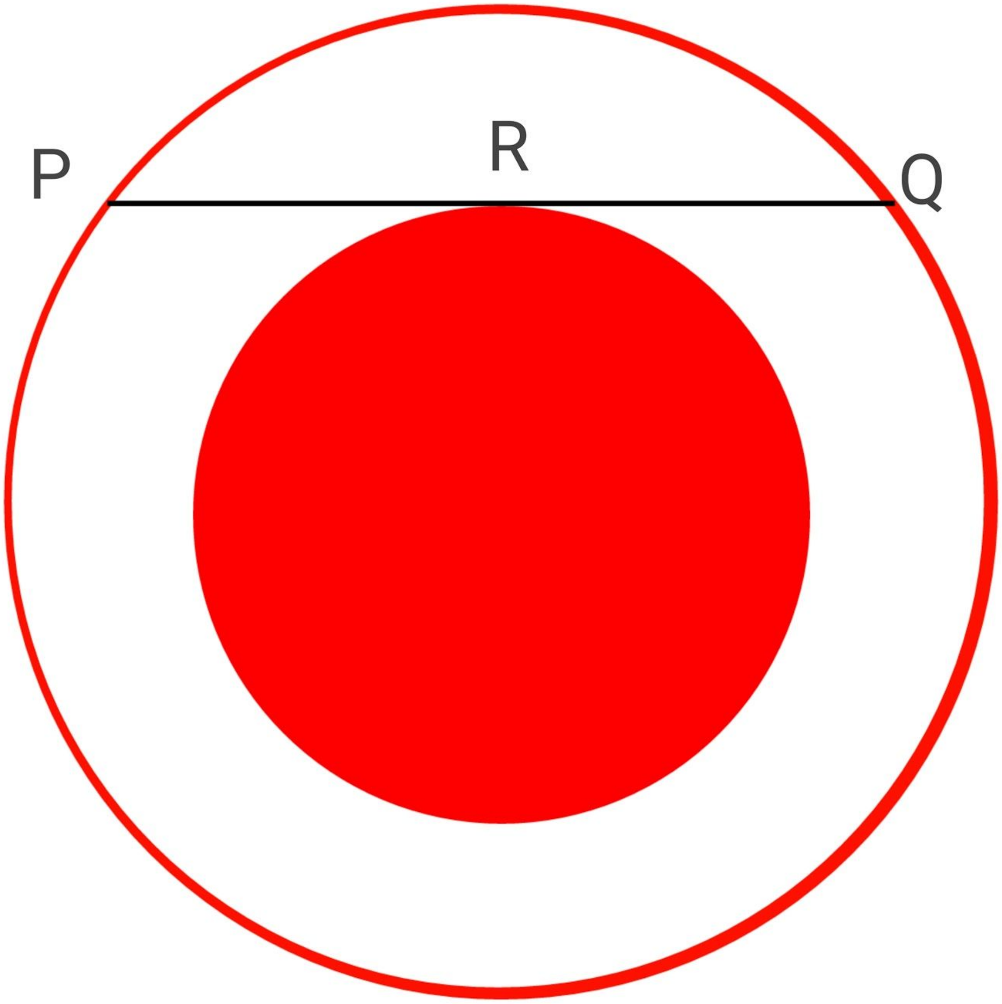


Q8• In this diagram, three circles are given with MN the common tangent to the two given smaller Circles inside the larger one. MN = 16 units. Find the area outside the smaller circles but inside the large circle.

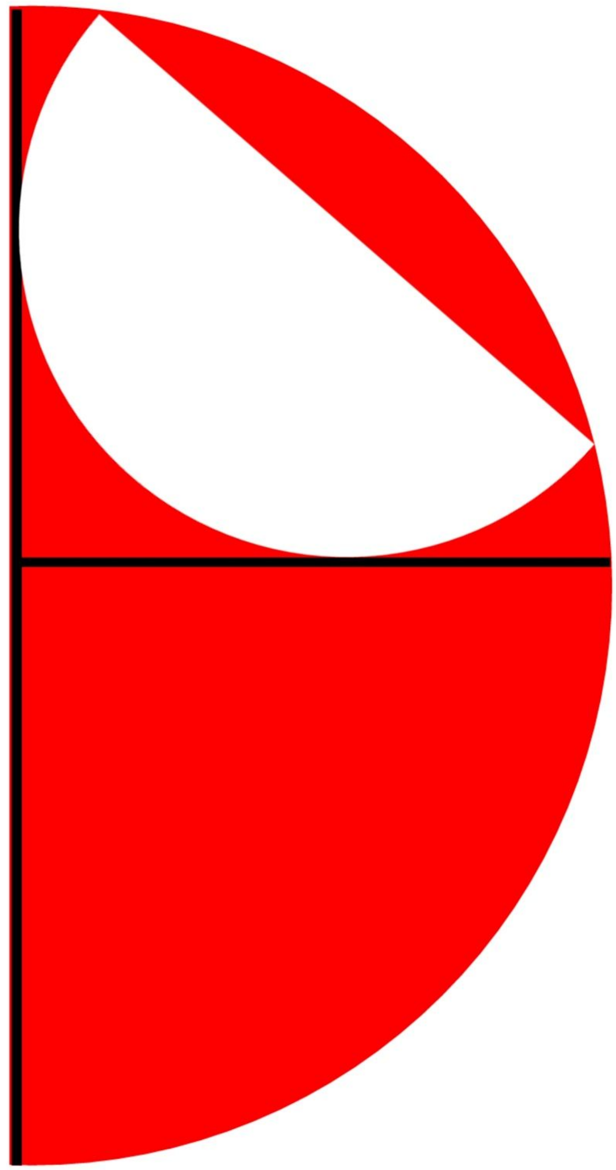


Q9• Given two concentric circles with a Chord PQ of length 8 units which is tangent to the smaller circle. Find the area outside the smaller

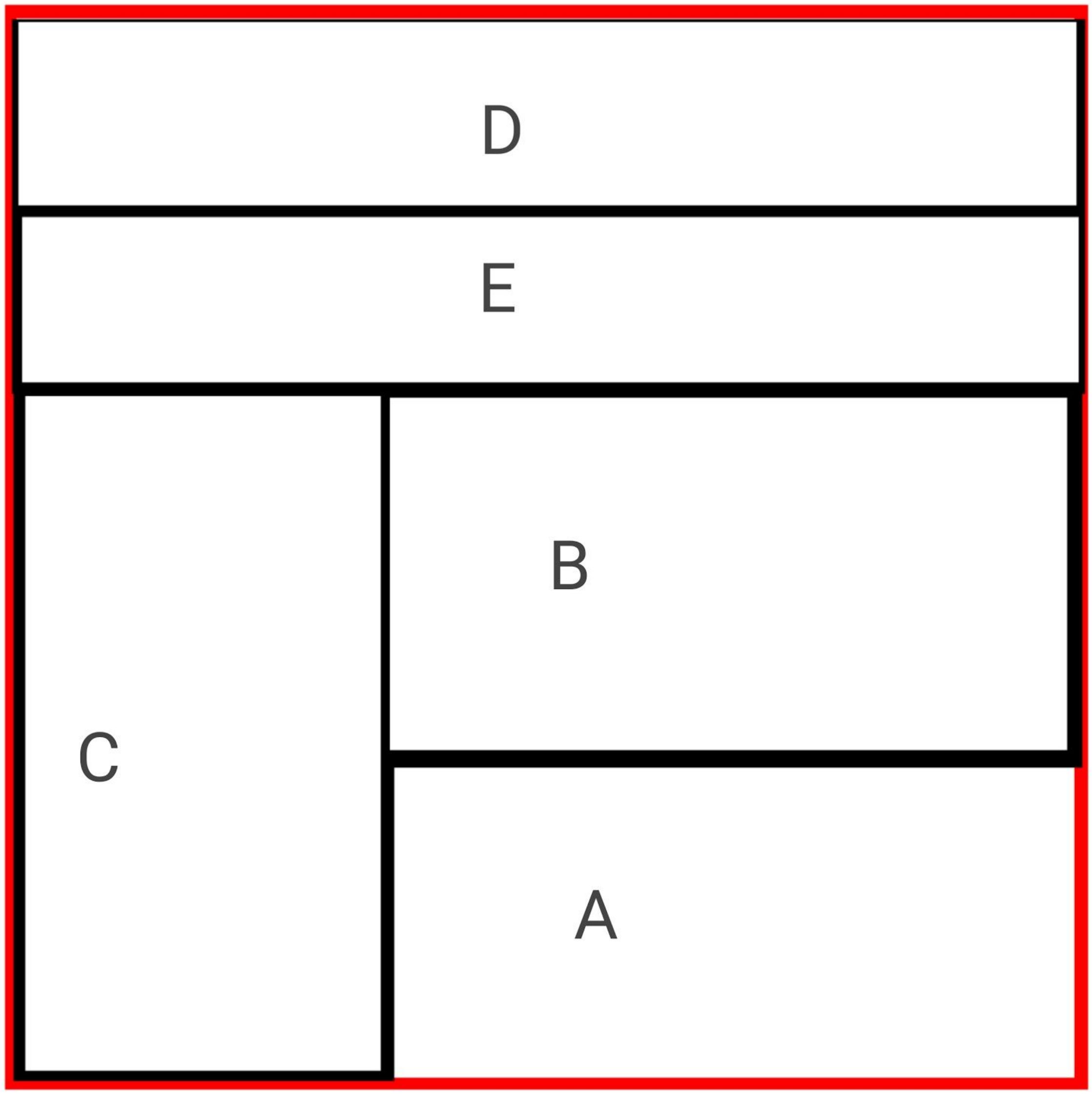
circle but inside the large circle.



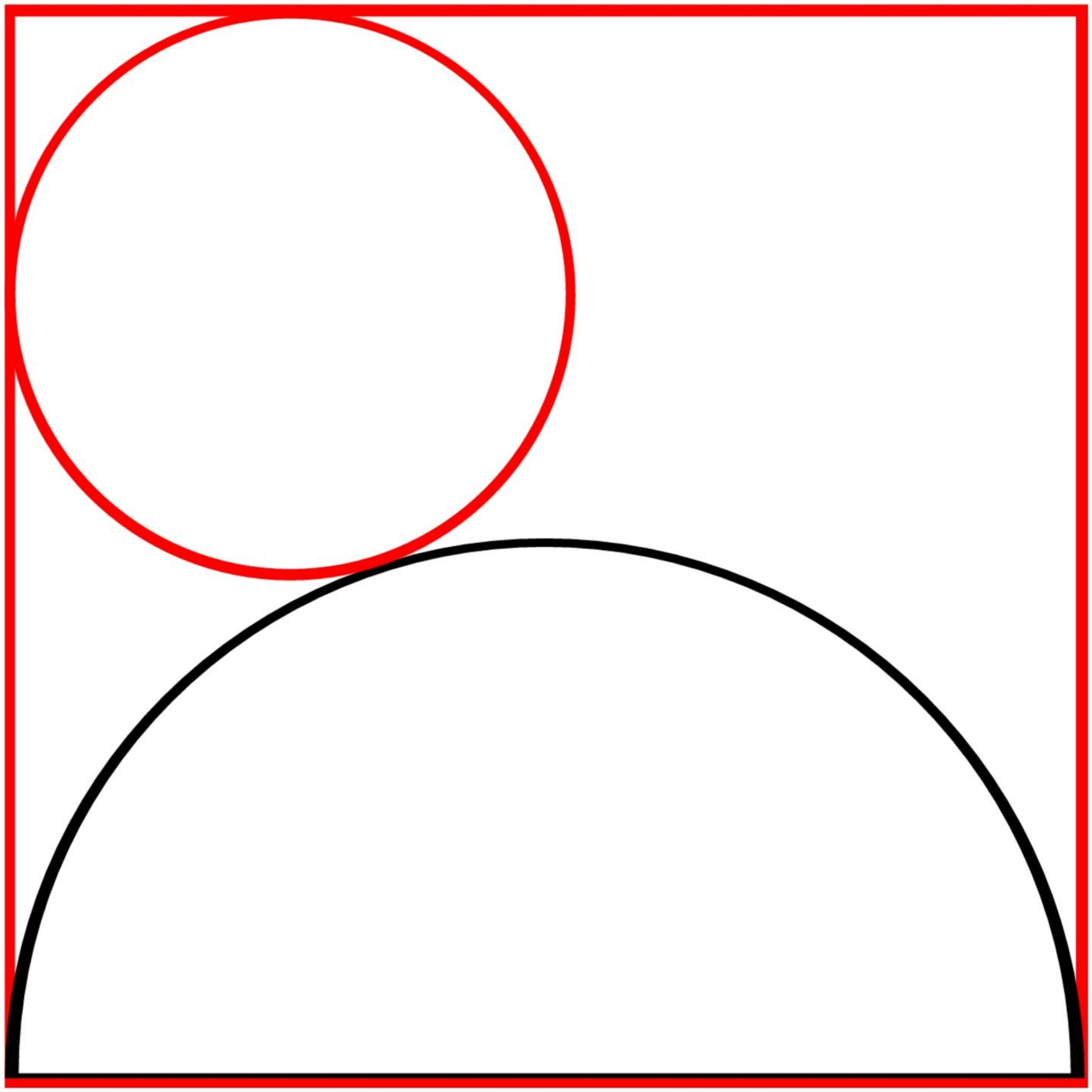
Q10• The smaller Semicircle is what part of the larger Semicircle in terms of area ?



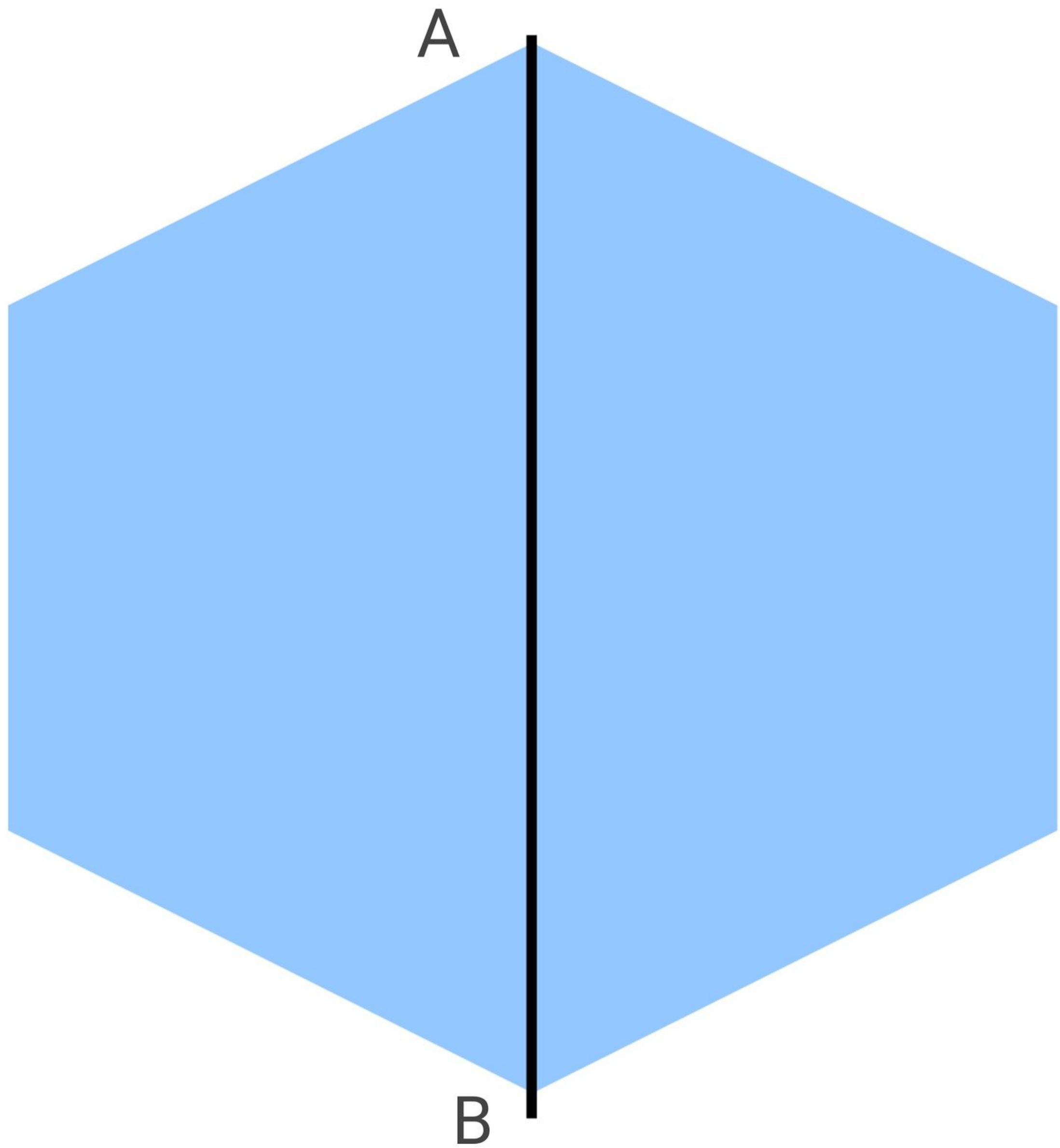
Q11• Following is a Square of length 2 cm. It has been divided into 5 rectangles, each of equal area. Find the perimeter of Rectangle E.



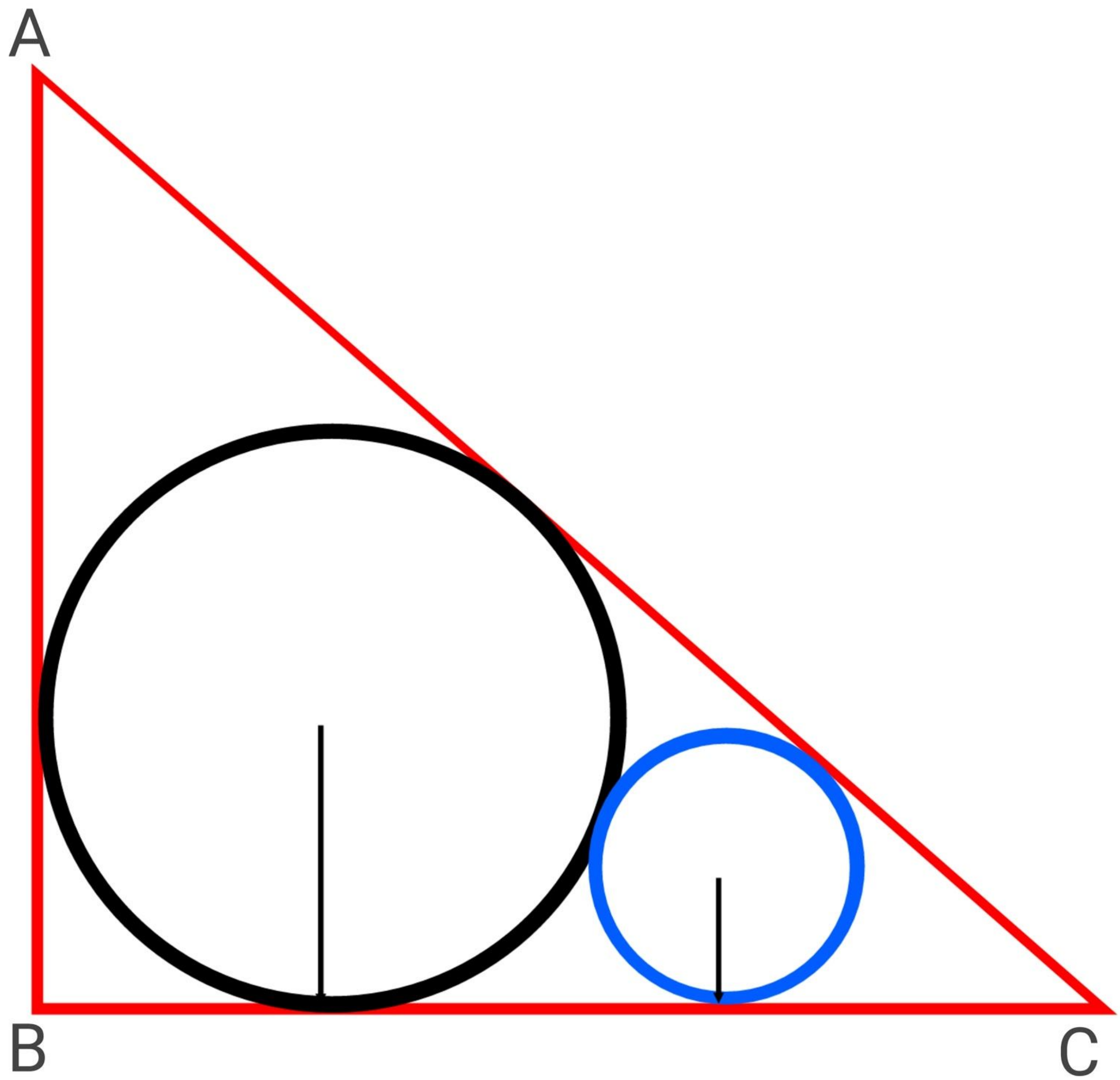
Q12• Find the radius of the smaller circle if the length of the square is 2 cm.



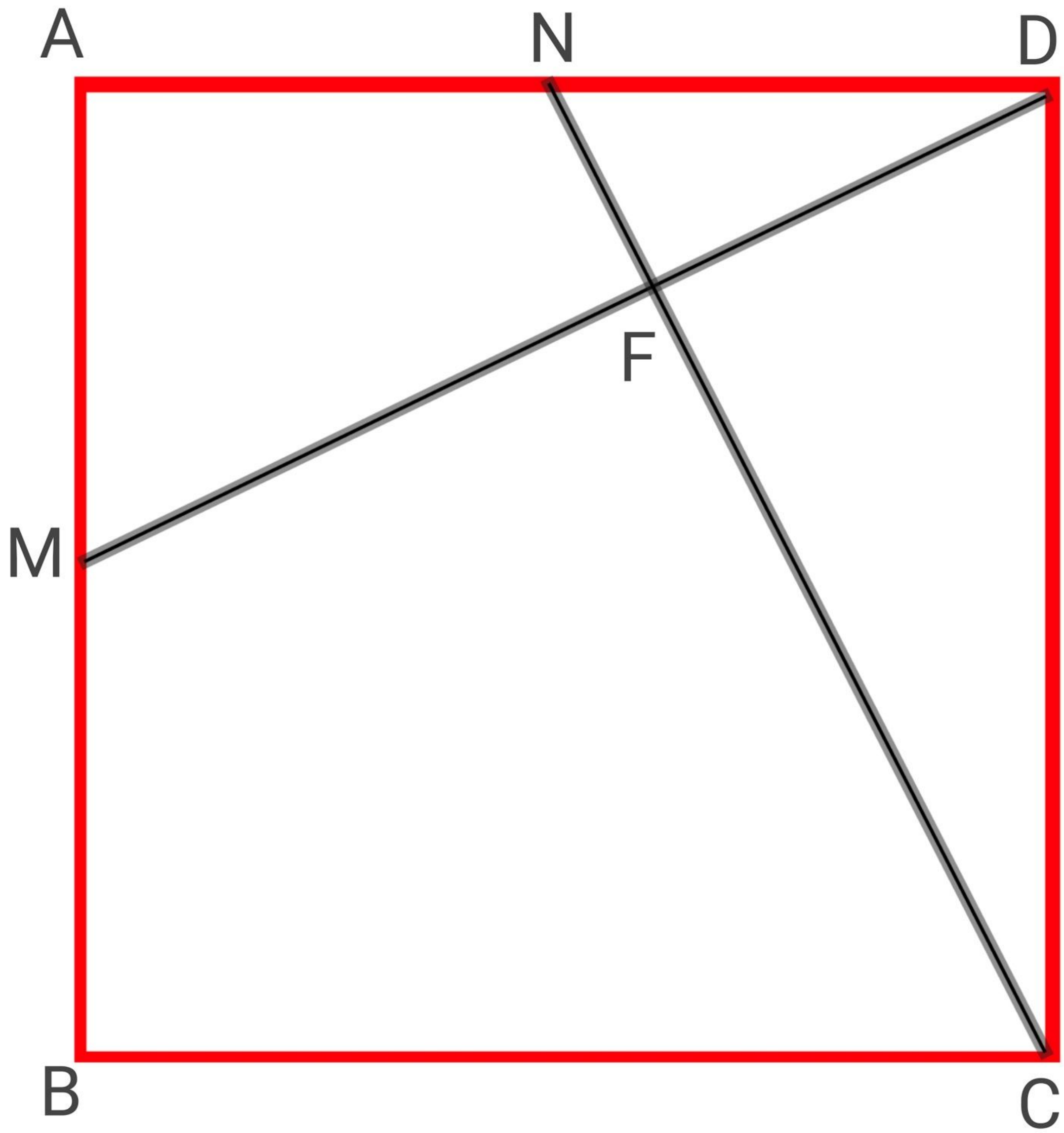
Q13• Given Hexagon of side length 2√2 cm is rotated about the line AB. The volume it will encompass is ?



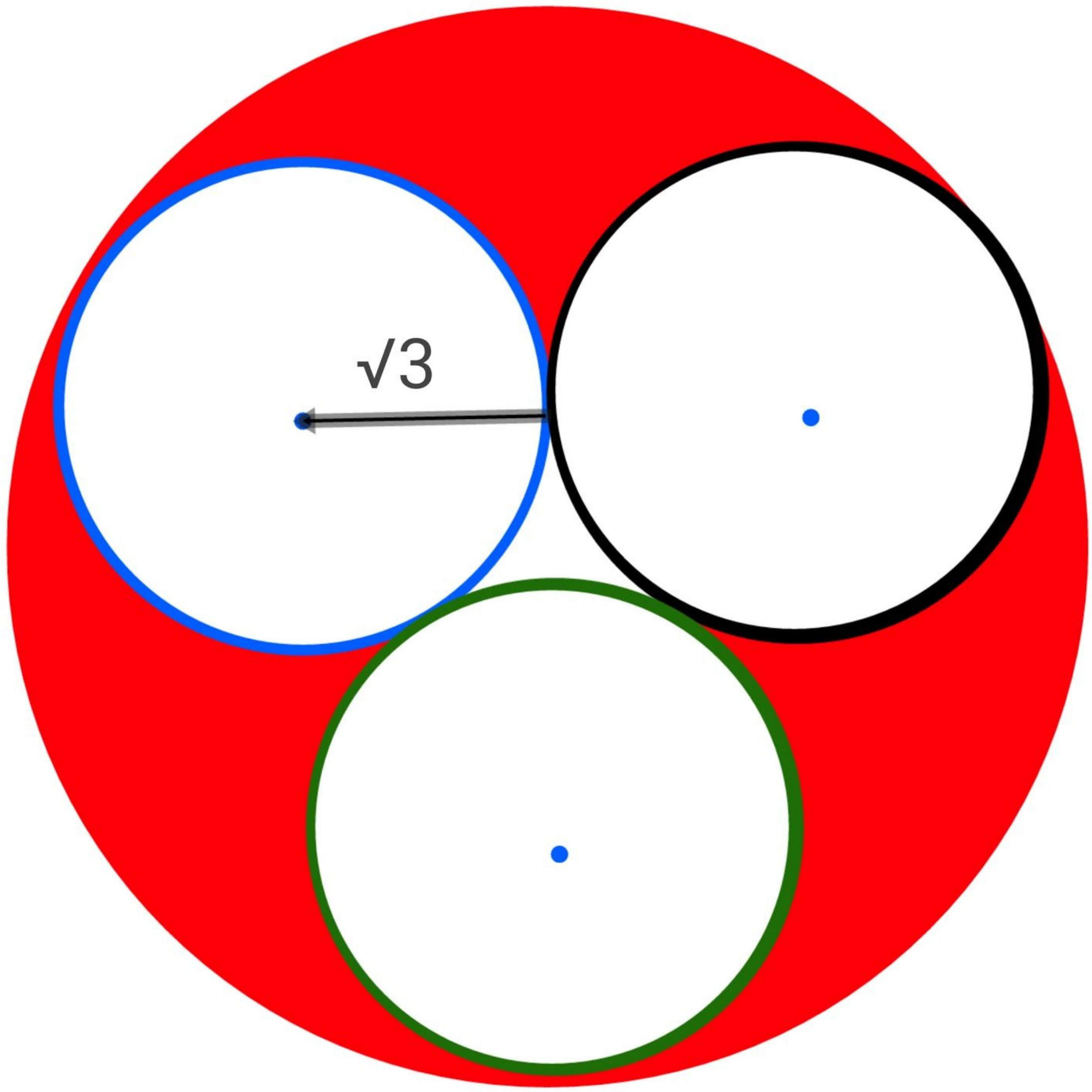
Q14• The radius of larger circle is 4 cm and smaller one has a radius of 1cm. Find the length of AB.



Q15• In a Triangle PQR, PD bisects angle QPR. PQ = 4 , PR = 3, and angle QPR = 60°. Find the length of the bisector PD.
Q16• Following is a Square of side 1 cm. M and N are the midpoints. Find the area of Quadrilateral BMFC.

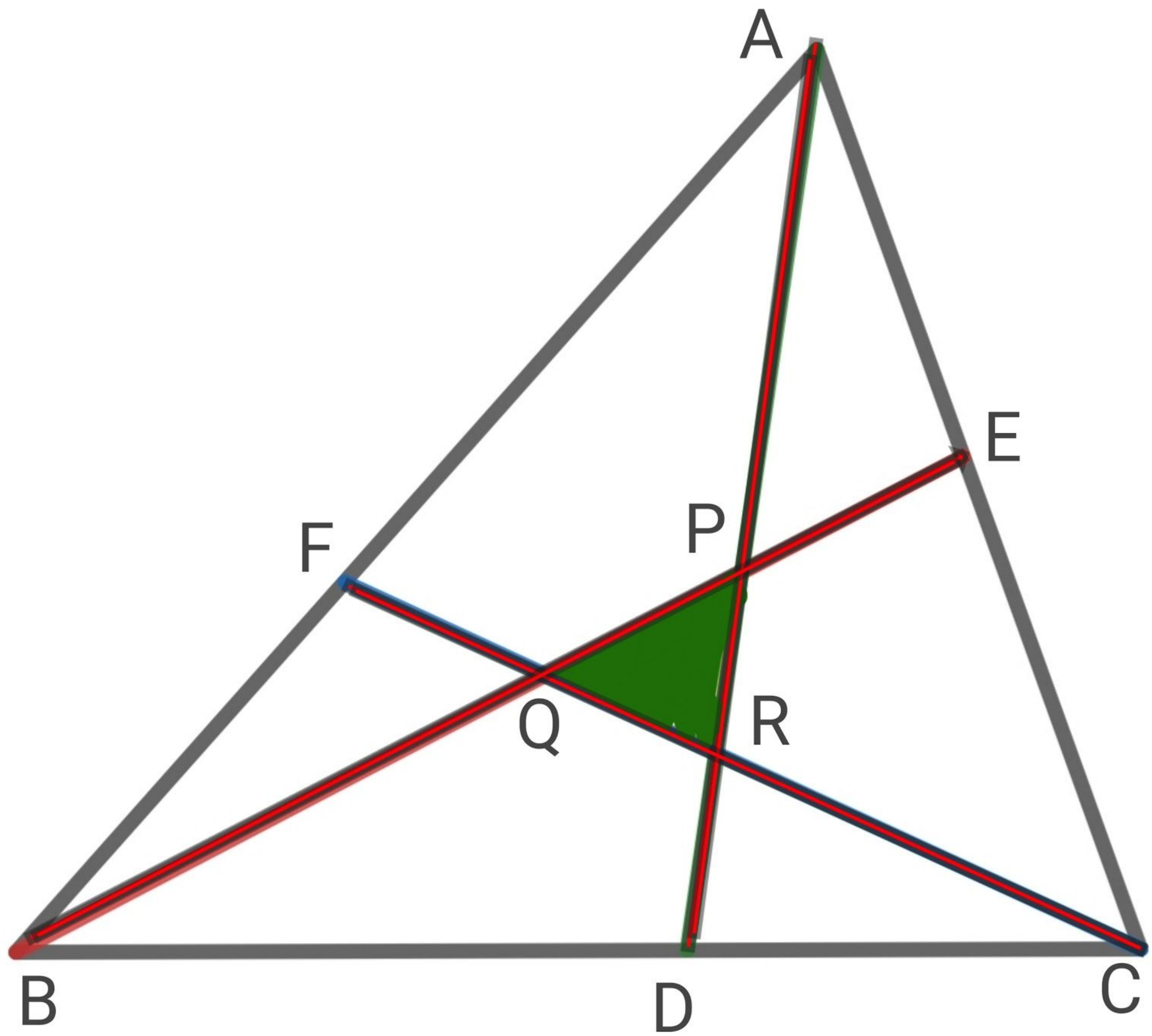


Q17• What would be the maximum volume of a Cube, that can be chiseled out of a Cone of radius √2 cm and height 6 cm ?   Q18• Following diagram has three small Circles of radius √3 cm in one large circle of radius unknown. Find the area of shaded region.



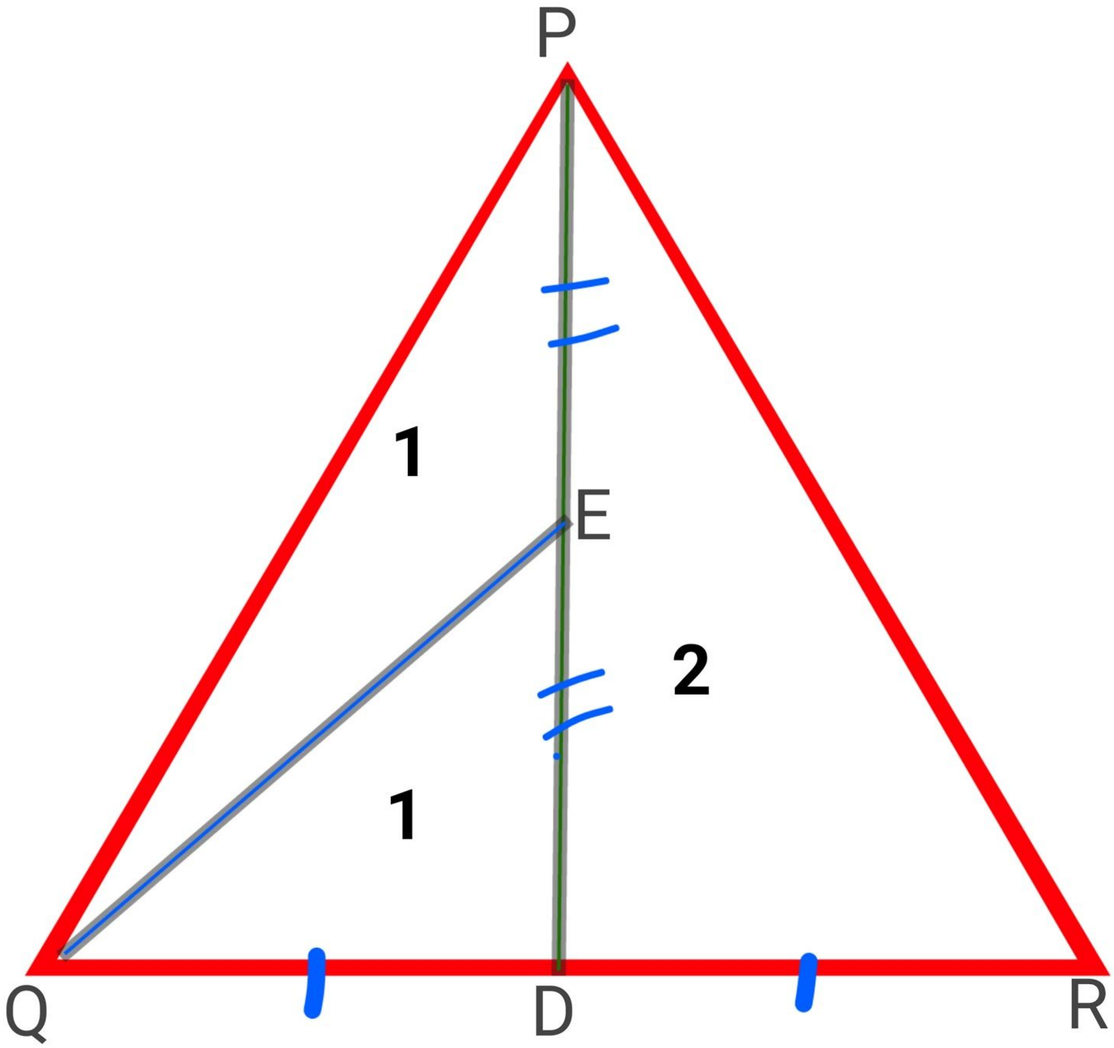
Q19• A right circular cone of height 20 cm is divided into 3 parts by two planes parallel to the base at heights of 4 cm and 10 cm from the base. Find the ratio of volume of all three parts of cone. (i.e, V 1 : V 2 : V 3).

Q20• In Triangle ABC , AD bisects CF at R , BE bisects AD at P, CF bisects BE at Q. Find the area of triangle PQR, if area of Δ ABC = 1 cm².



Solutions (Easy Questions):

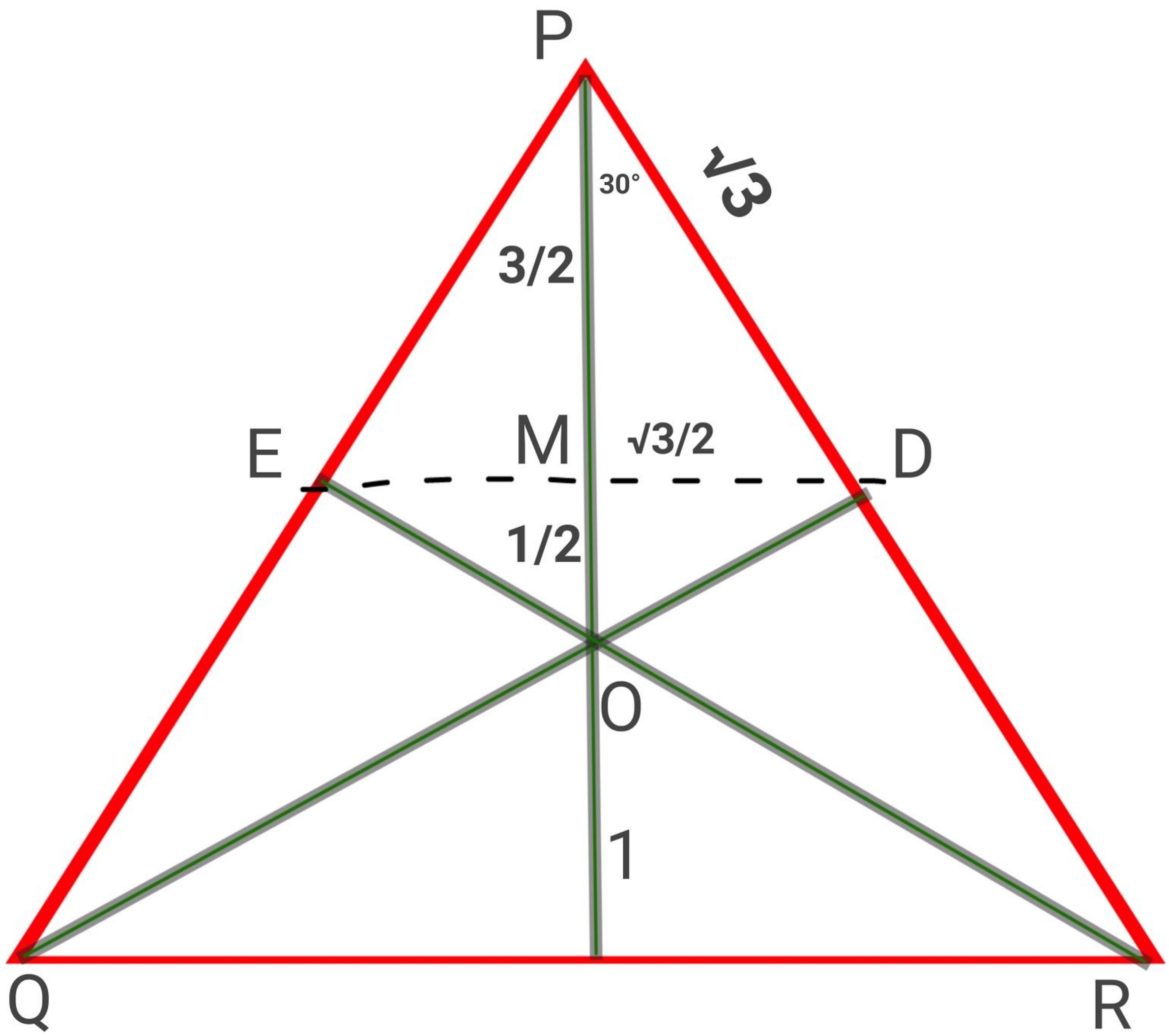
A1• Suppose Area of Δ PQR = 4cm².



Clearly, ΔQEP/ΔPQR = ¼ , by the property of triangles having common bases.

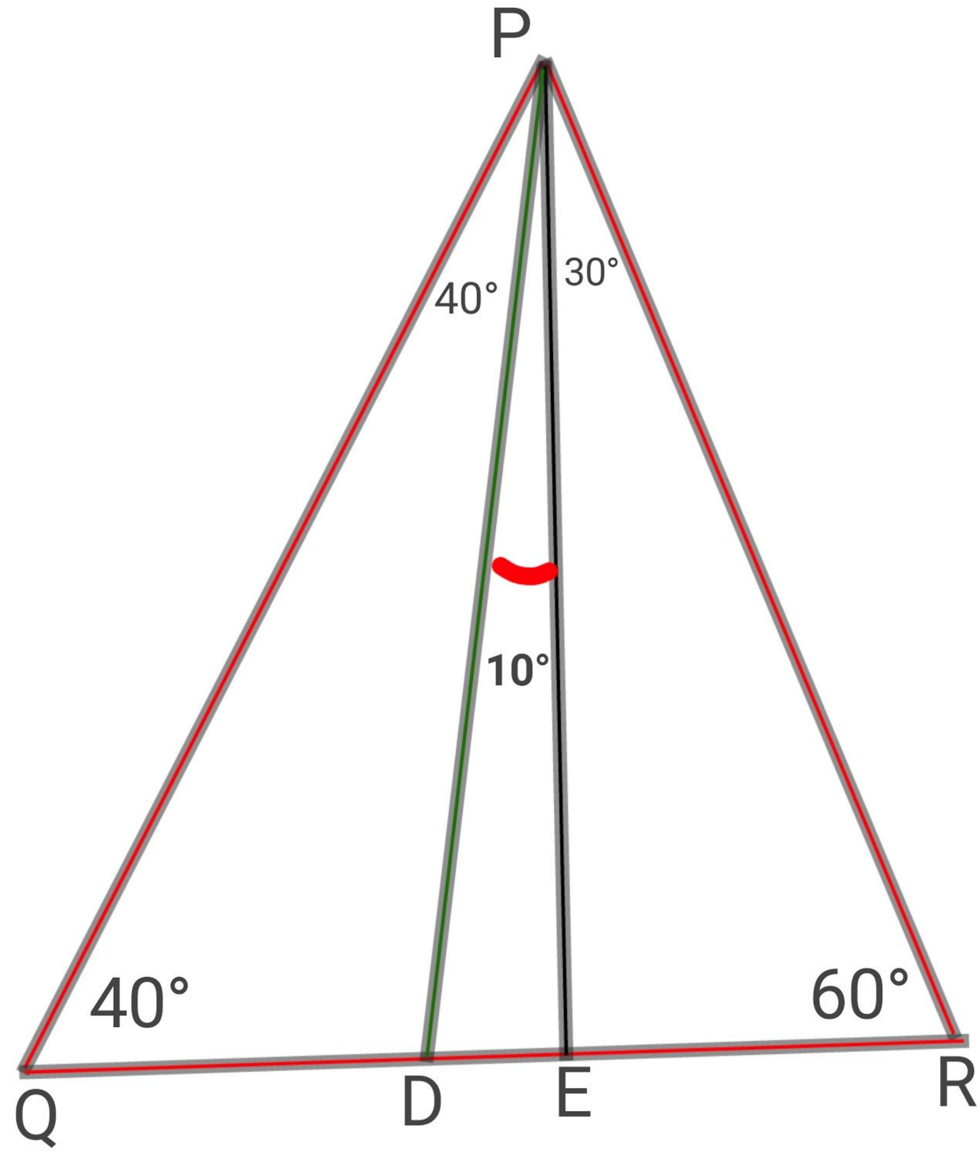
A3• Well, this one's pretty easy, the angle at the incenter would be , angle QIR = [90+ P/2] = 90+30 = 120°.

A2• I have assumed the triangle to be an equilateral triangle of side 2√3. Not to worry because the result would be universally accepted for all types of triangles



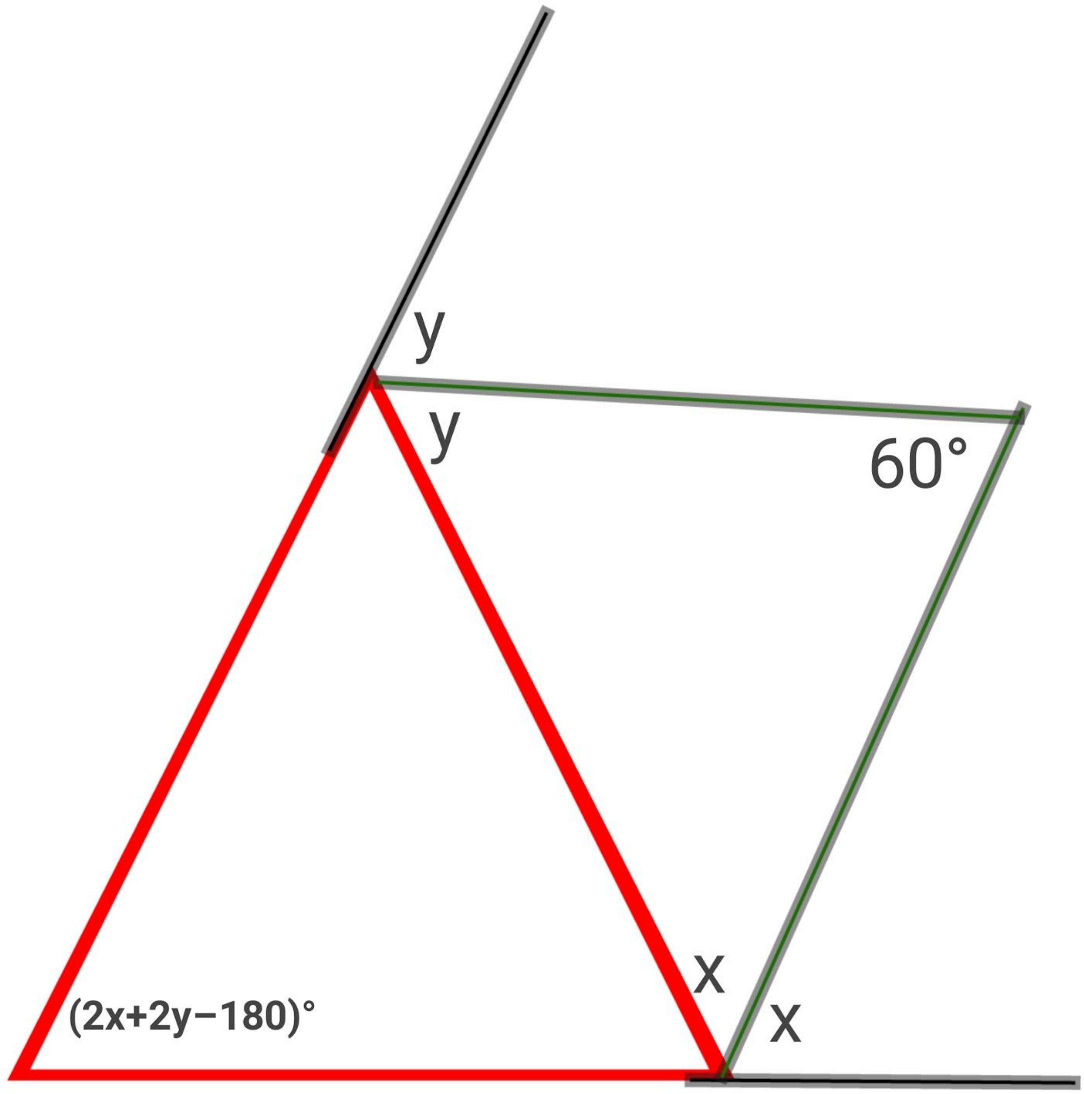
Hence, PM:MO = 3:1.

A4•



Hence, Angle DPE = 10°

A5• Let's assume one external angle as '2x' and other '2y'.



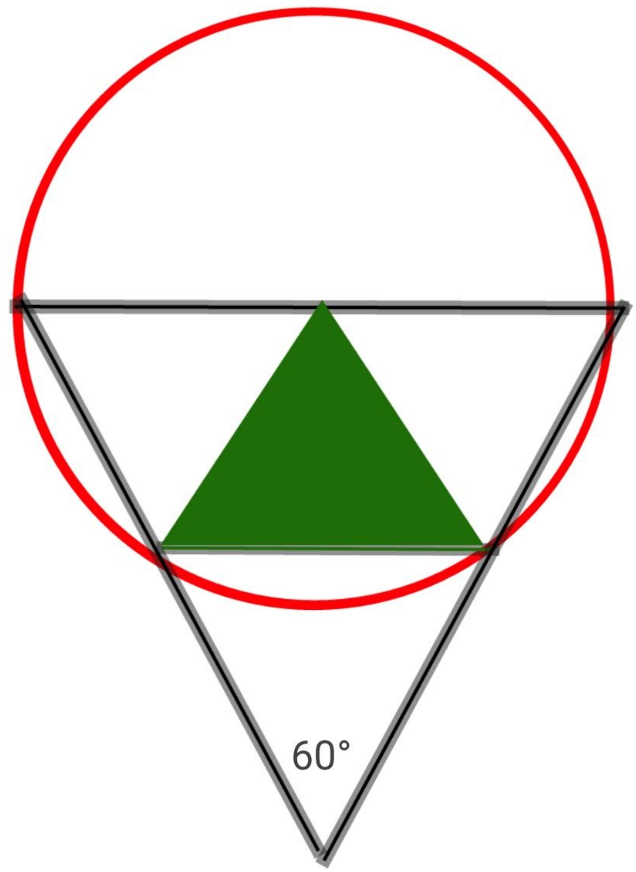
From the above figure, x+y = 120 , thus third angle = 2x+2y–180 = 60° .
A6• The centroid divides the median in the ratio 2:1. Thus if PO = 10 cm , then OD = 5 cm and PD = 15 cm.

A7• From Apollonius Theorem and Pythagoras theorem, we get a standard result
$PQ^2 + PR^2 = 5QR^2$
Thus, QR = 13 cm.
A8• Equating Area of triangle for various bases and heights,

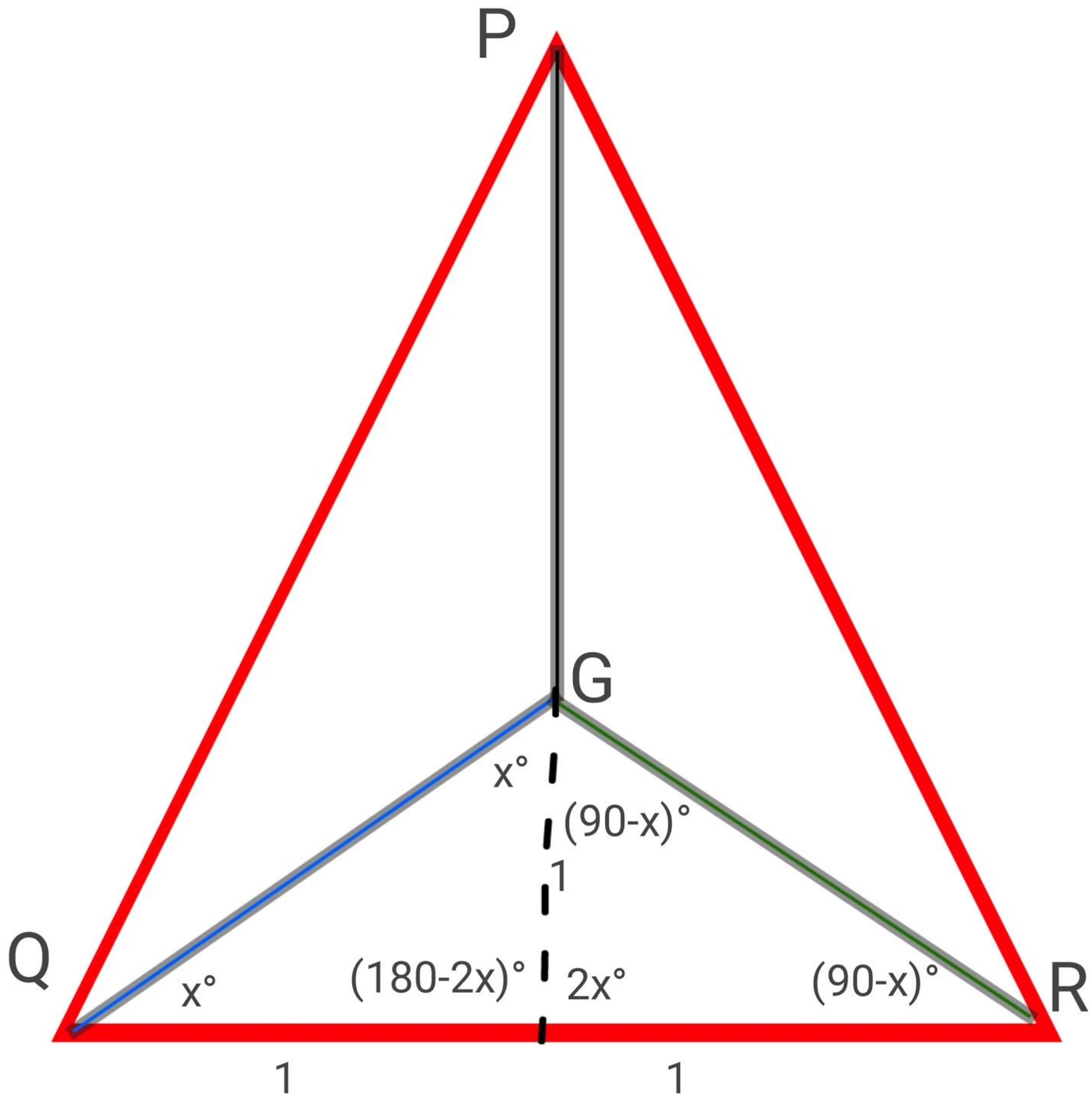
PD =1 , QE = 2 , RF = 3
½×PQ*RF = ½×QR*PD = ½×PR*QE , thus the ratio of sides = 2:6:3
A9• Angle at the Orthocenter, is (180–P) = 180–70 = 110°.

A10• In an equilateral triangle all the Heights = Medians = Perpendicular bisectors = Angle Bisectors. Here, easy peasy Option A.

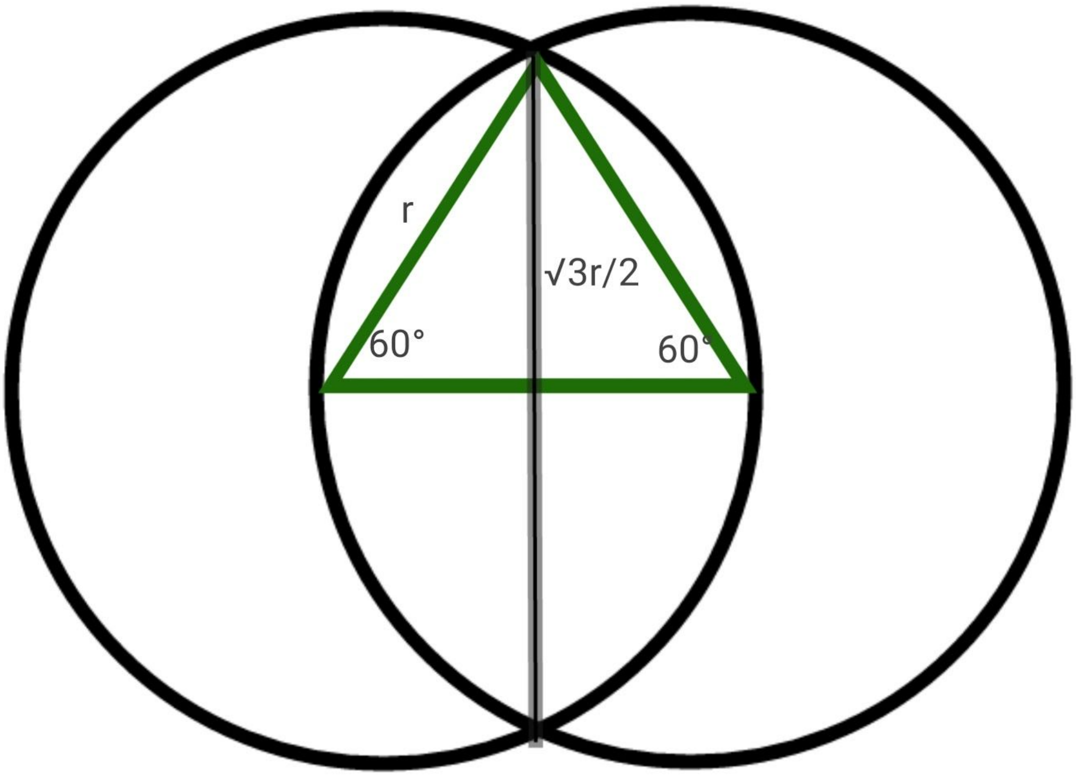
A11• Angle Bisector theorem, AB/AC = BD/DC, thus QC = 18 cm.
A12• Angle MOP = 60° , means Angle MPO = 30°. Angle MPN = 2MPO , thus MPN = 60°.
A13•



since Chord = Radius. The angle PMQ = 60°.
A14• Angle QGM = x+90-x = 90°

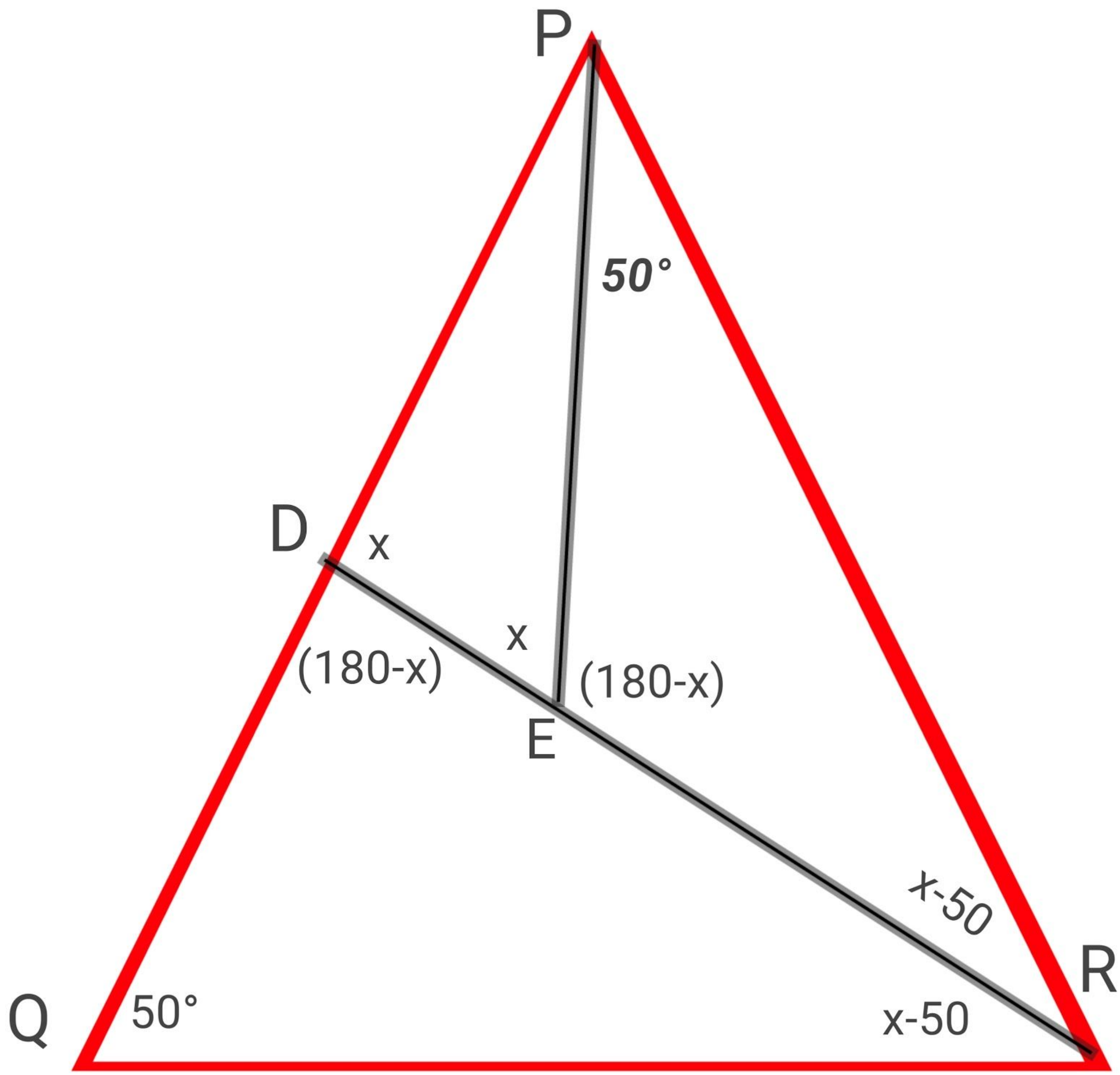


A15• I've drawn a figure and you can see the equilateral triangle between these two circles.



Length of common chord = r√3 units

A16• This one's easy again, AMO would be 68/2 = 34°. Join AB and you'll get MAB = 90-34 = 56° .
A17• Just a little Maths and you get angle RPE = 50°. Check the figure below

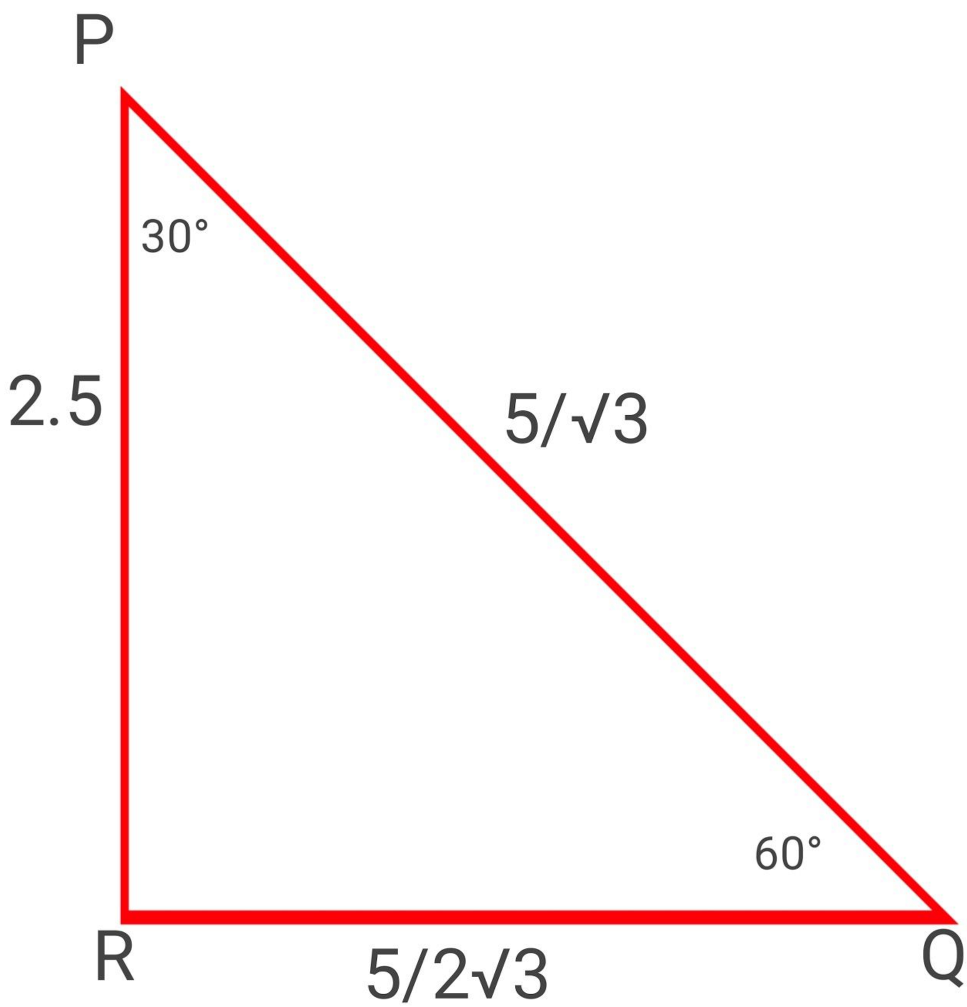


A18• Mark the angles carefully and you'll get ΔPQR and ΔPRA are similar. Thus 9/15 = PA/12 , thus, PA = 7.2 cm.
A19• Three different triangles with different heights but base same as equilateral triangle's side = X. Thus, Area of equilateral triangle = $X^2\sqrt{3}/4 = \frac{1}{2}\times[6X + 8X + 10X]$.

Thus, X = 16√3 cm.

A20• This one is pretty easy. Since QN is Perpendicular to PR, we can say that, PQ×QR = PR×QN , thus, QN = 4.8 cm. Now, using Pythagoras or similarities NR = 6.4 cm , PN = 3.6 cm. PN:NR = 9:16 .

A21• Since DE/QR =⅖, ΔPDE/ΔPQR = 4/25 , and hence ΔPDE/ΔDERQ = 4/21.
A22• Centroid divides median in the ratio 2:1. Hence PG = 8.
A23• Applying similarity in triangles PQR and DQP, we get QD = 1.
A24• Refer the diagram below,

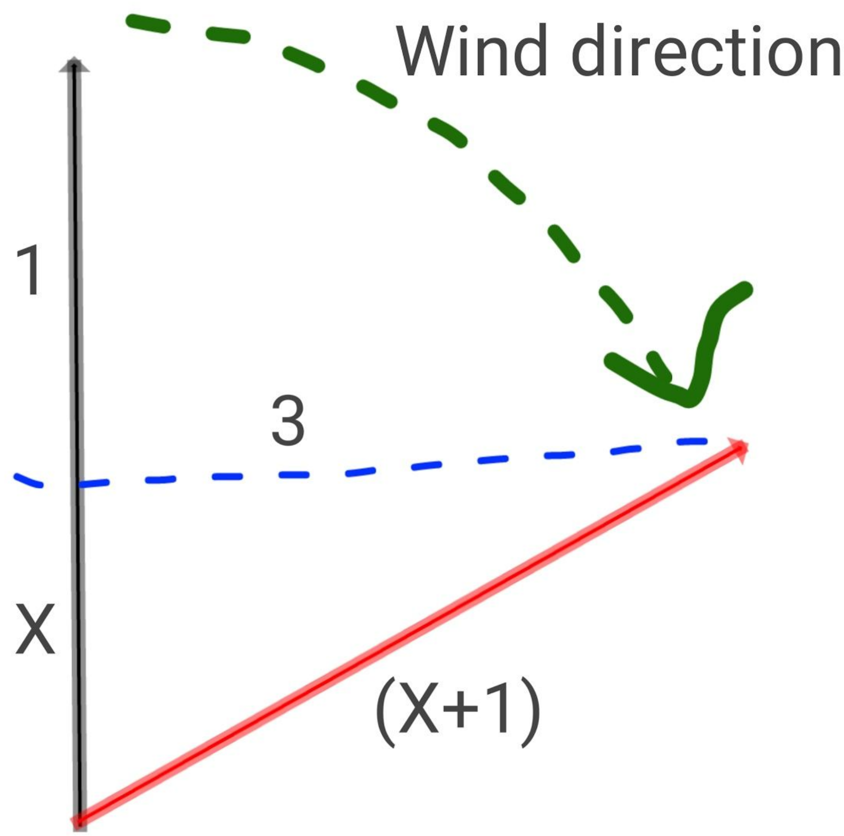


Hence, PQ = 5/√3.

A25• Length of the common tangent is given by $L = \sqrt{d^2-(R-r)^2}$, here d = Distance between the centers = R+r = 13 cm. Hence $L = \sqrt{13^2-5^2} = 12$. Hence, AB = 12, area of a square with side AB = 144 cm^2.

Solutions to Tough Questions:

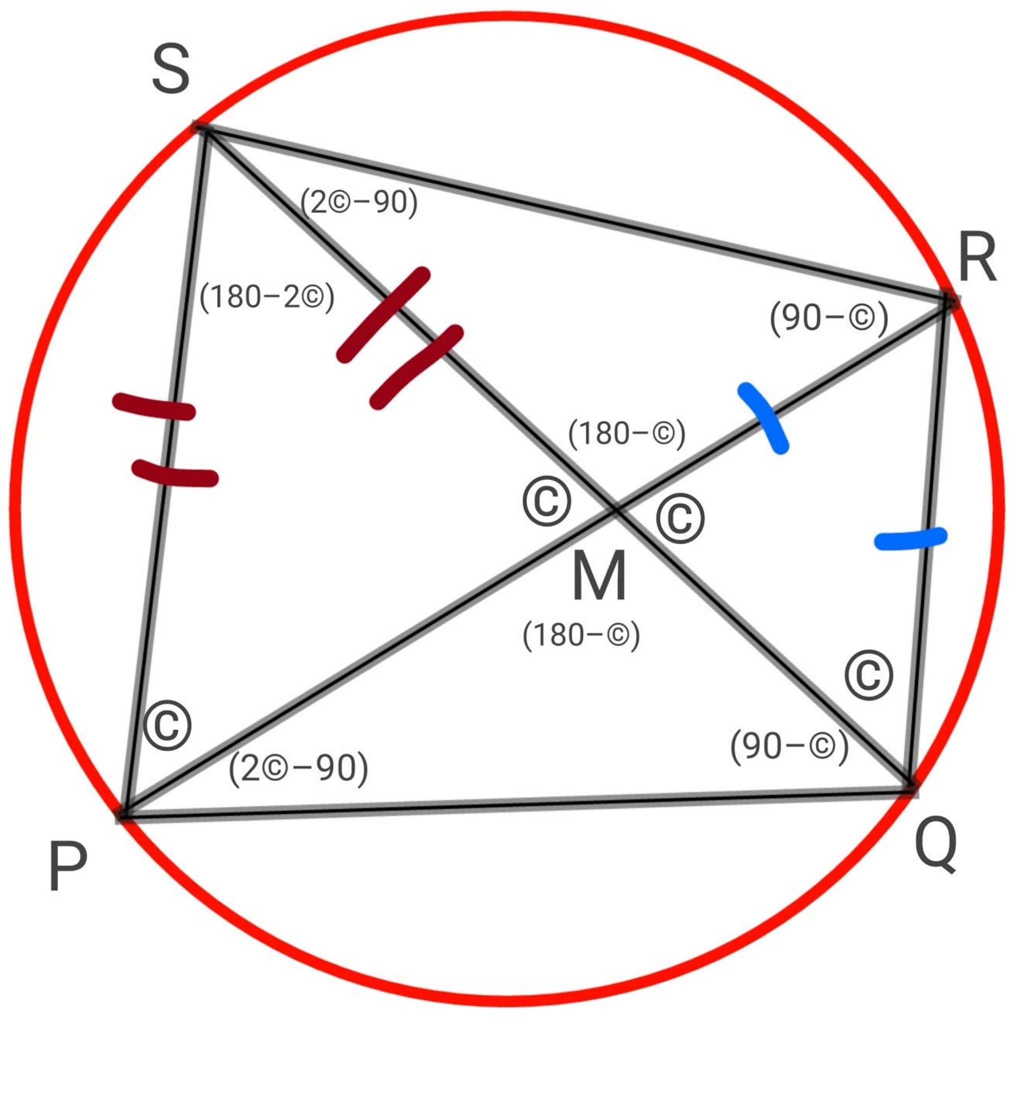
A1• Applying similarity in triangles PQR and PDE , we get PQ = 5X. Subsequently, QE = 5X–8. Given, PD = QE , so, X = 2. A2• From the detailed diagram below,



$(X+1)^2 = X^2 + 3^2$, thus X = 4 cm.

A3• Δ FQD is congruent to Δ FRD by S.A.S test. Thus, QF = 7 cm. Now, apply Similarity in Triangles PQF and PQR, we get $PQ^2 = 16 \times 9$, PQ = 12 cm. Thus, Area Δ PQF = $14\sqrt{5}$ cm^2.
A4• Let the side of the square = X cm. Apply similarity in Δ ASD and Δ DRQ , (X–AP)/X =3/5, AP = 2X/5. Now, Δ GAP and Δ GRQ are similar. Thus, AG/(AG+8) =2/5, AG = 12 cm.

A5• This question is a bit lengthy , not tough but lengthy. Here's the solution figure,



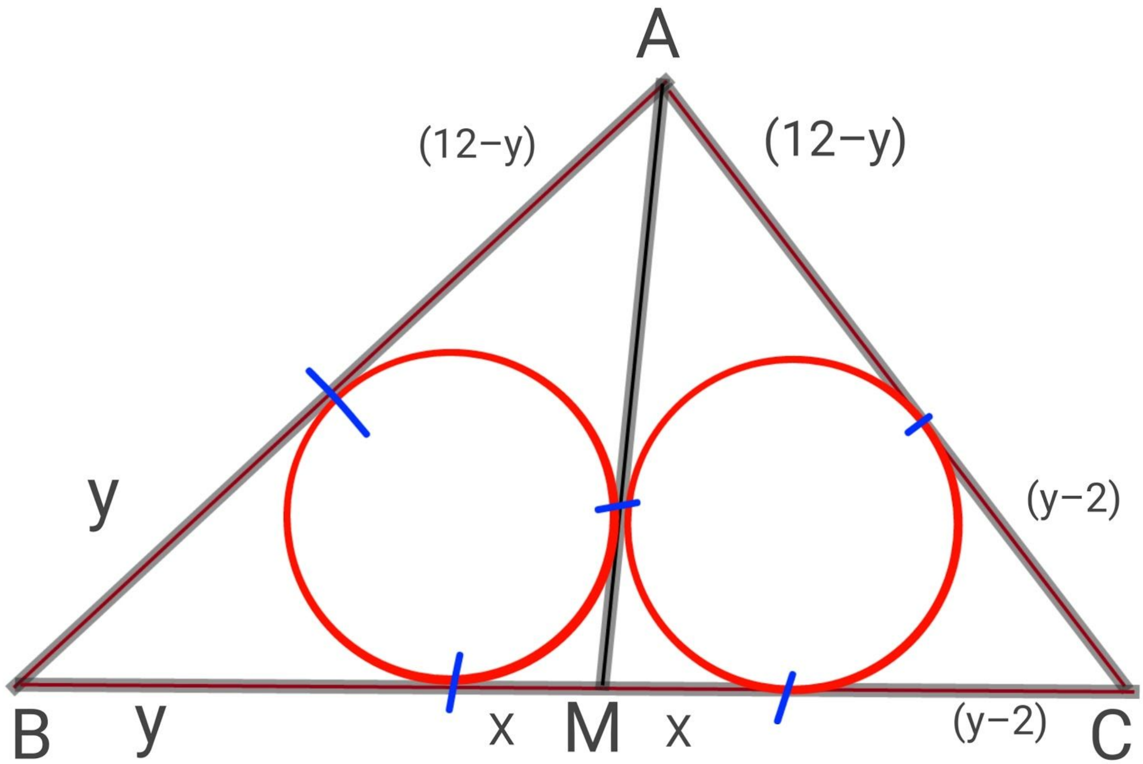
Let PM = X , SM = Y. Also, note that PR the diameter subtends right angles at S and Q respectively. Now, applying similarity in Δ PMS and Δ RMQ, we get Y/(125-X) = X/(117-Y). By Pythagoras theorem, $SR^2 = (125^2–Y^2)$ and $PQ^2 = X(250–X)$.

Also, Δ PMQ and Δ SMR are similar, and we get SR/PQ = Y/X. Thus, $Y^2/X^2 = (125–Y^2)/X(250–X)$. From these two equations we get one equation in X, which is

$585\sqrt{5} = \sqrt{X}[(375–2X)/\sqrt{2}]$. On solving we get , X = 90 cm and Y = 75 cm. Hence, PQ = 120 cm.

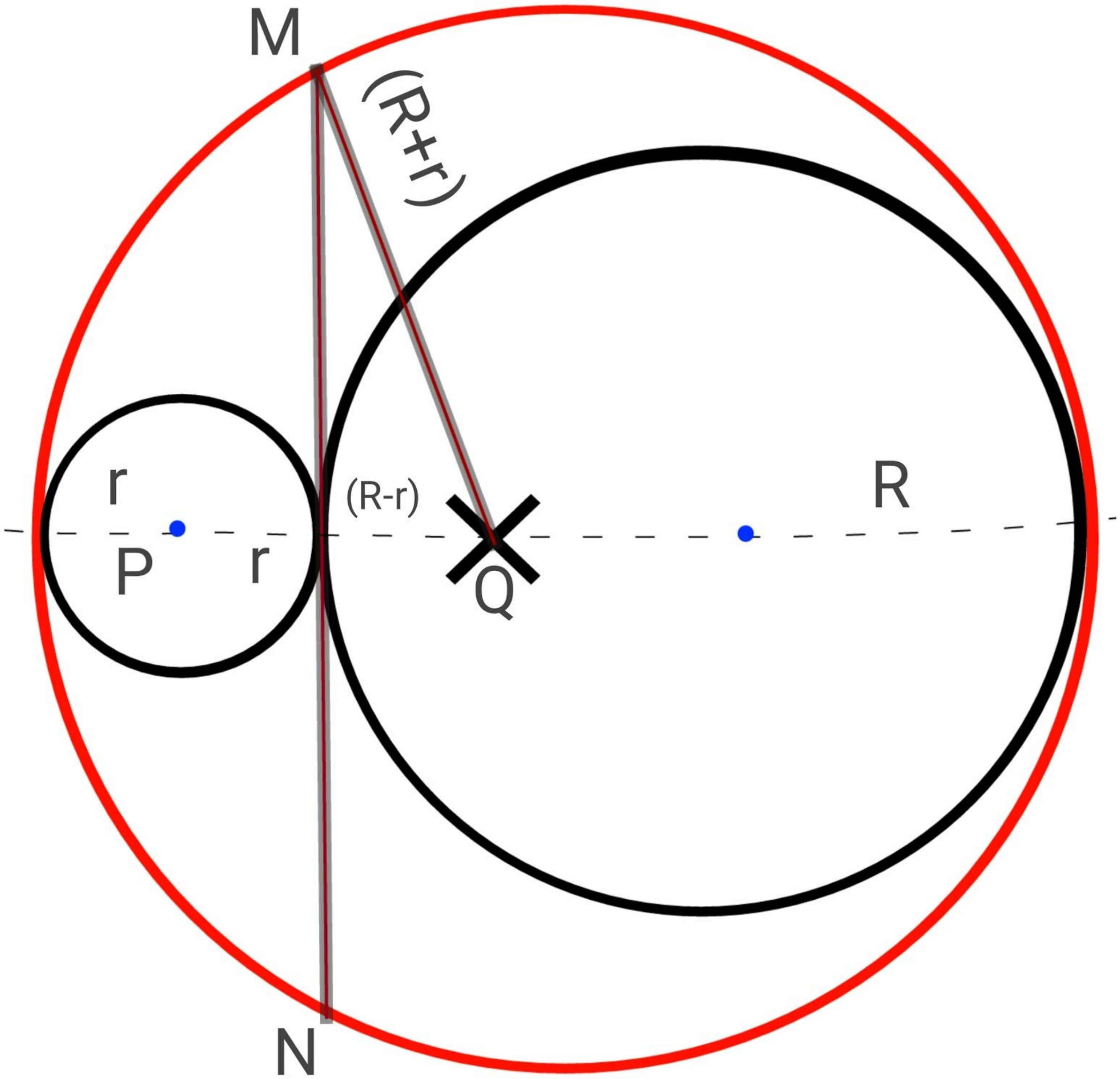
A6• Area of ABC = $8\sqrt{21}$ cm^2. Here I have used ,Area of a Triangle = ½×a×b×Sin(C).
So, Δ APS/ Δ ABC = 5×6/10×10 = 3/10 = 24/80 ,
Similarly, Δ QBR/Δ ABC = 10/80 , Δ CRS/Δ ABC = 12/80.

Remaining area out of 80 = 80–(24+10+12) = 34/80 which is area of Quadrilateral PQRS with respect to Δ ABC. Thus Area of PQRS = $17\sqrt{21}/5$ cm^2.
A7• It would be clear from the diagram below,



Since, BC = 18 = 2x+2y–2, and length of BM = y+x, we get BM = 10 units.

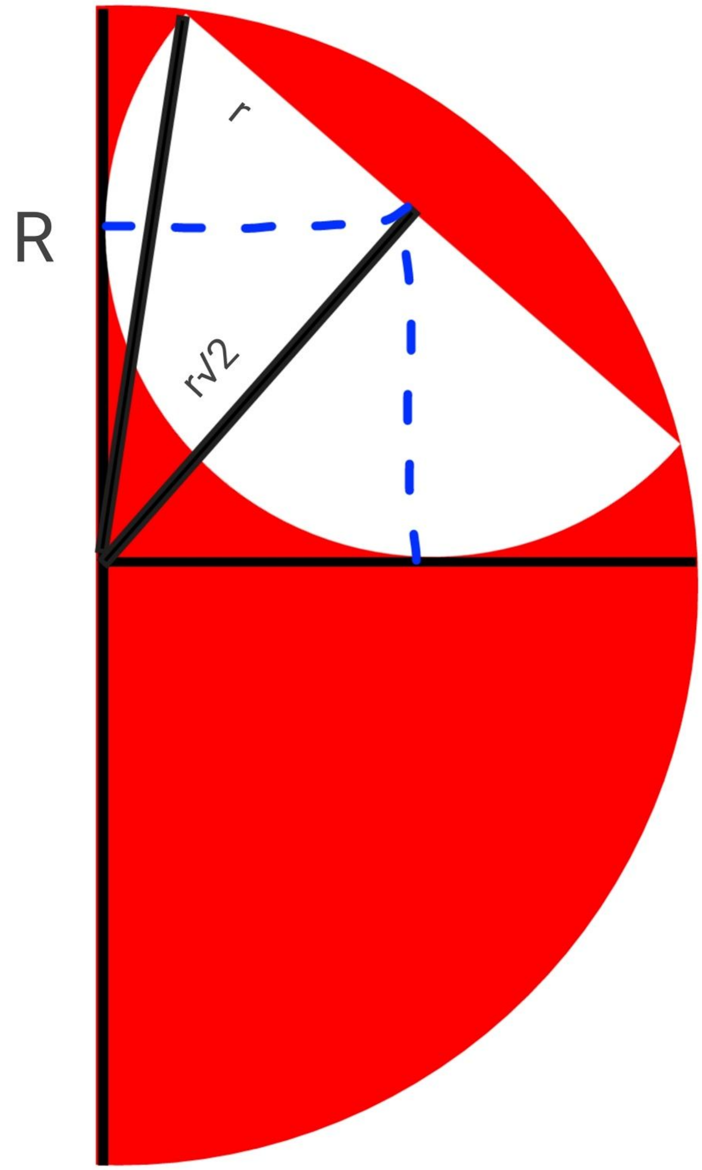
A8• Let the radius of medium circle be ‘R’ , and small circle be 'r’. Thus, radius of large circle Would be (R+r).



Now, area of the part that has been asked would be,
$\pi(R+r)^2-\{\pi R^2+\pi r^2\} = 2\pi Rr$.

Now, by Pythagoras theorem, $Rr = 16$. Thus area = 32π cm². A9• Area outside small circle but inside large circle = $\pi\{R^2-r^2\}$. And by Pythagoras theorem, $R^2-r^2 = 16$. Thus , Area = 16π cm².

A10• Let the radius of large Semicircle be 'R' and smaller one be 'r'.

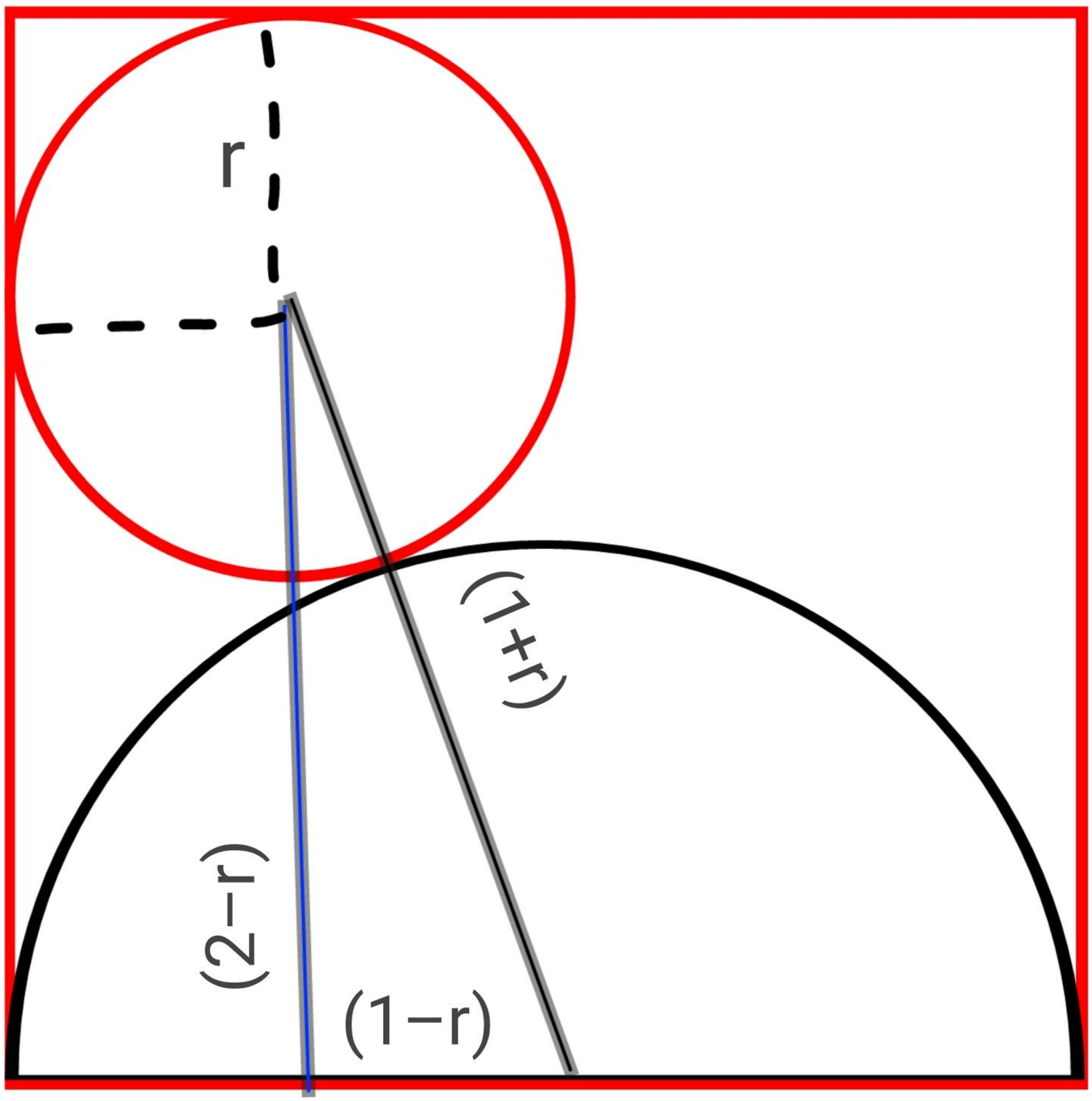


by Pythagoras theorem, $R^2 = 3r^2$.
Area of Smaller Semicircle = $\pi r^2/2$ and Area of larger Semicircle = $\pi R^2/2 = 3\pi r^2/2$. Ratio = ⅓.

A11• Easiest in this slot, let the width of Rectangle E be 'Y'. As we know all rectangles have equal area, hence area of the square = 2^2 = 4 will be divided into five equal parts i.e, $\frac{4}{5}$. So, area of Rectangle E = $2 \times Y = \frac{4}{5}$, thus $Y = \frac{2}{5}$. Now, perimeter of Rectangle E = $2\{\frac{2}{5} + 2\} = 24/5$

cm.

A12• The solution would be clear from the diagram below,



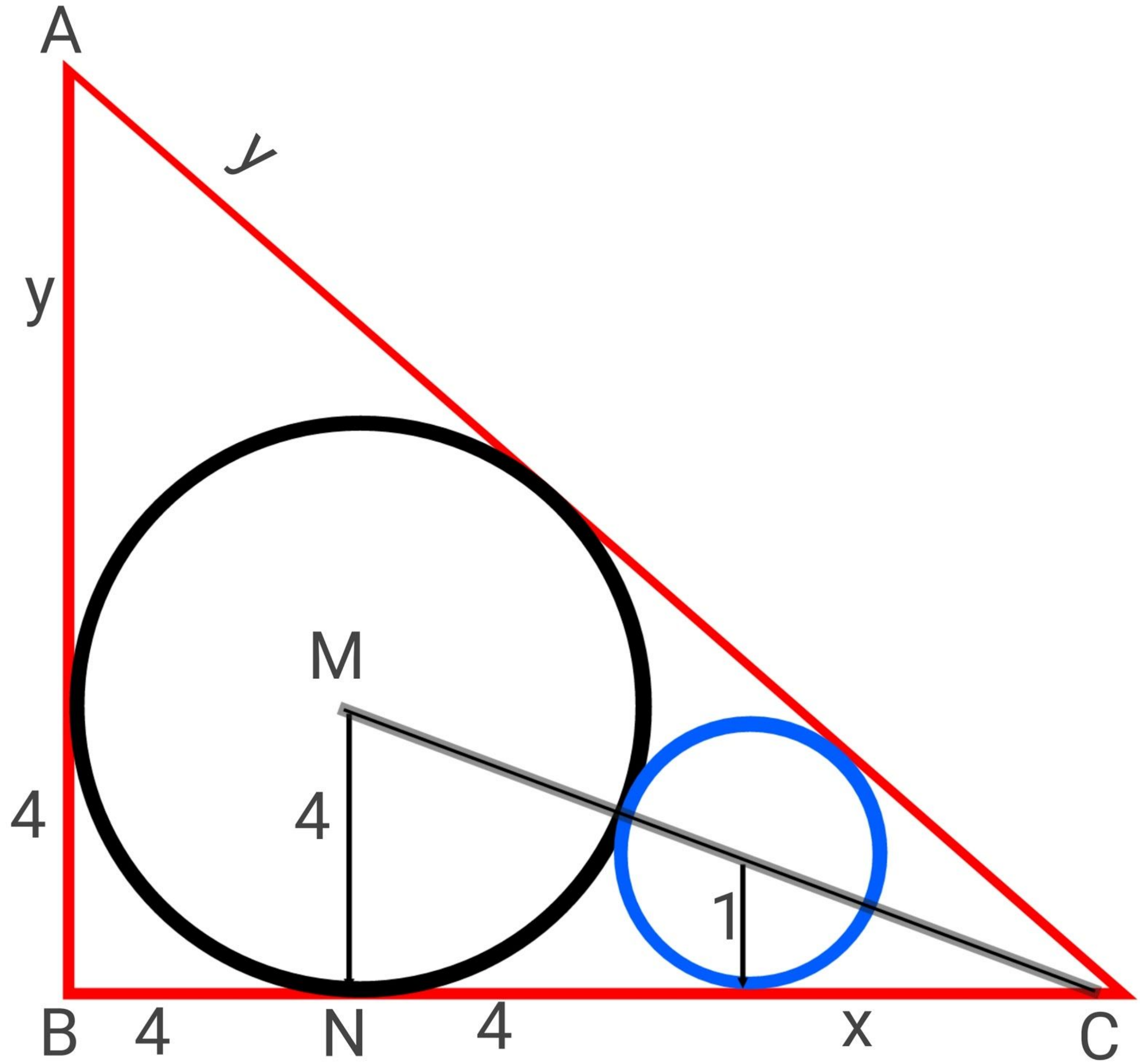
Solving by Pythagoras theorem, $(2–r)^2 + (1–r)^2 = (1+r)^2$,

$r = 4–2\sqrt{3}$.

A13• You get two cones of radius = $\sqrt{6}$ cm ($a\sqrt{3}/2$) and height = $\sqrt{2}$ cm ($a/2$), and one Cylinder of radius = $\sqrt{6}$ cm and Height = $2\sqrt{2}$ cm.

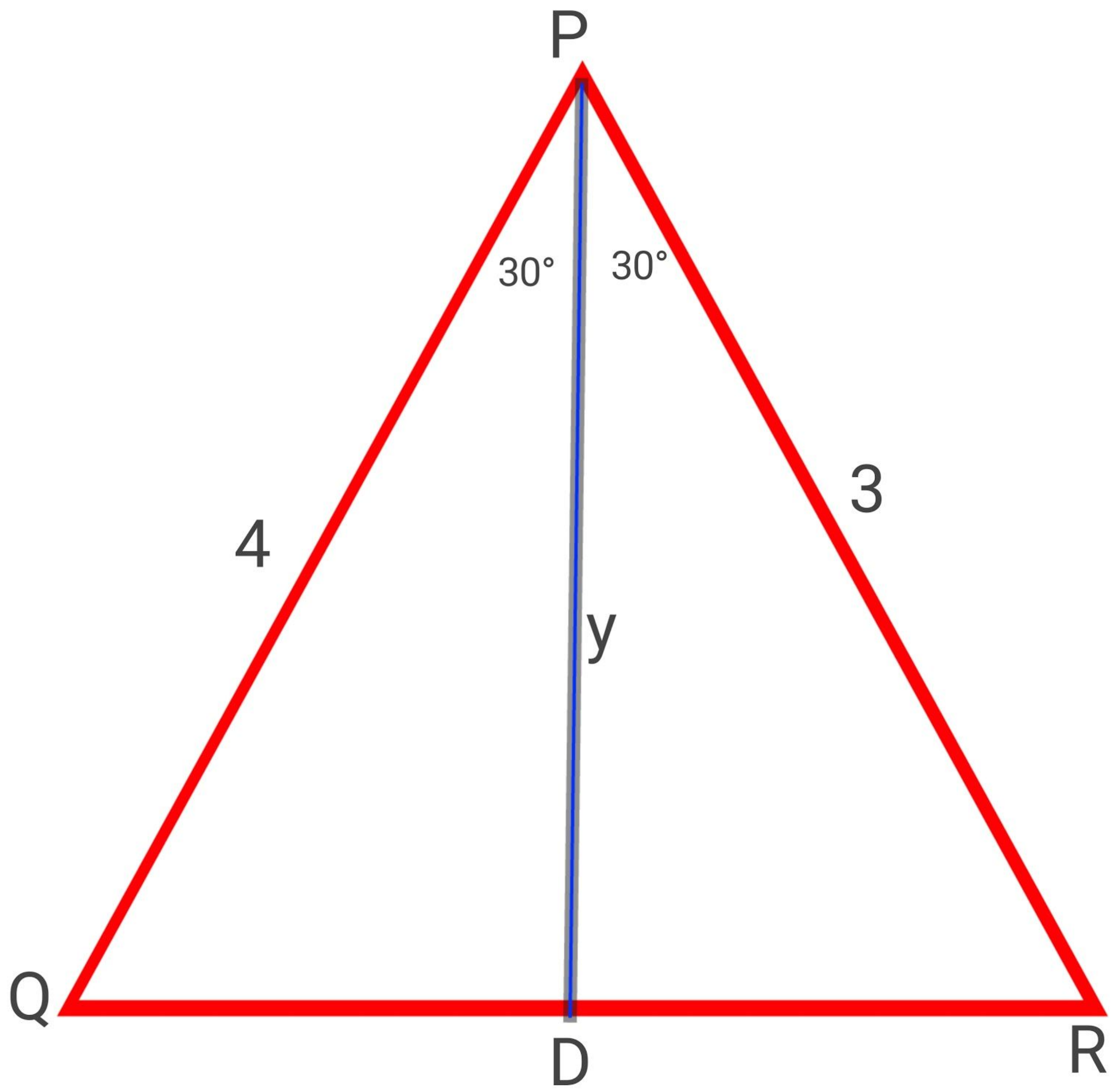
Volume of a cone = ⅓×π(√6)²×√2 = 2√2π , two cones = 4√2π cm³. Volume of cylinder = π(√6)²×2√2 = 12√2π cm³. Total volume encompassed by the Hexagon = 16√2π cm³.

A14• Using Δ = r×s , we get ½×(y+4)(x+8) = 4×(x+y+8) ,

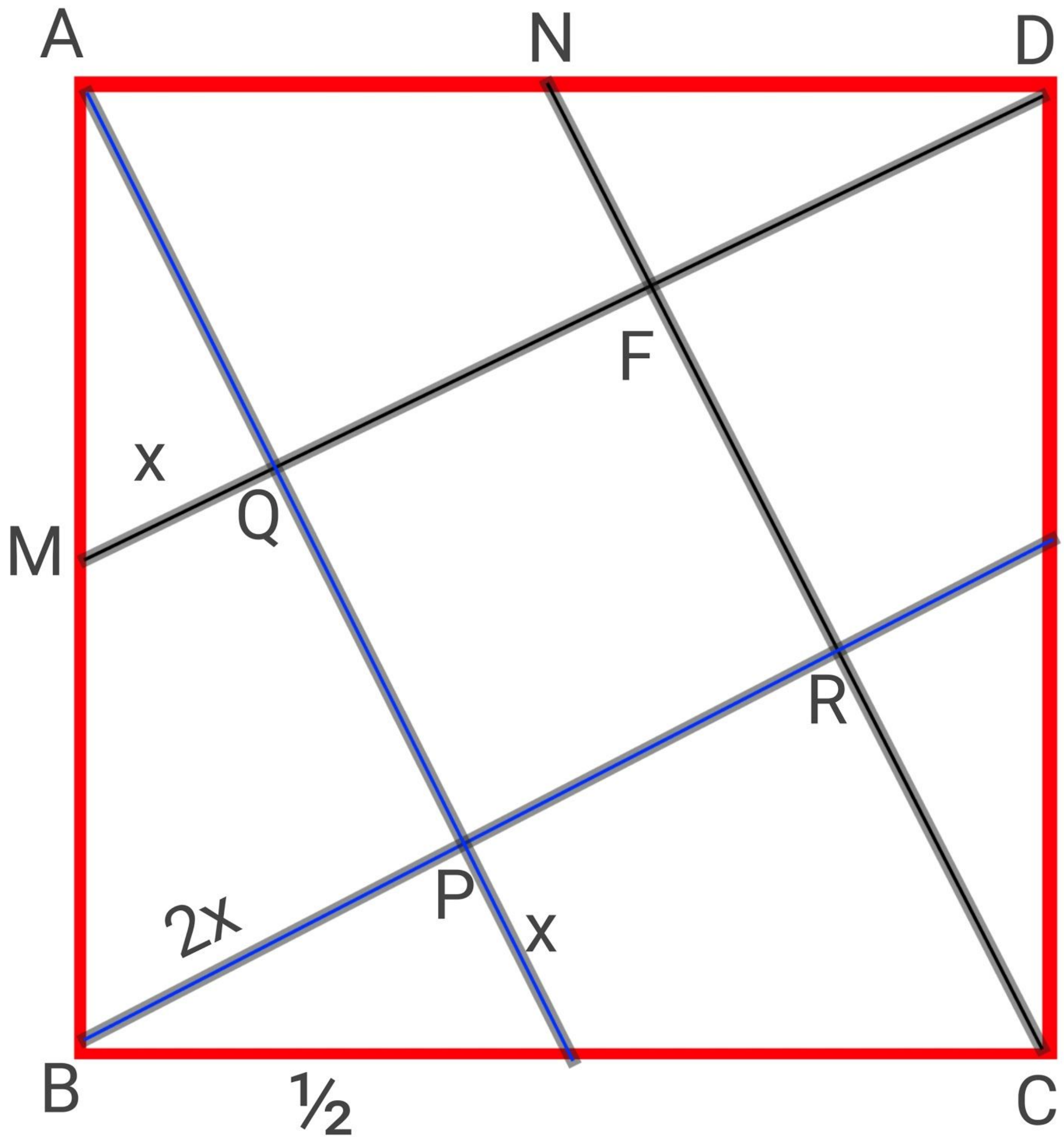


On solving this, we get, x(y–4) = 32. Also, Applying similarity in Triangle MNC, we get x/(x+4) = ¼ , thus x = 4/3. Substituting this above we get, y = 28 cm. And AB = 28+4 = 32 cm.

A15• Here we'll use A = ½×abSinC.



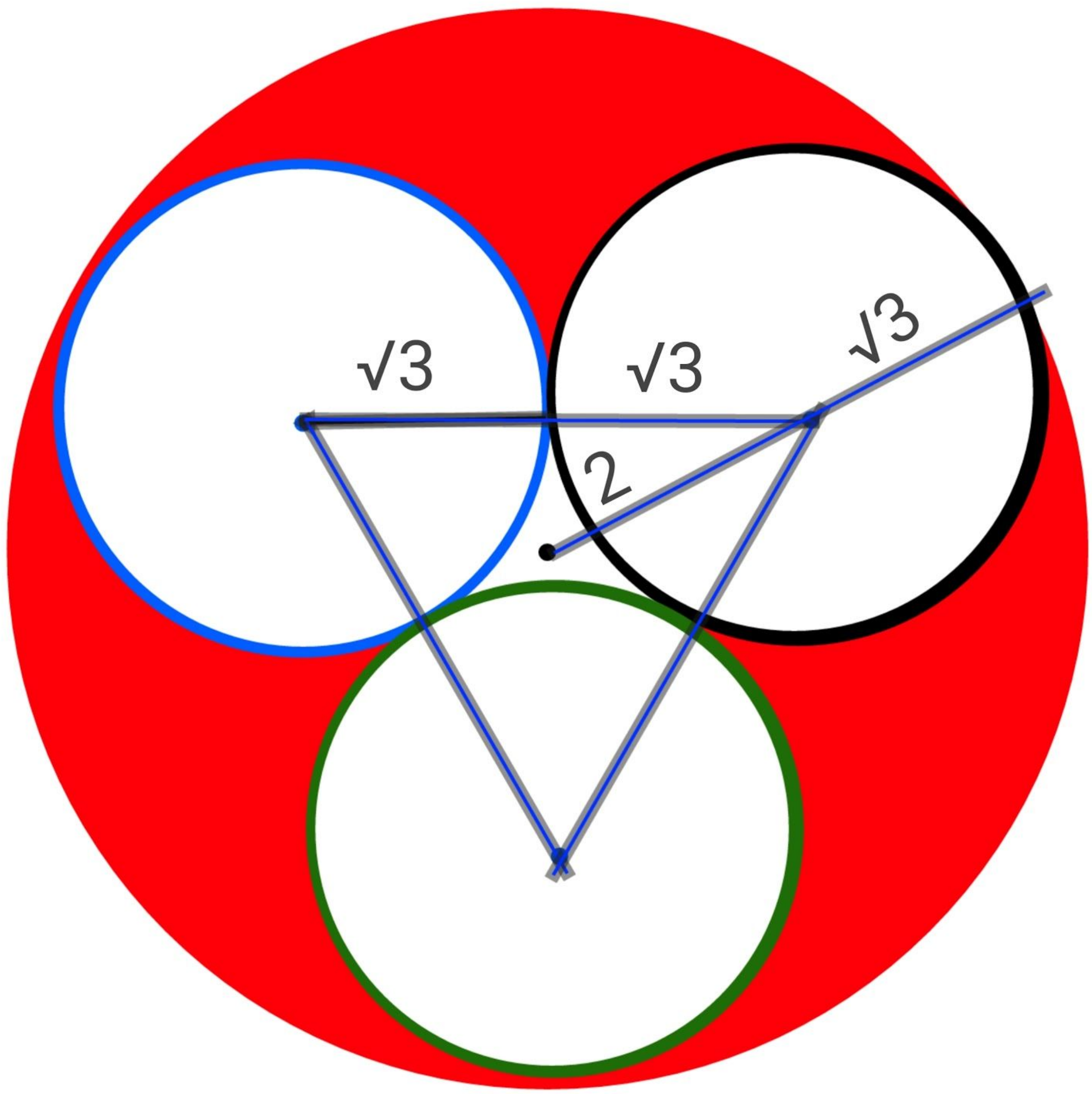
Area PQR = ½×4×3×Sin60 = 3√3.
Area PQD = ½×4ySin30 = y,
Area PRD = ½×3ySin30 = 3y/4.
y+3y/4 = 3√3 , thus y = 12√3/7 cm.
A16• Construct the lines as below,



We get , $4x^2+x^2 = \frac{1}{4}$, thus $x = 1/2\sqrt{5}$.
Area of Trapezium MQBP = $\frac{1}{2}(3/2\sqrt{5})\times 2/2\sqrt{5} = 3/20$.
Area of Triangle = $\frac{1}{2}\times 2x^2 = 1/20$.
Area of central square = $4x^2 = 4/20$. Total required area = $2\times 3/20 + 1/20 + 4/20 = 11/20$ cm^2.

A17• Maximum Side of the cube which can be chiseled out of a Cone = $h\times r\sqrt{2}/(h+r\sqrt{2})$.
Here, $h = 6$, $r = \sqrt{2}$, hence, side = 3/2. Volume = 27/8 cm^3.

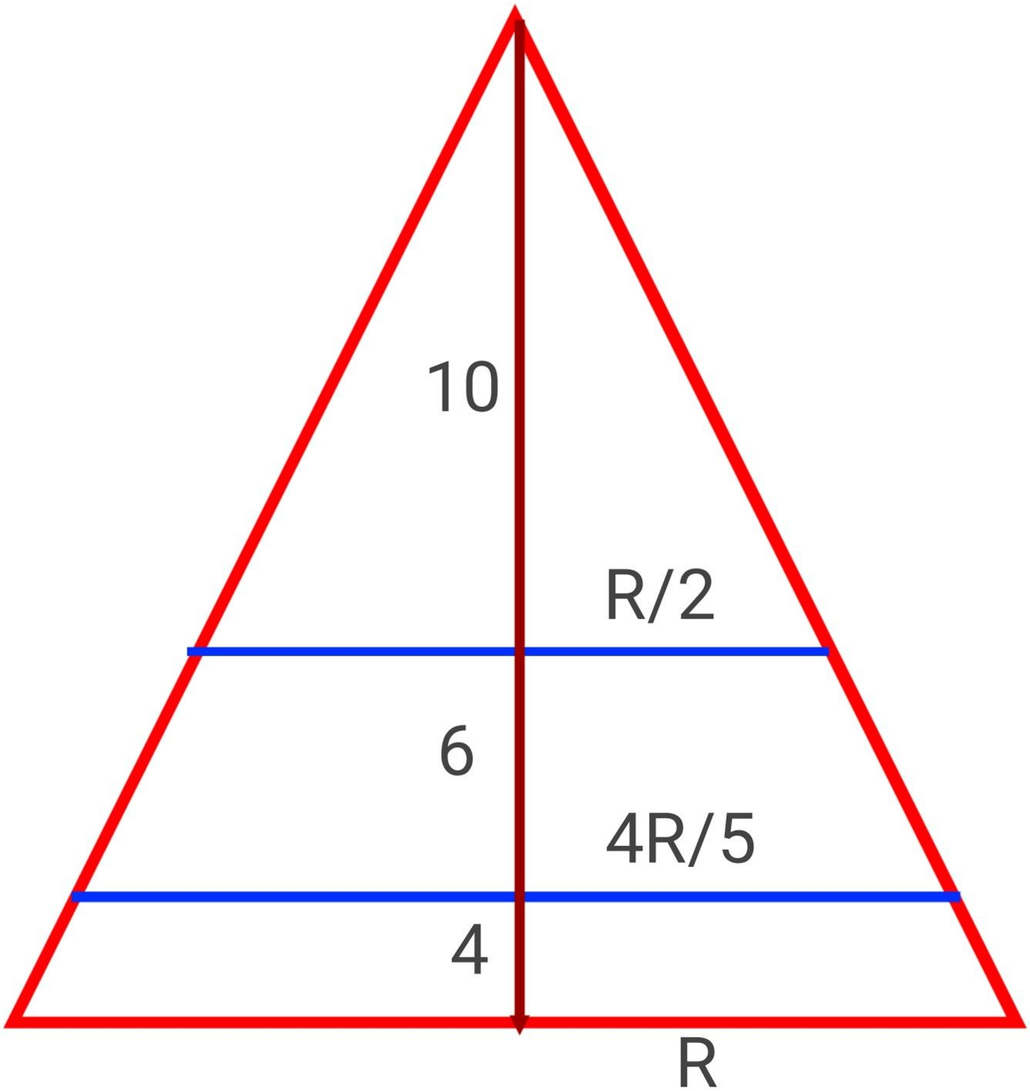
A18• Connect the three centers of small circles and you get the following figure,



Radius of large circle = $(2+\sqrt{3})$ cm. Area of shaded region = {area of large circle – area of Δ – $(3\times5/6\times$ area of small circle) }

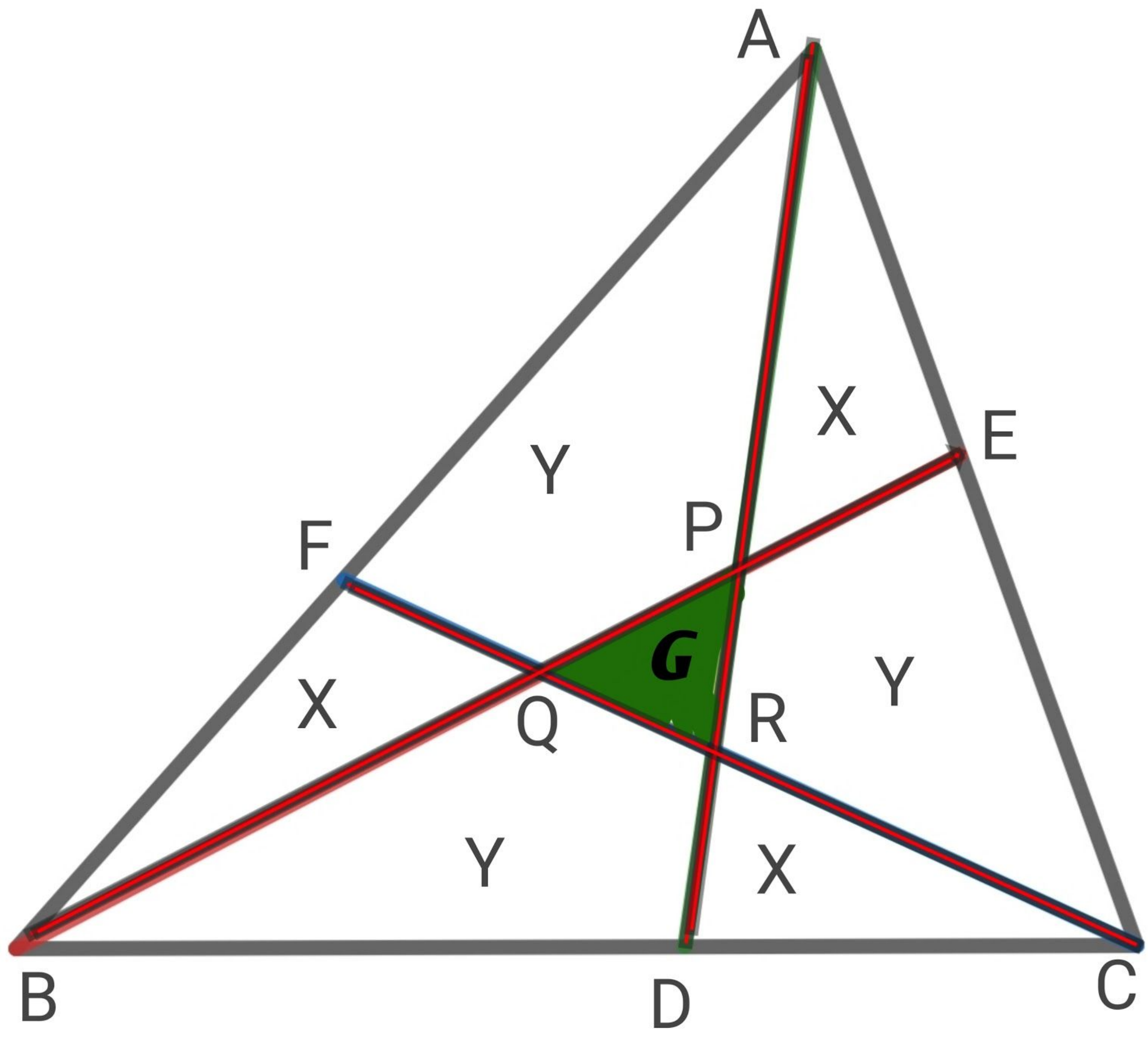
$= (2+\sqrt{3})^2\{(3\wedge⅛ - 1)/6 - \sqrt{3}\}$ cm^2.

A19• By applying similarity, we get the heights and radii,



Hence Origin Volume , V =½πR²×20, Volume of top part = ⅓π(R/2)²×10 = V/8 = 125V/1000.
Volume of middle frustum = Volume of mid Cone – Top cone = ⅓π(4R/5)²×16 – V/8 = 256V/500 –V/8 = 387V/1000.
Volume of Lowest part = V – 256V/500 = 244V/500 = 488V/1000.
Ratio of all three parts = 125:387:488.

A20• This is a beautiful question, but i tamed it. Let's assume this as an equilateral triangle.



I have named the areas as X, Y and Required area = G. Now, total area = 3X+3Y+G = 1, thus (X+Y) = (1–G)/3. Also, (X+Y) = (Y+G) , thus X = G. Now, we apply the extended Ladder Theorem for figure FBCEQ with respect to ABC.

$1/1 + 1/(X+Y) = 1/(2X+Y) + 1/(2Y+X)$, substituting each in terms of G, we get a quadratic equation,
$4G^2 - 14G + 1 = 0$. Thus, on solving we get , $G = (7 - 3\sqrt{5})/4\ cm^2$.

Thank You for Enjoying this Book !!