The Concept of Motion in Ancient Greek Thought

Foundations in Logic, Method, and Mathematics

Barbara M. Sattler

THE CONCEPT OF MOTION IN ANCIENT GREEK THOUGHT

This book examines the birth of the scientific understanding of motion. It investigates which logical tools and methodological principles had to be in place to give a consistent account of motion, and which mathematical notions were introduced to gain control over conceptual problems of motion. It shows how the idea of motion raised two fundamental problems in the fifth and fourth century BCE: bringing together Being and non-Being, and bringing together time and space. The first problem leads to the exclusion of motion from the realm of rational investigation in Parmenides, the second to Zeno's paradoxes of motion. Methodological and logical developments reacting to these puzzles are shown to be present implicitly in the atomists, and explicitly in Plato, who also employs mathematical structures to make motion intelligible. With Aristotle we finally see the first outline of the fundamental framework with which we conceptualise motion today.

BARBARA M. SATTLER has taught at the University of St Andrews and is now Professor for Ancient and Medieval Philosophy at the Ruhr-Universität Bochum. She works mainly on metaphysics and natural philosophy in the ancient Greek world, with a particular focus on the Presocratics, Plato, and Aristotle.

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For a long time the fate of this book resembled that of Achilles' competition with the tortoise – while the point of accomplishment was clearly in sight, it seemed the finishing line would never be crossed. The length of the race is evident from the fact that some crucial ideas go back to my PhD thesis.[1] Some chapter sections overlap with articles I have published in the meantime, but these articles explore or develop individual points.[2] A full view of my subject can be found only here.

While the project seemed to move with the speed of a tortoise, I was lucky in receiving a wealth of support, for which I am immensely grateful. First and foremost, I want to thank those people who committed the time and effort to read the entire manuscript: Sarah Broadie, whose comments always pushed me to dig deeper into the philosophical problems; Ken Winkler, who is perhaps the most careful and subtle reader I have ever known; and Michael Della Rocca, the most generous monist around.

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[1] Material from chapters 7, 8, and 9, as well as the second half of Chapter 3, and the last part of Chapter 5. My doctoral thesis, titled "The Emergence of Motion", can be read on microfiche at the library of the FU-Berlin and at a couple of other German libraries.

[2] Section 1.3.2.1.1 in Chapter 1 and parts of chapter 2 overlap with Sattler 2011; section 3.6.4 in Chapter 3 with Sattler 2015 and a subsection of 3.6.1 with Sattler 2019b; section 7.2.1 in Chapter 7 with Sattler 2019a; and sections 1.4.2 in Chapter 1 and 8.1 and 8.2.3 in Chapter 8 with Sattler 2017a.

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Introduction

Overview of the Project

The main object of this book is to study how the understanding of physical motion in ancient Greek thought developed before and up to Aristotle. It investigates which logical, methodological, and mathematical foundations had to be in place to establish a fully fledged concept of motion that also allows for comparing and measuring speed.[1] Given that physical motion is the core concept of natural philosophy, this study thereby also seeks to reconstruct in rough outlines how natural philosophy came to be established as a proper scientific endeavour in ancient Greece.[2]

According to a prevailing picture, scientific investigation of physical motion and change started properly in the West with Aristotle but only achieved its true form in modern times, with the overthrow of central Aristotelian doctrines. In the early modern period, so runs the narrative, Aristotelianism was rejected and the basis laid for what today we consider the science of physics.[3] This account is at least doubly misleading. Undoubtedly, great achievements were realised in early modern times, but if we take a step further back in history, we can also discern an alternative narrative. This broader perspective allows us to see, first, that Aristotle marked a high point in an extended investigation of motion that started a long time before him and, second, that when this earlier way of doing science is included in our perspective, there is strong continuity between Aristotle and modern natural philosophy and science. Many basic concepts that Aristotle introduced in reaction to earlier natural philosophy remain fundamental for how science is done today (for example, the idea that time and space can be treated as structured in similar ways). This continuity on the basic conceptual level is too often overlooked,

1 I will only be able to cover some of these foundations of motion, by no means all of them. And these foundations are not necessarily explicitly formulated in the thinkers discussed, but sometimes only implicitly used.

2 As a first pass we should understand natural philosophy as being concerned with the theoretical basis for doing physics in general. For a discussion of the notion of natural philosophy, see Chapter 1.

3 So, for example, Theodor Gomperz 1912, p. 108 and Alexandre Koyré 1968, pp. 90–1.

however, as a result of our focus on the important 'paradigm shifts' in the conception of nature that occurred after antiquity and mainly since the Renaissance.

Readers who do not follow the prevailing picture and do not think that scientific investigation of motion started only with Aristotle, holding that natural philosophy had already been established by the Presocratics, for example, may well wonder why I would even claim that it took until Aristotle for natural philosophy to be established as a 'proper scientific endeavour'. We will see, however, that for the study of the natural realm to become what we might call a 'scientific enterprise',[4] not only certain logical, ontological, and methodological developments were required but also the integration of central mathematical notions into philosophical discussion. These developments and this integration have become part of the fundamental framework with which we conceptualise nature as an object of science today, but they were first formulated in the way that is familiar to us by Aristotle.

The basic conceptual framework for natural philosophy was essentially shaped by the philosophers in the fifth and fourth centuries BCE on whom I will concentrate in this book: Parmenides of Elea and his fellow Eleatic Zeno, followed by the atomists Leucippus and Democritus, and finally Plato and Aristotle. For reasons of space I will have to leave out other important thinkers who contributed to the development of this framework in the period investigated, individuals such as Heraclitus, Empedocles, Anaxagoras, and Gorgias. Nor will there be space to look at the Milesian thinkers Thales, Anaximander, and Anaximenes, who endeavoured to explain natural phenomena in rational terms in a manner traditionally associated with the birth of both philosophy and science.

I begin with Parmenides for two reasons: first, Parmenides introduced strict criteria for philosophy and science that made them possible as truly rational endeavours;[5] and, secondly, Parmenides was the first philosopher to develop a system of basic logical or conceptual tools that implicitly determine the domain of possible objects for rational inquiry.[6] To begin with Parmenides means to begin with his questioning of the very possibility of natural

4 I will show what we may understand by a 'scientific enterprise' in Chapter 1.

5 Hence, we can say that Parmenides is also the first philosopher where we find second-order thoughts about philosophy, thoughts like 'What counts as a proper inquiry and why?' Kahn 1994 has argued for understanding Anaximander as the inventor of models of explanation and Gregory 2016 claims that we find such second-order thoughts also in Anaximander, for example, in the decisions he makes about what he takes as evidence. But this is implicit in Anaximander. There is no hint of any explicit discussion of such questions, as we find, I argue in the second chapter, for example, in Parmenides' fragment 7.

6 I will explain in the first chapter what I understand by logical tools and the broad notion of logic at work here.

philosophy and science, however.[7] In their response to this challenge, his successors laid the foundations for a scientific investigation of nature.

One centrally important criterion that Parmenides employed systematically is consistency. This requirement both imposed a central condition of rationality on inquiry into nature and became a central engine of Parmenides' challenge to the possibility of such rational enquiry. Parmenides and Zeno after him, through his generation of a series of well-known paradoxes, argue that motion, time, and space – essential to any science of nature – cannot be given accounts which satisfy the requirement of consistency. This is not to say that Parmenides or Zeno deny that we have experience of things as changing, enduring, and located. What they do deny, I argue, is that these phenomena that we experience are available for rational enquiry.[8] Denying this possibility is the reason that Parmenides and Zeno end up creating important challenges for the development of a natural science. One crucial reason why Parmenides and Zeno cannot accommodate motion, time, and space within their requirement of consistency is that, as I will demonstrate, the logical framework they established, though an important start, is too narrow to form a basis for natural science.

The next act in the story I reconstruct thus calls for significant expansion of this underlying methodological and logical framework. This takes place, I argue, in two separate stages. On the one hand, the logical apparatus and the criteria for philosophy themselves are expanded by distinctly articulating aspects that were run together in Parmenides.[9] On the other hand, mathematical concepts begin to be imported into this logically inspired framework, as a result of recognising that Parmenides' conceptual framework on its own cannot give us the terms we need for an analysis of nature. First in the work of the atomists and later in Plato's *Sophist* we find the necessary expansion of the underlying logical and methodological framework that allows for the development of a natural science and philosophy. However, only once this development is combined with mathematical notions that are brought into the description of natural phenomena are we close to having a real foundation for natural philosophy to capture the phenomena of time, space, and motion. This

[7] With Parmenides we will see how the specific form taken by his criteria and his logical tools contributes to ruling out natural philosophy as a field of strict and systematic inquiry. That it is worthwhile even so to investigate Parmenides for a discussion of the beginning of natural philosophy can also be seen from Aristotle's discussion of Parmenides in *Physics* I, chapters 2 and 3, and Aristotle's explicit claim in 185a17 ff. that while Parmenides and Melissus do not investigate nature as such, they nevertheless raise problems for natural philosophy.

[8] Similarly, some of us may not want to deny the existence of certain astrological, naturopathic, or theological phenomena, but also may not think that they are proper objects for scientific enquiry.

[9] For example, Parmenides does not separate operators and operands, as we will see in the first two chapters.

combination happens, I argue, in Plato's *Timaeus* and, partly in response to Plato, in Aristotle's *Physics*.

The central focus in our analysis of the conceptualisation of motion in this period will be speed. Certainly, ancient Greeks, Parmenides and Zeno included, could determine who won a race at Olympia. But this context for considering speed – namely, determining which competitor is the fastest at the Olympic Games – crucially controls two of the complex of notions that make up our modern understanding of speed: both distance and start time are fixed. If these factors are not controlled – if, for instance, the distance that each competitor covers is different – then they cannot measure and compare the two speeds. For speed involves a relationship between distance covered and time taken to cover that distance, but laying the conceptual basis for such a relationship (and not just for time and distance each on their own) is, as we will see, highly problematic for most of the period I am investigating.

By using a logical apparatus and criteria of inquiry that leave motion, space, and time outside the realm of rational enquiry, Parmenides and Zeno challenge the very possibility of conceptualising speed. A framework for the conceptualisation of speed requires an account of time and space[10] in which they form a relationship that is quantifiable (i.e., that admits of measurement) for we want to answer the question how fast something is moving, that is, how much space is covered in how much time. The conceptual foundation for a quantifiable relation between time and space was the mathematical notion of a continuum and its incorporation into accounts of both time and space; that step, as we shall see, is taken by Aristotle,[11] who provides the end point of the development under investigation.

This book thus deals with a crucial stage in the long process that was the birth of physics as a science of motion. As such my project examines factors that shape how we still approach the natural sciences today, offering a philosophical explanation as to why mathematics and logic are intimately connected in our picture of science. In giving an account of the historical process that established the connections between these different realms that characterise our enquiry into nature, I show that our understanding of time and space as related in the notion of motion is not a given, but rather an achievement. This picture of motion as a unity of time and space was not available in the early Greek tradition. By demonstrating the extent to which the conceptualisation of complex notions such as speed depended on developments of the criteria used for philosophical investigation,[12] on innovations in

[10] I will talk about 'space' as a shorthand, while often we only need an account of place or of the distance covered. I offer a detailed discussion of the relationship between the different spatial notions in my book manuscript *Conceptions of Space in Ancient Greek Thought*.

[11] At least we only have systematic evidence for Aristotle doing so, though Eudoxus may sometimes be in the background; see chapters 8 and 9.

[12] As, for example, changes in the understanding of the law of non-contradiction.

logic, and on the introduction of mathematical notions into the philosophical framework, I show how antiquity prepared the path for the manner in which we conceive of speed today, and for our ability to calculate speed and perform mathematical operations within the field of natural philosophy and science.[13]

In this account of the concept of motion I will not be able to do any justice to neighbouring notions such as the those of cause[14] or force. There are also some more general concepts that may arise in an investigation like the one I envisage, to which, again, I will not be able to give the space they deserve, such as truth or knowledge.[15] In general, I will only look at basic foundational issues in natural philosophy and not be able to deal with a lot of the metaphysical and epistemological underpinnings that are in play here.[16] No doubt some readers will be disappointed not to see these notions or other thinkers discussed in this book. Their absence should not be read as a verdict of insignificance, but be taken simply as evidence that we are finite beings who can deal only with a finite number of things.

Methodology, Treatment of Sources, and Relationships of Thinkers Investigated

In Chapter 1 I will say much more about what I understand by the criteria for philosophy, logical operators, and the mathematical notions introduced into natural philosophy, providing systematic coverage of all the main concepts that are of importance for this book. (It would therefore be helpful to read the first chapter before turning to the chapters on individual thinkers.) But before I move to the first chapter, let me first, in this current section, address my relationship to the scholarly literature and how I shall handle the ancient texts, before, in the final section of this introduction, providing a brief overview of the chapters that follow.

(a) Methodological Remarks and Treatment of the Sources Some readers may be surprised by the argumentative and logical tools I will use for my

13 This is not to say that there were no other interesting streams that were lost – I deal with some of these in my book manuscript *Ancient Notions of Time*.

14 Even though the notion of a cause of a motion is obviously important for an investigation of motion also in antiquity, causation can only be hinted at in the chapters on Plato and in the investigation of the principle of sufficient reason.

15 Aristotle's theory of knowledge and of demonstration, for example, seems to be important for his idea of the possibility of natural philosophy, but I will only be able to hint at it in the chapters on Aristotle. The distinction between *epistêmê* and *doxa*, and what their respective objects can be, will to some degree come into the discussion of Parmenides and Plato.

16 I will also not debate the distinction between what is often seen as Plato's quantitative account of physics versus Aristotle's qualitative account; indeed I will be dealing with aspects of Aristotle that are very much on the quantitative side.

analysis of the ancient thinkers. They are meant to help translate the seemingly familiar, but in fact often rather different conceptual frameworks of the ancient Greek thinkers into a language and terminology that is accessible to a modern reader. Our modern, substantially enlarged, toolbox for doing philosophy may, if used prudently, allow us to figure out what is going on in these ancient texts in a clearer way.

Using in part modern logical and argumentative tools to understand ancient thoughts bears the risk, however, of altering or even distorting the ancient views, as these tools may include assumptions that the ancient thinkers do not share. And this may feed into a dangerous tendency in the scholarship of the history of philosophy to make the ancients less unwieldy and to assimilate them simply to our own thinking – a tendency that I think is harming us not only as historians of philosophy but also as philosophers, since it reduces our investigation to looking for confirmation, rather than for alternative ways of understanding the world.[17]

The use of modern tools often seems necessary, however, to make ancient thoughts understandable for us (and if we do not make the modern tools we use explicit, so much the worse, for the chances are high that they will creep in implicitly). Thus, we will have to think about these tools, what alternatives to them there might have been in ancient contexts, and accordingly, we will often work with a somewhat wider or different understanding of these tools than contemporary philosophers would. And if we do this in a conscious and responsible way, we may thus also learn something about how our modern toolbox came into being and why certain distinctions may be distinctions on which, deep down, we still base our philosophical activities.[18]

The ancient sources we will look at are of very different kinds – from Plato's dialogues, where we possess a (comparatively) safe and complete textual basis, to fragments of Parmenides, Zeno, and the atomists. Especially with the atomists we often have only snippets of their original works or have to rely on the summarising accounts of other, not necessarily sympathetic, thinkers. One problem that thus arises concerns the methodology of how to deal with these sources, especially the fragments.[19] In general, I will treat the sources we have very seriously and believe them, if possible – an approach I would regard as methodological carefulness. A source may be

[17] Cf. Sattler 2014.

[18] I will thus try to combine what are sometimes called historic and rational reconstructions of ancient thought; cf. Makin 1988.

[19] For a fuller treatment of the problems with which we are faced when dealing with Presocratic fragments, cf. Mansfeld 1999, Runia 2008, and Sattler 2013. In the current book, all fragments are numbered according to Diels-Kranz. In addition, other collections of the Presocratic fragments will be used if they contain more evidence, as, for example, Lee's edition of Zeno's paradoxes, or Taylor's collection of the atomist fragments. Citations of editions, translations, and commentaries are to editors', translators', and commentators' names only, without dates.

deemed questionable with respect to a particular fragment, however, if it gives conflicting reports about a theory without explicitly making it clear that the author reported on does indeed hold conflicting views.[20] While this criterion is, I assume, relatively uncontested, the situation is more problematic when a source reports a view that conflicts with a view reported by another source.[21] In such cases the first step is to see whether the different accounts may hold in different respects or on different levels (for example, the phenomenal level and the level of what truly is for the atomists). Only if this step is unsuccessful will I proceed to a discussion of which source is more likely to be confused and thus should not be followed.

Plato (to some degree), Aristotle, and their commentators remain our earliest and most important sources on which most other sources rely. We therefore need very good reason not to trust their *report*.[22] Their status does not mean, however, that we must necessarily follow their *interpretation* – not that it is always easy to distinguish report clearly from interpretation.[23]

Ever since Harold Cherniss, there has been a tendency to dismiss Aristotle as an untrustworthy witness of Presocratic philosophy and to take the accounts of Presocratic philosophy that were written before Cherniss as uncritically Aristotelian.[24] Against this trend, in general I take Aristotle very seriously (though not uncritically) as a witness, because I do not think that we have been shown real alternatives. After all, it is not as if we can turn to an authorised edition of the Presocratics, and without Aristotle and his tradition very few reports would be left for us. Additionally, more often than not, it seems to me, such a general suspicion of Aristotle is based, at least partly, on misunderstanding him. In the chapters that follow I argue in specific instances that Aristotle should be taken as a serious witness.[25]

20 According to Makin 1993, p. 63, we find such a case with Aetius. Another case may be Simplicius' report on the partlessness of atoms: in *In Phys* 82.1 he reports that the atoms of Democritus have parts, while in 925.14 ff. he tells us that they were seen as being partless.

21 An example of this would be the testimonies on weight in the atomists.

22 Curd and Graham 2008, for example, reject Plato and Aristotle as reliable witnesses because the ancient reports of Parmenides as being a monist of sorts do not fit Curd's and Graham's understanding of Parmenides as not rejecting change and plurality (cf. also Osborne 2006, p. 227). While it seems uncontroversial to me that the ancients may have seen Parmenides with other eyes than we see him, turning the ancients thus into unreliable witnesses on such a fundamental point seems to me too high a price to pay for making sure that Parmenides does not violate contemporary preferences for pluralism.

23 One example where we can clearly make such a distinction is Plato's *Symposium* 187a, where we are given a report of Heraclitus' fragment B51 first and then (in a consciously humorous form) a rather idiosyncratic interpretation and correction of it by the character Eryximachos.

24 Cherniss 1935. This tendency seems to have become even more of a trend recently with Palmer 2009.

25 Cf., for example, Chapter 4, where I deal with Sedley's 1982 claim that Aristotle's testimony is of little historic value for the atomists' notion of a vacuum.

(b) The Relationships between the Thinkers Investigated To date we have no agreed overall narrative about how motion, change, and processes came to be established as proper objects of scientific enquiry in antiquity. We do have overviews of the development of ancient philosophy as a whole,[26] and there is a fairly standard chronology for the main thinkers – Parmenides, Zeno, the atomists, Plato, and Aristotle – that I will follow.[27] But there is nothing specific on the development of the concept of motion all the way from the Presocratics through to Aristotle.[28]

We do, however, have accounts of parts of this story[29] and of the broader relationships between some of its actors. With respect to the Presocratics, I will go against two current trends in the scholarly literature to some degree:[30]

A) I will group Parmenides, Zeno, and Melissus as the 'Eleatics'. Although that grouping has been questioned in recent years, I will show the extent to which Zeno (and to some degree Melissus too) can be deemed to have developed Parmenides' main line of argumentation.
B) The relationship between Parmenides and his non-Eleatic successors has been variously interpreted. Did Parmenides issue a challenge to his successors? Or did his successors continue Parmenides' thought? Or is their relationship characterised by indifference, with Parmenides' successors not influenced by him? Traditionally, Parmenides' philosophy has been conceived as a challenge posed to natural philosophers, to which the thinkers who succeeded him responded.[31] Recently, however, this interpretation has been questioned, and the current trend is to place greater

[26] Found in histories such as Guthrie 1962–81 or Überweg 1983–2018.
[27] Where exactly to place Melissus is somewhat more difficult, see Chapter 4. I should also note that I take Philolaos to be earlier than Democritus.
[28] Such investigation as there has been regarding ancient Greek conceptions of motion, space, and time (for instance, in Sorabji 1983 and 1988) has not integrated accounts of time, space, and motion and has not paid sufficient attention to the increasing incorporation of mathematical notions into philosophy.
[29] Books that deal with part of this story tend to concentrate either on the Presocratics (as, for example, recently Curd 1998 and Graham 2006), on Plato (recently Gregory 2000 or Broadie 2012), or on Aristotle (Bostock 2006), each treated individually; or on the relationship between Plato and the Presocratics (for instance, Dixsaut and Brancacci 2002 or Palmer 1999), between Aristotle and the Presocratics (Cherniss 1935), or between Plato and Aristotle (Cherniss 1944). But the continuity within and stages of the development all the way from the Presocratics up to Plato and Aristotle has not been the object of a single, unified philosophical study.
[30] What I give here is a general overview – I will deal with the secondary literature on individual thinkers in detail in the individual chapters.
[31] Guthrie 1965, for example, sees Parmenides as dividing Presocratic philosophy into two halves and the philosophers following him as reacting to his anti-cosmological move. McKirahan 2011, p. 157 understands Parmenides as introducing a different philosophical style (including rigorous proofs) and different conclusions.

stress on the continuity from Parmenides to his non-Eleatic successors[32] – either by seeing Parmenides as less revolutionary than he was seen before,[33] or by understanding his successors as more Eleatic. Rejection of the traditional narrative has also led to claims that Parmenides had no considerable effect on his successors at all.[34]

In this book I want to show that there is both a challenge put up by Parmenides for the natural philosophers succeeding him and also important continuity. On the one hand, the post-Parmenidean thinkers do indeed endorse important aspects of his philosophy. However, these thinkers are endorsing not findings on the monism/pluralism front, as is often assumed, but rather Parmenides' criteria for philosophical investigations, in some sense his basic logical operators, and, to an important degree, also most of the main features (*sêmata*) that he claims are possessed by what truly is. On the other hand, Parmenides' austere ontology and his rejection of natural philosophy did indeed set up a challenge for succeeding natural philosophers to which these thinkers did react.[35] By showing how the atomists responded while at the same time taking up essential criteria and operators introduced by Parmenides, I will also demonstrate why the third possibility, that Parmenides did not have any effect on succeeding Presocratic philosophers, appears to me indefensible.[36] Even if Parmenides' successors during the period under investigation did not react to him identifying him by name,[37] we see enough of his basic thoughts and arguments taken up and modified to appreciate that his non-Eleatic successors dealt with his position

32 So, for example, by Sedley 2008. Palmer 2009 even sees Parmenides in continuity with both the Milesians and Plato's account in *Republic* V (for my assessment of this claim, see Sattler 2014).

33 For example, by understanding him as a pluralist in the way Curd 1998 does.

34 So Osborne 2006.

35 Even scholars who are deeply committed to the continuity thesis usually see what continuity there is in metaphysics, not in natural philosophy. Such a clear distinction between cosmology on the one hand and the metaphysical realm on the other is new with Parmenides.

36 Osborne 2006, p. 224 thinks that in the traditional story the post-Parmenideans meet Parmenides' challenge by positing a plurality, which contradicts his monism. But, according to Osborne, given that they provide no systematic argument to defend it, nothing of what Parmenides said had any effect on them. Curd seems to deal with this problem by making Parmenides himself not a real monist; but then, as Osborne holds, there is nothing revolutionary about him. I will challenge Osborne's position by showing how the atomists defend their pluralism by taking up and extending the logical operators and criteria of Parmenides, which shows that his philosophy did indeed have an important effect on them. I should, however, mention that Osborne's scepticism mainly holds for Empedocles and Anaxagoras, about whom I will not make any claims here.

37 Osborne 2006, pp. 244–5 herself points out that philosophical interaction need not have taken place in the way contemporary philosophers expect.

intensively. We will see that the main reactions to Parmenides from the atomists seek to show that natural philosophy could still be done, but that it required, as Parmenides demonstrated, a new method and rigour.[38]

However, Presocratic interaction is only part of the narrative I give here, which also includes Plato and Aristotle. The breadth of my account brings additional interpretative problems – the relationship between Plato and Aristotle, for example, or that between Plato and Parmenides.[39] In the latter case recent literature has adopted two extremes: on the one hand, Parmenides has been seen as closely prefiguring Plato,[40] on the other hand, Plato has been seen as misunderstanding and distorting Parmenides.[41] I select a middle way between these two extremes, suggesting that while Plato did understand Parmenides quite well, he saw that Parmenides' position lacked a middle ground for contingent things, those things that are in some ways but are not in others, which Parmenides cannot conceive with his logical tools. We will see how Plato decisively develops Parmenides' logical tools; the lovers-of-sight-and-sound passage in *Republic* V can be understood as precisely such a development.[42]

In this book, I aim not only to show previously unrecognised connections and developments over the whole period I am considering, but also to offer novel readings of the work of each of the actors in my story, as will be evident in the overview of the chapters. Let us thus move on to this outline of what I seek to achieve in the individual chapters.

[38] For some readers, the Presocratic part of the story I tell may sound comparable to some parts of Guthrie's *A History of Greek Philosophy*. While I am sympathetic with the broad outline of Guthrie's account of the Presocratics, this book can be read completely independently of the reader's stance on Guthrie for at least three reasons: (1) while I think Guthrie is right in understanding Parmenides as a watershed in Presocratic philosophy, I will not rely on this; (2) none of the main points I follow in the development of this story – the logical operators, the criteria of philosophy, and the introduction of mathematical concepts – are found in Guthrie; (3) finally, my interpretation of the individual thinkers differs markedly from Guthrie's – to give just one example, Guthrie takes Parmenides to claim that everything apart from the One Being is mere appearance, a position I explicitly argue against in Chapter 2.

[39] It may also be seen as a problem that the atomists, in contrast to the Eleatics, are materialists, and that we are moving from a mechanistic account of motion in the atomists to a teleological one in Plato and Aristotle. While these different starting points for understanding motion pose different requirements on the explanation of motion, I will concentrate on the basic structures that are relevant to all these positions.

[40] So Palmer 2009, who understands Parmenides' threefold division of necessary, contingent, and impossible being as immediate preparation for the lovers-of-sight-and-sound passage in *Republic* V.

[41] So Cordero 2011, who claims that the assimilation of Parmenides to Plato has led to a wrong ordering of the fragments and that Plato himself did not understand Parmenides.

[42] *Republic* 476a ff.

Overview of the Chapters

Chapter 1: Conceptual Foundations

The point of the first chapter is to provide an overview of the central notions this project employs. I give a brief overview of the problems the concept of motion raises in the period investigated and discuss what will be understood by 'natural philosophy' in this book. The main bulk of the chapter will then sketch what I understand by the criteria or standards established for philosophical inquiry, and discuss the roles of logic and mathematics in establishing natural philosophy. In this first chapter I show in broad outlines, both systematically and historically, what will appear in each of the following chapters separately with respect to one thinker (or one school, in the case of the atomists). Thus this chapter should serve as a first orientation for the project as well as a reservoir for consultation if questions concerning the basic concepts employed arise during the reading of the whole book.

Chapter 2: Parmenides

In the second chapter I present Parmenides' account of the object of rational enquiry and the challenge it represents for natural philosophy. I show how Parmenides' specific interpretation of the criteria he establishes for rational enquiry and his logical operators heavily influence his understanding of Being (i.e., what there is and can be thought) and lead to the exclusion of natural philosophy from rational enquiry.[43] This reconstruction of the reasoning for Parmenides' account of what can be an object of rational enquiry offers a new interpretation of the basis of his monism.

I argue that Parmenides establishes explicit criteria for rational investigation – logical consistency, what I call 'rational admissibility', and a principle of sufficient reason. Parmenides tries to show that his philosophy alone satisfies these criteria. His philosophy is based on three fundamental notions: Being (what truly is); Sameness (understood as self-sameness); and Negation (roughly understood as a one-place operator non-A that denotes the polar opposite of A, and does not allow for difference only in certain respects). The notion of Sameness works as the only connection operator, that of Negation as the only separation operator. Being, Sameness, and Negation are systematically intertwined in such a way that any change of one conception would necessitate changes in the conceptions of the others.

Though prima facie Parmenides' account maintains logical consistency, the way in which it is set up explicitly excludes the main components of natural

[43] Thus, while his philosophy might seem counterintuitive, I want to show how his exclusion of the realm of change and motion from philosophy is a logical result of his philosophical framework.

philosophy: time, space, motion, and change. I show how this exclusion results not only from his understanding of the negation operator, but also from the fact that the number of fundamental concepts in use in his philosophy is too small to sustain an account of nature. In addition, the concepts themselves are too indeterminate – the basic concepts are used to signify basic entities ('Being') as well as what I shall call 'logical operators' ('is'). And there are less basic concepts in the background, such as Parmenides' notions of unity and of a whole, that are never explicitly clarified.

This reconstruction prepares us for how Parmenides' successors could, and indeed had to, respond to Parmenides' challenge: they reacted to Parmenides' rigorous standard of knowledge by developing his criteria and operators further and by distinguishing between weaker and stronger standards of knowledge.

Chapter 3: Zeno

In the third chapter I show how Zeno takes up and advances Parmenides' criteria for philosophy. For example, Zeno adopts the principle of non-contradiction in the way established by Parmenides and uses it as a basis to develop his paradoxes of plurality, place, and motion,[44] which start with his opponent's position and then show how this position leads into inconsistencies. With the help of these paradoxes, in particular those of motion, Parmenides' challenge is further spelled out. Zeno's paradoxes of motion confront any natural philosopher with two kinds of problem: mereological problems and spatio-temporal problems. Zeno's paradoxes thus also provide a first indication of what it would be to account successfully for time, space, and motion. Hence they can be taken as a touchstone for determining whether later natural philosophers meet the Eleatic challenge. Parmenides' criteria for philosophy are thus not only taken up and developed but also implicitly complemented by Zeno's paradoxes which function as a criterion specific for natural philosophy.

What is distinctive about my approach to the interpretation of Zeno's paradoxes is that it offers what I call a 'conceptual reading': it starts with a careful analysis of the concepts of time, space, and motion and their implications as employed by Zeno in his paradoxes, and then investigates which features of these notions contribute to the paradoxical result. By contrast, the dominant view in the secondary literature so far takes the notions implicitly employed in Zeno's paradoxes for granted, without further analysis, and only

[44] Parmenides' poem is the first place where the principle of non-contradiction is methodically employed as a criterion for reliable knowledge (see Chapter 2). And Zeno's paradoxes would not work if the principle of non-contradiction was not already firmly established.

considers how we might deal with or solve the paradoxes (for example, by employing modern mathematical tools). This literature also reduces the paradoxes of motion to paradoxes arising for any continuous magnitude.[45] In contrast, my conceptual reading allows for a reconstruction according to which they are truly paradoxes of *motion* and examines whether the notions Zeno uses are sufficiently developed and indeed appropriate for grasping what is specific for motion. Thus understood, Zeno's paradoxes show that the task of giving an account of motion is made problematic not only by the relation between part and whole but also by the relation between time and space.

With respect to the relation of part and whole, I show that Zeno inherited from Parmenides an ambiguous notion of wholeness, which leads to a confusion of two basic notions in his paradoxes: the notion of a whole that is the sum of its parts and thus posterior to its parts, and the notion of a whole that is prior to any parts that might be gained from the whole. One reaction to his paradoxes required of subsequent thinkers is to establish a coherent account of whole and part that is suitable for understanding time, space, and motion.

Zeno's paradoxes can be understood as one of the most severe explicit attacks on a conceptualisation of motion. Another attack is that of Melissus, which claims that there can be no motion since there is no void and motion is only possible if there is void. This claim by Melissus will be taken up briefly in Chapter 4.

Chapter 4: The Atomists

My fourth chapter focuses on the atomists Leucippus and Democritus as a clear example of the first wave of reactions to Parmenides and Zeno by subsequent natural philosophers. The atomists illustrate my contention that exploiting the logical potential inherent in one of Parmenides' concepts by giving it a new interpretation necessitates changes to all intertwined concepts.

My starting point is the physical interpretation the atomists give to the basic notions of Parmenides, interpreting non-Being as void and Being as physical atoms. What is new about my interpretation is that I show how this physical interpretation permits the atomists to take two groundbreaking steps: (1) for the first time in Western thinking the realm of experiences is explained systematically in terms of a distinct realm of what truly is – the idea of ontological grounding thus originates with the atomists, and (2) the logical possibilities of the Eleatic foundation are expanded in crucial ways. This

[45] I am using the term 'magnitude' here as referring to any physical magnitude; thus, it is not restricted to spatially extended magnitudes. In our discussion of Aristotle's usage of *megethos*, however, we will see that he predominantly uses it for spatial magnitude. See also Mueller 1970, p. 168.

second step is possible because their physical interpretation of Being and non-Being allows the atomists not only to set up a precursor of a conception of space, but also to understand non-Being as a basic concept on a par with Being. To understand non-Being as on a par with Being necessitates an implicit distinction between understanding Being and non-Being as entities and understanding them as operators. In this way the atomists increase the complexity of the logical system in use. 'Non-Being' plays two roles: (1) it denotes an 'entity', the void, and (2) it signifies an operator, 'is not'. In contrast to Parmenides' negation operator, this atomist operator indicates not strict opposition but difference, for example, the difference between atoms. Likewise, 'Being' plays two roles: (1) it signifies an entity, atoms, and (2) it denotes an existence operator, 'is'. This existence operator can also be applied to the void, and hence to non-Being.

To an extent, this atomist approach prepares the way for a logical framework for a philosophy of nature. It cannot, however, as yet give a consistent account of motion, time, and space. Zeno's paradoxes – the test for the conceptual coherence of a natural philosophy – cannot be answered in the atomist system for at least two reasons. First, the atomists react only to the mereological problems of Zeno's paradoxes, but not to the spatio-temporal problems. Secondly, while the atomists no longer confuse different part-whole relations but work with a single conception of a whole as the sum of its parts, the part-whole relation they work with can be shown to be insufficient to avoid Zeno's paradoxes.

Chapters 5 and 6: Plato

In the fifth and sixth chapter I set out the first climax in the historical progression I chart in the book: the advancement of logical concepts in Plato's *Sophist* and the integration of mathematical concepts into the logical framework in Plato's *Timaeus*. In his employment of mathematical concepts, Plato takes up, but also significantly revises, certain features of the Pythagorean account of nature in mathematical terms which had developed independently of the logical developments that have been my focus so far.

In Chapter 5, I show Plato's *Sophist* to be the first explicit investigation of the systematic connection between the concepts inherited from the Eleatics. This investigation leads to crucial changes in the logical system by making explicit the 'difference operator' implicitly used by the atomists and by giving to the notion of sameness a broader interpretation than its Parmenidean understanding as self-sameness. Both this difference operator and this new notion of sameness are required for a consistent account of the relation between Being and non-Being that is a logical precondition of the conceptualisation of motion. Furthermore, we will see how in connection with this development of the separation and the connection operators Plato also develops further the principle of non-contradiction, the principle of sufficient reason, and the understanding of rational admissibility.

In the *Sophist*, Plato has the Eleatic stranger investigate five greatest kinds or Forms. The fact that motion and rest are among them suggests, I argue, that these logical investigations are also undertaken with a view to a possible natural philosophy. And the criteria of philosophy will be understood as criteria that natural philosophy also has to meet to a certain degree.[46] However, Plato's logical achievements and his development of the philosophical criteria are not sufficient for a consistent account of physical motion as they are not specific to natural philosophy. Recognition of this insufficiency is, I argue, a crucial reason why Plato finally takes up features of a mathematical approach to natural philosophy in the *Timaeus*.

In Chapter 6, I analyse how Plato employs mathematical structures in a systematic way in order to explain the natural world. The most important mathematical notions that Plato imports into his natural philosophy are the number series, proportions, and geometrical forms. They allow for conceptualising features of nature that would otherwise seem unintelligible. For Plato, geometrical forms underlie the basic building blocks of the appearances (such as fire and earth) and thus also the spatial order of objects; proportions determine the order according to which the revolutions of the heavens are set up; and the number series allows us to give an account of the motions of perceptible objects. These motions can be assigned to numbers and hence be measured with the help of time – motion thus can be shown to be intelligible to a certain degree.

However, time is connected to the number series, whereas space is connected to geometry. This allocation of different mathematical fields to time and space seriously hinders the establishment of a framework for natural philosophy and science: time and space are understood as having fundamentally different structures that do not allow for measuring them in the same basic way such that their quantities could be assigned to a single set of numbers. Hence, they cannot be combined in an account of motion. Speed, as understood in terms of space covered over a time taken, cannot be conceptualised in this framework. Plato's account of motion is given only in temporal terms; Zeno's paradoxes of motion still cannot be dealt with.

Chapters 7, 8, and 9: Aristotle's Physics

Aristotle's *Physics*, considered in the last three chapters of this book, represents the final point in the development of the basic concepts traced by my investigation. The focus of my discussion in Chapter 7 is Aristotle's notion of a

[46] We will see that these are the positive criteria which an *eikôs mythos* has to fulfil.

continuum as developed in his *Physics*. Continua are Aristotle's answer to the problematic part-whole relations underlying Zeno's paradoxes: continua are wholes that are prior to their parts; (potential) parts are acquired by dividing the whole. I show that the notion of a continuum as understood by Aristotle is heavily influenced by an implicit mathematical understanding of continua which is modified in some respects so as to make it the fruitful concept for the investigation of nature that it becomes in Aristotle's work.

In Chapter 8, I show how this notion of a continuum enables Aristotle to deal with a central problem raised by Zeno's paradoxes and left unresolved by Plato's *Timaeus*: how to combine time and space in an account of motion and speed – a problem Aristotle solves by understanding time and space as continua and thus, independent of any specific differences, as possessing the same basic structure. In order to measure motion, time and space must be related to numbers in the same way – Aristotle lays the foundation for this in his *Physics* and *Metaphysics*. These changes in the conceptual scheme are part of what allows Aristotle to combine time and space *in practice*, when analysing Zeno's paradoxes, and thus to prepare the ground for central features of the fundamental framework that will persist through to modern investigations of nature.

In Chapter 9, I argue, however, that ultimately the implications of the mathematical concepts that Aristotle takes over, as well as some of his metaphysical convictions, prevent him from offering an *explicit account* of motion sufficient for natural philosophy and science. One important example of such an implication is the principle of homogeneity employed by the geometrical proofs of Aristotle's time, which requires that the elements of a proof be of the same dimension. This principle, I argue, is part of the reason why Aristotle ultimately fails to conceptualise motion as a *relation* between two different dimensions – time and space in the case of locomotion, time and some quality in the case of change – and instead falls back on an account of motion only in temporal terms. However, while Aristotle does not define motion in terms of distance covered over time taken, he makes enormous progress in bringing all the necessary elements together and thus prepares the conceptual ground for understanding motion in this way.

1

Conceptual Foundations

This chapter provides an overview of notions crucial to the present project: these include the notions of the 'methodological foundations' (i.e., the criteria or standards established for philosophical inquiry), of logic, and of the role of mathematics in establishing natural philosophy. Each notion is usually sketched both systematically and historically: the systematic approach clarifies the conceptual basis on which the individual discussions in the chapters build, whereas the historic approach outlines the overall development of each notion during the whole of the period investigated. While this chapter is meant to give a first orientation for the whole project, the detailed reconstruction of the arguments and reasons for my readings of the individual texts will be found in the following chapters. Accordingly, the claims of this road map can only be supported in the individual chapters.

Before we turn to the conceptual foundations, that is, the logical, methodological, and mathematical notions, I will outline what is understood by *kinêsis* (the Greek term for motion and change) and the philosophical problems this concept raised in ancient Greece. I will also briefly consider what we can understand by 'natural philosophy' and its relationship to the concept of *physis* (the Greek term for what we call 'nature').

1.1 The Concepts of *Kinêsis*, *Physis*, and Natural Philosophy

*1.1.1 The Concept of Motion (*Kinêsis*)*

The concept of motion is central in ancient Greek natural philosophy. In fact the very idea of nature itself is fundamentally dependent on that of motion. For example, it is a basic point for Aristotle that things are by nature (*physei*) only if they have a principle of motion (*kinêsis*) and rest in themselves.[1] *Kinêsis* can be understood in an active or a passive sense here – as caused by an agent or as suffered by a patient.[2]

[1] *Physics* 192b13–14, cf. also 200b12–13.

[2] For the passive sense see *Physics* 255b30–1 and Anagnostopoulos 2017, p. 172. On the other hand, the word *kineitai*, that is the grammatical passive of the cognate verb, can often be translated as 'it moves' (in an intransitive sense).

Given that our focus is natural philosophy, not every kind of motion and change will be addressed in this study. Intellectual motions, for example, although sometimes important, will not be discussed.[3] Which kinds of motion will be investigated then? The object of investigation could be characterised as motions and changes that take place in time and space.[4] However, since the establishment of time and space as constituents of or a clear framework for certain kinds of motion is one element of the great task undertaken by the individuals examined in this book, we must not presuppose that the ancient Greeks studied here had a clear notion of processes *in time and space*. Only with Aristotle – and, in some aspects, only after Aristotle – are time and space seen as quantities on the same footing, with the result that we can start to understand the motions we are talking of as motions in time *and* space.

Another alternative is to understand the processes investigated by natural philosophy as motions and changes of sensible things, as the motions and changes of what can be perceived. This is roughly Aristotle's line, but it will get us into trouble with the atomists, for they distinguish what is perceptible (the arrangement of a sufficient number of atoms) from what is bodily (atoms), yet atomic movement seems to be a clear case of something that should be part of natural philosophy and that in fact Aristotle also discusses in his natural philosophy. Similar problems hold for Plato's *Timaeus*. We will therefore use *bodily motions and changes* as our working definition of the kind of motion and changes investigated. We must remain aware, however, that we are looking at a period when the very notion of bodies also underwent significant changes.[5] And we usually understand bodies as at least spatially extended, so again we need a notion of space that was not yet available at the beginning of the thought process under consideration here.[6]

[3] Nor are human actions part of what is investigated, as they raises problems in addition to the basic problems of the intelligibility of processes.

[4] Motion and change *in* time and space are here meant to cover not only locomotion, but all processes that we think of as taking place in time and space, including alteration, growth, and so forth. For Aristotle, alteration and growth presuppose spatial contact, which is a reason why locomotion is the paradigmatic form of *kinêsis* for him. In most of the book, I will also focus on locomotion.

[5] Cf. Sattler 2016.

[6] Given these problems, it might seem to be more appropriate to talk about ancient philosophy of science rather than natural philosophy. The problem with this suggestion is that we are dealing with a time when philosophy and science were not yet clearly differentiated. Apart from mathematics, and probably medicine, there were no other sciences around as sciences independent of philosophy. Furthermore, what I will do in this book is 'classical' natural philosophy in the sense it came to be established on Aristotelian grounds, as looking at the basic notions that a philosophy of nature investigates, most prominently motion and change, time and space, infinity, continuity, and measure. Since the eighteenth and nineteenth centuries there has been a distinction between philosophy of science, as philosophical reflection on methods and concepts of

The Greek term that is meant to be captured by 'motion' is *kinêsis*, a term that can have a wider meaning than the contemporary English word, for it can refer to all kinds of changes, alterations, and processes, like changing colour, or growing. This broad understanding is probably most explicit in Aristotle's *Physics*, where he explains that *kinêsis* and *stasis*, used in his account of what is "by nature", are said in respect of place, growth and diminution, and alteration (192b13–23).[7] Although *kinêsis* has this broad meaning, it is often translated as 'motion' in the sense of locomotion, and I will follow this tradition, since locomotion is a, if not the, paradigm for *kinêsis*.[8]

In ancient times, the conceptualising of locomotion raised two main problems: (1) the ontological challenge that *kinêsis* requires Being and non-Being to be connected; and (2) the problem that an understanding of locomotion requires time and space to be brought together.[9]

With both problems we see that relations are central – the relation of Being and non-Being in the first problem, and the relation of time and space in the second. Let me spell out the centrality of relations more specifically: if something moves or changes, it is first at place p_1 or in state s_1 and then at place p_2 or in state s_2. It is therefore essential to motion and change that after the motion or change has occurred (or after *some* of the motion or change has occurred), the thing moving or changing is no longer in the place or condition from which it started out – for otherwise no motion or change has yet occurred.[10] So in order to capture motion and change, we have to understand that it relates being at place p_1 and then no longer being at p_1, or being in state s_1 and then no longer being in s_1, which means it can be characterised as a relation between some Being and some non-Being.

Similarly, we have to think of locomotion as a relation of time and space. In order to show this, let me take as a simple example a tortoise that moves from the apple tree in my garden to my garden shed in three minutes. This example shows that in order for there to be any motion, we have to deal with different places (the apple tree and the garden shed) and different times (before and

and for the sciences, and natural philosophy as the inquiry into nature in a way not done by the sciences, as explaining and understanding nature.

7 For the relationship between *kinêsis* and *metabolê*, which are sometimes, but not always, used interchangeably in Aristotle, see Waterlow 1982, p. 93 ff. For the understanding of *kinêsis*, see also Kelsey 2003.

8 With other changes, like changing colour, we need to bring together time and some state or condition, like colour. While I will concentrate on locomotion, other changes can in many respects be seen as parallel to locomotion if we just substitute space with state or condition (as Aristotle makes clear, for example, in *Physics* 227b23–6).

9 I talk about 'space' here, but, as mentioned before, most of the time all we need is an account of place or of the distance covered.

10 Also an object moving in a circle will be in a different place than that from which it started for at least part of the motion. And an object spinning in the same spot will have its parts in different positions at different times.

after, which today we would capture as the moments when the tortoise's journey begins at t_1 and when its journey ends at t_2). If there weren't different places in space or points in time, no process would take place. Additionally, the different times and spaces need to stand in some relation to each other, since if one had nothing to do with the other, that is, if the different places/times were completely unrelated, then again there would be no motion (if place p_1 and place p_2 are not connected by something moving from p_1 to p_2, then the mere fact that there are these two places could also hold in a world in which no motions and no processes whatsoever take place).

Motions and changes involve at least the following three relations: (1) the relation between different times (for example, the relation between the starting time t_1 and the finishing time t_2 of a process, labelled R_1 below); (2) the relation between different places or conditions (for example, the relation between a place p_1 at which a process starts and a place p_2 at which a process ends, R_2 below); and (3) the relation between the times and the places/conditions (R_3):

<table>
<tr><td>time
R_1 (t_1, t_2)</td><td>space/condition
R_2 (p_1, p_2)</td></tr>
<tr><td colspan="2">speed
R_3 (R_1, R_2)</td></tr>
</table>

R_3, the relation of R_1 and R_2, gives us the speed of the process, that is, the relation between times and places/conditions.

The relationship of Being and non-Being with respect to motion appears as problematic in Parmenides, whereas Zeno shifts the focus to the relationship between time and space, as we will see in chapters 2 and 3. While the relationship of Being and non-Being is dealt with successfully by Plato and Aristotle, as we will see in the course of the book, the relationship of time and space is not treated satisfactorily until the very end of the period we are considering.[11] We will see both the importance of this relation for an understanding of motion and the problem its conceptualisation raises in the last two chapters of the book.

Let me now sketch separately and more fully each of the two problematic relations involved in accounting for motion, first the relationship of Being and non-Being and then the relationship of time and space.

1.1.1.1 Motion as a Relationship of Being and Non-Being

The fact that *kinêsis* connects Being and non-Being seems to be the reason why motion was seen as indefinite or indeterminable in early

[11] As I will concentrate on locomotion in the following, I will only deal with the relationship of time and space, and not with the possible relationship between time and some other condition or state which is, however, structurally similar.

Greek thought or accounted for in terms of difference or inequality.[12] For the Eleatics, *kinêsis* cannot be part of the realm of what can be known, since motion and change are understood on the basis of generation, and Parmenides rejects generation as a possible object of knowledge. Motion requires bringing together Being and non-Being within Parmenides' framework: the condition in which something 'is' ceases to be, thus it 'is not', and the new condition comes into being – a way of thinking about *kinêsis* that we also find described in Aristotle's *Physics* VIII, 3: "it seems to almost everybody that being moved/changed is the coming into being and passing away of something".[13] Thus change is understood as the destruction of one property and the generation of another – accordingly, Parmenides derives the unchangeability of Being (its being *akinêton*) from the refutation of generation.[14] Similarly, with locomotion, the place at which something starts its motion ceases to be its place and a new place 'comes into being' as the place where this thing is now. This connection between motion and generation is an inheritance that Parmenides' successors will contest.

For the atomists, non-Being qua void is integrated into the basic ontology and connected with Being – what is (the atoms) needs the void (what is not), as it guarantees the atoms' separateness and thus their plurality as well as the possibility that the atoms can move. The phenomena we perceive are understood as arrangements of the indestructible but moving atoms. Thus motion is no longer understood in terms of generation; on the contrary, the generation of the things we perceive is understood as the locomotion of atoms.

While for the atomist (and Melissus) non-Being qua void is a necessary condition for motion, Plato shows in his *Sophist* that non-Being is, so to say, a constituent of motion, since motion itself has to be thought of as a relationship involving Being and non-Being. Such a connection is, however, unproblematic if Being and negation are understood in an appropriate way. Furthermore, everything else that we conceive of will require us to connect Being and non-Being as well, for whatever 'is' also 'is not' all other things.

Finally, Aristotle moves the understanding of *kinêsis* from its seemingly uncomfortable position between Being and non-Being to another metaphysical position, one central for him, namely between potentiality and actuality.[15] For Aristotle, *kinêsis* cannot be classed solely with what is potentially or solely with what is actually, but has to be understood as the actuality of what is potentially, in so far as it is potentially. Aristotle even defines motion in terms

[12] See, for example, Aristotle *Physics* III, 201b18 ff.

[13] 254a11–12.

[14] In fr. 8, lines 26–8.

[15] Which, however, may be seen in close relationship to Being and non-Being.

of potentiality and actuality in book III of his *Physics*, a definition to which we will return briefly in Chapter 7.

1.1.1.2 Motion as a Relationship of Time and Space – Speed

The second problem concerns the conceptualisation of motion as covering a certain distance in a certain time. The specific relation of the distance covered and the time taken gives us the speed of a motion – the greater the distance I cover in the same time, or the less time I need to cover the same distance, the faster I am moving.

For us the notion of speed is rather unspectacular. We have no problem, for example, with talking of driving at 100 kilometres an hour, a speed we can easily check on our speedometer. I want to show in this book, among other things, that the idea that we can measure speed in units of space (kilometres) per units of time (hour) is not a natural given and that such a conceptualisation of speed was a huge achievement, the result of a long process started in ancient times. In the beginning, however, speed seems to have been tied more to practical concerns, such as performing a task while a certain opportunity is available, wondering whether an army will get to a certain place by a certain time, and so forth.

Our modern understanding of speed requires time and space to be brought together as two entities that can be treated on the same footing, which was not possible at the beginning of ancient Greek philosophy. As we shall see, Zeno's motion paradoxes and the atomists both neglect time, while Plato's account of motion neglects space; accordingly, they all have problems with the notion of speed. With Aristotle we find most of the major steps required for a modern conceptualisation of speed, but for mathematical and metaphysical reasons he never fully conceives of speed in this way, as we will see at the very end of this book.

At times we will look at astronomical accounts of motion and speed, not because I assume that astronomical accounts could be used for sublunary motions without any modifications[16] or that these accounts were necessarily given with a view also to explain non-celestial motions, but because often these are the few places where we find any discussion about the speed of motion at all.

Understanding speed as kilometres or miles per hour (distance covered in a certain time) seems so natural to us that it can be hard to imagine that it took hundreds of years to be able to do so. You may wonder, What's so difficult about bringing these two different entities, time and space, together? After all, a cook at that time could have thought, 'I need two eggs per person for this meal', or he could have reasoned, 'I'll use six roses to decorate each ivy garland'. Surely, eggs and people, roses and ivy garlands are each different

[16] We will see that dealing with angular velocity in the case of planetary motions faces different requirements than the sublunary speed we are concerned with here.

entities. But eggs, people, roses, and garlands come as individual units – they are discrete entities – while units of time and space do not come individually prepacked. How then do we get spatial and temporal units in the first place? You may think, the same problem arises when our cook plans minced meat for our party, which does not come in given units – he purchases it in one piece and will simply chop it into pieces afterwards. But here is our second problem, for time and space are not entities given to us in the same sense as minced meat is given to our cook. Time and space are not tangible in the way minced meat is, and indeed time and space were only established as determinate and distinct 'things' in the time of Plato and Aristotle.

The ancients up to Aristotle[17] obviously had some idea about being faster and slower. They knew that a ship may cover the distance from Athens to Sicily in different times depending on the season and winds, and, as we noted in the Introduction, they could compare two runners in a single race at Olympia and decide who the winner was. There were, however, problems with comparing the speed of two ships sailing different routes or two competitors running different races – not just for practical reasons, but rather for conceptual ones. While the difference in speed between two things may be expressed as ratios of distances and times, this normally required either the times or the distances covered to be the same.[18]

But just a minute: in his account of the so-called Achilles paradox, Aristotle mentions Achilles as "the quickest runner in legendary tradition" – don't his words imply that the ancients could compare runners across different races? No, because they had no conceptual basis allowing them to measure that Achilles was in fact faster than whoever came after him. During his lifetime (if indeed there ever was an Achilles) or through Homer's stories, Achilles acquired a reputation as the fastest runner in the ancient world, and he retained that reputation thanks to the stories about him, not because his speed was *measured* against the speed of other runners; certainly not after his time.

When we compare the motion of two different bodies, there are at least three conceptual stages to understanding speed:[19]

(1) The first is a comparison of two objects that are moving simultaneously, starting at the same point and covering the same distance. Here we can simply say that the object that arrives first at the end point is the faster.[20]

[17] I am not looking here at later developments in ancient thought that we may find in Apollonius of Perga, Ptolemy, and others.

[18] The times or distances covered must *be* the same, and not just *be expressible* in the same way, as we will find in Chapter 8.

[19] I owe the suggestion to describe it as three separate stages in this way to Sarah Broadie.

[20] We will see in Chapter 9 that this is the basic case also for Aristotle when he compares different motions – he always assumes that they move in the same time though in fact they need not have started at the same time.

(2) A slightly more elaborate conception is required when the two objects compared cover the same distance but, because they start at different times, do not move simultaneously. Then the faster ship need not necessarily arrive in the port before the slower ship. Rather, depending on when the two ships set out from their port of origin, the faster ship will arrive before or after the slower ship. Yet, whether it arrives first or second, the faster ship will still arrive 'sooner' in the sense of 'in less time' – that time need not be a sub-stretch of the actual time in which the slower boat travelled. Accordingly, in this case we need a means to measure time, while in the first case we need only to see which ship arrives before the other.

(3) Finally, if we want to compare the speed of two ships that not only start at different times, but also cover different distances, one from Athens to Sicily, the other from Athens to Crete, say, we need to measure not only the time taken in each case but also the distance covered; and we also need to be able to relate time and space in such a way that we can actually measure the ships' speed. To measure speed, it is not sufficient to be able to say that one ship covered greater distance and the other ship needed less time; those statements alone will not enable us to tell which ship is faster.[21] To determine speed we need an understanding of speed as a relationship between distance covered and time taken. (We usually capture this relationship as 'distance covered *over* time taken', but by 'over' we do not necessarily have to think of a division, since the right kind of relationship between distance covered and time taken may be expressed in different ways.)

In (2) we can compare the same distances or the same tasks (for example, this person is building a house more quickly than that person, or this person convinced the jury in less water clock units than that person). The distance or task is fixed and we only compare the time taken in each case.[22] Accordingly, this method does not allow us to compare different distances or different tasks, as in (3). Thus, in both (1) and (2) speed can be reduced to the amount of time

[21] See also Chapter 8.

[22] The Greek thinkers we will look at would know in general that a merchant ship takes more time to get from one place to another than a trireme, but again this does not yet give us a conception of speed as it does not allow us to say how much faster the trireme was on this particular journey compared to the merchant ship on some other journey. You may object that we know that a trireme is faster than a merchant ship even if we cannot quantify the difference between them. Presumably then we think the trireme is faster from cases of (1), where we have seen them cover the same distance starting out at the same time and the trireme coming into the port earlier. There could, however, be a case where a trireme is really slower, because the rowers are taking it easy, so that a merchant ship is in fact faster – our general idea about triremes being faster than merchant ships would not allow us to deal with such a case.

taken – either the amount of time perceived (as in case (1)), or the amount of time measured (as in case (2)). But neither account understands speed as a relation of distance covered over time taken. Rather, when something is talked of as 'being faster', what is actually meant is what we might call 'arriving before' the other or 'arriving sooner'; 'soon/later' and 'before/after' thus seem to be more basic. And the distance traversed does not seem to have been considered in its own right, but is rather seen as a task to be accomplished.

Furthermore, the ancient Greek thinkers we will look at could not give an account of speed that was not comparative – while they may think that you are running quite fast, this would always be thought of in comparison to some other people running (or to the time you usually take). But we do not find a non-comparative account of your speed simply as how much distance you covered in how much time (independent of any other runs). As we will see with Zeno's paradox of the moving rows, in Chapter 3, and with Aristotle's notion of measure, in Chapter 8, the ancient Greek thinkers looked at here did not have a non-comparative, complex measure of motion that would allow them to measure speed.

Thus we see that conceptualising speed is not simply a practical problem and not a problem that stems from the ancient Greeks not having speedometers, or sufficiently precise watches.[23] If the concept of speed had been available, it would have been possible to use a water clock to measure time, measure out length, and then calculate speed.[24] The problem is conceptual – for reasons we will discuss in this book, the idea of measuring a complex quantity like speed, which combines time and space, is problematic. While we do find a comparative account of speed along the lines of (1) and (2), we do not find step (3) or a non-comparative account of speed in ancient times up to and including Aristotle.[25] None of the assumptions necessary for conceptualising speed as a certain distance traversed in a certain time[26] is, however, completely foreign

[23] Stephen White 2008, p. 122 points out this practical problem with respect to time measurement in Anaximenes: he "has no way to measure time precisely enough to determine velocities". But, as mentioned in the main text, I do not think that this is the fundamental problem. We also know from Ptolemy that he had more precise means of measuring time than he cared to employ (cf. Jones 2019, which shows that while Ptolemy's method of measuring time in *Almagest* book 9, chapter 10; book 10, chapters 4 and 8; and book 11, chapters 2 and 6 would give a precision of approximately four minutes, in fact the times are expressed in equinoctial hours to a precision of a quarter-hour at best).

[24] As Koyré 1968, p. 94 has pointed out, Galileo's experiment of letting a bronze ball roll down a sloping wooden groove to figure out the relationship of time taken to space traversed in the *Discorsi* used a timing device (a vessel filled with water) that was even less accurate than Ctesibus' constant-flow water clock introduced in the third century BCE. Thus lack of accurate timing devices is not itself a reason for the lack of an explicit measure of speed (even though it obviously is a reason for the lack of exact measurement).

[25] Thus some readers may prefer to talk about 'being fast', 'fastness', or 'swiftness' rather than 'speed' in an ancient context.

[26] The idea of instantaneous motion is a derivative notion, see Chapter 3.

to ancient times: most are established at a certain point in Greek thinking, although in some instances only the preparation is in place.

What, then, are the requirements for conceptualising speed as a relation between the distance covered and the time taken to cover that distance? An understanding of the complex notion of speed (as expressed, for example, as kilometres *per* hour)[27] requires that:

(1) Time and space are identifiable. Time and space must be clearly distinguishable from each other (and from other possible 'things', like matter). I am unable to explore this criterion here, but I do so elsewhere, making evident that it is not nearly as unproblematic as it may seem. It is often assumed that an understanding of time and space are just given to us.[28] By contrast, I have tried to show that while some basic aspects may be simply given, a full-blown concept or notion of time and space is not; rather, such concepts require lots of intellectual work, which is part of the conceptual development we see taking place in ancient Greece.[29]
(2) Time and space are quantifiable. Time and space can be measured by means of measurement units.
(3) Time and space can be related to each other, that is, they are of such a character that it makes sense to combine a quantum of one with a quantum of the other.[30]
(4) The time and space involved in different motions are universally comparable (for example, by using the same ruler to measure different distances). This then allows different motions to be compared with each other.[31]
(5) Time and space possess a certain order. Different points in space and time must have a certain fixed order with respect to each other; they are either before or after each other.

In early Greek times, before the thinkers we shall consider here, there were problems with the identifiability and universality of time and space. Plato's account of time in the *Timaeus* shows to a certain degree the universality, quantifiability, and identifiability of time and probably of space for the first

[27] Again, 'per' is not the only way to think of the right kind of relationship between time taken and distance covered, but it is a way that is familiar to us.
[28] E.g., as a basic part of our epistemic apparatus, as we find in Kant.
[29] I address the concepts of time and space in two manuscripts that are currently in preparation, *Ancient Notions of Time* and *Conceptions of Space in Ancient Greek Thought*. In the present book I can only look at time and space strictly insofar as they are indispensable for accounting for motion.
[30] The same holds true for changes: here we need the relation between time and a second element, for example, colour, if we look at the speed in which something changes its colour. In the following I will, however, only focus on locomotion.
[31] Today we would simply say that the same time and the same space can be involved in all kinds of motion, but this is a stronger assumption that is not necessarily fully applicable to the period under investigation.

time. Accordingly, his account may sound very much like our understanding of speed. But Plato does not give any account of (3), how time and space can be brought together. And while Aristotle prepares the ground for combining time and space in the notion of speed, as we will see, he too has problems with (3) and cannot define motion in terms of time and space.

Aristotle may not define motion in terms of time and space for good reasons: such an account would not in itself give the unity of a motion, that is, it would not give criteria for identifying one motion in contrast to another, in which Aristotle seems to be interested in his *Physics*, as we will see later in this book. Being in one condition/place A at one time and in another condition/place C at another time would not tell you whether the motion from A to B and from B to C is one and the same motion. But Aristotle also *cannot* conceptualise motion in terms of time and space, as we will see in the last chapter.

1.1.2 *The Ancient Greek Conceptions of* Physis *and Natural Philosophy*

The project of this book is an investigation of the concept of motion in ancient Greek thought as the core concept of *natural philosophy*. This is a modern way of putting it, however. That we cannot just presuppose our own understanding of natural philosophy for the thinkers we are investigating already becomes clear from the fact that the natural realm – the supposed object of natural philosophy – is initially not carved out as we understand it. For example, for the early Greeks, nature (which is how we translate the Greek term *physis*) is not strictly distinguished from the theological realm or even from the social realm.[32] The modern idea of natural philosophy as philosophy dealing with the natural world is a product of a certain development of the notion of *physis* that took place in the ancient world.

For the purpose of this book, I use Aristotle's understanding of *physis* as a guideline, since it integrates important aspects of his predecessors and was also highly influential for our modern notion of natural philosophy. As we saw above, for Aristotle something is by nature (*physei*) if it has a principle of motion and rest in itself. He can use *physis* to indicate the natures of individual things, but at times also *the whole* of everything that has come into being on its own, the whole of all natural phenomena.[33] In this latter case, *physis*

[32] For the claim that the natural and divine realms are not clearly distinguished, see, for example, Sarah Broadie 2012, p. 272: Plato has "to guard against the easy (for fourth-century Greeks) conflation of nature and divinity". For the lack of a clear division between the social and natural realms, cf. Macé 2012 and Kahn 1994, p. 192.

[33] The clearest example for this usage is probably *Eudemian Ethics* 1235a9, but several scholars have also seen *De caelo* 268b11 and *Metaphysics* 984b9 as examples, cf. Hager (1984). *Physics* 185a18 and *Metaphysics* 984a31 also seem to use *physis* in this global sense.

distinguishes a realm that seems to be close to what we would call nature from other realms. By contrast, in early Greek writing[34] the word *physis* seems primarily to refer to the natural constitution of some *individual thing*, either as the result of that thing's growth[35] or as an innate quality.[36] But it is not used to refer to what we would call 'natural things' in contrast to things artificially made and it cannot refer to all things of a particular realm, a usage found clearly in Aristotle. We find some preparation for understanding *physis* as referring to a particular realm in some works of the *Hippocratic Corpus*;[37] and also one fragment in Euripides and some passages in Plato point in this direction.[38]

When we look at how thinkers before Aristotle understood *physis*, we can roughly identify three principal meanings:[39]

(1) *Physis* can indicate the fundamental characteristics of something, its 'true nature'. Such is its first usage in a philosophical text, which we find in Heraclitus.[40] Although this understanding has no specific ties to natural philosophy, it is a philosophically interesting understanding of *physis* that we also find in Aristotle to indicate the nature or essence of something in general.[41]

(2) *Physis* can indicate that which has grown and come into being.[42] Such is the use of the term we find in the fragments of Parmenides, who seems to have been the first to restrict *physis* to things of a certain field – the field where things come into being, grow, and pass away – things he deals with

34 At the very beginning, the word is rarely used: Hesiod does not use *physis* at all, and Homer only once, when in *Odyssey* X, 303 the *physis* of the moly plant is pointed out to Odysseus. See Macé 2012, for the development of this notion.

35 See, for example, *On the Nature of the Child* XVI, Aeschylus *Persians* 441, Parmenides fr. 10, and Naddaf 2008, pp. 25–7. In Empedocles DK31 B8, line 1, *physis* even seems to be equated with coming into being.

36 In Pindar it means the innate quality of an individual, which is given by birth and cannot be changed. For example, in *Olympia* 2, 86 we learn that only the person who knows much due to his *physis* is wise, while what we acquire will only allow us to resound with emptiness. An aristocratic personality especially is due to *physis*: see, for instance, *Pythia* 8, 44.

37 Cf. Macé 2013.

38 Euripides fr. 910 Nauck. For Plato see, for example, *Lysis* 214b, where Socrates refers to the philosophers who talk about *physis* and the whole (*hoi peri physeôs te kai tou holou*), or *Phaedo* 96a8, where Socrates talks about *peri physeôs historian*, which interested Socrates in his youth.

39 The first two senses reflect the meaning of the verb *phyô*.

40 See frr. B1, 106, 112, 123.

41 See, for example, *De anima* 403b24–5. It is also a meaning prominently found in Plato, for example, when in *Timaeus* 35a he talks about the *physis* of the Different and the Same.

42 It thus refers to the result of growth; but there are also passages where *physis* can mean the process of generation or, as in Plato's *Sophist* 265d, include the origin of generation.

only in the *doxa* part of his poem.[43] Parmenides does not use the term to refer to the whole of this field, however.

(3) *Physis* can refer to what is given to us, in contrast to what is made or set up by human beings. Such appears to be a frequent use of *physis* in the so-called *nomos-physis* debate of the fifth and fourth centuries BCE.[44] In this debate, *physis* usually appears in opposition to *nomos*, a contrast fleshed out in a variety of ways; and both *nomos* and *physis* can receive very different evaluations. What all these different understandings usually share, nevertheless, is that *nomos* is seen as in some sense artificial, which can be either positive or negative,[45] referring to the statutes and regulations that human beings give to themselves. *Physis*, by contrast, is given to human beings or to an individual human being, not made by human beings, and thus is to some degree independent of us.[46]

The second and third meanings listed here, which define *physis* as that which has come into being and grown and as something that is given to us, are important aspects of Aristotle's understanding of it, even if he develops them further. Aristotle clearly distinguishes between nature and what is made by *technê*, human productivity. He also clearly distinguishes natural philosophy from theology when he introduces natural philosophy (*philosophia physikê*), as the name of a specific philosophical enterprise that, together with theology (*theologikê*) and mathematics, constitutes the theoretical sciences (*epistêmai* or *philosophiai theôrêtikai*, 1026a18 ff.).[47] Natural philosophy investigates those things that are perceptible by our senses in so far as they are moving and can be grasped conceptually. Aristotle (and then his school) attempts to show us that most of the earliest Greek philosophers were his precursors in this project.[48]

43 See fr. 10, lines 1 and 5; fr. 16, line 3; and fr. 19, lines 1–2; and what we would call a 'counterfactual' in fr. 8, line 10.

44 Cf. Heinimann 1945 and McKirahan 2011, ch. 19.

45 Positively as that which allows for forming civilisation; see, for example, Critias DK 88 B25; negatively as that which is used by the weak to suppress those who are naturally strong, as Callicles in Plato's *Gorgias* assumes. Cf. also McKirahan 2011, pp. 397 and 403–4.

46 This can, but need not, imply that *physis* is unalterable, as the example of Antiphon DK87 A44 shows. And often *physis* is seen as given to human beings by the gods (thus assuming a close connection between the 'natural' and the divine), but again this is not always the case. The contrast between *nomos* and *physis* seems to be taken up also by Democritus with his contrast between what is by *nomos* (sweetness, colour) versus what is in truth, *eteê* (atoms and void). Cf. Makin 1993, p. 70 f. and Taylor 2007, p. 8.

47 There are, however, divine things in nature for Aristotle, for example, the heavenly bodies. And he seems to distinguish *theologia*, as what the poets do, from *theologikê*, as a – and in fact the most eminent (*timiôtatê*, line 21) – philosophical science.

48 It seems likely that περὶ φύσεως became a standard title for many works of Presocratics in Aristotle's school (see Schmalzriedt 1970, p. 106 ff. and Huby 1973). This standard title probably helped to see the works from the Milesians all the way to Anaxagoras as one

Some may argue that this position produces a distorted view of the Presocratics; but, while it leaves out other significant ways of looking at them, it remains one important way of thinking about the Presocratics and the development of philosophy.

1.1.3 The Concept of Being

Another theme that plays into our investigation is the development of the understanding of 'Being'. As we have just seen, the notion of motion connects Being and non-Being – something *is* here now, but later on it *is not* here any longer. We will consider the negative element in 'non-Being' by looking at the negation operator below. To consider what is understood by 'Being' would also be an important step, but in light of the size and complexity of that enterprise, I will not be able to advance down that path.

Some of the main problems with the notion of Being are tied to the fact that the verb *esti* can be used in very different ways. Aristotle points out that it can have different meanings: in *Physics* I, for example, he complains that Parmenides takes 'being' as if it were used in only one sense when in fact it is employed in many ways (*pollachôs legetai*).[49] Today we distinguish between syntactic and semantic differences: *esti* can have distinct syntactical functions – it can be used in a complete or an incomplete way[50] – and it can be understood semantically in existential, veridical, predicative,[51] and identifying ways; that is, it can indicate the existence or reality of something (existential), that something is the case (veridical), that something is some F (predicative), or that some *x* is the same as some *y* (identifying). Debate surrounds which of the two distinctions, the semantic or the syntactic, deserves more attention and, within the syntactic and semantic fields, which were the ancient possibilities and which of those possibilities were dominant.

I do not have space in this book to participate in this debate. All I shall do here is show that Parmenides' position rests not on a confusion in the notion of

tradition, the περὶ φύσεως tradition. Following Aristotle's account, περὶ φύσεως seems then to be the tradition dealing mostly with what we would call natural phenomena. For possible problems with such an understanding of the establishment of the περὶ φύσεως tradition, see Schmalzriedt 1970.

49 186a22 ff. In Aristotle we find the distinction between substance and other categories on the one hand (*Metaphysics* Γ, 2) and the fourfold division into accidental, substantive, veridical, and potential/actual on the other hand (in *Metaphysics* Δ).

50 See, for example, Brown 1994.

51 Kahn 1966, p. 19 claims that the distinction between existential and predicative fuses "a syntactic and a semantic criterion into a single antithesis". But Aristotle himself draws a difference between predicative (being something) and existential usage (being *haplôs*), for example, in *Posterior Analytics* 89b32–5 (while in other passages, 'being *haplôs*' means simply 'being without qualification'). According to Brown, however, Aristotle also allows the inference from '*x* is F' to '*x* is'.

Being (i.e., on an illegitimate, or perhaps not so illegitimate,[52] inference from '*x* is F' to '*x* is'), but rather on his logical operators and his understanding of the criteria for philosophy. The subsequent history of philosophy does not therefore simply react to a confusion in the notion of Being, or so I argue.

A quick note on translation is appropriate here: I will translate (*e*)*on* usually as 'Being' and not as, as is often the case in English, 'what is', using the uppercase *B* to indicate that we are dealing with an object of inquiry rather than with a participle that is part of the grammatical construction of my own sentence.[53] 'Being' may sound somewhat artificial in English, but this artificiality may itself be helpful: it encourages us, for example, to avoid leaping to interpret the object of investigation in Parmenides' case as ordinary perceptible things in the world without first undertaking careful investigation.[54]

1.2 Criteria of Inquiry

The possibility of developing a *conception of motion* relies not only on a certain understanding of time and space, but is also influenced by our understanding of the criteria for philosophical inquiry, of basic logical operators, and of notions, such as those of continuity and infinity, that were informed by the mathematicians – and to these requirements we will now turn. We will start with the criteria for philosophical and scientific inquiry. Such criteria can be understood in different ways, and they can work on different levels.[55] This book will employ fairly general criteria and principles for all kinds of rational inquiry – only once philosophy and science became more specialised, with

52 According to Brown 1986 and 1994, we can move rather freely between '*x* is F' and '*x* is', and between '*x* is not' and '*x* is not F' in ancient Greek.

53 What I mean here is that 'being' could also just be part of a sentence I construct, as, for example, in the sentence '(*e*)*on*, being ambiguously formulated, never becomes fully clear in Parmenides'.

54 Cf. also Rowe's translation, p. xliv ff. (judging from personal communication, Rowe seems to have concerns similar to mine, but thinks they are better dealt with if we use the translation "what is").

55 For example Vlastos 1975c, p. 36 understands scientific inquiry as inquiry working with scientifically ascertained facts and lists as criteria for these that (1) they are established by observation and are hence derived by the use of the senses; (2) they have theoretical significance, which means they provide answers to questions posed by our theory; and finally (3) they are sharable and corrigible (investigators are aware of possible sources of error and can repeat or vary the observations to reduce suspected error). If we look at criteria of inquiry in Leibniz, on the other hand, we are told that there are "two great principles" on which all reasoning is based, namely the principle of sufficient reason and the principle of contradiction (see *Monadology* sections 31–2). While Leibniz's criteria are meant to hold for all kinds of reasoning, on the most basic level, the criteria named by Vlastos are specific for a science based on observation and work on a less basic level. The criteria I am interested in here are (with one exception) much more Leibnizian than Vlastosian: they are the most basic criteria for any kind of inquiry.

different sub-disciplines, did the criteria become more specialised, intended for different fields. In this section, I will give a brief systematic and historic account for each criterion involved.

Most of the standard formulations of the criteria I discuss in the following can first be found in Aristotle; this is the reason why I will say least about them in the chapters on Aristotle and why we will often encounter a wider understanding or less exact formulation among the thinkers before Aristotle. But the core ideas of these criteria were already there before Aristotle; it may just be more difficult to recognise them immediately.

Parmenides will be understood as the first thinker in the West to introduce clear criteria for philosophical inquiry. This claim may provoke two objections: first, you may doubt that Parmenides does indeed introduce criteria for inquiry; and second, you may doubt that Parmenides is the first thinker to do so. In response to that first possible objection, I will give a full account of the criteria he introduces in his poem in the next chapter. But note that I am not claiming that Parmenides calls the principles he works with in his investigation 'criteria',[56] nor that he is always explicitly reasoning about these principles. Rather, I am claiming that he introduces these criteria (sometimes explicitly, sometimes implicitly by employing them in his reasoning) and uses them systematically in such a way that something which does not fit these principles cannot be part of the philosophical investigation.

As for the second possible objection – perhaps you wish to point out that Anaximander had famously already employed something like the principle of sufficient reason and that surely consistency must have been used by Parmenides' predecessors. What then is so special about Parmenides? Parmenides is special because he is much more explicit in employing these criteria and, more importantly, he firmly sticks to them as criteria for his 'way of truth [*alêtheia*]' and thus strictly draws from them the consequences for his philosophy.

The main criteria we will be discussing are the principle of non-contradiction, the principle of excluded middle, a criterion I call 'rational admissibility', and the principle of sufficient reason. These criteria are crucial for Parmenides, and they are taken up and developed by his successors. In addition, natural philosophers after Parmenides introduce a criterion more specific to natural philosophy, namely that of 'saving the phenomena'. Let us turn now to look briefly at each of these criteria.

1.2.1 The Principle of Non-Contradiction

We normally understand the principle or law of non-contradiction as one concerning propositions. For our purposes, however, we will use a wider

56 Usage of the word κριτήριον seems to start only with Plato, see *Republic* 382a and *Theaetetus* 178b.

understanding, such that a thing cannot be A and not-A, leaving open for the time being the category of expression to which 'A' refers (so A could refer to a proposition or to a term or predicate).[57] Such an understanding is supported by Aristotle, who sometimes understands the principle of non-contradiction as claiming 'not (p and not-p)', but at other times as 'not (Being and Non-Being)'.[58] For this study, I want to understand the law of non-contradiction as a feature of actual and possible things and as a law for how we can think – based not on psychological grounds, however, but on logical grounds. Accordingly, while we can violate this law or principle in speech (and can claim A and not-A), it cannot be violated in thinking (for then we are no longer thinking) nor in being.[59]

57 The distinction between names and propositions was made first by Plato, but we will also look at thinkers before him. In fact, if we deal only with propositions, some more restrictions would be necessary to exclude the possibility that one of the propositions is itself a disjunct that would get us an inconsistent triad, so cases where p is inconsistent with (q and r) since they together give us not-p.

58 You may, however, object to such wider usage of the principle of non-contradiction by pointing to Aristotle himself and understanding him as Horn 2018 does: "As Aristotle explains in the *Categories*, the opposition between contradictories – 'statements opposed to each other as affirmation and negation' – is defined in two ways. First, unlike contrariety, contradiction is restricted to statements or propositions; terms are never related as contradictories. Second, 'in this case, and in this case only, it is necessary for the one to be true and the other false' (13b2–3). Opposition between terms cannot be contradictory in nature, both because only statements (subject-predicate combinations) can be true or false (*Categories* 13b3–12) and because any two terms may simultaneously fail to apply to a given subject." For the thinkers before Aristotle, however, it will make good sense to allow also for terms to be understood as contradictory – by this I mean that if they and their opposite were employed in two, otherwise identical, statements, one of the two statements would be true and the other false. And both opposites cannot simultaneously fail to apply to a given subject in such a statement.

59 *Reductio ad absurdum* proofs seem to call into question such a claim, since they finish with something like 'if p, then A and not-A' in order to dispose of p. So by the time I finish the proof, in some sense I'll be thinking both A and not-A, and if I weren't, the proof wouldn't have succeeded. A closer look at such *reductio ad absurdum* proofs in fact supports my claim, however. Working through such a proof we can think A and not-A in the sense of entertaining both simultaneously and in the same respect, but we cannot think or claim that both are true simultaneously, for then it is no longer clear what I am claiming. *Reductio ad absurdum* proofs would not work if in fact we thought that both can hold at the same time – it is precisely because we cannot think both to hold at the same time that such a *reductio* moves us to give up p. Alternatively, you may think that the principle of non-contradiction should just be normative – we shouldn't violate it in our thinking but in fact our thoughts often involve inconsistencies (such as those that Socrates attempts to expose in the beliefs of his interlocutors in many of the early Platonic dialogues). In this case, we are working with a wider understanding of thinking than I suggest in the main text, a notion of thinking that is closer to just entertaining a thought. While we may entertain contradictory thoughts, it seems to me that we cannot be committed equally to contradictory thoughts (at least not if we are fully aware that two

You may think that such an understanding is only a precursor to the principle of non-contradiction, since it does not involve propositions, may sometimes involve opposites in a broader sense than just contradictions, and is only spelled out as a principle by Plato and Aristotle. If you think it is only a precursor, then whenever I talk about the 'principle of non-contradiction', just read 'precursor to the principle of non-contradiction', 'principle of non-contradiction*', or, inspired by Julia Annas' terminology, 'principle of non-conflict'.[60] I will refer to it as 'principle of non-contradiction' nevertheless, for I believe that the core idea of this principle was already on the table with the early Greek thinkers.

In the very beginning of the period under investigation we find the principle of non-contradiction in a form that we may think not sufficiently specified: we find 'something cannot be A and not-A', and do not yet find 'something cannot be A and not-A *in the same respect*'. This qualification as to respect is a development explicitly spelled out by Plato and Aristotle.[61] Perhaps some will deny that without respect this can indeed be the principle of non-contradiction, since that qualification matters precisely because it is not a contradiction for something to be F at one time and not-F at another time, or for something to be F with respect to one part but not with respect to another. If we are using a principle that proscribes not only agreed contradictions but also things that are not contradictions, the argument would then run, this would seem to be not a principle of non-contradiction, but something else. However, such a principle contains at least *in nuce* the very idea of the principle of non-contradiction – it is just that idea has not yet found its fully appropriate expression. To teach somebody what the principle of non-contradiction is, we may very well start with exactly this core idea: we could say first that this principle means something cannot be F and not-F, and only then specify further that it cannot be F and not-F at the same time or with respect to the same thing, etc.[62]

thoughts, or their implications, are inconsistent) – and this is what I want to understand by claiming that we cannot violate the principle of non-contradiction in our thinking.

60 Annas 1981, p. 137; cf. also Robinson 1971, p. 38 ff., who calls it the 'principle of opposites'.

61 But see also Gorgias *Palamedes* DK82 B11a25, which clearly seems to include respects: "how can one trust a man of the sort who, speaking the same *logos* before the same men says the most inconsistent things about the same subjects?" We will also see the atomists implicitly using this qualification.

62 This is also how conversations may unfold – I may tell you, 'J. is a total fool' and two minutes later I may claim, 'J. is not a fool', and only if you object, 'He cannot be a fool and yet not a fool' may I specify, 'Oh, he is a complete fool in these kinds of conversation, but he is very clever in those other kinds of situations'. Cf. also Wedin 2004, p. 234, who formalises the ontological version of the law of non-contradiction in Aristotle without explicitly putting in any respects: "$\neg\Diamond(\exists x)(Fx \wedge \neg Fx)$".

Let us now look at the history of how this principle developed. We may assume it was already at play with Heraclitus, since his paradoxical statements concerning opposites seem to make sense only against the background of a principle of non-contradiction (i.e., only then are his statements paradoxical). In light of his claim that the same thing produces opposite effects on different perceivers, Heraclitus may also be seen as introducing respects.[63] However, Heraclitus seems to be more interested in the paradoxical formulation that the same thing is F and not-F than in the idea that employing different perceivers allows us to make the claim consistent.[64] Only with Parmenides does the principle of non-contradiction appear as a principle *guiding* philosophical inquiry.[65] It is thus a criterion for philosophical inquiry – if a claim violates this principle, it can be dismissed immediately.

As we will see in the next chapter, the particular version of the principle of non-contradiction to which Parmenides subscribes is more austere than we are used to after the developments it underwent with the atomists, Plato, and Aristotle. As just discussed above, we usually understand the principle of non-contradiction to mean something like 'not (p and not-p) at the same time or in the same respect'.[66] Parmenides, however, understands it as 'not (F and not-F)' without allowing for taking into account that something may be F and not-F at different times, or in different respects. Thus, for Parmenides, what we would call the principle of non-contradiction is captured as 'not (F and not-F) simpliciter'.[67]

Furthermore, for Parmenides, the principle of non-contradiction has to be understood in a way that is intimately connected with the principle of excluded middle; he is not just committed to both of them, but both are simultaneously applied in philosophical inquiry: 'something cannot be F and not-F [principle of non-contradiction], but yet it has to be either F or not-F [principle of excluded middle]'. So for Parmenides the laws of non-contradiction and excluded middle are conjoined: (1) (x) not (Fx and not-Fx) and (2) (x) (Fx or not-Fx).

Zeno then uses the principle of non-contradiction extensively for his paradoxes. Paradoxes give an apparently sound proof of an unacceptable

63 See, for example, DK22 B61, 102, and 111.

64 DK22 B49a employs a contradiction ("in the same river we step and do not step, we are and are not"), though Heraclitus often seems to employ contraries, for example in DK22 B67 and 88.

65 Simplicius *In Ph.* 117.4–13 quotes fr. 6, lines 1b–9 as evidence that Parmenides held that contradictories cannot be true simultaneously.

66 Alternatively, we may think that p is a fully specified statement or respect so that this reference to respects is redundant, because built into p. This is not how I understand p in the following, nor F as a fully specified property in this sense.

67 I am talking about 'F' rather than 'p' here in order to make it clear that with Parmenides we should not restrict this guiding principle to propositions.

conclusion, and the conclusion is normally unacceptable because of an inconsistency: either it is inconsistent in itself, or it is inconsistent with some state of affairs, or with some other principle or conviction we hold. Thus, a paradox can only work if it is clear that getting entangled in a contradiction is a proof that the reasoning is not sound; that is, the principle of non-contradiction must be understood as a crucial criterion for our inquiries and accounts.

It is with the first philosophers from outside of Elea who react to Parmenides that we find the principle of non-contradiction used as *implicitly* including different respects. So in Empedocles we find: "Thus insofar as [*hêi men*] one has learnt to come into being out of many, and when one disintegrates many are formed, to that extent they are becoming and have no stable life. But insofar as [*hêi de*] these things never cease from their continual change, to that extent they are forever unmoved in a circle".[68] Empedocles here introduces different respects: in one respect, things seem to be unstable and subject to generation, while in another they always seem to be and to be unmoved. The atomists also employ this principle in such a way that it implicitly includes different respects, as we will see in Chapter 4.

In Plato's *Republic* and *Sophist* we will then find formulations of the principle of non-contradiction that *explicitly* include differences of respect.[69] Aristotle's *Metaphysics*, finally, is the *locus classicus* for the principle of non-contradiction familiar to us.[70] There we find formulations of this principle that have been understood as ontological,[71] psychological,[72] and logical.[73] Aristotle also attempts to *prove* that the principle of non-contradiction is a necessary principle: as it is a most basic principle, it cannot be derived from another, more basic principle, so Aristotle attempts to give a negative proof by showing that even those people who deny that the principle of non-contradiction holds, have to employ the principle themselves in order to claim that it does not hold.[74]

If something satisfies the law of non-contradiction, we think of it as being consistent. But we can talk about consistency on different levels. Today,

[68] Fr. DK31 B26, lines 8–12.

[69] See *Republic* 436a–437e and *Sophist* 230b7 ff.

[70] *Metaphysics* Γ, 3–8, 1005b ff.

[71] "It is impossible that the same thing belongs and does not belong to the same object at the same time and in the same respect, and all other specifications that might be made, let them be added to meet local objections" (*Metaphysics* Γ, 1005b19–22).

[72] "It is impossible for anyone to believe that the same thing is and is not, as some consider Heraclitus said" (*Metaphysics* Γ, 1005b23–5). The psychological formulation is the one I am least interested in in this book.

[73] "The opinion that opposite assertions are not simultaneously true is the firmest of all" (*Metaphysics* Γ, 1011b13–14). For the distinction between the ontological, psychological, and logical formulations, see Łukasiewicz 1910.

[74] For they want to claim that the principle does not hold – they do not want to claim that it does and does not hold.

consistency is usually understood as a syntactic feature of a set of sentences such that the set is consistent if we cannot derive both statements p and not-p from it. When we look at the early Greek thinkers, however, a sense of consistency that is not as technical as this understanding and that allows for a semantic understanding seems preferable. At least three such understandings of consistency are possible:

(1) We can require the specific content of a concept (our account of something) to be consistent, that is, for example, its definition and *propria*.[75]
(2) We can require the usage of the concept to be consistent, that is, we have to avoid any uncontrolled changes. This includes avoiding using our concept in a way that is not consistent with its definition[76] or applying it in a wrong way – for example, in a realm in which it is not applicable.
(3) We can require a system of concepts to be consistent, that is, the concepts must fit together in such a way that no implication of one concept is inconsistent with the implication of the others.

This last notion of consistency is rather strong as it entails that the concepts of such a system are not independent of each other: for the system to remain consistent, a change in one concept would normally have to affect the other concepts too. We will see that Parmenides' philosophy seems to attempt to satisfy all three understandings of consistency and that Plato understands natural philosophy to meet only the first two understandings, but not the third (and in some cases there may be problems even with the first).[77] One may want to include a possible fourth understanding, one requiring consistency of expression, that is, that we can consistently express our philosophical view or that our philosophical view is consistent with the language employed. We will see a potential problem with this form of consistency in the next chapter with Parmenides' attempt to express his understanding of *eon*.

1.2.2 The Principle of Excluded Middle

The principle of excluded middle essentially says that of any *x*, we must either say that *x* is F or that *x* is not F; there is no further, third, possibility. Aristotle expresses this principle in his *Metaphysics* Γ, 1011b24: "Of any one subject, one thing must be either asserted or denied". The principle of excluded middle also

75 I am not distinguishing here between unsatisfiable and inconsistent concepts.

76 The point here is not linguistic consistency in the sense that we always use the term 'pullover' rather than 'sweater' when referring to a certain garment, but rather that we always keep the definition of our referent constant (in this case, that it is a garment for the upper part of the body that is drawn over the head).

77 Note that I am not claiming that Plato or Parmenides would explicitly distinguish between these different levels of consistency, but only that we can analyse their accounts with the help of these levels.

marks the distinction between contradictory and contrary opposites: while both are governed by the principle of non-contradiction, only contradictory opposites are also governed by the principle of excluded middle (for example, either *x* is hot or *x* is not hot).[78] By contrast, contrary opposites allow for a middle (*x* need not be either hot or cold, it could also be neither, for *x* may be lukewarm, or *x* may be something for which statements about temperature do not make sense, such as a geometrical proof, which is neither hot nor cold).

The first formulation of what we would call the principle of excluded middle seems to be found in Parmenides' poem, fr. 2, where we are told that there are only two ways of inquiry: either that it is and that it is impossible not to be, or that it is not and that it is necessary not to be.[79] While Parmenides' account of what truly is[80] is governed by the principle of excluded middle, mortals in their *doxa* wrongly include a middle;[81] we will see in the next chapter that we may even conceive the difference between the way of truth and *doxa* in terms of whether the principle of excluded middle holds.

As a consequence of Parmenides' strict adherence to the principle of excluded middle for what truly is, we find that on the way of *alêtheia* the principle of non-contradiction is understood in such a way that the principles of excluded middle and non-contradiction are joint commitments undertaken by anybody who predicates anything, even though mortals do not realise this. There is no sign that Parmenides thought of his formulation of the principle of non-contradiction as conjoined with another principle; rather this version of the principle of non-contradiction seems to be the basic formulation we get with him. While Parmenides also employs the principle of excluded middle to some degree on its own,[82] his understanding of the principle of non-contradiction on the way of truth is such that it is necessarily conjoined with the principle of excluded middle.[83]

Whereas in Parmenides the principles of excluded middle and non-contradiction are in fact not clearly differentiated, with Aristotle we find

[78] This is a standard way of describing the difference between contrary and contradictory opposites, that we also find, for example, in Horn 2018. Since the principle of non-*contradiction* is meant to govern also *contraries*, some people may find it more apt to talk about the principle of non-conflict; cf. above.

[79] There has been some debate about whether Parmenides in a later fragment does introduce a third way (see Chapter 2). But in general the way of truth seems to be governed by the principle of excluded middle. I will say more about the modality involved here in the next chapter.

[80] While Parmenides only talks about 'Being' (*eon*), we may call it 'what truly is' in contrast to what mortals posit; 'what truly is' will also be the label the atomist use for Being.

[81] See frr. 9 and 12.

[82] For example, it seems to be due to the principle of excluded middle that Parmenides derives the conclusion in fr. 8 that coming into being can only happen from Being or non-Being.

[83] We will discuss the reasons for this in Chapter 2.

them as clearly distinguished principles and as basic principles of his philosophy.[84] The thinkers who came between Parmenides and Aristotle seem gradually to have distinguished the two principles, even if for the most part implicitly. Among the atomists, for example, a common form of argument, whose structure we will discuss in more detail below, is to claim that there is no more reason to assume something to be F than not to be F so it is G, a *tertium quid*. Accordingly, the atomists do not assume the principles of non-contradiction and excluded middle to entail each other.

1.2.3 The Principle of Sufficient Reason

In its most basic form, the principle of sufficient reason claims that everything that is or happens does so for a reason that is sufficient to explain *that* something is the way it is or happens and *why* something is the way it is or happens. Evoked in a negative version,[85] denying that there is a sufficient reason for an object *x* or for a state of affairs *s*, the principle forms the basis for what is called 'indifference reasoning' in modern discussions and *ou mallon* reasoning in ancient debates.[86] *Ou mallon* is Greek for 'no more' or 'not rather'; as a principle it claims that there is no more reason to assume *x* than *y*, no more reason to assume *s* than not-*s*. While in its positive form the principle of sufficient reason is often seen as a rationalist principle, *ou mallon* reasoning eventually became an important tool for ancient scepticism, as can be seen most prominently with Sextus Empiricus. However, *ou mallon* reasoning need not lead to scepticism, and we will see that Greek thinkers before Plato evoked the negative form of this principle also in order to argue for positive positions.[87]

Before we analyse further what counts as a sufficient reason, let us first look briefly at the way in which the principle was introduced and how it was dealt with in the history of philosophy in ancient times. Anaximander is often credited as the first thinker in the Western tradition to use something like the principle of

84 See *Metaphysics* Γ, 7. See also Horn 2018, who understands "the law of non-contradiction (LNC) (also known as the law of contradiction, LC) and the law of excluded middle (LEM)" as the "twin foundations" of Aristotle's syllogistic (which Horn calls "Aristotle's logic").

85 I call it 'negative version' here, since it starts with the supposed lack of a sufficient reason, see also below.

86 For a more formal account of indifference reasoning, see Makin 1993, ch. 5. We should bear in mind that in the period I am investigating, many thinkers did not clearly distinguish between reasons and causes.

87 Some people think that Democritus, who uses the *ou mallon* principle most extensively, was himself a sceptic. The view that Democritus was not a sceptic is, however, also supported by Sextus, who distinguishes between his own, sceptical use of this principle and what he sees as Democritus' 'dogmatic' use.

sufficient reason:[88] according to a testimony from Aristotle, Anaximander argues that the earth is in equipoise for there is no more reason (*mallon outhen*) for it to move one way than another. The background premise is, as Aristotle points out, that the earth is seen as situated at the centre of the cosmos and as equally related to the extremes.[89] While people before Anaximander seem to have asked what prevents the earth from falling, Anaximander turns the order of explanation around and points out that the earth would fall only if there were a sufficient reason or cause for it to fall. Because there is no such reason for the earth to fall, given its position in the universe, it stays still. The principle of sufficient reason is employed here in a negative way – there is no sufficient reason for motion, so motion does not occur.

While the testimony just discussed is often seen as the *locus classicus* of the principle of sufficient reason, Anaximander also seems to have pointed out that Thales' suggestion to posit water as the underlying principle that explains all phenomena is not sufficient. For Anaximander's assumption of the *apeiron* as the basic principle appears to react to the problem that water is one specific element or kind of stuff, so it does not seem qualified to have everything come out of it: if we take water itself to be cold and wet, for example, it is unclear how its opposite, what is dry and warm, fire, can be derived from it in any way. There is no sufficient explanation in Thales' account of why all stuff would come out of water. Accordingly, instead of one element being assumed as the basic stuff out of which all the others come into being, for Anaximander we need something that is none of the elements so that all can come from it equally and that by its very nature would explain how all the other elements could originate from it.[90]

Unlike the principle of non-contradiction, the principle of sufficient reason is used rather sparsely by Parmenides in his poem.[91] He does employ it, however, for a central argument that has been very influential in the later tradition: Parmenides argues that there is no sufficient reason for Being to come into being from non-Being sooner or later, so there is no sufficient reason for generation to occur at any particular time; yet it would have to occur at a

88 Most scholars share this assessment; see, for example, Kahn 1994, p. 77 and Melamed and Lin 2016. Furley 1989, pp. 17–19, however, claims that Aristotle reads this back from the *Phaedo* into Anaximander, since for 'likeness' (the earth's position in the middle) to explain that the earth remains at rest, we need to assume a "centrifocal dynamics", which would be "nothing less than the abandonment of the archaic world view". While Furley may be right that a centrifocal dynamics would have to accompany Anaximander's indifference reasoning for the position of the earth, I do not think we have any reason to assume that Anaximander would have conceived of any theory of dynamics in order to hold this view. See also McKirahan 2001, pp. 64–5 and Gregory 2016, p. 165.

89 See Aristotle *De caelo* 295b10–296a23; Hippolytus *Refutation of All Heresies* I, 6, 1–7; Barnes 1982, pp. 23–8; Makin 1993, pp. 101–5.

90 See Aristotle's *Physics* 204b22 ff. and Simplicius *In Phys*. 479,33. We will see an echo of this consideration in Plato's account of the receptacle, see Chapter 6.

91 For an account that, however, sees it used ubiquitously, see Della Rocca 2020, ch. 1.

particular time in order to occur at all. In addition, Parmenides' poem is also thought to point out that there is no sufficient reason for Being to come into being from Being, since this would lead to something *beyond* the *Being* that is there in the first place. Since coming into being from Being (what is) and coming into being from non-Being (what is not), as the only possibilities, are both shown to be inconsistent, there cannot be a sufficient reason for generation.[92]

While here the principle of sufficient reason is used with regard to the occurrence of processes (or the lack thereof), Parmenides also uses it for a state of Being in his move from homogeneity to indivisibility.[93] In the later instance, his argument seems to be that if there are no internal differences, there is no reason for Being to be divided; and since Being is completely homogeneous, it is thus indivisible. The argument for homogeneity is made by pointing out that no sufficient reason prevents Being from being completely homogeneous – neither is there a Being nor a non-Being that would do so.[94]

The latter argument is elaborated by Zeno in his plurality paradoxes, and accordingly scholars have argued that he too uses reasoning based on the principle of sufficient reason.[95] Zeno seems to support Parmenides' inference from homogeneity to indivisibility by spelling out further the indifference argument that is in the background. His claim that if something is divisible then it is divisible everywhere appears to hold only against the background of indifference reasoning along the following lines: since there are no internal differences, there is no more reason for something to be divided here than there; if it is divided here, it is therefore also divided there, and there, and there, *ad infinitum*.[96] Then the argument can be made that since we cannot have a magnitude that is divided at every possible point, Being is not divisible.

While with Parmenides the principle of sufficient reason has strong argumentative power, but is seldom used, with the atomists the principle in its *ou mallon* form is dominant.[97] It is central to the argument for the basic features of atoms – at least when it comes to the infinity of shapes (there is no more reason for an atom to have this shape than that shape), and perhaps also for the variety of sizes.[98] The *ou mallon* principle also leads the atomists to the

92 See fr. 8, line 6b, which asks, "which generation could you find for it?" in order to argue that neither generation from Being nor from non-Being would be consistent.

93 In fr. 8, line 22 ff.; see Chapter 2.

94 See chapters 2 and 7, and Sattler 2019a.

95 See especially Makin 1993, p. 24.

96 See also Makin 1993, p. 156.

97 Makin 1993, p. 49 even claims that the basic atomic theory is generated by repeated application of indifference reasoning. While Makin uses this form of reasoning more often in his reconstruction of Democritus' position than our fragments do, he is certainly right that Democritus seems to have used indifference reasoning systematically.

98 The doxographers do not agree on whether there are in principle atoms of all kinds of sizes (it just so happens that there are none of the size we could perceive in our world), or whether the size of an atom is necessarily beyond what is perceptible for us.

assumption of infinitely many worlds. It is generally used to infer that neither F nor not-F is the case, or to posit a *tertium quid*.

While *ou mallon* considerations are dominant in the atomistic account of atoms and worlds, on the level of the mid-sized phenomena we ordinarily perceive, we have sufficient reasons for changes and generation of phenomena (in terms of arrangements of atoms that explain these phenomena and their changes). Thus the atomists seem to employ the principle of sufficient reason in a positive way too.

Investigation of what counts as a sufficient reason is the next substantial step in the history of philosophy, a step taken by Plato. In his *Phaedo*, Plato attempts to give criteria for sufficient reasons.[99] Furthermore, the *Phaedo* and then more fully the *Timaeus* show that what had been understood as sufficient reasons should be divided into necessary reasons or causes and rational reasons or causes (that which has been crafted by *nous* or is due to something intelligible). Plato understands necessary reasons to be those that stem from the stuff from which the cosmos is formed.[100] In accounting for the phenomenal realm, the necessary reasons include the receptacle, the atomistic triangles, and the bodies formed out of them, while the rational reasons are the Forms and the demiurge. The necessary reasons are responsible for things changing, moving, and having a location, and set constraints on the teleological reasoning of the demiurge. By contrast, the rational reasons account for something's being what it is, for it being as good and orderly as possible, and for guaranteeing that locomotion in the created cosmos is rational.

With Aristotle, finally, it is not clear that we still find universal acceptance of the principle of sufficient reason – Aristotle may be understood as allowing, for example, for spontaneous generation, and for him coincidences probably do not have sufficient causes.[101] We do, however, get an elaboration of what can count as an explanation or cause, with the famous division of reasons and causes into the four introduced in book II of Aristotle's *Physics*: the formal, the material, the moving, and the final cause. Reasoning that draws on the principle of sufficient reason is also involved in Aristotle's famous demonstration that there must be eternal unmoved movers to guarantee the eternal motion of the world.[102]

[99] See Sattler 2018a.

[100] While Plato sometimes uses the notion of necessity for logical necessity, the notion of necessity in play here refers to the limitations imposed by the basic 'material' from which the cosmos is formed, as Cornford, Johansen, and others have pointed out, cf. Sattler 2012, p. 166. Plato's distinction between necessary and rational causes is one of the forerunners to our distinction between necessary and sufficient causes.

[101] See Sorabji 1980, ch. 1. Perhaps also accidents exist without sufficient reason for their existence.

[102] See *Physics* VIII, 5–6. In general, *Physics* VIII draws heavily on the principle of sufficient reason, as we can see, for example, in ch. 1.

After this brief historical summary, let us now look at claims that there is (or is not) a sufficient reason for something from a more systematic viewpoint. We will look at (1) what counts as a sufficient reason, (2) what it means that a reason is *for* or *against* something, (3) what is in need of a sufficient reason, and (4) what these statements of the principle of sufficient reason amount to.

1.2.3.1 What Counts as a Sufficient Reason

This section will consider first what counts as a reason and then what counts as a sufficient one. When Parmenides explicitly employs the principle of sufficient reason in fr. 8, lines 9–10 – "What need [*chreos*] would have made it [Being] grow, coming from non-Being, later or sooner?" – it seems clear that he is asking for a rational explanation as a reason (if there is a rational explanation). But asking for something that would have made Being grow also seems to be asking for a cause or, more precisely, for an efficient cause. So a reason may be an explanation as well as a cause.[103] In the following we will work with such a broad notion of reason that can include explanations as well as causes.

Understanding reasons as explanations as well as causes also fits the atomists, who on the one hand refer to the atoms and their motions as reasons in the sense of causes for phenomena to occur and change while on the other hand dealing with explanations when they employ the *ou mallon* principle in order to argue that we have no more reason to assume F than not-F.[104] Some scholars have seen a combination of both, of cause and explanation, in Plato's account of the Forms.[105] However we understand Plato's Forms in general, his distinction between necessary and rational *aitia* in the *Phaedo* and the *Timaeus*[106] seems to be the first moment in history to provide an explicit discussion of what can count as a reason, and a first step towards distinguishing between causes and explanations.

But when is a reason – be it a cause or an explanation – sufficient? The answer seems to depend on the context, and the reason will have to meet psychological, metaphysical, or epistemological criteria for what is required for an adequate account. We should be cautious, however, when employing necessity. Not every sufficient reason must be a necessary reason,[107] and giving

103 There are different ways to understand the difference between cause and explanation. See, for example, Falcon 2015 and Hankinson 2001, p. 4. The way I distinguish them here is that an explanation can be something merely conceptual (or linguistic), while causes necessarily possesses reality beyond the mere conceptual (or linguistic).

104 Within the atomistic framework, if there were more reasons for F to occur than for not-F, that would be a sufficient reason for F.

105 While Forms clearly seem to be explanations, there is debate over whether Forms are also causes; see, for example, Sedley 1998 for an argument in favour of understanding Forms also as causes, and Vlastos 1969 for an argument against this assumption.

106 For example, *Phaedo* 98d ff. and *Timaeus* 46e.

107 By 'necessary reason' I do not understand here a reason that necessitates – reasons that are not necessary can also necessitate – but a reason that has to obtain or else a certain

a reason for a fact does not make that fact necessary. Furthermore, a necessary reason may not yet be a sufficient reason.[108] Additionally, what is explained could in itself be either necessary or contingent.

1.2.3.2 What It Means that a Reason is *For* or *Against* Something

While the principle of sufficient reason in its standard form is formulated in a positive way – claiming that there is a positive reason for something – we have already seen that it can also be evoked in a negative way – there is no negative reason against something. The positive standard formulation of the principle of sufficient reason is something along the following lines:[109]

(1) if *x* exists or it is reasonable to assume that *x* exists, then there is a sufficient reason for *x*.[110]

While this is the standard account of the principle of sufficient reason, in our investigation we will also find the principle invoked in two other forms, in a negative form (which also relies on the principle of non-contradiction):

(2) if *x* exists or it is reasonable to assume that *x* exists, then there is no sufficient reason against the existence of *x*,

and in the form of an *ou mallon* reasoning:

(3) there is no more reason for *x* than for *y*, so either (a) neither *x* nor *y* exists, or it is not reasonable to assume that *x* or *y* exists, or (b) both *x* and *y* exist, or it is reasonable to assume that both *x* and *y* exist, or (c) neither *x* nor *y* but a tertium quid, *z*, exist, or it is reasonable to assume that neither *x* nor *y* but a tertium quid, *z*, exists.

What I call the negative form (2) is not the same as the *ou mallon* principle, since the latter is always a comparison (there is no more reason for *x* than *y*, or for p than not-p) and can have a positive or negative consequent. By contrast, the negative form does not compare two things or claims but concentrates

effect can never be brought about. Cf. Makin 1993 in general for a discussion of the notion of reason and modality in play with indifference arguments.

108 Rain can be seen as a sufficient reason for the streets to be wet, but it is not a necessary reason, since a different reason could also lead to the same effect, for example, a street-cleaning machine may have caused the streets to be wet. And there could be a necessary cause without it being a sufficient cause, like a female mammal for conceiving offspring – if no male mammal of the same kind is available, no offspring will be produced.

109 I am talking about *x* and *y* in the following, rather than p and q, since for the ancient context it is not only propositions and state of affairs corresponding to propositions that may or may not have a sufficient reason, but also individual things.

110 The statement '*x* exists' is meant to be a metaphysical claim, while 'it is reasonable to assume that *x* exists' is meant as an epistemic claim. Additional qualifications may be needed for all forms, e.g., by assuming 'becoming' rather than 'existence', or kinds rather than particulars.

solely on the occurrence or being of one, and simply states that there is no sufficient reason against *x* being or occurring. While (1) and (2) have a positive antecedent (they start with the existence of a certain thing or state of affairs), (3) starts from a negative antecedent (and then can have either a positive or negative consequent).

Which formulation of the principle of sufficient reason will be employed depends on what is seen as natural and not in need of explanation – whether there is a background assumption that *x* will occur unless there is a reason for it not to occur, or whether it is assumed that *x* occurs only if there is a reason for it to occur.

1.2.3.3 What Is in Need of a Sufficient Reason

A sufficient reason needs to be given for that which is in need of a reason. But is it always clear what is in need of an explanation or cause? We saw with Anaximander that he changes the subject that requires an explanation: for Anaximander, what is in need of a reason is not the current status of the earth, that is, its motionlessness, but rather a change or motion, should one occur. Usually, what is in need of explanation will simply be what is not seen to be the default assumption: if the default assumption is F, then not-F requires explanation.

In general, the principle of sufficient reason is employed to account for all kinds of motions, changes, and processes (unless they are the default assumption), and the notion of a reason as a cause seems to be prominent for understanding them. In addition, the principle of sufficient reason is also employed to account for the state of affairs or the being or state of something, and here the notion of a reason as explanation seems more prominent.

1.2.3.4 What These Principle of Sufficient Reason Statements Amount to

The principle of sufficient reason could be called a metaphysical principle as well as an epistemic principle.[111] The conclusions drawn with the help of the principle of sufficient reason by the philosophers we will look at are sometimes epistemological, sometimes metaphysical. Some arguments amount to claiming that it is *reasonable to believe* that *x* occurs or exists, others to claiming that there is *reason* for *x* or a change of *x to exist or take place*. Today many scholars may be more comfortable with epistemological claims rather than metaphysical claims resulting from these kinds of argument. Many of the arguments we find in the Greek texts do, however, make metaphysical claims. Throughout

[111] If we understand the principle of sufficient reason in its basic form as 'if *x* exists, then there is a sufficient reason for *x*', we see that the antecedent makes a metaphysical claim. The consequent makes what can be seen as an epistemic claim to the effect that *x* is intelligible and can be understood.

the history we will be looking at, we will find metaphysical and epistemological uses of the principle of sufficient reason.

1.2.4 Rational Admissibility

By 'rational admissibility' I understand that an account of some *x* can be given which is based on what we would call rational analysis and can thus withstand rational scrutiny. As a consequence, we can conceptualise *x* in such a way that *x* is in principle understandable for every other rational being and comparable to other objects of investigation. It makes us independent of any authority, chance, or individual experience with respect to this kind of knowledge.

So we can say that an account of *x* is rationally admissible if and only if (1) the standard for judgement is reason, (2) there is no authority for the judgement beyond reason, and (3) the account can be generalised.

If the principle of sufficient reason claims that everything that is or takes place has a reason, the principle of rational admissibility claims that this reason can withstand rational scrutiny. Rational admissibility only requires that our reasoning is the standard for judgements; it does not require that we can always give a reason or an explanation why a state of affairs obtains or why a statement is true.

We might call rational admissibility a rationalist criterion. However, as we will see below, rational admissibility is also a criterion for the atomists and Aristotle, whom few would want to call rationalists.[112] The atomists and Aristotle complement rational admissibility with what might be seen as an 'empiricist criterion', namely 'saving the phenomena'. Yet this latter criterion is also important for Plato, whom, in turn, not many people would want to call an empiricist. Accordingly, I suggest we put aside ideas about rationalism and empiricism and concentrate on the criteria.

Rational admissibility is first introduced, as I argue in the next chapter, by Parmenides, in his fr. 7: having given a first evaluation of 'the well-rounded truth' and the 'opinions of the mortals', the goddess asks the mortal listener to test with (his) *logos* what she has said. We are expected to accept Parmenides' philosophy as true and trustworthy not because this account is presented by a goddess, but because we can test it with our own reasoning and get to the same result as the goddess.[113] Our own reason not only verifies what the goddess has

[112] Cf. also Della Rocca 2014 on showing how understanding the principle of sufficient reason as the heart of rationalism undermines the traditional rationalist/empiricist contrast.

[113] Heraclitus' claims that "everything comes about according to *logos*" (*kata ton logon*, fr. 1) and that we should listen not to Heraclitus but to the *logos* (fr. 50) may seem like a predecessor of this criterion in Parmenides. However, neither is it made clear what it means that something comes about *kata ton logon*, nor is this claim related to us proving what we are told with our own human rational capacity, as we find it in Parmenides. And while Parmenides claims that our reasoning can in principle get to the same results as divine reasoning (see fr. 7 and the next chapter), Heraclitus in fr. 78 explicitly claims that

said, but is meant to trump any other authority, like the authority of a goddess. And the account we thus gain can be generalised – a thorough examination of a claim by you and by me (and, indeed, by a Greek goddess) should lead to the same result.

As we will see in the third chapter, this criterion is strengthened by Zeno's paradoxes, which go one step further: not only do their results not fit our everyday experiences, but they show that our everyday experiences are no adequate criteria for judging an ultimate explanation of what there is. For Zeno, as for Parmenides before him, we cannot rely on our senses to judge what is an adequate object of knowledge and how we can understand it; we can rely only on our reason. Rational admissibility understood in this Eleatic way excludes sense perception.[114] By contrast, for many of the Eleatics' successors, such as the atomists and Aristotle (and to some degree also for the later Plato), rational admissibility and sense perception can work together or complement each other.

The atomists implicitly take over the criterion of rational admissibility, at least to a certain point, when Democritus distinguishes what we know through the senses (as bastard knowledge) from what we know through thought. For Democritus, only the latter is genuine knowledge, since only this second form of knowledge unerringly recognises the truth.[115] What the senses give us is not necessarily false, but it is also not ultimately true, since only atoms and void truly exist, which cannot be grasped by the senses. Genuine knowledge requires reason. At the same time, the atomists claim that our account of what truly is also has to fit our experiences, that is, it has to explain why we experience the world the way we do. But this does not mean that the criterion of rational admissibility is no longer in force. On the contrary, the atomists' theory, by assuming atoms and void, clearly responds to the criterion of rational admissibility. This is obvious from the fact that for us to object to the atomists that we do not have any sensory experience of atoms would only show that we have not understood the status of their assumption. The atomists also develop the criterion of rational admissibility in such a way that it fits with another criterion they introduce, the 'saving the phenomena' criterion I

only divine, not human, nature possesses *gnômê*. Accordingly, I do not think that we find rational admissibility as a criterion in Heraclitus.

114 As can also be inferred from Parmenides' treatment of sense perception. Rational admissibility, together with the principle of non-contradiction, helps us to distinguish true knowledge from mere opinion at least in two respects: If I do not hold myself to consistency, I myself may contradict what I say (only in being consistent can I be sure that I am going on with the same argument and I am not undermining what I have said so far, cf. also *Sophist* 241e). And if what I say cannot be checked by reason and is hence not rationally admissible, an opposite claim by somebody else may contradict me without this dispute being resolvable. In both cases I am not dealing with true knowledge.

115 See, for example, DK68 B11, Taylor D22.

discuss below. For the atomists, an account of what there is, of the true *onta*, must not only be rationally admissible – once the Eleatic logic was developed further, what is rationally admissible would in fact have allowed for too many possibilities – but also give an explanation of our sense experiences of the world.

It is in this form, as further developed by the atomists, that the criterion of rational admissibility is also taken up by Plato and Aristotle. For Plato in his *Timaeus*, as for the atomists, what can be experienced by our senses can be explained in terms of something more fundamental that meets this criterion (in terms of atoms and void for the atomists; in terms of the intelligible model for Plato). In addition, according to Plato's account, the phenomena themselves acquire what we might term their own intelligibility, in so far as they are mathematically structured and thus rationally admissible accounts of them are possible – we will see in Chapter 6 that, according to the *Timaeus*, the world is set up with the help of arithmetical and geometrical structures. For the notion of motion, it will be crucial that motion can be shown to be measurable (i.e., that it can be connected to the number series in a controlled way). The notion of a measure thus becomes central in the *Timaeus* – it extends the criterion of rational admissibility to the sensible world.[116]

That Plato adheres to rational admissibility as a criterion for establishing natural philosophy can be seen in the *Timaeus* in his encouragement to others to improve his account of the physical world:[117] they are not asked to provide more sensory data, but rather to improve the account in rational terms, for example, by giving a more thorough account of the basic triangles, which are not perceptible.

Finally, Aristotle will be shown to pursue the Platonic path further: he employs mathematical notions to help make the natural realm intelligible and thus subject to statements that are rationally admissible. And he takes up the thought that a measure allows motions in the sensible realm to be quantified – such a quantification in turn makes possible statements about sensible motions that every rational being can test by using her own reason.[118]

[116] I am not claiming that Plato is the first to see the importance of the notion of a measure; but he seems to give it a centrality for natural philosophy that we do not find in the thinkers before him.

[117] See, for example, *Timaeus* 54a–b.

[118] Algra 1999, reacting to Popper, argues that Presocratic hypotheses were not testable or falsifiable. This is certainly right, if we understand testable in the modern sense that we have a model which we test against our empirical evidence. However, our account of what there is rationally testable from Parmenides onwards, in the sense that we can prove with our own reason whether our account can possibly hold.

1.2.5 Saving the Phenomena

While the criteria discussed so far are general criteria for philosophy, the saving-the-phenomena criterion is specific to natural philosophy. Since it seems that it could come in degrees (saving more or less), it may not seem to be a criterion in the strict sense. However, for the non-Eleatic successors of Parmenides it is nevertheless a standard that their theories aim to meet and thus can also be called a criterion of sorts.

The formulation of this criterion as we know it stems from Simplicius' commentary on Aristotle's *De caelo*. According to Simplicius, the saving-the-phenomena criterion is introduced as a task set by Plato to the mathematicians:

> καὶ εἴρηται καὶ πρότερον, ὅτι ὁ Πλάτων ταῖς οὐρανίαις κινήσεσι τὸ ἐγκύκλιον καὶ ὁμαλὲς καὶ τεταγμένον ἀνενδοιάστως ἀποδιδοὺς πρόβλημα τοῖς μαθηματικοῖς προὔτεινε, τίνων ὑποτεθέντων δι᾽ ὁμαλῶν καὶ ἐγκυκλίων καὶ τεταγμένων κινήσεων δυνήσεται διασωθῆναι τὰ περὶ τοὺς πλανωμένους φαινόμενα, καὶ ὅτι πρῶτος Εὔδοξος ὁ Κνίδιος ἐπέβαλε ταῖς διὰ τῶν ἀνελιττουσῶν καλουμένων σφαιρῶν ὑποθέσεσι.
>
> And, as I have said previously,[119] that Plato without hesitation assigned to the heavenly motions circularity, uniformity, and order and put forward to the mathematicians this problem: by making what hypothesis about uniform, circular, and ordered motions will it be possible to save the phenomena presented by the planets? And I have said that Eudoxus of Cnidos first proposed the hypotheses using the so-called counteractive spheres.
> (Simplicius *In de Caelo* 492.31–493.5; translation by Ian Mueller with alterations)

According to this, 'saving the phenomena' is a task set for the mathematicians in the context of astronomy, which was seen as a part of mathematics. The mathematicians are meant to save the phenomena by coming up with a set of circular, uniform, and regular motions whose combination will amount to the revolutions of the different heavenly bodies. But what exactly these phenomena are that are to be saved and what exactly 'saving' means have been matters of dispute; debate has also flourished over whether the criterion was understood as astronomical alone or whether it was extended further.

According to G. E. R. Lloyd,[120] 'saving the phenomena' was not one clear programme, and therefore we find different answers to these three questions.

[119] *In de Caelo* 488, lines 18–24: "And, as Eudemus recorded in his second book of his astronomical history (and Sosigenes took this over from Eudemus), Eudoxus of Cnidos is said to be the first of the Hellenes to have made use of such hypotheses, Plato (as Sosigenes says) having created this problem for those who had concerned themselves with these things: on what hypotheses of uniform and ordered motions could the phenomena concerning the motions of the planets be preserved" (translation by Ian Mueller).

[120] Lloyd 1978.

Let me briefly sketch the main possibilities that were offered in response to each of these questions (even though each answer is not independent of the other two).

As for the range of this criterion, the immediate challenge issued in the passage cited above is to save astronomical phenomena.[121] There were, however, also tendencies to include more or all of the perceptible phenomena,[122] which meant that the criterion was seen as not just an astronomical criterion but also a general criterion for natural philosophy. We may therefore ask whether the phenomena to be saved are only the astronomical phenomena or a broader set of perceptible phenomena.[123] Furthermore, are the astronomical phenomena to be saved the seemingly irregular motions we perceive in the sky or the imperceptible "uniform, circular, and ordered motions", a combination of which can be understood as leading to the irregular motions we perceive?[124] We might rework that second question to ask whether the uniform, circular, and ordered motions are what truly exists and should be inferred from their seemingly erratic effects, or whether the uniform motions are only hypotheses introduced to give an intelligible explanation of the perceptible motions.[125]

The differences in how the phenomena are understood will affect what is meant by 'saving' them:[126] if by 'phenomena' we understand the perceptible heavenly motions, then 'saving the phenomena' can mean one of two things: either we show that these seemingly erratic motions are intelligible in so far as they can be explained by means of a hypothesis of regular motions, or we show

121 This is, for example, how Duhem 1908 understands Simplicius.

122 Mittelstrass 1962, p. 147 talks about two tendencies, one including "alles zu dem die *aisthesis* Zugang hat", and another including only "einen ausgezeichneten Teil der *aistheta*". On p. 1 of the same work he assumes, however, that the phenomena in question were originally heavenly phenomena, and that it was only with the "Programm der neuzeitlichen Naturwissenschaft", starting with Galileo, that everything perceptible was included. With this claim Mittelstrass wants to oppose the neo-Kantians, who saw a similarly broad understanding of phenomena already at work in Plato, while for Mittelstrass Plato was not even the author of the "saving the phenomena" programme (no matter how narrowly understood). Instead, Mittelstrass assumes Eudoxus to be the starting point, while according to Simplicius, Eudoxus is the first to meet this challenge.

123 In both cases they will, however, not include everything there is for perception. Smith 1981 points out the preselection of the phenomena: phenomena are usually only what is deemed to be salvageable, e.g., the movements of the planets, but not of comets or shooting stars.

124 This question became more prominent due to the discussion of it in Simplicius, and the twentieth-century debate in Duhem, Lloyd, and others. I will leave to one side the debate about instrumentalists and realists that Duhem and Lloyd connect to this discussion.

125 Simplicius *In de Caelo* 488.25 claims that the assumption of several motions for the orbit of each planet is only a hypothesis, and not the truth; but, according to Lloyd 1991, p. 251, mathematicians took mathematics to be true of the world; cf. also Duhem 1908, p. 19.

126 See also Lloyd 1991, p. 252 and Mittelstrass 1962, pp. 1 and 141.

that the erratic motions are only seemingly erratic and can in themselves be understood as regular.[127] But if by 'phenomena' we understand the imperceptible, uniform, circular, and ordered motions that present themselves (in erratic clothing), then 'saving the phenomena' means showing that these motions are the true reality underlying the erratic motions we perceive. In all three cases, the phenomena are saved because we can connect the seemingly erratic motions we perceive with something intelligible that can be grasped mathematically. Thus 'saving' means that they are explained by something that can be traced back to mathematical structures; in this way they are saved from the accusation of displaying anomalies, and that means from attacks against their intelligibility. Hence the saving-the-phenomena criterion also supports a claim we will discuss in the section on mathematics: that what the mathematicians can describe or reconstruct mathematically is 'saved' in the sense that it is a possible object of scientific inquiry. These attempts to save the phenomena need not be tied to any form of calculation, however.

We see that this criterion found a variety of interpretations. I will treat it in a way that allows us to see some continuity between the Presocratic and the classical thinkers. Accordingly, I will treat it as a broader criterion that is not restricted to astronomy and is concerned with giving a coherent explanation of the perceptible realm as it appears to us. It requires an account of the basic constituents of the world in such a way that it is thus also explained why and how the phenomena we perceive occur, phenomena such as the planetary revolutions and experiences of qualities, motions, and changes.

Anyone made uneasy by possible historical inaccuracy because such a broadening only started after Plato[128] (although I will show its earlier importance for the atomists), should read it as a different criterion, as something like 'explaining what we perceive in the sensible world'.

Let us take a brief look at the history of a saving-the-phenomena criterion understood thus. It is not a criterion for the philosophy in which Parmenides is interested, for his investigation of Being;[129] Parmenides and Zeno do not seem

127 Regular motions are not necessarily simple motions. Rather, all motions which are or are composed of one or several uniform and circular motions are seen to be regular, since we can find a rule to describe them. Accordingly, motions including epicycles (which are thus composed of more than one motion) and eccentric motions (where we are not at the centre of their uniform circular motion) can both in principle be regular. By contrast, if we cannot find such a rule to describe a motion, it is seen as erratic.

128 Some scholars have also doubted whether the evidence concerning Plato is indeed to be trusted, since, as the Simplicius passage in n. 119 above reveals, Simplicius seems to have gathered this information only from Sosigenes, not from Eudemus; cf. Knorr 1990; Zhmud 1998; Bodnár 2012, p. 270.

129 I cannot discuss the question of whether a form of this criterion can be found already at the beginning of philosophy, with the Milesians, and then also with the Pythagoreans, who frequently come up in the discussions of the literature on the saving-the-phenomena criterion.

to think that the perceptible phenomena can (or should) be saved by philosophy or natural science.[130] It becomes very prominent, however, with the non-Eleatic successors of Parmenides, who tried to reintroduce natural philosophy into the framework of the criteria for philosophy established by Parmenides. Most importantly, it seems to be a guiding principle for the atomists.[131]

The atomists try to explain the appearances as they present themselves to us in terms of their ontological foundation, atoms and void, and in this way attempt to 'save' them. They save the phenomena from Parmenides' attack against natural philosophy by giving an explanation of them that does not consider what we perceive in the natural world around us as anomalies but rather as something that can be traced back to the behaviour of atoms in the void. The atomists explain not only heavenly phenomena[132] but also perceptible qualities and the sensible things that possess these perceptible qualities. They connect the saving-the-phenomena criterion with the other criteria of philosophy which they take over from Parmenides, and in so doing they partially modify understanding of the other criteria. For example, if the basic ontological constituents not only have to be consistent, adhering to the principle of sufficient reason, and rationally admissible, but also have to explain the phenomena in terms of what truly is,[133] then, contrary to Parmenides' claim, reason does not specify the ontological foundation sufficiently; for some questions, we simply need to consider the phenomena.[134]

The next big step in the development of this criterion is found in Plato. For our inquiry it does not matter whether Plato was indeed the author of the programme described in Simplicius,[135] or not.[136] What is important is that Plato, not in the *Republic*[137] but in the *Timaeus*, lays out a programme for

130 Parmenides gives some account of natural phenomena on the way of *doxa*; but, as we will see in the next chapter, saving perceptible phenomena is not an aim that would affect his metaphysics in any way such that we could see it as a criterion he employs for his philosophy.

131 See my paper "What Is Doing the Explaining? An Atomistic Idea".

132 For their explanation of heavenly phenomena see, for example, fr. DK67 A1 (Taylor T77a); DK68 A78, 86–92, 96–9 (Taylor T84, 86–8, 90–4, 97, 101–2); and DK59 A78 and 80–1 (Taylor T85, 91–2).

133 See also Sextus VII, 140 (DK68 A111, Taylor T179a) according to which Democritus claimed the appearances as criterion (*kritêrion*) for the apprehension of "things hidden" (i.e., the theoretical basis).

134 Thus, it seems to have been the infinity of phenomena that led the atomists to assume an infinity of different shapes for the atoms (but cf. also Chapter 4).

135 As not only Simplicius has it, but also Proclus, and as Duhem 1908 stresses.

136 As, for example, Knorr and Mittelstrass claim (though the latter does not take the *Timaeus* sufficiently into account).

137 Lloyd 1991 p. 252 distinguishes two forms of saving appearances – one in which appearances are finally rejected as mere appearances in order to arrive at the "true underlying intelligible realities" – this he understands as the Platonic programme – and one in which the original data, what was first perceived as an anomaly, can be seen as

saving at least the heavenly phenomena, in the sense that for him an account of the cosmos has to explain the phenomena we can perceive. In so doing, he goes beyond the atomists. For Plato, saving the phenomena does not mean only that what we experience with our senses can be explained in terms of something more fundamental that is itself rationally admissible, adheres to the principle of sufficient reason, and is consistent – as was the case for the atomists. Rather, as we shall see, for Plato the phenomena themselves are mathematically structured and thus can be saved.

In Aristotle, finally, we find two further developments. First, his whole method of doing natural philosophy emphasises that we must first observe the phenomena.[138] Secondly, he introduces an additional consideration that helps us deal with the potential problem that each of several hypotheses may equally well save the phenomena: the specific nature or essence of the thing under consideration.[139] For the heavenly bodies, for example, this nature is the fifth or celestial element, which is eternal and unchanging, and our attempt to save the heavenly phenomena must also take this nature into account.

1.3 The Role of Logic

Usually, scholars do not talk about logic before Aristotle, or possibly Plato,[140] but only because in Aristotle we get a *system* of logical structures[141] and

conforming to laws and need not be revised. While Lloyd's account of Plato may be a fair description of Plato's treatment of the heavenly phenomena in book VII of the *Republic*, we will see, *pace* Lloyd, that in the *Timaeus* Plato's attempt to save appearances does not result in rejecting them as mere appearances. Johansen 2004, p. 175 claims that Plato is "a rationalist at heart with respect to astronomy" and thus, for Plato, perception is ultimately not important for doing (mathematical) astronomy. However, while this may be true for the *Republic*, where the perceptible phenomena seem to be put aside, it does not hold for the later Plato, who gives an account of the perceptible astronomical phenomena in the *Timaeus* and the *Laws*, as we will see in Chapter 6.

138 For astronomy, see, for example, *De partibus animalium* I.1. Cf. also Bodnár 2012, p. 280, who claims that "Aristotle propounded an account throughout his different works which took account of a theoretical procedure which, even in Aristotle's own terminology, could be legitimately called *sôzein ta phainomena*." (However, Aristotle sometimes uses a much broader notion of 'phenomena', not only covering what is empirically observable, but also *endoxa*; cf. Irwin 1990).

139 See Duhem 1908, pp. 6–10.

140 See, for example, Horsten and Pettigrew 2014, p. 2. Kneale and Kneale 1962 discuss some pre-classical beginnings, however, with geometrical demonstrations, dialectical and metaphysical arguments (like Zeno's *reductio ad absurdum*), and eristic and sophistry. Smith 2002, p. 11 claims that Greek philosophical logic originates with Parmenides – though without showing which features of Parmenides' account are meant to justify this claim.

141 Cf. also Kneale and Kneale 1962, p. 12.

inferences for the study of reasoning for the first time.[142] However, before the classical period, people had also worked in a non-erratic way with what we can call basic logical notions. As we will see in this book, such logical notions play a crucial role for the possibility of establishing natural philosophy all the way from Parmenides to Aristotle regardless of whether they are explicitly reflected upon.[143] And while some readers may fear that looking at the role of logical notions in the period investigated may result mainly in diagnosing faults and confusions, my main aim here is rather to show how the conceptual possibilities were enlarged in this period and that we are missing crucial points in understanding natural philosophy at that time if we do not also look at the developments of the logical understanding.

The ancient logical notions we will look at do not necessarily map directly onto modern notions. As a result, understanding of what can be counted as a logical notion within an ancient context may be wider or narrower than what we count as a logical notion in a modern context. The field of ancient logical notions may seem narrower than that of modern notions, for we possess more logical tools and can thus in some cases make finer distinctions than the ancients (distinctions that can sometimes also help us understand what is going on in an ancient context). The field of ancient logic may also seem wider, since, as certain distinctions were not yet made, notions we now think of as not logical but belonging to some other field are mingled together with logical notions.

Additionally, not only is it possible that the field of ancient logic appears narrower or wider, but particular logical notions may not have been understood exactly as we understand them. At several points in this study we will come across logical functions that seem similar to functions we use in modern logic but will have to be understood in a somewhat different way if we are to grasp the ancient thinking. We should not simply assume that the ancient thinkers investigated here started out understanding these notions and operators as we do. The understanding with which we work is the product of a long philosophical development, as becomes clear from the fact that we see Plato and Aristotle explicitly labouring to determine how we should understand these notions.

For our purposes the areas in which ancient logic is narrower than our modern logic are less important than those in which it is wider. And so the notion of logic at work in this book will be very broad. It includes principles, structures, and terms that denote the kinds of thing that would feature in a

[142] Kneale and Kneale understand logic as "reflection upon principles of validity" (1962, p. 1) – not all the thinkers we will be looking at necessarily reflect explicitly upon logical principles of validity, though we will see that some do so.

[143] It is a different debate, and one I cannot enter here, whether the ancient thinkers we will look at should be understood as logicians (a debate that suffers from the problem that different scholars understand different things by 'being a logician').

modern logic (for example, a negation operator), but it also includes features that denote what today we consider to be metaphysical rather than logical notions (for example, different understandings of 'being'). What is important is that these principles, structures, and terms are so general that they apply no matter the topic or subject matter, yet they implicitly shape our understanding of the world by determining basic features required of all possible subjects. Accordingly, the logical changes during the period I am considering had consequences not only for natural philosophy but also for other realms of philosophy. In this discussion, however, I will only be able to look at the effects for natural philosophy.

1.3.1 Operators and Operands

When I talk about logic in a broad sense here, some readers may expect to hear something about syllogisms, inference rules, proofs, true and false statements, classifications of types of sentence, investigation of syntax and semantics of sentences, types of arguments, and so forth. To make the investigation of the logical changes manageable for the current project, I will concentrate on something that may seem quite small and unimportant, but is in fact a crucial aspect of the development of the logical tradition, namely the development of what we would call logical 'operators' and 'operands'. We may also think that the notion of operators and operands is a modern notion. But I think the notion of operators and operands that I will be using in the following can capture something that is also essential for the ancient thinkers – a way to think about the separation and connection of thoughts, concepts, and notions.

We will see that in order to understand the ancient thinkers, it will be helpful to have a broader notion of what can count as an operator than many may feel comfortable with today.[144] For the purpose of this book, we will understand an operator as a logical tool that is applied to some operand, that is, to some argument[145] or object, to yield some output that is related to the original input in a systematic way.[146] An operand is, accordingly, that which can function as a possible argument or input (or as an output for that matter). For the sake of simplicity, I will assume that the operators we look at are themselves not used also as operands on which operates another, higher-level operator, even though this is in principle possible. Logical operators can be expressed linguistically, but I will concentrate on logical operators, not linguistic ones.

[144] It is certainly broader than the understanding of operators in Gentzen 1969.

[145] I am using the term 'argument' in the section here to mean roughly 'what is fed into a function'.

[146] This distinction could also be captured as a distinction between copula and substance, but describing it in terms of a distinction between operator and operand is a more general way of capturing the problem, as not all operators need to be copulas.

The field of operators includes connectives, quantifiers, modal operators, and so forth, but I will concentrate on connectives.[147] We will see below that our understanding of connectives will have to be slightly different from the mainstream modern understanding that captures general operators of separation and connection.

In addition, I want to leave open the possibility that some ancient thinkers also treat predicates in a way that suggests they understood them as what we would call operators. We may, for example understand Aristotle's distinction between (first) substances and members of other categories as a distinction between operators and operands: (first) substances function as operands of which we say something but which are not said of something else, while the members of other categories, like quantities or qualities, function as operators. The input in this case would be a substance, the output a sentence.[148]

For the earliest thinkers we will look at, the operators of separation and connection on which we will focus are closely tied to assertion and denial: if something is asserted of something, we have a basic connection, and if something is denied of something, the two are separated. Within Parmenides' framework we cannot distinguish clearly between separation and denial on the one hand, and connection and assertion on the other. Only if we distinguish the content of a claim from the claim of its truth can we assert that it is the case that *x* is F (connection) or that it is the case that *x* is not F (separation), asserting a connection and a separation, respectively.[149] Without such an understanding, '*x* is F' is a connection and at the same time an assertion, and '*x* is not F' is a denial as well as a separation. The differences between assertion and connection, on the one hand, and denial and separation, on the other, are gradually developed in the thinkers who succeed Parmenides.[150]

[147] Although questions of quantification and modality will factor into this investigation, they cannot be given the attention here they would deserve.

[148] See *Categories* ch. 2 and *Metaphysics* Γ, 2; cf. also Palmer 2009, pp. 129–33. Alternatively, a predicate could on some accounts be seen as referring to a universal that is understood as an entity. What works as a tool operating on a subject nevertheless in these cases is not the predicate as such, but rather the copulative aspect of a predicate. We either have an explicit copula or a negative copula (in sentences such as '*x* is F' or '*x* is not F'), or we have a predicate (in 'Fx') that can be transformed in such a way that we get a copula (we can transform 'Fx' into '*x* is F'). So again the input here is '*x*' and 'F', while the output is a statement, '*x* is F' or '*x* is not F'.

[149] For what truly is, Parmenides does not distinguish between truth and intelligibility, i.e., between something being true and its being intelligible, and between something being false and its being incomprehensible.

[150] For a clear distinction see Aristotle *Metaphysics* Δ, 7, 1017a30 ff: "'Being' and 'is' mean that a statement is true, 'non-being' that it is not true but false – and this alike in affirmation and negation; e.g., 'Socrates *is* musical' means that this is true, or 'Socrates is not-white' means that this is true; but 'the diagonal of the square *is not* commensurate with the side' means that it is false to say it."

One other twist we find in the treatment of logical operators by the early Greek thinkers is that connection is expressed as identity or sameness and separation as negation.[151] Affirmation and denial, connection and separation may prima facie have nothing to do with identity and negation. But if we start with connection and separation (assertion and denial come later as they demand the separation of truth from its content), the straightforward way to express a separation is a negation (*x* is not F) and a rough first way of expressing a connection can be a form of identification (*x* is F).[152] This idea is reinforced in Plato's account of the cognition of the World Soul in *Timaeus* 37a7 ff., where sameness is understood as the basis for assertions and difference for denial, since '*x* is F' is understood as 'something is the same as something else' and '*x* is not F' as 'something is different from something else'.[153] Furthermore, as we will see in Chapter 5, in *Sophist* 260a ff. Plato introduces non-Being as a separation operator and identity or sameness as a connection operator and distinguishes the two operators from denial and affirmation by showing that they only come into play once the operators are connected to statements and thoughts.

1.3.2 Negation and Identity as Operators

1.3.2.1 Negation

1.3.2.1.1 Possible Understandings of Negation Today we may be inclined to understand negation in philosophy as an operation on a proposition that takes truth to falsity and vice versa.[154] Such an understanding was influenced by the Stoics and Frege, so we should not simply presuppose it in our attempt to understand the early and classical Greek philosophers.[155] Where negation is understood as separation (which is

151 The latter may seem less odd to us.

152 Since Frege 1918–19, the orthodoxy has been that denial should be understood in terms of negation, to deny a content is just to assert its negation (cf. Ripley 2011, p. 622). Rejectivists think, however, that our understanding of acceptance and rejection is prior to that of negation; see, for example, Price 1990, who claims that asserting that p equals denying that not p.

153 I discuss this in "Thinking Makes the World Go Round", currently in preparation. However, Plato also clearly distinguishes between separation and denial, and between affirmation and connection, since the world soul declares (*legein*, it asserts) that something is the same as something else or different from something. So it asserts some connection or separation.

154 I will understand negation as an operator in the following, since it operates on a certain input; cf. also Horn's talk about the "negative operator".

155 The Stoics seem to have been the first to employ systematically the idea of an external negation ('Not: *x* is F'), cf. Horn 2001, p. 21. For Frege, negation takes 'the True' (as an object) to 'the False', while for the Stoics negation takes a true proposition (which is more like Frege's thought) to a false proposition.

sometimes seen as analogous to physical disassociation), the operands are naturally thought of as thing-like, not proposition-like. Furthermore, the earliest thinkers we will look at do not explicitly introduce or discuss their understanding of negation; it is not even clear that they have an explicit understanding of negation. I want to show, however, that the particular notion of negation employed by these thinkers is crucial for understanding central claims of their philosophy and that the semantic value of the expression to which negation is applied changes in a systematic but, for us, not straightforwardly familiar way. Accordingly, it will be useful to look briefly at a variety of possible understandings of negation, which will make it easier to grasp how the philosophers I look at in this book use negation and how far their understanding of it differs from our modern usage.

Today negation tends to be studied as a matter of either logic or language or both. For the present project I will concentrate on the logical possibilities of negation, rather than the linguistic possibilities – though they are often interconnected – and I will also have to put to one side certain particularities of the Greek language concerning negation.[156] Furthermore, we should be prepared for the possibility that negation can be seen as having an ontological basis or counterpart, or ontological implications – as is evident in Plato's *Sophist* and in Aristotle's *Categories*. Aristotle points out that not only is the affirmation 'he is sitting' opposed to the negation 'he is not sitting', but this opposition between propositions corresponds to an opposition we find in the world: his sitting is opposed to his not-sitting (at a given time, he can in fact only be either sitting or not sitting).[157] Such ontological implications can also be found in modern

[156] For example, the Greek language uses iterations of negations which might be used to strengthen the negative sense (an example of a strengthening of negations in Parmenides we find in fr. 7, lines 1–2: *ou gar mêpote*, never); but under certain circumstances they cancel each other out, e.g., if a simple negation follows a compound one, see Kühner and Gerth 1904, p. 205. Most notably, Greek uses two different terms and their compounds for different forms of negation, *ou* and *mê*. One well-known way this distinction has been understood is that *ou* negates objectively, i.e., facts, while *mê* negates subjectively, i.e., demands, wishes, and conditions that are willed or thought; see Smyth 1956, section 2688 and Kühner and Gerth 1904, 2. Teil, 2. Band, p. 179. However, other ways of understanding these different forms of negation have been suggested; for example, Horn 2001, pp. 447–8 understands them as the distinction between declarative (for example, indicatives) and non-declarative (for example, subjunctives) types of embedded speech acts. A further way to express negation in ancient Greek is the use of the prefix *a*(*n*)-. While it plays a major role for Aristotle by inspiring his opposition of privative/positive, I will not be able to discuss this prefix in this book.

[157] *Categories* 12b. For a debate about existential import in ancient times, see Alexander's commentary on Aristotle's *Prior Analytics* 402 ff. In the course of defending Aristotle's account of negation against a Stoic one, Alexander discusses the case of "Socrates died", where assuming that the name Socrates has existential import would undermine the very statement in question; cf. Barnes 1986.

debates on negation, in discussion of the existential import of propositions in logic, for example, or in discussion of the possibility of negative facts.[158]

It is important not only for a negation operator to have what we may think of as a 'machine', the negation, that turns a certain input into a certain output, but also for the input (*x*) to be connected to a certain property that this machine turns into its opposite with the output. For some of us, the property of *x* would be T or F, but for a thinker like Parmenides it would be comprehensibility or incomprehensibility, if we want to put it like this. Thus for Parmenides, when we put Being, which has the property 'comprehensible' or 'conceivable' or 'knowable', as input into the negation machine, we get as output non-Being, which has the property 'incomprehensible' or 'inconceivable' or 'unknowable'.[159]

In order to figure out systematically a broader (if rough) range of possible understandings of negation that is suitable for this project, I will look first at what a negation operator can take as 'argument', that is, what things can be negated, then at the relation between the original input and the result,[160] and, finally, I will consider the possible results of a negation on their own.

We can distinguish different kinds of negations according to what is negated. From a logical point of view, what is negated can prima facie be a term (subjects and predicates as well as parts of subjects and predicates) or a proposition.[161] General terms, like man and mortal, seem to be prime examples for term negation; but I do not want to rule out for our present purposes that negation might be combinable with singular terms. In principle, there is the option to negate the different parts of a statement (S is P) – the subject term (not-S is P), the predicate term (S is not-P), and the copula (S is not P) – as well as the whole statement (not: S is P). If a sentence has a quantifier or a modal operator, it can also be negated.

The relation between the result of a negation in a broad sense and the original input is either a contradictory or a contrary opposition.[162] With

158 See Horn 2001, pp. 55 ff.

159 For a discussion of the way in which Parmenides describes Being with the help of negation, see Chapter 2.

160 Readers steeped in the Fregean tradition might want to think of this as the relation between value and argument, as long as they do not employ from this framework the idea that value and argument have to be the True and the False. At the moment I want to be as neutral as possible about what the input and results of a negation might be – they could be an expression, or the things referred to, etc.

161 So we should not rule out that we can negate non-sentential components of sentences, which is possible on an Aristotelian view (or that of modern linguists who allow for constituent negation).

162 Given that we do not know yet what the early Greek thinkers are in fact negating, it seems to be reasonable to allow also for the possibility that (at least general) *terms* are seen as contradictory or contrary. As mentioned above, contradictory terms can be understood as terms which, if both are applied to whatever subject, will necessarily produce one true

both kinds of opposition, the opposites cannot both be true, but one of the opposites of a contradictory opposition has to be true and the other false, while both opposites of a contrary opposition can be false – either because there is a third possibility in addition to the two poles or because we are dealing with things that cannot be characterised by either opposite. Both kinds of opposition are governed by the principle of non-contradiction; in addition, contradiction is also governed by the principle of excluded middle. While hot and cold can be understood as contrary opposites, for there are things that are neither, hot and not-hot can count as contradictory opposites,[163] one of which must hold true of everything.[164]

Contrary opposites also allow for what we might call different 'degrees' or 'intensities'. If we have a contrary opposition that allows for a *tertium*, like black and white, then negating 'S is white' could either mean that S is black or that S is some colour in between black and white, let us say some shade of grey. The former case can be termed an extreme negation as it excludes white completely and produces the polar opposite of the positive – it produces black.[165] The latter case can be termed a moderate negation.[166] The result of a moderate negation is different only in some sense from what is negated; the result is a simple, not polar contrary.[167]

In principle, the result of the negation will be the same kind of thing as the original input (the 'argument'), that is, also a term or proposition, etc. If we are negating a proposition, the result will have a different truth value from that of the original proposition. In addition, the result will be more or less determined or restricted, depending on whether it is situated within a restricted or an unrestricted domain. In the case of a contrary opposite, we are usually dealing with a restricted domain: only within a restricted domain,

and one false statement, while the application of contrary terms to whatever subject can produce two false statements and cannot produce two true ones.

163 See, for example, *Metaphysics* I, 1056b9–11, where Aristotle makes it clear that everything must be equal (*ison*) or not equal (*ouk ison*), while not everything must be equal or unequal (*anison*) but only that which belongs to what is receptive of equality.

164 We should be aware, however, that these logical differences need not always correspond to the different linguistic expressions used here, like the difference between 'cold' and 'not-hot', which we understand here as referring to a contrary and a contradictory opposite of being hot, respectively. If we encounter an expression like 'not-hot', we will have to look at the context to figure out which logical possibility is employed.

165 This is what Aristotle seems to understand by a contrary opposition (*enantiotes*) in *Metaphysics* I, 4 while he does seem to allow for what I call a 'moderate opposite' in the following in places like 1055b2–3 but does not pay it further attention.

166 For the distinction between extreme and moderate negation, compare Aristotle's distinction between complete and incomplete privation. See also Plato *Sophist* 257b–c and the discussion of this passage in Chapter 5. Sometimes we can also derive the result of an extreme negation from what we may think of as 'ordinary' negation, simply if the context is such that there are only two elements.

167 As Horn 2001, pp. 38–9 calls it.

such as, for instance, the realm of living beings, will the negation of healthy lead to the opposite ill; only in such a clearly defined realm must everything be either ill or healthy and the negation of the proposition '*x* is healthy' mean that *x* is ill.[168] Outside this realm of living beings, let us say with respect to numbers, healthy and ill is not a reasonable opposition; the number four is neither healthy nor ill. But we can say that the number four is not healthy, if by this we are referring to the contradictory opposite of being healthy (and the same holds true of subjects that do not exist).[169] In the case of a contradiction, the result of a negation will often belong to an unrestricted domain and thus not be clearly defined. Everything is either healthy or not healthy; whatever is not living will count as not-being healthy, while living things will be either healthy or ill.[170]

Thus the product of a negation is situated in either a restricted domain (as is a contrary opposite)[171] or in an undetermined domain. If the realm over which the negation ranges is specified (a group like the realm of living beings that contains two or more elements, for example), the negation produces something that is (relatively) well determined – if I know that Socrates is not healthy in the sense of being ill, I will send him best wishes for a speedy recovery. If, on the other hand, the realm is not determined at all, then the negation will produce something indefinite, since we know of the result only that it is not the positive argument (term or proposition, for example) out of all possible things in the universe.[172] If Socrates isn't healthy in the sense of the contradictory opposite of healthy, then I had better wait with my best wishes, for he might be no longer with us (or he may have turned into a number).

168 I am leaving aside problems of vagueness and borderline cases. And if we understand, for example, not-white as a contrary negation of white, it will also be at least relatively determined, since we then know that it is some colour, either black or something in between white and black. The same holds for all other kinds of term negation.

169 While I understand '*x* is not-healthy' as contradictory opposite to '*x* is healthy' and '*x* is ill' as contrary, for Aristotle '*x* is not-healthy' and '*x* is ill' both yield contraries *vis-à-vis* '*x* is healthy', since if *x* fails to exist or is a number, both '*x* is healthy' and '*x* is not-healthy' are false. But '*x* isn't healthy' (with different word order in Greek) is true also in cases of vacuous subjects as well as category mistakes. See Horn 2001, pp. 9 and 15. We will exclude the problem of non-referring subject terms for the time being (which is not to exclude that we might have to come back to a similar problem, if 'non-Being' in Parmenides or later thinkers turns out not to denote).

170 Cf. also Aristotle *Categories* 13b.

171 It is not necessary that this realm is always *explicitly* defined, if it is clear from the context what the realm is. In *Sophist* 257b–c, the fact that the *mê mega*, the not-large, refers to the small or medium-sized, but not, let us say, to the green, makes the domain restriction on the negation obvious – it is situated within the range of everything that can have a certain size. See Chapter 5.

172 They can be understood in a way similar to Kant's 'infinite' judgements in the *Critique of Pure Reason* B97–8.

1.3.2.1.2 History Let us now sketch the development of the understanding of negation in the period relevant to this book. For Parmenides, negation on the way of truth[173] indicates the absolute opposite of the positive: he uses negation as a one-place operator that works as an 'extreme negation', which produces the polar opposite of the original input. This understanding of negation makes differences of respect impossible, for it means that 'S is either completely P or it is completely not-P'. As a consequence, Parmenides understands the principle of non-contradiction as 'not (P and not-P) simpliciter'. The relation between the original argument and the product of negation has features not only of a contrary opposite – they are extreme opposites (which, as we have seen, are the end points of a potential range of options) – but also of what we would call contradictory opposites – one of them must be true and the other false. This last feature means that the principle of excluded middle must also hold for any pair of opposites, and therefore the principle of non-contradiction and the principle of excluded middle are conjoint commitments: 'not (S is P and S is not-P), and S has to be either P or not-P'.[174] We may wonder why Parmenides understands negation as extreme negation, but I think we have to bear in mind that we do not find any explicit reflection on negation before Plato – it is not as if Parmenides had a range of options to choose from. Parmenides himself seems to have a systematic, but not an explicit, understanding of negation.

With the atomists we find a notion of negation that is crucially different from Parmenides' and that allows for a significant expansion of the underlying logical framework, as is obvious from their understanding of non-Being. The atomists understand Being in terms of the full and of body and non-Being in terms of the empty and of void. Their understanding of non-Being implicitly means that negation cannot be understood as an 'extreme negation' in the strict sense, producing the polar or extreme opposite of the original input. For what is negated is not understood to be the absolute opposite of the result of the negation – this would lead to a non-Being that is not only not full in any way (which seems to be true of the void), but also to a non-Being that is in no way, while, as the atomists point out, non-Being also *is*. The atomists' new understanding of the negation operator allows for respects to some degree; for example, it allows for difference in time or perceivers, as we will see in Chapter 4. Accordingly, the principle of non-contradiction is already changing implicitly with the atomists.

[173] In Chapter 2, we will see that on the way of *doxa* Parmenides operates with a different form of negation.

[174] We will see in Chapter 2 that Parmenides' argument forbidding any difference to be ascribed to Being rests on an understanding of negation as extreme negation and would not go through otherwise.

While the negation operator so far has been used as a one-place operator (Parmenides' non-Being, 'not-*x*'), that is, it is applied to one input and produces a result that has a semantic value opposite to that of the original input, in his *Sophist* Plato uses negation in a way that could be seen as two-place, grounding the difference between two operands. Negation becomes a relation, a relation of difference, as found, for example, in Plato's claim that "Motion is not the Same" (*Sophist* 256a5), '*x* is not *y*'.[175]

Furthermore, in his *Sophist* Plato demonstrates explicitly that negation can be understood as denoting not absolute opposition but simple otherness. This demonstration forms the basis for allowing for respects, since the result of a negation need not be the complete opposite of the original input, and thus need not exclude the original input. It can be different to the original input only in some way, in some respect. Accordingly, this understanding of negation is the basis for reformulating the principle of non-contradiction as 'not (P and not-P) in the same respect'.

While Plato keeps the possibility of absolute negation for what he considers contradictories, like motion and rest, he concentrates on understanding negation as otherness. For Aristotle, in contrast, both understandings of negation are equally on the table. Furthermore, Aristotle seems to restrict contradictions to statements: while healthy and ill are understood as contrary opposites, only pairs of sentences of the kind 'S is healthy' and 'S is not healthy' are seen as contradictory.[176]

1.3.2.2 Connection Operator: Identity and Predication

The counterpart to the separation operator is the connection operator, which allows for connecting an object to another object or to a property to make a positive claim. The connection thus allows for ascribing something to some object. In the period considered, such connection includes what today we would call 'identity statements' as well as predication.[177]

When we talk about identity here, it will be helpful to put aside problems that guide contemporary discussions of identity, such as questions of personal identity or identity over time. Our focus is on identity as a way in which a subject (logical as well as linguistic) can be connected to something or as a relation a subject holds to something.

Today we understand identity in many contexts as the relation each thing has to itself and to nothing else, which is sometimes called 'strict identity' or, more precisely, 'numerical identity'. Numerical identity can be characterised

[175] The important point here is that negation is dyadic, we are not dealing with concerns of symmetry.

[176] See *Categories* 13a–b.

[177] Cf. also Owen 1970, p. 254 for the idea of a "connective use" of the verb "is" that is "distributed between identity and predication". As we will see in chapters 2 and 5, in the beginning identity seemed to be the way how some *x* could be connected to some F.

by reflexivity and Leibniz's Law[178] and thus entails three characteristics: transitivity (if $x = y$ and $y = z$, then $x = z$), symmetry (if $x = y$, then $y = x$), and reflexivity ($x = x$).[179] Numerical identity can be distinguished from qualitative identity, which is usually understood as a sharing of properties, as similarity. Qualitative identity can have different degrees, since more or fewer properties can be shared. I will take up this distinction in a slightly modified way and with modified terminology.

For our purposes let me take up the idea of strict or numerical identity as what I call 'absolute identity': if we understand '$x = y$' as expressing absolute identity, then there is no difference in any aspect between x and y,[180] and if translated into an exact language, the x on the left-hand side of the equation can replace the y on the right-hand side. We shall distinguish absolute identity from what I call 'partial identity' or 'sameness', used for any form of likeness that involves a difference between x and y in *at least* one respect.

The second kind of identity, partial identity, is normally two-place, a relation. And this is also how identity is understood in a Fregean scheme – as a binary function, xRy. However, identity can also be treated as a unary function, if we understand it as taking an argument x to the property Ry (so the function Ry, i.e., 'being identical to y', is applied to x). Interestingly, such an understanding of identity as a property and thus as one-place can be found in Parmenides, although it is used to express absolute identity.

For us, identity and predication may seem completely different things – after all, predication is not transitive and symmetrical.[181] In the history of philosophy, however, identity and predication were often taken to be very closely related or not clearly distinguished.[182] Thoughts about identity relations even

[178] By 'Leibniz's Law' I understand here only the weaker version which assumes the indiscernibility of identicals, not a stronger version which in addition also assumes the identity of indiscernables.

[179] Cf. Williamson 1998. The three characteristics themselves are necessary, though not sufficient for identity.

[180] Very much as Leibniz defines identity as having all properties in common.

[181] If we claim that 'Dr Jekyll is Mr Hyde' then we must also agree that 'Mr Hyde is Dr Jekyll' – identity is clearly symmetrical. But if we claim that 'human beings are animals' this does not entail that 'animals are human beings' (only some animals will be human beings) – predication is normally asymmetrical. Cf. the dispute between Ackrill 1957 and Cornford about whether the blending of Forms in Plato's *Sophist* should be understood as a symmetrical or asymmetrical notion. For Aristotle, however, some predications are symmetrical (particular affirmative predications) and some are transitive (universal negative predications, for example).

[182] So, for example, in Ockham or Hobbes. In the Port-Royal Logic it seems that the copula functions as a sign of identity. It does not identify the idea of the subject and the idea of the attribute in their proper extensions; but it identifies the extension of the subject with the extension of the attribute as restricted by the subject, so that in the statement 'all humans are animals', "the word 'animal' no longer signifies all animals, but only those animals that are humans" (Arnauld and Nicole 1996, pp. 130–1); cf. Pariente 1985 (I owe

seem to have been the starting point for the conceptualisation of predication, as will become most obvious with the late learner's paradox in Plato's *Sophist*.[183]

Furthermore, there is linguistic evidence that for the early Greeks the predicative sense and identity sense of *einai* (being) were not clearly separated. Indeed, grammatically, predication and identity are the same thing.[184] While the exact understanding of the Greek term *einai* is hotly debated and depends on the period under consideration, it seems that most, if not all, of these meanings are compatible with the idea that *einai* can be used as a copula as well as to express identity without identity being a separate aspect of the meanings of *einai*.[185] Separate words that mean 'identity' seem to appear rather late in Greek.[186]

We know that Frege still felt the need to point out the difference between identity statements and predication when he emphasised that in a sentences such as 'x is green', 'is' is used in a copulative way, whereas in a sentence such as 'x is Venus', the 'is' only seems to have the same role as in the other sentences, for in this sentence 'is' is indeed used as an identification.[187]

It may seem strange at first to put both forms of identity, absolute and partial, as well as predication, under the heading 'operator'. However, predication can clearly be understood as an 'operator' if we think of it in terms of the copula that connects the subject to the predicate, very much as we find it in Aristotle's *Analytics*. And we can think of identity as an operator as well, since it is a logical tool that operates on an object in a way that can be grasped independently of any knowledge of the content of the object. The input here may be 'Cicero' and 'Tulli', and the output would be an identity relation between the two.[188]

the reference to the Port-Royal Logic to Kenneth Winkler). Not distinguishing between the two usages also seems to have been a background for Lykophron's idea to exclude the usage of the word 'is', an idea Aristotle reports in *Physics* I, 2, 185b27–8.

183 And the idea of partial identity may have allowed moving from identity to predication.

184 See also Kahn 2009, p. 201.

185 Kahn 1966 p. 246 claims that there is no strict notion of identity connected with *esti*, but thinks that what he calls the *einai* of whatness corresponds in part to the modern 'is' of identity (p. 263). Mates 1979, p. 218 assumes that there is a more basic notion of being underlying both; cf. also Cherniss 1957, Vlastos 1973, and Ackrill 1957, p. 1. Kahn 2004, p. 385 claims that "Both in the *Sophist* and in the *Republic*, then, we can say that Plato has only one concept of Being, expressed by *einai*, *ousia*, and *on*, a concept that will cover the notions of existence, predication, identity, truth, and perhaps more."

186 We find *tautotês* in Aristotle's *Nicomachean Ethics* 1161b31 and *Metaphysics* 995b21, and *autotês* in Sextus.

187 See Frege 1892. On Frege's account, if we deal with terms of the same logical type, we are dealing with identity, if with terms of different logical types with predication.

188 While with the negation operator we usually have one item as input, with the connection operator there are usually two. If we wanted to understand identity as an operator with a truth function, it would be one that takes T to T and F to F.

We will see in the course of this project that the three forms of connection – predication, absolute identity, and partial identity – are not always clearly distinguished, and the development of the connection operator can be read as increasing clarification of the differences between these three forms. For example, in Plato's *Sophist*, important steps are taken to distinguish identity statements from predication, while, as we will see, absolute and partial identity are sometimes merged.

Let us turn to the historical development. Our story will start out with Parmenides' connection operator, which expresses absolute identity. Given that for Parmenides in truth there is only the One Being, it would be difficult to connect the One Being with something else. The only connections he can truthfully employ are identity statements claiming the identity of one thing with itself ($x = x$). All further characteristics that his account of Being implies are presented as *sêmata* of Being, which are not, as we will see in the next chapter, to be understood as normal predicates. For the ascription of those characteristics to Being by using a predication (or by employing any kind of partial identity) would imply a difference between the two relata and, according to Parmenides, non-Being.

Predication, as well as partial identity (thus all proper connection that is not strict self-identity), presupposes plurality and the logical means to conceive of plurality in a consistent way. Accordingly, this form of connection is not found with Parmenides.[189] This kind of connection is possible, however, with the atomists: there we can say of the atoms that they are extended, for example, or they are of different shapes. We saw above that the atomists changed the understanding of the negation operator so that what is negated is no longer understood to be the absolute opposite of the result of the negation.[190] Corresponding to this change, absolute identity, that is, identification with no exception, is no longer the only connection operator, or the only way to ascribe something to something.

While the atomists are no longer reduced to identity statements and instead use predication for what truly is, this change in the connection operator is not explicitly discussed. Only in Plato's *Sophist* do we find a first explicit discussion of possible understandings of a connection operator. In spite of this explicit discussion, in Plato's *Sophist* the connection operator is much less developed than the separation operator, and we find Plato switching implicitly from absolute to partial identity.[191]

189 Strictly speaking, if we think there is a distinction between an operand and an operator (even if the latter expresses absolute identity) in Parmenides, this already seems to presuppose a plurality at least in expression – a problem we will come back to in Chapter 2.

190 Non-Being qua void may be seen as the absolute opposite of the atoms in so far as it is not at all full and there is no third possibility, but it is not the absolute opposite in so far as it also has being.

191 See Chapter 5.

One highly important step for the development of logic we do find in the *Sophist*, however, is Plato's distinction between the different forms of connection as predication and as identity, respectively, a distinction that is originally introduced in order to deal with the late learner paradox.[192] And in the *Timaeus*, Plato is trying to substitute understanding references to things in the world as *identifying* with understanding such references as *attributing*[193] – instead of identifying this (*touto*) thing we perceive as water, we attribute water to it (it is *toiouton*, such-like, i.e., water-like).[194]

Finally, in Aristotle we can find the difference between identity and predication explicitly spelled out. And in his *Metaphysics* (Δ, 1018a7), we get an account of identity as a unity (*henotês*) of what is – either with respect to several things (what we called 'partial identity') or with respect to one thing, if it is somehow seen as a plurality. What we found as absolute identity in Parmenides is thus shown to be internally complex, an internal plurality.[195]

1.4 The Role of Mathematics: The Connection between Mathematics and Natural Philosophy

1.4.1 The Use of Mathematics for Science in General

1.4.1.1 Contemporary versus Ancient Accounts of the Relation between Mathematics and Science

It is often claimed that the use of mathematics for investigating motion and the natural realm is specific to modern times – there is some foreshadowing of this practice in ancient mechanics, optics, and astronomy (which, however, employ geometry, not arithmetic), but that is all there is.[196] Such a claim is only true, however, for a certain kind of mathematical usage. For, as we will see in the course of this book, some employment of mathematics in a broad sense already plays a crucial role for ancient natural philosophy.[197] Indeed, the usage of mathematical notions for the description of time, space, and motion is central for getting a real foundation for natural philosophy and science in antiquity.

192 In *Sophist* 251b ff.

193 *Timaeus* 49c7–50b5, at least in the traditional reading of this passage, which since Cherniss 1954 has been called the "much misread passage".

194 Cf. also Silverman 2002, p. 259. This distinction seems to be already present, though not explicitly discussed, in earlier dialogues of Plato, like the *Euthyphro* and the *Meno*, where we find the distinction between saying *poion esti* and *ti esti*.

195 See also Aristotle *Topics* I, 7, 103a6 ff. What Aristotle calls sameness in number there corresponds to absolute identity, while what he calls sameness in kind and in genus is included in partial identity.

196 See, for example, Duhem 1969, p. 3 for this foreshadowing in astronomy.

197 See also Lloyd 1987, p. 279, who claims that "the application of mathematics to the understanding of natural phaenomena of various kinds was one of the most important and fruitful preoccupations of ancient science".

Today we tend to accept as a matter of course that the natural sciences use the language of mathematics. No matter *why* we can successfully employ mathematics for the purposes of science – whether it is because mathematics is a general language of quantitative patterns or whether there is some closer link[198] – without doubt this employment contributes to the success of the sciences. We understand the regularities of the natural world as laws of nature that can be formulated in mathematical terms, for example, expressed as equations.[199] This modern idea of the physical world as mathematisable is due to a development that started with Galileo, Descartes, and their successors.[200] Descartes formulated three laws of nature – in essence they are laws of bodily motion – which then became a seed for Newton's laws of motion.

The ancient Greeks only gradually developed what we would call the idea of a law of nature[201] and in the period investigated did not express these regularities as mathematical equations.[202] But this is not the only way in which mathematics can be connected with the physical realm. Mathematics can play a crucial role for natural philosophy in other ways, as we will see with some of the ancients thinkers investigated here. One important way is that these ancient thinkers take over concepts which the mathematicians explicitly or implicitly use, as we will see, for example, with Aristotle's employment of a mathematical understanding of continuity. Another way is understanding

198 For the first possibility, see, for example, Resnik 1997. But the reason why we can give a mathematical account of physical processes could also be that there is some structural identity between mathematics and the physical world or, as Galileo 1933, p. 232 claims, that the universe "is written in the language of mathematics". It is in any case remarkable that the mathematical virtues of a physical theory speak strongly in favour of employing it (for example, string theory is a physical theory that was developed on the basis of its success in mathematics).

199 See, for example, Gregory 2016, ch. 11.

200 See Descartes' claim in correspondence with Mersenne: "my entire physics is nothing but geometry" (27 July 1638, AT II 268, CSMK III 119). But see Hatfield 1990, p. 113 for the ambiguity over whether this is a claim about method or content; cf. also Garber 1992. In any case Descartes is usually seen as one of the founding fathers of the notion of laws of nature.

201 In *Gorgias* 484a–b Plato talks about what we can translate as law or rule of nature (*to tês physeôs dikaion*), which is, however, understood in a normative sense (we get to it by going against the *nomoi* that are *para physin*). In the West, the term 'laws of nature' seems to go back to Roger Bacon, who in his *De multiplicatio specierum* I, 6 refers to Avicenna *Metaphysics* VI as talking about a law of universal nature, cf. Crombie 1996, pp. 75–6. While today the laws of physics are regarded more like mathematical laws, as they seem to share the same kind of necessity, in the very beginning of philosophy, social laws also seem to have been taken as paradigms, as we can see, for example, in Heraclitus fr. 94 (cf. also Gregory 2016); the ancients do not seem to have talked about 'mathematical laws'.

202 Mathematical equations only appear with Pappus.

mathematical structures, such as numbers and geometrical figures, to indicate the rational set-up of the universe, as we will see in Plato's *Timaeus*.

In this way, mathematical structures did indeed play a crucial role in establishing the sensible realm as a proper object of science, even if these mathematical structures are neither identical to nor employed in the same way as the mathematical structures we employ today.[203] Accordingly, the ancients do not draw the distinctions between the physical realm and the mathematical realm in quite the way we do, or at least do not necessarily mark this distinction as we do.[204]

I will have to leave out here the more practical sides of the intersection of the natural realm and mathematics, what Aristotle sometimes calls the more physical (*physikôtera*) branches of mathematics,[205] such as mechanics, optics, and harmonics.[206]

1.4.1.2 The Beginnings in Ancient Times of Employing Mathematics for an Account of the Natural Realm

Mathematics or, more precisely speaking, some parts of mathematics, seems to have been understood as a secure body of knowledge by the time covered by this study, or at least at the time of Plato and Aristotle, which is when it becomes most relevant for our story. If whatever is known in mathematics is secure knowledge, then it would appear a good idea to figure out whether mathematics can also help establish secure knowledge in another realm, such as the realm of natural philosophy.

The first Greek philosophers to use mathematics for an account of the world seem to have been the Pythagoreans,[207] that hard-to-grasp school of like-minded philosophers with a special interest in mathematics. Some Pythagoreans, at least according to Aristotle, identified the phenomenal world in some way with numbers.[208] Today, we find it natural to describe

[203] For example, Plato uses mathematical structures in order to show the rationality of the natural world, but not predictability (Netz 2002, p. 256). However, we seem to find some form of predictability, also based in part on mathematical structures, with respect to eclipses in ancient cosmology.

[204] As we can see, for example, in Plato's account of the receptacle, which seems to show features of a physical as well as a mathematical space, without Plato making any distinction between physical and mathematical space, cf. Sattler 2012.

[205] *Physics* 194a ff. See also *Posterior Analytics* 78b35 ff. and *Metaphysics* 997b20, and 1078a14.

[206] You may think that employing mathematics for the set-up of the cosmos, as we see in Plato's *Timaeus*, is still applied mathematics (so, for example, Broadie 2012). Nevertheless, mathematics is not used here for practical human purposes, as in fields like mechanics (Burnyeat 2005 may nevertheless be right that the demiurge uses practical reasoning when setting up the world).

[207] Though there may be some traces in Anaximander, see Gregory 2016.

[208] For a somewhat more detailed account of what the Pythagoreans thought and a distinction between different groups of Pythagoreans, see Chapter 6.

physical facts and processes within a mathematical framework – we might, for example, depict a process as a graph in a coordinate system. Where we are normally aware that a point on this graph is not the same as a physical moment in this process, however, Aristotle thinks that the Pythagoreans did not distinguish between mathematical points and physical elements or constituents, and considered numbers as what physical things are made of.[209] Such overenthusiasm in bringing together mathematics and the natural realm may strike us as strange. But it shows that the Pythagoreans assumed a strong connection between the natural world and the world of numbers (in this case, that they are the same). Even if most of us would be unlikely to subscribe to such an identification, the Pythagoreans at least seem to have started a debate over how mathematics may be related to the physical world. By contrast, before the Pythagoreans, mathematics in Greece seems to have been connected with the physical world largely for more practical things, such as a tool for commercial exchange[210] and for construction.

As early as Philolaos – the first Pythagorean for whom we have a secure basis of fragments – numbers were not simply identified with the natural world (Philolaos' new principles are the limiters and the unlimited, not numbers, but numbers still play an important role for him). Rather, Philolaos assumes numbers to be a necessary condition for knowledge – nothing can be known without number.[211]

Plato seems to have used mathematical structures, and thus what belongs to secure knowledge, to establish natural philosophy as a kind of scientific investigation, after investigation of the natural realm (in the form of cosmologies) had come under philosophical fire from Parmenides.[212] Plato does not equate the natural realm with numbers;[213] instead he shows how the basic

209 *Metaphysics* M, 1080b16 ff. and 1083b10 ff. Cf. Tannery 1930, Lee's notes on Zeno, and Cornford's commentary on the *Timaeus*: they assume that the notion of a point and of a unit were also not clearly distinguished in ancient times. They think that this lack of distinction also contributed to Zeno's paradoxes (cf., e.g., Lee, p. 34), which Chapter 3 will attempt to prove wrong.

210 At least if we can use Aristoxenus: "Pythagoras most of all seems to have honoured and advanced the study concerned with numbers, having taken it away from the use of merchants and likening all things to numbers", fr. 23, Wehrli. Huffman points out that this fragment only shows that Pythagoras assigned special prominence to mathematical relationships that were in general circulation, not that he discovered or proved anything himself; but this is all we need here. The Near Eastern discussion of mathematical structures also seems to be tied in part to the context of commerce, and in part to cosmology.

211 As we are told in fr. 4. Testimony A29 may even suggest numbers are a sufficient condition when it claims that we can know the world insofar as it is guided by number.

212 For the restrictions by which natural philosophy is nevertheless limited according to Plato, and a discussion in how far Plato would see it as a science, see Chapter 5.

213 Though at least some people in the early Academy seem to have either equated his Forms with numbers or reduced them to numbers, see Burkert 1972.

structure of the phenomenal cosmos can be understood as mathematical – either as arithmetically structured, as in the case of the set-up of the heavenly motions, or as geometrically structured, as in the case of the basic elements and thus of extended things.

Aristotle takes up neither a Pythagorean equation of numbers with the sensible things in the cosmos nor Plato's idea that the cosmos is at root mathematically structured. But we will see that he imports into natural philosophy concepts that are employed (implicitly or explicitly) in the mathematical realm. And both Aristotle and Plato use mathematical structures and concepts to demonstrate the intelligibility of the natural world.

1.4.1.3 Problems with the Introduction of Mathematics into Natural Philosophy

With both Plato and Aristotle we will also encounter some of the problems that the introduction of mathematics into the realm of natural philosophy can cause. When Plato discusses the basic structure of the elements, we will see how solely mathematical considerations are decisive for positing some fundamental metaphysical structure underlying the physical world.

Aristotle, by contrast, is very careful to distinguish mathematics from physics.[214] In *Physics* Book II, 2 he provides the famous example that the natural philosopher who investigates a snub nose is interested in the nose and its material as well as its being curved, while the mathematician who investigates a snub nose is interested only in its curved form. In his *Metaphysics* he points out that mathematical method should not be used universally and is not the general method for doing natural philosophy.[215] Following this hint from Aristotle, it would seem useful to distinguish between the introduction of mathematical concepts and the introduction of mathematical methods. I will argue here that Aristotle very clearly does the first: he introduces mathematical concepts into natural philosophy, most prominently a mathematical understanding of continuity. To some degree, however, he also seems to introduce mathematical methods, not for pursuing an investigation within the field of natural philosophy, but for structuring his presentation in the *Physics*.[216] Aristotle's awareness of the differences between mathematics and physics in general means that he is cautious about how to transform these mathematical concepts so that they are adequate for the physical realm. As we will see in the last chapter, however, in the end implications from the realm of mathematics

[214] Cf. also Guthrie's commentary on his translation of *On the Heavens*.

[215] *Metaphysics* 995a15 f.: "The minute accuracy of mathematics is not to be demanded in all cases, but only in the case of things which have no matter. Hence the method is not that of natural science; for presumably the whole of nature has matter."

[216] See Chapter 7.

contribute crucially to preventing Aristotle from giving an account of motion in terms of time and space.

Our investigation of how mathematical concepts are introduced into natural philosophy and how fruitful the results are in the history of philosophy does not mean that in this book we will interpret the ancient natural philosophers as dealing with mathematical problems as such, an interpretation attempted most prominently for Zeno's paradoxes.[217] Mathematical interpretations of the kind we encounter with Zeno's paradoxes may seem appropriate, since mathematics deals with the most abstract objects – accordingly, whatever is true for those objects should also be true for physical objects. Such interpretations face, however, the problem that mathematical interpretations cannot show something specific to a physical thing – while some mathematical structures may also hold true of physical objects, in deploying such structures to interpret a physical object we may lose what is specific to the physical object.

So far we have looked at a few specific problems generated by introducing mathematical notions into the field of natural philosophy. Speaking more generally, introducing concepts from one realm into another can lead to at least three problems. First, implications that these concepts have in the original field may still be there in the new field, where they can come into conflict with the implications of other concepts in the new field. For example, when Plato uses the perfect geometrical bodies as the basis for the physical elements in his *Timaeus*, he faces the problem that there are five such perfect geometrical bodies but only four physical elements.[218] Second, the consequences of such an introduction may not be sufficiently clear, for example, Plato's geometrisation of the elements, starting from basic triangles, raises the problem of whether three-dimensional bodies can be made up of two-dimensional surfaces. Plato thinks they can (*Timaeus* 54c–d), which seems to be supported by mathematical constructions;[219] Aristotle, on the other hand, thinks they cannot, as three-dimensionality in the physical world can never come from something two-dimensional. Third, if there is a problem in a particular field, it is not always clear which concepts from another field should be chosen in order to solve it. For example, both Plato and Democritus employ geometrical features to explain the physical characteristic of something being very mobile, as is fire, but Democritus sees this job as best performed by a spherical body, while Plato choses a pyramid.[220]

217 See Chapter 3.
218 See Chapter 6.
219 For example, if a pyramid is constructed as if made out of its faces.
220 Cf., for example, Aristotle *De anima* 405a11 and Kirk, Raven, and Schofield 1983, p. 427.

1.4.1.4 Using Measurement to Bring Mathematics to Bear on Locomotion

Leaving aside the problems just mentioned, the introduction of mathematical structure from arithmetic and geometry into the field of natural philosophy seems fine if we want to give an account of something that is static and spatially extended (for example, the basic building blocks of the cosmos in Plato). But how can mathematical concepts or structures be used to describe motion, given that motion does not appear in arithmetic and geometry? This very question leads Aristotle to claim in his *Metaphysics* that the mathematical principles of the Pythagoreans are more abstruse than those of others in the *peri physeôs* tradition, since they are ill-suited to explaining the natural world and especially inappropriate to explaining motion.[221] For motion to be explained we need some account of how mathematics can help to understand it.

The most obvious way to solve this problem, that is, for mathematical structures to be used to understand motion, is through employing a measure, to which we turn in the following section.

1.4.2 How to Do Things with Numbers: Measurement and Countability

Demonstration that a motion can be measured is an important way of showing that motion is intelligible in at least one aspect – if motion can be quantified, it can be related to numbers and thus grasped as something intelligible. While we will be concerned with measurement in Plato and Aristotle in the main part of the book,[222] here it will be useful to give a brief general account of measurement against the background of modern measurement theory.[223] Introducing general structures with the help of a contemporary context may initially seem an unnecessary anachronism, but we will see that the basic structure of the notion of measurement I sketch here is the same as that with which Plato and Aristotle are concerned for their own purposes. Hence, the following brief

221 *Metaphysics* A, 989b29 ff.

222 We do not find any discussion of measurement in what is handed down to us from the Eleatics and the atomists. There may, however, have been some discussion of measurement in connection with natural philosophy in Anaximander, who uses the earth's radius or circumference as a measure for the distances between the earth, moon, and sun. See O'Brien 1967; White 2008, p. 105; and Gregory 2016. Also Heraclitus B 31 and B 94, Diogenes of Apollonia B3, and Philolaos' basic pair of limiters and unlimiteds may be related to a measurement context; see Philolaos frr. 1, 2, and 9 A in Huffman and Lloyd 1987, pp. 276–8.

223 My account will be in accordance with the standard work of measurement theory, Krantz et al. 2006, and with Ellis 1968. I will not, however, simply take up their accounts, since they are too abstract for our purposes; instead, I will extract three essential features of measurement that allow us to compare Plato and Aristotle with a modern account.

account will show the kind of measurement structure with which natural philosophy deals, it should make Plato's and Aristotle's accounts more accessible, and it will help explain the extent to which problems of measurement may have arisen in ancient times.

Let us start with a general question: what is the function of a measure?[224] While measuring allows us to connect the physical world to mathematical structures, measuring is not about assigning numbers randomly to the empirical world. A measure enables us to quantify empirical things, processes, and states of affairs systematically and it provides one way of understanding the physical world as something intelligible. In order to quantify something, we first have to decide which *aspect* of a thing we want to quantify – for example, do we want to measure its weight, its length, or its density? How much of that aspect a thing possesses can then be measured by assigning it to numbers with the help of measurement units. When I want to measure the length of my desk, I fetch a measuring tape that gives me a measurement unit, a centimetre, for example. I then see how many times I have to use this measurement unit to get from one end of the desk to the other. The number of times I have to use the measurement unit in order to cover the whole length of the desk is the number I assign to the length of my table. And I assign a number to the length with the help of a unit in a *systematic way*, that is, were I to cut off a piece from my table, I would have to assign a smaller number to the table, and, if I added an extension, a larger one.

From this brief description we see that we have to take three elements into account in order to measure something:

(1) The respect in which something is to be measured must be determined, that is, we have to decide whether we are going to measure the weight or rather the length of our table. That respect is what in the following I will call the *dimension* measured.[225] The thing to be measured with respect to this dimension (that is the thing qua possessing a certain feature) will be called the *measurand*. So when we measure the length of a table, the dimension measured is length, and the measurand is the length of the table.

(2) The measurand must be quantified by *assigning* it to the number series in a systematic way.

(3) For the quantification of a certain dimension, *units* have to be defined. Usually, a certain amount of the dimension in question is taken as a unit,

[224] The following account of measurement overlaps with and is developed more fully in Sattler 2017a.

[225] As is common in the measurement literature, following Fourier's introduction of the notion of physical dimensions. In fact, the English word 'dimension' derives from Latin *dimetiri*, which means 'to measure out'; see the OED entry on 'dimension'.

for example, a metre or a foot can be taken as a unit for length, and the measurand determined as a multiple of that unit.

Let us examine each of these three requirements in more detail:

(1) As suggested above, I will understand 'dimension' here simply as the aspect under which something is regarded in the process of measuring. 'Dimension' is thus not understood merely in a spatial sense, capturing length, breath, and width, but rather can refer also to any quality that is quantified (including, of course, length, breath, and width). For example, a tortoise can be regarded in light of its mass, its length, the force of its kick, or the amount of soup that can be made from it – each is a different dimension. Which respect is taken into account depends on what we want to examine, and that respect must be chosen before we can start measuring (there is no point in fetching my thermometer if we are going to measure time). Determining the respect in which something is to be measured involves determining whether the dimension is simple (length, for example) or complex (speed, for example, which requires that we deal with two dimensions, time and distance covered).[226] If we are dealing with a complex dimension, we will need to determine the relation of the different dimensions (for example, for our modern notion of speed we *divide* the space covered by the time taken, $v=s/t$, it is displacement *per* time, while in order to calculate electric charge we *multiply* amperes by seconds, $c = A \textit{ times } s$). If we are interested in the force of the kick of our tortoise, our measure will be the product of mass and acceleration ($F = m \bullet a$). After a specific aspect of the tortoise has been measured, we can compare the result of this measurement with the result of a measurement of the same aspect of another tortoise or of the same tortoise at a different time.

This understanding of dimension could suggest that determining the dimension to be measured is a trivial matter. In fact, many modern conceptions of measurement presuppose dimensionality without discussing it.[227]

[226] Modern measurement theory often talks about 'fundamental' and 'derived' measures, quantities, or scales instead of 'simple' and 'complex' ones, where derived measures depend on fundamental ones. See, for example, Krantz et al. 2006, ch. 10. However, as Ellis 1968 makes clear, derived measures could also be derived from just one fundamental measure, while the complex measure which is of interest for our project necessarily builds on two (or more) other dimensions.

[227] There are accounts of measurement which do not seem to refer to any dimension, for example, the first characterisation of measurement given by Krantz et al. 2006. However, they presuppose dimensionality in that they understand "the measuring of some attributes of a class of objects or events" as the process of associating "numbers (or other familiar mathematical entities such as vectors) with the objects in such a way that the properties of the attributes are faithfully represented as numerical properties" (p. 1). The measurands here are "some attributes of a class of objects or events", which is equivalent to our notion of dimension.

Dimensionality can, however, be difficult to deal with, as will become obvious when we investigate the measure of motion in Plato and Aristotle.

(2) In order to measure something we have to assign a certain aspect of it (the dimension we are measuring) to a mathematical structure, that is, to the natural number series or to a certain set of numbers. Hence Krantz et al. understand measurement as the "construction of homomorphisms . . . *from empirical relational structures* of interest *into numerical relational structures* that are useful".[228]

This construction has to fulfil certain conditions that we called measuring in a 'systematic way' above and that Ellis characterises as follows: "Measurement is the assignment of numerals to things according to a determinative, non-degenerate rule", "determinative" meaning that "provided sufficient care is exercised the same numerals (or range of numerals) would always be assigned to the same things under the same conditions", and "non-degenerate" meaning that the rule "allows for the possibility of assigning different numerals (or ranges of numerals) to different things, or to the same thing under different conditions".[229] Hence, the rule of assignment guarantees that under the same conditions the same assignments will be made and that under different conditions different assignments are possible.

This assignment used for measurement purposes does not necessarily mean that one physical element or part is assigned to one numerical element. The assignment we are dealing with is a homomorphism, that is, a structure-preserving map,[230] but not necessarily an isomorphism, that is, a homomorphism which is bijective (it is both one-to-one and onto). So while the structure is preserved in this mapping, we do not necessarily get a one-to-one correspondence of elements, as we would get in a bijection.[231]

In modern science, this systematic homomorphic mapping transfers the empirical realm to an Abelian group of real numbers[232] and thus we may use our knowledge of the arithmetical structure to infer information about the homomorphic empirical structure.[233] For example, we know that 900 is three

228 Krantz et al. 2006, p. 9; emphasis added.

229 Ellis 1968, p. 41.

230 To express 'preserving structure' in slightly more formal terms: we have a domain of physical objects, a concatenation function ($\circ$) and a relation (bigger than) on the one hand, and for the codomain the real numbers with addition (+) and a relation (>). Then a homomorphism is a function from the structure of what is to be measured into the real numbers, such that $F(g1 \circ g2) \rightarrow F(g1) + F(g2)$, $g1 \gtrsim g2 \rightarrow F(g1) \geq F(g2)$, etc. For Krantz et al. 2006, F is essentially not a surjection. I owe this clarification to Henry Mendell.

231 For this see Krantz et al. 2006, p. 8. In a modern, but not in an ancient, context, isomorphism would be too strict a requirement, and the weaker homomorphism gets us everything we need for measurement.

232 That is, to a group where commutativity holds, so that the result of an operation does not depend on the order in which the elements of the group are written.

233 As Suppes 2000, p. 549 writes: "What we can show is that the structure of a set of phenomena under certain empirical operations and relations is the same as the structure

times as much as 300; so if it turns out that the weight of your tortoise is assigned the number 900 in our measurement procedure (it weighs 900 g), while the weight of mine, measured with the same measurement tool, is assigned the number 300, then we know that with respect to weight your tortoise is three times heavier than mine.

(3) What we measure will usually be a continuum.[234] Consequently, its structure cannot be assigned a numerical structure straight away, since that would require given discrete chunks that we can assign to numbers. Instead, we need basic units to mark off parts. These basic units have to be constant,[235] for otherwise we cannot make sensible comparisons.

For many dimensions the units used can be defined once and then kept for further measurement procedures. For example, in order to measure weight, we can define a stone as our unit and then use the very same stone for measuring weight over and over again. Such is also the case for measuring length: I can use the very same centimetre on a ruler to measure out different lengths. What is specific to time measurement, which is crucial for measuring motion, is that here the measurement units always have to be 'produced' anew – at each moment when we want to measure some time we need a motion or change going on from which we can derive a unit (we cannot use a past second to measure a time right now). This process of creating units must be regular (for instance, the regular vibration of a quartz crystal in our watches) so that the units are constant.[236]

of some set of numbers under corresponding arithmetical operations and relations." We should bear in mind, however, that the ancient Greek understanding of numbers is not the same as ours – Pritchard 1995, p. 17 thinks that we should not even translate *arithmoi* as 'numbers' but rather as 'numbered groups', which already shows that numbers are connected to the physical world.

234 Whatever our metaphysics of continua is on the microscopic level. I will not get involved in measurement in quantum mechanics here.

235 The standards for constancy can, however, vary from context to context: when baking a cake, for example, our measuring cups can vary somewhat. The units just have to be constant and regular enough to not spoil the cake. In the context of technical measurements, on the other hand, we usually require more constancy.

236 While the units used to measure and the process used to gain measurement units must be regular, regularity of the thing inquired into is not a necessary condition for countability or measurability. For example, I can measure the duration of a motion even if it changes direction or speed. And counting or measuring something does not 'produce' a regular order, either. Besides, there can be regularity without countability/measurability, for instance, if we take the ancient problem of the incommensurability of the diagonal with the side of the square. If we use the rational numbers, there is no way of assigning the diagonal and the side of the square to a multiple of the same unit. Thus, although we construct the square and its diagonal in a regular way, they are not uniformly measurable with the help of this number system.

Furthermore, the units used for measurement must be chosen in accordance with the aspect to be measured. What is to be measured and the units chosen to measure it must be of the same dimension (we cannot measure the weight of a table with centimetres). They also need the same degree of complexity: for example, as the measure of the force of the kick of a tortoise is complex – it is the product of mass and acceleration ($F = m \bullet a$) – the units must be complex as well, namely $kg \cdot m/sec^2$.[237] In Aristotle we will see this requirement spelt out as a 'homogeneity' requirement.

The units used for measuring are in principle arbitrary in the sense that we can use units of different size – I can use centimetres as well as inches to measure the length of my table, the one is no more natural than the other. Centimetres will divide the length of my table in different (potential) parts than inches, but both will allow me to make reliable comparisons equally.

By understanding the arbitrary choice of a dimensional unit as a vital feature of measurement, we are distinguishing measurement from mere counting where what is quantified is given as discrete elements. Counting can be understood as determining the cardinality of a plurality of given discrete things by coordinating two procedures: the operation that allows us to consider each element of the plurality singly, no matter in which order, is coordinated with the operation that takes us through the series of the natural numbers. This has to be done in such a way that whenever I take up a new element I move ahead one step in the number series; for example, when counting my chickens, I take up the first chicken and start with the first element in the number series, 1, then I take up the next chicken and move ahead one step in the number series, to 2, etc. In the procedure of counting we do not need to break down a continuum into parts that can then be assigned to numbers, as we do when we measure something; rather, with counting there are already given discrete parts. Furthermore, with measuring we use arbitrary units to 'divide' the continuous quantity into parts, while with counting, the unit with which we count is given (as, for example, the unit 'chicken'). This unit cannot be made smaller or bigger arbitrarily, since then we are counting something else.[238] And with mere counting, like counting up to a hundred, no dimension whatsoever is required,[239] since we usually do not understand numbers to have any

[237] Taking 'kilograms' as a unit of mass, not of weight, and neglecting for purposes of simplicity that force and acceleration are vector quantities.

[238] For example, chicken wings instead of chicken. In case of counting, that is, of quantifying given discrete things, as in Aristotle's example of quantifying horses in *Physics* 220b19–22, the dimension is expressed by the very unit – both are 'horse'. When we measure and thus quantify continua, on the other hand, dimension and units are not the same thing: for example, the dimension measured may be time, in which case our units will be a certain amount of time, like seconds or minutes.

[239] In Aristotle's example of counting horses in *Physics* 223b13–14, however, dimension does play a role, since if we do not determine the dimension 'horse' first, we may as well

dimension of their own,[240] which makes them suitable for operations on all sorts of dimensions.[241]

There are, however, authors like Patrick Suppes who treat counting as a measuring procedure, taking it as an absolute scale.[242] Their approach seems to be supported by the fact that we talk about two groups being equal in number just as we talk about them being equal in other qualities. This rationale makes good sense to the extent that one can compare two groups with respect to the number of their elements. (For reasons to do with his ontology, Aristotle also belongs in this group of authors who, at least sometimes, understand counting as a form of measuring.) For our purposes, however, it will be useful to distinguish in principle between counting and measuring.[243] Both Plato and Aristotle face the problem of how to measure and compare continua where discrete elements or parts are not given but have to be established before the actual process of measurement.

While the conception of measurement discussed so far is suitable for a philosophy of bodily motion, it may seem less helpful for understanding Plato, since Plato seems to be interested in normative measures, that is, in measures telling us how something *should* be. Yet while we will find this tendency to understand measure in a normative way in the *Timaeus* – a reflection of his connection of ethics to cosmology – we will see in Chapter 6 that Plato is indeed concerned with the kind of measure sketched above. Indeed, conceptualising a measure seems to be a problem of the time, for Plato's contemporary Eudoxos probably established the first steps for a theory of measurement.[244]

With this overview of the methodological, logical, and mathematical foundations of the concept of motion let us now turn to the individual thinkers and their respective treatment of motion.

be counting owls and eagles. In contrast to measuring, however, there is no arbitrary unit involved – while I can count animals instead of horses, normally I cannot use a slightly smaller unit than horses in the way that I may use millimetres in some cases, rather than centimetres or inches (I may count human beings by counting legs or hands and dividing by two, but I cannot make up an arbitrary unit).

240 Understanding number as Russell 1972, p. 116 does, as the class of similar classes.

241 For the difference between counting and measuring, see also Ellis 1968, p. 15.

242 Suppes 2000. Ellis 1968, pp. 152–9, is quite reluctant to call counting a measuring procedure, however, thinking it lacks arbitrariness as there is no choice of unit with counting.

243 Against understanding counting as measuring with an absolute scale, see Berka 1983, p. 109.

244 It is often thought that Eudoxus' theory of proportion, which forms the basis of book V of Euclid's *Elements*, is the first step towards a measurement theory. See the Scholium on book V; Suppes 2000; Menn 2018. The need for a conception of measurement in Plato's time can also be seen from the discussion of the incommensurable, see *Theaetetus* 148a–b; the explanatory note on it in Burnyeat's edition of the dialogue; von Fritz 1970; and Mendell 2007.

2

Parmenides' Account of the Object of Philosophy

2.1 Introduction

This chapter spells out the challenge that Parmenides' philosophy poses for natural philosophy. This challenge arises not so much from explicit reflections on natural philosophy and on earlier cosmologies as from what we can call Parmenides' logical, metaphysical, and methodological reflections: it derives from the criteria Parmenides establishes, in part implicitly, for any rational or scientific investigation in conjunction with the logical operators available to him. The field of investigation that is thus methodologically prepared excludes natural philosophy, since what is subject to change and motion – the object of natural philosophy – cannot be rationally grasped with the help of Parmenides' criteria and operators.

Natural philosophy, understood as the rational investigation of bodily motions and changes, was a principal interest of Parmenides' predecessors. By contrast, for Parmenides, such an investigation is no longer the summit of scientific inquiry.[1] This becomes clear, among other things, from the fact that it is set out not on what is commonly called the way of truth,[2] but of *doxa*, which deals with that of which no fully reliable account can be given, as the goddess explicitly tells us.[3] And the rules of natural philosophy – if indeed it has rules –

[1] Thus we possess a much bigger chunk of the metaphysical part of Parmenides' poem than of his cosmology, since this part was quoted much more extensively than the cosmology, which seems not to have enjoyed the same degree of respect in ancient times. And, with the exception of frr. 5 and 6, all fragments dealing with the *alêtheia* part are handed down in more than one source, while of the cosmology only fr. 16 is given by more than one source. I will discuss the status of Parmenides' cosmology below.

[2] Mourelatos 1970, p. 67 has rightly pointed out that the goddess herself calls it "the path of conviction", but since "way of truth" has become the common way to refer to the only way the goddess finally thinks we can pursue and *alêtheia* is announced in fr. 1 as what we will hear about on this way, I will also use this term.

[3] See fr. 1, line 30 and fr. 8, lines 51–2. This is a point on which most interpreters of Parmenides agree, even though they seem to disagree on almost every other aspect of his poem. Osborne 2006, pp. 237–8 claims that an ancient reader reading the whole of Parmenides' poem might be left uncertain about its message, since the first part seems to demonstrate that cosmology is impossible, while the second part then

are not the rules of the kind of inquiry Parmenides attempts to establish as the core of philosophical investigation, which is an account of truth – what we can call a logically inspired metaphysics or ontology.

Eventually, I will put forward the stronger claim that Parmenides establishes rules for philosophy that require him to dismiss from philosophical investigation concepts that have been seen as crucial for natural philosophy, at least since Aristotle[4] – motion and change, time and space. What is left is metaphysics or ontology.[5] Parmenides is therefore a difficult but very fruitful starting point for an investigation of natural philosophy in ancient times: the point at which natural philosophy begins to come apart from metaphysics, another central field of philosophy, is also a natural point for reflecting on what counts as rational investigation, as what we call philosophy.[6]

provides a cosmology. However, if this ancient reader carefully reads the goddess's assessment of the cosmology just mentioned, she would know that the cosmology, in contrast to the metaphysics, is not to be taken as trustworthy. While Aristotle does refer to Parmenides' cosmology, he makes it explicit that he does not count Parmenides among the philosophers of the *peri physeôs* tradition (see, for example, *Physics* 184b25 ff. and especially 185a17–20) – this restriction also suggests that Aristotle sees Parmenides as being mainly concerned with metaphysics. Aristotle distinguishes clearly between Parmenides' metaphysics (which he claims to be monistic) and his cosmology, which he rightly reports to assumes two principles, without this affecting Parmenides' metaphysics.

4 For the Presocratic 'natural philosophers', these concepts were part of the general mix of what is investigated.

5 Some scholars have recently argued that Parmenides' philosophy does not challenge natural philosophy but rather continues it, for example, Patricia Curd, Catherine Osborne, David Sedley, and John Palmer. However, for the most part they do not doubt that Parmenides' main achievement is not his cosmology, but his metaphysics. For Palmer 2009, Parmenides' main insight is his claim that we must distinguish between necessary, contingent, and impossible being and that we should focus on necessary being, into which the *alêtheia* part inquires. This, however, raises the question of the status of natural philosophy. When Palmer 2009, p. 223 claims that "Parmenides never argued that there is no plurality, change, or motion *tout court*, but, much more narrowly, that what must be must be eternal and unchanging, unique, and homogenous", it is unclear how he can see this as not posing a challenge to natural philosophy, which at least would be guilty of focusing on that of which there can only be wandering thought, and no real knowledge. Against Osborne's 2006 claim that nothing of what Parmenides has said has any effect on the post-Parmenidean Presocratics, we will see in the chapter on the atomists how they defend their pluralism by extending Parmenides' logical operators and criteria; see also the atomism chapter for a reply to Sedley.

6 This is not to claim that the Presocratics themselves thought of what they did as philosophy, nor that they would conceive philosophy in our sense as distinct from science. Not only philosophy, but also science, can be understood as a kind of rational investigation, and it is doubtful that we can distinguish the two in Greece at the time of Parmenides. But the rational investigations that will be the focus of the following inquiry are concerned

Parmenides' account provides for the first time criteria for philosophical investigation that are clearly spelt out.[7] But we also learn why, according to Parmenides, natural philosophy does not meet these criteria, and why the core assumptions of natural philosophy may be problematic. Thus, to begin with Parmenides is to begin with his challenge to natural philosophy and science – a challenge not only to the natural philosophy that preceded him, but also to the very possibility of any natural philosophy to come. By spelling out this challenge, the current chapter will set the stage for those that follow, which consider how Parmenides' successors respond to this challenge and thereby gradually build the foundations for a philosophical account of the motions and changes of the sensible world.

By reconstructing the logical and methodological challenges which Parmenides' poem posed for natural philosophy, this chapter hopefully also changes our outlook on Parmenides. His austere philosophy appears to be the result of an attempt to meet the criteria for philosophy he has established while operating with the limited logical operators at his disposal. His account of Being and non-Being is not a starting point, as we will see, but rather a logical consequence of his logical operators and his specific interpretation of the criteria for rational enquiry.

Accordingly, Parmenides does not worry about the senses and their potential unreliability to the same extent as his successors when they seek to reintegrate the phenomenal world into the realm of philosophy. Rather, he worries about the logical structure of what can be thought, which excludes, among other things, what can be perceived. Allowing a little exaggeration, we could say that he is worrying not about the world of the senses (the conditions of perception), but rather about the world of thought (the conditions of thinking).[8]

Prima facie my interpretation may sound in some ways similar to the so-called 'logical tradition' in the interpretation of Parmenides. This line of interpretation[9] sees Parmenides' poem driven by one specific logical consideration, namely, by the problem of negative existentials: how can we talk about something that does not exist?[10] Parmenides is seen as claiming that we cannot talk about what does not exist, which explains his statement in fr. 2 that non-Being cannot be thought or talked about – whenever we speak or think, we speak or think about something, and so our thoughts and words need a

with basic concepts and notions, like that of motion, which what we understand as a science distinct from philosophy would just take for granted.

7 And these criteria survive in actual fragments; they are not solely, like Anaximander's hint at a principle of sufficient reason, to be found in testimonia.

8 We may find it difficult to imagine how someone could get to this idea of thought independent of the senses without extensive worrying about the reliability of the senses, but all we find in Parmenides in this respect is his comparatively brief criticism of the senses in frr. 6 and 7.

9 It started with Russell 1945 and was further developed by Owen 1960 and Barnes 1982.

10 Russell 1945, p. 50: "if a word can be used significantly it must mean *something*, not nothing, and therefore what the word means must in some sense exist". Cf. Owen 1960, p. 57.

referent. Accordingly, what can be talked or thought about needs to exist. For Parmenides, Being – the subject of the way of *alêtheia* – is "whatever can be thought of or spoken of",[11] or, in Barnes's version, "whatever we inquire into".

While I agree that logical concerns have to be taken into account if we are to understand Parmenides, in my opinion these concerns do not focus on negative existential statements.[12] My understanding of logical concerns here is more comprehensive than that of the logical tradition – for interpreting Parmenides I take as important his *criteria* for what can be thought of and the *logical operators* (such as, for example, negation) that are employed for giving an account of what is.[13]

In order to reconstruct Parmenides' challenge, in this chapter I first show that he establishes clear criteria for rigorous philosophical inquiry and then analyse the logical operators with which Parmenides works. These criteria and the logical operators are systematically tied to each other in such a way that, as becomes clear in the next step, Parmenides' monism and his exclusion of natural philosophy follow naturally. The final section spells out in detail the challenges natural philosophy thus faces.

2.2 Parmenides' Criteria for Philosophy and His Logical Apparatus

2.2.1 Criteria for Philosophy

The goddess of Parmenides' poem tells the mortal whom she has received right at the beginning that she will reveal to him the well-rounded truth or reality (*alêtheia*, fr. 1, line 29) as well as the opinion of mortals (*doxa*) in which there is no true trustworthiness (fr. 1, line 30). Establishing trustworthy knowledge, in contrast to mere opinion, can be seen as a major aim, perhaps even *the* major aim, of Parmenides' poem.[14] Such an enterprise requires determining how reliable knowledge can be differentiated from mere belief, and thus criteria for knowledge are needed. The three criteria we find in Parmenides' poem for differentiating opinion from knowledge are consistency, the principle of sufficient reason, and what I call 'rational admissibility'.

In the current section, I briefly reconstruct the three criteria as introduced by Parmenides in his poem. Let us start with rational admissibility.

11 Russell 1945, p. 49.
12 The so-called logical tradition also allows for Parmenides to be a numerical pluralist, while my reconstruction will show him to be a monist. Below I will show the differences in the assessment of the *doxa* part.
13 In contrast to some 'anti-logical' sentiments in the literature, I think that the logical basis established by Parmenides is indeed an enormous achievement, which does not require him also to be concerned with empirical truth, as Kahn dismissively claims, for instance, when stating that Parmenides could not be concerned with "what can be thought in some pseudological sense of the 'thinkable' as what can be conceived coherently or without contradiction". Kahn 1969, p. 712.
14 See also Kahn 1969, p. 704.

2.2.1.1 Rational Admissibility

As we saw in the first chapter, 'rational admissibility' is understood as giving an account of some *x* that is based on what we would call rational analysis and can thus withstand rational scrutiny. As a result, we should get an understanding of *x* that can be shared by all rational beings and that allows for comparing *x* to other objects of investigation. This account makes us independent of authority, chance, and individual experience with respect to knowledge of *x*. This idea of rational admissibility is what Parmenides introduces in fr. 7:

The goddess has started her account of knowledge by laying out two ways of investigation: "that [it] is and that [it] is impossible that [it] is not" [fr. 2, line 3] and "that [it] is not and that it is necessary that [it] is not" [fr. 2, line 5]. After the goddess has argued that only one of these two ways – the way that it is, the way of Being – can be investigated and that non-Being cannot be investigated, she invites the mortal to

> κρῖναι δὲ λόγωι πολύδηριν ἔλεγχον ἐξ ἐμέθεν ῥηθέντα
>
> examine with *logos* the much-contending *elenchos* spoken by me (fr. 7, lines 5–6)

At first, it seems puzzling that the goddess reveals the truth but then asks the mortal to whom she reveals it to examine (*krinai*) on his own what she has said. If a divinity reveals the truth, why would we still need to examine it (and not simply proclaim it, as Homer and Hesiod seem to do)?[15] Or, if we are to examine it with our own reason, why then let it be presented by a divinity and not, as the Ionian thinkers seem to have done, leave out the reference to divinities? Why do we get both with Parmenides – divine revelation and human inquiry?[16] One fruitful way to read this combination (a way reflected at the beginning of the cosmology in Plato's *Timaeus*) is to look at its upshot: in the end the divine revelation can also be examined by human beings on their own. Hence, divine reasoning and human reasoning are akin, at least to a certain degree.[17]

In the philosophical tradition before Parmenides, the Ionians focused on human knowledge, whereas Xenophanes assumes that only divinities (or the one God) can have true knowledge.[18] Parmenides now shows that divine and

15 Cf. Hesiod *Theogony* lines 23 ff.; West's commentary on line 32 on divine revelation; Homer *Iliad* II, 484 ff.

16 We may have one precedent (or perhaps contemporary occurrence) for this combination in Heraclitus' *Logos* (frr. 1 and 50), which on the one hand seem to be Heraclitus' own *logos*, and on the other is described as something that is clearly beyond him.

17 Cf. also Lesher 2008, p. 476: "what more fitting way to signal the transition from a god-centered to a human reason-centered understanding of the route to knowledge than to have a goddess declare that what can be known about the nature of what is what can be established through the use of rational argument".

18 According to a widespread interpretation; against it, however, Lesher 2008.

human knowledge can be brought together in such a way that human beings do not simply have to believe what the gods reveal, but can genuinely know it.[19] We will see, however, that the realm of knowledge which we share with the divinities, while secure, is rather small.

In Parmenides' poem, the goddess is not merely an authority who lends *gravitas* to what is presented. Rather, she encourages us not simply to believe what she has said, but to examine it ourselves. Accordingly, Parmenides conceives of our acquisition of true knowledge as a rejection of mere reliance on authority and as an examination on our own, human, part. But what, then, does he understand by such examination?

I have argued elsewhere[20] that the goddess' invitation to examine with *logos* the much-contending *elenchos* spoken by her should be understood as an invitation to form a well-founded judgement[21] about her refutation of the possibility of non-Being with the help of our reasoning. This examination is contrasted with the mere naming by the mortals.[22] The objects of the goddess's *elenchos* are the *mê eonta*, the non-Beings, mentioned at the beginning of the fragment. This is clear from the sentences immediately preceding the passage quoted, where we learn that the *mê eonta* constitute the path from which Parmenides should hold back his thinking. Why should he hold back from the path of non-Being? The possibility of investigating non-Being is refuted in frr. 2 and 3,[23] where non-Being is excluded as a possible object of investigation because it cannot be thought, which we will see to mean that it is not consistently conceivable. This refutation of the assumption of non-Being is then summed up in the first line of fr. 7 – "For never will this be forced that non-Being is".

Considering that what is to be judged is a refutation and that we get a clear contrast between *logos* (fr. 7, line 5) and the senses (lines 4–5), it seems that *logos* is best translated as some form of reason, rationality, or rational faculty[24]

19 Cf. also Kahn 1969, p. 706.

20 Sattler 2011.

21 *Krinein*, understood as distinguishing, interpreting, and examining, is the basis for such a judgement. See Bryan 2011, pp. 86–93 for the forensic undertones of *krinein* and *elenchos*.

22 See fr. 8, line 53 (mortals name two forms); fr. 9, lines 1–3 (they name all things night and light); fr. 19, line 3 (they name the things that have come into being and will pass away).

23 Verdenius 1942, p. 64 understands the aorist *rêthenta* in fr. 7, line 6 as making it clear that the *elenchos* has already been given before; similarly Tarán, p. 81. James Lesher (in email correspondence) questions whether the aorist participle unambiguously indicates a state or event that has already taken place; he wants *rêthenta* to mean just 'spoken', and not specifically 'has been spoken'. However, the action of the aorist participle in general is antecedent to that in the main verb (see Smyth 1956, section 1872). So while the aorist participle need not necessarily refer to a state or event that has already taken place, it seems to be most natural to read it like this here, as looking back at a past action.

24 This is also how most of Parmenides' translators take it. See, for example, Diels-Kranz; Mansfeld 1964; Barnes 1982; Kirk, Raven, and Schofield 1983; Cordero; Conche. Lesher 2008 and Coxon understand it as 'discourse', but Coxon describes it in his commentary as

that excludes sense perception. We are invited by the goddess to examine, to verify for ourselves with the help of reasoning the refutation spoken by her – just as a rationally demonstrable statement should allow us to do. We should be dependent neither simply on the authority of the goddess nor on chance or other human beings. Rather, *logos* on its own should allow us to test whether something somebody has said can hold. Thus, we are independent of any individual experience which others, or indeed we ourselves, happen to have. Such a judgement includes, as the goddess points out in fr. 7, not being forced by habit (*ethos*, line 3) to follow the wrong path of thinking, but only being forced by reasoning, which will lead us the right way. Accordingly, the goddess asks us to agree or disagree with a theory or statement not because of an authority on which we base our theory or fail to base it, but rather because we prove with our own reason what was stated. Use of our reason is contrasted with both using our sense perception and simply relying on others, be it the goddess or other human beings.

I talk of 'we' here because in principle other human beings, in addition to Parmenides,[25] should be capable of using their reason in the same way, even if at the moment they are still confused crowds (*akrita phyla*, fr. 6, line 7). Otherwise, why would the goddess ask Parmenides explicitly to keep what he has heard from her (fr. 2, line 1), which we can see his poem doing, if it is not preserved so as to be available for others? Furthermore, if we were meant to accept the poem as a revelation, why is it presented as a revelation of the goddess that is then tested by Parmenides?[26] If we 'other human beings' use our *logos*, we should be able to judge for ourselves in such a way that we can understand a thing or a state of affairs the same way as somebody else, provided we are also using our reason. Intersubjective agreement – based not on some arbitrary consent we have reached but on our ability to judge something by reason – should thus be a consequence of rational admissibility.

Summing up this analysis, we see that rational admissibility as introduced by Parmenides has three aspects: an account of x is rationally admissible if and only if (1) the standard for judgement is reason – it is by using *logos* that we can test whether and why a statement holds true; (2) there is no authority for the judgement beyond reason – reason trumps any other authority, including mere divine revelation; and (3) reason is a standard that can be generalised – human

'reason' and Lesher thinks that Parmenides' understanding of *logos* here "contributed to the emergence of an understanding of the term as referring to our capacity to reason". For Parmenides, perception does not lead to any knowledge, only rational thought will do so. This is supported by fr. 4, where the goddess asks us to consider the *apeonta* with *nous*. As absent things are obviously things which cannot be perceived but only thought of (or imagined), what we are concerned with is nothing perceivable.

25 I am talking about Parmenides as the receiver of the goddess's revelation, since for the philosophical purposes here it does not matter who exactly we should understand to be the person enlightened by her.

26 Which suggests that we should also test what Parmenides tells us.

and divine beings share the same *logos*, and a thorough examinations of a claim by different people should lead to the same result.[27]

Parmenides is asked not simply to believe but rather to examine what the goddess has said, and through his writing, the same is requested of us. We are meant to judge the main claims of his poem – his philosophy – with the help of our rational capability. Rational admissibility is thus established as a criterion for what can count as reliable knowledge, for philosophy.

For Parmenides, using reason as a standard is closely linked to consistency, as well as to giving sufficient explanations – what we would call a principle of sufficient reason. Let us look at consistency first.

2.2.1.2 Consistency

Consistency can be understood as satisfying the principle of non-contradiction. That consistency is necessary according to Parmenides is clear already from the goddess's refutation that we are meant to judge, as that refutation relies on a principle of non-contradiction:

> εἰ δ' ἄγ' ἐγὼν ἐρέω, κόμισαι δὲ σὺ μῦθον ἀκούσας,
> αἵπερ ὁδοὶ μοῦναι διζήσιός εἰσι νοῆσαι·
> ἡ μὲν ὅπως ἔστιν τε καὶ ὡς οὐκ ἔστι μὴ εἶναι,
> Πειθοῦς ἐστι κέλευθος· Ἀληθείηι γὰρ ὀπηδεῖ,
> ἡ δ' ὡς οὐκ ἔστιν τε καὶ ὡς χρεών ἐστι μὴ εἶναι,
> τὴν δή τοι φράζω παναπευθέα ἔμμεν ἀταρπόν·
> οὔτε γὰρ ἂν γνοίης τό γε μὴ ἐὸν (οὐ γὰρ ἀνυστόν)
> οὔτε φράσαις.
> τὸ γὰρ αὐτὸ νοεῖν ἐστίν τε καὶ εἶναι.
>
> Come now, I will tell you – and you preserve the word that you have heard – which ways of enquiry are there only to be thought: the one [way of enquiry], that [it] is and that it is not possible not to be; this is the way of persuasion (for it follows truth).
>
> The other [way of enquiry], that [it] is not and that it is necessary that [it] is not. This I will show you to be an altogether unknowable trail; for neither could you think of non-Being – that cannot be done – nor say anything about it. (fr. 2)
>
> for what is for thinking and for being is the same. (fr. 3)[28]

The second half of fr. 2, lines 5–8, and fr. 3 together can be understood as a conclusion that argues that non-Being cannot be known, thought of, or talked

[27] If an account is rationally admissible then, arguably, the thing it accounts for is rationally admissible.

[28] Diels and many others understand fr. 3 as immediately succeeding fr. 2 and I follow them. There is an extensive debate about how to translate especially fr. 3, with which I cannot engage here. I will give reasons for my own understanding of this line later on in this chapter.

about since everything that can be conceived *is*;[29] so it is necessary that I always conceive of Being. This refutation of the possibility of any knowledge of non-Being rests on the following premises:

(1) What is for thinking and what is for being are the same (which for the time being we can understand as 'that which is, Being, is the same as that which can be thought').

(2) Being and non-Being are contradictory, so only one of them can hold true – this becomes clear from the fact that there are only two ways or routes of inquiry, "that [it] is and that it is not possible not to be" and "that [it] is not and that it is necessary that [it] is not"; the first is later translated into the way of Being (fr. 6, line 1), the second into the way of non-Being (fr. 2, line 7). One of the two has to be true, otherwise it could not be inferred in fr. 8, lines 1–2 that the refutation of the path of non-Being has shown that the path of Being is indeed left.[30] And fr. 8, line 16 makes the contradictory relation explicit by claiming that either "[it] is or [it] is not".

Conclusion: non-Being cannot be thought.

[29] We will see below that this 'is' is not to be understood in the way the logical tradition has taken it, namely as 'exists'.

[30] Things are somewhat complicated by the fact that in fr. 6 Parmenides seems to introduce a third path over and above the only two conceivable ones named in fr. 2, the path of the double-headed who take Being and non-Being to be the same and not the same. Hence, we get two different distinctions: in frr. 1 and 6 we seem to get a contrast between what truly is (or the way of what is) and the unreliable path of mortals. And in fr. 2 we get the distinction between the way of inquiry that it is and must be, and the way of what is not and cannot be. There seem to be three ways to interpret the relation between these two sets of distinctions:

(1) There are two ways of inquiry, the path of Being and of non-Being, and the latter one includes the way of mortals and the cosmology. This interpretation has been supported by a change dealing with the lacuna in fr. 6, line 3 so that the goddess is no longer understood as proscribing two ways of inquiry but rather as pointing out two ways the inquiry has to take, i.e., the way that it is and the way that it is not (see Nehamas 1981 and Cordero 1979).

(2) There are three ways of inquiry, the paths of Being, of non-Being, and of Being and non-Being, and it is on the third path that the way of mortals and the cosmology can be found (for example, Reinhardt 1916 and Palmer 2009).

(3) There are two ways of inquiry, the path of Being and of non-Being, but mortals do not even understand that these are the only two conceivable ways, and thus mix up the two paths so that we *seem* to have a third path.

The last interpretation is the one which I think gives us the best reading of the text; it is close to that proposed by O'Brien. Palmer 2009, p. 69, n. 56 raises two objections to O'Brien's reading: first, a combination of the first two ways is still another conceivable way to go and, second, the goddess herself refers to this path as a *hodos dizêsios*. However, this confused third path is not conceivable according to the logical framework belonging to the path of truth, as we will see below, but only according to the mistaken framework of mortals. Palmer himself points out that only the first two ways are introduced by the goddess as ways for *noêsei*, for understanding, but not the allegedly third path in fr. 6 (see

In order to draw the conclusion from the two premises above, however, we see that a third, implicit premise needs to be supplied:

(3) A principle of non-contradiction holds.

Being as that which can be thought is contradictory to non-Being, and a principle of non-contradiction is at work. Non-Being has to be excluded as a possible object of investigation, because it cannot be thought; and we will see that it is unthinkable in the strict sense that it cannot be conceived of consistently. While we may not wish to go as far as to read fr. 2 as an explicit formulation of this principle, it is indubitable that Parmenides uses consistency as a crucial criterion to investigate Being.

We saw in the first chapter that consistency can be required on different levels: (1) for the content of a concept (our account of something), (2) for our usage of a concept, or (3) for the connection of different concepts. It seems Parmenides' philosophy implicitly attempts to satisfy all three understandings of consistency: (1) when he determines Being as the object of his enquiry in fr. 8, he attempts to ensure that his specific concept of it, its content, is consistent. Each *sêma* of Being has to be consistent in itself for otherwise the concept of Being would not be consistent. Lines 6–49 of fr. 8 show that no description of Being leading to a contradiction can be true, and that thus the opposite description has to be true. For example, Parmenides claims that Being cannot come into being, as it would have to come into being either out of what already is or out of what is not. Since both avenues lead into a contradiction (the contradiction that Being is not), we have to make the opposite assumption, that Being cannot come into being. Furthermore, all the different *sêmata* of Being have to be consistent with each other so that all can hold jointly of Being. This meaning of consistency is at play also in frr. 2 and 3, when the route "that it is not" is excluded from inquiry because it leads to inconsistencies – that it is not "cannot be thought [*gnoiês*] nor pointed out" (fr. 2, lines 7–8).[31]

(2) For Parmenides to be successful in this investigation, he also has to ensure that he is not using 'Being' first in one sense and then in another – thus

Palmer 2009, p. 73). This suggests, I think, that we should take 'way' when it is applied to the opinion of mortals in fr. 6 as being in inverted commas – "what mortals would see as a 'way'", but what is not there as a way for understanding, since it is inconsistent and thus not a possibility (similarly Palmer 2009, pp. 65, 104). That there are only two paths for understanding is also clear from the fact that the recapitulation in fr. 8, lines 15–18 mentions only two paths. Thus, we can say that Parmenides learns two different things from the goddess: (1) that there are two ways of enquiry; (2) the confused idea of mortals, who mix up the paths. Cf. also Cordero 2004.

[31] In principle, something may also be unthinkable, because it is too terrible to be grasped, unlawful, or has never been heard of; but given the role which consistency plays for Parmenides and what we will learn about the understanding of non-Being from Parmenides' understanding of negation, non-Being here seems to be unthinkable because it is inconsistent. This is also how Plato understands it, as we will see in Chapter 5.

ensuring consistency of the second kind mentioned above. (3) Finally, the third kind of consistency is presupposed by fr. 5, where the goddess claims that from where she begins is a common point (*xunon*) for her as she will come back to it.[32] Such systematic connection implies that the meaning of each fundamental concept is only fully determined and kept stable with respect to the other concepts, and any change in one concept would necessitate changes in the other concepts. We will see below that Parmenides' account of Being cannot be understood without understanding his other basic notions. Parmenides' description of truth as well-rounded (*eukuklos*; fr. 1, line 29) may also refer to this third kind of consistency – a systematic connection of concepts can be seen metaphorically as well-rounded, in the sense that there is a direct connection between all elements: we can start from any concept and get the shape of the whole out of it. Meeting this third kind of consistency connects with satisfying the principle of sufficient reason: the basic notions that are connected in such a way that they satisfy the demand for the third kind of consistency are 'mutually implicatory' in that the sufficient reason for assuming each one of them lies in the assumption of all the others.

Although Parmenides' poem is the first place where consistency and thus a form of the principle of non-contradiction are methodologically employed,[33] he seems to be surprisingly well aware of this principle. But the particular version to which he ascribes is more austere than we are used to after the developments it underwent with Parmenides' successors. By the principle of non-contradiction we normally understand something like 'not (p and not-p) at the same time or in the same respect'. Parmenides, however, understands the principle of non-contradiction as 'not (F and not-F) simpliciter',[34] without allowing that something may be F and not-F at different times, or in different respects. For him, the principle of non-contradiction has to be understood in such a way that it is co-joint with the principle of excluded middle: 'not (x is F and x is not-F) regardless of respect, and x has to be either F or not-F'.

A crucial reason for this austere understanding of the principle of non-contradiction is Parmenides' understanding of negation, which we will spell

32 See also Reinhardt 1916, p. 60, who thinks that coming back to the starting point testifies to a "Bewußtsein der Systemzusammenhänge". Mourelatos 1970, p. 3 even claims that "relations of logical dependence are at times overstressed". Whether or not *over*stressed, it seems clear that Parmenides wanted to emphasis the intimate logical connections of his basic thoughts.

33 See Kahn 1969, p. 707. While it is clear that Parmenides subscribes to this criterion, he does not state it explicitly, as we find Plato and Aristotle doing.

34 Parmenides does not yet seem to distinguish between respects and properties – so being F at one time is just a way of being F and excludes being not-F. Strictly speaking, characterising x in terms of F is already problematic within a Parmenidean framework, as we will see below.

out below. But let us first look at the last of the three important criteria for philosophy in Parmenides, the principle of sufficient reason.

2.2.1.3 Principle of Sufficient Reason

Like his predecessor Anaximander, Parmenides uses the principle of sufficient reason in a negative version. More precisely, in its most prominent usage in the poem he raises it as a question: "What need would have made it grow, coming from non-Being, later or sooner?" (fr. 8, lines 9–10). Parmenides raises the question of what need (*chreos*) could have made Being come into Being from non-Being later or sooner. *Chreos* can also mean, more generally, 'matter' or 'affair'; we can understand it here as the reason or explanation why Being comes into being sooner rather than later. Parmenides' question expects the answer 'no sufficient reason can be given for why it grows later or sooner', since this question is part of the deduction to show that Being cannot have come into being. If there is no sufficient reason for Being to come into being from non-Being sooner or later, then, the argument claims, there is no sufficient reason for generation to occur at any particular time. Yet it would have to occur at a particular time in order to occur at all and so Being has not been generated from non-Being. Moreover, at least on a widespread reading of the poem, Parmenides also points out that there is no sufficient reason for Being to come into being from Being, since such a situation would mean that there is already Being and thus something beyond Being, which is already there in the first place, would come into being.[35]

Since the only possible alternatives for generation to occur – coming into being from Being (what is) and coming into being from non-Being (what is not) – are shown to be inconsistent, there cannot be a sufficient reason for generation. Parmenides shows that we would need to give a sufficient reason for generation in two respects: first, we would have to answer sufficiently whether what came into being came into being from what is or from what is not (and why it came into being from the one rather than the other). Secondly, we would have to answer why what came into being came into being now and not at some other time.

In addition, Parmenides employs a principle of sufficient reason for his move from homogeneity to indivisibility in fr. 8, lines 22 ff., which present the following argument:

(1) If there are no internal differences, there is no reason for Being to be *diaireton* (divided/divisible);[36]

[35] See, for example, Tarán 1965.

[36] The Greek word *diaireton* can mean divided as well as divisible – a problem we will come back to in chapters 3 and 7. Parmenides' Being does not just happen not to be divided, but cannot be divided, given that it is completely homogenous. Thus, it seems better to understand the argument as attempting to show that Being is not divisible, rather than that it is not divided.

(2) Being is completely homogeneous, that is, there are no internal differences;
(3) Conclusion: Being is thus not *diaireton*, which for Parmenides means it is indivisible.

The second premise, that Being is homogenous, is argued for in fr. 8, lines 42 ff., by pointing out that neither of the two reasons that could prevent Being from being completely homogenous obtains – neither is there non-Being that would prevent this, nor is Being more here or less there.[37] While the first occurrence of the principle of sufficient reason deals with a process (there is no generation, since there is no sufficient reason for it), this second occurrence is tied to the question of what something is like (Being is not divided, since there is no sufficient reason for it). In both cases, Parmenides seems to make a metaphysical claim – he infers that no generation occurs and that Being is not divided – and not only the epistemological claim that it is not reasonable for us to believe that generation occurs or that Being is divided.

We have seen that it is essential for knowledge to be consistent, rationally admissible, and not lacking sufficient reason. Opinion does not satisfy any of these criteria for Parmenides. Fr. 6, lines 4–9 shows those who believe to lack knowledge and belief to be inconsistent – the belief of the double-headed mortals claims Being and non-Being to be the same and not the same, going against the requirement of consistency. In fr. 8, lines 40–1 we see the mortals simply positing different kinds of change, and in lines 53–4 merely naming basic principles: "they have decided to name two forms, of which it is not allowed to name only one – that's where they have gone astray". They *have decided* to *name* and *posit* principles which turn out to be not true – obviously, unlike the goddess, the mortals do not bother to give an account that is based on rational analysis and that would give sufficient reasons for what they posit. Their belief can be proven wrong solely by rational scrutiny: the goddess does not refer to any empirical state of the matter to prove them wrong, but instead shows by mere rational inquiry that their assumptions cannot hold. Hence, their belief is not rationally admissible.

Let us now move on to Parmenides' logical operators.

2.2.2 Logical Operators

We saw in the first chapter that, for the purposes of this book, a logical operator is understood as a logical tool that is applied to an argument (an input) to yield an output that is related to the original argument in a systematic way. This section will give a brief outline of Parmenides' main operators that are of philosophical importance, namely his separation operator (tied to a specific form of negation) and his connection operator. I do not seek to imply that Parmenides had a theory

[37] For a more detailed discussion of this argument, see Chapter 7.

introducing such operators, but rather that these operators, which we will have to reconstruct from his fragments, are used in Parmenides' poem in a way that is consistent and philosophically significant. The reconstruction of these operators will allow us to understand essential features of his view,[38] for Parmenides' use of negation has a strong impact on his ontology.[39]

2.2.2.1 Parmenides' Separation Operator: Negation in Parmenides

Fr. 2 seems to introduce "that [it] is" and "that [it] is not" as basic notions. Since the first 'path', "that [it] is", investigates *eon*,[40] the second path, that (it) is not, may be understood as investigating *mê/ouk eon*. Thus, it may seem that Parmenides treats *mê/ouk eon* as a second basic notion besides *eon*. However, "that [it] is not" and *mê eon* are excluded from rational investigation as non-intelligible,[41] and therefore non-Being cannot, after all, be a basic notion for Parmenides. Nevertheless, he implicitly uses negation, as expressed by *ou*, *mê*, and the alpha privative, as a crucial notion. Negation is crucial for the right account of Being since almost all *sêmata* or marks of Being are at some point expressed in negative form (for example, *agenêton*, or *oude diaireton*, which takes up *oulon*).[42] And the reasons that Parmenides offers for why these are the *sêmata* of Being are often themselves put in negative form; for example, Being is ungenerated (*agenêton*)

38 To make it clearer why I think looking at his operators will be philosophically enlightening, consider the following example: I may explicitly articulate a particular theory T, and I may implicitly always understand x as x^*, when x is used in my theory. Then, depending on what x is, it may be important for you to realise that by x I understand x^* to understand my theory T, even though I do not have an explicit theory about how to understand x.

39 Of course we could look at the usage of negation (at cases of grammatical negatives) and at what people mean when using negation in all kinds of texts, even in cookbooks. But while the writer of a cookbook might have an interesting way of using negation as well, it will (usually) not influence his ontology – for example, it will not influence his assumptions on what kinds of ingredients there are in the world. By contrast, we will see that looking at Parmenides' usage of negation will help us to say something about his ontology.

40 There is a debate in the secondary literature on the subject of "that [it] is". The logical tradition of interpreting Parmenides suggests 'whatever can be thought and talked about', against which several scholars have objected that this is not a subject a reader of the poem could know at this early point. I am following Cordero and others in assuming that the safest subject here is simply Being, *eon*. *Eon* is explicitly only used in fr. 6, which takes up the results from fr. 2, but it seems to be a subject that can be easily supplied by a reader (if only as some indeterminate thing), in contrast to 'whatever can be thought and talked about'. Owen objected to understanding Being as the subject of fr. 2, pointing out that this risks a tautology for the first way and a contradiction for the second. However, risking a tautology seems better than presupposing too much here; and the second way is excluded at the end of fr. 2 as not thinkable. Furthermore, even Owen agrees that the subject of investigation in what follows in the poem is *to eon*.

41 It cannot be thought or said, fr. 2, line 7.

42 That the negation operator is used as a central way to prove the *sêmata* was noted already by Fränkel 1962 p. 402, n.12 and Austin 2002, p. 96.

because all ways of understanding it as generated turn out to be inconsistent;[43] as we would express it: Being is not-F because it cannot be F.[44]

As mentioned in the first chapter, for the purposes of this book, it is best to understand negation from a logical perspective and in a broader sense than is often the case today – that is, we should not restrict it to propositions. The relation between the result of a negation and the original input forms an opposition that is either contradictory (exclusive and exhaustive; one of the two has to be true, the other false) or contrary (exclusive but not exhaustive; both cannot be true together, but both can be false). In a contrary opposition that allows for a *tertium*, a negation can be either extreme or moderate. An extreme negation produces the polar opposite of the positive (negating 'S is white' indicates that S is black), while a moderate negation produces something that is simply different from what is negated (negating 'S is white' indicates that S is a colour other than white).

If it is specified over which realm the negation ranges, the negation produces something that is (relatively) well-determined (if we talk about the realm of colour, my claim that '*x* is not white' tells us that *x* possesses some colour that is not the colour white). But if the realm is not determined, the negation will produce something indefinite, since we know of the result only that, out of all possible things in the universe, it is not the positive argument (*x* may be a number and thus not possess any colour at all).

With this reminder in mind, let us now look at the passages involving negations in Parmenides' poem. We will first look at the linguistic occurrences of negation, but only in order to find out how negation works as a logical operator for Parmenides.

2.2.2.1.1 Evidence of Negation in Parmenides Negation occurs more than forty times in Parmenides' poem. Most of these negations are in the *alêtheia* part, on which I will concentrate. What is negated is mainly terms and modalities, but in fr. 2, Parmenides seems to negate propositions when he claims "that it is not that [it] is not" (*hôs ouk esti mê einai*) in line 3, and "that [it] is not" (*hôs ouk estin*) in line 5. There are good reasons, however, for

[43] Fr. 8, lines 6–21.

[44] Apart from the above structure, we find the following forms of a negative proof structure:

1) *Eon* is neither G nor H because it is F (for example, neither was it nor will it be, since it is now, fr. 8). Here we are dealing with three elements, F, G, H, and *eon* is neither of the latter two because it is F (this argumentative structure is still consistent with Parmenides' commitment to the principle of excluded middle on the way of truth, since it just reiterates this principle: *eon* is either F or not-F, and if it is not-F, then it is either G or H).

2) *Eon* is not F, neither by being G nor by being H (for example, Being is not cut off from Being, neither by being dispersed nor by being contracted, fr. 4). In order to be F, *eon* would have to be either G or H (so G and H make up F), but it is neither G nor H. Cf. also Austin 2002, pp. 96–8.

following the traditional translation of line 3, "that it is impossible that [it] is not".[45] When thus understood, the first negation of this line negates a modal rather than the proposition in the scope of the modal: $\sim\Diamond$ (it is not). And in lines 6–8 Parmenides translates the second negation of line 3 and the claim of line 5, that "[it] is not", into *mê eon*, non-Being, when he writes that the suggestion from line 5 is an altogether unknowable trail "for neither could you think of non-Being [*mê eon*] – that cannot be done – nor say anything about it" (lines 7–8). What seems to be a negation of a predicate or proposition Parmenides takes up as the negation of a term, the result of which – "non-Being" – he claims to be not thinkable and not sayable. In lines 6–8, Parmenides thus claims not that the hypothesis from line 5 is intelligible but false, but rather that it is not even intelligible,[46] that its words are meaningless.[47] It is sayable in the sense that the words can be uttered, but not in the sense that a meaningful thought is thus conveyed. If he had negated a proposition in these lines – as would have been the case had he claimed that 'it is not the case that it is' – we would be dealing with a claim that was false, but one we could talk and think about.

That Parmenides is negating terms rather than propositions is also supported by the fact that the goddess does not make any statements such as, 'It is not the case that this path is in any way knowable' (and it is doubtful whether the goddess could have expressed anything of this kind in normal Greek syntax in Parmenides' time). Rather, she says that she wants to show that this path is utterly unknowable (fr. 2, line 6) or that she wants to hold him back from this path (fr. 6, lines 3–4), or that he shall hold back his thought from this way (fr. 7, line 2).

Apart from the negations referring to some form of *estin*, the most important negations we come across in Parmenides' poem are the explicit negations of modalities in frr. 2 and 8 and the extensive predicate-term negation used to characterise Being in fr. 8. Fr. 2 claims that there are only two possible paths of investigation, the first of which will turn out to be necessary, the second impossible.[48] In his attempt to show that the second is impossible,

[45] A modal translation of *ouk esti* is suggested by the fact that the statement in question is part of a *men* clause that is answered by the explicitly modal *chreôn* claim in line 5 as its *de* clause (I owe this point to Verity Harte). This is also how most translations render *ouk esti* here. And we will see in the course of the investigation of Parmenides' notion of Being that this modal interpretation is indeed fitting.

[46] Cf. also Palmer 2009, p. 95.

[47] One may wonder how we get this all from negating a term, given that, for example, 'not-red' is not meaningless. There are two crucial steps here: first, as we will see below, Parmenides is negating absolutely, so we get 'not being F in any sense'. Secondly, Parmenides is not negating any old F here, but Being. We will see that by "Being" he understands what can consistently be thought, so non-Being is what cannot be thought of consistently at all.

[48] In fr. 8, line 16 we seem to get a kind of second-order necessity, the necessity to abandon the one path and take the other, necessary one.

Parmenides negates the *possibility* of non-Being in line 3 ("it is not possible not to be"). And in lines 6–8 he negates the assumption of non-Being and its *necessity* from line 5. The result is in both cases clearly determined – impossibility.[49] So we can understand the relation between the original input and the result of the negation as contradictory in one case (possibility and impossibility) and contrary in the other (necessity and impossibility, not everything is either necessary or possible; for example, if it is contingent, it is neither).[50]

We also find an indirect negation of modality with the negation of some of the terms used to characterise Being in fr. 8. Some *sêmata* are verbal adjectives (ending in -*tos*), which can indicate either a possibility or a passive resulting state (having the force of a perfect passive participle).[51] From the context in which they are used[52] and from Parmenides' prominent employment of impossibility in fr. 2, we can infer that the negations of these verbal adjectives are not simply negations of a resulting state but of the *possibility* of whatever is expressed by the verb, the result of which is the impossibility of the original input.

The result of the negations discussed is usually clearly defined or restricted, as is obvious in the case of explicitly negating possibility and necessity. But even in cases that seem, prima facie, to produce an indeterminate or unrestricted result, like *akinêton* in line 26, we see on closer inspection that the result is clearly determined. *Akinêton* might refer to everything in the world apart from something movable,[53] but Parmenides is using the term in referring to an extreme (and hence contrary) opposite, for it is clearly equated with resting (*keitai*; lines 29–30). What that means for the case of non-Being we will see below.

Prima facie, the relation between the original argument and the result of the negation sometimes seems to be what we would call contrary (as we just saw in

[49] The negation of the second path is not, as we might expect, the contingent fact "that [it] is not" (which would understand the negation of necessity as possibly not), but impossibility ("for *neither could* you think of non-Being – that *cannot* be done – nor say anything about it"). See also fr. 8, lines 9–11, where from the lack of any necessity that would have made Being to come into being sooner or later, Parmenides derives that it *cannot* have come into being.

[50] I am not claiming here that Parmenides would allow for the contingent on this level, but only that for us the opposition between necessary and impossible that Parmenides works with would be an example of a contrary opposite, since the contingent would be neither.

[51] Like *akinêton* (line 26) or *oude diaireton* (line 22).

[52] For example, *oude diaireton* seems to have the sense 'being not divisible', not simply 'not divided', since the background assumption seems to be that division is only possible where there are differences and Being does not allow for any difference. An echo of this thought may be found in Plato's *Phaedo* 78c–d, where it is claimed that only what has parts can be split up again, while what is non-composite will not be affected by any change.

[53] Or, as we find it expressed in Aristotle's *Physics*, to that of which motion and rest cannot be sensibly predicated.

the case of *akinêton*, which is equated with resting), and sometimes contradictory (as, for example, in fr. 2, line 3, the relation between the possibility and the impossibility of non-Being – everything is either possible or impossible). However, Parmenides usually employs features from what we would see as contrary as well as contradictory oppositions. For the result is treated as an extreme or complete opposite of the original argument, as what we would think of as a contrary opposite.[54] Yet Parmenides also makes it clear that of the two, the original input and the result of the negation, one must be true, the other false – a feature we would see as belonging to a contradictory opposition.[55]

That Parmenides treats the result of the negation as the complete opposite of the original input, and hence as contrary, can most clearly be seen from fr. 2, lines 5 and 6: claiming that "it is not" (which in line 6 is translated into assuming non-Being) is by necessity a *panapeuthea atarpon*, an altogether or utterly unknowable trail. The *pan* ("altogether") indicates that we are dealing with a complete opposite, an extreme negation: not only is non-Being not necessary – the contradictory opposite to the necessity of non-Being would be that non-Being is possibly not – but this assumption cannot even be thought, and hence is impossible. Also the two occurrences of negation in fr. 1 fit this reading: when the goddess claims in line 26 that "it's not a bad lot" (*outi moira kakê*) that has brought Parmenides to her, the fate is not just different from being bad (as, for instance, a neutral fate would be), but also the polar opposite of bad, not bad at all; it is a good fate – as the goddess herself claims in line 28, it is "right and justice".[56] And in line 30 we hear about the opinion of mortals, "in which there is no true reliability" (*ouk eni pistis alêthês*). Again, the result is understood as the contrary opposite of what is negated – it is not left open whether there is some other reliability that is just not true; rather, mortal opinion is not to be trusted at all.[57]

Although we are dealing with extreme opposites, Parmenides always assumes that one of the two poles has to be true, the other false.[58] His position becomes most obvious in fr. 8, line 11, which claims it to be necessary that Being either is

[54] Contraries and thus complete opposites are tied to a restricted realm, for example, only within the realm of colour can the negation of white be understood without further specification as referring to the complete opposite black.

[55] Only if we understand not-white as potentially referring to everything apart from the colour white can we claim that everything in the world either has to be white or not-white.

[56] *outi moira kakê* may also simply be read as a familiar colloquial understatement. But being received by the goddess is not just 'pretty good' for a mortal; rather, it is presumably the highest form of revelation.

[57] While we may think that excluding a third option here could also be derived from some other form of negation, not only extreme negation, the important point is that this passage does not speak against Parmenides using an extreme negation on the way of truth.

[58] Thus it is clear that Parmenides is not simply working with what Boethius called an immediate contrary pair, i.e., a contrary pair that does not allow for a *tertium*, like odd and even. For both elements of an immediate contrary pair can be false in cases of vacuous

or is not wholly (*pampan*), that is, either it is in every respect or it is not and is in no respect.[59] For us, both poles of the complete opposite 'being in every respect or in no respect' could be false (namely, if something were in some respect but not in others), while the contradictory opposite to being in every respect would include not being in at least one respect. We see how Parmenides thus combines features of what we would call contrary and contradictory opposites.

Parmenides' initial field of inquiry is a group of two elements that are complete opposites, one of which must be true: we start out in fr. 2 with two possible ways: first, that it is and is impossible not to be, and second that it is not and is necessary not to be (necessary Being and non-Being).[60] It turns out that the second must be excluded as not viable, since this way assumes as an object of inquiry 'what is not Being in an absolute sense', and thus what is not at all.[61] But the way of non-Being must be listed initially as a way in fr. 2, since Being and non-Being (in their respective modalities) are the only possibilities we can conceive of in Parmenides' framework.[62]

We may think that Parmenides is actually not combining features of contrariety and contradiction, but rather simply using a contradictory opposite at some times and a contrary one at others. However, his very notion of non-Being shows that he is in fact combining features of both – non-Being is the absolute opposite of what is, of Being (so a contrary opposite) and at the same time one of the two, either Being or non-Being, must hold.[63]

Features of contrariety and contradiction are found again in fr. 8, line 33: "it is not lacking [in anything], for if it were [lacking in anything] it would be

subjects or subjects to which the predicate does not apply; for example, outside the realm of numbers, not everything is either odd or even, but could be neither.

59 I read *pampan* (completely) in connection with *pelenai* (be) as well as *ouchi* (not be), which is supported by the argument in fr. 2, lines 6–8. Kirk, Raven, and Schofield 1983 also read it like this when they translate "it must either be completely or nothing at all"; similarly the translations of Tarán and Coxon.

60 Similarly, when in fr. 8, line 26 he claims *eon* to be *akinêton*, hence negating that it is moved, he equates *akinêton* with resting in lines 29–30, as we saw above, rather than with being unmoved (in the sense in which it is said about something of which motion and rest cannot be sensibly predicated). He takes it for granted that we are dealing with a group of two elements, 'moving' and 'resting', one of which must always be true. That this is so can be seen from fr. 2 and its recapitulation in fr. 8, lines 15–16, where we are told that either it is or it is not, and it is not is not (because it is unintelligible), therefore it is; see also Gallop, p. 7.

61 Parmenides' absolute negation, working with a set of only two elements, also ensures that double negation equals affirmation (thus in fr. 8 he assumes that claiming Being not to be incomplete shows it to be complete). We will see, however, that the second element, non-Being, turns out to be itself not a single element, but rather a plurality.

62 While we cannot conceive non-Being, we can conceive that it is impossible to conceive non-Being.

63 According to fr. 2. We will see that in the *doxa* part he is consistently employing a contrary opposite as allowing for a third element.

lacking in everything".[64] For Parmenides the negation of "not lacking [in anything]" amounts to "lacking in everything" and one of the two must hold. We would usually think that Parmenides must decide: either the negation of "not lacking [in anything]" results in 'lacking in at least one thing (or respect)', if one of the two opposites must necessarily be true (Parmenides seems to assume this, for otherwise his argument in lines 30–3 would not work). Or, if we take the negation of "not lacking in anything" to produce "lacking in everything", Parmenides cannot require that one of the two must be true.[65]

2.2.2.1.2 Parmenides' Understanding of Negation Summing up the evidence we have seen so far, we can say that Parmenides uses an extreme negation as a one-place operator that takes terms and modalities as arguments and produces results that are clearly defined – they are the polar opposites of the original input. The relation between the original argument and the product of negation shows features of both contradictory opposites (one must be true, the other false) and contrary opposites (they are the extreme poles of a certain domain).

What are the immediate consequences of Parmenides' way of understanding negation? Let us spell out first the logical consequences and then the ontological ones. Parmenides' understanding that '*x* either is completely F or it is completely not-F' makes differences of respect (like *x* being F at one time, but being not-F at another) impossible.[66] Accordingly, Parmenides must understand the principle of non-contradiction as 'not (*x* is F and *x* is not-F) regardless of respect';[67] or, in language more fitting for Parmenides, 'not (*x* is F and *x* is not-F) simpliciter'.

Furthermore, Parmenides' employment of negation within the realm of *alêtheia* does not distinguish between what we would call contrariety and contradiction within the realm of *alêtheia*; rather, it employs one kind of negation that shows features of both. His negation does not fit a clear separation between the principle of non-contradiction and the principle of excluded middle; rather, in Parmenides these are conjoint principles – on the way of

[64] For possible readings of this line given the problems of the text handed down to us, see Tarán's commentary, pp. 114–15.

[65] We may think that in the case of a set of just one element, lacking something is the same as lacking everything, for there is just one thing that can be lacked, Being; cf. Tarán p. 119. However, the fact that Parmenides ends up dealing with a set of only one element is due to his very understanding of the negation operator.

[66] Parmenides' extreme negation does not allow for any respects as we saw in fr. 8, line 11; in line 5 he even explicitly excludes the possibility of any temporal respects.

[67] This does not mean that the principle of non-contradiction is determined by Parmenides' logic, but that his logic restricts which interpretation of it is possible. A negation of only one or another aspect of *x* would presuppose a difference in respect and allow for understanding the principle of non-contradiction as 'not (F and not-F) in the same respect'. The logical basis for this possibility we will find in Plato's *Sophist*, where the negation operator is redefined to denote not absolute opposition but rather otherness.

alêtheia the principle of non-contradiction is a commitment that essentially includes the principle of excluded middle: 'not (x is F and x is not-F) simpliciter and x has to be either F or not-F'.

Let us now spell out the ontological consequences. The central role we saw the negation of modality playing indicates that Parmenides is interested not in what happens to be the case factually, but rather in what must be the case and what cannot be the case.[68] Accordingly, non-Being seems to be what cannot be and what is in no way: it not merely is not, but also cannot be. The impossibility of non-Being is explicitly stated in fr. 7, line 1, "for never can it be forced that non-Being is", and becomes clear from Parmenides' negation of modals and verbal adjectives.

Being the result of an extreme negation, non-Being is not at all.[69] We are not dealing with a non-Being that also in some sense is, that is just different from Being itself. Rather, non-Being is the polar opposite of Being. Accordingly, non-Being seems to be clearly determined – it is determined as that which is in no way. However, since one of the two, Being or non-Being, must be true, as is claimed in fr. 2, lines 3–5, and what is true is Being qua what is completely, everything that we may characterise as not being completely should also belong to non-Being.

Yet complex elements and differences are explicitly supposed only in the realm of *doxa*, where another kind of negation is employed, one that is not permitted in the realm of true Being. This negation operator wrongly, in Parmenides' eyes, allows for the merging of extreme opposites. *Tantia* (fr. 8, line 55) refers to the basic opposites of light and night, which are blended for mortals.[70] And in fr. 6, a double merging is going on: x and not-x are F and not-F (Being and *ouk einai* are the same and not the same; lines 8–9). In the world of *doxa* there can be what we would call contraries that do not show features of contradictory opposites.

Parmenides' combination of features of contradiction and contrariety on the way of truth also has consequences for a question debated in the literature concerning whether there are two or three ways of inquiry according to

[68] That Parmenides understands non-Being in this modal way seems to be supported also by his tendency to use the negation *mê* when talking about non-Being, and thus the negation particle that in the Greek language normally expresses that something cannot be, may not be, or should not be the case. However, this is only a tendency to support this logical point also linguistically, since Parmenides once also uses *ouk eon* (in fr. 8, line 46), and the use of *mê einai* in fr. 2 could also be due to the fact that in Greek the infinitive commonly takes *mê* instead of *ouk*. In principle, I am sympathetic to a modal interpretation of Parmenides as Palmer 2009 develops it. However, Palmer claims that the modal account is the very *starting point* of Parmenides and non-Being as such is just a shorthand for "non-Being must not be", while I think the impossibility connected with non-Being is a *result* of Parmenides' understanding of negation and Being.

[69] See also Aristotle's claim that for Parmenides something different from Being, like being white, is not just some non-Being, but completely (*holôs*) non-Being: *Physics* 186b6–10.

[70] Cf. also frr. 9 and 12, and testimonium 35.

Parmenides' poem. If the ways of Being and non-Being are contradictory opposites, there can be only two ways; if the ways of Being and non-Being are contrary, there may be three. Fr. 2 gives us the way of Being and the way of non-Being as the only alternatives. These two are exclusive possibilities for the realm of truth. Thus a third way in between those of Being and non-Being cannot be a proper way of enquiry. The realm of mere opinion, however, seems to ask for something in between, something that Parmenides' logical operators of the way of truth cannot allow for. Accordingly, while no third path can be conceived on the way of truth, the demands of opinion for a third way may help explain why Plato felt the need to introduce explicitly what is and is not as a third domain in *Republic* V.[71]

2.2.2.2 Parmenides' Connection Operator

We have seen that Parmenides works with one operator for separation. This separation operator is required to deny wrong assumptions about Being.[72] What is needed in addition is an operator for connection that allows for positive characterisations of Being.[73] We saw in the first chapter that such a connection operator can be what we would call a copula for predication, or an identity operator.

Given his framework, the only kind of connection operator Parmenides can employ is *absolute identity*, the identity of one thing with itself ($x = x$). Only such a connection allows the x on the left-hand side of the equation sign to replace the x on the right-hand side in all contexts and thus avoid differences in any respect. By contrast, *partial identity* or *sameness* as well as *predication* involve a difference between x and y in *at least* one respect. If Parmenides employed partial identity or predication, he would have to state that while x is F, x and F are not completely the same; in at least one respect x would be not-F. But according to the negation operator sketched above, saying that x is not-F (in one respect) is the same as saying that x is not F at all. Hence, Parmenides can employ only absolute identity, rather than have some feature or property ascribed to Being. His framework does not allow for partial identity or predication, since both assume some form of difference between x and F. If Parmenides were absolutely consistent, he could claim only 'Being is Being', just as Plato's late learners acknowledge only that the good is good.[74] Within

[71] 476a ff. For the possibility that Parmenides may mention a third path in fr. 6, see above.

[72] This is stated in a way *we* would describe it. For Parmenides these assumptions can only be uttered but not thought.

[73] Within Parmenides' framework, we cannot clearly distinguish between separation and denial, on the one hand, and connection and affirmation on the other. See the first chapter.

[74] Plato *Sophist* 251b–c; see Chapter 5. In a way this seems to be what Parmenides is doing when he describes the path of inquiry as *hopos estin*, fr. 2, line 3: "that [it] is" seems to be the only possible true statement.

Parmenides' logical framework, the absoluteness of his negation corresponds to the absoluteness of identification, that is, identification with no exception[75] – or, as we may say, in every respect, if Parmenides could allow for respects.

That absolute identity is the only connection operator fitting his framework also explains why Parmenides does not employ any proper predication concerning Being in the *alêtheia* part; all the positive characteristics of Being (with one exception to which we will come back shortly) are called *sêmata*, signs (fr. 8, line 2). These signs hint at how we should think of Being, but they can be neither identified with Being nor simply ascribed to it – which is one reason why they are signs.[76]

Identity is usually thought of as a (particular) kind of relation, '*x* is identical with *y*' or '*x* is identical with *x*'. Parmenides, however, also seems to think of identity as a property complete in itself, as a monadic property, '*x* has the property of being identical' (i.e., '*x* is identical'). Accordingly, identity is employed in two different forms in Parmenides' poem: as a one-place, monadic property[77] and as an identity relation.[78] The former version can be found in fr. 8, lines 29–30, where *tauton* is used to determine *eon* further: "Being as the same and remaining in the same it rests in accordance with itself[79] and remains firmly here." Here we hear that 'Being is the same', not that it is 'the same as . . .'. I understand this line to claim that Being rests according to itself (not following anything else) by remaining the same and in the same condition (i.e., not changing in any way).

Identity as a relation can be found in fr. 3 and its variant in fr. 8, line 34. There Being, *einai*, is identified with thinking, *noein*.[80] Only this positive characteristic of Being is not seen merely as a sign. However, given that we understand identity, *tauton*, as absolute identity, do we not get the uncomfortable consequence that in fr. 3 *einai* and *noein* are absolutely identical?[81]

[75] A simple positive predication will not do since it could not exclude differences between Being and its characteristics or properties. Cf. also Zeno's paradox against plurality in Philoponus *In Phys*. 42.9, DK A 21, which claims (in a rather anachronistic example) that one thing, like Socrates, having many predicates, like pale, philosophic, pot-bellied, and snub-nosed, can thus not truly be one.

[76] There are *sêmata* also in the mortal realm but, as Mourelatos 1970, p. 250 points out, these are *posited* by mortals, while the *sêmata* of Being are something *given* to us. Since the *sêmata* on the way of truth are given by the goddess, they may also carry the connotation of divine signs that the notion of a *sêma* can have in Homer. See Bryan 2011, pp. 82–6.

[77] Alternatively, we could also understand it as a property in the sense that xRx is treated as a unary function taking x to the property Rx, to 'being identical with x'.

[78] If indeed we read *to auto* in this fragment as an operator and not as an operand; cf. below.

[79] I translated *en* as "in" and *kath* as "in accordance with" here (understanding "in accordance with itself" as indicating something like being autonomous, not being constrained by something else), while Hölscher and Barnes 1982 take both *kath* and *en* to mean 'in'.

[80] However, here we seem to have a single referent approached via two senses, that of *einai* and that of *noein*.

[81] While *auto* is often translated as operand ("the same [*to auto*] can be thought and be", e.g., Hölscher), according to the interpretation suggested, it should be understood as operator:

This claim may sound crazy when something can easily be without being thought, and when something might be thought without existing (at least in the reality we ordinarily take ourselves to exist in). But this absolute identification of *einai* and *noein* seems less crazy when we keep in mind that, as we will see below, Parmenides' Being is not an empirical object but rather a logical object. Accordingly, if it is thought, it also exists, since a logical object exists (also)[82] as the content of a thought. It cannot be thought without being in this sense.[83] Fr. 8, line 34, "thinking and the thought that it is are the same", can be read accordingly: thinking something also means thinking this something as existing, at least in the sense that it is a logically consistent structure. If we push this thought a little further, we can suggest that if the logical object in question is nothing other than the consistent content of a thought, that logical object cannot be independent of the thought, for then *it is thought* and hence identical with the thought, and Parmenides can claim *to gar auto noein esti kai einai*.[84]

2.3 Parmenides' Logical Apparatus as Intimately Tied to His Ontology

We have seen that Parmenides' specific understanding of the criteria for philosophy is closely connected to his logical operators. We will now look at the impact of the latter and of his criteria on his ontology.

noein and *einai* are the same (*auto*). Kahn 1969, p. 723 suggests that it is always *nous* or *noein* that is identified (and for him thus reduced to) its object, not the other way round, so that identity in Parmenides is asymmetric.

[82] I supply the parenthetical "also" here as a cautious first step, since one may think that a logical object also exists in some other way than as the content of a thought. However, the subsequent suggestion in this paragraph should show that this parenthesis can indeed be abandoned.

[83] Cf. Frege 1918–19, p. 144: "Wenn das Sein eines Gedankens sein Wahrsein ist, dann ist der Ausdruck 'falscher Gedanke' ebenso widerspruchsvoll wie der Ausdruck 'nichtseiender Gedanke'". Even though Frege eventually denies the antecedent of this statement, since a wrong thought can make sense in a question, in a hypothetical construction, and in a negation, this statement demonstrates the identity we might claim between the Being of a thought and its truth, which seems to be in the background of fr. 3.

[84] It may seem as if the connection operator works in the very same way in the realm of mortals when it is claimed in fr. 8, lines 57–8 that fire is "everywhere identical with itself but not identical with the other". However, this formulation "but not identical with the other" shows that the self-identity of fire, in contrast to that of Being, is claimed from the standpoint of a plurality. And since for mortals Being and non-Being are the same and not the same (fr. 6, lines 8–9), obviously they claim more than the identity $x = x$ to which Parmenides restricts the way of truth. Furthermore, mortals commit a double mistake: not only do they identify the strict opposites Being and non-Being, they also identify and do not identify them.

I will not argue in detail here for how best to understand Parmenides' notion of what is, of Being.[85] But three guidelines drawn from Parmenides' poem will get us quite some way: Being is characterised (1) by the lack of any differences (which rests on his negation and connection operator), (2) by conceivability (which is tied to the criteria of consistency and rational admissibility), and (3) by necessity (only necessity grants sufficient reasons). Let us start with Parmenides' assumption that Being cannot allow for any differences.

If we look in a systematic way at the *sêmata* or signs of Being with which Parmenides provides us in fr. 8, we see that these signs exclude all possible differences – there are neither internal differences[86] nor external differences that would allow for any complexity.[87] And this is exactly what will result from Parmenides' understanding of negation, as we have seen – his 'extreme negation' does not allow for any difference in any respect.

Of special importance for our project is Parmenides' explicit claim that neither temporal nor spatial differences apply to Being, which is *akinêton*, unchanged. At the beginning of fr. 8, we are told that Being "never was, nor will be, since it is now altogether". The claim that what truly is never was nor will be, since it is now altogether, can have two meanings: (1) that it is only now, in the present, and will not be in the future and was not in the past, or (2) that it is in a now that has neither a past nor a future and is thus beyond common temporal extension.[88] The first alternative – which makes Being something of a dayfly – implies that Being has come into being (otherwise it would not be true of it to say that it 'was not') and will pass away (otherwise it would not be true of it to say that it 'will not be'), possibilities that Parmenides strictly denies: the very first *sêma* of Being is that it is not generated and imperishable. With this first possibility excluded, the second remains, which claims that Being is not in time in the sense that there are no temporal differences – no 'was' or 'will be' – that would apply to Being. Thus Being is not stretched out in time, temporal differences cannot be applied to

85 I have done so in Sattler 2011.

86 Being is ungenerated, complete; it is *homoion* and everything that is based on this, being *suneches*, indivisible, etc.

87 It is single/whole and also excludes the possibility of any plurality; see below.

88 Some scholars like Tarán, p. 179 and Palmer 2009, p. 140 ff. have suggested as a third possibility that this passage indicates eternal temporal duration. According to Palmer, we should understand the two rejected possibilities of "neither was it nor will it be" as "What Is was once but is no longer" and as "What Is will be at some time in the future but is not now". However, in order to derive this interpretation of the two possibilities, Palmer requires a rather unnatural reading of the line just quoted, on the translation of which he accordingly elaborates extensively. Moreover, such a reading would still allow for saying that Being was (in the sense of it having existed before) and will be (in the sense of it continuing its existence in the future), and thus would allow for temporal differences, and hence for some form of non-Being. For a more extended discussion of the literature, see Sattler 2019a.

it, since these would endanger the absolute homogeneity of the one Being. Being is what we would call eternal or atemporal, or at least Parmenides seems to be pointing in this direction.[89] By contrast, in fr. 19 we hear about temporal differences in the realm of the mortals, who think that things are now because they have come into being and will pass away in the future; the world of becoming seems to be spread out into was and will be.

As regards space, Parmenides uses the image of a well-rounded sphere for his characterisation of Being, which some scholars have understood as an indication that he thought of Being as spatially extended. However, Parmenides explicitly introduces this sphere as an analogy, and specifically as an analogy that shows the homogeneity and completeness of Being: *like* (*enaligkion*) the bulk of a well-rounded sphere, Being is everywhere complete and homogeneous (fr. 8, line 43). An assumption of spatial differences – which a literal sphere would introduce simply by virtue of its being spatially extended, with one part here and another there – would lead to inconsistencies for Parmenides' philosophy, as it would imply that there is one part that is not the other, which, given Parmenides' notion of negation, means that it is not at all. Accordingly, Parmenides must also exclude spatial differences, and his analogy employing a well-rounded sphere does not speak against him doing so.

Given that it allows for no temporal or spatial difference, indeed for no difference whatsoever, Being cannot be an ordinary physical or sensible thing; this would require internal or external differences, for at the very least it would have to be extended in space or persist over time.[90] And we are not given any perceptible thing on the way of truth; empirical phenomena only come in once we are dealing with the realm of *doxa*. Some scholars have claimed that Parmenides' Being is extended in time and space. But such claims are usually based simply on the assumption that before Plato people allegedly could not conceive of something that is not spatially and temporally extended, usually without any reason given.[91] Furthermore, even scholars who understand Parmenides' Being as extended in time and space cannot ascribe to it any of the temporal or spatial differences that are characteristic of the objects with which natural philosophy deals.[92]

That the object of investigation is not a sensible thing is also supported by the fact that perception does not play any role in the knowledge "that

89 We will find such atemporality explicitly employed in Plato's *Timaeus*.

90 Even if this is not enough to make something a sensible thing, i.e., a thing available for perception, as spatially extended geometrical bodies show.

91 See, for example, Palmer's 2016 statement about Parmenides' Being, "[o]n the assumption, *inevitable at the time*, that it is a spatially extended or physical entity, certain other attributes can also be inferred" (emphasis added), without giving any reason why we should think this assumption to be 'inevitable'.

92 For example, Palmer 2009, p. 322 understands the temporal attributes of Being as eternity and changelessness, and the spatial attribute as complete homogeneity.

[it] is"[93] and that we judge what the goddess has said with our *logos*, not with our *aisthêsis*.[94] Understanding Parmenides' Being not as a sensible thing also squares nicely with the goddess's encouragement to focus on things that "while absent are present to thought [*noôi*]" in fr. 4.

So we know that for Parmenides, Being, what truly is, is not a sensible thing;[95] rather it seems to be solely an object of thought, which explains why in fr. 3, as we saw above, Parmenides can claim Being to be the same as what can be thought.

The second guideline tells us that only Being is conceivable: Parmenides explicitly states that something cannot be known or conceived if it is not.[96] Accordingly, for him, Being is the only thing that can be thought of – conceivability is a criterion for what is. Given the criteria for philosophical investigation we discussed above, what is conceivable has to be at least consistent and rationally admissible. This second guideline shows us that the important distinction between Being and non-Being is not that one exists or occurs in our world and the other does not, but that one is fully intelligible and hence an object of knowledge and the other is not.

But can we not rationally conceive many things, not only Being, even things that do not exist, like unicorns? However, for Parmenides, as we saw, what is, which means what can be rationally grasped, must be without temporal and spatial difference, which is not the case for things that are either sensible or spatially imagined. And given Parmenides' understanding of negation, something's being not fully intelligible means it cannot be thought at all.

Guideline three tells us that Being is necessary – in fr. 2 Parmenides states that the way of truth, the only way that can consistently be thought, amounts to the claim 'that it is' and that 'it is not not to be', that is, it is necessary that it is. This statement is supported by the beginning of fr. 6, where Parmenides claims

[93] This also fits Aristotle's claim in *Metaphysics* A, 5 that for Parmenides what is one in account (*kata ton logon*), while Melissus claims it to be one materially (*kata tên hulên*).

[94] In the first part of the poem, perception occurs only negatively, as the aimless eye, deaf hearing, etc. (see fr. 6, line 7 and fr. 7, lines 4–5); and mortals are deaf and blind, not because they do indeed have troubles with their sense organs but, as the continuation of fr. 6 shows, because they cannot judge, and thus take Being and non-Being to be the same and not the same.

[95] By a sensible, physical, or empirical thing, I understand what we can encounter in the physical world as individual things with our senses. In this sense, e.g., the square root of two is not a sensible thing. For while we might encounter lots of things that, given a certain measurement unit, we might claim to possess the size of the square root of two, the square root itself is not something we might stumble upon in the empirical world. Nevertheless, we can refer to it since it has a certain logical structure and can thus be seen as a part of a logical realm, whether this logical realm is created merely by abstraction or not. It is not necessary for this interpretation that Parmenides would or could make an explicit distinction between the empirical and logical realm. Rather, I want to show that from the way Parmenides argues, what we would call physical objects are not what his object of knowledge refers to.

[96] Fr. 2 and fr. 8, line 34.

that "it is necessary [*chrê*] to say and to think that Being is".[97] Fr. 6 thus also gives us the modal complement to the impossibility of thinking or saying anything about non-Being as stated in fr. 2 ("for neither can you think of [*gnoiês*] non-Being nor say anything about it").

While in fr. 2 Being itself is necessary, in fr. 6 it is saying and thinking that Being is that is necessary. But these two forms of necessity are closely linked, and both are connected to the principle of sufficient reason as employed by Parmenides:[98] In fr. 8 we are told that no sufficient reason can be given for generation since, among other things, generation would imply contingency (the contingency that something comes into being at a particular time rather than at some other time before or after, when it could equally well come into being). Only something that is necessary does not suffer from a lack of sufficient reason. And Parmenides' Being is necessary. But so too are our thinking and speaking about it, because if we think and speak at all (and do not just utter words or remain silent), they must be about what can be consistently thought and talked about, and this is only Parmenides Being. What can we say about this Being? That it is.[99]

Summing up, we can say that Being for Parmenides is what is necessary, completely simple, and the only thing that can be conceived – this alone is what can consistently be thought, is rationally admissible, and for which there is a reason that and how it is. As the polar opposite of Being, non-Being is, accordingly, not necessary, not simple, and thus inconceivable with the help of Parmenides' logical tools. Hence, non-Being is not intelligible – as Parmenides states in fr. 2, non-Being is not thinkable and not sayable (we might utter something, but we would not communicate a consistent thought and cannot point it out). Non-Being as what cannot be thought thus is not, as the so-called 'logical' tradition assumed, something like unicorns or the current king of France, which *are not* in the sense that they do not have a referent in our world.[100] Rather, we might compare Parmenides' non-Being to 'the

[97] On understanding *chrê* as necessity and for Being as necessary, see Palmer 2009, pp. 98–100.

[98] Some form of necessity can also be found in the *doxa* part: in fr. 1 the goddess promises to teach also how that which mortals believe must be (*chrên*) approved and pervades everything; and in fr. 10 we learn that necessity directs and binds the heavens. But in both these cases, necessity is not the logical necessity that we find in the *alêtheia* part, where negating this necessity leads to inconsistencies. In fr. 1, by contrast, we deal with what mortals posit as necessary and the necessity of fr. 10 seems to be close to what we will see Plato employing as necessary causes in the *Timaeus*, that is, as restrictions due to the way the physical world is set up.

[99] Being is the only thing that can consistently be conceived, and it is necessary; thus, it could not be otherwise. And, given Parmenides' criteria and operators, there could not be more than one necessary Being.

[100] Thus, I do not think that Parmenides dealt with the problem of negative existentials, which is how he was understood by Russell and many scholars in the twentieth century.

rational square root of two',[101] which, while utterable in words, is inconsistent and thus not a meaningful thought.[102]

The specific understanding of the criteria for philosophy and of the logical operators available that we find in Parmenides has a strong influence on his metaphysics and epistemology: it allows only for an absolutely simple Being[103] – any form of complexity or plurality would imply difference, which is inconsistent within Parmenides' logical framework. Only such a simple thing guarantees reliable knowledge, as no contradictory statements about it seem to be possible. Thus we see that Parmenides' logical monism – only the one Being can be conceived and hence known – is the logical consequence of his framework: it is not an arbitrary starting point or premise, but rather the necessary result, if Parmenides' criteria for philosophy and his basic logical operators are taken seriously;[104] for then all other positions must be excluded.

The ironic implication of this result is that this sole object that can be talked about seriously according to Parmenides is actually very hard to talk about; we saw that, strictly speaking, no normal predication about Being is possible. Accordingly, Parmenides' strategy to explain Being to his audience, how we should think of Being and its necessary existence, is either undertaken with the help of signs (the *sêmata* in fr. 8) or by arguing for the exclusion of other possibilities, as he does in fr. 2, like a *via negativa*:[105] he establishes the path of

[101] While the rational square root of two can be part of a negative statement like 'there is no rational square root of two', it is impossible to affirm its existence.

[102] This also fits the Greek expression *ouden legein*, which means talking nonsense. But it may seem difficult to square with Parmenides' negation of modalities: similar to the negation of a proposition ('it is not the case that p'), the negation of necessity seems to have a structure that leaves that whose necessity is negated intelligible ('it is not necessary that p'; or, since for him the negation of necessity equals impossibility, 'it impossible that p'). However, for Parmenides, 'it is impossible' equals 'it is unintelligible', as fr. 2 makes obvious. He does not claim that 'it is unintelligible that p', but rather '*x* is unintelligible', so that we are back to uttering something without communicating a consistent thought.

[103] On their own, the criteria for philosophy which Parmenides establishes do not imply any monism – as is obvious from the fact that we do, or at least can, employ them for many sciences that are in no way monistic. However, the specific interpretation they are given in Parmenides' poem, in accord with his operators, leads to his logical monism.

[104] If we ask what motivates Parmenides' usage of these operators, we should bear in mind that this is the very beginning of logical investigations in Western thought. So while *we* may say that his negation operator rests on a kind of merging between contrariety and contradiction, we should not forget that this is a time before contrary and contradictory opposites were clearly distinguished. I do not think that Parmenides had positive reasons for choosing this operator over another, but rather that this is the way negation seemed to work for him if he wanted to be as precise and strict as possible. And the way the negation operator is set up in turn influences his connection operator.

[105] I think this is the reason why fr. 2 calls our enquiry a *hodos*, a path or route: it is a route because even though Being is characterised as what can be thought and said, we cannot, strictly speaking, talk about Being; rather we need to be shown *the way to* Being in order to grasp it in a non-discursive way. But grasping it in a non-discursive way does not

estin by arguing against the alternative path, *ouk estin*. And we find many more negative descriptions of Being than positive ones in the poem. In fact, of the basic characteristics of Being, only "being one" is not negatively expressed[106] – which thus already stands out from all the other *sêmata*. Being one is also not further argued for, as the other *sêmata* are in the deductions of fr. 8.[107] Rather, being one seems to be an implicit result of the deductions of the other *sêmata* of Being,[108] which demonstrate the impossibility of any difference and thus a form of monism as the only tenable position.

However, monism can take different forms, and just what kind of monist Parmenides is has been vehemently debated in the recent literature.[109] Some kinds of monism allow for the existence of a plurality of Beings, and for that reason are sometimes called 'generous' monisms.[110] One such generous monism is predicational monism,[111] which claims that there is a plurality of things, but each is simple such that only one predicate holds true of it. The 'logical monism' to which Parmenides appears to adhere according to my

mean that we can kick away the negative process by which we got there, like a Wittgensteinian ladder we have climbed, since we still need it to protect us from the claims coming from the realm of *doxa*. Using a *via negativa* may, however, turn out to be problematic. For the general argument is '1. P or Q, 2. Not P (because P is impossible). 3. Therefore Q'. However, Q may also prove impossible.

106 Also 'being continuous' is not negatively expressed; it will, however, turn out not to be a basic *sêma*, but based on others in the course of the deduction, see Sattler 2011 and Chapter 7.

107 See also McKirahan 2008, p. 191.

108 Cornford understands it as a premise for Parmenides' overall argument, but since it is introduced only in fr. 8, after it has been established that Being is and that it is necessary that it is, I do not think it can be a premise.

109 For different types of monism, see Rapp 2006.

110 See Palmer 2009 pp. 38 and 42–4. No further explanation is given of what exactly is a 'generous monist'. But it seems to me that by 'a generous monist' scholars usually understand any monist who allows for more than one thing to exist truly (usually some additional reason is given why, despite this pluralism, monism is still claimed). The distinction between generous and what I call 'meagre' monism in the following seems to resemble the distinction between priority and existence monism made by Schaeffer 2007, p. 178: priority monism means that "one concrete object (the world) is basic", but there are other concrete objects (the parts), which "exist derivatively, as fragments of the whole", while existence monism holds that "there exists one and only one actual concrete object" (the world). However, the plurality of objects of the generous monist do not have to be parts of a whole, as with priority monism. And the one object of the meagre monist is not necessarily a concrete object, as with Schaeffer.

111 Prominently put forward by Curd 1998; it is based on Mourelatos' 1970 idea of 'speculative predication'. Sisko and Weiss 2015, p. 41 distinguish between generous monism, as a token monism of things of a specific type, from type monism of the nature of each kind of thing, under which predicational monism falls for them. This distinction is less important for me than the one between understanding Parmenides as a pluralist of sorts (which both Curd's predicational monism and Palmer's generous monism do) and understanding him as a numerical monist.

interpretation means that, given the logical operators available, Being must be completely simple. Logical monism shares this feature with predicational monism; however, logical monism also implies what is often called numerical (and what may thus be seen as 'meagre') monism[112] – there is no more than one Being, since if there were a plurality of internally simple beings, each would nevertheless have to be different from all others, not be any of the others. Plurality would thus introduce non-Being (*x* is different from and hence not *y*), and accordingly is not conceivable and consistently thinkable within the Parmenidean framework.[113] However, for the sake of the current project, I will focus on the absolute simplicity of what is, since this is the main point for the remainder of the book; I will leave numerical monism to one side for the most part. And while I think Parmenides is a numerical monist with respect to true Being, his position does not imply, as we will see below, that sensible phenomena must be mere illusions, in the sense that they are merely invented by our senses rather than occurring in our mortal world. We will see that Parmenides can be 'generous' in this respect.

That Parmenides' operators and criteria for philosophical investigation lead him to a monistic position also shows the close connection between the different notions of his philosophy. These notions depend on each other in such a way that a change in one concept would necessitate changes in the others, which is part of the tight systematic connection we already saw hinted at in the section on consistency.

Parmenides' notion of what is and his logical operators fit together roughly as follows: an extreme negation of non-Being leads to an absolutely simple Being, since only such a Being excludes any differences and hence non-Being. If Being is absolutely simple, then a negation of it will be a negation of any Being, and hence an extreme negation. The only way to ascribe something to such an absolutely simple Being is by absolute identity, which is the counterpart to extreme negation.

Furthermore, Parmenides' philosophy seems to meet his criteria for rational investigation: given how his notion of Being and his logical operators are tied to each other, they seem to fulfil the requirement of consistency on a systematic level.[114] And we are given the premises and arguments needed to prove the statements the goddess makes about Being (with the help of these two operators) – so Parmenides' philosophy itself also seems to satisfy the criterion of

[112] Being is not the unity of a plurality; its unity is in no way compromised by any complexity or relation.

[113] We will see in Plato's *Sophist* how the assumption of plurality also implies that each one can no longer be thought of as being absolutely simple.

[114] For Parmenides it seems that only such a minimal set of concepts – one operand, Being, and two operators – can fulfil the requirement of consistency. We will see in the chapter on Plato's *Sophist* how this set is enlarged.

rational admissibility. Finally, we saw that everything for which there is no sufficient reason was expelled by the goddess from the realm of *alêtheia*.

Parmenides' criteria make philosophy a more rigorous discipline than it had been for thinkers before his time in the sense that with Parmenides we now have a much clearer idea of what counts as a good argument – namely, what meets the criteria of consistency, rational admissibility, and the principle of sufficient reason. Let us now spell out the consequences of his criteria and operators for natural philosophy.

2.4 Problems for the Very Possibility of Natural Philosophy

If I am right that Parmenides' reasons for his monism are logical reasons, then there cannot be any Parmenidean natural philosophy: not only is there nothing apart from the one Being that is worthy of our philosophical attention, but also the logical apparatus available would not allow us to capture the complexity and plurality which natural philosophy must necessarily deal with. Parmenides' logical apparatus, in connection with the criteria for philosophy, leads to at least four major problems for what we would call a philosophy of nature: (1) the logical universe is restricted in such a way that there are not enough basic concepts (operands and operators) available for natural philosophy; (2) it does not allow us to differentiate between operators and operands (entities); (3) the status of the less basic concepts is never explicitly clarified, and a certain indeterminacy is thus inherent to Parmenides' philosophy; and (4) this indeterminacy, together with the rigid restriction of logical possibilities, leads to the exclusion of all complexity and relation, which means that time, space, motion, and change must be expelled from the realm of philosophy.

While the problems just named are general logico-metaphysical problems raised by Parmenides' philosophy, we will now look more specifically at how these problems affect the possibility of pursuing natural philosophy in particular.

2.4.1 The Absence of Adequate Basic Concepts for Natural Philosophy

Parmenides' philosophy does not provide sufficient basic concepts for a philosophical explanation of motion and the natural realm. We find some discussion of temporal notions in fr. 8, lines 5–6, where we are told that 'was' and 'will be' are not on the same footing as 'is', so that when we use what can be interpreted as the present tense form of 'being', we may express something not subject to temporal differentiations. But this discussion is a negative investigation into time, claiming that something that could be taken as being in time should in fact be beyond any temporal differentiations.

In contrast to the exclusion of temporal extension and differences, Parmenides seems to employ spatial extension with his spatial image of a well-rounded sphere (fr. 8, lines 42–9). However, as we saw above, he explicitly introduces the sphere as a *simile* for the completeness and homogeneity of Being. According to our line of interpretation, neither spatial nor temporal differences can be conceived with respect to what truly is, and thus motion cannot be part of it. Consequently, Parmenides claims what truly is to be *akinêton*.[115]

Some scholars have assumed that Parmenides' Being is temporally and spatially extended, as we saw above. However, they cannot allow for the temporal and spatial *differences* needed to account for motion: space, even on their interpretation, must be fully homogenous, as Parmenides claims in fr. 8, and time is simply eternally extended without allowing for any change.[116]

Some notions specific to natural philosophy are given in the second part of the poem, where Parmenides deals with the heavens, the *physis* of the heavenly bodies, and fire and night as the two basic principles. In the fragments that survive, we do not get an account of motion, but we do find some ideas about the way human beings come into being. However, all these ideas are found in the *doxa* part, which is clearly said not to meet the criteria for true knowledge. There we also get the notion of an 'is' that is part of the temporal series that includes 'was' and 'will be'. But the temporal series applies to the perceptible cosmos, which has come into being (fr. 19), not to what truly is. Thus, no investigation of the temporal series is part of the truly philosophical investigation.

Fundamental notions of natural philosophy such as time, space, and motion are not developed so as to count among philosophical notions (rather than being just mortal names) according to Parmenides' criteria. Furthermore, these notions could not be introduced into what Parmenides would count as rigorous investigation, since this would result in two problematic consequences. First, for Parmenides, the introduction of another basic notion

[115] Fr. 8, line 26. This, however, does not mean that Parmenides does not use metaphors that employ connotations of motion and becoming, e.g., in fr. 8, line 25. I discuss the spatial language that Parmenides uses, which I take as one way to express new logical structures, in my manuscript in progress, *Conceptions of Space in Ancient Greek Thought*.

[116] See, e.g., Palmer 2009, p. 322. Such an interpretation gets Palmer into two problems: first, if 'what is' and 'what is and is not' (the things dealt with by *doxa*) are both temporally and spatially extended, we get into the problem of the co-presence of both, a problem which Palmer acknowledges, but does not solve (p. 180 ff). Secondly, it is the mistaken reliance on their senses which, according to Palmer 2009, p. 323, makes mortals assume that only contingent being exists. However, if Being is also temporally and spatially extended, then we would need a reason for why Being is not accessible by sensation, and thus why mortals are wrong to rely on their senses (a reason of the sort we find, for example, with the atoms of the atomists).

besides Being would raise the question of whether it could be different from Being without thus 'being' non-Being?[117] Second, the basic notions of natural philosophy are such that they require differences as part of what they capture – a notion of time needs to capture at least the difference of before and after, a notion of space at least the difference of here and there, and a notion of motion at least the difference between an earlier and a later phase of this motion, let alone much more complex differences.[118] I will deal with this second problem below, in Section 2.4.4, and concentrate on the first problem here, that Parmenides could not allow for the introduction of any further notion.

Parmenides' logical apparatus prevents the introduction of any concept of difference. To conceive of difference would require bringing together Being and non-Being – if *x* is different from *y* then *x is* but it *is not y*. Within the Parmenidean framework, however, Being and non-Being must be thought as extreme or polar opposites: if '*x* is', then x is simpliciter;[119] if '*x* is not', then according to Parmenides' negation operator, *x* is not at all. Even Parmenides must assume that Being as his basic notion is *different* from his logical operators and thus allow for some difference. However, this allowance is made implicitly, and the impossibility of capturing a notion of mere difference consistently within Parmenides' framework does not allow for the introduction of any further basic concepts. Moreover, the explicit introduction of other basic entities or any change of one of the basic operands and operators would undermine the tight connection between Parmenides' basic concepts.[120] Changing the understanding of negation in such a way that it no longer expresses extreme negation, for example, would change the understanding of non-Being from a polar opposite of Being to a non-Being that in some sense is. But this latter position does not fit with Parmenides' understanding of Being as excluding any form of non-Being, and it would undermine his argument in fr. 2 for the exclusion of non-Being from philosophical investigation.

In Plato's *Sophist* we will see an explicit reaction to this set of problems: Plato shows that Parmenides' set of concepts is too small and that a minimal set to account for the basic relations between concepts must contain at least five concepts (by establishing motion and rest as two further concepts that are part of such a minimal set, Plato derives his five

[117] This is also a major problem for every pluralistic interpretation of Parmenides. See Austin 1986, pp. 20–1.

[118] See also the first chapter on relations as necessary conditions for conceptualising processes.

[119] It does not allow for the further question 'What is *x*?', since that would imply some complexity already (*x* would be whatever we take the answer to 'What is *x*?' to be and it would *be*) – a problem we find taken up in Plato's *Sophist*, see Chapter 5.

[120] That is, the alleged consistency between these notions.

megista genê). In addition, Plato also reinterprets Parmenides' negation operator so as to establish a separation operator that can express simple difference. And he shows that the reinterpretation of Parmenides' concepts and the introduction of new concepts necessitate changes in the notions of Being and non-Being and in the operators.

2.4.2 No Distinction between Operators and Operands

The restriction of basic concepts does not allow for distinguishing basic entities and logical operators.[121] Note that not having a distinction between entities (operands) and operators is not the same as not distinguishing between one- and two-place usages of 'is', or not distinguishing between the 'is' of identity and the 'is' of predication, about which Parmenides scholars like to argue. Both these latter cases would count as possible confusions among different operators.

But why should we think that the distinction between operators and operands is important when we assess Parmenides' philosophy with a view to the possibility of pursuing natural philosophy? This distinction is important because its lack is a central factor in the inability to integrate other and more complex notions needed for natural philosophy. We need to distinguish between the basic thing there is – Being, in Parmenides' case – and what can be ascribed to that basic entity.[122] Although there is only one Being for Parmenides, in principle several things could be ascribed to the one Being: that it 'is' one, homogenous, and continuous, and so on (but for this we would need to make the distinction between a thing and what is ascribed to it). And if we do not distinguish between the entity 'non-Being' and the separation operator 'is not', then we cannot deny something of Being (for example, that it has not come into being and will not pass away) without running the risk of understanding it as the entity non-Being. We will see that operators and operands sometimes are still run together in Plato's *Sophist*.

But can we indeed recognise the absence of such a distinction in Parmenides' poem, and if so, where? The lack of this distinction surfaces in fr. 2, with the very introduction of the two ways of investigation. The first way – the only way that can, in the end, be thought – is introduced as *hopôs esti*, "that [it] is", in line 3. This introduction leaves open what we should expect from this way – does it introduce an entity ("Being") or an operator ("is", which can operate on

[121] There are in fact two possible points for the blending of operators and operands: (1) the basic concepts are not sufficiently determined and are thus used to refer to entities as well as to operators; (2) Parmenides cannot provide the grounds for employing, on the one hand, operators in his philosophy, negation and identity, and, on the other hand, an entity to which they refer, Being.

[122] For an explanation of why I think we can group predicates under 'operators' here, see Chapter 1.

operands not yet named)? Accordingly, we find in the secondary literature scholars who understand this way as referring to a certain 'object', like Being, and other scholars for whom it introduces a function that includes essentially an operator, like 'speculative predication'.[123] Fr. 6 talks about the participle *eon*, which again could either be understood as an operator or as an entity expressed by a verb form. Given the *sêmata* referring to *eon* at the beginning of fr. 8, I think it is clear that it is understood as an entity, but the lack of a systematic distinction between operators and operands allows for both interpretations in fr. 2.

Similarly, the introduction of the second way of investigation as *hôs ouk estin*, "that [it] is not" (fr. 2, line 5), leaves open the possibility that this way deals either with the entity 'non-Being' or with an operation 'not being'. In lines 6–8 Parmenides translates the claim of line 5, "that [it] is not", into *mê eon*, "non-Being", when he writes that the suggestion of line 5 is an altogether unknowable trail "for neither could you think of non-Being [*mê eon*] – that cannot be done – nor say anything about it". Again we seem to end up with an entity (the impossible entity non-Being), but the first occurrence introduces what sounds more like an operator.

The absence of a distinction between operators and operands also gives an additional reason, besides Parmenides' connection operator, for why that which should be ascribable to Being is not actually predicated of Being in fr. 8, but is rather called *sêmata*, signs of Being. If you do not clearly distinguish between operand and operator, you cannot predicate these *sêmata* of Being, since it is not clear that doing so would not generate new entities (for example, the entity homogeneity).

The absence of such a distinction is less surprising if we bear in mind that the distinction between nominal expressions and verbs does not seem to have been made before the very end of Parmenides' lifetime, at the earliest.[124] And the effortless nominalisation of verbs in Greek allows for an easy switch between entities and operators. But there are also philosophical reasons for not keeping operators and operands clearly apart: doing so requires some

[123] See Mourelatos 1970, p. 55, who understands *esti* here as a copula with "both the subject and the predicate-complement left blank". Since Mourelatos assumes that what Parmenides is concerned with are predications answering the question 'what is it?', *esti* functions grammatically as a copula but logically as indicating identity; this is what Mourelatos 1970, pp. 57–8, calls "speculative predication", which specifies what it is to be the nature or essence of a thing.

[124] Protagoras seems to have been the first to introduce such a distinction, at least implicitly, given that he was the first who distinguished explicitly between the three genders of nouns and the tenses and moods of verbs (according to Aristotle, *Rhetoric* 1407b6–8), and it seems hard to make these distinctions without distinguishing between nouns and verbs. The careful, almost tedious, way in which Plato introduces the distinction between nominal expressions and verbs in the *Sophist* suggests that this distinction was still not a very familiar one in his time (which does not mean that people would not implicitly have made such a distinction correctly in their usage of verbs and nominal expressions).

complexity (and plurality) – for example, our claim that '*x* is blue' (using the predicate/operator 'being blue' of an entity/operand) does not mean that *x* is the blue, or that it is what it means to be blue. Rather, *x* is some entity that, while being blue, potentially also has other features. Differentiating between operator and operand means, at least in principle, dealing with the possibility that the operator could apply to a plurality or that the outcome of the operation could in principle be quite different from the original input. Under these conditions, Parmenides cannot sign up to a clear difference between the entity 'Being' and the logical operator 'is'.[125] He cannot strictly distinguish 'is not' as standing for an entity, non-Being, and as used as an operator for determining Being. That he does not clearly distinguish between operator and operand also becomes obvious in the scholarly discussion about how best to translate *to auto* in fr. 3 (τὸ γὰρ αὐτὸ νοεῖν ἐστίν τε καὶ εἶναι). Should we read it as an operator (so that we would understand the claim as "being and thinking are the same") or as an operand ("the same is for thinking and for being", i.e., the same can be thought and be)?

Some cases in Parmenides can be read as keeping operators and operands implicitly apart,[126] but only with the atomists is such a distinction based on the basic philosophical framework: Democritus makes this difference an important and provocative sounding point when he claims in fr. 156 that Being is no more than non-Being,[127] so both *are*. In this way the operator 'is' must be distinguished from the entity 'Being'; similarly, 'non-Being' qua void is distinguished from 'is not' qua difference operator.

2.4.3 The Indeterminacy of Background Concepts

Less basic background concepts work as implicit premises in Parmenides' philosophy but are never explicitly clarified. Such concepts include not only his notion of a whole,[128] but also his notion of being one, homogeneity (being *homoion*), and continuity (being *suneches*) – notions that are also important for natural philosophy, as we will see. Parmenides' notion of a whole, for example, is meant to be independent of any parts and in fact excludes parts, but it is never clarified in which sense this whole is indeed a whole. It seems to be a

125 There are occasions when the different verb forms require referring to different things, e.g., the finite verb form refers to an operator, the infinite one to an operand (see fr. 3). But Parmenides' philosophy does not allow for a systematic separation.

126 For example, those lines of fr. 8 which tell us that Being is not-*x* in order to prevent a wrong account of Being.

127 Literally, he claims that thing (δέν) is no more than nothing (μηδέν), but it is clear from other fragments that we can equate 'thing' with 'Being' and 'nothing' with 'non-Being'. See Chapter 4.

128 Plato's *Sophist* 244c ff. already points out problems with the Parmenidean notion of a whole.

whole in the sense of being complete, but completeness and being a whole are listed as two *sêmata* in fr. 8, line 4. We will see in the next chapter that blending two different notions of a whole is central to Zeno's paradoxes of motion. And in Chapter 7 we will see Aristotle respond to this lack of conceptual clarity with his own understanding of how something homogenous and continuous can be a whole.

2.4.4 Problems with Relations

A consistent account of relations is crucial for any natural philosophy. Time, space, motion, and change cannot be thought without the notion of a relation between two (or more) elements, as we saw in Chapter 1: every process involves the relation between different times (for example, the relation between before and after or, for Aristotle, between two nows – or, as we would capture it, between the starting time t_1 and the finishing time t_2 of a process), as well as different places or conditions (the relation between two different places or conditions that are connected by the process). Finally, a process includes the relation between the times and the places/conditions, which gives us the speed of the process.

Parmenides' philosophy does not allow for a consistent account of relations. In order to conceive of a relation, the relata must differ from each other in some aspect. With some relations, a difference in the logical roles or functions of the relata is sufficient;[129] but with time, space, and motion the relata themselves must be different – here we return to the second problem mentioned under Section 2.4.1. For example, in order to be sure that we are dealing with distance (and not just with one point), any two points d_1 and d_2 which I choose must be different points. If I want to make sure I am indeed dealing with motion, any two times t_1 and t_2 in which this motion takes place must be different, and not only their function, for otherwise no motion has occurred. For Parmenides' framework, the basic problem is that the relata of the relations in question (for example, t_1 and t_2) must be understood as being different, and that means that the one must be understood as not being the other, while simultaneously both are (and, in our example, are both times and are connected in one temporal series). But for Parmenides this difference means that they are both Beings and non-Beings, they require Being and non-Being to be brought together – a relation which must be thought of as contradictory within Parmenides' framework. 'Generous monist' interpretations have the additional problem that Parmenides cannot account for the relations between elements of a plurality.

[129] For example, the function $f(x)=x^2$ is satisfied if x and $f(x)$ both equal 1; but 1 plays two different roles here, the role of an argument and the role of the value of the function.

Developing a consistent account of relations remains a challenge throughout the period covered by this study – as can be seen, for example, from Aristotle's account of *pros ti* in chapter 7 of his *Categories*, which requires a second start after the first attempt seems to encompass items of other categories.[130] But the main problem Parmenides encounters with relations, namely that Being and non-Being must be combined, could be overcome by the thinkers dealt with later in this book, the atomists, Plato, and Aristotle.

We will see in the later chapters that the notion of continuity also becomes closely tied to the notion of relation. Continuity plays a central role in capturing motion, time, and space. In Aristotle, continuity is precisely what allows for any possible relation to hold between two different moments in time or points in space. By contrast, Parmenides understands continuity as excluding any differences and as granting the indivisibility of what is.[131] The notion of continuity is still employed in this sense by Zeno, but once relations are (at least in part) consistently accounted for, the notion of continuity also changes.

Summing up, we can say that (1) the absence of adequate basic concepts for natural philosophy is based on the problem of conceiving any basic concept over and above Being. (2) The absence of a distinction between operators and operands and (3) the indeterminacy of background concepts show that Parmenides' basic concepts are not sufficiently clarified for a consistent account of motion in the sensible realm. (4) The problems with relations, finally, show that the notion of relation central for motion cannot be fitted into a Parmenidean framework.

The concepts crucial for giving an account of the realm of natural phenomena must therefore be excluded from an Eleatic framework because of conceptual and logical difficulties, but not because the natural phenomena are mere appearance (i.e., just a deception for which we mortals fall). The still influential strand in the literature according to which Parmenides understands everything apart from the One Being as mere appearance,[132] did not surface here as a reason for expelling nature. The difference that is explicitly drawn between the first and the second part of the poem is the difference between *alêtheia* and *doxa*, not a difference between Being and the appearances of Being.[133] Accordingly, we do not have good reason for assuming that what Parmenides

[130] See Chapter 9.
[131] See Chapter 7.
[132] Prominently found in Guthrie 1965.
[133] The very idea of 'mere appearances' does not make sense within Parmenides' framework: (1) 'mere appearances' cannot be dependent on our human form of cognition, in, e.g., a Kantian way, since we are meant to judge the truth that the goddess shows us with our own reason – so the things spoken of in the *doxa* are in some way 'there' for the goddess, too; (2) it cannot depend on our perception, for fr. 6 shows that our senses do not grasp enough, not that they produce too much, like appearances; (3) the idea that 'mere appearances' are mere names established by mortals, as frr. 8 and 19 might seem to suggest, requires something to which they can apply these names, even if this something may be a mere illusion; and then we are back to possibility (1) and (2).

excludes from the realm of philosophy is rejected because it is merely apparent, in the sense that, for instance, motion does not really take place but merely seems to be, like a hallucination or an illusion.[134] Rather, the natural phenomena are excluded because the concepts required to grasp them do not satisfy the criteria for rational investigation: they cannot be accounted for consistently, they are not admissible to rational demonstration, and they seem to violate the principle of sufficient reason. But this inability does not yet imply that they are a mere illusion invented by our senses. Natural phenomena cannot be conceived of consistently since they get us caught up in contradictions. Accordingly, they cannot be given a true account. But they may nevertheless 'exist' in the sense that they do occur (not only seemingly) in our world, even if they do not exist in the sense in which what is in the *alêtheia* part exists, which is what is intelligible.

2.5 Relation to the *Doxa* Part: The Role of Cosmology

If Parmenides' philosophy does indeed exclude the very possibility of natural philosophy, why then, picking up on a standard question in the literature,[135] would he himself provide us with a cosmology in the second part of his poem? Does not this discussion show an appreciation of the sensible world that speaks against the interpretation put forth in this chapter? And is he not here developing a natural philosophy after all?[136]

This second part of the poem demonstrates that the problems for the fundamental presuppositions of natural philosophy raised by the first part cannot be solved simply by giving a different or better cosmology. The explicit reason for giving a cosmology is stated in fr. 8, lines 60–1:

> I tell you this fully [*panta*] fitting [*eoikota*] ordering of the world [*diakosmon*],[137] so that no judgement of mortals shall ever outstrip you.

[134] Palmer thinks that what mortals are concerned with is not inexistent; rather it is what is contingent. For problems with Palmer's 2009 claim that Parmenides is the first to distinguish between what must be, what must not be, and what is but need not be, see Sattler 2014.

[135] See, for example, Guthrie 1965, II, p. 5, and Long 1963.

[136] One explanation on offer that is compatible with my interpretation so far is the tradition established by Owen 1960, for whom Parmenides' cosmology is a mere dialectical device showing the vacuousness of temporal and spatial distinctions by employing them in a proof. However, employing these concepts in order to show their internal inconsistency is actually a method of argumentation we first find in Zeno's paradoxes. There we see a methodical way of demonstrating time, space, and motion to be inconsistent by showing how employing these notions entangles us in inconsistencies, as, for example, that the moving arrow is resting. But we do not find anything like this in the remaining fragments of the second part of Parmenides's poem (see Chapter 3). While Zeno shows motion to be internally inconsistent, Parmenides shows it to be inconsistent with his own framework, and thus externally.

[137] For a brief discussion of this clause, see the beginning of Chapter 5.

The goddess instructs the mortal in cosmology so that the explanations of the world given by other mortals will never outdo him, so that he will be able to argue against them.[138] Thus, the goddess's account of cosmology makes sure that the seductive force of mortal belief will not overcome the truth of knowledge. Giving his own cosmology is not a matter of asserting it, but a resource for being sceptical about others.[139] In this way Parmenides' cosmology supports the way of truth, as it shows what a most adequate cosmology looks like and where even such a cosmology goes astray, namely in naming two principles.

The cosmology is introduced as mortal opinion, "they decided to name two forms, of which it is not allowed to name only one[140] – that's where they have gone astray" (fr. 8, lines 53–4). "They decided to name two forms" – who are "they"? So far in Parmenides' text all employment of plurality and names was done by mortals, the double-headed, who take Being and non-Being to be the same and not the same. But what follows in the remaining *doxa* fragments is at least in part a rather sophisticated cosmological account.[141] True, some features – the astronomical objects of fr. 10, for example – are just conventional ingredients of a cosmology of the time. But Parmenides' cosmology does not seem to be merely conventional in its explanation of these ingredients; for example, frr. 14 and 15 are the first instances where Greek thinking claims that the moon gets its light from the sun.[142] And in ancient doxography, Parmenides is credited with both postulating the earth's roundness and

[138] See Barnes 1982, p. 157.

[139] My explanation seems to be called into question by Dorter 2012, p. 50, who points out that since in frr. 2–8 the goddess has given an a priori argument against multiplicity, motion, and change, "there is nothing to be gained by refuting an individual cosmology on its own terms". While Dorter is right that an a priori argument does not need to go through individual instances, it can be perceived as stronger if it is exemplified with the help of an individual paradigmatic case. And while the goddess has already rejected motion as incoherent in the first part of the poem, it is only the second part that shows explicitly how a cosmology must rely on the notions rejected as inconsistent in the first part.

[140] This line has been interpreted in different ways. I think there are three main possibilities. If we see it as a prohibition established by mortals, it reads, "it is not allowed to name only one", i.e., since there are two basic principles, it is a mistake to name only one. If, on the other hand, it is the goddess's own commentary on mortals' decision to employ two basic principles, it could either be understood as "it is not allowed to name even one", i.e., both are wrong principles and accordingly even naming one of them is false. Or it reads, "only it is not allowed to name one", in the sense that only one of the two principles is wrong (namely darkness or non-Being), while Being, the right principle, on this reading is equated with light. For different readings, see also Mourelatos 1970, p. 80 ff.

[141] Even though Parmenides suggests turning to metaphysics instead of natural philosophy, he obviously knows the natural philosophy of his time very well.

[142] See Graham 2006, pp. 179–82 and Mourelatos 2011.

identifying the morning and evening star.[143] Furthermore, Parmenides' cosmology is highly systematic in building everything from two principles.[144]

Given this sophistication, it seems that "they" cannot refer to average mortal men, typical educated Greeks. But nor does "they" seem to refer to Parmenides' predecessors, since the cosmology given cannot be identified as one of theirs; it is, for instance, not the cosmology of one of the Ionian thinkers.[145] This cosmology, then, seems to be exactly what the goddess explicitly claims it to be – a most suitable and fully fitting or likely cosmology, *panta eoikota*. In light of the mindset of mortals as outlined in the poem so far, if they were in a position to develop the most convincing cosmology possible, it would look just like this. It develops everything systematically out of two basic principles, the absolute minimum of a plurality of principles required for the complexity and plurality of the world. Hence, the cosmology is most fitting, for it is as similar as possible to the truth, which is based on one principle, Being.[146]

But even such a convincing cosmology cannot satisfy the criteria for reliable knowledge.[147] This the goddess makes clear when she claims that the assumption of two principles is the reason this approach has gone astray (fr. 8, lines 53–4) and shows how everything in this cosmology is based on these two principles.[148] Besides, this cosmology must employ operators different from

[143] See DK A1; Furley 1987, p. 53; Mourelatos 2011 and Cerri 2011, p. 89 ff.

[144] It seems to be clear in Parmenides' poem that in order to explain all the phenomena and their processes, at least two principles are needed, thus a minimal plurality. This is explicitly expressed in Plutarch *adv. Col.* 1114b (testimonium ad fr. 10). By contrast, the Ionian thinkers, who are often seen as some kind of substance monists, do not seem to realise that a cosmology requires more than one principle. For this point of contrast cf. Aristotle *Metaphysics* A, 984b1–4. Curd 1998 claims that there could be a rational cosmology acceptable to Parmenides, if there is no change and the basic entities follow the predicational monism outlined in the *alêtheia* part. But the cosmology Parmenides gives is not such a cosmology, since it clearly employs generation (see fr. 11) and its basic entities, fire and night, are not ontologically sound (as the nature of each is not to be the other). If a rational cosmology acceptable to Parmenides were indeed possible, as Curd claims, then it would be very strange that he does not give it in the *doxa* part, but instead a cosmology that is unacceptable to him.

[145] While Zeller 1879 and Burnet 1930, p. 185 both wanted to understand it as a sketch of a Pythagorean cosmology, the basis for their interpretation has since been called into doubt, see for example, Long 1963, p. 90.

[146] Cf. Wilamowitz-Moellendorf 1899.

[147] Cf. Kirk, Raven, and Schofield 1983, pp. 254–257. Fr. 10 seems to speak against such a reading, since it promises the interlocutor of the goddess that he will *eisêi* what concerns the aether (line 1), and *eidêseis* what concerns the *ouranos* (line 5). But it never talks about *noein* or *gignôskein* as fr. 2 does. Aristotle *De caelo* 298b18 ff. asserts that Parmenides was the first to see that there cannot be knowledge without an unchanging entity (presumably as the object of knowledge).

[148] For the idea that fire and night may function only as an example for a basic opposition of two principles that could take on different forms, see Simplicius *In Phys.* 30, 21–2 and 31, 4–7; Fränkel 1955, pp. 180–1; Long 1963, p. 102.

those employed in the realm of truth; they bring together what in the realm of truth would have to be conceived of as uncombinable opposites: in the realm of *doxa* the negation operator *tantia* in fr. 8, line 55 allows for a blending of the basic opposites of light and night.

If even this best possible cosmology cannot satisfy the criteria for philosophy, like consistency and so on, then a fortiori other cosmologies cannot satisfy it either, and Parmenides need not worry about them. Parmenides' cosmology demonstrates that natural philosophy is excluded from what counts as rational or scientific inquiry or philosophy not because the cosmologies before Parmenides were not good enough but because the realm of natural phenomena is per se not a possible object of rational investigation. This understanding of natural phenomena already indicates that the Eleatic exclusion of natural philosophy ultimately cannot be overcome solely with the help of a more elaborate cosmology; it can be overcome only with the help of serious changes in the metaphysical and logical frameworks.

The fact that Parmenides nevertheless gives us an advanced cosmology in his poem, which he, however, claims to be part of the *doxa* of mortals, rather than part of the truth,[149] suggests that for Parmenides natural philosophy should in principle be subject to the same criteria he establishes for metaphysics. This idea we will see prominently taken up by Plato in his *Timaeus*. In Parmenides' eyes, however, natural philosophy is unable to meet these criteria.[150]

The discussion above in Section 2.4 shows not only four major problems for founding a philosophy of nature on Parmenidean ground, but also that the concepts he employs for his metaphysics are themselves too indeterminate to count as elements forming a consistent doctrine. They do not appear to be inconsistent in themselves, as Parmenides thinks concepts like time and motion are. However, Parmenides' basic concepts turn out to be too indeterminate not only for a consistent account of the processes in the sensible realm but also for a rationally admissible account of Being as he wants to give it.[151]

[149] I think it is clear that fr. 8, lines 50–2 claims everything that follows, and this is the cosmological fragments, to be what mortals wrongly assume. For a different view, see Cordero 2011.

[150] Johansen 2016 claims that the objects of *doxai* are to some degree and also meet the standards of intelligibility, since they display features resembling the *sêmata*. However, the similarities pointed out allow for plenty of differences and do not seem enough to show that Parmenides assumes the cosmos to resemble Being or that the cosmology can indeed meet scientific standards.

[151] It will turn out that the simplicity of Being and the absoluteness of his operators cannot be maintained consequently (as we will see in Chapter 5). The restriction of Parmenides' logical tools means that in the end he cannot express his philosophy in a consistent way (at least not discursively, as this brings with it differences and distinctions). Even if Parmenides' own account cannot allow for being differentiated and determinate, our own account of his position must employ differences that cannot be captured within his framework in order to explain his position.

They need further specification also for his own project, as his successors will show. While Parmenides establishes standards for scientific inquiry and uses them in his own philosophy, later thinkers showed that his philosophy does not in fact fully meet these criteria. Accordingly, his successors took to heart his concerns about accommodating natural philosophy to rational demands, much more so than his particular ontological conclusions.

Thus, the important legacy Parmenides leaves for the natural philosophers to come is twofold. On the one hand, he has established fundamental criteria for rational investigation, which made it possible for philosophy, and thus also for a possible natural philosophy, to be raised to a new level of rationality and rigour. On the other hand, Parmenides has begun to develop the conceptual problems faced by any attempt to make nature a respectable object of science, a step which Zeno will take further in a very important way. Accordingly, Eleatic philosophy puts forward the most serious challenge for the very possibility of natural philosophy – whether the sensible realm, its motions and changes, can ever be conceptualised so as to satisfy the criteria for rigorous philosophy, and hence, whether natural philosophy is indeed possible. Parmenides' challenge includes a rigid restriction of the logical realm as well as indeterminate concepts, and it is at these points that his successors start to attempt to meet his challenge. Their responses include developing further his logical tools and his criteria for knowledge in order to keep both the new rigour introduced by Parmenides' philosophy as well as the realm of motion and change as an object of philosophical investigation. Parmenides' successors will also take up some features from the *doxa* section, like the notion of contraries, and show why they can be part of true philosophical investigation.

3

Zeno's Paradoxes of Motion and Plurality

3.1 Introduction

We saw in the previous chapter that Parmenides' criteria for philosophy, together with his logical apparatus, imply that nature as the realm of bodily motion and change cannot be a proper object of philosophical inquiry. Zeno, Parmenides' eminent student, does not change this negative assessment. In fact, Zeno does not introduce any new doctrine. But he spells out in detail the problems any natural philosopher faces, and thus shows what it would be to account for time, space, and motion. In this way his famous paradoxes not only deepen the challenge for establishing any natural philosophy, they also prepare the ground for a more thorough analysis of these central concepts for natural philosophy: they provide us with criteria for what it would mean to give an account of plurality, time, space, and motion that is much more specific than what we could derive from Parmenides' exclusion of these notions. For example, Zeno's paradoxes seem to show that giving an account of a motion over a finite extension will entangle us in problems with covering infinitely many parts in a finite time, if we assume that finite extensions are divisible. Or they show that in order to give an account of motion, time and space must be understood to a certain extent as magnitudes that are not independent of each other, but that at the same time we run into problems when we understand them as simply depending on each other. Thus Zeno's paradoxes can serve as a touchstone for determining whether later natural philosophers can indeed meet the Eleatic challenge: from Zeno onwards, serious natural philosophy must show how it can avoid his paradoxes without losing their insights. Moreover, we will see how Zeno's introduction of the genre of paradoxes into philosophical discourse also provides a further methodological development important for the analysis of the basic notions of natural philosophy.

Given the focus of the book, this chapter will concentrate on Zeno's paradoxes of motion. In contrast to the dominant secondary literature so far, I will suggest what I call a *conceptual* interpretation of these paradoxes. By this I understand that we must start with a careful analysis of the concepts of time, space, and motion as employed by Zeno in his paradoxes: what are the implications of the concepts used and which of their features contribute to the paradoxical result? These questions must be asked prior to any suggestions on how to solve the paradoxes. The dominant literature usually simply takes for granted the notions that are implicitly employed in Zeno's paradoxes, without analysing them any further.[1]

The vast majority of these interpretations can be situated along a spectrum of possible positions that range from those which regard the paradoxes of motion as mathematical problems to those that consider them to be problems of a physical-empirical nature.[2] An empirical interpretation seems unviable to me, however, since the problem is not whether we do in fact experience motion (as I tried to show in the previous chapter),[3] but rather whether we can give a consistent account of this experience. This is what is done in a conceptual reconstruction which asks whether the concepts used by Zeno for describing motion are adequate for an account of motion in terms of time and space.[4]

Mathematical interpretations seem to be more promising.[5] The basic idea is that Zeno got into these paradoxes because he did not yet have the necessary mathematical tools to deal with motion, which were only developed in the nineteenth century: things like the calculation of the sum of an infinite series, mathematical functions since Cauchy, and the limit of a function. Such a mathematical treatment is often not distinguished from what I call a conceptual rethinking. However, I want to show that conceptual and mathematical interpretations do indeed work on very different levels and will get very different results for Zeno's paradoxes.

A mathematical treatment merely asks which mathematical tools are necessary for the concepts employed to work smoothly. It does not ask whether the concepts employed are indeed adequate to account for the problem in question. A concrete mathematical treatment should thus only be a possible second step, which needs an adequate conceptual analysis as a first step.[6] And this first step is precisely what a conceptual treatment, in my understanding, provides – asking

1 While Hasper 2006 does indeed not simply take the notions employed in Zeno's paradox for granted, I will show below why I think that he nevertheless does not sufficiently analyse the concepts used.

2 The latter can, for instance, be found in Ferber 1995, who does not want to solve the paradoxes, but rather tries to eliminate them with the help of new empirical results (p. 2).

3 If the problem were whether we do in fact experience motion, Diogenes' reply to Zeno's motion paradoxes – to stand up and run – would indeed be adequate. See Simplicius *In Phys*. 1012.22.

4 This is also what Ferber 1995 ends up doing to a certain degree in his third chapter.

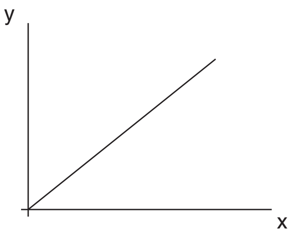
5 As we find them, for example, in Grünbaum 1968 and Salmon 1980.

6 Code 1982a and 1982b points out problems with the mathematical notion of a continuum as such. I will not be able to get into this here; all I need for my purposes is to show that even if the mathematical notions are unproblematic, they should only come in once we have dealt with the conceptual questions which Zeno's paradoxes raise.

first of all whether an account of the notions employed is correct, or whether, instead, it may have analysed them in the wrong way or not thoroughly enough.

We will have to ask, for example, which understanding of time and space Zeno actually employs in his motion paradoxes and whether these notions are sufficiently developed and appropriate for grasping motion.

We will see that such a conceptual interpretation will allow us to get results for Zeno's paradoxes that are quite different to those of the literature so far. Most importantly, it will allow us ultimately to reconstruct Zeno's motion paradoxes as true paradoxes of *motion*, while modern mathematical interpretations treat them as paradoxes of any continuous magnitude. In order to make this difference clear, let me give a simple example showing a common way in which we use mathematical tools to give an account of motion nowadays. Let us assume we have the following graph representing Zeno's latest race:



Taking the x-axis to represent the time line and the y-axis to represent the distance covered, this gives us a good idea of Zeno's run. But now take the x-axis to represent length, and the y-axis to represent height: this can be seen as an adequate depiction of part of the mountain on which Zeno was running. The very same graph can be used as an adequate mathematical representation of a continuous motion, as well as of a static mountain. So while a mathematical treatment allows us to do certain things with motion, it does not give us anything specific for motion.[7] That the notion of motion is correctly understood must simply be presupposed by the mathematical interpretation. And this is exactly what is happening with mathematical interpretations of Zeno's paradoxes.

By contrast, the conceptual interpretation I am suggesting will allow us to understand the paradoxes of motion not only as paradoxes that Zeno could have raised for any continuum, motion or mountain, but as paradoxes specific

[7] We may think that it does supply us with something specific to motion given that it allows us to calculate features like speed. However, what is behind this calculation is a standard way of quantifying the relation between two magnitudes, in this case of time and space. The questions of which understanding of time and space must be presupposed in order to allow for such a relation and whether this is indeed an adequate relation of time and space fall outside the realm of mathematics and require a conceptual investigation.

to motion.[8] Understanding them as genuine paradoxes of motion (also) means understanding motion as possessing temporal and spatial features that are not reducible to one another. We will see how the different paradoxes point to different inadequacies in accounting for both features of motion: motion is either understood in temporal *or* spatial terms, but never in both equally. Given the way that Zeno implicitly conceptualises motion, time, and space, there are indeed severe problems for combining time and space in an account of motion.[9]

Note that I am not claiming that Zeno *explicitly* works with certain accounts of time, space, and motion that then lead to his paradoxes. Rather, he attempts to attack any account of motion with his paradoxes. But in reconstructing the paradoxes, we can show which features of time, space, and motion he does indeed implicitly presuppose, and that these are features which at least some of his successors try to change in order to give a consistent account of motion. We will see in chapters 7 and 8 that Aristotle's reaction to Zeno's paradoxes is in large part a rethinking of the concepts used in setting up the paradoxes in the way just described, which allows him to react to the problems raised by Zeno.

Furthermore, this conceptual understanding will show that while there is also a certain part-whole difficulty involved in the motion paradoxes, it is not a difficulty specific to motion, but a problem to which motion, time, and space are subject in so far as they are continua. And the problem is not solely the part-whole relation usually noted to be difficult in the secondary literature, namely that a finite whole is made up of infinitely many parts. Rather, we will see different part-whole relations employed by Zeno.

Finally, our conceptual reconstruction will allow us to understand Zeno as continuing Parmenides' project, which attempts to show that, given the criteria for rational inquiry, no account of the realm of motion and change can be

8 There have been other accounts arguing against a mathematical interpretation of Zeno's paradoxes, most clearly Code 1982a and 1982b and Hasper 2006. Hasper opposes mathematical treatments of Zeno's paradoxes and wants to understand them as conceptual problems instead. However, we will see that he reduces Zeno's problems to part-whole problems not specific to motion. Code's main point of argument is that a mathematical approach presupposes the assumption that "the real mathematical continuum accurately and exhaustively reflects the physical continuum" (1982b, p. 45), but that there is "no authority . . . to warrant the supposition that mathematical structures are completely isomorphic to physical structures" (p. 54). However, Code's main interest, the problem of the applicability of mathematical concepts to physical notions, is rather different from dealing with Zeno's paradoxes; and consequently it is not important for him "to know what the historical Zeno actually said" (p. 59). Code 1982b focuses exclusively on "problems of extension" in the motion paradoxes, claiming that the dichotomy, like the Achilles paradox, "contains confusing and irrelevant references to time and motion" (p. 46; cf. also p. 59).

9 For instance, on the basis of the dichotomy paradox, it seems to be unclear whether Zeno considers that time can be divided in the way distance can, and the arrow paradox might indeed indicate that it cannot.

provided, because the concepts used for such accounts will eventually lead to inconsistencies.

In the present chapter, I will try to reconstruct how the paradoxes work and show their implicit assumptions about time, space, motion, infinity, parts, and wholes. Different possible solutions will only be hinted at here, but more fully discussed in the following chapters, especially in chapters 7 and 8.[10]

The paradoxes of motion will be the main focus of this chapter. We will begin, however, with a brief account of the paradoxes of plurality, since plurality must be assumed by anybody dealing with motion – at least in the most basic form where what is moving must be differentiated from that in which it is and moves (its space or place), and different times and places are presupposed when something is moving such that it is here now and there afterwards.[11] Let us start with a quick account of the general aim of the paradoxes and their relationship to Parmenides' philosophy.

3.2 The General Aim of Zeno's Paradoxes

At least since Plato's dialogue named after Zeno's teacher Parmenides, the idea has been widespread that Zeno's motivation for setting up his paradoxes was to defend Parmenides' philosophy. The Socrates and Zeno of this Platonic dialogue name as the target of proof of the paradoxes a defence of Parmenides' ontology;[12] Zeno's account of plurality and movement is meant to show the unintelligibility of the world of *doxa*.[13] This view has sometimes been challenged, notably by Barnes, who asserts that "a proof of monism and a

[10] For the problem of distinguishing between reconstructing the paradoxes and solving them, cf. Hasper 2006, p. 49.

[11] For reasons of space, I will not be able to look as Zeno's *topos* paradox here; I do so in my manuscript in progress *Conceptions of Space in Ancient Greek Thought*.

[12] See Sedley 2017 for a possible different reading of Zeno's aim.

[13] 127e ff. Sedley 2017 takes the paradoxes referred to in Plato's dialogue to be the paradoxes Zeno dealt with in his youth, not including, for example, the motion paradoxes. Although only paradoxes of plurality are explicitly mentioned in Plato's dialogue, it seems very likely to me that all paradoxes share in this basic aim, for they are all closely connected. The paradoxes of place are connected with those of motion, since without a consistent concept of place no locomotion can be thought. For the connection of the paradoxes of plurality and that of motion see below and Ross, p. 72: "from a famous passage of the *Parmenides* it is possible to infer that they [the paradoxes of motion] were directed against people who ridiculed Parmenides' denial of the existence of plurality. The apparent fact of motion, involving the occupation of different places at different times, is a *prima facie* evidence of plurality, and therefore Zeno tried to deprive pluralism of this apparent support by proving the non-existence of motion". Lee, p. 38 follows Philoponus: "As to its purpose, Philoponus is probably right when he says (513.8) that clearly by showing the conception of place self-contradictory Zeno would *a fortiori* be making a pluralistic position untenable." Finally, the dilemma of the millet seed not only questions the relation of part and whole, which is also a crucial point in the paradoxes of plurality

refutation of pluralism" do not "come to the same thing. Zeno's pupil Gorgias was well aware of that: he was, notionally at least, a nihilist".[14] Barnes is, of course, right that excluding pluralism will not necessarily give us monism. However, Zeno explicitly claims it to be a support for monism: "But, if it is impossible for there to be a plurality, yet it is necessary for there to be either one or a plurality, but it is impossible for there to be a plurality, we are left with the conclusion that there is one."[15]

Furthermore, Barnes understands Zeno's arguments as attacking monism as well as pluralism. And some fragments, like fr. 5 (DK29 A21), seem to support this account that Zeno is equally attacking both assumptions. But we will see below why the fragments that seem to attack monism should in fact be read as attacking only the conception of the one that the pluralists employ,[16] not Parmenides' One.[17]

Thus Barnes's objections do not seem decisive. Furthermore, the fact that we find two kinds of paradox of plurality with seemingly different aims also speaks for Plato's interpretation: there are some in which Zeno only attacks the assumption of plurality, but there are others in which he seems to establish something positive himself, something like a Parmenidean One (notably frr. 1–3). This speaks in favour of understanding Zeno as supporting Parmenides. And it also supports the idea, as we will discuss below, that those paradoxes in which Zeno attacks not only plurality but also a one are using a different notion of a one.[18]

Plato's interpretation is not only supported by the ancient commentators Simplicius and Philoponus,[19] but can still be understood as one standard view among modern scholars.[20] I will also take this interpretation as a working

and of motion, but is treated as a puzzle of what causes a certain form of alteration; for reasons of space, I cannot discuss this last paradox here.

14 Barnes 1982, p. 235.

15 Lee fr. 8, DK29 A21 (I will use Lee's edition for the numbering of Zeno's fragments, and, if available, provide concordances to the Diels-Kranz numbering). This understanding is also what we should expect from the previous chapter, which showed that arguing against pluralism within the Parmenidean framework is indeed understood as support for monism.

16 See also Lee, p. 24 for the interpretation that fr. 5 and others are attacks on a one which the pluralist would assume, a one of which there are many.

17 As, for example, Solmsen 1971 assumes.

18 See also Lee p. 26 on this last point.

19 Simplicius 1895 *In Phys*. 102,30 and 134,4; Philoponus 1887 *In Phys*. 42.9 ff. and 80,23–5; cf. Lee, pp. 7 and 65.

20 See, for instance, Kirk, Raven, and Schofield 1983, p. 277; Russell 1970, p. 46; Lee, pp. 7 and 65; Salmon 1980, p. 31. Against it, however, see Solmsen 1971. Curd 2011 claims that scholars have reached agreement that they "should resist thinking that there is a unified 'Eleatic Theory' that is embraced by all of the so-called Eleatics: Parmenides, Zeno and Melissus". But this in itself does not show that Zeno did not support Parmenides, and the only support Curd gives for her claim, Rapp 2006, does not do so either. Rapp explicitly

hypothesis, but I will extend it further: I want to show that Zeno tries to reinforce Parmenides' legacy not only in content but also in methodology. Zeno strengthens the content by showing that the assumption of a plurality, of time, space, and motion as proper objects of philosophical investigation, leads to problems that the assumption of the single One does not; and he confirms Parmenides' methodology, since it is the very criteria and mode of investigation that Parmenides set up in his poem which allow Zeno to build paradoxes which entangle the opponent in an untenable position.

3.3 Parmenidean Inheritance

3.3.1 Advancing Parmenides' Criteria

Zeno's paradoxes not only presuppose the criteria Parmenides established for philosophy but also develop them further. Let us start with consistency before examining the role of rational admissibility and of the principle of sufficient reason in Zeno's paradoxes.

3.3.1.1 Logical Consistency

Zeno seems to be the inventor of philosophical paradoxes in the form we know them today. One necessary condition for setting up paradoxes is that the law of non-contradiction (in one form or another) must hold. The structure of a paradox can be said to be an apparently sound proof of an unacceptable conclusion. But why is the conclusion of a paradox unacceptable? Either because it is inconsistent in itself, or because it is inconsistent with some state of affairs or with some other principle or conviction we hold. So for a paradox to work it must be clear that getting entangled in a contradiction is a proof that our reasoning is no longer valid. And Parmenides' poem is the first place where a version of the principle of non-contradiction is methodically employed as a criterion for reliable knowledge.[21]

Zeno would not be able to set up *paradoxes*, if the principle of non-contradiction were not already firmly established.[22] Thus Parmenides' poem seems to be an important basis for Zeno's use of the principle of non-contradiction as a criterion that automatically excludes everything from philosophical inquiry which does not satisfy this principle.

But Zeno's paradoxes not only rest on the principle of non-contradiction and reinforce it by taking it as a criterion. He also develops the immediate

prefers not to commit himself on the question of whether Zeno's paradoxes also challenge Parmenides (p. 181).

21 As in the preceding chapters, I understand the principle of non-contradiction here in a broad sense so that it does not entail any implications about propositional logic.

22 This does not mean that it must be explicitly laid out, only that it is systematically used and generally accepted.

argumentative use of this criterion further. In Parmenides' poem we find the following argument structure: Being is not-F because it cannot be F: Parmenides argues for a certain understanding of Being by showing that the contrary assumption leads to absurdities. We can see this, for example, from his argument in fr. 8, lines 3–14 for the assumption that Being is ungenerated:

> Being is ungenerated and indestructible . . .
> For what generation will you seek for it?
> How, whence did it grow? That it came from what is not I shall not allow
> You to say or think – for it is not sayable or thinkable
> That it is not . . .
> Nor will the force of conviction allow anything to come to be from being,[23]
> beyond itself; wherefore neither to have come into being
> nor to perish has Justice loosened her fetters to permit it.

The way this argument works is that it examines the two possibilities of generation (and destruction) – what has come into being could come either from what is not or from what is.[24] By showing that both possibilities will get us into trouble with the results so far, it is demonstrated that Being cannot be generated.

Prima facie, this argumentative structure may seem like Zeno's paradoxical method, and so Parmenides' poem may seem to show features similar to Zeno's paradoxes. However, Parmenides in fact claims that "you cannot think p because this conflicts with q, which I have established before". He does not, like Zeno in his paradoxes, start from the position of the opponent, take up the opponent's claims, and then show that accepting these claims entangles the opponent in a contradiction. Thus Zeno goes one step further than Parmenides by showing that some assumptions, like plurality, are not only inconsistent with respect to some Eleatic position, but are inconsistent in themselves.[25]

What about Parmenides' treatment of the realm of *doxa*, however? Does this not show the opponent's position to be inconsistent, even if it is left to the reader to draw this conclusion? Yes, it does, but again the realm of *doxa* is shown to be inconsistent with what Parmenides has established in the poem before, in the realm of *alêtheia*; it could not be understood as being inconsistent if this truth had not been sketched beforehand.[26] Zeno, by contrast,

[23] I am following the editions of Karsten and Reinhardt, who emended *mê* to *tou* in line 12.

[24] As we saw in the preceding chapter, the full argument also includes the principle of sufficient reason.

[25] This is the general way Zeno's paradoxes work. However, in some of the paradoxes of plurality we will see that from this 'neutral' starting point he wants to establish an Eleatic One.

[26] Accordingly, we should expect the way of truth to come before the way of *doxa* in Parmenides' poem, as fr. 8 makes clear this is the case. The account of the two-headed

attempts to show that, independent of what was established by Parmenides in his poem, the assumptions of plurality and motion do not get off the ground, because assuming plurality and motion results in inconsistent claims and the opponent of Eleatic philosophy thus contradicts himself.[27]

3.3.1.2 Rational Admissibility

We saw in the previous chapter that this criterion implies three claims: (1) the standard for judgement is *logos*, reason: it is by using our own reason that we can test whether a statement holds true; (2) reason trumps any other authority; (3) reason is a standard that can be generalised, so that a thorough examination of a claim by you and by me should lead to the same result.

This criterion is strengthened by Zeno's paradoxes, since they show that if somebody uses only his or her reason in giving an account of plurality, motion, or change – not building on any authority or belief – they will have to admit that we get into the inconsistencies which Zeno's paradoxes point out. But Zeno also provides us with a further development of rational admissibility: not only do the results of his paradoxes not fit our everyday experiences, they also show that our everyday experience is actually not an adequate criterion for judging an ultimate explanation of what there is (which must be such that it can be known). Fr. 8 (DK29 A21) tells us that: "Those who introduce plurality put their confidence in its self-evidence; for there exist horses and men and a variety of individual things, and the aggregation of these produces plurality. This self-evidence Zeno therefore attempted to overthrow sophistically."[28] So yes, our senses and our beliefs tell us that there is plurality and motion. And as I tried to show in the previous chapter, this is actually not doubted by the Eleatics. But if we try to give an *account* of plurality and motion, if we try to explain how we can understand the world of many changing things, we get into all kinds of inconsistencies. For example, Zeno attempts to overthrow the self-evidence of our knowledge of motion by showing that in order to cover a finite distance we seem to have to cover infinitely many sub-distances, which he understands as a contradiction. Accordingly, he tries to demonstrate that we cannot rely on our senses and our beliefs as a judge of what is an adequate object of knowledge. We can rely only on our reason.[29]

mortals in fr. 6 may be closest to what Zeno does in his paradoxes, since there the mortals themselves seem to claim that Being and non-Being are the same and not the same. However, it is from Parmenides' strict account of what we should understand by Being that their claims seem to confuse what Parmenides considers to be Being and non-Being.

27 For the assumption that Zeno did not want to argue from a particular position in order to give as strong a refutation as possible of the notion of movement, see Lee, p. 9. Some of Zeno's arguments can be seen as full *reductio ad absurdum* arguments.

28 Philoponus *In Phys.* 42.9; cf. Lee, p. 17.

29 Hence, I think Curd 1998, p. 172, n. 118 misjudges the force of Zeno's argument when she writes that his arguments against motion "do not seem to have been of serious concern to other Presocratic philosophers of Zeno's time. The obvious possibility of motion (as reported by the senses) provides evidence that Zeno's arguments are unsound". The mere

The fact that Parmenides' philosophy might be in conflict with our experience is not a problem for the Eleatics.[30] Rather, Zeno's paradoxes make it explicit that rational admissibility is fundamentally what counts in philosophy; refutability by sensory experience cannot be an ultimate criterion. And Parmenides' austere account of what there is – only the One, absolutely simple Being – certainly seems not to fit our everyday experiences. Rather, it is *para-dox* in the original Greek sense of the word[31] (i.e., strange or shocking, not fitting common opinion or expectation, and thus needing explanation). It is the opposite of what is *endoxos* (i.e., what is generally approved or acknowledged). But Parmenides and Zeno show that what seems to be *endoxos* is what is truly paradoxical.

3.3.1.3 The Principle of Sufficient Reason

The principle of sufficient reason is the one criterion employed by Parmenides which is least clearly taken up by Zeno. Parmenides uses it most famously for his argument against generation. In this context we do not see it taken up by Zeno at all (on generation Zeno is interestingly silent). But Parmenides also uses this principle in order to infer the indivisibility of Being from its homogeneity. This argument is clearly in the background of Zeno's paradoxes concerning divisibility: Zeno's claim that if something is divisible anywhere then it is divisible everywhere can be best understood against the background of Parmenides' reasoning that since there are no internal differences, there is no more reason for something to be divisible here rather than there. This reasoning is exactly what Zeno takes up in his paradox, combining the lack of differences with troublesome consequences if we consider division not indicating a possibility but a resulting state: if something is divided here, it is then also divided there, and there, *ad infinitum*. However, as we will see below, Zeno's focus in these paradoxes is not that divisibility will get us into trouble because there is no sufficient reason for division. Rather, he tries to show that no matter how we understand the possible parts of Being, we will get into problems. Zeno thus assumes Parmenides' sufficient reason argument for indivisibility and supports it by pointing out the paradoxical consequences the assumption of divisibility would have;[32] but he is not developing the sufficient reasoning argumentation further.

possibility of motion 'occurring' is not what is at issue, but rather whether motion can be made understandable. Furthermore, Curd's own argumentation later in the same book speaks against excluding Zeno's paradoxes of motion as being of concern for the atomists, since she herself thinks that sense experience is something that is often not acknowledged as a valid criterion for what is true by Presocratic philosophers.

30 See also Taylor's 1999 edition, p. 74.

31 As we find the word used, for example, by Xenophon *Instit. Cyri* VII, 2, 16, and Demosthenes *Orat.* 3, 10.

32 See also Aristotle *De gen. et corr.* 316a24–34 for the problems which the assumption of actual infinite division seems to raise.

3.3.2 Deepening of the Challenge Parmenides Poses

Parmenides challenges the natural philosophers by trying to establish that only the one simple Being is, while everything required for natural philosophy, such as time, space, motion, and change, cannot be accounted for consistently given the Parmenidean criteria for philosophy. This is what I call the 'challenge' of Parmenides, part of which becomes explicit in Parmenides' fr. 8, where we are shown that we cannot think of temporal and spatial differences and that the One must be *akinêton*, as well as in Parmenides' evaluation of the world of *doxa*. This challenge is deepened by Zeno, who shows that these notions crucial for any kind of natural philosophy are not only inconsistent from the point of view of the Parmenidean framework. Even if we put the Parmenidean doubts about them aside, they will turn out to be internally inconsistent and thus not intelligible – at least this is what Zeno aims to demonstrate. By developing Parmenides' methodology further (employing and widening his criteria for philosophy), Zeno also deepens the challenge of the crucial notions of natural philosophy, and thus of its content, which Parmenides launched.

In his paradoxes of plurality, Zeno shows that pluralistic assumptions, which are necessary for any kind of natural philosophy, are inconsistent in themselves and that, as a consequence, only a Parmenidean One can consistently be thought.[33] This conclusion presupposes something we saw that Parmenides also presupposes: that we are working with a set of two elements, the one simple Being on the one hand, and plurality, including time, space and motion, on the other; *tertium non datur*.

By showing that time, space, motion, and plurality are inconsistent in themselves, Zeno makes it even harder for natural philosophers to prove that they are legitimate objects of philosophy. However, he thus also provides the basis for a further analysis of these notions. For the paradoxes show that if we want to introduce time, space, and motion into our philosophical investigation, we must be able to conceive the relationship between a certain time, a certain space, and a certain motion and the parts thereof, as well as of the relationship between different motions. While all of these seem equally to lead into trouble, they nevertheless give us some idea of which aspects and relationships might be crucial for a notion of motion, time, and space to take into account.

Let us look more closely now at the surviving paradoxes of Zeno.

3.4 The Fragments, Their Sources, and Their Connection

There obviously existed numerous paradoxes of Zeno,[34] but only a few have been preserved in our times. They can be divided into three series, the

[33] Frr. 1–3 in Lee.

[34] See DK29 A15; Kirk, Raven, and Schofield 1983, pp. 264–5; Barnes 1982, p. 233; Sorabji 1983, p. 321.

paradoxes of plurality, the paradoxes of motion, the paradoxes of *topos*, and, in addition, the single paradox of the falling millet seed.[35] Our main sources for these fragments are Aristotle and his three commentators Themistius, Simplicius, and Philoponus, of whom only Simplicius claims access to a work of Zeno.[36] Of Zeno's own words, we only have a few sentences with the paradoxes of plurality in Simplicius, and a single sentence with the paradoxes of *topos* in Diogenes.[37]

Many scholars have tried to fit the different paradoxes of Zeno into a systematic programme. Owen sought to reconstruct one system for all the Zenonian paradoxes we know: assuming, just as Parmenides did, that only the single One exists, Zeno's programme, according to Owen, was "to work out an exhaustive list of possible ways of dividing things [in space and time] and to set about refuting all the possibilities separately".[38] Against this scheme, Barnes argues that not only do we not have any ancient testimonies that Zeno's arguments were meant to be parts of one big overarching argumentative structure, but also that it would make for an argumentative building full of holes.[39] Other scholars have restricted their system to the four paradoxes of motion.[40]

But there seems to be a good reason why antiquity did not pass on Zeno's paradoxes as part of one overarching argumentative structure, for in one way or other all these schemes are wanting in their interpretations of the paradoxes. This should become clear in the following analysis of the paradoxes and in the reconstruction of Aristotle's reaction to them in the last chapters. For the time being, let me simply refer to Kirk, Raven, and Schofield, who comment on one very popular scheme for the four paradoxes of motion according to which they

> formed two pairs: one pair (the Stadium and the Achilles) assumed that space and time are infinitely divisible, the other (the Arrow and the Moving Rows) that they consist of indivisible minima; and in each pair one argument produced absurdities in the idea of a body's motion considered just in itself, the other in the idea of its motion considered relative to the motion of another body . . . Such schemes are undoubtedly

[35] The paradoxes of *topos* can be found in DK29 A24 and B4, and Lee frr. 13–18; the paradoxes of plurality in DK29 B1–3 and A21–3, and Lee frr. 1–12; the paradoxes of motion in DK29 A25–8, and Lee frr. 19–36; and the paradox of the falling millet seed in DK29 A29, and Lee frr. 37–8. For the division see Lee, p. 9.

[36] See Simplicius *In Phys.* 140.27 and Lee, p. 3. Aristotle is our primary source for the paradoxes of motion, place, and the millet seed, and Simplicius is the primary source for the paradoxes of plurality.

[37] See DK29 B1–4. Cf. Kirk, Raven, and Schofield 1983, p. 266; Lee, p. 29; Sorabji 1983, p. 334.

[38] Owen 1957–8, p. 200.

[39] Barnes 1982, pp. 233–4; cf. also Stokes 1971, pp. 189–91 for problems with Owen's scheme.

[40] See, for instance, Salmon 1980, p. 35 or Eberle 1998.

> attractive, but none has any ancient authority, nor have they withstood critical scrutiny very well. (Kirk, Raven, and Schofield 1983, p. 265)[41]

Accordingly, I will not try to fit the four paradoxes of motion into one of the systems suggested, nor will I try to give an overarching architecture for all of the paradoxes. I will, however, demonstrate that the paradoxes of motion form a systematic unity in so far as there are two basic problems underlying all the different paradoxes: how can the relation between whole and part be thought in the case of continua like movement, space, and time? And how can we conceive of the relation between time and space with respect to motion? The paradoxes of plurality examine various versions of the fundamental problem of how something can be both one and many, which includes part-whole problems.

3.5 The Paradoxes of Plurality

Roughly speaking, we can understand the paradoxes of plurality as arguing against two main ways one can be a pluralist:[42] either we postulate one thing which has many parts, being what we can term an 'internal pluralist', or we assume many individual things that are not themselves parts of one thing, being what we can call an 'external pluralist'. The fragments showing that the assumption of divisibility is paradoxical are mainly directed against internal pluralists; these are frr. 1–3 and 7 in Lee's edition. Let me give fr. 1 (in Simplicius' version) and fr. 2 as examples:

> καὶ ὁ Θεμίστιος δὲ τὸν Ζήνωνος λόγον ἓν εἶναι τὸ ὂν κατασκευάζειν φησὶν ἐκ τοῦ συνεχές τε αὐτὸ εἶναι καὶ ἀδιαίρετον "εἰ γὰρ διαιροῖτο, φησίν, οὐδὲ ἔσται ἀκριβῶς ἓν διὰ τὴν ἐπ' ἄπειρον τομὴν τῶν σωμάτων".
>
> And Themistius says that Zeno's argument tries to prove that what is, is one, from its being continuous and indivisible. "For" runs the argument "if it were divided, it would not be one in the strict sense because of the infinite divisibility of bodies".
>
> (fr. 1, Simplicius, *In Phys.* 139.19–22; Lee's translation)
>
> For, he argues, if it were divisible, then suppose the process of dichotomy to have taken place: then either there will be left certain ultimate magnitudes, which are minima and indivisible, but infinite in number, and so the whole will be made up of minima but of an infinite number of them; or else it will vanish and will be divided away into nothing, and so be made up of parts that are nothing. Both of which conclusions are absurd. It cannot therefore be divided, but remains one. Further, since it is

[41] This scheme is employed, for instance, by Salmon 1980 and Heath 1921, I, p. 275.

[42] In Lee's edition, which also takes up passages not included in Diels-Kranz, we find twelve paradoxes of plurality.

> everywhere homogeneous, if it is divisible it will be divisible everywhere alike, and not divisible at one point and indivisible at another. Suppose it therefore everywhere divided. Then it is clear again that nothing remains and it vanishes, and so that, if it is made up of parts, it is made up of parts that are nothing. For so long as any part having magnitude is left, the process of division is not complete. And so, he argues, it is obvious from these considerations that what is indivisible, without parts, and one.
> (fr. 2, Simplicius, *In Phys*. 139.27–140.6; Lee's translation)[43]

The paradoxes against internal pluralists show that if we assume a one to be divisible and thus to have some kind of parts, this one will be at the same time many.[44] But being one and many are mutually exclusive notions, due to the Eleatic understanding of the philosophical criteria and the logical operators available. It is only with the adjustment of the principle of non-contradiction in Plato that their conjunction need no longer be seen as a contradiction.[45]

Zeno's use of negation may be seen as a first step in this direction, but he does not yet seem to draw any consequences from this. While the lack of his own words transmitted to us make it difficult to assess Zeno's usage of negation, the claims that have come down to us as his own wording in general negate specific things (like "not smaller", "not larger", "neither moving in the *topos* where it is nor in a *topos* where it is not") and thus suggest that Zeno is not using an extreme negation. This also fits fr. 1, quoted above, where he is not simply negating "being one", but "being one in the strict sense" so that the outcome seems to be "being one in a less strict sense". However, we will see below that Zeno seems to have simply equated "not moving" with resting in his arrow paradox, and thus treats motion and rest, like one and many, as contradictories.

The paradoxes against internal pluralists aim to show that if we assume internal plurality (or complexity) – in its simplest form, a whole that has parts – and that the whole can be identified with its parts,[46] we will always come up against the problem that one is also many. But this is, as fr. 8 makes explicit, *adunaton*, impossible. Fr. 1 points out that only if the one is

[43] Porphyry attributes fr. 2 to Parmenides, but Alexander and Simplicius think it more likely to be by Zeno; see also Lee, p. 12. This ascription to Zeno rather than Parmenides is also clearly supported by our results.

[44] Cf. also Plato *Parmenides* 129a2–3 and Lee 2014, p. 264.

[45] See Chapter 5. Hence, Socrates' playful demonstration in Plato's *Parmenides* 129c–d that his being one man and at the same time many parts is no problem, does not aim to show solely that only the Forms do not have opposite features. Rather, it is also an ironic reminder that one thing being simultaneously many was indeed inconceivable within Zeno's and Parmenides' framework and required the developments we find in Plato's *Republic* (see Chapter 5) in order to be thought unproblematic.

[46] Identifying the whole with its part seems to be a necessary condition for these paradoxes of Zeno that the one is also many.

continuous and indivisible[47] will we be dealing with what is really, *akribôs* (in the strict sense) or *bebaiôs* (firmly), one.

Furthermore, the assumption that the one is divisible and divided[48] also shows that we cannot conceive these parts in any consistent way (see fr. 2). For we can assume (I) that the process of division *ad infinitum* has taken place, then either the parts (Ia) must be divided until there is nothing left (but then we will have to assume that these parts of nil extension make up an extended whole, which fr. 2 claims to be absurd) or (Ib) we have reached indivisible minima which have some extension; then, according to fr. 2, the one finite whole will be made up of infinitely many minima, which seems equally "absurd" (*atopos*). Here an infinite division seems to lead to infinitely many extended parts; and given that these parts are extended (and are not of an ever-decreasing size, as they are in a convergent series), when added back together we seem to get an infinitely extended whole. Infinite divisibility thus seems to lead to infinite extension.[49]

Alternatively, (II) the process of division has not been performed *ad infinitum*; in which case we end up with parts having some extension that are in principle further divisible. But this only means that the division is not yet complete, and our parts are undetermined because of this.[50]

All three possible routes of dealing with the division of a continuous one, (Ia), (Ib), and (II), seem to be problematic, but all three are endorsed as viable in the later tradition. Route (Ia), where we have parts of nil extension which make up an extended whole, seems to be just what current mathematics assumes, as Grünbaum points out: a line, and thus something extended, consists of extensionless parts. Since extension is simply a feature of the set making up the line, not of any of the individual members of this set, there is, according to Grünbaum, no paradox. Route (Ib), where there are indivisible minima, is the route the atomists will pursue. And finally (II), that with physical continua we will always end up with parts of some extension and the division will never have been performed completely, is what Aristotle claims to be the necessary outcome of such divisions. We will come back to all three traditions.

47 Zeno here takes up Parmenides' *sêmata* of being *oulon mounogenês* (in the form it took in the deduction of Parmenides' poem, namely as indivisible) and *sunêches*, in order to prove that what is one.

48 In Zeno's paradoxes something's being divided is immediately inferred from something's being divisible, an inference that will be put under certain constraints in Aristotle, as we will see in Chapter 7.

49 On Zeno's assumption that infinitely many minima as parts would lead to infinite extensions, see also Themistius *In Phys.* 91.29–30 and Simplicius *In Phys.* 492.16–19.

50 See the penultimate sentence in frr. 2 and 3, and also Aristotle's version of this dilemma in *On Generation and Corruption* I,2, 316a14 ff. Cf. also Owen 1957–8, pp. 201–11. Alternative (Ib) is problematic, but in fact not inconceivable.

Zeno's immediate attack against external pluralists – those who assume a plurality of individual things – shows that they get into either 'quantitative' or 'qualitative' paradoxical results. As for quantitative paradoxical results, there is what we can call a 'discrete' version:[51] it claims that if there is a plurality, it must be both finite (the number of things they are) and infinite (for in order to be separate things, there must always be some other thing in between).[52] And there is what we can call a 'continuous' version in frr. 9 and 10, which claims that the assumption of a plurality of things implies that they are both large, so as to be infinite in magnitude, and small, so as to have no magnitude at all.[53] The qualitative paradoxical result is reported to us only as a result in Plato's *Parmenides* (127e1–4): the assumption of plurality entails that things are like and unlike.[54]

Assuming a plurality of things does not relieve the pluralist of the burden of assuming a one, since a plurality is nothing but many ones, as is shown by frr. 6 and 8. If the pluralists dismiss Parmenides' idea of a One as basic, they must be cautious not to undermine their own position: they themselves must assume a (indeed many) one(s), for the unit is also the basis for every plurality; without the notion of a one, we could not have the notion of a plurality.[55]

From the attack against the internal pluralist, it is already clear that the one the external pluralists need to employ as a basis for plurality cannot be divisible, since otherwise this one will not be a unit, but again a plurality: "If there were no indivisible unit, there could be no plurality, for plurality consists of a plurality of units" (fr. 3).[56] On the other hand, if the basic ones of the pluralists are really indivisible, it seems they can have no size, for otherwise they are at least theoretically divisible.[57] But if such a one has no size, then, as frr. 4 and 5 point out, it cannot be the one needed for the plurality of spatially extended things, which the pluralists assume, since it would not add or subtract anything. Hence it could not exist in the way a one that is the basis for a physical plurality needs to exist;[58] such

51 Lee fr. 11; DKB3.

52 Cf. also Owen 1957–8, pp. 210–11. This latter requirement – in order for there to be many things, there must be something that separates them – is one reason for the atomists to introduce the notion of a void. Aristotle attempts to solve this problem with the help of distinguishing a thing from its limits, see Chapter 7.

53 See Hasper 2006, pp. 51–8 for a reconstruction of this paradox.

54 Fr. 12 in Lee's edition (Plato *Parmenides* 127e1–4); cf. also Plato's *Phaedrus* 261d. For possible reconstructions of this paradox, see Lee, p. 32; Cornford, p. 68; Lee 2014.

55 As Owen 1957–8, p. 200 expresses it: "if you want to say that there are a number of things in existence, you have to specify what sort of thing counts as a unit in the plurality. If there can be no such individuals as you claim there can be no such plurality either".

56 In Chapter 8 we will see that Aristotle deals with this problem by claiming that a one must be *treated* as indivisible, but not necessarily be indivisible.

57 See also Owen 1957–8, p. 210.

58 I understand frr. 4–5 as employing assumptions that Zeno's opponents would make; so the claim that whatever does not have spatial extension does not exist is an (at least implicit)

a one can only fulfil its function as a basis for extended things if it is itself extended. As we have seen in the previous chapter, Parmenides' One is not only not part of a plurality, but it is also not a physical thing, so the problem sketched does not arise for Parmenides' position.

All the paradoxes of plurality point out problems in the relation between the many assumed by the pluralists and a one, which the pluralists must also assume – either because they assume one whole with many parts, or because a plurality of things is nothing but multiple ones. The only assumption that does not lead to these inconsistencies is that of a single One, which is simple (fr. 8) and indivisible (frr. 1–3) – Parmenides' One.

Mathematical interpretations of the plurality paradoxes assume that the paradoxes concerning internal pluralism do not raise any genuine problems, since in the realm of mathematics it is unproblematic to understand extended things, like lines, as consisting of parts with zero extension. Such an assumption is unproblematic because extension is simply a feature of the set making up the line, not of any of the individual members of this set; and for the whole to have the property of extension we do not need every member of it to possess this property.[59] According to Grünbaum's mathematical interpretation, Zeno's paradoxes only show that we need modern set theory in order to deal with these alleged problems of divisibility. Thus Zeno's account of whole and parts is simply taken up in the mathematical interpretations without any further analysis.[60] It is not clear, however, that what holds true of mathematical

assumption of the pluralists against whom Zeno is arguing, not a premise of Zeno himself. For Zeno's fr. 8 clearly distinguishes between a one as a unit of a plurality and the One which is not such a unit. That Zeno could employ different notions of a one in his argument is also clear from fr. 1, which, as we saw, distinguishes a proper notion of a one from a one that is not *bebaiôs* or *akribôs* one (according to fr. 7 these latter ones are *mêden tôn ontôn*, "nothing of the Beings"). See also Makin 1993, p. 21 for an interpretation according to which Zeno's argument does not threaten the One defended by the Eleatics.

59 See Grünbaum 1968, ch. 3.

60 If we look back at the three possible ways resulting from the divisibility of something continuous above, we see that Grünbaum adopts the route on which the extended whole would have to be made up of unextended parts, and points out that there is a way to allow for such part-whole relations in mathematics. He thus assumes that the same holds in the physical realm without undertaking any further analysis. By contrast, the atomists, who chose the route on which the continuous whole is made up of indivisible minima, arrive at this choice by making it clear that in order to separate one thing from another in the physical world, we need not employ another thing (as Zeno seems to assume not only in fr. 11, but also when claiming that infinite divisibility would lead to infinitely many non-converging minima), but can actually employ the void. This can be seen as an implicit analysis of Zeno's assumption of how physical things can be separated from each other, and a suggested alternative. Finally, Aristotle in his *Physics* explicitly points out the conceptual problem with Zeno's assumption that the unrestricted divisibility of something continuous implies that the process of division can indeed come to an end.

things can also be said of physical things – that an extended physical thing can *consist* of unextended physical parts.[61] An alternative view would be that such phrasing is merely a mathematical description. Nor is it clear how to understand the problems connected genuinely to *external* pluralism as problems that simply require modern mathematics.[62]

In order to understand the *conceptual* problems raised by Zeno's plurality paradoxes, we will briefly analyse a few central notions which Zeno inherited from Parmenides. Looking back at Parmenides' *sêmata* in his fr. 8, we can see that Zeno prominently takes up being *oulon* (being a whole), *adiaireton* (divisible/divided), *suneches* (continuous), *hen* (one), as well as *homoion* (homogenous)[63] – concepts which play a significant role in the background of Parmenides' philosophy. However, they were never clarified and, as a consequence, are ambiguous. Let us see how Zeno deals with some of these notions.

Zeno's paradoxes make explicit an assumption we have already seen at play in Parmenides' account of the One: because the One is *homoion*, everywhere alike, it is indivisible. Zeno gives a further argument for this inference, by showing the consequences that dividing a homogenous One would have: since such a One is everywhere alike, if it were divisible, it would equally be divisible everywhere.[64] And it seems that if it were divisible everywhere, it could be divided everywhere. Also the Greek verbal adjectives ending in *-tos* may suggest that we can infer the state of being divided from the state of being divisible, since they can denote a possibility (what can be V-ed) as well as a passive resulting state (having the force of a perfect passive participle: what has been V-ed). But being divided everywhere leads to absurdities – the absurdity that the parts resulting from division could not have any extension (for otherwise the one would not be divided everywhere), implying that something extended would have to consist of non-extended parts. So Zeno concludes that the one must be indivisible. As we will see later on, Aristotle agrees with the assumption that if what is *homoion* and divisible, it must be divisible everywhere alike, but nevertheless tries to show that we do not have to draw this devastating consequence. For this consequence follows only if it is

[61] So even if mathematics gives us a consistent theory for dealing with physical magnitudes on some level, this does not mean that it can capture what is specific for physical magnitudes.

[62] This lack of applicability to the problems of external pluralism is probably one reason why Grünbaum calls his chapter on the plurality paradoxes "Zeno's Metrical Paradox of Extension", thus giving it a title that refers to the problems raised against internal pluralists only.

[63] Although this last one is not explicitly called a *sêma* in Parmenides.

[64] So the second part of fr. 2; a principle of sufficient reason is at work here.

assumed that the possibility of being divisible everywhere entails the possibility of being divided everywhere – an inference that will be questioned by Aristotle.

With regard to the notion of *hen*, Zeno seems to introduce something new into the Eleatic framework. The idea that the One needs to be continuous and indivisible in order to be *bebaiôs* or *akribôs* one introduces a distinction between different kinds of ones, one in the strict sense and one in a less strict sense.[65] This distinction can also be seen as a first step on the way to distinguishing operators from operands. For if something is one but not strictly speaking, it is clearly not the One, but it is some *x* of which we can also say that it is one. So 'one' becomes a predicate, 'being one', that can be said of different things (of whatever the pluralists claim to be their basis). But as a predicate it is no longer the name for a basic entity – the function that 'One' seemed to have in Parmenides[66] – but rather something operating on entities. Zeno introduces here part of the grounds for what will ultimately lead to a distinction between basic entities and operators.[67] We saw in the chapter on Parmenides that keeping operators and arguments apart would require a notion of difference unthinkable within Parmenides' philosophy and that the lack of this distinction led to severe problems. Zeno does not change the basic Eleatic concepts explicitly, but his distinction between the One and one in a non-strict sense is a distinction that will ultimately contribute to a change of the Parmenidean framework.

Of crucial importance for our story too is Zeno's usage of the notion 'whole and part'. Parmenides argues for Being to be *oulon* (whole) by arguing that *oulon* is not *diareton*, divided or divisible, obviously into parts. And this is justified by claiming that Being is all alike, not allowing for any difference as this would introduce non-Being, not being some *x*. The whole Parmenides envisions is a whole without the possibility of any parts. But it is never clarified to what extent we can think of it as a whole, if it is neither the whole of parts nor a whole against other wholes (as there is only One). It might be called a whole, then, in the sense of being complete, not missing anything, being in its entirety. But given that 'being complete' is actually mentioned as a separate *sêma* in the poem and that Parmenides leaves the notion of a whole indeterminate, his philosophy seems to allow for different senses of what can be understood by a

[65] The one in the less strict sense, the one which gets us into trouble according to Zeno, seems to be what people take to be one, but is not genuinely one. It may be prefigured in Parmenides' way of *doxa*, fr. 8, lines 53–4, where we are told that mortals "decided to name two forms, of which it is not allowed to name only *one*"; but it is only in Zeno that we find an explicit distinction between one in a strict and in a less strict sense.

[66] While 'one' is named as a *sêma* of Being in Parmenides' poem (fr. 8), we saw in Chapter 2 that being a *sêma* is clearly distinguished from being a predicate. Furthermore, 'one' is not the result of any deduction in fr. 8 (as are the other *sêmata*), but rather seems to refer to Parmenides' Being as such.

[67] Even if today we do not necessarily think of predicates as operators.

'whole' and, consequently, what might be understood by 'parts' of a whole (if they were possible); possible part-whole relations are never specified. This ambiguity is what allows Zeno implicitly to employ two mutually exclusive understandings of whole and part, which partly account for the trouble we get into with some of the paradoxes of plurality as well as with some of the paradoxes of motion.

If we look, for example, at the paradoxes given in frr. 2 and 3, we see that on the one hand, the whole is presupposed to be prior to the parts, since it is only by dividing the one that we gain the parts. On the other hand, the one is meant to be constituted by the parts, since the puzzle is either how non-extended parts can *make up* something extended or how infinitely many parts can *make up* something finite. You may think that this switch in the relation of whole and part is unproblematic. After all, if I take apart my lyre, I should be able to put it back together to form the original whole. In the case of the lyre, this works because the whole was originally built from discrete parts, each of which has its unique function (like the strings, the two arms, and the cross-bar). But if we look at a whole that is not originally made out of (functionally) different parts, at continua, like a piece of wood, things look different: if we cut a continuum into parts, we may not get a whole out of it again in the same easy way (even if we use glue, it does not have the same unity as before).[68] The parts of wood do not constitute a whole in the way they do in the case of the lyre, and thus cannot necessarily be taken as the building blocks for the whole. With continuous things, parts are posterior to the whole, while the idea of parts making up a whole presupposes parts prior to the whole. Thus, while Zeno raises problems for the relation of whole and parts with respect to continua, he himself can be shown to work with an ambiguous notion of whole and part, a notion of *oulon* that is insufficiently clarified: one kind is whole because it has not been divided, the other because has been put together from prior parts. We will see how this ambiguity also leads to problems with Zeno's paradoxes of motion.

3.6 The Paradoxes of Motion

In accordance with our conceptual interpretation, I will begin by reconstructing Zeno's paradoxes of motion – probably the strongest rejection of the intelligibility of movement in ancient times – in a way that clarifies the notions which produce inconsistencies in each paradox. The first two paradoxes demonstrate problems which arise once we try to divide movement into

[68] Continua like water or syrup will indeed give us back the original whole, but this fact seems to be specific to liquid continua and things like moulding dough, and does not apply to continua over which a locomotion will take place, like wooden planks, earth, or hard asphalt.

parts,[69] while the third shows the difficulties of composing a whole movement out of given parts. In addition, these paradoxes will turn out to be problematic because time and space are not equally taken into account. This last problem is also the basis for the fourth paradox of motion.

3.6.1 The Dichotomy: Passing Infinitely Many Segments in a Finite Time

When Aristotle, our main source for Zeno's paradoxes of motion, deals with Zeno's paradoxes in *Physics* VI,9, he does not really set out the dichotomy paradox, but only gestures towards it (239b11–14). So this paradox must be understood with the help of his reference back to 233a21–6 and its resumption in 263a4–11:[70]

> τέτταρες δ' εἰσὶν οἱ λόγοι περὶ κινήσεως Ζήνωνος οἱ παρέχοντες τὰς δυσκολίας τοῖς λύουσιν, πρῶτος μὲν ὁ περὶ τοῦ μὴ κινεῖσθαι διὰ τὸ πρότερον εἰς τὸ ἥμισυ δεῖν ἀφικέσθαι τὸ φερόμενον ἢ πρὸς τὸ τέλος, περὶ οὗ διείλομεν ἐν τοῖς πρότερον λόγοις.
>
> There are four arguments of Zeno about movement which cause problems for those who want to solve them: first, the one about the 'not-moving', on the grounds that before the moved thing gets to the end it must have arrived at half of it; this we have analysed in the preceding investigations [namely in 233a21 ff.]. (Aristotle *Physics* VI,9 239b9–14)
>
> διὸ καὶ ὁ Ζήνωνος λόγος ψεῦδος λαμβάνει τὸ μὴ ἐνδέχεσθαι τὰ ἄπειρα διελθεῖν ἢ ἅψασθαι τῶν ἀπείρων καθ' ἕκαστον ἐν πεπερασμένῳ χρόνῳ.
>
> For Zeno's argument turns out to be wrong [in assuming] that it is not possible to go through the infinite or to touch each single [part] of the infinite in a finite time. (Aristotle *Physics* VI,9 233a21–3)
>
> τὸν αὐτὸν δὲ τρόπον ἀπαντητέον καὶ πρὸς τοὺς ἐρωτῶντας τὸν Ζήνωνος λόγον, καὶ ἀξιοῦντας, εἰ ἀεὶ τὸ ἥμισυ διιέναι δεῖ, ταῦτα δ' ἄπειρα, τὰ δ' ἄπειρα ἀδύνατον διεξελθεῖν, ἢ ὡς τὸν αὐτὸν τοῦτον λόγον τινὲς ἄλλως ἐρωτῶσιν, ἀξιοῦντες ἅμα τῷ κινεῖσθαι τὴν ἡμίσειαν πρότερον ἀριθμεῖν καθ' ἕκαστον γιγνόμενον τὸ ἥμισυ, ὥστε διελθόντος τὴν ὅλην ἄπειρον συμβαίνει ἠριθμηκέναι ἀριθμόν· τοῦτο δ' ὁμολογουμένως ἐστὶν ἀδύνατον.

[69] There are no given parts; rather they are 'produced' by division.

[70] As the paraphrases of the paradox in Simplicius, Philoponus, Themistius, and *De Lin. Insec.* are very close to Aristotle's report, they are not cited here. *Topics* 160b7 also seems to refer to this paradox; but apart from providing us with one name for it, "the stadium", it does not give us any information about the paradox and can thus be left out. As the name "the stadium" is also used sometimes to refer to the fourth of Zeno's paradoxes about masses moving in a stadium (see Barnes 1982, p. 261), I will stick to the name 'dichotomy' in order to avoid confusion and because of its common use in the discussion, even if it might not be historically correct. See Vlastos 1975a, p. 215, n. 2.

> In the same way one has to reply to those who put forth Zeno's puzzle and claim that, if it is always necessary to go through the half [before one can traverse the whole distance],[71] and these [halves] are infinite [in number], then it is impossible to go through the infinite. Or as the same argument is asked by some in a different way, at the same time as the movement starts across the half, one could count each single half which occurred before, so that we get the result that if something has travelled the whole [half], we have already counted an infinite number; but this is admittedly impossible. (Aristotle *Physics* VI,9 263a4–11)

In order to reconstruct the problem raised by Zeno accurately, we will simply start by going through the given quotations: if something moves over a certain stretch, for example, a runner wants to cover a certain (finite) distance, he must first cover half of this distance. Aristotle does not explain Zeno's argument any further at 239b9–14, but with the help of the two other passages, the paradox can be completed as follows: of the second half of the given distance, the runner must again cover the first half of the remaining distance and then again the first half of the still remaining distance, and so on. So he will have to pass an infinite number of spatial segments before reaching the end. In 233a21–6 Aristotle states that these infinitely many spatial segments cannot be covered in a finite time. In 263a4–11 this paradox is presented in two forms, which are often called 'progressive' and 'regressive' in the literature.[72] The first variant, the progressive form, is just the given state of affairs. The second, regressive version seems to intensify this paradox as it shows that the runner must have already gone through an infinite number of spatial segments to cover even the first half of the distance.[73] So while in the first variation a runner can never reach the end of his track, in the second he

[71] Something like these words in brackets must be understood as the background to make the argument understandable, and is added in all translations. Aristotle does not talk of distance here, only of "something infinite". But he mentions distance a few lines below – in 263a18, where he is referring back to our passage, he asks how the question would change if *mêkos* (length) were not taken into account but only time. Distinguishing *mêkos* from time here, Aristotle seems to talk about the only other magnitude involved, namely spatial magnitude. Thus, talking about distance in the passage cited above is justified by this later passage; it makes the problem clearer, and no translation works without it.

[72] See, e.g., Salmon 1980, pp. 32–3; Vlastos 1975a, p. 201.

[73] Wagner, p. 620, takes the first passage cited to present this regressive form; Barnes 1982, p. 262 claims that while the ancient commentators of the *Physics* favoured the regressive version, Aristotle himself preferred the progressive one. In his edition of the *Physics*, Ross, pp. 658–9 gives reasons for both understandings of this first passage. Whichever interpretation we choose, the paradoxical problem stays the same. The objection that Aristotle cannot mean the regressive form here, for he talks about counting and counting always has a first, starting number, can be met by understanding the process of counting as starting with the first half of the whole distance, 1. And then we go on counting the parts which are already included in this first half: before one can travel the first half, 1, one must travel half of this half distance, 2, and before that, half of that, 3, etc. This seems to be the

cannot even get started. In logical terms, we do not get anything new with this second version, however, for both forms seem to raise the following three logical problems:[74]

(1) Covering a *finite distance*, a runner must cover an *infinite* number of *spatial parts*, which, according to Zeno, cannot be done.
(2) This *infinite* number of *spatial parts* must be covered in a *finite time*, which seems to be impossible.
(3) Finally, the finite process of running as a whole seems to involve the completion of an *infinite* number of *actions*, namely to *run* through infinitely many spatial parts, which seems to be an impossible task.

A crucial question here is how to deal with the infinity raised in this paradox, and the relationship between infinity and finitude. Trying to conceptualise movement consistently will thus imply the task of looking for a notion of infinity (and the concepts related to it) which does not lead to the apparent impossibilities mentioned. The first problem (1) will turn out to be a general problem with all magnitudes that are continua; resolving it is only a necessary condition for understanding motion. The second problem (2) is the problem specific to motion and will therefore be our focus. Finally, the third problem (3) may actually be a non-problem – it is highly questionable whether it is a problem facing Zeno at all.

In the following, I will deal with (1) as the necessary condition for understanding motion first, before looking at (2), the problem specific to motion. And finally I will quickly point out why (3) is actually not a problem that Zeno faces with his paradoxes.

(1) The paradox states that before a moving thing reaches the end of some distance, it must cover half of this distance, and half of the half, *ad infinitum*. Talking about the end (*telos*) which the moving thing must reach, and a particular half (or part) which is to be covered, implies that we are dealing with something finite, a limited magnitude. The first problem arises because this finite magnitude seems to contain an infinite number of parts. We are not explicitly told how to spell out this problem further, but with some help from the plurality paradoxes we can understand this problem in two different ways, depending on whether the parts are understood to be infinitely many in the sense that it is always possible to divide them further and further (I) or as being actually infinitely many parts (II).

I. The parts of this whole can be divided further and further without any given end *ad infinitum*. This is problematic because it poses the question of how to conceive of a whole which has an end, and is thus determined in its

process of counting Aristotle is thinking of when he says "one could count each single half which occurred before" in the third passage above.

74 This is not to claim that Zeno himself would have distinguished these three points.

extension, but at the same time is divisible without end, and hence undetermined as its parts and the boundaries between them cannot be fixedly determined. The parts of this whole seem to be arbitrary in number and extension; the whole can be divided into parts in whatever way we like. Going on and on with the division, there is no last part which could be determined – we are confronted with a sequence with no end, in a whole which has an end. Movement understood as such a magnitude necessarily seems to be undetermined for although the whole is determined, its parts are not. These parts are neither defined fixedly in their extension, which can be freely chosen, nor in any qualitative sense.[75] Aristotle's report that movement and change were assumed by his predecessors to be indeterminate is thus not surprising.[76]

II. Alternatively, all the infinitely many parts of this whole may be presumed as given, so that they are all, infinitely many, determined. This seems to avoid the problem of indeterminacy but it poses the question of how these parts can be thought. It seems that if each half of such a whole has been divided infinitely many times so as to get all possible parts, these parts can neither be without extension nor possess any extension: (IIa) these parts could not be without extension for it seems that extensionless parts, that is, points, can never make up an extended whole.[77] (IIb) If, however, they are understood to be extended, why do all of these infinite (non-converging) parts joined together not make up a whole with infinite extension?[78]

Similar to the plurality paradoxes, the dichotomy also seems to suggest that assuming a whole to be always further divisible leaves us stuck with three equally untenable accounts of parts: (α) we must assume parts with nil extension, which raises the question of how even an infinite plurality of unextended parts can ever lead to something extended; or (β) we assume the parts to have some extension, in which case it seems unclear why an infinite number of these parts does not lead to an infinitely extended whole; or (γ) we do not commit ourselves to either possibility, but claim that we could just go on with the division *ad infinitum*, in which case we are facing the problem that the parts are left indeterminate.

In the secondary literature, it seems to be agreed that a part-whole problem in one version or the other is stated by Zeno as *the* problem of the paradox.

75 Every part of a distance will be of the same quality, a spatial part; and while it may be to the right or left of some other part, both parts are not thus given as we may equally choose a half of them as parts.

76 See *Physics* III, 201b24 ff.

77 The relation between an extended whole and an unextended part will also be one of the problems in the third paradox of motion.

78 This is the way that, for example, Salmon 1980, p. 36 understands the paradox, as he eventually reduces the dichotomy to the question: "how can infinitely many positive intervals of time *or* space add up to anything less than infinity?" Cf. also White 1992, p. 10.

Opinions differ only in their assessment of whether this is an inadequate or an adequate description of the state of affairs. The former assessment we find in Hasper, who claims that this paradox is driven by the idea that "a whole of parts is nothing more than these parts together", for which we need a "conceptual cure limiting our tendency to apply this part-whole principle".[79] The latter assessment, that this is an adequate description, can be found in mathematical interpretations, which assume Zeno simply lacked some modern tools that would explain why this adequate account does not actually lead to inconsistencies.

But the part-whole relation sketched is not a problem specific to movement. Rather, it is a problem which motion raises insofar as it is a magnitude which we would call a continuum. We saw essentially the same problem with the paradoxes of plurality, which also demonstrates that the problem with the part-whole relation cannot be specific to motion. This problem, which I call the 'continuum problem' underlies the dichotomy paradox as a structural problem insofar as movement is a continuous magnitude (and the internal plurality paradoxes in so far as we deal with internally continuous things). But the difficulty which movement qua movement introduces is the second main problem mentioned above, the question of how an infinite number of *spatial segments* can be covered in a finite *time*.

That the first problem is not specific to motion can also be seen by looking at the lines following the third passage quoted above. While in the part quoted Aristotle deals with movement and thus with time *and* space simultaneously, he subsequently deals with the problem of whole and part solely in connection with time, and thus changes his topic. With time he discusses a problem which we get with every continuum (not merely with a complex continuum like movement, where we must take into account two different magnitudes, time and space). In his inquiry Aristotle starts with what is specific to motion before switching to the abstract part-whole problem, which the discussion of movement leaves as an open problem. Clarifying the part-whole relationship in question is the *conditio sine qua non* for understanding all continua, and thus also for the 'true nature' of time and space each taken by themselves:

> ἀλλ' αὕτη ἡ λύσις πρὸς μὲν τὸν ἐρωτῶντα ἱκανῶς ἔχει (ἠρωτᾶτο γὰρ εἰ ἐν πεπερασμένῳ ἄπειρα ἐνδέχεται διεξελθεῖν ἢ ἀριθμῆσαι), πρὸς δὲ τὸ πρᾶγμα καὶ τὴν ἀλήθειαν οὐχ ἱκανῶς· ἂν γάρ τις ἀφέμενος τοῦ μήκους καὶ τοῦ ἐρωτᾶν εἰ ἐν πεπερασμένῳ χρόνῳ ἐνδέχεται ἄπειρα διεξελθεῖν, πυνθάνηται ἐπ' αὐτοῦ τοῦ χρόνου ταῦτα (ἔχει γὰρ ὁ χρόνος ἀπείρους διαιρέσεις), οὐκέτι ἱκανὴ ἔσται αὕτη ἡ λύσις.

[79] Hasper 2006, p. 51. Hasper's reduction of this paradox to one not specific to motion can already be seen from the fact that he understands it as working in the same way as the ninth plurality paradox (DK fr. 2).

> But, although this solution [that time is structured in basically the same way as space] is adequate as a reply to the questioner – the question asked being whether it is possible in a finite time to traverse or reckon an infinite number of units – nevertheless, as an account of the fact and explanation of its true nature, it is not sufficient. For suppose the distance to be left out of the account and the question asked to be no longer whether it is possible in a finite time to traverse an infinite number of distances, and suppose that the inquiry is made to refer to the time taken by itself (for the time contains an infinite number of divisions): then this solution will no longer be sufficient.
> (Aristotle *Physics* VIII,8 263a15–22, translation by Hardie and Gaye with modifications)

The secondary literature understands the passage cited to show that Aristotle reduces his first solution, which works with the relation of time and space, to a solution which is only concerned with one dimension, and thus only with the question of how a finite whole can be thought of as containing infinitely many divisions or parts.[80] Thus the secondary literature assumes that once this part-whole problem is solved, the problem of motion is solved as well.[81] It is true that Aristotle thinks that the solution specific to motion is not sufficient for the investigation of only one dimension, as for example time (there will still be a problem left when we consider one continuum on its own). We will see, however, that Aristotle does not *reduce* the solution for movement to the solution which is needed for all continua. Rather, both solutions are necessary to solve the dichotomy paradox.[82] And we cannot substitute one solution for the other. The solution specific to motion is not adequate if we are only talking about a single continuum like time. By contrast, the solution for the part-whole relation is only a necessary condition of conceptualising movement. But with this alone we are not able to solve the problem specific to movement, which is how time and space can be connected consistently.

According to 263a11–18, Aristotle considers his first solution of the dichotomy paradox (233a24–34) to be an adequate solution for an investigation of movement. But it is not a complete answer in so far as movement is also a continuum, as are time and space. Therefore, in order to give a full reply, the problematic part-whole-relationship that belongs to the concept of a continuum in general must be dealt with. Aristotle discusses the part-whole problem using the example of time – a clear indication that it is not a problem specific to movement. But this does not mean that Aristotle's previous solution – that time is structured in the same way as the distance covered – was only *ad hominem* in the sense that it is a solution inadequate to

80 See, for example, Ferber 1995, p. 7.
81 See, for example, Salmon 1980, p. 36.
82 See chapters 7 and 8.

the problem of motion.[83] The problem Aristotle is taking up in 263a15 ff. is one which underlies every problem we may have with a specific continuum, since it concerns the basic structure of all continua. So instead of calling Aristotle's first solution *ad hominem*, we could call it *ad motum,* and the second *ad continuum*. In order to clear up the paradox completely, both answers are necessary.[84]

Furthermore, the secondary literature assumes that the main problem raised by Zeno's paradox is the part-whole relationship in one of the versions (α)–(γ) mentioned above, or that a whole of parts is nothing more than the sum of the parts so that it requires a final part if it is to be limited.[85] However, if we analyse how Zeno derives this problem, we see that he in fact employs two incompatible part-whole relationships: on the one hand, Zeno determines parts as being gained from a preceding whole (for instance, as being a half, a quarter, etc. of the initial whole). The whole finite distance is given, and we derive the parts by dividing it. On the other hand, Zeno assumes the same parts as given elements whose addition (via being covered in the run) is what constitutes the whole. Accordingly, Zeno's argumentation rests on two mutually exclusive premises: (a) that the whole is given and logically prior to the parts, since the parts are merely the result of dividing the whole; and (b) that the parts are given and logically prior to the whole, since it is their sum that constitutes the whole. It is this ambiguity in the understanding of the relation of parts and wholes which constitutes the 'continuum problem' here.

(2) The actual paradox, in so far as it is a paradox of *motion*, arises from the fact that the infinitely many spatial parts, which something moving seems to be forced to cover, must be traversed in a finite time. No matter how the infinity of the parts is understood, the paradox of "not-moving," as Aristotle calls it at *Physics* 239b11–12, is that an infinite distance must be covered in a finite time. If the problem were only that infinity cannot be dealt with, it could be a mere paradox of distance as well, the paradox of the 'not-distance'. As we have seen, the problem of how to deal with infinity applies to all continua, so this would not make it a paradox of *motion*. But with Zeno's paradoxes we are facing the problem that of the two magnitudes which are necessary to determine movement, namely time and space, the latter is thought to be infinite and the former finite.[86] Thus, the time available always seems to be too short to cover

[83] This is, for instance, how Ferber 1995, p. 7 understands 233a21–3.

[84] Cf. also Ross, pp. 73–4.

[85] So, for instance, Hasper 2006, who claims that Zeno's main mistake is that a whole of parts is nothing more than the sum of the parts of which it consists, so that the parts have ontological priority (p. 74), and that the whole thus needs to have a final part if it is to be limited (p. 78).

[86] See also Wagner's commentary on the second passage, 233a21 ff., which claims that the argument "spricht nur der Strecke, nicht aber auch der Bewegungszeit Infinitesimalstruktur zu" (p. 638).

something infinite, while in an infinite time it might be possible to traverse something infinite. It seems that, as a matter of principle, we cannot go through something which is infinite because we are *finite* beings and thus have only a finite time available. Hence, a problematic connection between an infinite spatial magnitude and a seemingly finite time seems to be unavoidable. Note that the problem we are dealing with here is not that because of infinite divisibility we must cover an infinitely extended magnitude. Rather the problem is how an infinite distance can be traversed in a finite time. We can also gather that this is the paradox of motion from Aristotle's formulation of the paradox in the second passage quoted, which is where he first discusses it: "it is not possible to go through the infinite or to touch each single (part) of the infinite in a finite time" (233a22–3).[87]

In order to solve this motion problem and to conceptualise movement consistently, we must clarify the connection between (seemingly finite) time and (seemingly infinite) spatial magnitude.[88] Hence, in the context of the second passage quoted (233a13–21), Aristotle wants to prove that if the spatial magnitude covered is continuous and infinite, so is time, and vice versa. What leads to the motion problem thus seems to be the fact that Zeno does not take time sufficiently into account. For, although time is explicitly mentioned in the paradox when asking for the possibility of covering the infinitely many parts in a *finite time*,[89] it is ignored in the process of division. Otherwise, if

[87] Also the third passage cited above (263a4–11) refers to the relation of something finite and something infinite: "so that we get the result that if something has travelled the whole (half), we have already counted an infinite number; but this is admittedly impossible". Why is it impossible that we have counted an infinite number? Apparently, because we are finite beings, and thus we would have to count an infinite number in a finite time. As Aristotle is not talking about an infinite spatial magnitude here, but rather about infinitely many numbers (which can be assigned to the spatial pieces counted), this passage shows that for mental movements as well, such as counting, it is the relation of something finite and something infinite which seems to be problematic. Note, however, that Aristotle makes it clear in this passage that he is now no longer answering Zeno, but others who seem to have raised a similar question. And, as Aristotle clarifies in 263a23 ff., the mental movement of counting is not a continuous movement, which for Aristotle only contains a potential infinity; rather in the case of counting, if we were able to go through an infinity of numbers, and Aristotle thinks we cannot, then we would be dealing with an actual infinity, which would require an infinitely extended time for its performance.

[88] It is not enough, as Ferber 1995, pp. 6–8 claims, that the paradox can be understood in a spatial *or* a temporal sense. Ferber deals only with the problem of how a continuous finite whole, be it spatial or temporal, could have infinitely many parts. He never deals with the problem of how seemingly infinitely many spatial parts can be covered in a finite time, which is what makes it a genuine paradox of motion.

[89] According to Vlastos 1975a, p. 203, it is not Zeno himself who brings in time in 233a21–3, rather, this is an Aristotelian addition, a reading which I do not consider convincing. As was shown above, it is not only 233a21 ff., but also the formulation of the paradox in 263a4–11 which requires us to assume the impossibility of covering something infinite in a *finite time*. Furthermore, in 263a Aristotle explicitly distinguishes the motion problem

Zeno had considered time as well as space and as a magnitude like space[90] during the whole description, he would have put the division somewhat like this: first the runner must cover half the racecourse in half the time. He must then cover another half of the remaining distance in half of the half time, and so on *ad infinitum*.[91] If Zeno had really considered the correlation of time and space, whenever a spatial part is marked off, a corresponding temporal part would have been marked off.

In contrast to a conceptual interpretation, which looks at what is specific to a paradox of motion, mathematical interpretations, like Salmon's, do not account for the motion problem. Salmon reduces our paradox of motion to the general continuum paradox which also underlies movement, since for him it is a problem of time *or* space rather than of time *and* space.[92] For Salmon, Zeno's paradox confronts us with the task of conceiving an infinite division which has led to an actual infinity of parts. The appropriate mathematical tool to answer this problem is, according to Salmon, the concept of the limit of an infinite series. So Salmon accepts Zeno's conceptualisation of movement and merely asks how, from a mathematical point of view, we can deal with a thus produced 'actual' infinity.[93] By contrast, a conceptual interpretation, as we will find also in Aristotle, not only examines the genuine motion problem but also tries to inquire what kind of infinity is needed here, if infinity is needed at all, and how one can conceive of an infinite division.

Furthermore, understanding this division, as Salmon does, as producing actually infinitely many parts, leaves the connection of these elements unexplained. For it is not clear what makes them still parts of one whole motion. Starting out with marking off distances (½ of the original distance, ¼ of it . . .) and putting these distances in a one-to-one correspondence with the positive integers, Salmon then passes over to counting only numbers, as if these

from the continuum problem, and I do not see any reason to assume that Aristotle simply invented this second problem. Finally, why would this paradox have been transmitted as a paradox of motion if indeed it concerned every continuum?

90 The problem is not simply to take time into account – Zeno might have said that what is moving is *first* here, *later* there, and *before* that at another place without this changing the paradox at all – but to take it into account as something which can be divided in the same way as the distance covered can, and which *must* be divided whenever the distance of a movement is divided. However, this presupposes a basic similarity in the structure of time and space – an idea we do not find before book VI of Aristotle's *Physics*.

91 We will see in Chapter 8 that this is exactly the relation Aristotle employs when comparing two motions of different speed in 232b20–233a31.

92 Salmon 1980, p. 36. Mathematical interpretations could in principle take into account the combination of two magnitudes, for example, in a two-dimensional coordinate system. However, they would have to treat them in the same way they treat a stationary mountain, as we saw in the introduction of this chapter, and could not take into account anything specific to these two physical magnitudes. The mathematical interpretations prevalent in the secondary literature do not even consider the combination of two magnitudes.

93 See Salmon 1980, pp. 36, 47–8.

distances were points on a line which converge; he simply equates this number of converging points with motion. The only connection Salmon mentions between these 'parts' is the addition of the terms.[94] But this does not explain why they add up to *one* movement rather than to a sum of different movements. We will see later on that giving an account of the unity of a motion is a difficult conceptual problem which Aristotle tries to deal with in his *Physics*.

Thus with Salmon the dimension of movement is reduced twice: first he assumes that it is not the complex dimension of movement, time *and* space, which the paradox shows to be problematic, but rather time *or* space; and in the course of the investigation it is only space which remains. Secondly, Salmon reduces the (spatial) distances to points and thus to zero dimension. He does not explain which thoughts or concepts lead to the paradox, assuming simply that the problem put forward by Zeno needs only a mathematical reformulation, as the problem of a convergent series, in order to be solved mathematically with the idea of a limit. And that is indeed what Salmon does: he gives a mere reformulation of Zeno's paradox in a precise mathematical way.

(3) Proponents of the supertask debate understand the dichotomy paradox as raising the third problem mentioned, that running a finite distance in a finite time seems to require completing an *infinite* number of *actions*. Thus, Zeno seems to put forth the first supertask.[95] A supertask is commonly understood as an infinite sequence of actions or operations carried out in a finite interval of time.[96] To illustrate such a supertask, let us look briefly at the first so-called infinity machine (i.e., a machine which is supposed to perform such a supertask), Thomson's lamp: this lamp has one switch to turn the light on and off. Over the period of one minute we are meant to imagine the following scenario: after ½ a minute the switch is pressed to turn on the light, it will be pressed again to turn the light off after another ¼ of a minute, pressed again to turn it on after ⅛ of a minute, etc. Thus, after one minute, the task will have been performed infinitely many times and we will not be able to tell whether the lamp is on or off.

Judging from paradigmatic examples like this one, and accounts we find in Black, and Laraudogoitia,[97] a task or action (and thus a part) involved in such a supertask has a definite beginning and end, and is clearly distinguished from the next task – by a change of direction, by an alternating state, or by a period of rest. Accordingly, no task can be chosen arbitrarily.

94 Salmon 1980, p. 37.
95 See Weyl 1949, p. 42, who links the dichotomy paradox to a machine's completion of an infinite sequence of distinct acts, and the start of the supertask debate in the journal *Analysis* in the 1950s between Black 1951, Taylor 1951, Watling 1952, and Thomson 1954. Cf. also Salmon 1980; Moore 2001, pp. 3–4; and Laraudogoitia 2016.
96 See, for example, Laraudogoitia 2016 and Thomson 1954, p. 2.
97 Black 1954; Laraudogoitia 2016

Can we understand the dichotomy as such a supertask then? A run as discussed in the dichotomy paradox does not show any of the features of a supertask just named, since it does not have any clearly given, discrete parts. There is no change of direction or something similar which would definitely separate one part of the run from the next, so that there are noticeably different tasks to carry out in one run. If we mark off different parts of the run as we please in our mind when discussing Zeno's paradox, this does not affect the run qua run; it provides us with parts in our mind that can easily be changed again. But it seems difficult to define the run portrayed in the dichotomy paradox as a supertask when not even a single task is clearly marked within a motion.

Nevertheless, many scholars, including scholars of ancient philosophy,[98] interpret the dichotomy as a supertask and consequently treat movement as if it consisted of discrete parts. This idea has probably been given its strongest expression with Grünbaum's introduction of a staccato runner who pauses after the completion of half of the distance, again after a quarter of the remaining distance, and so forth.[99] But this modification of the situation raises the question of why we should still regard such a motion, riddled with pauses, as *one single* motion, rather than as a number of different motions. With a continuous run, beginning and end are determined but the parts can be chosen in whatever way we please, arbitrarily, but with a supertask it is the other way around: each single task and thus each part is determined, while the beginning and the end of the whole supertask are underdetermined. For the end can only be determined by a certain time (not by a certain task) one chooses for the supertask, and no important feature of the supertask changes if we start one task earlier or later (if, let us say, we started with pressing the switch of the lamp one minute earlier). Thus, the unity of a supertask must be defined quite differently from the unity of a run: it is based on the rule that a specific task must be performed in a constantly diminishing time, beginning and end determined externally by the time fixed. The unity of a run, however, is not defined by any rule determining a series of fixedly given tasks, but rather by the standstills which mark the beginning and end of the movement.[100] No parts are given nor is their unity fixed once some parts have been freely chosen, since there is no better reason to determine a chosen segment as a part than to take, for instance, the halves of this segment as parts. By

98 See, for example, Barnes 1982; Kirk, Raven, and Schofield 1983.

99 Grünbaum 1968, p. 79. The supertask debate usually deals with the so-called 'progressive' version of the dichotomy paradox (see above), but the "Paradox of the Gods" invented by Benardete 1964, pp. 259–60 seems to take up the regressive version.

100 Cf. the discussion of the unity of a motion in Aristotle's *Physics* V and in the chapters on Aristotle in this book.

contrast, with supertasks there can be pauses between each task, since a task is not the opposite of rest and thus excluded by it, as movement is.[101]

This comparison with supertasks shows how the parts of a movement cannot be thought, namely as given discrete parts, in the way the single tasks of a supertask are. Besides, the comparison makes it obvious that the kind of unity required for an account of a continuous motion is very different from that a supertask requires.[102]

3.6.2 Achilles: A Variation of the Dichotomy Paradox

> The second paradox is the so-called Achilles; it states that the slowest will never be overtaken by the fastest in a run, since the pursuer must first get to where the pursued started, so that necessarily the slower is always ahead. This is the same argument as the dichotomy, but it differs in that the magnitude which is added is not divided into halves. It follows from this argument that it does not happen that the slower is overtaken, but it proceeds along the line of the dichotomy (for in both a division of the magnitude in a certain way leads to the result that the goal is not reached, though the 'Achilles' goes further in that it affirms that even the quickest runner in legendary tradition must fail in his pursuit of the slowest), so that the solution must be the same. And the axiom that that which holds a lead is never overtaken is false: it is not overtaken, it is true, while it holds a lead, but it is overtaken, nevertheless, if it is granted that it passes through a finite distance.
>
> (239b14–29, translation by Hardie and Gaye with modifications)

The problem sketched in this paradox is the following: when Achilles, the fastest runner of the ancient world, starts behind the slowest in a race, a tortoise as later authors have it, it seems impossible that he will ever overtake the tortoise. For, first, Achilles must cover the distance from his starting point, A, to the starting point of the tortoise, B. In the meantime the tortoise has moved on to another point B_1, and when Achilles reaches this point B_1, the tortoise will have moved ahead again to B_2. Achilles, being faster than the tortoise, will reduce the distance between them, so the distances between the successive Bs will increasingly diminish, but, nevertheless, no matter which B_n Achilles has reached, the tortoise will always have moved on to B_{n+1} in the meantime.

The distance which is divided in this paradox is that between Achilles' starting point A and the possible point C, where Achilles and the tortoise are level; this last point does not belong among the points reached by the

[101] For a more detailed comparison, see Sattler 2019b.

[102] Of course, there may be series of discrete jumps, for example at the subatomic level; but this is not the continuous motion we are concerned with here.

construction sketched in the paradox. From the infinite division within this finite distance, Zeno infers that Achilles cannot reach a point outside this distance, namely their meeting point. This invalid inference seems to be what Aristotle refers to when he states that what is ahead will not be overtaken as long as it is ahead (during its travel from A to C), but nevertheless will be overtaken after covering a finite distance, namely at C.

Aristotle points out that this paradox is a variation of the dichotomy, and many modern scholars follow his assessment.[103] Aristotle himself mentions two differences between the two paradoxes: first, while it could be anybody or anything moving in the dichotomy, the Achilles paradox employs the fastest runner known to the ancient world (239b24–5), which makes the problem more dramatic. Second, the distance in the first paradox is always divided by two, but each time by some other number in the second (239b19–20), as it is divided by the movement of the tortoise.

In addition, the Achilles paradox is somewhat more complicated than the dichotomy. We also have a finite distance here, namely the distance from Achilles' starting point to the point where he and the tortoise are level, a point which can be calculated once we know the speed of both competitors. However, the end point, for which Achilles is aiming, is not fixed for the time being, but rather moving. Thus, it seems that it is not one single movement we must look at but rather the relation of two movements, one performed by Achilles and another by the tortoise. From a logical point of view, however, the paradox can be reduced to the problem of the tortoise, and thus again to a single movement. Achilles, it is true, is needed in this paradox to produce the end point (which is simply given in the dichotomy), and he motivates division at certain points. But for the actual division of the track we would only need the tortoise. In the same way as in the dichotomy, the thing moving (here the tortoise) is dividing the distance *ad infinitum*; Achilles only 'inherits' the divided distances from the tortoise. Thus the basic structure of the paradox stays the same in both cases: what is moving over a finite distance seemingly must go through an infinite number of parts and thus cannot reach the end point. Furthermore, these infinitely many spatial parts must be covered in a finite time, which seems to be impossible. Hence, for our conceptual reconstruction of the paradoxes, we will not need to deal with the Achilles paradox independently of the dichotomy.

3.6.3 The Flying Arrow: Motion as a Sequence of Rests

> Ζήνων δὲ παραλογίζεται· εἰ γὰρ αἰεί, φησίν, ἠρεμεῖ πᾶν [ἢ κινεῖται] ὅταν ᾖ κατὰ τὸ ἴσον, ἔστιν δ᾽ αἰεὶ τὸ φερόμενον ἐν τῷ νῦν, ἀκίνητον τὴν φερομένην εἶναι ὀϊστόν. τοῦτο δ᾽ ἐστὶ ψεῦδος· οὐ γὰρ σύγκειται ὁ χρόνος ἐκ τῶν νῦν τῶν ἀδιαιρέτων, ὥσπερ οὐδ᾽ ἄλλο μέγεθος οὐδέν.

[103] See Barnes 1982, p. 274; Kirk, Raven, and Schofield 1983, p. 272; Ferber 1995, pp. 8–10.

> But Zeno reasons falsely: for if, as he says, everything rests [or is in motion] whenever it is in/against what is equal, and what moves is always in the now, the moving arrow is unmoved. But this is wrong. For time is not composed of indivisible nows, nor is any other magnitude.
>
> (Aristotle *Physics* 239b5–9)

> . . . τρίτος δ' ὁ νῦν ῥηθείς, ὅτι ἡ ὀϊστὸς φερομένη ἕστηκεν. συμβαίνει δὲ παρὰ τὸ λαμβάνειν τὸν χρόνον συγκεῖσθαι ἐκ τῶν νῦν· μὴ διδομένου γὰρ τούτου οὐκ ἔσται ὁ συλλογισμός.

> . . . but the third [argument] is the one just mentioned, that the flying arrow is standing still. This results from the assumption that time is composed of nows. For if this is not granted, the conclusion will not follow. (Aristotle *Physics* 239b30–3)

This paradox is scarcely understandable at first glance, and different scholars have given different reconstructions of it.[104] But before we examine the logical structure of the paradox, a short philological note is necessary: 239b5–6 as it stands – ἠρεμεῖ πᾶν ἢ κινεῖται ὅταν ᾖ κατὰ τὸ ἴσον – does not make any sense for the subsequent conclusion. Two readings seem to be possible: either we follow Ross and Barnes[105] in leaving out ἢ κινεῖται ("or in motion"), which is not to be found in Themistius. As ἢ κινεῖται seems to have been there in the manuscripts, however, as well as in Simplicius' and Philoponus' texts, the alternative would be to assume that something has dropped out after ἢ κινεῖται, and to insert something like Diels's οὐδὲ δὲ κινεῖται. Then we would get: "for if, as he says, everything is either at rest or in motion, but nothing is in motion whenever it is in/against what is equal".[106]

Against this reading, Ross objects that it would only make sense "if Zeno had argued disjunctively that because the arrow is not in motion it is at rest; but since his conclusion is ἀκίνητον . . . he does not seem to reason thus".[107] Ross's objection is, however, undermined by the fact that Zeno infers in the conclusion that the arrow is unmoved from premises giving the conditions for something being at rest. Accordingly, we must assume that Zeno equates 'being at rest' and 'being unmoved'.[108] Thus the second alternative[109] simply spells out a presupposition with which Zeno is working anyway, namely that motion and rest are contradictory opposites, everything is either in motion or

[104] See, e.g., Barnes 1982, pp. 276–9; Owen 1957–8, p. 156 ff.; Kirk, Raven, and Schofield 1983, p. 273; Vlastos 1975b; Makin 1998.
[105] Ross, p. 658 and Barnes 1982, p. 277
[106] See Lee, pp. 79–80.
[107] Ross, p. 658.
[108] For the equation of "being unmoved" with "resting", see also Parmenides' poem, fr. 8, lines 29–30 and Chapter 2. It seems we find both first distinguished in Plato's *Parmenides* 139a–b. Cf. also Aristotle's *Physics* 202a5 and 228b3, and Vlastos 1975b, p. 190 ff.
[109] See Simplicius 1895, *In Phys.* 1011.19 ff.; Lee, pp. 52–3.

at rest – this disjunction is exhaustive, *tertium non datur*. Even if, for the sake of simplicity, we choose the first reading, we nevertheless must assume this exhaustive disjunction between movement and rest at least implicitly in order to derive the conclusion about the arrow being unmoved from a premise talking about rest. Aristotle's understanding, that in the now things neither move nor rest, but are rather unmoved, was not available to Zeno – Zeno seems to have inherited Parmenides' assumption that opposites are to be thought of as mutually exclusive and exhaustive.

These philological concerns apart, the main problem with the arrow paradox is that the premises are formulated in a way which makes it difficult to understand how they lead to the given conclusion. Together with the already mentioned disjunction, the three premises explicitly stated are:

> Everything is either at rest or in motion
> Everything rests[110] whenever it is in/against what is equal (*kata to ison*)
> What is travelling is always in the now
>
> And the conclusion:
> The travelling arrow is unmoved.

The premises are fairly unclear, especially the formulation *kata to ison* in the second. But it is commonly agreed, by ancient commentators as well as by modern scholars, that "something is in/against what is equal" means that something is in an equal space (i.e., in a space equal to its own size).[111] The implicit reason for the last premise, that everything is travelling (i.e., moving) in the now, seems to be that everything is moving in time and time is a succession of nows.[112] That Aristotle takes this to be the reason for the last premise is clear from his solution of the paradox: "It follows from taking time to be composed of nows; for if one does not concede this, there will not be that conclusion." Time seems to be thought to 'consist' of nows, presumably because everything which is happening in time is happening in the present (was happening in the then-current present and will be happening in the future present), and the present moment is the now.

But in order to get from the premises mentioned to the conclusion, another premise is needed which is not stated in the Aristotelian text. This premise

[110] This is a positive reformulation of "nothing is in motion", which seems to best fit the structure of the paradox.

[111] See, e.g., Barnes 1982, p. 278, who points out that for this understanding there are two additions to Aristotle's *kata to ison* to be made, which were added by most of the ancient commentators: *kata to ison* is glossed as "occupying an equal space" and this is taken as being elliptical for "something occupies a space which is equal to its own space [its own volume]". Some scholars prefer to talk about 'place', others to talk about 'space' for the reconstruction.

[112] For the fact that they must be indivisible nows, see below.

must be something like: everything is occupying a space of its own size in the now.[113] Thus the argument can be reconstructed as follows:

(1) Everything is either at rest or in motion.
(2) Everything is at rest whenever it is in a space equal to its own size.
(3) Everything is in a space equal to its own size in the now.
(4) What is moving is always moving in the now.
Conclusion: Whatever is moving, for example, a moving arrow, is moving in the now. But in the now, it is occupying a space equal to its own size and thus it is resting. Hence, what is moving is unmoved.

The reason for the additional premise (3) is the need for a premise to connect the now with a space equal to its own size in order to derive the given conclusion. This now must then be thought as indivisible and without extension so that what is moving can be connected with a space equal to its extension.[114] For in an extended period of time, there would be different places which are occupied by something moving.[115] Only if the now in (3) is understood as indivisible and without extension is it necessary that *everything* be in a space equal to its own size. If this understanding of the now were explicitly spelt out, however, (4) would not seem obvious but rather highly disputable. Thus the paradox rests on an understanding of the now that is not sufficiently determined, as our conceptual interpretation shows.

Aristotle's analysis that the now in this argument must necessarily be thought of as indivisible, and a period of time as the sum of indivisible instants, is doubted by Kirk, Raven, and Schofield. In contrast to Aristotle's understanding, they think that for his inference Zeno only assumes that "what is true of something *at every moment* of a period of time (whether or not moments are indivisible instants) is true of it *throughout* the period".[116] However, the

[113] See, e.g., Ferber 1995, p. 10; Lear 1981, p. 91; Lee, p. 80, where he claims: "The nerve of the argument must be, that what is κατὰ τὸ ἴσον is at rest, and that what is ἐν τῷ νῦν is κατὰ τὸ ἴσον, and therefore at rest." In order to get this fourth premise, Diels inserts πᾶν δὲ κατὰ τὸ ἴσον ἐν τῷ νῦν after φερόμενον ἐν τῷ νῦν.

[114] See below and Lear 1981, pp. 96–7. What is resting can of course be connected to a space equal to its own size in an indivisible instant as well as in an extended period. But since for something moving this connection is only possible in a now which is indivisible and without extension, '*everything* is in a space equal to its own size' can only be said of a now thus determined.

[115] Let us assume that our arrow has a length of 20 cm and is moving at 1 m/s. If we consider a period of one second, and take as a part of it half a second, then obviously the arrow is covering and thus also occupying 50 cm during this half second, which is more than its own length. The assumption that in every part of a period of time the arrow is always occupying no more space than its own thus works only for parts without extension. See also *Physics* VI,3 234a24–31, where Aristotle shows that if the arrow moved in the now, the now would necessarily be divisible.

[116] Kirk, Raven, and Schofield 1983, p. 273.

inference from the parts to the whole alone does not lead to the paradox. Everything depends on how these parts are conceived of. For if we think of the now not as being an indivisible instant, but rather as divisible at least into A and B, we would have to assign one place, A_1, to A and another, B_1, to B as the moving arrow goes on flying during the now AB. But when different places are connected to different parts of one now, the arrow is not in a space equal to its own size in the now, but in something bigger, and so we can no longer speak of rest.[117] Accordingly, Ross also claims that if the now has duration, "there is no reason why movement should not take place in it; if it has not, then since no number of moments not having duration can make up a time that has duration, the fact that movement cannot take place in the moment is no reason why it should not take place in the time".[118] Thus, whether the inference from every moment of time to the whole period of time – this is what Kirk, Raven, and Schofield take as the sole basis of the paradox in their interpretation – will ever lead to a paradox depends on how the 'moments' are understood. If they are understood as indivisible instants, then the inference from what happens in each of them to what happens during the whole period will produce a paradox. But if they themselves are assumed to be divisible periods,[119] then it will be right to infer from what happens in these different small periods (say, movement) what happens during the whole period, namely also movement. In this latter case, Zeno would not be able to state that during such an extended and divisible moment the flying arrow is at rest. Only in an indivisible now can the arrow in its flight be thought to be in a place equal to its own extension, so that a fixed space can be assigned to it. Hence, it is not Aristotle's own understanding of the now which requires it to be thought of as indivisible, as Ferber and Vlastos falsely claim.[120] Rather it is the construction of the paradox itself which does not work without this assumption.

In contrast to Kirk, Raven, and Schofield, Salmon's mathematical interpretation takes the indivisibility of the nows into consideration as he conceives of movement as the sum of discrete paired points: "the motion itself is described by the pairing of positions with times alone . . . the motion consists in being *at* a particular point *at* a particular time".[121] Thus, Salmon's conception of

[117] Not connecting different places to the beginning and end of the divisible now AB would mean to assume the conclusion – that the moving arrow is always at rest – already in the premise, assigning the same place to each possible part.

[118] Ross, p. 80. Cf. also Ferber 1995, p. 11 and Barnes 1982, pp. 284–5.

[119] If the nows were assumed to be indivisible but extended time atoms, then premise (4), that what is moving is always moving *in* the now, would not hold.

[120] Ferber 1995, p. 11 and Vlastos 1975b, p. 187. Vlastos's reconstruction on p. 189 assumes that the now is superfluous (similarly Barnes 1982, p. 278). However, the analysis provided by Vlastos and Barnes does not explain why something moving should always be in a place equal to itself; see also Lear's 1981 reply to Vlastos.

[121] Salmon 1980, p. 41; emphasis original.

movement confirms Zeno's description, since it also leads to the result that "there is no distinction between being at rest at the point and being in motion at the point". Movement and rest are distinguished, Salmon tells us, by looking at the *surroundings* of a point: "aside from *being* at the appropriate places at the appropriate times, there is no *additional* process of *moving* from one to another. In this sense, there is no absurdity at all in supposing motion to be composed of immobilities".[122] Thus all we have is one point and some points surrounding it. There is no "additional process of moving" from one point to another, as Salmon tells us.

This understanding has proved to be a useful way for modern mathematics to handle movement. But it is not satisfying for a conceptual understanding of the arrow paradox. For mathematics simply provides us with a function to deal with movement in a mathematical way. It is, however, not concerned with movement as a kind of process and not interested in giving us a criterion to differentiate movement from rest. Rather than treating movement as the change of places in a time, it treats movement just like rest, namely as occupying one particular place at one particular time.[123] Starting from a point at which movement and rest cannot be differentiated, we will never be able to infer from it what happens over some period or distance, as the analysis of the paradox shows.

Owen's reference to instantaneous velocity and speed is no solution to this problem either.[124] For although we can talk about movement at a point nowadays, the way we derive instantaneous movement and speed is to take an extended period and distance over which a movement takes place and make the period and distance shorter and shorter so as to make them converge to the limit of the initial period and distance. Our notion of instantaneous speed thus also depends on extended periods and distances.[125] And the mere fact that there are different points of space in the surroundings of the initial space-point does not guarantee that there is continuous movement going on, for they could also imply indicate instantaneous changes of place, a mere staccato.

Salmon precludes the question of how the arrow can get from one point to another, since this question implies that there is something in addition to the points. Mathematics cannot answer this question, since it is only interested in whether some point belongs to the function we call movement; it is not

[122] Salmon 1980, p. 41; emphasis original.

[123] We may assume that in talking about immobilities, Salmon is not conceptualising movement as a sequence of rest, but rather as a sequence of something which is neither movement nor rest. Although this is a differentiation which is important for the conceptualisation of movement (see Aristotle's reply to the arrow paradox), the crucial point here is that Salmon thinks of movement as being *additively composed* of these immobilities, so there is no genuine motion involved in his account.

[124] Owen 1957–8; see also the chapters on Aristotle below.

[125] Even if the limit is distinct from any member of the series; cf. Lear 1981.

interested in what is specific to motion. Hence, movement as getting from one point to another (in contrast to the mere pairing of time- and space-points) is exactly what Salmon gets rid of in his 'solution' to the paradox; accordingly Ross notes of the 'at/at-theory' that "it leaves us still with the problem [of] how the moving body gets from one place to another, which yet it does".[126]

Furthermore Salmon does not distinguish the way mathematics conceptualises its calculation (t_1/x_1, t_2/x_2) from the physical understanding of movement. Hence, from the mathematical analysis Salmon infers the physical structure of movement, as is made clear by his switch from talking about how "motion itself is *described*" to "motion *consists*" in the quotation above. From the fact that in mathematics motion can be *described* as the "pairing of positions with times", he infers, without justification, "motion *to be composed* of immobilities", ascribing the mathematical concepts to motion itself.[127] What a physical interpretation takes to be an external point of measurement, Salmon (and Zeno) treats as a point of which movement consists – once again, a mathematical interpretation simply takes up the concepts used in Zeno's paradoxes without analysing their appropriateness first.

Summing up the difficulties raised by the arrow paradox, the main problem seems to be that Zeno infers what happens during a period of time from what happens in the now. He can do this because he treats the now as part of the whole 'time'. The now is thought of as an indivisible instant without extension in which everything occupies a space equal to its own extension, and a period of time as the mere sum of these indivisible instants. As we cannot think of a movement in such an instant, we consequently cannot think of movement over a period of time consisting of these instants. Zeno presupposes that it is possible to infer the characteristics of the whole, the period of time, from the characteristics of what he takes to be a part, the now. But such an inference requires that whole and part have the same basic features on which this inference relies. The nows are not extended, however, while the period of time is, so whole and part do not share the same basic features. Accordingly, it is not possible to infer what happens during a period of time from what happens in the now.

In the following chapters we will see not only that the inference from the premises to the conclusion is doubted, but also that the premises come under pressure. In his *Physics* Aristotle points out that we cannot differentiate between movement and rest in an indivisible instant. Accordingly, all we can

126 Ross, p. 81.

127 When Cantor explains a continuum as a "unendliche Punktmannigfaltigkeit" he is not suggesting that something physical *consists* of this *Punktmannigfaltigkeit*, but only that for certain purposes it is useful to *describe* it thus. Similarly, for certain purposes it might be useful to describe the body of a tortoise as segments of a sphere, five cylinders, and a cone, but we'd better be careful not to think of the tortoise as consisting of these mathematical forms.

say about something in a particular instant is that it is unmoved, which Aristotle clearly distinguishes from moving and resting. Hence premise (1) must be changed to 'Everything is either at rest or in motion or unmoved'.

Premise (2), that everything is at rest whenever it is in a space equal to its own size, is only true, on an Aristotelian account, if it reformulated as 'Everything is at rest whenever it is in a space equal to its own size *over a certain period*". Without this reference to a period of time, the mere fact that something is in a space equal to its own size is not sufficient for determining rest. We need information about the thing's behaviour at more than one point in time, so from some time t_1 to some time t_2, as we get with a period of time. Motion as well as rest needs to be defined with respect to a period of time. But once this is made clear, premises (3) and (2) will not get us the result that everything is resting in the now.

The understanding of premise (3), that everything is in a space equal to its own size in the now, is closely connected to the understanding of the second premise. After the necessary change in premise (2) just mentioned, we can either still understand the now in premise (3) as unextended, in which case the now has nothing to do with the definition of rest from the revised second premise, which is concerned with rest over a certain period, and thus does not support the whole conclusion. Or we do not understand the now of premise (3) as unextended. But then premise (3) no longer works for something moving – either what is moving will not be in a space equal to its own size in an extended now, if the now is divisible, or it will not be moving *in* the now, as premise (4) claims, if we assume indivisible but extended time atoms.

Premise (4) – that what is moving is always moving in the now – equally comes under pressure from Aristotle's argument that something can only be moving in a period of time. This period may be called 'now', but such an extended and divisible now will not work as a premise in support of Zeno's conclusion, since something moving will not be in a space equal to its own size in such a now. If we stick with an unextended now, however, there cannot be any motion in it (and we saw above that the notion of instantaneous velocity will not change this).

The arrow paradox focuses on the temporal dimension of motion; the spatial aspect is only mentioned in order to define rest. And even the temporal aspect is extremely reduced. Time is not taken into account in a way which could be used to account for motion – not only because the second premise is not true without reference to a period of time, but also because if the now is something indivisible and without extension, it cannot be used to define motion in its temporal respect. Hence, what will become obvious in the tradition to follow is that this paradox, as well as the two preceding it, did not sufficiently take time into account. The 'continuum problem' in this paradox is that something extended seems to consist of parts which are not extended.[128] Underlying this problem are, again, two incompatible part-whole relationships, which are employed simultaneously: on the one hand, parts are assumed to be such that the whole can be inferred from them – from what happens in a now, we infer what happens over a period of time. On the other hand, the parts and whole employed are such that the similarity required for such an inference is lacking – parts are unextended while the whole is extended.

The arrow paradox presents the problems we seem to get into when we try to construct a motion out of parts of motion, while the following paradox deals with the problems arising once we try to construct motion out of units of time and space.

3.6.4 The Moving Rows: Double the Time Is Half the Time

Zeno's fourth paradox is usually seen to be the one which is most difficult to reconstruct in a philologically accurate way,[129] although philosophically it seems to be the least interesting.[130] In accordance with this opinion, it takes some time to reconstruct fully the situation discussed. But we will see that it contains a piece of philosophy that is absolutely crucial for the overall point of this project – the difficulties in conceptualising movement.[131]

3.6.4.1 The Textual Reconstruction of the Paradox

(I) τέταρτος δ' ὁ περὶ τῶν ἐν σταδίωι κινουμένων ἐξ ἐναντίας ἴσων ὄγκων παρ' ἴσους, τῶν μὲν ἀπὸ τέλους τοῦ σταδίου τῶν δ'ἀπὸ μέσου, ἴσωι τάχει, ἐν ὧι συμβαίνειν οἴεται ἴσον εἶναι χρόνον τῶι διπλασίωι τὸν ἥμισυν.

(II) ἔστι δ' ὁ παραλογισμὸς ἐν τῶι τὸ μὲν παρὰ κινούμενον τὸ δὲ παρ' ἠρεμοῦν τὸ ἴσον μέγεθος ἀξιοῦν τῶι ἴσωι τάχει τὸν ἴσον φέρεσθαι χρόνον. τοῦτο δ' ἐστὶ ψεῦδος.

(III)

(1) οἷον ἔστωσαν οἱ ἑστῶτες ἴσοι ὄγκοι ἐφ' ὧν τὰ ΑΑ, οἱ δ' ἐφ' ὧν τὰ ΒΒ ἀρχόμενοι ἀπὸ τοῦ μέσου τῶν Α, ἴσοι τὸν ἀριθμὸν τούτοις ὄντες καὶ τὸ μέγεθος, οἱ δ' ἐφ' ὧν τὰ ΓΓ ἀπὸ τοῦ ἐσχάτου τῶν Β, ἴσοι τὸν ἀριθμὸν ὄντες τούτοις ὄντες καὶ τὸ μέγεθος, καὶ ἰσοταχεῖς τοῖς Β.

[128] This can be seen as spelling out one of the paradoxical alternatives in the dichotomy (alternative α above), which suggested that the infinitely many parts of the finite whole have no extension at all.

[129] See, for example, Pellegrin's translation of the *Physics*, ad loc. and Wagner's commentary on this passage, p. 639.

[130] So Eudemus fr. 106b Wehrli. Simplicius 1895, *In Phys*. 1019.32–1020.2, following Eudemus in his assessment of this paradox, calls this argument "most silly". See also Barnes 1982, pp. 290–1.

[131] For a more detailed discussion of the philosophical problems which this paradox raises, see Sattler 2015, with which the discussion here partly overlaps.

(2) συμβαίνει δὴ τὸ πρῶτον Β ἅμα ἐπὶ τῷ ἐσχάτῳ Γ εἶναι καὶ τὸ πρῶτον Γ ἐπὶ τῷ ἐσχάτῳ Β, παρ' ἄλληλα κινουμένων.

(3) συμβαίνει δὲ καὶ τὸ Γ παρὰ πάντα τὰ Β διεξεληλυθέναι, τὰ δὲ Β παρὰ τὰ ἡμίση Α· ὥστε ἥμισυν εἶναι τὸν χρόνον· ἴσον γὰρ ἑκάτερόν ἐστι παρ' ἕκαστον, ἴσον χρόνον παρ' ἕκαστον γινόμενον τῶν Β ὅσον περ τῶν Α, ὥς φησι.

(4) ἅμα δὲ συμβαίνει τὸ πρῶτον Β παρὰ πάντα τὰ Γ παρεληλυθέναι ἅμα γὰρ ἔσται τὸ πρῶτον Γ καὶ τὸ πρῶτον Β ἐπὶ τοῖς ἐναντίοις ἐσχάτοις, διὰ τὸ ἀμφότερα ἴσον χρόνον παρὰ τὰ Α γίγνεσθαι. ὁ μὲν οὖν λόγος οὗτός ἐστιν, συμβαίνει δὲ παρὰ τὸ εἰρημένον ψεῦδος.[132]

(I) The fourth [argument] is the one about the moved [bodies] in the stadium, when equal bodies are going along equals in the opposite direction, the one [starting] from the end, the other from the middle of the stadium; [both] with the same speed. From this he thinks it results that double the time equals half the time.

(II) The fallacy consists in assuming that the passing of a moving body and of a resting body of equal size with equal speed will take the same time. But this is wrong.

(III)

(1) For example, let there be the equal-sized resting bodies, the AA, and the BB are those starting from the middle of the As, the same in number and size as those [the AA]; and the CC [start] from the end of the Bs, the same in number and size as those, and having the same speed as the Bs.

(2) Then it follows that, as they are moving past each other, the first B is at the end of the Cs at the same time as the first C is at the end of the Bs.

(3) It turns out that C has gone past all the Bs, but B only past half the As; so that the time is only half; for each of them is for the same [time] alongside each, as it takes the same time [to pass] each of the Bs as each of the As, so he says.

(4) At the same time it follows that [the first] B passed all the Cs. For the first C and the first B will be simultaneously at the opposite ends, because both are for the same time alongside the As. So this is the argument and it follows from the aforementioned fallacy.

(Aristotle *Physics* VI,14 239b33–240a17)

[132] The text of Zeno's paradox of the moving rows is usually judged to be a mess, full of manuscript variations none of which yields a satisfactory paradox. In my paper, "Reconstructing Zeno's Fourth Paradox of Motion", currently under submission, I attempt to establish a clear and unambiguous text and the text I give here is that defended in this article.

In order to reconstruct the paradoxical situation in this text, I have numbered the different steps we can find in Aristotle's account of this paradox. There are three main parts, which I have marked with Roman numerals in the text:

(I). First, Aristotle very briefly names the paradox – half the time equals double the time[133] – without showing how the problem arises.
(II). In a second step, Aristotle determines the fallacious conclusion on which, according to him, the paradox rests, namely the assumption that if something moves with constant speed, it will need the same time to pass something moving as to pass something resting.
(III). Finally, the construction of the paradox is set out with the help of an example.

In order to reconstruct Zeno's paradox, we must start by considering this third part to find out how the construction given in this passage leads to the paradox named in the first part. If we follow the lines of Aristotle's text, we can find four steps in this third part, which I have marked with Arabic numerals: the first explains the starting position of the bodies in the stadium, the second the end position. Steps three and four provide an account of the motion of the rows of bodies that leads to the paradox. Let us go through the four steps in turn:

(1) In the beginning, there are three different rows of bodies in a stadium, As, Bs, Cs, all of the same size and number.[134] In the beginning, the Bs are in the middle of the stadium as well as in the middle of the As; the Cs are at the end of the stadium and at the end of the Bs.[135] We can represent the starting position as follows:[136]

						A1	A2	A3	A4		
				B4	B3	B2	B1				
								C1	C2	C3	C4

[133] "Half the time equals double the time" could either mean $t/2=2t$ or $t/2=t$. The latter interpretation is not only in accordance with "einem häufigen griechischen bzw. arist. Sprachgebrauch" (so Wagner, p. 640), but also turns out to be the one which the logic of the argument employs.

[134] Different manuscripts preserve different numbers of As, but what is essential is that there is the same number in each row. I have chosen four letters for the simple reason that four letters give a clear diagram and have been frequently used in the tradition commenting on this paradox; cf. also Wagner, p. 640.

[135] Although both the Bs and the Cs start from the middle of the As, the text does not state it in this way, because only the Bs are characterised with respect to the As, while the Cs are characterised solely with respect to the Bs.

[136] I discuss the problem of how to understand the starting position literally in my article "Reconstructing Zeno's Fourth Paradox of Motion".

While row A stands still, rows B and C move at the same speed in opposite directions. Given the form of ancient stadia, it seems that one row, let us assume the Cs, have already passed the turning point we suppose to be on the right-hand side of our sketch, and are hence moving away from it to the left, while the Bs are travelling towards it and thus move to the right. The As seem to have a breakdown, and are thus resting in between the Cs and Bs.

(2) As the Cs and the Bs move past each other, the first B will be at the (opposite) end of the As at the same time as the first C. We can visualise this end position as follows:

						A1	A2	A3	A4		
						B4	B3	B2	B1		
						C1	C2	C3	C4		

So far we have a situation which may be difficult to reconstruct, but which poses no logical problems. It is the following conceptual description of this state of affairs which turns it into a paradox. The text shows this in two steps, which I have labelled 3 and 4, respectively:

(3) We can understand the third step as a conclusion with three premises, one of which is stated only after the conclusion is given:

 (a) C has passed all the Bs (premise)
 (b) B has passed only half the As (premise)
 (c) Thus the time (which B needs for its movement past the As) is only half (the time C needs to move past the Bs) (conclusion from (a), (b), and a missing premise).

The missing premise can be found at 240a12–13: "for each of them [the Bs and the Cs] is for the same time alongside each [of the *ongkoi*]". And the reason why they are alongside each for the same time is that "it takes the same time [to pass] each of the Bs as each of the As, so he says", since they are "equal masses going along equals" (239b34), that is, the As, Bs, and Cs are all of equal extension. The crucial assumption for the conclusion is that each body represented by a letter is of the same size or extension, so it seems to take the same time to pass any of them. Accordingly, it should take twice as much time to pass four Bs than two As. The apparent basis of this premise was already given at the very beginning in part (I) – "equal masses are going along equals", without, however, spelling out the implications of this equality.

Up to this point it seems that to pass all the Bs, it took C double the time that it took B to pass half the As, for C has passed twice as many Bs as B has passed As. Given the fact that the As are resting while the Bs are moving, the problem already implied is made explicit in the following step:

(4) While the first B has passed half the As, it has also passed all the Cs. This follows from the characterisation of the end position: the first B is at the end at the same time as the first C. And this simultaneous arrival in turn follows from the fact that "both are for the same time alongside the As" (i. e., B and C have passed the same number of As at – as stated above – the same speed). With the starting position we have assumed, the first B and the first C will be at opposite ends of the A row simultaneously. So all Zeno is doing in step (4) is cashing out the implications of the initial set-up of the paradox:

a. The first B and the first C each start in the middle of the As (premise)
b. The Bs and Cs move at the same speed (premise)
c. The Bs and Cs are alongside the As for the same time (from a, b)
d. The first B is at the end (of the As) at the same time as the first C (from a, c)
e. While the first B has passed half the As, it has also passed all the Cs (from a, d)

Steps (3) and (4) in Aristotle's text together fully unfold the paradox: the first B seems to have needed only half the time for its movement past the As that the first C needed to pass the Bs, for B passed only half as many As as C passed Bs. Simultaneously, it appears that it took B double the time of this half time, since it passed twice as many Cs as As.[137] Hence, we arrive at the paradox stated at the beginning that double the time is equal to half the time, $t/2 = t$.

3.6.4.2 The Crucial Point of the Paradox

Even before Aristotle describes this paradoxical state of affairs, he points out the mistake of this argument: "the fallacy of the reasoning lies in [the assumption] that the passing of a moving body and of a resting body of equal size at equal speed will take the same time. But this is wrong" (240a1–4). According to Aristotle, Zeno does not take into account whether the thing passed is itself moving or resting.

However, Zeno points to the difference in the bodies passed – that one group is moving, the other at rest – at the very beginning of the paradox; it is unlikely that he would simply forget this later on. Such an oversight would also stand in stark contrast to Zeno's other paradoxes, in which he never simply ignores what he himself has set up. Nor does he seem to fail to notice that this difference between the bodies being passed makes a difference to the assessment of the time taken for the motion past them. Rather, Zeno demonstrates that while we are well aware that moving past something resting and past something moving should take different times, there seems to be a convincing

[137] The same problem arises with the Cs passing twice as many Bs than As.

account of such a scenario which shows that the same time seems to be half and double.

I have given a full discussion of the philosophical problem presented by this paradox elsewhere,[138] and will thus only give the gist of it here. The crucial point on which this paradox rests is that Zeno *implicitly* assumes that the time a motion takes is dependent on the distance traversed. This seems to be an unproblematic assumption, but only if it is confounded with the true assumption that the time is dependent on the motion performed (which in part is determined by the distance covered). Under certain circumstances both assumptions can be seen as coinciding. But Zeno's paradox skilfully picks out a case where they do not coincide and where confusing the two assumptions is in fact problematic. Zeno makes another, unproblematic assumption, namely that time as such, in which all motions take place, is not dependent on space. Initially, both assumptions – (1) time is dependent on the space traversed, and (2) time is not dependent on the space traversed – seem to be contradictory. However, when we see that (1) deals with the time of a particular motion and (2) with time as such, we understand that they are in fact not contradictory assumptions. Nevertheless, we will see that they are incompatible, since assuming both together is what leads to the conflict the paradox sketches. Let us deal with each assumption in turn.

(1) Let me start with time's dependence on space. By a 'dependent magnitude' I mean a magnitude whose value can be immediately derived from the magnitude it is dependent on; for example, in the function $y = x^2$, the value of y can be immediately derived from x. Likewise in the paradox, time is treated as a magnitude whose value (i.e., the amount of time needed for the movement) can be immediately derived from the value of space (i.e., the amount of distance covered by the motion);[139] time is treated as if it were a mere function of space. This dependence relation is an asymmetrical relation[140] in which space is the only relevant factor in determining time. Hence, the amount of time required is determined solely by the amount of distance travelled (i.e., by the number of bodies passed). This becomes obvious at 240a10–12 (quoted above): from the fact that C1 has gone past all the Bs, while B1 has passed only half the As, it is concluded that B1 must have needed only half the time for its motion that C1 took for its motion. The time needed is determined solely by the distance travelled, the number of As or Bs: covering one space unit always requires one unit of time. Accordingly, while Zeno seems to take into account

138 Sattler 2015.

139 In Chapter 8 we will see that the basic understanding of a unit of measurement in Greek thought is an invitation to this fixed connection of time to space, since it impedes thinking of magnitudes as having a complex dimension.

140 This asymmetry can also be seen from the example given above: a function is asymmetrical, since the value of x cannot be immediately and unambiguously derived from the value of y (for example, if y has the value 36, x could either be 6 or -6).

both time and space, as he mentions the distance passed as well as the time needed, he does not take time and space into account *each on its own*.

Prima facie this seems to be an unproblematic assumption since, if C1 performs a motion past twice as many elements as B1 does, shouldn't this motion go on for longer? However, in Zeno's account the time is in fact not conceived of as dependent on *motion*, but on *distance covered*. This is clear from the fact that from the mere number of bodies passed (not from the motion performed in passing them), it is inferred that B1 must have taken half the time for its enterprise of covering half the As by comparison with the time it took C1 to cover all the Bs. Covering more distance will naturally also mean performing more motion, and this is the reason why Zeno's assumption does not strike us as wrong prima facie. However, in making the amount of time dependent on the amount of space covered, Zeno's assumption is merely in the neighbourhood, as it were, of the correct assumption that the amount of time is dependent on the amount of motion performed.

To see this, compare a slow walker to a brisk one, both setting out from a common point at the same time. In the same time the brisk walker will cover more distance, so covering more space does not necessarily mean taking more time. But it would necessarily require more time if we were to assume that the amount of time needed is always dependent solely on the space covered. And this wrong, but not un-intuitive, assumption is what we see at work in Zeno's paradox. For, in his framework, the movement past a body cannot vary in speed, but must always take the same time.

You may object that Zeno explicitly tells us to consider rows moving at the *same speed*. We are not dealing with a case of slowly walking Bs and fast racing Cs. If we work with a set speed, then the two assumptions – that the amount of time is dependent on the amount of motion performed, and that the amount of time is dependent on the amount of distance covered – seem to coincide. There should thus be a fixed relation between the distance covered and the time required.

However, assuming that B and C move at the same speed is not the same as assuming that time is completely dependent on space. If a body is in uniform motion, then the time it takes to travel a certain distance will stay the same as long as all the circumstances of the motions considered stay the same. By contrast, a dependence relation would also hold if these circumstances changed. Even if we assume uniform speed, we must nevertheless take time and space into account as independent magnitudes, so that in principle the speed could vary. This shows once the circumstances change, as is the case either if the moving body begins to speed up or the extension covered itself begins 'to move'. The latter is the situation in Zeno's paradox.

If our slowly strolling Bs walk past a moving chariot, then the time needed to pass it will not depend only on the Bs' speed and the distance passed – the size of the chariot – since the time itself will vary with the speed of the thing being passed. But, on the interpretation I am proposing, Zeno cannot take into account the speed of the thing being passed for, were he to do so, time would no longer be treated as dependent solely on the distance passed. The presence of the resting As in Zeno's paradox lulls us into thinking that time is simply dependent on space, since if this is all that we look at, we will not get any false results. However, the fact that the Bs are also passing the Cs, which are moving in the opposite direction, reveals that the two assumptions that seem to be equivalent in the resting case – the assumption that the amount of time is dependent on the amount of motion performed and the assumption that the amount of time is dependent on the amount of space covered – in fact come apart.

It is in these cases, where a fixed connection between time and space is not sufficient to account for the motion performed, that we encounter an apparent paradox. In contrast to the motion past a resting magnitude, where treating time solely as dependent on space does not emerge as a mistake, the moving row scenario makes it clear that motion must be understood as a relation between time and space in which both relata can change independently of each other. In this case, where the thing to be passed itself moves towards the thing passing, the speeds of both add up. If they share the same speed, the passage will occur twice as fast as if the thing passed were at rest.

(2) We also find the assumption that time as such is understood as a magnitude that is *not* dependent on space, that is, as a magnitude whose quantity cannot be immediately derived from the quantity of distance covered in Zeno's paradox. This second assumption is required in order to understand as simultaneous Bs' motion past the As, Bs' motion past the Cs, and Cs' motion past the Bs. If time were merely understood as dependent on the distance travelled, then the motion of the Cs past the Bs and that of the Bs past the As would be two completely independent events and not comparable. Likewise, the motion of the Bs past the As and that of the Bs past the Cs would be independent of each other. True, the Bs would still be the 'movers' in both. But, we only get a paradox if we claim that the Bs are *simultaneously* passing the Cs and the As. B1's passing of two As may be completely unrelated to its passing of four Cs. Why, then, should the time taken by the one be in any conflict with the time taken by the other? B1 might perform both motions at the same speed, but unless we also claim that it requires the same amount of time to carry out both enterprises, there is no paradox. (Strictly speaking, both motions need not take place *at* the same time, but only *in* the same time, i.e., requiring the same amount of time – simultaneity is just a simple way of making sure that we talk of the same amount of time.) What allows us to compare the different motions is the fact that time as such (i.e., not the time a particular motion takes) is treated as a magnitude not dependent on space, that is, as a magnitude that is not simply a function of the distance covered, so that we can say that the two

motions (covering different amounts of distance) happen *simultaneously* or 'in the same time'.[141]

The paradox thus arises as a result of two assumptions. The time of a motion is treated as dependent on the distance travelled, so that it can be inferred that the passing of each body requires one time unit (1). On the other hand, time as such is treated as an independent magnitude, when it is stated that *at the same time* B passes two As and four Cs (2). If time as such were also dependent on space, these motions would not take the same time. If we look at the motions of the Bs and Cs in relation to time as a magnitude not dependent on space, we see that both move for the same amount of time. But when time is understood as determined by the distance travelled, and the movement of the Bs is seen with respect to the As and the movement of the Cs with respect to the Bs, the Bs seem to have moved for a shorter period than the Cs; and the motion of the Bs past the As seems to have taken a shorter time than their motion past the Cs. Assumptions (1) and (2) work together to generate the paradox.[142]

These two assumptions about time are not made explicit. Both seem to have some intuitive appeal and both seem unproblematic so long as they are not assumed simultaneously. The second assumption, that time is seen as an independent magnitude, we usually do not take to be problematic but rather to be true. And the first assumption is unproblematic as long as we look at continuous motions past something resting.

The scholars who assumed this paradox to have any philosophical worth at all have understood it as either resting on atomistic assumptions or as failing to take into account some essential relativity of motion.[143] The atomistic interpretation in its most widespread form understands the paradox along the following lines: Zeno assumes that there are smallest, indivisible time atoms, and that one body will be passed in one indivisible time atom. But if we look at the time atom in which B1 passes A3, we find that during that same indivisible time atom, B1 has passed two elements of the C row, C1 and C2. According to the premise, however, the passing of each element is connected to one indivisible time atom, so that double the time – the two time atoms required to pass C1 and C2 – equals half the time – the one time atom which is necessary to pass the one A3.

Against such atomistic interpretations, it must be pointed out that we will be caught up in the paradox whether we start from the assumption that time (and space) are structured continuously or from the assumption that they are structured discretely. We saw in the reconstruction above that we do not

141 This is an assumption we usually make and thus I do not spell it out as an extra step in Sattler 2015.

142 Note that the distinction we find later in Aristotle between time being dependent on a particular motion and time being dependent on there being some motion does not help with this paradox.

143 For a much fuller discussion of the secondary literature, see Sattler 2015.

have to assume any time (or space) atoms; the crucial point of the paradox is not the assumption of atoms. Rather, it is the assumptions that time is completely dependent on the distance travelled, and that time is a magnitude not dependent on space; both assumptions are also made by the atomistic interpretations.

Without these assumptions, we get the following atomistic scenario: B1 passes A3 in one time atom, but at the same time it also passes two Cs. So B1 passes one C in half a time atom although a time atom is by definition indivisible. Salmon, who takes up the atomistic account, offers an alternative: B1 passes two Cs in one time atom in such a way that it is never actually opposite the first C. In both these versions, atomistic interpretations wind up reconstructing a paradox with a different punchline from the one handed down to us as Zeno's: they demonstrate either that what is posited to be indivisible must in fact be conceived of as divisible to accommodate motion, or that, when passing some extension, we are never actually opposite it, so that motion occurs in jerks.

Furthermore, if one wanted an atomist paradox, much less machinery would suffice. All that is needed is Aristotle's question in book VI of *Physics*, "if A passes B in one time atom, what happens if A moves twice as fast?" Why, then, should Zeno build up this rather elaborate scene in the paradox (which, judging from the manuscript tradition, confused at least some of the scribes) in order to make a point that he could have made in a much simpler way?

Relativistic accounts assume that Zeno fails to take into account some essential relativity of motion by ignoring the difference between a motion past something moving and past something resting. However, Zeno points to the difference in the bodies passed – that one group is moving, the other at rest – right at the beginning of the paradox. He does not seem to fail to notice that this difference between the bodies being passed makes a difference to the assessment of the time taken for the motion past them. Rather, he points out that if we try to give an account of such a scenario, we end up with the paradoxical situation that the same time seems to be half and double.

Aristotle himself, our source for this paradox, is extremely brief in his account of the philosophical problem of the paradox in his *Physics*; his actual analysis consists in the three lines we have seen above (240a1–4). And, in contrast to the runner paradox, this paradox is not taken up again anywhere else. Part of the reason why Aristotle dismisses this paradox so quickly is that, as we will see in the last two chapters, avoiding this paradox requires something also Aristotle cannot do in full – give an account of speed in which time and space are brought together as two independent yet correlated magnitudes. Time, which Zeno's paradox assumes to be dependent on space, must be conceptualised as *independent of the distance*[144] and *dependent on the*

[144] Time is nevertheless correlated with the distance covered in a motion, for, if everything else stays the same, the greater the distance covered the more time is needed.

movement in question. While Aristotle undertakes important steps in this direction, we will see in the chapters to come why he nevertheless cannot give such an account of time, space, and motion.

3.6.5 *The Basic Problems of All Paradoxes of Motion*

Our conceptual interpretation shows that the four paradoxes of motion are founded on two basic problems; the first is a problem for all continua, while the second is a problem specific to motion:

(1) The continuum problem: How can the relation between whole and part be thought of in the case of continua like motion, time, and spatial magnitude where a finite whole seems to contain infinitely many parts? Can we infer from what holds true of the parts what holds true of the whole? How do we know what a proper part is, if there are no given parts and the division between parts can be done as we please? And how can these parts be determined if not via their extension?

(2) The motion problem: How must the relation of time and space be conceived if we want to give an account of motion? On the one hand, a certain time and a certain distance are correlated in a certain motion and they need to share some features, for example, it cannot be the case that one is infinite and the other finite. On the other hand, they must be understood as distinct and independent entities.

The arrow paradox is based on the first problem, since it raises the questions (i) what can be considered a proper part of a period of time and (ii) under which conditions is it possible to infer what holds true of the whole from what holds true of what we take to be a part. The paradox occurs because the nows are treated as parts of time in such a way that we are meant to infer from the indivisible and extensionless now, 'in' which motion is impossible, what is the case in an extended period of time, namely that motion is impossible over the whole period. Furthermore, the arrow paradox can also be seen as touching upon the second question, for it defines the now as a part of time in a way which creates problems for defining movement in its temporal respect.

The paradox of the moving rows is based on the second problem, as it raises the question of how to understand the relationship of time and space when we consider motion. The paradox arises because time and space are not treated as independent magnitudes; rather, parts of time are fixedly assigned to spatial parts.

Finally, the dichotomy and the Achilles paradox rest on both problems, since they raise two questions: (1) How can a finite whole, for instance a run, consist of infinitely many parts? and (2) How can an infinite number of spatial segments, which seem to be contained in a finite race track, be covered in a finite time?

Due to these two basic problems, any account of motion seems to be inconsistent: a certain movement is a whole and has parts, but nevertheless the relation between whole and part cannot be consistently conceived. And although the paradoxes seem to take time and space into account, it is not clear what the relationship between them is when we try to give an account of motion. The following chapters will demonstrate that the two basic problems are not independent of each other. As will be shown, the time taken and the distance covered by a motion can be divided by each other mutually, which raises the question of how the parts thus produced relate to their whole.[145]

Our conceptual interpretation, according to which we analysed the notions employed in the paradoxes and asked whether these notions are developed sufficiently, consistently, and adequately for the realm in question, has allowed us to reconstruct Zeno's paradoxes of motion as genuine motion paradoxes. Furthermore, it has shown that these paradoxes not only raise the two problems sketched above – the motion and the continuum problems – but also that Zeno's paradoxes simultaneously employ different part-whole relationships, which leads to inconsistencies: he works with a part-whole relationship that presupposes the priority of the whole over (potential) parts, and, simultaneously, he understands this part-whole relationship as presupposing the priority of parts over the whole that is made up of them. Moreover, for Zeno time is understood as independent of space, while the time of a motion is seen as completely dependent on the distance covered.

Zeno's paradoxes can thus be seen as a touchstone for determining whether later natural philosophers can indeed meet the Eleatic challenge: whether they can clarify the central notions involved in understanding motion – notions like time, space, whole, part, and infinity – in such a way that they avoid these paradoxes. Only a natural philosophy that can avoid getting into Zeno's paradoxes has shown that motion is not inconsistent and that we can indeed have a scientific understanding of motion. Such a natural philosophy must show that although time and space must be independent of each other, the two are correlated when we deal with motion. And with respect to the part-whole relationship, natural philosophers will have to opt for the priority either of the parts or of the whole. We will see the atomists choosing the first alternative, Aristotle the second.

Zeno's paradoxes demonstrate the inconsistencies that seem to arise if we try to give an account of plurality and movement. If the problems they raise cannot be resolved, the concepts of plurality and motion can have no place within philosophy and science, since their application would lead inevitably to inconsistencies. In that case, assuming an unmoved single One – the position of Zeno's teacher Parmenides – might indeed seem to be the only tenable position.

145 These parts usually exist merely for the purposes of measurement, not as physically separate parts, see Chapter 7.

4

The Atomistic Foundation for an Account of Motion

4.1 Introduction

This chapter will focus on the atomists Leucippus and Democritus as a particularly clear example of the first wave of philosophers' reactions to the Eleatic challenges for natural philosophy.[1] These challenges are, as we saw in the two preceding chapters, Parmenides' exclusion of time, space, motion, and change from the realm of philosophical enquiry and Zeno's paradoxes, which show that any assumption of plurality and motion leads into inconsistencies.

The atomists' reaction is not simply a rejection of the Eleatic position. In fact, the atomists remain within the Eleatic framework to a considerable extent in their understanding of the fundamental reality. However, they take an important step towards the development of a genuine natural philosophy thanks to two main points: (1) they adopt a physical interpretation of the fundamental entities and (2) they claim that a crucial role for the fundamental entities is to explain sensible phenomena.

The view that the atomists[2] accept crucial parts of the Eleatics' argumentation while trying to do justice to the sensible phenomena, which they take as an important part of their theory, has been around at least since Aristotle's account in *De generatione et corruptione* 325a23–9:

> Leucippus thought that he had a theory which would grant to perception what is generally agreed, and would not do away with coming into being and passing away or motion or the plurality of things. In those respects he agreed with what seems to be the case, but to those who proposed the

1 Anaxagoras' reaction to Parmenides seems less direct, and Empedocles has several additional agendas, apart from doing natural philosophy, that influence his particular approach. Hence the atomists are the clearest example for our project. Osborne 2006 and Palmer 2009 have recently expressed doubt that Empedocles and Anaxagoras react to the Eleatics. Both scholars have, however, excluded the atomists from this doubt to some degree. I will not have space here to look at other atomistic positions such as that of Xenocrates or Diodorus Cronus.

2 I will not be able to go into possible differences between Leucippus and Democritus here. Scholars who think that Leucippus was primarily interested in cosmology and not in questions of Being and Non-Being should read this chapter simply as an account of Democritus.

> theory of the One he agreed that there can be no motion without void and that the void is not, and that nothing that there is not; for what really is a total plenum.[3]

Modern scholars mostly agree with Aristotle that the atomists somehow react to the Eleatics, but their exact understanding of this reaction varies in their assessment of the degree of influence of Eleatic thinking, how critical they assume the atomistic reaction to be, and which of the three Eleatics they take to have had the most decisive influence.[4] In this chapter, I will show that the atomists are influenced by the Eleatics much more broadly than often assumed and in fact by all three Eleatics, and that we can find both 'anti-Eleatic' and 'neo-Eleatic' elements in the atomists. We will see how their metaphysics allows the atomists to keep the Eleatic criteria for philosophy while at the same time attempting to give an account of the sensible phenomena.

The atomists also illustrate my point that any attempt to exploit the logical potential inherent in Parmenides' concepts by giving them a new interpretation, necessitates changes of all the other connected concepts. In Parmenides, the three basic concepts systematically connected are Being, negation as a separation operator, and identity as connection operator. The understanding of these three notions determined also that non-Being must be understood as the polar opposite of Being and as what is in no way. With the atomists, we will see how the introduction of a new notion of non-Being changes the understanding of negation, as well as of Being and their connection operator. The reappropriation of the Eleatic philosophy is crucially based on the physical interpretation the atomists give to the logical notions of Parmenides, interpreting non-Being as the empty and void and Being as the full and atoms.

My reading of the atomists is novel in that I show how this physical interpretation permits them to take two groundbreaking steps: (1) for the *first time* in Western thinking they explain the realm of phenomena *systematically* in terms of a distinct realm of what truly is and thus establish an ontological grounding relation;[5] and (2) they implicitly expand the logical

[3] In this passage, the echoes of Eleatism seem to be more obviously related to Melissus' arguments and vocabulary than to Parmenides', as is shown, for example, by the reference to the void as a necessary condition for motion. And it has been argued that it is only Melissus who had a doctrine of the One, not Parmenides. I will show, however, that the atomists clearly also reacted to Parmenides; Chapter 2 should have shown why I think that Parmenides also had a doctrine of the One.

[4] For example, Sedley 2008 has argued that Democritus should be seen as "not anti-Eleatic but as neo-Eleatic", but does not think that Zeno played much of a role for them; while Osborne 2006 downplays the Eleatic influence on the atomists altogether.

[5] While Empedocles does indeed show how the realm of appearances can be seen to be compatible with Parmenides' claim about what truly is, we are not given an explanation in

possibilities of the Eleatic basis in crucial ways, for their physical interpretation allows for the understanding of non-Being as a basic concept *on a par* with Being. That is, both Being and non-Being can be clearly characterised and thus both can be thought, and, ultimately, both *are* in some sense.

We will see that this understanding of non-Being as *on a par* with Being necessitates an implicit distinction of Being and non-Being as entities from being and non-being as operators. In this way the atomists increase the complexity of the logical system in use, which we saw to be a precondition for establishing natural philosophy as a proper scientific endeavour. But while the atomist approach prepares a logical framework for a philosophy of nature to some extent, it cannot give a consistent account of motion, time, and space. Zeno's paradoxes – our test for the explanatory power of a natural philosophy – cannot be sufficiently dealt with in the atomist framework.

4.2 Eleatic Inheritance in the Atomists

Metaphysically speaking, the fundamental notions the atomists start out with are the full or solid (*to plêres*) on the one hand, and the empty (*to kenon*) on the other. They seem to have called the full also 'thing' or 'Being', and the empty also 'no-thing' or 'non-Being'.[6] The two main ingredients of their theory – what I will refer to here as simply 'atoms' and 'void' – are captured in terms of full and empty, and that means in terms that come from the physical and bodily realm. In addition, they seem to have taken up from the phenomenal realm what Zeno has explicitly attacked in his paradoxes as inconsistent, namely motion and plurality.

Already from this brief sketch we see that while certain features sound like an echo of Eleatic ontology, there are also important changes. What I will do in the current section is sketch the aspects where the atomists seem to be firmly Eleatic – in their adoption of the Eleatic criteria for philosophy. I will then, in the next section, show how their ontology and logical operators introduce important changes to the Eleatic base.

Let us now look at the role played for the atomists by Parmenides' basic criteria for philosophy – consistency, a version of the principle of sufficient reason, and rational admissibility.

terms of what truly is for how or why certain phenomena do come about. For Anaxagoras, everything that ever will be is already in the original mix out of which the world comes into being; thus there are not two realms that are clearly differentiated from each other (DK59 frr. 1 and 4). I offer a more detailed account in my paper "What Is Doing the Explaining?"

6 See Aristotle *Metaphysics* A, 985b4–20 (= DK67 A6); T45. If available, I give references to Diels-Kranz, otherwise I follow Taylor's edition; Democritus' fragments are cited as "D", testimonies as "T" in Taylor.

4.2.1 Rational Admissibility

Parmenides establishes the criterion of rational admissibility in fr. 7 of his poem. It amounts to giving an account of some *x* that is based on a rational analysis and can thus withstand rational scrutiny. As a consequence, such an account of *x* should in principle be intelligible to every other rational being and comparable to other objects of investigation. Zeno's paradoxes strengthened this criterion by showing that our everyday experiences are not an adequate criterion, if we want to judge what truly is; only our reason can be such a judge.

The atomists implicitly take over this criterion, as can be seen, for example, from Democritus, frr. 9 and 11:

> "νόμωι" γάρ φησι "γλυκύ, καὶ νόμωι πικρόν, νόμωι θερμόν, νόμωι ψυχρόν, νόμωι χροιή, ἐτεῆι δὲ ἄτομα καὶ κενόν".
>
> For he says, "by convention sweet and by convention bitter, by convention hot, by convention cold, by convention colour; but in reality atoms and void". (Sextus *Against the Mathematicians* VII. 135, also in Diogenes Laertius and Galen, DKB9)
>
> ἐν δὲ τοῖς Κανόσι δύο φησὶν εἶναι γνώσεις· τὴν μὲν διὰ τῶν αἰσθήσεων τὴν δὲ διὰ τῆς διανοίας, ὧν τὴν μὲν διὰ τῆς διανοίας γνησίην καλεῖ προσμαρτυρῶν αὐτῆι τὸ πιστὸν εἰς ἀληθείας κρίσιν, τὴν δὲ διὰ τῶν αἰσθήσεων σκοτίηνὀνομάζει ἀφαιρούμενος αὐτῆς τὸ πρὸς διάγνωσιν τοῦ ἀληθοῦς ἀπλανές. λέγει δὲ κατὰ λέξιν·"γνώμης δὲ δύο εἰσὶν ἰδέαι,ἡ μὲν γνησίη, ἡ δὲ σκοτίη· καὶ σκοτίης μὲν τάδε σύμπαντα, ὄψις, ἀκοή, ὀδμή, γεῦσις, ψαῦσις. ἡ δὲ γνησίη, ἀποκεκριμένη δὲ ταύτης". εἶτα προκρίνων τῆς σκοτίης τὴν γνησίην ἐπιφέρει λέγων·"ὅταν ἡ σκοτίη μηκέτι δύνηται μήτε ὁρῆν ἐπ' ἔλαττον μήτε ἀκούειν μήτε ὀδμᾶσθαι μήτε γεύεσθαι μήτε ἐν τῆι ψαύσει αἰσθάνεσθαι, ἀλλ' ἐπὶ λεπτότερον . . ."[7]
>
> But in his *Canons* he [Democritus] says that there are two sorts of knowledge, one through the senses and the other through thought; he calls knowledge through thought 'genuine', testifying in favour of its trustworthiness in the judgment of truth, and he names knowledge through the senses 'bastard', denying it inerrant recognition of the truth. His own words are: "There are two kinds of judgment, genuine

[7] Sedley 1992, pp. 41–2 suggests to read the two Democritean excerpts in this passage as continuous (ignoring the gloss in between), to delete the comma after αἰσθάνεσθαι, and to accentuate the following word, ἀλλ', on the alpha, so that the last sentence would read: "The one which is genuine, but separate from this one, is when the bastard one is no longer able either to see in the direction of greater smallness, nor to hear or smell or taste or sense by touch other things in the direction of greater fineness." This reading would allow for a qualitative distinction between perception and thought, but would not fit with Aristotle's claim that Democritus understands thought on the model of perception, see below.

> and bastard. To the bastard form belong all these, sight, hearing, smell, taste, touch, but the genuine is separate from this." Then he continues, ranking the genuine above the bastard form, "When the bastard form can no longer see anything smaller or hear or smell or taste or perceive by touch, but to a finer degree . . .". (DKB11 plus context from Sextus *Against the Mathematicians* VII.135–40; Taylor's translation)

Fr. 9 draws a contrast between what is by convention and what is truly. The qualities we gather via the senses are what they are only by convention, *nomô*. The sweetness conveyed to us by the senses is not what truly (*eteê*) is. This distinction between convention and truth mirrors the Eleatic contrast between the realms of *doxa* and *alêtheia*: what is by convention resembles what is relied upon in the realm of *doxa*, and it is clearly separated from what truly is, which seems to correspond to the realm of *alêtheia*.[8] Furthermore, fr. 11 states that all sense perception belongs to the bastard kind of judgement. Accordingly, genuine judgements cannot be derived from perception, but are based on reason – the claim that sweetness is only by convention must come from thought, as fr. DK68 B125, T179c, makes clear.[9] So it is by reason that the atomists have derived knowledge of the truth, that there are only atoms and void. Atoms and void are the basis for the phenomena of perception, but they are not themselves perceptible.[10] Accordingly, what counts as knowledge has to be admissible by reason.

But how does treating the senses with suspicion fit with Aristotle's account, quoted above, that the atomists want to *explain* the perceptible phenomena, which is also supported by testimonia that claim the truth of appearances?[11]

The two seemingly contradictory groups of fragments can be reconciled if we take into account that what truly is also the basis for the explanation of sensory phenomena. The atomists do not doubt the phenomena the senses give us, since the phenomena are a central aspect which the atomic theory is meant to explain. However, Sextus' testimony tells us very clearly that the atomists do not understand the appearances to be true in the sense that claims about atoms and the void are true. That is, appearances are not ultimate constituents of

[8] While Parmenides called the fundamental reality simply "Being" (*eon*), the atomists also ascribe some form of being to non-Being and the phenomena and thus refer to what ultimately is as what truly is.

[9] There are different possibilities for understanding *nomos*. Taylor 2007, pp. 9–11 takes it that something's being F νόμωι can mean either what is in fact F but only because humans have agreed or decided that it is to be regarded as F, or what is in fact not F but merely called F. Galen interprets what is νόμωι as what is relative to us qua perceivers; see DK68 A49, Taylor T179d. Barnes 1982 understands what is νόμωι as what is mind-dependent. Here, however, the exact understanding of νόμωι does not matter; all that matters is that the qualities we experience via the senses are not what truly is.

[10] See also T180 (= DK68 A110), T182a (= DK68 A59), b–d, and f.

[11] See T42a (= DK67 A9) and b, and 183b.

reality independent of any perceiver; this understanding is supported by another piece of testimony from Galen:[12]

> . . . all perceptible qualities are brought into being relative to us who perceive them, by the combination of atoms, but *by nature* nothing is white or black or yellow or red or bitter or sweet. (DK68 A49, T179d, Taylor's translation; emphasis added)

Appearances are true in the sense that we try to explain them as they present themselves to us. They provide the *explanandum* that the theory must explain, and thus they are a control on our explanation. The main interest of the atomists is not whether what we perceive as green is instead red or what we perceive as sweet is instead bitter, in other words, whether phenomena differ from what they seem to be. Rather, they are principally concerned with how we can account for our perception of something as red or sweet. Accordingly, the atomists should be regarded neither as complete sceptics with respect to sensory knowledge[13] nor as relativists who claim, as Protagoras does, that all perceptions, even apparently contradictory ones, are true.

For the atomists, the appearances are nothing but certain arrangements of atoms as they show themselves in the sensible world. The appearances have an objective side – they are arrangements of atoms – as well as a subjective side, for these arrangements are to us perceptible qualities.[14] Furthermore, it seems the arrangements of the atoms must explain not only the perceptible qualities, but also the things that have these qualities, as for example a table that possesses a certain colour or a tart with a certain sweetness.

Hence, we can say that the atomists take over the criterion of rational admissibility for what truly is. But they do not adopt it without modification. The most important change is that the basic ontological constituents must not only be rationally admissible but also explain the phenomena.[15] Thus, the atomists modify the Parmenidean scheme by bringing together the realm of truth and the realm of *doxa* in such a way that the former is also meant to account for the appearances of the latter. This change introduces a new criterion, the *criterion of saving the phenomena*. This addition seems to strengthen the basic Parmenidean approach by giving it greater explanatory power: the account of what truly is can also explain the phenomena in the sensible realm. From a Parmenidean perspective, this strengthening of the explanatory power may come at a cost, however, as the atomists must say that reason can take us only so far. For example, reason cannot tell us how many

12 And by Sextus' claim in T182a–e that for the atomists only intelligible, but not sensible, things are true.

13 As Barnes 1982, p. 559 ff. assumes.

14 The perceptible side also depends on the physical condition of the perceiver, according to DKA135.

15 See also T179a.

atomic shapes there are; for that, we need to consider the phenomena. This significant limitation on reason means that it underspecifies the ontological basis. In addition, the account of the difference between sense perception and thought is no longer qualitative, as in Parmenides, but quantitative for the atomists: perception is concerned with larger phenomena, thinking with smaller phenomena[16] – a distinction that also seems to be the basis for Aristotle's claim that Democritus supposed knowledge (*phronêsis*) to be perception.[17]

4.2.2 Consistency

The second criterion that Parmenides introduced in his poem, consistency, is used by his immediate successors without much ado, and the atomists seem to do likewise. There are, however, testimonies suggesting that the atomists assumed contradictories to be true of the same thing, which would not fit with employing consistency as a fundamental principle:

> This [i.e., that opposites are true of the same thing] follows also in the case of those who say that what appears to be and what is are the same, e.g., Democritus, Protagoras, and their followers. (T183b, Alexander *Commentary on Metaphysics* 271.38–272.2)

When we look in more detail at the way in which opposites are regarded as being true of the same thing, however, we find the examples claim either that the same thing appears opposite to us and to other animals[18] or that the same thing appears opposite to different persons.[19] And in Aristotle *Metaphysics* Γ, 1009b8–9 we are told that the same individual does *not always* arrive at the same judgement about what appears, so the atomists also seem to allow for temporal differences in the appearances as they present themselves to one individual. Thus, rather than abandon the principle of non-contradiction, we can understand the atomists as developing that principle further: by introducing the distinctions of different perceivers and different times, they implicitly appeal to differences in respects which were lacking in Parmenides' account. Therefore, the principle of non-contradiction at work seems to be 'not (F and not-F) for the same perceiver at the same time'. Plato and Aristotle will make explicit this important modification and generalise it.

The understanding that the atomists do not abandon the principle of consistency is supported by the fact that the consequence they draw from the

[16] This is what DK68 B11 suggests in claiming that bastard judgements can discern features only up to a certain point of fineness.
[17] *Metaphysics* Γ, 1009b7–17.
[18] *Metaphysics* Γ, 1009b7–8.
[19] Sextus *Pyrrh*. 1.213–14; cf. also T179d above.

occurrence of opposite appearances of the same thing to different perceivers is not that something can be both F and not-F, which would call into question the principle of non-contradiction, but rather that in truth neither F nor not-F holds simpliciter.[20] But it is only with Plato that the important changes in the understanding of the principle of non-contradiction we see with the atomists are made explicit.

4.2.3 The Principle of Sufficient Reason

With their *ou mallon* reasoning the atomists take up Parmenides' principle that a sufficient reason is required for something to happen or to be. In Parmenides, the principle is focused on the argument against generation and supports homogeneity, while the atomists use it in a very broad way. *Ou mallon* means 'no more' or 'not rather' and claims that there is no more reason to assume this than that, F rather than not-F, which can be understood as an inversion of the principle of sufficient reason. As we saw in the first chapter, Anaximander claimed that the earth is resting since, given its place in the cosmos, there is no more reason (*mallon outhen*) for it to move one way rather than another. This first *ou mallon* reasoning may have been an inspiration for the atomists, given that they take up the principle of sufficient reason in this negative way. Or at least this negative way is dominant, for we find a positive formulation in the only surviving fragment giving us a full sentence from Leucippus (DK67 B2): "Nothing happens in vain, but everything in accordance with reason and by necessity."[21] But in all other cases the principle is used in a negative way.

The atomists derive different conclusions from *ou mallon* reasoning, positive and negative ones. Either they conclude simply that since there is no more reason for assuming something to be F than not-F, it is neither F nor not-F. This is in cases were F and not-F are assumed to be incompatible, for example, in a passage from Sextus (testimonium T178a): there is no more reason to assume honey to be sweet than bitter, since it appears sweet to one and bitter to another; so honey is neither sweet nor bitter. Alternatively, the atomists argue that there is no more reason for assuming something to be F than not-F, so it is G, a *tertium quid*, which shows that for the atomists the principle of non-contradiction no longer implies the principle of excluded

[20] We saw in Chapter 1 that the result can be that something is F here/now/for *x* but not-F there/later/for *y*, or, simply, that in truth it is neither F nor not-F. For more options see below.

[21] According to Schofield 2002, our sources suggest that the *ou mallon* principle is a distinctively Democritean contribution to atomic theory. This would fit with finding Leucippus using only a positive version, but is harder to square with Simplicius *In Phys.* 28.4 (DK67 A8 and DK68 A38), which suggests that Leucippus used *ou mallon* reasoning for the assumption of infinitely many shapes of atoms.

middle.[22] Finally, there is a version of this principle that seems to have the structure 'there is no more reason for there to exist some *x* than there to exist some not-*x*, so if *x* exists, not-*x* also exists', where *x* and not-*x* are assumed to be compatible. We find such a version in testimonia T80c and d from Aristotle and Philoponus, respectively: since there is no more reason for a world to be here than there in the void and there is one here, there is one in all the void, and thus there are infinitely many worlds.[23]

Makin points out that the general premise of an indifference argument has the form "there is no more reason for p than for q"[24] and can be read either epistemologically (there is no more reason to say/assert/judge that p than q) or non-epistemologically (there is no more reason for p to be true than for q to be true). The conclusions drawn can also be epistemological or non-epistemological. We saw above that there are clear examples of non-epistemological conclusions with the atomists, such as the existence claim of infinitely many worlds or the claim that honey is neither sweet nor bitter, which shows a rather strong principle of sufficient reason at work.

So far we have seen that the atomists take over Parmenides' criteria for philosophy to a considerable degree. In order to see how the atomist can establish a natural philosophy in spite of this Eleatic inheritance, we must now look at the changes they introduce to the Eleatic framework.

4.3 Atomistic Changes

4.3.1 What Truly Is Must Explain the Phenomena

The atomist theory can be read not only as responding to the problems for natural philosophy raised by the Eleatics, but also as a systematic development of the Eleatic fundamentals such that they prepare the way for establishing natural philosophy. The starting point for this development is not simply that the atomists want to give an explanation of the sensible world that implies plurality and motion. Had they wished to do so, they could have followed the model we saw in the *doxa* part of Parmenides' poem. Rather, the crucial starting point is the idea that what truly is must itself in some way be responsible for the appearances of the phenomenal world. That is, given that

[22] So, for example, in testimonium 99c by Ps.-Plutarch: according to Democritus (and Parmenides), there is no more reason for the earth to move in this than in that direction, so it does not move. (Aristotle claims in *De caelo* 294b13 ff., however, that the atomists explain the resting of the world by the resistance of the air beneath it.)

[23] Note that I am not claiming that the atomists had an explicit understanding of these different argument types, but only that we can in principle distinguish these types in the arguments handed down to us.

[24] Makin 1993, p. 131 ff. Makin talks about different propositions p and q, while the examples from the atomists above seem to talk about different predicates F and not-F, but the following distinction can hold for both.

the appearances are how what truly is appears to us, what truly is must be such that its features can also explain how these appearances come to have the features they have.[25]

4.3.2 A Physical Theory

The way the atomists choose to explain the appearances in terms of what truly is makes a crucial assumption: since the sensible world is bodily, a plurality, and in motion, these features must belong also to what truly is. There is no logical necessity for what truly is to share these features with the appearances. But the assumption that they share these features makes for explanatory economy,[26] since no complicated account of how something non-bodily can appear as bodily is thus needed.[27] But how can an account according to which what is truly is bodily, a plurality, and in motion – all features expelled from the realm of what truly is by Parmenides – be in any way compatible with Parmenidean criteria?

First, any *generation* on the fundamental level of the atoms is excluded – generation is what seems to have been most bothersome for the early Greek thinkers and what the atomists thus explain on the level of phenomena with the help of an ungenerated atomistic base. Further support is gained from the introduction of the void into the discussion – presumably first by Melissus, though he seems to have done so only to reject the assumption of its existence:[28]

[25] The starting point of the atomistic explanation has been interpreted in different ways. While Kirk, Raven, and Schofield 1983, pp. 412–13 think that the atomists give a priori arguments for atomism and confirm those with reference to perception, Graham, p. 625 thinks that what we experience gives general evidence of a pluralistic and changing world, which is sufficient to start the atomistic argument.

[26] But if we start out with bodies in order to account for the bodily world we perceive, why do we not also need to start out with what can be perceived? The idea in the background here may be that we are familiar with the possibility of something getting smaller and smaller until it ceases to be perceptible (for example, a turtle crawling away from us), so that we can imagine a sum of such non-perceptible things to be perceptible; but we do not find a similar phenomenon for bodies.

[27] However, given that the atoms do not possess most of the qualities that the phenomena display, such as colour, the atomists will still have to assume that such perceptible features in the phenomenal realm come from the arrangements of atomic bodies, while each atomic body on its own does not possess these qualities.

[28] This innovation makes sense as an immediate reaction to the atomists; alternatively, it could come out of an intra-Eleatic discussion of Parmenides' claim that the One is limited. If this claim is understood in physical terms, then we face the problem that this physical thing cannot be limited by another Being, since there is only one Being. The alternative is limitation by the void (DK30 A8), which would thus also somehow exist. Melissus' claim that Being is unlimited would then be intended as a means of avoiding the void.

> οὐδὲ κενεόν ἐστιν οὐδέν· τὸ γὰρ κενεὸν οὐδέν ἐστιν· οὐκ ἂν οὖν εἴη τό γε μηδέν. οὐδὲ κινεῖται· ὑποχωρῆσαι γὰρ οὐκ ἔχει οὐδαμῆι, ἀλλὰ πλέων ἐστίν. εἰ μὲν γὰρ κενεὸν ἦν, ὑπεχώρει ἂν εἰς τὸ κενόν· κενοῦ δὲ μὴ ἐόντος οὐκ ἔχει ὅκηι ὑποχωρήσει.

> There is nothing void. For the void is not, and thus non-Being cannot be. And no Being can move.[29] For it has nowhere to give way, but is full. For if there were void, it would give way into the void. But since there is no void, it has nowhere to give way. (fr. DK30 B7, lines 23–9)

According to this fragment, the void is not, which has two consequences, one for non-Being and one for Being: if "the void is not", then "non-Being cannot be". The void is thus either a condition for the existence of non-Being or quasi-equated with non-Being.[30] In addition, if the void is not, Being cannot move. The idea here seems to be that if void is intimately connected with non-Being, then Being is the opposite of the void, it is what is full. And what is full can only move if there is something that is not full into which it can move, the void. In fr. 8, line 24 Parmenides had claimed that what is full of Being, which we understood as rejecting any internal differences that would prevent its complete homogeneity. Also for Melissus, the fullness in question is the guarantor of homogeneity. In addition, it seems to get him close to a physical interpretation of Being, as we find in the atoms of the atomists, where fullness is thought as the fullness of a physical body[31] – a physical interpretation of Being complementing the physical understanding of non-Being as (or as conditioned by) void. In this way the introduction of the notion of the void prepares a shift of the discussion about what ultimately is from an intelligible realm in Parmenides to the physical realm of bodies and their motion, and thus prepares the way for a physical interpretation of Eleatic metaphysics.

The void is not introduced as something bodily itself, but as a necessary condition for the motion of bodies. As a necessary condition for the motion of Being, it can serve as a basis for a notion of space or place, as that in which motion can take place.

But Melissus' fragment not only demonstrates the void as a necessary condition for motion and thus as a spatial notion; we can also read him as distinguishing between being void and *the* void. While 'the void' can be understood as spatial, as what allows for motion, 'being void' can be

[29] I follow Diels-Kranz in understanding this sentence to be claiming that nothing that is can move. Kirk, Raven, and Schofield 1983 understand it as claiming that the empty cannot move, but the following explanation for the non-moving, also in their translation, is that it is full, which cannot be said of the empty.

[30] For the atomists, the void is explicitly equated with non-Being. See Aristotle *Metaphysics* A, 985b4 ff.

[31] While fr. 7 would make best sense if the fullness was the fullness of a body, fr. 9 explicitly denies that the One Being can be understood as a body. Thus I talk about Melissus getting close to a physical interpretation of fullness, but not of him actually giving one.

understood as the internal rareness of something that is not full, as a micro-void, a kind of negative substance.[32] Furthermore, in the Pythagorean tradition the void takes up the role of a separator, between numbers as well as things.[33]

All three functions of the void – as a spatial notion,[34] as a negative substance,[35] and as a separator that guarantees plurality[36] – are attested for the atomists. Understood as a separator, the notion of the void allows the atomists in principle to block one of Zeno's plurality paradoxes, which claims that if there is a plurality, it would have to be both finite (namely the number of things they are) and infinite (for in order to be separate things, there must always be something in between, *ad infinitum*).[37] Aristotle will answer this paradox with his notion of an external limit. But the atomistic void also seems to be a powerful tool to stop this infinite regress, for all we need between two things is a void, which is itself no other thing but yet is able to separate one Being from another.

The introduction of the void on its own breaks much important ground for the assumption of plurality and motion. But given the Eleatic constraints on knowledge, if the atomists want to meet these criteria, they must shown that this notion of a void can indeed be known. Parmenides had excluded non-Being as not thinkable and sayable from philosophical discourse. How can void be thought, if non-Being is indeed not conceivable, as the Eleatics claimed?

4.3.3 Change of Logical Operators[38]

We have seen that the alleged inconceivability of non-Being rests not only on the criteria established for philosophy, but also on the logical operators and on Parmenides' basic notion of Being. Understanding non-Being as void

[32] This is also taken up in the continuation of the fragment.

[33] See DK58 B30. For more details see my manuscript *Conceptions of Space in Ancient Greek Thought*, chapter 3.

[34] Aristotle and Simplicius repeatedly provide testimony for the atoms moving in the void, which thus must be understood as a spatial notion; in his commentary on *De caelo* 294.33 ff., Simplicius even identifies *topos* with the atomistic void.

[35] For Sedley 1982 this is the central idea the atomists want to convey. However, Sedley cannot exclude that Leucippus and Democritus *also* speak of *to kenon* as empty space. He does not consider that understanding to be their core notion, but Aristotle's and Simplicius' testimonies suggest otherwise. Sedley claims that Aristotle's testimony is of little historic value. But the reason he gives for this dismissal – that Aristotle takes void as empty space in some discussion and in others as negative substance – does not take into account that Aristotle's different treatments of *kenon* may simply mirror the different notions connected to this idea in the Presocratics, which Sedley himself grants (p. 179, n. 10). Cf. also Algra 1995, pp. 46–52.

[36] We hear that the atomists regard the void as that which prevents the bodies from being continuous; see Aristotle's *De caelo* 275b 29 ff. and *Physics* 213a31 ff.; Simplicius' commentary ad loc.; Themistius *Commentary on Physics* 123.18–20.

[37] DK29 B3.

[38] For reasons of space, I cannot discuss here whether Melissus prefigured this change.

implicitly changes the interpretation of the logical operators as well as of Being. As void that is indispensable for what is, non-Being is employed as a basic concept *on a par* with true Being – true Being for the atomists is the atoms that require non-Being (i.e., the void) in order to move. The atomists thus understand non-Being as a non-Being that in some sense is:[39]

> διορίζεται μὴ μᾶλλον τὸ δὲν ἢ τὸ μηδὲν εἶναι, δὲν μὲν ὀνομάζων τὸ σῶμα, μηδὲν δὲ τὸ κενόν, ὡς καὶ τούτου φύσιν τινὰ καὶ ὑπόστασιν ἰδίαν ἔχοντος.
>
> [Democritus] determined that thing is no more than nothing, calling 'thing' body and 'nothing' the void, since that too has a nature and reality of its own. (DK68 B156)

"Thing" (δὲν) in the text quoted is a play on words by Democritus, cutting off the negation particle μη from the common word μηδὲν, 'no-thing'. Thus, in this fragment the very construction of the word 'thing'– even if it is introduced as a joke[40] – already shows the centrality which Democritus gives to non-Being, for only somebody who knows the word μηδὲν would understand δὲν. δὲν, thing or Being, is understood as body, while μηδὲν, nothing or non-Being, is understood as void. Most importantly, Democritus explicitly claims that Being is no more than non-Being.[41] He supports this claim in B156 with the idea that both Being and non-Being have their own φύσις and ὑπόστασις, nature and reality. From an Eleatic perspective, only Being can be given a positive explanation, the *sêmata* of what it is, while non-Being would be accounted for, at best, in a privative way. However, from an atomistic perspective, we can give an account of both in a positive way – of Being in terms of body, of non-Being in terms of void. On the linguistic level, Democritus makes Being even derivative of non-Being with his wordplay on μη-δὲν. Understanding non-Being as void is especially suited for the transition to a positive account of non-Being, because it seems immediately understandable that it is not – it is not the bodies – and that yet it somehow, nevertheless, *is*, since there needs to be something in which the bodies are and move.

This understanding of non-Being implicitly requires an understanding of negation that is different from the 'extreme negation' we found in Parmenides that produces the polar opposite of the original input. The central point of the quotation just given is that it is not the case that non-Being is in no way or has no being whatsoever. Rather non-Being, too, "has a nature and reality of its

39 In the quotation from Aristotle above (*De generatione et corruptione* 325a23–9) void is claimed by Leucippus not to be. Here, however, we see that this not-being of the void is, at least with Democritus, not understood as non-existence, as Melissus argued, but rather as a complementing the being of the atoms.

40 And linguistically not fully correct; see Moorhouse 1962, p. 235.

41 In the background of Democritus' claim that thing is no more than nothing may be Gorgias who, in order to show that nothing exists, claims that non-Being is no less than Being (*On Melissus, Xenophanes, and Gorgias* 976a25–7).

own". Thus the μη in μηδὲν indicates some form of difference. The result of the negation is not the absolute opposite of the positive here, a μηδὲν that in no way is.[42] And absolute identity (i.e., identification with no exception) is no longer the only connection operator, or the only way to ascribe something to Being, to what truly is.[43]

We have seen that because of his negation operator, the principle of non-contradiction in Parmenides does not allow for any respects. By contrast, the new understanding of the atomists' negation operator allows for respects, taking into account differences in time or perceiver, as we saw above.

In accordance with these changes by the atomists, Being must be understood in two ways: (1) as the thing (operand) that truly is, the atoms, the full, and (2) as something that can be said of the operand Being as well as of the void and which thus embraces Being and non-Being as an operator applicable to both: an 'is' operator.[44] Similarly, 'non-Being' is (implicitly but systematically) divided into (1) denoting the operand 'the void', the empty, and (2) denoting an operator 'is not'. Interestingly, however, the atomists claim only that *non-Being is*. They do not yet seem to say that *Being is not*, as Plato will in his *Sophist*.[45]

The distinction between the operators and operands named[46] shows that the integration of the void into the basic furniture of the world requires the logical system inherited from Parmenides to be made more complex. But this is not yet enough, it seems, to grant the phenomena some form of being;[47] this step is explicitly taken only in Plato's *Republic*, when discussing the lovers of sight and sound. There the phenomenal realm, what Plato calls the realm of Becoming, which for him is more than just the appearances of what truly is, is said to be

42 This is an insight that we will see explicitly spelled out in Plato's *Sophist*.

43 See, for example, Simplicius' commentary on *De caelo* 294.33–295.36, where we read that atoms "*are* so small as to escape our notice", and "some of them *are* uneven, some hook-shaped, some concave", etc. This is only a testimony, not a fragment, but it seems clear that the atomistic framework requires such a connection operator that is not simply indicating absolute identity – otherwise the atomists could not even conceive of a plurality of interacting atoms that are connected but not identical.

44 This difference seems to anticipate Aristotle's distinction between subjects and what is said of subjects; however, since what will be called 'subjects' is of two kinds for the atomists – atoms (subjects that are entities) and void (a non-entity possessing a different kind or grade of reality than the atoms) – I think it is safer to talk about the difference between operator and operands.

45 It is not clear, however, whether the atomists can avoid assuming that *Being is not* when they say this atom is not that atom.

46 I am not claiming that we have explicit evidence for such an distinction in the fragments handed down to us, but I think this distinction must be assumed as an implicit background to understand the metaphysical changes the atomists make in a charitable way.

47 It is only claimed that phenomena are true in some sense – according to Aristotle *De generatione et corruptione* 315b9–10, the atomists understand them to be τἀληθές, while atoms are, as we saw above, what is in reality (ἐτεῇι).

and not to be. The phenomena are then no longer just the way in which an aggregate of what truly is appears, but entities of their own, which nevertheless derive their being from participation in what truly is.

Let us now investigate how the changes just discussed affect the notion of what truly is.

4.3.4 The Atomistic Account of What Is

As we saw above, the atomists keep the Eleatic criteria for philosophy. In addition, the atomists want what truly is to function as the basis also for the physical and bodily realm. The way they make what truly is fit this task is by understanding it as bodily with physical attributes. They keep almost all of the Parmenidean *sêmata* we saw in Chapter 2, even if now they are features of each individual atom. The atoms are not generated (*agenêton*) and imperishable (*anôlethron*); they are not *diaireton*, and indeed indivisibility is a core feature of the atoms. For Parmenides indivisibility results from being continuous, which includes showing that being is all alike – this also holds of the atoms. The atoms are what we can call 'locally continuous' (i.e., an atom is completely continuous within itself), while there is no global continuity across a phenomenon that is made up of different atoms.[48]

The atoms also seem not to be incomplete in the sense that they are full and do not need anything else for their existence, just as Parmenides' One was full of Being and needed nothing else. And each of the atoms is one. Oneness in a strict sense remains a distinctive feature of what truly is. The atoms are strict unities, whereas the perceptual phenomena possess only a weak unity – a difference we saw already prefigured in the two kinds of oneness in Zeno. Parmenides' *sêma* that "it never was, nor will be, since it is now altogether", is translated by Melissus into everlastingness,[49] which is how it is taken up by the atomists: the atoms are everlasting, rather than outside of time (as we understood this *sêma* in Parmenides in the second chapter).

The only *sêma* not taken up by the atomists is, unsurprisingly, that of being *akinêton*, for motion is the very feature the atomists set out to introduce into the realm of what truly is. Parmenides had dismissed motion, *kinêsis*, as a feature of Being on the grounds that motion would imply generation and corruption: if Being were to undergo *kinêsis*, the quality it possesses (in the case of alteration) or where it is (in the case of locomotion) would necessarily perish and a new quality or place would come into

[48] On the local continuity of the atoms, see Simplicius' commentary on *De caelo* 303a ff., 609.19 ff; for the lack of global continuity, see his commentary on *Physics* 213a22 f., 648.11 ff.

[49] See, for example, fr. B2.

being.[50] Empedocles, Anaxagoras, and the atomists turn this argument on its head. In a first step, the post-Parmenidean thinkers separate *kinêsis* from generation and corruption – they agree with Parmenides that there cannot be generation simpliciter,[51] but do not take this to prove the impossibility of *kinêsis*. And once they allow for *kinêsis*, they can also allow for generation of something on the level of appearances – the atomists do so by understanding such a generation as the new arrangement of the moving atoms, thus reversing the initial relation between locomotion and generation and corruption.

Motion may still seem to involve non-Being, either because we understand motion as something's being here now that was not here before, or because we think we need some non-Being into which Being can move. The first kind of non-Being is accounted for with the new operator is-not, which indicates mere difference, and the second kind of non-Being is understood as void; both kinds of non-Being seem to be conceivable.

As the atoms are situated in a void, they can, in contrast to what we saw Melissus claim, be limited, at least against the void.[52] The infinity of extension (*megethos*) which Melissus introduced for Being qua unlimited is taken up by the atomists for the void.

We see that the atomists integrate all the *sêmata* of Being, apart from being *akinêton*. In addition they also introduce a minimal set of new, physical *sêmata* which answer, at least in part, the requirements introduced by an explanation of the phenomenal realm.

4.3.5 New Physical Features and Their Functions

The atomists seem cautious not to furnish the atoms with too many features, or features that have an opposite and thus could be seen as subject to change.[53]

50 See Aristotle *Physics* 254a11–12; Makin 1993, pp. 38–9; and chapters 1 and 2.

51 While this may have been a view also held by all Presocratic philosophers preceding Parmenides (as the basic idea that things do not simply come spontaneously from nothing), it is only with Parmenides that we get an argument for this idea; and in the succeeding philosophers we find implicit reactions to his argument.

52 The question remains whether the atoms could also be limited against each other or whether any contact between them would in fact mean that they fuse and become a single atom. Makin 1993 and Taylor think that atoms cannot be in contact, since it is the lack of any void that guarantees their indivisibility. However, in fr. DK68 A42 (= T44b) we get a separate argument for the impossibility of atomic fusion, in addition to the argument for the indivisibility of the atoms (cf. also Bodnár 1998).

53 Employing features like dry and wet – characteristics of the traditional elements fire, air, water, and earth – would have raised the problem that in fr. 8 Melissus claims these perceptible qualities to be subject to generation and corruption (what is wet becomes dry, etc.). As the basic constituents of the atomists are thus not differentiated by perceptible

The most basic features named[54] are indivisibility, size, and shape,[55] and these physical features each have a clear explanatory role to play: the different shapes (in their different arrangements and positions) are meant to account for the diversity of phenomena we experience, while the size of the atoms explains their imperceptibility, at least according to one prominent account. In addition, the indivisibility of the atoms is interpreted as a physical feature that explains their unity and thus indivisibility is one guarantor of plurality. However, given that this unity is no longer backed up by monism, the separator between the different unities, the void, has a decisive role to play for the individual unities.

Shape and size must be basic features since the atoms are bodies. If we try to reduce bodies to their basic characteristics, they must have some extension, and hence must have size and shape. They need not have a taste, or a smell, and in a world without light they might not have a colour, but unless they have some extension, they are not bodies.[56] Indivisibility, by contrast, is not a necessary implication of the atoms being bodies, but rather a feature guaranteeing their unity.[57] It also allows for avoiding some of the problems that infinite divisibility seems to create according to Zeno's paradoxes, since this assumption blocks the possibility that divisibility can go on infinitely.[58] But it is not clear how the atomists account for the indivisibility, and what kind of indivisibility the atomists assume.

As for the first question, the indivisibility of the atoms is reported to be due to their continuity, to their small size, or to their inability to be affected, their impenetrability; and the idea of atoms being impenetrable is sometimes connected with assuming the atoms to be solid, hard, or partless. But each of these

qualities, a new kind of differentiation must be found between them to ensure the basic plurality (otherwise it would just be the One continuous Being), the atomistic void.

54 Aristotle claims that the most basic features employed by the atomists are shape, arrangement, and position (*Metaphysics* A, 985b13–19). Some scholars have understood these differences as differences of the atoms (so Taylor in his edition). Others have understood them as differences of the phenomena, i.e., of the compounds of atoms. We can infer that some of them are only features of compounds of atoms from Aristotle's example for arrangement: AN differs from NA in arrangement, so this is a difference that only comes in once we deal with at least two atoms, A and N. And given that the void itself does not seem to have any internal structure, 'position' seems to be the position of the atoms that can be determined only with respect to other atoms. See also Bodnár 1998, p. 37, n. 5.

55 For size, see Simplicius *In de Caelo* 242; for shape, see Aristotle *De generatione et corruptione* I,1.

56 It is not clear whether atoms also possess weight according to the atomists: Aetius claims that they do, but Aristotle and Simplicius do not discuss weight when they talk about the basic properties of atoms (DK67 A6 and A14 and DK68 A37).

57 As the atomists assume a plurality of ultimate entities, they must explain how the atoms can be distinct from each other.

58 Cf. Aristotle *De generatione et corruptione* I, 2.

possibilities displays some problems, and no consensus has been reached on how to account for their indivisibility in the scholarly literature.[59]

As for the second question, modern scholars do not agree on whether these atoms are thought to be indivisible in a physical or in a theoretical sense (and there are different versions of what is understood by being divisible theoretically).[60] Physical indivisibility is often understood as not allowing for the possibility of contiguous parts that are separated by a spatial interval,[61] while theoretical indivisibility prohibits divisibility by the mind.[62] I will give reasons here for understanding the atomists as assuming not solely physical, but also a form of theoretical indivisibility.

Some scholars have argued against even framing the debate in terms of this distinction between physical and theoretical, which has been seen as vague[63] and external to the ancient context.[64] They are right in cautioning that this distinction is not explicitly drawn in the ancient discussions of atomism. But the question remains of how *we* should understand the indivisibility of the atomists.[65]

Many scholars take Leucippus and Democritus to be solely physical atomists.[66] But if we assume them to be simply physical atomists who claim that it is impossible to divide something physical after we have reached a certain point, atomism boils down to a merely empirical question: can sensible things be split without end? This characterisation seems to be at odds with the very starting point of atomistic theory, which assumes atoms

[59] For a discussion see Makin 1993; Bodnár 1998; Taylor; and Gregory 2013.

[60] Furthermore, some scholars distinguish between the theoretical impossibility of dividing an atom physically and the theoretical impossibility of dividing an atom conceptually. See Taylor, p. 165.

[61] Most clearly perhaps in Furley 1967, pp. 4–5.

[62] Makin 1993 thinks that the notion of theoretical divisibility only makes sense if we specify a theory that gives content to "theoretical". Given that "both the impossibility, and what is to count as a division, need to be specified by some theory", Makin thinks that "*all* indivisibility is theoretical in this sense". However, this makes the distinction unnecessarily blurry. While Makin is right that we must define what we understand by a division, with physical divisions there is a fact of the matter – can I practically cut this thing further into parts or not? Makin's example – it is impossible for a runner to run at 100 mph if we have an anatomical theory in mind, but possible if we have a physical theory in the background – overlooks that there is a fact of the matter, a human being cannot run 100 mph, even if this may be a physical or metaphysical impossibility for the biologist, while for the physicist it may be a contingent fact.

[63] So Makin 1993.

[64] So Sedley 2007.

[65] Sedley 2007 himself comes down on the side of understanding the indivisibility in question as a physical indivisibility, since he thinks that theoretical divisibility must hold for anything apart from what is altogether without size.

[66] See also Furley 1967, pp. 86–99. If Aristotle (*De generatione et corruptione* 325a30) is right that Leucippus argued for the indivisibility of atoms due to their smallness, this would indeed suggest that Leucippus assumed physical indivisibility.

to exist despite the fact that we cannot empirically experience them – how then could the basic feature of atoms' indivisibility simply be an empirical observation?

Assuming theoretical indivisibility seems to be a weak position, however, since in principle we could divide anything extended in our mind. For example, the notion of a half, and thus a theoretical division, makes sense for each magnitude, even the smallest and by no means physically divisible body. And many scholars simply presuppose theoretical divisibility owing to the infinity of different atomic shapes. A strong argument in favour of theoretical atomism, however, may be that the idea of unrestricted divisibility seems to lead into problems, as Zeno attempts to show in his dichotomy paradox and in some of his plurality paradoxes.[67] Thus the theoretical divisibility must be restricted in order to avoid certain inconsistencies – like the inconsistency we find ourselves in when trying to conceptualise a continuous body divided up to the last possible part.[68] Thus, there is good reason for stopping the division at a certain point and for thinking that we have thus reached ultimate entities. Furthermore, some of Aristotle's testimonies also support an understanding of the indivisibility of the atoms as more than mere physical indivisibility. For example, *De caelo* 306a26–b2 suggests that Aristotle thought Democritus' account of atoms brought him into conflict with mathematical theorems, which goes against the idea that Democritus assumed unlimited conceptual divisibility.[69]

4.4 Consequences of the Atomistic Changes for Natural Philosophy

4.4.1 Reply to Eleatic Problems

In the second chapter, we saw the devastating consequences of Parmenides' account for natural philosophy: it leads to four logico-metaphysical problems

[67] See Chapter 3.

[68] Pointing out that I can still conceive of half of such a theoretically indivisible atom cannot refute this kind of atomism, since it would be a mere exchange of conceptual problems (the arbitrariness of stopping the division at a certain point although I can conceive of a part of this atom at least in my mind, versus the inconsistency caused by divisibility *ad infinitum* shown by Zeno). Such an objection would have real force only if it were combined with a conception of infinite divisibility that can avoid Zeno's paradoxes – and this is what, in Chapter 7, we will see Aristotle setting out to do in his *Physics*.

[69] See also Furley 1967, p. 86 ff. and 1987, p. 129 ff.; Makin 1993, p. 89, n. 53. Mendell offers a different reading of Aristotle's testimonium in the unpublished manuscript *Democritus on Mathematical and Physical Shapes and the Emergence of Fifth-Century Geometry*. Democritus seems to have been engaged in mathematical questions, however, like conic sections, as DK68 B155 shows.

that prevent the very possibility of natural philosophy. To what extent, then, does the atomistic framework permit a response to these problems?

(1) Because of its restricted logical basis, Parmenides' philosophy does not allow for basic concepts of natural philosophy such as motion, time, and space. For all such concepts require working with differences (the difference between two times, for example), and differences are inconceivable within the Parmenidean philosophy.

What about the logical basis for time, space, and motion with the atomists? We saw above how the atomists implicitly changed the logical basis available: their new understanding of negation and Being allows in principle for a notion of difference, and thus for the introduction of new basic concepts, such as motion. The atomistic notion of the void not only prepares the ground for logical changes, it is also itself productive for developing natural philosophy. Introduced as a physical interpretation of Parmenides' non-Being, it is the basis, together with the atoms, for all the phenomena we experience. The void is employed as separating what is, atoms, so as to guarantee plurality; and as that in which the full atoms can move, it provides a spatial basis enabling motion. Thus, we have at least a basis for space and separation, crucial for any natural philosophy, introduced into a heavily Eleatic-inspired framework.

Time, by contrast, seems hardly to be developed in the atomistic theory. All we learn is that the atoms are everlasting and that time cannot have come into existence.[70]

(2) The restriction of basic concepts within Parmenides' philosophy does not allow for distinguishing between basic entities (like 'Being') and logical operators (like 'is'). By contrast, the atomists prepare the first steps for distinguishing between entities and operators, which they implicitly distinguish in their understanding of Being and non-Being. On the one hand, they employ 'Being' as referring to entities, the atoms; on the other, they use 'is' as an operator when they state that Being is no more than non-Being (μὴ μᾶλλον τὸ δὲν ἢ τὸ μηδὲν εἶναι) – claiming that 'non-Being is' requires a clear difference between operator (is) and operand or entity (Being). Similarly, non-Being is treated as an operand, the void, which itself is not Being, but on a par with Being. And 'is not' can be reconstructed as an operator – 'some entity is not *x* or F' claims that this entity is different from some other entity *x*, or that it does not possess some property F.[71]

(3) We have seen that within the Eleatic philosophy there are less basic concepts in the background that work as implicit premises but are never

[70] Aristotle *Physics* 251b14–17.
[71] 'Is not' may also be understood as an operator in statements like '*x* is not' in order to indicate that *x* does not have full reality in the sense the atoms do.

explicitly clarified, such as the notion of a whole, of unity, homogeneity, and continuity. The atomists clarify the notion of a whole to some degree.

Parmenides argued that Being is *oulon* by arguing for it being not *diareton*, divided or divisible, obviously into parts. This position is justified by claiming that Being is all alike and thus does not allow for any difference (required for division). It seems then to be a whole not of parts, nor against other wholes, since there is only One Being, but in the sense that it is intact, that nothing has been taken away by any division. Melissus seems to take up Parmenides' notion of a whole qua intact in fr. 9 when he argues against parts. However, in fr. 6, Melissus employs a different notion of a whole: by introducing the idea that Being must be without limits, he is proposing a notion of a whole in the sense that there is nothing outside it – a whole qua being all there is. Furthermore, in Zeno's first paradox of motion we saw that two different relations between whole and part are employed simultaneously, thus leading to trouble: a whole as preceding its parts, which are only 'produced' out of the whole, and a whole that is made up of given parts that thus precede the whole.

With the atomists, we see the beginnings of a distinction between different notions of a whole, which are used to explain different features. The atomists give up Melissus' notion of Being as a whole qua having nothing beyond it. Each atom is a whole in the Parmenidean sense that it is indivisible and so intact. But, in addition, it is also a whole in the sense that it is delimited against others – at least against the void, probably also against other atoms. And each atom itself is a whole without any parts.[72] By contrast, all phenomena are wholes that are made up of their parts, the atoms.

In contrast to Zeno's simultaneous employment of two different part-whole relations, the atomists clearly commit themselves to parts that make up a whole and thus precede the whole for the macrocosm: the sensible phenomena are wholes that come into being out of their everlasting parts, the atoms; these wholes are the sum of their parts.[73] The atomistic position provides a first step towards solving the mereological problems of Zeno's paradoxes,[74] by showing that it is in fact not an infinite division that leads us to indivisible minima, but a

[72] There is no scholarly consensus on whether the atoms are indeed partless, but since Epicurus seems to have employed parts of atoms as a reaction to Aristotle's critique of atomism, it seems more likely to me that the early atomists did assume the partlessness of atoms.

[73] On how the atomists could in principle react to Zeno's dichotomy paradox, see Owen 1957–8, pp. 207–8. (Owen talks about the Academic atomists, but the principle is the same for both kinds of atomism.)

[74] And indeed a possible answer to the mereological problem of Zeno's dichotomy argument. Makin 1993, pp. 51–2 argues against assuming the motion problem to be significant for the atomists on the grounds that this would have required an atomic structure for space as well, which we do not seem to get in Democritus. However, the runner in Zeno's dichotomy paradox runs a racecourse, or down a street – things which would have an atomistic structure for the atomists.

finite one, and thus we need not put together our finite whole from infinitely many non-converging minima. Restricting infinite division to division only up to a certain point need not be arbitrary here, but can have a sufficient reason – that what is divided is made up of indivisible parts that can be separated. But the spatio-temporal problems of Zeno's paradoxes are still left open.

As for being *suneches*, the atomists seem to take over from Parmenides the idea that continuity and homogeneity is connected to indivisibility. They restrict the homogeneity to a local one, the homogeneity of each individual atom. There is no global homogeneity; the universe as a whole is not homogenous because of the void. But apart from singling out a certain level where continuity and homogeneity do indeed hold, the atomists do not flesh out this notion any further. By contrast, they develop the conception of unity further in that they employ two fundamentally different notions of a unity: a strong unity for the atoms, which is explained in terms of indivisibility, and a weak unity[75] for the sensible phenomena, which is explained in terms of entanglement. This distinction corresponds to the distinction we saw Zeno drawing in his paradoxes of plurality between something being One in the strict sense (Parmenides' Being, which is continuous and indivisible) and one in a less strict sense (which seemed to be what people in the world of *doxa* would call 'one'). But with the atomists this distinction of different kinds of unity is developed further, as can be seen from the fact that we get some explanation for both kinds of unity: the first kind of unity is based on continuity,[76] while the second kind is secured by the interlocking of atoms.

(4) We have seen that relations cannot be consistently conceived within the Eleatic framework. Since every process or motion involves a relation between different times (for example, its starting time t_1 and its finishing time t_2), as well as between different places or conditions (two places or conditions connected by it), processes and motions are not a possible object of investigation for the Eleatics.

The atomists do not explicitly respond to the basic challenge of establishing a consistent notion of relation, or to all the challenges of Zeno's motion paradoxes. But they attempt to reintroduce what we can call nature as a field worth philosophical attention. Widening the conceptual possibilities of the Eleatics and thus determining their basic concepts further, the atomists introduce the possibility of conceiving of difference and thus also of plurality. With the plurality of atoms, however, relations are in some sense back on the philosophical menu since the relations of atoms – their arrangements – are

[75] They may not have called it a unity, given their claim in DK68 A42 that it is impossible that "out of two a one comes into being or two of one", but it seems that the atomists' theory requires what we can call a weak form of unity in order to distinguish between one phenomenon and another.

[76] Or impenetrability, etc.; see above

what constitute the phenomena we perceive. And the new conceptual possibilities of the atomists also prepare the ground for the inclusion of time, space, and motion in principle. We saw them preparing the ground for including space above; let us now finish by looking at the kind of motion the atomists employ.

4.4.2 *Motion and Changes in the Atomistic Framework*

What motions and changes are the atoms and their arrangements subject to and which of the motions and changes that natural philosophy normally deals with can be explained in the atomistic framework? To answer these two questions, we shall look first at the atomic level, and then at the level of the arrangements of atoms.

(a) The Atomic Level
The atoms are not subject to any change, generation, or corruption. Atoms only undergo locomotion. Their locomotion is a constant motion,[77] and since it takes place in the void, which does not seem to involve any friction, the atomists need not account for the continuation of motion, just as we would think that there is no need for an explanation of the continuation of motion in a frictionless Newtonian universe. The atomists need only explain changes in the direction of locomotion. Such changes are accounted for by the basic set-up of the atoms and their motions in the void: the atoms move with extreme rapidity, as Pseudo-Plutarch tells us in T79a, and bump into each other; these collisions then lead to new paths for the atoms involved. Changes of direction are thus the product of interactions with other atoms. Collisions can lead to the atoms being "shaken away in any chance direction" (DK67 A14). But the atoms can also stay together after a collision, either because they are the same shape (or at least similar) or because they are shaped in such a way that they interlock.[78]

This purely mechanistic account of motion is based on the assumption of an infinite void in which the atoms can move without any restriction. The motions have no aim or *telos*, rather the path of each motion is determined by the last collision, and since the atoms and their motions are everlasting, there is always a prior collision that caused the specific direction of the current motion.

Whether the atomists thus give a sufficient account of the direction of the atomic motions is debatable.[79] Clearly they do not explain what causes the

[77] Hippolytus, testimonium T78 (= DK67 A10 and DK68 A40); Ps-Plutarch, testimonium T79a.

[78] For the second alternative, see DK68 A37; for the first, DK68 B164 and DK67 A14. Cf. also Berryman 2002, pp. 186–7.

[79] In line with Gregory 2013, we may say no, since we can always ask for a sufficient reason for the previous motion, without ever getting to some point where the question comes to

motion of the atoms in the first place, which accounts for Aristotle's complaint in *Metaphysics* A, 985b that "the question of the origin and nature of motion in things they [the atomists] too ignored, just as blithely as the others". For the atomists, locomotion seems to have been just a given, that was always there, like the full and the void.

(b) Motions of the Aggregates of Atoms

The collision of the atoms also leads to the separation of aggregates of atoms or the formation of new aggregates. The former leads to the destruction of sensible phenomena; the latter brings new sensible phenomena into being. Accordingly, the phenomena of the sensible world are subject to coming into being and passing away.[80] These aggregates can change, when parts are rearranged or split off, or when new parts join. And these aggregates can also move. So on the level of the phenomena and sense experience, the atomists assume that there is locomotion, change, as well as generation and corruption.[81] On the basic ontological level of atoms, however, there is solely locomotion of the ungenerated and unchanging atoms.[82]

The locomotion of the atoms is meant to explain all the motion, change, and generation of our phenomenal world. As the atoms move continuously, the sensible world is in constant change. What truly is, the atoms, remains unchanged, however, and in this respect satisfies the Eleatic requirement. But in order to preserve the unchangingness and unity of the atoms, what seems to our senses to constitute *one* phenomenal thing is understood as a combination of many unities, many atoms. According to atomist thinking, such phenomena are then not really one, as they can be divided – there is void in between the atoms – and thus they will one day also pass away. The atoms constituting a phenomenon are just entangled or aggregated so as to form a temporary and weak unity, which can be destroyed in one of two ways: either the entanglement becomes less and less strong (which seems to be the idea behind the notion of worlds in decay),[83] or there is a collision with some other aggregate of atoms (T78).

an end. Alternatively, we may think, with Berryman 2002, that the void can actually account for the occasion, extent, and direction of motion.

80 See *De generatione et corruptione* 325a30ff. In 315a34, Aristotle claims that the atomists are the only ones who gave a detailed account of coming into being and passing away, in terms of the shapes of atoms being joined and separated.

81 See also *De generatione et corruptione* I, 2.

82 See also Curd 1998, p. 181; Furley 1967, p. 5. For Melissus, fr. 7, however, such a rearrangement would be a form of generation and corruption – a corruption of the old arrangement and a generation of the new one, so that locomotion must be excluded. See also Barnes 1982, p. 433.

83 According to the atomists, there is a permanent generation and passing away of worlds; see T80a and b (= DK12 A17).

In this way, different forms of unity are connected to different kinds of processes – the weak unity of the sensible phenomena to changes and generation and corruption, and the strong unity of atoms to locomotion. An Eleatic philosopher may thus claim that the price one pays for allowing for change and generation is that what undergoes change or generation is not truly one – just as Zeno made it clear in his paradoxes of plurality that the individual ones of a plurality are not truly one.

4.4.3 Problems that Remain

We have seen how the atomists prepare the metaphysical and logical framework to some extent for a philosophy of motion, change, and nature. Their physical interpretation of non-Being allows for understanding it as a basic concept *on a par* with Being and thus leads to a crucial expansion of the logical possibilities. This physical interpretation also allows them to set up the basis for a conception of space. However, the atomists can react only to some aspects of Zeno's paradoxes. One reason for this limited response is the part-whole relation that the atomists employ for the phenomenal level. Unlike Zeno, the atomists no longer merge different part-whole relations when dealing with motion; they work rather with a single conception of a whole as the sum of its parts. But this part-whole relation turns out not to be sufficient to avoid Zeno's paradoxes of motion – as we will see in Chapter 8, it cannot give a full account of speed.[84]

This brings us finally to two problems concerning motion and change that the atomists did not tackle. While we find suggestions for what accounts for the unity of the entities on the different ontological levels, there does not seem to be any discussion about what makes for the unity of a motion – for example, what makes the motion of me walking through the room a single motion and when should we rather talk of several motions? The only unity a motion seems to be given in an atomistic framework, at least on the phenomenal level, is the unity of the motion that is tied to the moving entity – this is the motion of a human being, that is the motion of a river. But whether a human being performs one or more motions on a given occasion does not seem to be debated. The unity of a motion or a change seems to be discussed for the first time with the introduction of teleological accounts, and it seems to be first discussed in a systematic way in Aristotle.

[84] For example, as Aristotle points out, atomistic accounts do not allow for unrestricted comparability of different speeds, since from a certain point onwards time and space cannot be divided any further and thus things moving at different speed will cover the same spatial atom in the same temporal atom. While the void may be divisible *ad libitum*, phenomenal motion occurs by covering a certain path that is restricted in its divisibility by its atomic constituents.

Furthermore, motions and changes are not shown by the atomists to be intelligible in themselves, a project first undertaken in Plato's *Timaeus*. Time – a notion on which the atomists were so notably silent – will play the crucial role in this task by allowing us to understand motions as regular and rationally designed. So let us move on to Plato in the next chapter

5

The Possibility of Natural Philosophy According to Plato I: The Logical Basis

5.1 Introduction: The Investigation of the Natural World as an *Eikôs Mythos*

Plato's Timaeus famously begins his speech by pointing out the limitations of any investigation of the natural world:[1]

> ἐὰν οὖν, ὦ Σώκρατες, πολλὰ πολλῶν πέρι, θεῶν καὶ τῆς τοῦ παντὸς γενέσεως, μὴ δυνατοὶ γιγνώμεθα πάντῃ πάντως αὐτοὺς ἑαυτοῖς ὁμολογουμένους λόγους καὶ ἀπηκριβωμένους ἀποδοῦναι, μὴ θαυμάσῃς· ἀλλ᾽ ἐὰν ἄρα μηδενὸς ἧττον παρεχώμεθα εἰκότας, ἀγαπᾶν χρή, μεμνημένους ὡς ὁ λέγων ἐγὼ ὑμεῖς τε οἱ κριταὶ φύσιν ἀνθρωπίνην ἔχομεν, ὥστε περὶ τούτων τὸν εἰκότα μῦθον ἀποδεχομένους πρέπει τούτου μηδὲν ἔτι πέρα ζητεῖν.
>
> But if, Socrates, it is not possible to give accounts that are in all respects and all ways consistent and accurate in themselves, even though many have already said many things about the gods and the generation of the universe, be not surprised. Rather, we must be content if we can offer accounts that are no less likely than those of others, remembering that I who speak and you who are the judges are only human, so that it is fitting to accept an *eikôs mythos* on these matters and not strive for anything beyond this. (29c4–d3)

Of the sensible world we can only give an *eikôs mythos*, which has often been translated as a "likely account". It might seem that we are thus back to Parmenides' assessment of cosmology in his poem: there the Parmenidean goddess claims, after she has finished her trustworthy account and thought about the truth, that what she is now going to tell us is an fully fitting or likely world order (*diakosmon eoikota panta*).[2] The reason this account of the

[1] The investigation of the *Timaeus* is a cosmology, but it also contains parts which we can classify as optics, physiology, 'chemistry', and so forth. It is also the work which tells us most about Plato's general account of motion and change. See Sattler 2016.

[2] Fr. 8, line 60. The phrase *diakosmon eoikota panta* raises at least three problems for any translation and interpretation: first, *diakosmos* can refer to the ordering of the account or to the ordering of the subject matter, the world order. Secondly, it is not clear whether *panta* qualifies *diakosmon* or *eoikota* or both (Tarán goes with the first possibility, Fränkel

sensible world is given in the Parmenidean poem is so that "no judgement of mortals shall ever outstrip" the listener.[3]

As we saw in Chapter 2, Parmenides' goddess makes it clear that her account of the natural world is something that is fitting to the thinking of mortals, but ultimately deceptive;[4] in this sense her account is only likely or probable (*eoikôs*).[5] Similarly, Plato seems to restrict his account to mere likelihood (to an *eikôs* account) when it comes to speaking about the natural world. Like the Parmenidean goddess, Timaeus compares his account of the natural world with others out there, and claims that his is (at least) as likely or fitting (*eikotas*)[6] as any other to be found. And, like Parmenides' cosmology, the Platonic one is an investigation of what belongs to the realm of *doxa*.[7] So have we come full circle back to Parmenides? Is Plato's *Timaeus*, perhaps more sophisticated and anyway better preserved, a rerun of the *doxa* part of Parmenides' poem?

We are back to Parmenides in the sense that we are back to a fundamental discussion about the possibility of doing *natural* philosophy. And we are back in the sense that consistency, rational admissibility, and a principle of sufficient reason are still basic criteria for what counts as philosophy. But not only have these criteria been further developed (we will see this with consistency in Plato's *Sophist* in the next section and with rational admissibility and the principle of sufficient reason in the section after), the assessment of the realm of *doxa* has also changed: we saw that Parmenides himself does not think that his criteria for philosophy are met by the realm of *doxa*. Accordingly, he relaxes his stricture on the object of investigation in the *doxa* part, but he also makes it clear that dealing with the realm of *doxa* is not a strictly philosophical or rational investigation. We may think that in principle Plato could take this as a model for a relaxed account of an investigation when it comes to the sensible world studied in the *Timaeus*. However, Plato does not simply take over Parmenides' assessment of the sensible world in the *Timaeus*. Rather, he wants to show that to a certain degree Parmenides' criteria for philosophy are also applicable to investigations of the natural realm

1955 with the second). Finally, *eoikota* can be translated in significantly different ways. For a discussion of the main translations as similar, fitting, specious, and plausible/likely/probable, see Bryan 2011, p. 66 ff.; for Parmenides' assessment of cosmology, see the last section of Chapter 2.

3 Fr. 8, lines 60–1.

4 Fr. 8, lines 51–2.

5 Cf. Mourelatos 2008, p. 231 and Bryan 2011, p. 110.

6 What I say about the understanding of *eikôs mythos* below obviously also has repercussions on how we understand the occurrence of *eikotas* here.

7 Plato does not talk about a realm of *doxa*, but in 28a he clearly counts what is sensible as what can only be an object of *doxa*, and in 51d it is the distinction between *nous* and *doxa* which guarantees that there is a distinction between Forms and sensible things.

and that we can indeed investigate the sensible realm in a philosophical way (even if there are some restrictions in place).[8]

The fact that Parmenides gives us an advanced cosmology in his poem may be seen prima facie as a suggestion that natural philosophy should in principle be subject to the same criteria as metaphysics. But in Parmenides' eyes, natural philosophy is clearly unable to meet them.[9] By contrast, in the *Timaeus* Plato demonstrates how to conceive of the sensible world in a way that is, for the most part, in line with these criteria. In addition, Plato claims the cosmos, as the entirety of the sensible world, to be as beautiful and excellent as anything generated can be (29a) – a strong positive value judgement about the natural world of which we find no hint in Parmenides.[10] A reason for this positive understanding of the sensible cosmos seems to be the fact that in the *Timaeus* the good enters as a principle of explanation of nature,[11] while it seems to be absent from the explanation of nature in Parmenides. Accordingly, the status of investigating the natural realm has changed substantially *vis-à-vis* Parmenides' assessment.

A change, though of a different kind, is also indicated by the fact that while Timaeus claims his account to be at least as likely as any other out there, he does not use this as a way to guarantee that his readers will not be 'overtaken' by other accounts, as Parmenides' goddess tried. On the contrary, Timaeus frequently tells us that anybody who can come up with a better explanation of a particular point investigated will be seen as a friend, not as an enemy.[12] Timaeus presents his account of the natural world as one that is indeed improvable and, it is suggested, to improve it would be a worthy use of our philosophical time.

Most importantly, the *eikôs mythos*[13] we give of the sensible world is, as scholars have recently argued, not only a negative limitation, but also a positive

[8] Cf. Burnyeat 2005, pp. 153–4, who claims that Plato is both echoing and subverting Parmenides: echoing in the strict distinction between the rigorous reasoning of the way of truth and a *diakosmon eikota* (which he understands as an "appropriately ordered account of the sensible world"), subverting by aiming at an appropriately argued, not just appropriately ordered, account of the sensible world.

[9] See Chapter 2.

[10] The difference is not in their assessment of their own account – both Plato and Parmenides think they give the best possible (or available) account – but rather in their assessment of the natural world itself. For Plato this value judgement presumably has what we would call ethical as well as aesthetical connotations.

[11] Cf. Section 5.3.1 below.

[12] See 54a–b, and Broadie 2012, who understands these encouragements as Plato's way of launching a research programme on natural philosophy.

[13] Or *eikôs logos*. Plato seems to use *eikôs mythos* and *eikôs logos* more or less interchangeably in many places in the *Timaeus*: see 29d, 59c, 68d for *eikôs mythos* and 30b, 48d, 53d for *eikôs logos*. Cf. also Burnyeat 2005 and Aristotle's usage of *eikôs* in *Prior Analytics* 70a2–4 in the sense of a generally approved proposition. Betegh 2010, p. 218 thinks that the difference between an *eikôs mythos* and an *eikôs logos* is that with the former we also

description of the criteria that an account of the world of *genêsis*, change, and motion should meet.[14] We are told that an account is *syngenês* (of the same kind) to that of which it is an account: so an account of what is stable will itself be stable, and an account of an *eikôn* (an image or copy) is itself an *eikôs mythos*.[15]

For a long time an *eikôs mythos* was understood to mean that we can only give a likely account of the sensible world – either in the sense of giving a hypothesis and approximation to the truth in the way modern sciences do,[16] or in the sense that for Plato there can be no consistent account of the sensible world, since it is always changing.[17] In any case, it was seen as a mere restriction of the kind of explanation we can give of the sensible world[18] – until it was argued convincingly in recent debates that we should hear the positive ring in the term *eikôs mythos*, namely that there is a certain standard.[19] And it has been stated that Plato understands it as a standard which situates cosmology between irrefutable discourses about what is unchanging (which we deal with in metaphysics and mathematics) and discourses without any standard (which

"get a narrative in which we hear about the maker and the maker's reasons for creating the thing in question". For possible differences, see also Johansen 2004, pp. 62–4 and Bryan 2011, pp. 179–80.

14 Burnyeat 2005, p. 154 wants "to celebrate the Platonic insight that reasoning which lacks the rigour of mathematical proofs or Parmenidean logic might nonetheless have standards of its own by which it can be judged to succeed or fail". Osborne 1996 already gave it a positive sense, but did not discuss the question of how to understand the notion of an *eikôs mythos* as explicitly as Burnyeat, whose paper thus became the starting point for this debate. Other important contributors are Betegh 2010, Mourelatos 2010, Bryan 2011, and Broadie 2012.

15 Mourelatos 2010 thinks that making the account of an *eikôn* itself an *eikôs mythos* already shows that the account is *syngenês* to what is accounted for. The idea that an account of an *eikôn* is itself an *eikôs mythos* may be seen to be prefigured in Parmenides: in fr. 8, 52 the goddess talks about the deceptive *kosmos* of her words, which, as we learn in line 60, are about the *dia-kosmos*, the world mortals assume.

16 So Taylor's commentary, pp. 59–60.

17 So Cornford's commentary, pp. 28–32.

18 In fairness to these scholars, I should mention that Plato also uses *eikôs* in a different, and clearly pejorative, sense in his criticism of the rhetoricians in other works, most clearly perhaps in the *Phaedrus*, where Teisias and Gorgias are criticised for putting *ta eikota* before the truths (267a6–b2), and *to eikos* is characterised as nothing other than what seems true to the masses (273b2). In the rhetorical tradition, *eikôs* arguments seem to be predominantly those required in cases lacking eyewitnesses or other evidence – a reconstruction of how Plato reinterprets this tradition with his understanding of the *eikôs mythos* would require its own study.

19 See especially Burnyeat 2005. This was balanced by Betegh 2010, p. 214, who pointed out that it is indeed both, a restriction as well as a standard, and by Mourelatos 2010, who showed that the traditional reading of a restriction as well as the reading in the sense of a scientific hypothesis are in fact both compatible with Burnyeat's account.

would be the case if the world, counterfactually, were made according to a paradigm that has itself come into being).[20]

Such an understanding of the status of our study of the natural world can be seen as a forerunner to Aristotle's famous distinction of the degree of rigour that should reasonably be expected from different disciplines.[21] But what exactly does Plato's norm for the study of the natural world consist in? The negative side is clear: an account of the natural world will not be irrefutable, and it will not be absolutely consistent. That it is understood as not irrefutable can be seen from the fact that Timaeus continuously encourages his audience to improve his account.[22] And, as we saw in the passage quoted above, Timaeus explicitly tells us not to expect consistency and accuracy of every part of his account of the universe: we should not be surprised if it is "not possible to give accounts that are in all respects and in all ways consistent[23] and accurate in themselves" (πάντῃ πάντως αὐτοὺς ἑαυτοῖς ὁμολογουμένους λόγους καὶ ἀπηκριβωμένους, 29c). We will also see some restriction of rational admissibility.

That Timaeus' account is not fully consistent can be seen, for instance, in his account of female human beings and of mortal animals. On the one hand, we are told that all the mortal animals on earth, in water, and in the air are needed in order for all the living beings in the intelligible model to be mirrored in the created cosmos. On the other hand, we learn that all the animals are souls that were originally human beings, but, because of bad behaviour, were reborn as animals (exemplifying a particular aspect of their badness) – so we are given two different and, it seems, competing explanations of the creation of mortal animals that are not related in any way. Similarly, while female human beings are on the one hand seen as the first degradation that a badly behaved soul must take as punishment in its next life (90e), we are also told that human beings were originally created male and female (42a);[24] and, indeed, in order to get human

[20] So Broadie 2012, ch. 2.

[21] For example, as is made clear by the opening sections of Aristotle's *De anima* and *Nicomachean Ethics*, we should expect the greatest rigour in the study of the soul, but less precision in an examination dealing with human actions. Cf. also Plato's *Philebus* 55c–9d.

[22] His account is refutable in the sense that, as Broadie 2012, pp. 53–4 expresses it, "a better account could, in principle, be generated within the same basic framework". Broadie ibid., p. 54 distinguishes what is irrefutable from what is non-refutable: irrefutable are *logoi* that testing (*elenchos*) has shown to be immune to refutation – as in mathematics, where it is undisputed that we can attain this standard. By contrast, what is non-refutable is never a candidate for possible refutation, as are the starting points of a science that are not negotiable within that science.

[23] Literally "that are in every way in agreement with each other", which I understand as Plato's way of expressing consistency.

[24] This inconsistency in the creation of women would still make the *Timaeus* as likely as the account we find in Genesis, which has a similar twofold story, if indeed it had been available for Plato. Cornford, p. 145 assumes that males and females must always have existed, since he takes the creation only metaphorically. He attempts to explain the

reproduction going, there had to have been female human beings right from the start.[25] In the next chapter, we will have to investigate whether Plato's accounts of time and space, which to us seem to be of surprisingly different types, are in fact also inconsistent with each other or can be shown to be compatible.

The reason why an account of the cosmos does not allow for *full* consistency,[26] accuracy, and rational admissibility seems to be fourfold: the world is a product of divine intervention, but (a) the demiurge must take up what is given to him, which is not fully intelligible; (b) the product cannot be understood on its own but only with respect to the model according to which it was formed; (c) the world is a sensible thing and as such has come into being and always keeps changing; and, finally, (d) we as human beings are restricted in what we can understand. While (a), (b), and (c) show that cosmology as a discourse about the sensible world will not be fully consistent, accurate, and rationally admissible, because the world is not fully intelligible in itself, (d) shows problems for this discourse that come from our human side, from us who participate in this discourse:[27]

(a) The world was not created solely by reason, but also by the demiurge using what was already there to work with. What the demiurge worked with as the starting point for his activity – the receptacle and the traces of the receptacle in it – was not fully intelligible. In fact, the receptacle seems to be at the far end of comprehensibility. We are explicitly told that the receptacle can only be accessed by a "bastard reasoning" (52b) – that is, the receptacle is only to some degree rationally admissible. And it seems hard to give a rational account of such a thing.

(b) The object of investigation has not only come into being but is also a likeness of some original existence, the model.[28] Accordingly, it has only restricted being in the sense that its being depends on the being of the model

seeming inconsistency by understanding the claim that females are fallen males as: "all that is meant is that every soul that is at any time incarnated for the first time, is incarnated in male form". This would, however, require some assumption about certain souls being incarnated later than others, while 41e explicitly claims that the first incarnation was the same for all souls so that none would be disadvantaged, which would be hard to square with different souls being first incarnated at different times.

[25] Otherwise the gods must bring in the next generation, as is the case with Hesiod's first race in his *Works and Days*.

[26] For the lack of full consistency, cf. also Sedley 2007, pp. 104–5.

[27] The earlier secondary literature in the twentieth century derived the *eikôs* status of cosmology from the changeability of the subject matter (for example, Taylor and Cornford), while the recent secondary literature has focused on the *eikôs* character of the physical world (for example, Bryan 2011) or on our human restrictions (Broadie 2012). It seems to me that all the points mentioned above, (a) to (d), must be taken into account.

[28] Of course, within Plato's metaphysics everything that has come into being is in some sense a likeness of some Being (even if in earlier works Plato uses somewhat different language: for example, in *Phaedo* 74d ff. he talks about sensible things "attempting" to be like the Form in which they participate). So with (b) and (c) I am analysing two different aspects that Plato brings together in his account of the sensible world – (b) focuses on the

and will never outgrow this dependence. And it cannot be understood on its own.[29] Any account of it will always have to refer (also) to heavy metaphysics, to the model. Without this, natural science will not meet the requirements of rational admissibility and consistency, and it will be incomplete, falling short in accuracy – we cannot see natural science meeting the standards for scientific/philosophic investigation when looking only within natural science. While the model is an unchanging entity, the world as its image keeps changing – this brings us to the next point.

(c) An account of the sensible world is an account of the things that have come into being and thus of objects that, according to the Platonic Timaeus, cannot be completely understood.[30] For they continuously change – "they always become and never are" (28a). Thus what we can truly say about them changes as well, and if we think of knowledge in the strictest sense as a stable account of things, then we see that our account of something continuously changing cannot stay completely stable.[31]

The same relation we find between Being and Becoming we also find, according to the *Timaeus*, between reason (the way we can relate to Being) and belief (the way we can relate to Becoming), and between a (successful) account of Being and a (successful) account of Becoming.[32] Accuracy and rational admissibility in its full sense can only be expected for an account of Being, for what we would call metaphysics and for mathematics.[33]

(d) Because Timaeus, his audience in the dialogue, and we – the producer and the listeners of this account of the cosmos – are human beings,[34] we must

fact that they are likenesses and thus dependent beings, while (c) focuses on the fact that they have come into being and are always changing.

29 This allows for understanding the cosmos positively as a representation of the model, which Bryan 2011, pp. 144–7 points out. But it means that we cannot aim solely for a cosmology without any reference to metaphysics.

30 See also Zeyl 2014: "To the extent that the subject of the account is a thing that becomes rather than a thing that is, as well as a thing that is perceptible rather than a thing that is intelligible, the account will be no more than likely."

31 You may object that the natural sciences deal with generalisations whose truth value is stable, since change is only on the level of particulars but not of types. Similarly, it seems to be the case that the main features of the universe in the *Timaeus* do not change. However, this seems to me to be an understanding of the stability of an account, for which we only see the first steps in Plato. Cf. Sattler 2010.

32 Burnyeat 2005 rightly points out that the comparison here is between successful accounts, since unsuccessful accounts of Being need not stand in the same relationship to a successful account of Becoming. Cf. also Mourelatos's 2010 detailed account of this ratio.

33 Burnyeat 2005 claims that the ordering of a world is a case of practical reasoning which by its very nature cannot be as rigorous as theoretical reasoning, and so an account of this order will not satisfy the strict standards of theoretical reasoning.

34 Timaeus is of course a fictional human being, but he stands in for every human being giving an account of the cosmos (as the other figures in the dialogue stand in for any human listeners).

be satisfied with an *eikôs mythos*, as Timaeus explains in 29c–d.[35] It is clear that this epistemic fragility or restriction is not the same as the one we are used to from Plato's middle period, where it derives from the ontological status of the perceptible thing.[36] Here we are dealing with a limitation of human cognition of nature that was not relevant in the Divided Line example of the *Republic*.[37]

So much for the restrictions on an account of the sensible world and the reasons for them. What about the positive standard now? The secondary literature has been much less explicit on what exactly the positive standard of the *eikôs mythos* is and all we find explicitly in Plato's text is that in the same way that the cosmos has a ratio to the model of which it is a likeness, the *eikôs mythos* about the cosmos has a ratio (29c2) to the irrefutable *logoi* about the original – that is, the positive standard of the *eikôs mythos* must be such that it ensures there is such a ratio. Burnyeat understands an *eikôs mythos* as one "appropriately argued", and one that is probable relative to the starting point that the world has a supremely good designer, without, however, spelling out any criteria for appropriateness and probability with respect to the starting point.[38] Of more immediate help as a positive criterion are Burnyeat's and Broadie's claims that an *eikôs mythos* should be consistent within each block of investigation, though we will see that this understanding needs some modification.[39] Betegh presents perhaps the fullest positive account by claiming that a "successful discourse about an *eikôn* will be *eikôs* precisely in so far as it presents its subject as an *eikôn*, explaining what its model is, why it was created after that particular model, by whom, etc.".[40] While Betegh's account captures a lot of what is specific to an *eikôs mythos*, I want to show that there are also more general features required: the positive side of the *eikôs mythos* standard means that the criteria for philosophy which we have seen evolving so far – consistency, rational admissibility, and a principle of sufficient reason – can still *to some degree* be understood as applicable to the investigation of the natural realm. The reduced degree of the applicability of these criteria also explains what kind of ratio links the *eikôs mythos* to the irrefutable *logoi*.

The remainder of this chapter will look in some detail at how Plato develops the criteria for rational investigation and the logical operators in a way that is

[35] Broadie 2012 understands this claim as indicating that our reason is not the very same as (though it is similar to) the reason of the demiurge, so we cannot be sure about the intention of the maker, and thus about the ultimate structure that he gave the world.

[36] See Broadie 2012, p. 38. It seems, however, that we have some access to the maker's reasoning, since our reason works to some degree like his (very much as we find divine reasoning and human reasoning being akin to some extent in Parmenides' poem), as proven by the fact that Timaeus can in fact tell us a certain amount about the motivation of the maker, namely to make things as good as possible (30a).

[37] See also Betegh 2010, p. 220.

[38] Burnyeat 2005, pp. 153, 162.

[39] Burnyeat 2005 p. 155, Broadie 2012, p. 39, n. 32.

[40] Betegh 2010, p. 218.

also necessary for natural philosophy. The main part of it will focus on the *Sophist*, since this is the place where Plato deals with the logical problems raised by Eleatic philosophy. We will start by looking at the changes Plato introduces to the understanding of the separation and connection operators and the principle of non-contradiction. While in the context of the *Sophist* these changes are explicitly developed with respect to the Forms as the most fundamental realm where Plato's revised logic must hold, they will turn out to be changes that also prepare the logical ground for understanding motion and change in the sensible world and thus for setting up a natural philosophy.[41] The demanding nature of the *eikôs mythos* criterion makes it clear that Plato attempts to hold the realm of *doxa* to a higher standard than Parmenides assumed we could. This requires a more advanced logic, which Plato can transfer from the *Sophist* to his cosmology (at least to a certain degree).

At the end of the chapter, we will also look at the *Timaeus* in order to show how the principle of sufficient reason and rational admissibility are further developed. Chapter 6 will then try to show why changing the logical basis and criteria is a necessary but not a sufficient condition for establishing the natural world as a proper object of scientific inquiry and how Plato attempts to meet the sufficient condition by introducing mathematical notions into natural philosophy.

5.2 The *Sophist*

In the *Timaeus* Plato can be shown to use the standards that Parmenides developed for metaphysics for natural philosophy, albeit in a restricted form. But it is the *Sophist* where he explicitly develops further the logical basis for these standards, in a way that makes them suitable also for natural philosophy.[42] The atomists had, in a certain sense, already implicitly developed the logical basis. But with Plato we get an explicit reflection of how this can be done. We will see that this leads to a major revision of the logical operators and the criteria for philosophy we have looked at so far: it includes a new account of separation and connection, the former requiring a new understanding of negation which is prominently investigated in the *Sophist*. And the principle of non-contradiction will be captured as including respects explicitly, which the atomists had already assumed implicitly.

Let us start with Plato's new account of the basic operators.

[41] As Mourelatos 2010 has pointed out, in the *Timaeus* Plato cares about getting the cosmology right, not just about outperforming other cosmologies.

[42] I do not seek to make a claim about the chronological order between the *Timaeus* and the *Sophist*. I follow the general consensus that both are late works. Thus I assume that important thoughts from the *Sophist* are already in the background of the *Timaeus*, whether or not Plato had already written the *Sophist*. Some points of the second part of Plato's *Parmenides* would also be relevant to the discussion here, but cannot be discussed for reasons of space.

5.2.1 The Reinterpretation of Negation and the Connection Operator

5.2.1.1 Negation Operator

What is often seen as the inner part or core of the *Sophist* starts with the idea that phenomena like false beliefs or images – both brought up in order to give an account of the sophist – require Being and non-Being to be brought together (240b11): for example, an image of a tortoise *is not* what it seems to be – a tortoise – but it *is* nevertheless, namely, it is an image of a tortoise. And if we have a false belief, we believe what-is-not to be, or what-is not to be. For example, if we falsely believe that Theaetetus flies, we believe what is not – Theaetetus' flight – to be; or, if we falsely believe Theaetetus not to speak with the Eleatic Visitor, we believe what is – Theaetetus' conversation with the Eleatic Visitor – not to be. But, as the Eleatic Visitor of the *Sophist* shows, bringing together Being and non-Being is impossible if we follow Parmenides. For then we should understand non-Being as the absolute opposite of Being, something that cannot be applied to Being in any way (237c) and to which Being cannot be applied in any way (238a). In order to understand non-Being in the way discussed by the Eleatic Visitor here, we must understand negation in exactly the way we showed Parmenides to understand it in the second chapter, namely as indicating the extreme opposite of the positive following the negation sign.

This understanding of negation and subsequently of non-Being implies that non-Being cannot even be talked and thought about, as the Eleatic Visitor and Theaetetus point out in 236–9.[43] For not only do we always talk and think about something and so about some Being, but also if we wanted to speak about non-Being, we would have to talk about either a non-Being (in the singular) or non-Beings (in the plural) and thus ascribe some being, either oneness or plurality, to it (238b–239a).[44] Hence we undermine our own talking about non-Being: on the one hand, we claim that no being whatsoever can be ascribed to it but, on the other, we cannot even make this claim without ascribing some being to it. So not only is making any claim about non-Being self-defeating, since it requires us to improperly attach being to it, but also if we attempt to point out this problem with non-Being, if we want to claim that non-Being is "unutterable, unsayable, and inexpressible", we get entangled in inconsistencies.[45]

[43] Referring explicitly also to Parmenides fr. 7, lines 1–2.

[44] A further possibility in the Greek language is the dual, which ascribes twoness or pair-ness to something; accordingly, the Eleatic Visitor and Theaetetus also take the dual into account in this passage. They claim that number is thus ascribed to non-Being, because oneness, twoness, as well as plurality, are based on the possibility of counting something (even if we have not yet calculated the exact number of our plurality). Cf. Gill 2016 for a list of the further puzzles involved in attempting to talk about non-Being.

[45] 238c ff. See also Lewis, 1977, p. 90, who claims that by these same principles the very statement that non-Being is inexpressible is improper – thus all talk about non-Being is reduced to incoherence, Parmenides' included. Cf. Owen 1970 for an account of the different steps in this passage. There seem to be three possibilities for understanding "speaking of non-Being": to

We see that Plato's analysis of non-Being here supports our interpretation of Parmenides so far: non-Being as Parmenides understands it according to Chapter 2 cannot be talked about because any attempt to do so leads into inconsistencies.[46] Furthermore, Plato thought that the problems in which non-Being entangles us relate to the understanding of the negation operator in play,[47] which he thus explicitly discusses in the *Sophist*. The way Parmenides understands non-Being, also according to Plato's *Sophist*, is based on understanding negation as an extreme negation, a negation that leads to the complete opposite of what is negated: 'not-*x*' means 'not being *x* in any possible respect'. In the *Sophist*, Plato points out that this understanding is not the only one possible. Rather, we often use negation in a way that indicates a mere difference from what is negated (257b–c).

To be more precise, Plato 'divides' negation into an operator expressing difference (for example, between Motion and the Same) and one expressing mutual exclusion (for example, between Motion and Rest). In the *megista genê* doctrine – Plato's discussion of which kinds or concepts can be applied to which and to what extent with the help of five greatest kinds – he shows how difference and mutual exclusion differ from each other (255e–256d):[48]

speak of non-Being is (1) not to speak of anything; (2) to speak of a non-existent thing (for example, unicorns); or (3) to speak of what is not anything (has no properties, etc.).

46 I tried to show in the second chapter that speaking about non-Being for Parmenides does not simply fail to refer and thus is meaningless, but rather involves us in inconsistencies like talking about a round square. There is no indication that Plato in the context discussed is worried about fictional entities and the possibility of referring to them, as we should assume if Plato also followed the Russell line of interpreting Parmenides. See also Owen 1970, p. 299 f., who points out that the Eleatic Stranger's parity assumption, i.e., the idea that the notions of Being and non-Being must be clarified together, speaks against Plato understanding non-Being as what is non-existent. While the passage discussing whether a monist can allow for two names, 'Being' and 'One', raises problems of reference, namely whether a numerical monist can allow for different names (and ultimately for names at all) necessary for referring to things, this is not the main problem the investigation of non-Being encounters in the *Sophist*.

47 See also Owen 1970, p. 240.

48 In passages like 256c4–5 ("Motion is different from the Different, just as it is also other than both the Same and Rest"), it becomes clear that Plato understands negation as encompassing both the difference between Motion and the Different and the contradictory relation between Motion and Rest. This is also supported by the claim in 256e5–6 that what each Form is multiple, while what it is not is countless: the difference in number between what something is and what something is not can be explained by the fact that each Form is not (in the sense of 'is not identical to') all the things predicated of it, that is, which are different to it (ἕτερον ἐστιν) but which it also 'is'; however, it is also not all the things that cannot even be predicated of it, and which are thus completely separate from the Form (παντάπασιν ἕτερον). Pointing out that negation can also indicate mere difference does not mean that Plato gets rid of all the negative value judgement that Parmenides connected with non-Being. This can, for example, be seen from passages like 254a–b, where we are told that "the sophist runs off into the *darkness* of non-Being", while

'Motion is not Being' is understood to mean that Motion is different from Being (ἕτερον εἶναι τοῦ ὄντος) such that it also participates in Being and hence *is*. By contrast, 'Motion is not Rest' is understood to mean that Motion is *completely* different from Rest (παντάπασιν ἕτερον). For Plato this means that there is no way that Motion and Rest can blend with each other – as Plato puts it, Motion and Rest in no way participate in each other; or, as we would say, they cannot be predicated of each other in any way. Given that in the *Sophist* everything must either be in motion or at rest (according to 248c–249d), motion and rest are in fact understood as forming a contradiction, which is not how the relationship between motion and rest has generally been conceived since Aristotle.[49] For Plato, the claim that Motion is at rest or that Rest is moving is not simply false, but does not even make sense, since Motion and Rest cannot participate in each other in any way.[50]

It is the understanding of negation as difference that Plato concentrates on and that he wants to show should normally be at play when we talk about non-Being. He introduces this idea in a couple of steps. First he shows that when we express mere difference (i.e., the difference between the Forms that blend with each other),[51] we also use the term 'non-Being': when we want to claim that Motion is different from the Same we say that Motion *is not* the Same. So all that is understood by 'is not' here is 'is different from'. The Eleatic Visitor then shows that in the very same sense we say that Motion *is not* Being. Accordingly, he concludes, Motion is

the philosopher is so hard to see because of the "*brightness* of Being" with which he associates.

49 See the chapters on Aristotle.

50 For us, the claim that 'motion is at rest' may sounds strange, because we see a category mistake involved – we cannot say of motion itself that it is or is not at rest, but only of things moved. But once we correct this mistake, once we say 'the things moved are at rest' or 'what is at rest is moving', we seem to deal simply with a false claim, claiming something different from what is the case. Plato, however, considers motion and rest to be contradictories, so that for him this claim is not simply false, but undermines its very meaning.

51 Plato accounts for the difference between Forms that blend in principle not with the help of their own nature but with the help of their participation in the Form of the Different (since, for example, what it means to be motion is not the same as what it means to be different from the notion of Being). He also understands what we would take to be a dyadic relation of difference as triadic: Form A participates in the Different D with respect to another Form B. For example, 'Motion is different from Sameness' means that Motion participates in the Different with respect to Sameness. This difference relation can hold between Forms that also blend with each other in some respect. It remains unclear, however, how the participation in the Form of the Different can also ground the complete difference between Forms that mutually exclude each other, like Motion and Rest, as 255e seems to suggest. In this case, Motion and Rest would have to partake in Difference with respect to each other in a different way than, for example, Being and Motion do with respect to each other. What would then explain the difference in the two participation relations?

some non-Being, and non-Being is in the case of motion (256d);[52] and all that is meant by this is that Motion is different from the Form of Being. From this we can go on to Being: the Form of Being too is different from the other Forms, and thus *is not* these other Forms; so Being itself *is not* in so far as there are other Forms (257a).[53] We have already seen the atomists claiming that not only Being, but also non-Being 'is'; Plato here goes further and shows that Being, too, 'is not'.

Plato demonstrates in a further step that when we talk about non-Being (some non-*x*) we are actually just referring to something that is different from Being (different from *x*). He introduces this thought with the help of the example of largeness in 257b6–c3: when we say that something is not-large, we can be indicating that it is small or medium-sized.[54] This example is meant to show that any non-*x* need not indicate the complete opposite (ἐναντίον) of *x*, but only something different from that which is negated. In the same way in which 'not-large' refers to some not yet further determined element within the realm of size, so 'non-*x*' refers to some not yet further determined element within the realm of *x* (or within a realm where it makes sense to talk about *x*). For Plato, calling something not-large does not mean that it has no *megethos* whatsoever, and calling something non-Being does not mean that it has no Being whatsoever.

In the first chapter we saw that an opposite pair can either form a contradiction ('*x* is not-F' means that *x* can be anything in the world apart from F) or a contrary pair (within a certain domain, like size, *x* is something in this domain, but not F, not large). Contrary opposites can also allow for what we have called different degrees: the 'not-F' of a contrary pair can either refer to the complete opposite of F within the domain specified (in our case, to the small), if it is the result of what we have called an extreme negation; or it refers to anything in the domain specified apart from the one pole, F, if it is the result of what we have called a moderate negation. It is this moderate negation that

[52] Plato switches in this argument from the predicate 'not being' to the noun 'non-Being' and thus runs the risk of introducing an entity when he is in fact only dealing with an operator. (He may try to defend this substantivation by explaining that 'to be' and 'not to be' derive from participation in Being and non-Being so that every being and non-being presupposes the entities Being and non-Being).

[53] Parmenides could in fact agree with this statement and then claim that this is the reason we should not allow for any plurality – plurality would require allowing for difference and thus for an understanding of non-Being that does not fit Parmenides' philosophy.

[54] The Greek reads: Οἷον ὅταν εἴπωμέν τι μὴ μέγα, τότε μᾶλλόν τί σοι φαινόμεθα τὸ σμικρὸν ἢ τὸ ἴσον δηλοῦν τῷ ῥήματι. There are two ways of understanding this – either we understand *mega* as meaning 'larger' and thus what is not larger is smaller or of equal size (so here we are comparing the size of two different things, a reading that does full justice to the word *ison*). Or we understand *mega* as 'large', in which case we better translate *ison* as medium-sized. I will use the latter understanding here, but everything I say would equally hold if we used the former.

Plato wants to introduce into the philosophical discussion here: within the domain of size, the not-large can indicate not only the opposite pole of what is large, namely the small, but also the medium-sized, since the medium-sized too is not large.

With respect to Parmenides, we saw that within the realm of *alêtheia*, negation always expresses the complete opposite of what is negated, which seemed to indicate a contrary opposite.[55] But it does so in such a way that the opposites cannot mix and one of the two must be true and the other false – what we usually take to be an essential feature of a contradictory opposite. So Parmenides' understanding of negation in the case of non-Being shows features of what we would think of as contrariness as well as of contradiction.

However, in the realm of *doxa* we also saw a form of negation employed that did allow for merging opposites, like night and light.[56] This form of negation seems to prepare some form of real contrariety, that is, a contrary opposition that does not also show features of a contradiction, and potentially a moderate negation. While such a form of negation can be seen as prefigured in Parmenides, Parmenides employs it only in his account of the deceiving opinion of mortals. Plato now shows that such a notion of negation also holds within the realm of what truly is by focusing on moderate negation as applicable to Forms.

It is exactly this moderate negation that Plato needs in order to reintroduce non-Being into the philosophical discussion. For both contradictory as well as complete contrary opposites would leave Plato with a problematic understanding of non-Being: if negation could only refer to a contradictory opposite, no domain whatsoever would be specified (so the not-large could be anything in the world apart from what is large: mud, hair, or the number two) and non-Being would be completely indeterminate.[57] That the not-large refers to the small or the equal (but not, let us say, to the green) makes the domain restriction implied in Plato's understanding of negation obvious – the domain ranges over everything that can have a certain size. It is thus clear that we are not dealing with a contradictory opposite (since domain restriction is a sign

[55] We saw in the chapter on Parmenides that 'non-Being' is understood to refer to what possesses no Being whatsoever, and thus to the contrary opposite of Being. By contrast, if we assumed Parmenides to be working simply with a contradictory opposite 'not-F', 'non-Being' could refer to anything in the world apart from Being itself, but would not necessarily indicate what possess no Being whatsoever.

[56] Fr. 8, line 55 and fr. 6.

[57] But for Plato non-*x* is not completely indeterminate as it refers to some element other than *x* within the domain specified. We saw that Parmenides seemed to understand non-Being to be clearly determined as that which is in no way. However, since everything that is not completely should also belong to non-Being according to the logical basis of his poem, non-Being is in fact not clearly determined in Parmenides.

of contrary opposites).[58] If, however, non-Being indicated the opposite pole of a contrary opposite (i.e., if non-Being were the result of extreme negation), then it seems non-Being would be what is not at all, what has no Being whatsoever, and thus we would be back to a non-Being that cannot be talked about, as we saw in Parmenides. What is needed therefore is a negation of Being that only indicates difference, but difference within a determinate realm.

While it seems unproblematic to use a moderate negation within the realm of size or beauty (beauty is the second example introduced in the *Sophist*), it is not clear that such an understanding of negation can indeed be applied to the realm of Being or to the realm where we can talk about Being. In order for negation to be understood as a moderate negation in the case of non-Being, Plato must show two things: (a) that non-Being is (in some sense) a part of the realm of Being, just as the not-beautiful is a part of the realm in which we can talk about beauty; and (b) that the realm of Being/non-Being can be divided in the same way the domains of size or beauty can.[59] It is obvious that within the domain in which we can talk about beauty and ugliness there will be things that are neither especially beautiful nor especially ugly, but somewhere in between, and within the realm of size are not only the small and the large but also the medium-sized. However, it is not immediately obvious that within the realm of Being there is something in between the two opposite poles of Being and non-Being at all.[60] Does the realm of Being allow for a third thing in addition to these two, which is required for a moderate negation?

We may think of sensible things, which Plato describes in the *Republic* as what is and is not and which thus may seem to qualify as some third thing in addition to Being and non-Being at all. However, this is not the route Plato chooses here, since he seems to stay within the realm of the Forms in the *Sophist*. Instead, Plato shows that there is a myriad of different non-Beings that

[58] We saw in the first chapter that domain restriction need not always be explicitly defined if it is clear from the context what the realm is. See Crivelli 2012, pp. 184–96 for an overview and discussion of what exactly Plato is committing himself to in his largeness example in 257b6–c3 according to different interpretations.

[59] This second requirement implies that it can be divided, and that it can be divided into more than two elements (a least a third element is necessary if we want to have moderate negation). The example of the beautiful is important for showing that different realms of Being can in fact be divided in such a way. While largeness for Plato is always relational (i.e., something is large if it is larger than something else) and thus presupposes more than one element, he uses being beautiful as an absolute term, and hence does not presuppose more than one element.

[60] In the *Sophist*, 'the large' refers to the whole realm in which we can talk about largeness, as well as to one element in it (what has a particular size). In the same way, 'Being' refers on the one hand to the Form of Being itself, and on the other to the whole realm of what is. In the former role it is opposed to non-Being (though, as we will see, not as a contradictory opposite); in the latter it includes non-Being.

all are and thus can be seen as elements in between the two opposite poles of that which is not at all and the Form of Being.

Plato shows that both (a) and (b) hold for the realm of Being/non-Being by pointing out a structural analogy between knowledge and the Different: knowledge is a single, unified realm, but yet each 'part' of it, that is, each subfield relating to a particular thing, is marked off and has its own name, so that we rightly talk not only about knowledge in the singular, but also about many kinds of knowledge. Similarly, the Different can be understood as one single, unified realm, but different parts of this realm relating to particular things are marked off and opposed to some Being. For example, from those things where we can talk about beauty we separate off some part, oppose it to the beautiful, call it the 'not-beautiful', and understand it as something with respect to which we can talk about beauty but which is different to beauty.[61] In this way, Plato claims, we oppose one thing that is, one Being (the beautiful) to another Being (the not-beautiful), rather than something that 'is' to something that 'is not at all'.

The not-beautiful is a part of the Different as is every non-*x*.[62] But if all non-*x* are parts of the Different, and all parts of the Different are, because the Different is (as has been shown, when the Form of the Different was first introduced), then we can necessarily claim that non-Being also is. And we have shown also what it is, namely a part of the Different.[63]

Now it seems that of the two conditions necessary for a moderate negation, Plato has only shown that (a) holds – non-Being (in some sense) is and thus is a part of the realm of Being – but not (b) – that the realm of Being/Non-Being can be divided into multiple parts. For the analogy with knowledge, demonstrating that one realm of what is, knowledge, is divided up into different parts, does not show that the realm of Being/non-Being itself can be divided into multiple parts, but rather that the realm of the Different can.

[61] Plato writes: "By having another one marked off within one kind of those that are, and then again set over against one of those that are – isn't it in this way that the not-Beautiful turns out to be?" (257e2–4). According to this passage, we derive any particular non-Being, as, for example, the not-Beautiful, by separating off some indefinite different (cf. 256c4–5) from some realm of Being with the help of negation. Subsequently, we oppose this part of the difference to one Being from this realm, call it accordingly, and thus it becomes a definite part of the Different. Lee 1972, pp. 279–80 thinks that marking off some Different and opposing it to, say, the Beautiful does not represent two separate acts; rather by being opposed to the Beautiful a part of the Different is marked off.

[62] Every non-*x*, that is, which is a contrary opposite to *x*; contradictory opposites do not seem to be part of this investigation.

[63] This argument raises a problem, however. It could be meant as a universal argument that all non-*x* are part of the Different, whether or not they are in contrary or contradictory opposition to *x*. In this case we can infer that non-Being must also be. But we are faced with the problem that the contradictories Motion and Rest do not seem to fit the general pattern. Or this is just meant as a general, but non-exhaustive pattern, as Plato's understanding of Motion and Rest suggests. But then, if this pattern does not include all cases of non-*x*, we cannot be sure that the pattern also holds for non-Being.

However, Plato's inquiry into the realm of the Different shows that there are many non-Beings, since all the many non-*x* are non-Beings (the non-Beautiful, for example, is what is not beautiful).[64] And at the same time they all are. In this way, we see that the realm of what is, of Being, contains many elements, and 'non-Being' can indicate any of them.[65]

Summing up, we can say that Plato changed the Eleatic understanding of the negation operator in ways crucial for avoiding the paradoxes of non-Being and falsity that derived from Parmenides' position: the term negation that we find in Parmenides has become a predicate negation in Plato. In Parmenides, negation is a one-place operator that signifies the polar opposite of what is negated and thus works as what I term 'extreme negation' (though it also showed, as a feature of contradictory opposites, that one of the two poles had to be true and the other false). As a result, non-Being is understood as what has no being at all, as what is in no way. This is also how the Eleatic Stranger understands non-Being at the beginning of the investigation in the *Sophist*, as what is in no way (μηδαμῶς ὄντα, 240e2).

By contrast, the Eleatic Stranger ends up treating negation as a two-place relation ('*x* is not F')[66] that indicates some difference between the two relata.[67] The 'non' in 'non-Being' seems at first glance to work like a one-place negation as well, but in the *Sophist* 'non-Being' is in fact a short-hand for '*x* is not the Form "Being"'.

Plato understands non-Being not as the polar opposite of Being, but as something that is merely different from the Form Being, and thus compatible with it.[68] But if non-Being *is*, it is also utterable, and hence talking about non-Being no longer undermines itself.[69]

This is the parricide that the Eleatic Visitor has warned us our investigation will lead to: non-Being in some sense has Being, Being in some sense is not – the basic claims of Parmenides, that it is and cannot be that it is not, and that

[64] It is not beautiful, however, by being part of the realm where we can reasonably talk about beauty.

[65] It is, unclear, whether it can also refer to the Form 'Being': Being was also shown not to be other Forms, so it should in principle also be a non-Being. However, non-Being is what is divided off and opposed to that whose realm it is, so for this reason non-Being should not also cover the Form Being.

[66] For example, in 256a5: "Motion is not the Same."

[67] For Plato, negation is a two-place relation in the sense of a relation between subject and the negated predicate; additionally, the subject is also related to the Different, in which it participates in respect to the predicate in question (cf. above).

[68] At least this is the main understanding of non-Being that we find in the *Sophist*, for this dialogue offers three kinds of admissible non-Being (cf. also Lee 1972): (1) some Form, like Motion, not being the Form of Being (256e5–257a5); (2) some Form not being some other Form; and (3) some Form in connection with Difference constitutes the Being of something new, of a part of the Different, like the non-beautiful.

[69] In 240e5–6 the interlocutors describe false belief as believing that what is in no way is in some way or that what is in every way is in fact in no way (μηδαμῶς). Following the investigation of Being and non-Being, it has become clear that Being can in some sense

non-Being cannot be thought or talked about, has been overturned. But we will see that it has been overturned in order to save Parmenides' project of establishing philosophy as an investigation following strict criteria.

We saw in the second chapter that the three basic concepts of Parmenides' poem – Being, negation, and a connection operator – are exactly tailored to each other: they are systematically connected in the sense that they depend on each other in such a way that any change in one concept would necessitate changes in the others. In accordance with this systematic connection, we saw in Plato's *Sophist* that changing the understanding of 'not', so as to indicate not an extreme negation but only 'difference', not only changes the understanding of non-Being from a polar opposite of Being to a non-Being that in some sense is,[70] but also leads to a change of the understanding of 'Being' (and of all the Forms) in such a way that it is rendered internally complex: the Form of Being itself not only is, but is different from the other Forms and thus is not these other Forms.[71] In the next step we will see how the modification of the negation operator also leads to changes in the connection operator.

5.2.1.2 Connection Operator

We saw in Chapter 2 that the only connection operator that fits Parmenides' framework is an operator indicating what we called 'absolute identity', the identity of one thing with itself ($x = x$), for only such a connection avoids differences in any aspect. All connections that cannot be reduced to absolute identity – predication as well as what we called 'partial identity' – presuppose some form of plurality and thus require the logical means to conceive of plurality in a consistent way.

The atomists very clearly employ such connections presupposing plurality when, for example, they claim the atoms to have certain properties, like being

not be. So in order to prevent false beliefs, it is not enough to prevent having any non-Being ascribed to it; what is needed is rather to figure out in which way Being is also not. Furthermore, in order to show that false thought is possible, Plato must first investigate the structure of statements, which is what happens in the *Sophist* after the passages looked at here. There, Plato clearly distinguishes between the point of reference of a statement (the object it is referring to) and its truth. For example, the statement 'Theaetetus is flying' is taken to be a false statement, but it is nevertheless understood as a statement about some being, namely about Theaetetus. It is not, as suggested by the traditional puzzles that conflate the point of reference and the truth of a statement, a statement about non-Being (263a8–d5).

[70] This would obviously not fit with Parmenides' understanding of Being and would undermine his argument for the exclusion of non-Being from philosophical investigation in fr. 2.

[71] The so-called parity assumption in the *Sophist* – the assumption that after we have been equally confused by Being and Non-Being, clarifying one of them will help to clarify the other (250d–251a) – also shows that clarifying, and indeed changing, one basic concept will lead to a clarification and change of the other.

extended or of a certain size. However, their understanding of the connection operator is never discussed in the atomist fragments as handed down to us. It is only in Plato's *Sophist* that we find a discussion of possible connection operators and an explicit distinction between predication and absolute identity. With the so-called late learner paradox (251a–c) Plato explicitly shows how employing absolute identity as the only connection operator would restrict our speech to tautologies: the late learners, occupying a simplified Parmenidean position,[72] assume that if we connected one thing to many different 'names', it would in fact have to be many different things. They understand our common practice of connecting different properties to one and the same subject as undermining the unity of the subject:

> When we talk about a man we surely give him many different names by applying colours to him, and shapes and sizes and vices and virtues. In these cases and a million others we say that he is not only a man but also that he is good and indefinitely many other things. And similarly, on the same account we take a thing to be one, and at the same time we speak of it as many by using many names for it. (251a8–b3; White's translation with modifications)

Objecting to this common practice, the late learners claim, as the Eleatic Visitor points out, that

> it is impossible for that which is many to be one and for that which is one to be many. They evidently enjoy forbidding us to say that a man is good, and allow us to say only that the good is good or that man is man. (251b7–c2; White's translation with modifications)

Because the late learners understand the "is" that connects "man" and "good" as absolute identity, they think that the claim "man is good" would destroy the unity of man; it would make the one man into many things (one that is good, one that is man).[73] Strictly speaking, their position also rules out definitions, like "man is a rational animal", or non-trivial identity statements, like "the morning star is the evening star", where the same subject is given in different senses, since there are different terms on each side of the "is", and so the one subject seems to be made into a plurality. Hence the late learners restrict our speaking to those statements that capture indeed an absolute and simple identity, to statements where the right-hand side of the "is" can indeed always be substituted for the left-hand side, statements like "the good is

[72] I understand it as a *simplified* Parmenidean position, since the reduction to absolute identity as the only connection operator is not embedded in a general philosophical framework requiring this kind of connection operator, as we saw was the case with Parmenides.

[73] See also Ackrill 1957, p. 1: "There remain two other meanings of *estin*, as copula and as identity-sign. The assimilation of these had led to a denial of the possibility of any true non-tautological statements."

good":[74] nothing else is identical to the good, so we cannot say of anything else that it is good.

The ensuing discussion in the *Sophist* clarifies what we understand by this "is" when we call one thing many names, as in "man *is* good". In order to figure out which combinations are allowed, that is, which kinds or concepts can be applied to which and to what extent, the Eleatic Visitor and Theaetetus look at five greatest kinds, the *megista genê*, Motion, Rest, Being, the Same, and the Different. Early on in this investigation, the Eleatic Visitor distinguishes Being from the Same, which eventually allows him to distinguish between a connection operator expressing identity and one expressing what we would call predication:[75]

ΞΕ. Ἀλλ' εἰ τὸ ὂν καὶ τὸ ταὐτὸν μηδὲν διάφορον σημαίνετον, κίνησιν αὖ πάλιν καὶ στάσιν ἀμφότερα εἶναι λέγοντες ἀμφότερα οὕτως αὐτὰ ταὐτὸν ὡς ὄντα προσεροῦμεν.

ΘΕΑΙ. Ἀλλὰ μὴν τοῦτό γε ἀδύνατον.

ΞΕ. Ἀδύνατον ἄρα ταὐτὸν καὶ τὸ ὂν ἓν εἶναι.

VIS: But if Being and the Same do not refer to different things, then when we say that Motion and Rest both are, we will be labelling both of them as being the same.

TH: But certainly that is impossible.

VIS: So it is impossible for the Same and Being to be one. (255b11–c3)

Given that "being the same" is grammatically incomplete (it is elliptical for "being the same as *x*" or "being the same as itself"), it seems natural to read this passage[76] as also understanding "being" as grammatically incomplete here, as elliptical for "being F", since only if the two share the same grammatical surface structure is the substitution discussed in this passage possible. According to this understanding of the passage, then, the Same and Being must be different. Otherwise we could substitute "is" and "are" for "the same", and if we then claimed that motion and rest both are (some F), we would in fact be claiming that they are the same (as F), and if motion and rest were both the same as F, we could infer that motion is the same as rest (255a–c).[77]

The lack of a clear distinction between Being and Sameness contributes to several of the paradoxes earlier in the dialogue that can be resolved once this difference and what Plato calls the participation or blending relations are clearly established; it also contributes to the paradox we find in the discussion with the dualists: Plato starts his investigation of preceding ontologies and their

74 Strictly speaking, the late learners would also have to object to the article "the" (τὸ) in "the good is good" (τὸ μὲν ἀγαθὸν ἀγαθόν, 251c1).

75 While Plato explicitly distinguishes between absolute identity and predication, he does not explicitly draw a distinction between absolute and partial identity, as we will see below.

76 See also Owen 1970, p. 257.

77 Cf. Gill 2016.

paradoxes with a look at monistic and pluralistic positions, and investigates a dualistic position as the simplest form of pluralism. In 243d ff. he shows that the hot and the cold of the dualist – hot and cold seem to represent any two basic principles we like – cannot be said *to be*, for this seems to lead to one of three possible problems: it would mean either that Being is a third thing, over and above the hot and the cold, which undermines dualism; or that Being is the same as one of the two, either the hot or the cold, in which case it cannot be applied to the other, there is only one thing instead of two; or that both should be called 'Being', but then it is unclear why they are not the same.

Against the third alternative – that both should be called 'Being', but it is unclear why they are not the same – after the distinction between Being and Sameness has been drawn, Plato can now point out that 'the cold *is* or *has being*' does not, as has just been shown, mean that the cold is identical to or the same as Being. Rather, it means that the cold participates in Being, which we can understand to mean 'Being can be predicated of the cold'. It has being but is also different from Being itself. Plato embraces the first alternative – that Being is a third thing. Before the *megista genê* are discussed, this alternative cannot be embraced, not only because it undermines a dualistic position (a more 'generous' pluralist would not be bothered by this), but also because up to this point it seemed that some *x* can be F only if it is identical to F; identity seemed to be the only way in which *x* and F could be connected.[78] If *x* is not identical to F, it cannot be F at all (in a rather Parmenidean vein). So if Being is a third thing, then the cold and the hot would not be. What can be understood as confusion between identity and predication is at work not only with the late learners, but also in this dualism paradox.

By contrast, Plato's idea of *x* blending with, or participating in, Being means that *x* also possesses some F-ness without being the same as F. For example, if the cold is not the same as Being, it can still be. Once participation, and thus what we would call predication, is established as a consistent and reliable way of connecting two things, for example, a subject and a property, there is no longer any reason to follow the late learners and allow only tautological statements; now besides the good, men too can be good, and the cold *is*, without thus being identical to Being. (However, while Plato establishes the idea of blending and participating in the *Sophist* as a response to this general problem, in the case of motion and rest Plato actually sticks to understanding them as a contradictory pair which means that they cannot participate in each other at all.)

It is this distinction between Being and Sameness, between predication and identity, that also allows one to claim that "Motion is the same and Motion is not the same" at the same time. What would have seemed like a contradiction for the

[78] See, for example, 243e4–244a2.

Eleatic Visitor and Theaetetus before the *megista genê* investigation can now be seen to hold true simultaneously: we can make both claims about motion, because the 'is' in both statements is not understood in the same way. According to a widespread interpretation,[79] we understand "is" in "Motion is the same" as predicative, as indicating participation or blending, while the "is" in "Motion is not the same" is understood as a sign of identity. Both statements together claim that Motion participates in Sameness, thus it is the same (with respect to itself), but it is not identical to Sameness, thus it is not what it means to be the same.[80] (In this way predication seems to be more basic than identity for Plato, since also self-identity is based on something participating in or blending with Sameness.)

There is, however, at least one problem with Plato's account of Sameness in the *megista genê* inquiry, since he uses Sameness in two different ways here.[81] Plato first introduces Sameness as self-identity, the sameness of something with itself: "So each of them [that is, Being, Motion, and Rest] is different from two of them, but is the same as itself" (254d14–15). However, in the passage quoted above that establishes the distinction between Being and Sameness (where the Eleatic Visitor says, "But if Being and the Same do not refer to different things, then when we say that Motion and Rest both are, we will be labelling both of them as being the same"), 'the Same' cannot be understood as referring to self-identity, but must be understood as Sameness with something else (for example, "Motion is the Same as some *x*", understood to mean that this sameness involves a difference between Motion and *x* in at least one respect). Otherwise Plato's argument for distinguishing between the two concepts would not work, for there would be no problem with saying both "Motion is the same as itself" and "Rest is the same as itself". So under the heading of 'Sameness' Plato first introduces what we called absolute identity – the identity or sameness of one thing with itself – but he then moves on to partial identity or sameness.[82]

A further question, frequently discussed in the secondary literature, is whether Plato distinguishes not only an 'is' of predication and of identity but also an 'is' of existence. This is often framed in syntactical terms – does Plato distinguish between a complete usage of *esti* as a one-place predicate ("*x* is") and an incomplete usage of it as a two-place predicate ("*x* is some

[79] See, for example, Ackrill 1957, pp. 1–6. For a different interpretation, see Frede 1967 and Brown 2008. Anscombe 1981, p. 29 and other scholars have claimed that the distinction between the 'is' of the copula and of identity was already established earlier by Plato through his distinction between participating in φ and being φ itself; however, it seems that only in the *Sophist* is he making such a distinction a central philosophical point.

[80] For Plato, the fact that Motion is not identical to Sameness is due to Motion also being associated with the Different (with respect to Sameness).

[81] A further question, which I cannot pursue here, is whether Sameness must participate in itself, or self-predicate, in order to be the same as itself. I must also leave to one side most of the problems with Being.

[82] He keeps using both forms in the remainder of the *megista genê* doctrine.

complement")? A semantic distinction is usually seen as answering to the syntactic one: there are different versions of this, but generally in its complete use the one-place predicate is taken to mean 'exist' or 'is the case'. In its incomplete use it signifies the copula ("*x* is F") or the identity sign ("*x* is *x*").

Frede and others have argued against seeing this further step taken in the *Sophist*, among other things, because it would require a separate Form.[83] Brown has pointed out that in ancient Greek the distinction between a complete and an incomplete sense is not as sharp as some have thought, rather there is continuity between them.[84] However that may be, I will leave this discussion to one side; for our project, the distinction between predication and absolute identity is the important step, and Plato also provides two Forms for these two notions, Being and the Same.

Finally, the new understanding of negation and the connection operator also allows for a consistent relationship between Being and non-Being. We have already seen above that the new understanding of the operators brings with it a new account of what is, of the Forms. In the *Republic* and the *Phaedo*, the way they are characterised is reminiscent of Parmenides' Being: they are of one kind (μονοειδές, *Phaedo* 78d), each is one, and has neither come into being nor will pass away. They are simple and possess no internal complexity, and are not composites in any way (*Phaedo* 78c–d) – internal complexity in Plato is a feature only of things that are subject to generation and destruction, in the same way that in Parmenides internal complexity is only displayed by things in the world of *doxa*. The complete simplicity of Forms is also a necessary assumption for the affinity argument for the immortality of the soul in the *Phaedo*.[85] The *Sophist*, by contrast, provides the logical basis for understanding the Forms themselves as internally complex – they participate in other Forms and thus have different respects, they are and are not.[86]

Having looked at Plato's reinterpretation of the logical operators, let us now move on to investigate how he understands the principle of non-contradiction and its relation to the principle of excluded middle which Parmenides established as a necessary criterion for philosophy.

83 Frede 1992 and Owen 1970, p. 257.

84 On this view, the complete use of 'is' does not simply correspond to physical existence; rather anything describable exists, so Pegasus exists on this view, since we can describe what Pegasus is. Leigh 2008 has pointed out, however, that Brown has problems finding decisive passages for this continuity claim.

85 *Phaedo* 78b ff. In the *Republic*, we also find the idea that the Form of the Good pervades all the other Forms, but it is not explained how this fits with them being completely simple.

86 I am following a very widespread assumption in the secondary literature in understanding Plato's *genê* in the *megista genê* discussion as Forms, but I will not be able to argue for this assumption here.

5.2.2 *The Reinterpretation of the Criteria for Philosophy 1: The Principle of Non-Contradiction and the Principle of Excluded Middle*

The change of operators and the understanding of Being also leads to changes in the understanding of the criteria for philosophy. In the *Sophist* this becomes clear with respect to the principle of non-contradiction and the principle of excluded middle. We will see Plato's decisive changes to the principle of sufficient reason and rational admissibility below, when we turn back to the *Timaeus*.

We saw in the second chapter that Parmenides' understanding of negation and connection meant either that something is completely F or it is completely not-F; differences of respect are thus impossible. Accordingly, the principle of non-contradiction in its Parmenidean version must be understood as 'not (F and not-F) simpliciter' without allowing for taking into account that something may be F and not-F at different times, or in different respects.

It is in Plato's *Republic* that we find the first explicit changes to this understanding of the principle of non-contradiction (we already find implicit changes in the atomists). There Plato seems to keep Parmenides' idea of non-Being as what is not at all and Being as what is simpliciter (even though the characteristics of Being – that it is without any internal differences, eternal, and so forth – apply now to a plurality of Forms). But he also establishes Becoming as its own realm whose members partake in Being as well as non-Being. While Parmenides' austere version of the principle of non-contradiction might fit Plato's notion of Being and non-Being in the *Republic*, it is clear that this version is not suitable for understanding what is taking place on the level of Becoming. There things change from being F to being G and thus no longer be F, and are internally complex so that one part may be F while another may not be F; in short, things subject to becoming may be F in one respect and not in another.

So it is no surprise that Plato uses sensible things and states of affairs, like the motions of a human body, as examples to explain his version of the principle of non-contradiction that includes respects. But it is centrally employed to account for the soul:

> Δῆλον ὅτι ταὐτὸν τἀναντία ποιεῖν ἢ πάσχειν κατὰ ταὐτόν γε καὶ πρὸς ταὐτὸν οὐκ ἐθελήσει ἅμα, ὥστε ἄν που εὑρίσκωμεν ἐν αὐτοῖς ταῦτα γιγνόμενα, εἰσόμεθα ὅτι οὐ ταὐτὸν ἦν ἀλλὰ πλείω.
>
> It is obvious that the same thing will not be willing to do or undergo opposites in the same [part of itself], in relation to the same thing, at the same time. So if we ever find this happening in the soul, we will know that we are not dealing with one thing but many. (436b9–c2)[87]

[87] Translations from the *Republic* are from Grube.

Plato presupposes a version of the principle of non-contradiction that allows for respects in this passage in order to show that we must assume different parts of the soul – otherwise we cannot explain psychic conflicts in which we simultaneously want and do not want to do or undergo something. More precisely, Plato points out that opposites cannot be done or undergone (a) in the same (part of something), (b) in relation to the same thing, and (c) at the same time. Points (b) and (c) fall under what we would call the same respect. Time is, however, mentioned separately as the respect that is probably the most prominent, since things usually change from one time to the next. (a) makes sure that we are talking about the same agent or subject. I am following Grube in understanding κατὰ ταὐτόν as "in the same part" in 436b9,[88] because it is exactly the need to distinguish between different parts for different actions that Plato goes on to explicate in the lines immediately following:

> Is it possible for the same to stand still and move at the same time with respect to the same?
> Not at all.
> Let's make our agreement more precise in order to avoid disputes later on. If someone said that a person who is standing still but moving his hands and head is moving and standing still at the same time, we would not consider, I think, that he ought to put it like that. What he ought to say is that some part of the person is standing still and some other part is moving [τὸ μέν τι αὐτοῦ ἕστηκε, τὸ δὲ κινεῖται]. (436c6–d1)

Plato points out here that the same thing, for example, a person, may well be simultaneously at rest and in motion, but the same *part*[89] of the person cannot be at rest and in motion at the same time. Plato does not argue for an understanding of the principle of non-contradiction as involving different respects here; in his argument for different parts of the soul he seems to take the principle of non-contradiction for granted in the form 'not (F and not-F) in the same respect', in contrast to Parmenides' 'not (F and not-F) simpliciter'. Applying the principle of non-contradiction as involving different respects allows him to determine ultimately what counts as one and the same 'agent' (i.e., the soul part) – if the soul wants F and does not want F at the same time and in the same respect, we know that we must assume different parts of the soul.

[88] Even though κατὰ τὸ αὐτὸ seems to mean simply 'in the same respect' in 436e9. And Plato does talk about the *merê* of the soul once he has completed the arguments for the *tripartion* (in 442b10, 442c4, and 44b3). For a discussion of this point, see Sattler forthcoming.

[89] Plato seems to think of parts here since his argumentative aim is to show that the soul must have more than one part; but we can think of it more generally as the same respect that cannot be at motion and at rest at the same time.

Plato's formulation of the principle of non-contradiction in the *Republic* could be described as an ontological one, as it is a claim about things in the world. If we compare it briefly to Aristotle's ontological formulation in the *Metaphysics*, we can note that Aristotle generalises Plato's account and standardises it:

> τὸ γὰρ αὐτὸ ἅμα ὑπάρχειν τε καὶ μὴ ὑπάρχειν ἀδύνατον τῷ αὐτῷ καὶ κατὰ τὸ αὐτό (καὶ ὅσα ἄλλα προσδιορισαίμεθ' ἄν, ἔστω προσδιωρισμένα πρὸς τὰς λογικὰς δυσχερείας)· αὕτη δὴ πασῶν ἐστὶ βεβαιοτάτη τῶν ἀρχῶν·
>
> For it is impossible that the same at the same time both does belong and does not belong to the same thing and in the same respect (and all other specifications that may be made, let them be added to meet logical difficulties); this is the most secure of all principles. (*Metaphysics* 1005b19–23)

According to Aristotle's formulation of the principle of non-contradiction – "the most secure of all principles" – it is impossible that the same thing at the same time both belongs and does not belong to the same object and in the same respect. There are three points to note in comparison to Plato's account: first, Aristotle makes the modality involved very clear ("it is impossible . . . "), while Plato's first formulation is vaguer, talking about "willingness" (ἐθελήσει). That this formulation of Plato can, nevertheless, be understood as an implicit non-technical modal[90] is supported by Plato's second formulation, which also translates it into a modal claim – it is not *possible* (δυνατόν) for something to stand still and move at the same time with respect to the same.

Secondly, in Aristotle's formulation the modality is external to the whole proposition ('it is impossible that p and not-p'), while Plato's first formulation restricts the modality to the relation of subject to predicate ('the same S will not be willing to do or undergo opposite things in the same respect'). However, in the second passage of the *Republic* quoted, Plato also uses the modality (δυνατόν) as external to the whole formulation: "Is it possible for the same to stand still and move at the same time with respect to the same?" (Ἑστάναι, εἶπον, καὶ κινεῖσθαι τὸ αὐτὸ ἅμα κατὰ τὸ αὐτὸ ἆρα δυνατόν;). Again we find different possibilities in Plato that seem to have become standardised in Aristotle.

Finally, Aristotle's formulation is a generalisation *vis-à-vis* Plato's account: it talks in general about opposed properties applying to the same object, while Plato's first formulation refers to the agency of doing or undergoing opposites,

[90] Plato uses *ethelein* as a non-technical modal here; sometimes he also uses *boulesthai* in this way. That both terms can be understood in a modal sense becomes clear from examples in the *Timaeus*, where the subjects of *ethelein* and *boulesthai* are beings lacking mentality. See for example, 50b and 57b (I owe this point to Sarah Broadie).

since the agency of the soul is what Plato is interested in, and the following account is formulated with respect to motion and change only.[91]

We already find some kind of generalisation in Plato's *Sophist*, however. There Plato adds three points to his formulation of the principle of non-contradiction so far: (1) he supplements an epistemological formulation of the principle of non-contradiction; (2) he extends the principle (in the form in which it involves respects) to apply also to Forms and thus generalises it; and (3) he provides the logical basis for understanding the principle of non-contradiction as a principle that centrally employs the notion of respects, that is, for understanding it as '*x* cannot be F and not-F *in the same respect*'.

(1) In *Sophist* 230b ff. the Eleatic Visitor and Theaetetus are still making an initial attempt to define the sophist. They have now reached what will lead to the sixth definition, and here we learn the following about alleged sophistic activities:

> Διερωτῶσιν ὧν ἂν οἴηταί τίς τι πέρι λέγειν λέγων μηδέν· εἶθ' ἅτε πλανωμένων τὰς δόξας ῥᾳδίως ἐξετάζουσι, καὶ συνάγοντες δὴ τοῖς λόγοις εἰς ταὐτὸν τιθέασι παρ' ἀλλήλας, τιθέντες δὲ ἐπιδεικνύουσιν αὐτὰς αὑταῖς ἅμα περὶ τῶν αὐτῶν πρὸς τὰ αὐτὰ κατὰ ταὐτὰ ἐναντίας. οἱ δ' ὁρῶντες ἑαυτοῖς μὲν χαλεπαίνουσι, πρὸς δὲ τοὺς ἄλλους ἡμεροῦνται, καὶ τούτῳ δὴ τῷ τρόπῳ τῶν περὶ αὑτοὺς μεγάλων καὶ σκληρῶν δοξῶν ἀπαλλάττονται πασῶν [τε] ἀπαλλαγῶν ἀκούειν τε ἡδίστην καὶ τῷ πάσχοντι βεβαιότατα γιγνομένην.

> They ask questions on whatever someone thinks he's talking sense about when in fact he is talking nonsense; and then, because the people whose beliefs they are examining are continually shifting this way and that, their task is easy. They use their conversations to collect those beliefs together and put them side by side, thereby revealing them as opposed to one another at the same time on the same subjects in relation to the same things and in the same respects. When those being examined see what is happening, they are angry with themselves but become gentler towards others; and it is in this way that they are liberated from those great, obstinate beliefs about themselves – of all liberations, the most pleasant to listen to and the most securely based for the person who experiences it. (230b4–c3, Rowe's translation with modifications)

Here it is *opinions* that conflict with each other "at the same time on the same subjects in relation to the same things and in the same respects"; it is opinions that are inconsistent. Hence we are not dealing with an ontological claim, but an epistemological one. However, we have no reason to believe that Plato meant the epistemological understanding of the principle to replace the

[91] Nor does the following spinning top passage in the *Republic*, 436d–e, give us a formulation of the same generality as Aristotle's.

ontological one, given that he later also used ontological formulations;[92] rather, it seems that he is now also spelling it out explicitly on an epistemological level.

(2) The first steps established in the *megista genê* inquiry – prominently summed up with the example 'Motion is the same and Motion is not the same' – shows that the principle of non-contradiction in the version including different respects also applies to Forms, *genê* – the Form 'Motion' is the Same in one way or respect (in so far as it participates in the Form of the Same) and it is not the Same in some other way or respect (in so far as it is not identical to the Form of the Same). This extension of the principle of non-contradiction in its Platonic version is possible, since Forms, too, have been shown to be internally complex.

(3) The logical basis for this new understanding of the principle of non-contradiction is provided only in the *Sophist* with the newly developed understanding of negation and connection we saw above: Plato's new understanding of negation means that if *x* is not F, *x* does not have to be not F at all, as in Parmenides, the absolute opposite of F. Rather, if *x* is not F, *x* is different from F, and this means that *x* can still be F in some way (for example, Motion can differ from the Form of the Same, since it is not the Same, but it also is the Same, since it participates in the Same). But if such a negation only indicates difference, it allows for the principle of non-contradiction not to be infringed if the object is (said to be) F in one way and not F in another; and this can ultimately be generalised[93] to a formulation of the principle of non-contradiction as 'not (F and not-F) in the same respect'.

The new understanding of negation as allowing for differences not only admits a novel understanding of the principle of non-contradiction,[94] but also means that the principle of non-contradiction and the principle of excluded middle come apart explicitly. We saw that in Parmenides these two are necessarily connected on the way of truth: for any *x* and F, it must hold both that *x* is either F or not F (principle of excluded middle) and that *x* cannot be F and not-F (principle of non-contradiction). Thus the overall principle Parmenides is working with is "not (*x* is F and *x* is not-F) simpliciter and *x* must be either F or not-F".

92 See, for example, *Sophist* 249b, which gives an ontological formulation: "Do you think that without rest anything would be the same, in the same state in the same respects?"

93 The *Republic*'s examples tie differences in respect rather closely to parts of *x* and to different times, while in the *Sophist* Plato widens what can count as differences in respects, as the example of '*x* being and not being the Same' shows.

94 This is meant as a logical claim. Historically, it may very well be that it was by applying the Parmenidean version of the principle of non-contradiction and finding that some cases which it ruled out seemed perfectly possible or actual that people implicitly revised the notion of negation and thus developed a reformulation of the principle of non-contradiction.

Plato's understanding of negation as difference, by contrast, allows for a *tertium quid*, so that *x* does not have to be either F or not-F; rather, in being different from F (not being itself what it means to be F), it can also be F in some respect. Hence, the principle of non-contradiction and the principle of excluded middle are not necessarily tied together. It is only for what Plato understands as 'absolute opposites', like motion and rest, that the principle of excluded middle holds. In the fourth chapter we saw a move to understand the principle of non-contradiction as independent of the principle of excluded middle already among the atomists – especially when they employed indifference arguments of the kind that there is no more reason for assuming something to be F than not-F, so it is G, a *tertium quid*. But it is only with Plato's *Sophist* that the principle of non-contradiction is explicitly spelt out as including respects and no longer necessarily tied to the principle of excluded middle.[95]

5.2.3 Widening the Conceptual Possibilities

Plato's *megista genê* doctrine is the first explicit investigation of the systematic connection between the concepts inherited from the Eleatics: we have seen in the preceding sections how this investigation leads to crucial changes in the understanding of Being, the connection operator, and the negation operator. These changes lead to a new understanding of Being and non-Being and thus provide the logical basis for a consistent account of motion and change, since motion and change require Being and non-Being to be connected, as we saw in Chapter 1. The *megista genê* investigation also leads to another important change of the logical basis *vis-à-vis* Parmenides: to a widening of the basic conceptual scheme. Plato's *megista genê* mirror Parmenides' three basic notions: we have Being, a negation operator qua Difference, and a connection operator qua Sameness. However, in Plato we have a quintet, not a trio, as in Parmenides, and for good reasons. Five different concepts seems to be the minimum set necessary to show the different logical relationships possible between concepts: Sameness, Difference, and Being are concepts that are required in order to conceive of any other concept and thus 'blend' with everything, while one pair of absolute opposites (Motion and Rest) shows that there can be contradictory opposites that cannot 'blend' with each other at all. If we were to stick solely with the former three, it would mean that a crucial relation that can indeed be conceived, namely not blending with each other at all, is neglected; accordingly, Parmenides' set of three concepts turns out to be too limited.

Plato chooses Motion and Rest as complete opposites because they have already been central notions in the discussion. And the example of "Motion is and is not"

95 Plato's understanding of negation allows for 'not-*x*' qua being different from *x*, and 'not-*x*' qua not being *x* at all.

is very carefully chosen, as it is a first hint at how one can solve the traditional problem that motion requires bringing together Being and non-Being.[96] True, the traditional problem of motion and change brings Being and non-Being together in a different way than the example here in the *Sophist* shows. The former connects Being and non-Being in the way that something *is* first here and then *is not* here any longer, or it *is* first F and then it *is not* F any longer, while the latter understands by "Motion is and is not" simply that motion has some being but is not the same as (the Form of) Being. Nevertheless, with the example "Motion is and is not", Plato explicitly offers a way that one form of Being and non-Being can be connected and thus starts the discussion, taken up by Aristotle in *Physics*, of how the Being and non-Being required for understanding motion can be seen as consistently connected. Looking at the example in the *Sophist*, we saw that "Motion is F" is understood to mean "F can be predicated of it" and "Motion is not F" to mean that Motion is not identical to F. If we now look at change, say the change of x from white to black, we can say that at t_1 "x is white" in the sense that 'white' can be predicated of it and "x is not white" in the sense that x is not identical to 'being white'. If x has then changed to being black at t_2, we will again say that "x is not white", but in a different sense, namely in the sense that Plato prepares with the case of "Motion is not Rest"[97] – meaning that motion does not participate in rest or, in more modern parlance, rest cannot be predicated of motion. So "x is not white" at t_2 is understood in the sense that 'white' cannot be predicated of x, though x nevertheless 'is' (for example, black), and we need not claim that either 'being white' or x have gone out of existence (even if 'white' is no longer a feature of x). A crucial move for both cases is that non-Being is not understood as an absolute non-Being as it was by Parmenides, but only as not being some F. Plato can thus be understood as indicating the extent to which the logical developments of the *Sophist* provide a consistent basis also for natural philosophy, since they prepare the way for understanding the most basic notion of natural philosophy, motion, in a consistent way.[98] (But he points it out without yet showing us how this understanding can help us to grasp processes in particular.)

The same quintet of concepts will also play a special role in the *Timaeus*: Being, Sameness, and Difference are the ingredients of the World Soul, whose

96 For this traditional problem see especially Chapter 1 and Aristotle, *Physics* book I.

97 It is not the very same sense, however, since whiteness can be predicated of x at some other time, even if now x is not white, while motion and rest can never be predicated of each other.

98 Gill 2016 points out that the status of change and rest is problematic, since they are sometimes treated as what she calls categorical kinds (i.e., they possess categorical content so that they can be organised into genus-species trees; they are also called consonant kinds) and sometimes treated as what she calls structural kinds (i.e., most fundamental kinds that do not have categorical but only structural content; also called vowel kinds); see *Parmenides* 129d–e, 136a–c; Ryle 1939. But Gill also thinks that in the second half of the dialogue change and rest must be categorical kinds, since many arguments depend on them being contradictory.

motion is the basis for the regular revolution of the heavenly bodies. And Motion and Rest – core concepts for any natural philosophy – are essential, given that the *Timaeus* attempts to show the intelligibility of motion. In this way the quintet will also be *megista* in the *Timaeus*.

Plato, like Parmenides, treats Motion and Rest as contradictory opposites – everything must either be at rest or in motion.[99] As contradictories, Motion and Rest do not mix with each other (i.e., no meaningful statement can be derived from attempting to blend them, let alone a true one. Being, Sameness, and Difference, by contrast, blend with both sides of this contradictory opposite, and we get meaningful and potentially true statements out of these connections.

5.2.4 Possible Answers to Parmenides' Problems

If we now look back at the four major problems for natural philosophy that we saw Parmenides' logical apparatus leading to in the second chapter, we can summarise Plato's logical achievements as follows.

(1) Parmenides' logical apparatus leads to a restriction of the logical universe in such a way that time, space, and motion cannot be developed so as to count among consistent notions and there are not enough basic concepts for natural philosophy available. A major reason for this is that, on Parmenides' account, it is unclear how to guarantee that another basic notion besides Being would not simply be identical to Being, but yet different from it without thus 'being' Non-Being. In the *Sophist* Plato develops the logical operators in such a way that other concepts besides Being can be introduced as consistent with Being; and in the *Timaeus* we will see how these operators also allow him to develop notions of time, space, and motion.

(2) Parmenides' philosophy does not allow for a systematic differentiation between operators and operands; he cannot distinguish between the basic entity or thing that is, Being, and the operator 'is', that is, the idea that something can be ascribed to it; or between the 'entity' 'Non-Being', and the separation operator 'is not'. The atomists had already made this distinction implicitly when they claimed that "Being is no more than non-Being": Being, according to the atomists, has no more entitlement to be said to be than non-Being, which is to say that both Being and non-Being are. But in order to make sense of this claim, there must be a clear distinction between the operator 'is' and the entity 'Being'.[100] Again Plato makes explicit a distinction that is already

[99] We will see that only Aristotle changes this; for him things like justice are neither at rest nor changing. Accordingly, not everything is either in motion or at rest; motion and rest turn out to be contraries.

[100] Similarly, a possible 'entity' non-Being must be distinguished from the difference operator 'is not'.

systematically used by the atomists when he distinguishes between the Form of Being and participating in Being in the *Sophist*.[101]

However, while the *Sophist* takes important steps to distinguish operands and operators explicitly, sometimes we find that they are still run together by Plato. For example, this distinction seems to be lacking when in 256d the Eleatic Visitor infers from the claim that Motion is not Being itself that accordingly non-Being exists – so from the fact that Motion is distinguished from the entity (the Form) 'Being', the 'existence' of the entity 'non-Being' is inferred; a mere difference between two things seems to be turned into an entity. Similarly, in 256e it is claimed that because the nature of the Different makes every Form different from the Form Being, it turns them all into non-Beings. Plato uses these inferences here in order to introduce a notion of non-Being that is not the polar opposite of Being; but the way he does it is by not clearly distinguishing operators and operands. And when Plato first shows that we cannot talk about non-Being in 237–8, he switches without argument from the idea that 'non-Being cannot be applied to Being' to 'Being cannot be applied to non-Being', a switch that mixes up operators and operands.[102]

(3) We have seen that in Parmenides the meaning and status of less basic concepts, like wholeness, are never clarified – a point which Plato shows to be problematic. The notion of a whole which Plato first takes up is that of a well-rounded sphere, which Parmenides had in fact only introduced as an analogy for the complete homogeneity of Being. If Being were indeed a whole in the way that a sphere is, it would have a middle and an end and thus parts, as Plato points out (244e). For Plato, this shows that it would not be what it means to be a whole, it would only have the characteristic of being a whole, and thus "all things will be more than one" (for there is, on the one hand, the whole and, on the other, that which has the characteristic of being whole, similarly to there being the colour purple itself, which is what it means to be purple, and then also that which has the colour purple, that which only has the characteristic of being purple); hence there would be a plurality. If, to avoid this consequence, we wanted to claim that Being does not have the property of a whole, but it is the whole itself, then, according to 245c1–3, it is not Being itself (but rather the whole itself) and then it is non-Being, and so Being is lacking itself (ἐνδεὲς τὸ

[101] See, for example, 259a. This is a distinction we already find in earlier works of Plato, but it is in the *Sophist* that it is fully spelt out. This distinction is lacking also in the late learners, for if they clearly distinguished between operators and entities, they would have no reason to assume that if we connect one thing to many different names, this pluralises the one thing and makes it many.

[102] From the statement "'Being' is the operand to which the operator 'is not' cannot be applied", it is inferred that the operator 'is' cannot be applied to the operand 'non-Being'. However, this last case is given in an aporetic context, thus the switch may be deliberate.

ὂν ἑαυτοῦ, 245c2).[103] And, finally, if there is no whole at all, then there is neither being nor becoming of what is, since whatever becomes, becomes something, some whole thing (244d–245d).

(4) Finally, we have seen that Parmenides' logical apparatus leads to the exclusion of all complexity and relation. But every process involves the relation between different times (like its beginning and end), as well as different places or conditions, so that time, space, motion, and change must be excluded from the realm of philosophy. That the change of operators in the *Sophist* allows for relations and internal complexity becomes clear in the *megista genê* inquiry, where the relationships of five greatest kinds are investigated. And time, space, motion, and change are further analysed in Plato's *Timaeus*.

You may want to object, however, that even if Plato develops the logical operators and the principle of non-contradiction further in the *Sophist*, he does so to trace relations between the *megista gene*. Why should this carry over to Plato's cosmology? While clarifying Being, negation, identity, and the principle of non-contradiction is important for dialectic and the method of division, there may be no special need to apply this more precise logic to the sensible world. However, not only do the basic structures developed in the *Sophist* apply to all fields, and that means also to the field of natural philosophy, we will see in the following that in the *Timaeus* Plato wants to demonstrate that the sensible world is much more intelligible than some devoted readers of the *Republic* may have thought.

If Being and non-Being can be brought together in a conceivable way when investigating Forms, their connection need no longer be a problem when investigating sensible things and their processes either. Conceiving of Being and non-Being together was a problem that sensible things raised according to the *Republic*; it was the reason for their reduced ontological status. A sensible thing may be F at one time, though not at another, or F in one respect but not in another. However, even then, the problems raised by sensible things were less severe then those raised by motion. For with sensible things, we can concentrate on the time or respect in which they are F, and so cling to them being F. Thus, at least in one respect, a sensible thing may be understood as being F. With motion, however, if we just cling to one time or one point in space, we do not yet have motion. The very essence of motion and change is to be a relation between one time and another, first being F and then not, first being here and then there. Thus we can understand how processes intensify the

[103] This account follows White's translation. By contrast, Cornford translates this tricky passage as claiming that if being does not have the property of being whole, but if wholeness itself exists, then Being falls short of itself, since there is some thing (wholeness) which yet it does not include; Rowe's translation is similar to Cornford's. For our purposes, both ways of reconstructing the argument are in principle possible. I follow White here, since his reconstruction seems to fit the Greek better, though in many other places Cornford and Rowe are closer to the Greek text.

problem of bringing together Being and non-Being. Conceiving of Being and non-Being together is not only a deficiency with regard to motion, as it seems to be with sensible things in the *Republic*, but the very essence of motion. In the *Sophist* we are shown, then, that what necessarily holds true of Motion – that it is and is not[104] – turns out to hold true of all Forms. They all are and are not the Same.[105] Rather than being a reason for excluding Motion from the realm of philosophical investigation, this is a feature that can be shown to be true of all Forms.

Given that processes essentially involve the relation between different times, as well as different places or conditions, the understanding of negation developed in the *Sophist* is a precondition for making processes intelligible: it renders comprehensible changes like x is here at t_1 but not here at t_2, or x is F at t_1 but not F at t_2, without getting into inconsistencies. And given that the *Sophist* shows that Being and Non-Being can be brought together with the Forms that are the main object of thought, it is clear that this connection is indeed intelligible in principle.

We have seen how the *Sophist* changes the logical possibilities and prepares the logical ground for developing the criteria for philosophy further and for conceiving Being and non-Being as combined. When we now switch back to the *Timaeus*, we will see how Plato translates this logical basis into the physical sphere,[106] how he develops the criteria we have not yet looked at and thus complements the *Sophist*, and how, finally, in order to give as ‘likely’ an account of the physical world as possible, he employs mathematical structures. The current chapter will only look at the logical advances of the *Timaeus* that are relevant for our project, while Plato’s introduction of mathematical structures into the field of natural philosophy will be looked at in the following chapter. There we will also discuss the degree to which the logical changes and the mathematical structures allow Plato to give a consistent account of motion.

5.3 The *Timaeus*: Logical Advances

We saw in the preceding section how the *Sophist* prepares the logical grounds for a new understanding of the separation and connection operators, as well as of the principle of non-contradiction. And we saw before

104 We saw above that the way Motion is and is not in the *Sophist* is different from the way a moving object is here and then not here, but nevertheless the *Sophist* can be read as offering a way in which one form of Being and non-Being can be connected that could then pave the way for understanding physical motion.

105 Apart from the Form ‘the Same’ itself.

106 He does so in immediate (for example, when understanding the World Soul as made up of Being, Sameness, and Difference) and less immediate ways.

that within the *Timaeus*, and hence within natural philosophy, not all parts should be expected to be fully consistent, accurate, and rationally admissible. It has been suggested in the literature that we should understand the reduced degree of consistency to mean that while there may not be consistency across different blocks of the cosmology, there should at least be consistency within each individual block of investigation. However, it is not entirely clear what constitutes such a block,[107] and the example of the role of female human beings and of animals may also show inconsistencies within a single block (whatever constitutes a block). Accordingly, it seems to be more likely that the reduced degree of consistency does not necessarily apply to the relationship between different blocks, but rather to those theorems which are less essential to the overall account – if the essential bits were inconsistent, it is hard to see how Timaeus' account could be "no less likely than any other", as he claims in 29c. This understanding of the restriction of consistency is also supported by Timaeus' treatment of colour. Colour is a part of the natural world of which we cannot even give an *eikôs mythos*: we can give neither a necessary nor an appropriate account (μήτε τινὰ ἀνάγκην μήτε τὸν εἰκότα λόγον) of the exact proportions needed for the production of mixed colours (68b6–8) without this inability undermining Timaeus' general account of the cosmos. It seems then that the demands for consistency are more rigorous on the more basic level (for example, concerning the fact that we need rational and necessary causes), less so on the less basic level.

What about the other criteria for philosophy? Rational admissibility and the principle of sufficient reason also turn out to be criteria for a far more extensive realm than in Parmenides. When we look in more detail now at the ways in which Plato developed the understanding of these two principles in the *Timaeus*, we will see that they are crucial criteria for what counts as the standard to which Plato thinks natural philosophy should be held, for an *eikôs mythos*.

5.3.1 The Reinterpretation of the Criteria for Philosophy 2: The Principle of Sufficient Reason and Rational Admissibility

Timaeus starts out his account of the natural world with a basic metaphysical distinction familiar from other works, like the *Republic*: he distinguishes what always is and has no share in becoming, which is apprehensible by thought, from that which undergoes a constant process of becoming and never strictly

[107] Are these different blocks taken over from other thinkers, or from the pre-theoretical realm? Are we thinking about vertical blocks (for example, different blocks for the details of cosmology within the reason part) and horizontal blocks (for example, the part to do with the works of reason, on the one hand, and those dealing with the works of necessity on the other)?

speaking is, which is an object of belief and sensation.[108] Timaeus then proceeds in a less familiar way: what has come into being needs a cause, some reason why it came into being. This cause is understood as a maker,[109] a maker who uses a model according to which he forms his product: either he looks at a model that is itself unchanging, in which case he will accomplish something good, or he looks at a model that has itself come into being, in which case the result will not be good (27d–28b).

In a second step, this basic metaphysical clarification is applied to the realm of the cosmos: the world itself is sensible (it can be seen and touched, and it has a body) and thus must have come into being (28b–c). Subsequently, we encounter a straightforward *modus ponens* inference: if the world is good (and its maker is good) then it must have been made based on an unchanging and eternal model. The world is good. Hence, the world was made according to an unchanging and eternal model. But if it was made according to such a model, per the next step in the argument, it must be a likeness of it. It is such a likeness that we investigate when we do natural philosophy.

Already this very beginning of Plato's cosmology shows us that the principle of sufficient reason and rational admissibility play crucial roles. As for the principle of sufficient reason, Timaeus' account of the universe makes it clear that we should not assume any old cause for its coming into being, but rather a good cause (29a),[110] which in the course of the investigation seems to prove to be the best possible – given the supreme excellence of the effect, that is, the excellence of the cosmos, this is what we must assume. We are told that everything that has come into being must have a cause – for why would it have come into being otherwise? And this cause needs to be a good, ultimately the best possible, and thus a sufficient reason.

We see that Plato is not only working with the principle of sufficient reason in the realm of the generated cosmos, but that he also interprets it in an axiological way: a sufficient reason is not just adequate for explaining (for example, by assuming that much power in the cause to account for that much effect), but it must be good. It reminds us of the *Phaedo*, where Socrates gets excited about Anaxagoras' *nous* because it promises not only some explanation

108 Plato's use of language has become even more radical in his late works: becoming is now described as what *always* (*aei*) becomes and never is, so that the present tense of the verb being is never correctly applied to it (27d–28a). Proclus, Simplicius, and manuscripts F, W, and Y omit *aei* from 28a1, but it seems to me to fit the later claim in 38b1–2 that it is wrong to state that what is coming to be *is* coming to be; thus I follow manuscripts A and P, Eusebius, Rivaud, and Burnet in reading *aei*.

109 Cf. also Betegh 2010, p. 222 for this immediate translation of the cause into a personal agent.

110 Goodness here seems to indicate metaphysical as well as moral goodness. The cause is good in the sense of being rational, and the demiurge acts towards a morally good end.

of the world, but one that shows its goodness, that explains why this set-up of the world is best the way it is. While Anaxagoras' writing ultimately disappoints Socrates' hopes, we see that goodness in the *Phaedo*, as in the *Timaeus*, is a principle of explanation for nature. Something is only a sufficient reason for the natural realm if it shows that the world is better this way, that it contributes to making the world most excellent (for example, the assumption of the demiurge not only explains how the sensible cosmos can have been brought into existence, but also how the world thus turned out most excellently).[111]

As we saw in the second chapter, Parmenides assumes that there could be generation and corruption of Being only if there were a sufficient reason for it, and he claims that there is no sufficient reason to assume generation (or corruption) for what is. Plato agrees that there is no generation or corruption of what truly is, of the intelligible entities. But he goes further in claiming that also within the realm of *doxa* assuming generation or corruption requires a sufficient reason,[112] since the realm of *doxa* also fulfils the basic criteria of intelligibility. Any generation needs a cause that explains the effect which has come into being. The universe as a whole is such a thing that has come into being and for which we must thus find the cause.

Parmenides points out two questions to which a sufficient reason for generation must answer: first it must answer whether what came into being came into being from what is or from what is not (and how one or the other is possible without getting into inconsistencies). Secondly, it must answer why what came into being came into being now and not at some other time. The first question, Plato thinks, can be answered by showing that the universe came into being not from what is not (Plato agrees with Parmenides that there cannot be a creation *ex nihilo*) but from what is. At least it comes into being from what is in some way, from what is originally in a completely disorderly way, so that what actually comes into being is order and measure among the things that exist from the very beginning (53a). The second problem has a twofold answer: (1) there is a divine maker, who may have his reasons, and (2) the universe comes into being 'before' the introduction of time. No matter whether we take the latter claim literally or metaphorically, it is clear that the creation of the universe

111 Plato sometimes also uses goodness to help us decide between two possible assumptions: for example, whether there is one or several worlds.

112 28a4–6: πᾶν δὲ αὖ τὸ γιγνόμενον ὑπ' αἰτίου τινὸς ἐξ ἀνάγκης γίγνεσθαι· παντὶ γὰρ ἀδύνατον χωρὶς αἰτίου γένεσιν σχεῖν ("Again, everything that comes into being must come into being through some cause/reason, for it is impossible for something to come to be without any reason"). *Aition* in Plato is sometimes better translated as 'cause', sometimes as 'reason'. Plato uses both *to aition* and *hê aitia*. See Sattler 2018a.

as a whole is not the kind of thing where it makes sense to ask why the universe was created now rather than at some other time.

Furthermore, Plato not only shows that the two questions can indeed be dealt with in the case of the generation of the universe. He also develops the notion of a sufficient reason further, by showing in the *Timaeus* that for natural philosophy we have to distinguish between two kinds of *aitia*: necessary and rational ones. By rational causes of the universe he seems to understand the model, the Forms,[113] and the demiurge. The model is the teleological cause for the cosmos as a whole, the Forms are the reason for something being what it is (fire, for example, is what it is because it partakes in the Form of fire), and the demiurge, bestowing order onto what is given to him, is responsible for things being accessible to our understanding and reasoning. By 'necessary reasons', on the other hand, Plato seems to understand what is related to the stuff with which the cosmos is formed.[114] The necessary causes include the receptacle, the traces of the atomistic triangles, and the bodies formed out of them, and they explain why things change, move, and can be located. Thus Plato shows that for natural philosophy we need to be more specific what kinds of causes are required to explain the sensible realm: we need not only rational causes, as seemed to be sufficient for the kind of metaphysics Parmenides established, but also necessary ones.

Let us now move on to rational admissibility. According to Parmenides, cosmology is done by mortals, who simply *decide* to *name* and *posit* basic kinds and principles.[115] By contrast, on Plato's account, people investigating the natural realm can do what Parmenides' goddess did for the realm of *alêtheia*: give an account that is based on rational analysis and give a reason for what is posited. In fact, Timaeus seems to echo the introduction of the criterion of rational admissibility in Parmenides' poem (fr. 7), where the goddess asks us to prove with (our own) reason what she has spoken, when Timaeus claims at the beginning of his discourse that not only must we invoke the gods (as the preceding theogenies had), but also call on our own

[113] It is not clear exactly how the model and the Forms are related to each other and different interpretations have been proposed. The main point here is that Forms and the model are part of the rational causes, however we conceive of their relationship.

[114] Cornford, Johansen 2004, and others have pointed out this understanding of necessity in Plato. It is an understanding of necessity that goes back to a usage we also find in Parmenides' fr. 10, as we saw in Chapter 2. In the *Timaeus* these causes are necessary in the sense that the demiurge must take into account the limitations imposed by what is given to him as basic 'material' from which to form the cosmos, as a carpenter can only produce a good piece if she takes into account the characteristics of the wood she is using. The wood clearly makes a different contribution to the finished desk than the carpenter does. Accordingly, Plato assumes there are two different kinds of cause. As the craftsman gets the material ultimately to manifest his ideas, so reason *persuades* necessity to follow its idea (48a).

[115] See fr. 8, lines 40–1 and 53–4.

nous.[116] As in Parmenides' poem, we get the idea that divine and human reasoning are akin – we can understand what the demiurge is doing (at least to some degree) and that it is the best thing to do. Furthermore, we have seen that the world is made in the likeness of an unchanging and eternal model, and that means in the likeness of something that is accessible to reason. This idea already implies that rational admissibility is an important criterion for our investigation of the world for thus the world itself should also show at least some trace of accessibility to reason. And the fact that Timaeus' *logos* about the world is seen as a *logos* which is only refutable by reason, and that Plato even encourages others to improve this *logos*, makes it fully clear that rational admissibility is an important criterion. For these improvements or refutations are envisaged not in terms of getting more or better perceptible data, but in terms of better arguments, as Plato's examples show: we might give a better account of the most basic, indivisible elements by demonstrating them not to be triangles, but something even simpler, lines perhaps, or points – this is not something which, according to the *Timaeus*, we could in any way perceive. It is by rational analysis that we are meant to show which account is better and should be given authority (the one starting with triangles or the one starting with lines). The standard for this judgement is reason; it trumps any other authority; and it can be generalised so that we should all agree to this judgement.

But while it is by rational analysis that we should assess the strength of different accounts of the world, these accounts must also 'save the phenomena'. Plato takes up the criterion of rational admissibility in the way we saw it developed with the atomists: the basic ontological constituents must not only be testable by our own reason, but also explain the phenomena that can be explained in terms of what truly is. Plato shares with Parmenides and Zeno the idea that the mere fact that something is reported by our senses is not in itself a criterion for a good account of the world. But, contrary to the Eleatics, Plato does not simply refuse sense experiences any authority; rather, he shows how they can be traced back to something that is rationally admissible. For example, the apparently strange and irrational motions of the outer planets can be traced back to the combination of two rational circular motions – what he calls the motions of the Same and the Different.

5.3.2 An Eikôs Mythos

We have seen that Plato requires all the criteria for philosophy discussed so far to hold for the account of the cosmos given in the *Timaeus*: in claiming that

[116] ἀνάγκη θεούς τε καὶ θεὰς ἐπικαλουμένους εὔχεσθαι πάντα κατὰ νοῦν ἐκείνοις μὲν μάλιστα, ἑπομένως δὲ ἡμῖν εἰπεῖν (27c6–d1): "It is necessary that having invoked the gods and goddesses, we pray to them and that we explain the whole most of all in accordance with their *nous*, but also according to *ours*."

everything that came into being, and thus also the cosmos as a whole, needs a cause, he is demanding that the principle of sufficient reason holds. Furthermore, in the quotation at the very beginning of this chapter, Plato claims that we should not expect consistency in all respects for this account, which implies that we should at least expect consistency in some respects. And, finally, Plato employs rational admissibility as a criterion for the way in which he thinks we should improve on the account in the *Timaeus*. Giving an *eikôs mythos* of the cosmos thus means meeting these criteria to some extent.[117]

How exactly we should translate *eikôs mythos* is still difficult to say: *mythos* can mean a mere story in Plato, clearly so when combined with talk of children[118] or when contrasted with *logos*; it can then also be connected with falsehood.[119] But it can also mean a story illuminating the nature of something,[120] or a story in the sense of a serious supposition about an area in which we cannot have secure knowledge. It is used in this sense in the *Phaedo* when Socrates calls his account of what happens to human souls after death a *kalos mythos* (110b1) – a *mythos* is here the best kind of explanation we can offer, but it is, we are told, definitively worth giving it. This sense of *mythos* seems to be close to the one we find also in the *Timaeus* – where we are given a serious supposition about the natural world as a whole, about the sensible things which come into being, change, move, and pass away. About this realm we cannot have knowledge in the strict sense, that is, in the sense we can know about metaphysical fundamentals, but it is definitively worth trying to give a serious account. There is no connection between *mythos* and falsity in our context. And it is not just one part of the dialogue that is referred to as *mythos*, as in the *Republic* or the *Phaedo*, where the final section is called *mythos* and clearly contrasted to what has gone before, but rather the whole of Timaeus' speech.

117 Thus, Timaeus' cosmology also has a relation to the account of the intelligible world, which should meet these criteria without any restriction; and it fits Bryan's 2012 understanding of the *eikôs mythos* as "describing the cosmos as a likeness of the intelligible" (p. 148, cf. also p. 173). In 56b we find the seemingly strange wording that of the shapes constructed, the pyramid is the element and seed of fire according to the *right and likely logos* (*kata ton orthon logon kai ton eikota*). I take it that we get both a right and likely *logos* here, since we are dealing with the mathematisation of the physical world: the mathematical construction of the five Platonic bodies, and the pyramid among it, is *orthos*, while the connection to the particular element fire is *eikôn*. Cf. Gregory 2000, pp. 250–1.

118 So in *Sophist* 242c8, where the earlier philosophers are accused of having told (mere) stories about Being, as one does to children, or in the beginning of the *Timaeus*, where the Egyptian priest characterises the Greeks' description of their history as children's stories (παίδων μύθων, 23b5).

119 So in *Cratylos* 408c, where we hear that it is in the tragic realm that we find the majority of myths (*mythoi*) and falsehoods (*pseudê*).

120 The term *mythos* can also be used for a general rule set up by human beings: in *Laws* 773b we are given a *mythos* in the sense of a general rule about marriage.

Burnyeat suggests translating *mythos* as "story", but since this could be mistaken for a mere narrative, in the way that Critias tells a story,[121] I would prefer to stick with "account", or possibly "supposition".

The word *eikôs* is even more difficult to translate satisfactorily. There are at least two problems: first, standard translations of *eikôs*, such as 'likely' or 'probable', assume a modern framework which does not fit the Platonic setting. They assume either a probabilistic framework or a framework in which we can gradually get closer to the truth. The latter supposes that we can have an approximation to the truth, which we simply do not fully know yet, and the truth in this case is the fact of the matter. But for Plato there is no full truth in this sense concerning the sensible world, and accordingly Timaeus is not asking us to improve his account with more or better empirical data. So *eikôs* cannot mean aptness to truth in this sense.[122] A probabilistic framework in a modern sense assumes that we deal with all possible cases and then work out the probability of one individual case holding – not something we should expect in the ancient context. Secondly, Plato's usage of *eikôs* plays with the closeness to its cognate *eikôn*, 'image' or 'likeness', since the world as a whole is understood as an image of an eternal model.[123] So an *eikôs mythos* is meant to be an account that is appropriate for an *eikôn*, an image.[124]

These two considerations make it difficult, if not impossible, to find an adequate translation. I suggest we either simply use the Greek, or translate it as an 'appropriate account'[125] or a 'reasonable supposition'. More

[121] According to Broadie 2012, stories are just told over and over again, and audiences are meant to enter into them, while Timaeus' cosmology invites us to look at it critically, make suggestions for improvement, and judge it against what we know from the outside world.

[122] See also 59c and 68d.

[123] Osborne 1996 suggests that the account of the *eikôn* is done not by looking at the *eikôn* and describing it (third removed from the truth) but by looking at the model and creating an *eikôn* of it in words (second removed and thus on the same level of the hierarchy as the world); thus the description is an *eikôn* as is the world and the two match as two photos of the same object match. Osborne thus denies that the description is a description of that *eikôn*. While it is true that Timaeus suggests improving his account by reasoning rather than by looking at the world, Osborne's idea seems untenable to me. For an account that describes the model would be an account of something unchanging, and thus not an *eikôn* account, but an irrefutable one, while Timaeus explicitly deals with a discourse that is refutable (otherwise, there could be no stable and irrefutable accounts, which Timaeus clearly assumes when talking about the ratio of Being to becoming being the same as that of an account of Being to an account of Becoming).

[124] We should also bear in mind that, while some explanations in Timaeus' speech are only meant to give a possible account of the sensible world, others are meant to explain how it is possible that something is necessary; for example, Plato's account of the four elements is a possible explanation of why there are necessarily four elements.

[125] I do not understand 'appropriate' here in the sense it has often been taken to have in Parmenides' poem, as meaning appropriate to the kinds of cosmologies that mortals (wrongly) come up with, but rather as being fitting to its subject matter (which is our best

important than finding a translation for these words is to bear in mind which criteria such an *eikôs mythos* is meant to fulfil. These are the criteria for philosophy we have found in the tradition so far – in the particular form Plato gave them and only to some degree: with respect to the principle of sufficient reason, Plato not only assumes that there must be a sufficient reason for everything that has come into being, but also that we need to distinguish different causes for it, necessary and rational ones, since merely rational causes cannot explain the cosmos sufficiently. And since we are trying to give the best possible account of the cosmos, our natural philosophy must give a reason for its coming into being that suffices to explain the excellence of the cosmos – only a best possible reason will be sufficient in this case. But phenomena on a smaller scale, too, parts of what is going on in the cosmos, seem to be explained by sufficiently good reasons. The very fact the Plato explicitly employs such reasoning to hold for the realm of *doxa* shows that he thinks that the sensible realm is also in part intelligible.

The principle of non-contradiction, as employed by Plato, allows for respect. But it is not fully in play on each level of the cosmology, as Timaeus explicitly warns us at the outset. We saw in the first chapter that consistency can be understood as operative in at least three different ways: (1) the specific content of a concept (our account of something) is consistent, (2) the usage of concepts is consistent, (3) or a system of concepts is consistent (i.e., the concepts fit together in such a way that no implication of one concept is inconsistent with the implication of the others). We noted in the second chapter that Parmenides attempts to satisfy all three kinds of consistency with his ontology. We can now say that natural philosophy, for Plato, does not meet the third level of consistency – while it is still at work on the level of Forms, as we saw in the *Sophist*, with respect to the sensible realm not all the different *logoi* agree with each other in every respect. Furthermore, we saw that consistency is only required in the essential parts of the cosmology, so some, less important, concepts may also not fulfil point (1).

Finally, we have seen that the *eikôs mythos* is meant to meet the criterion for rational admissibility in a way the atomists employed it (i.e., such that the phenomena can also be saved). In addition, Plato goes a step further than the atomists: saving the phenomena in the *Timaeus* not only means, as the atomists assumed, that what can be experienced by our senses can be explained in terms of something more fundamental that meets this criterion (in terms of atoms and void for the atomists, and of the intelligible model for Plato): the phenomena themselves acquire their own

and most beautiful world) – and accounts of this subject matter should live up to certain (though not the strictest) scientific standards.

intelligibility and thus rational admissibility in so far as they are mathematically structured.[126]

While the logical advances we have seen so far allow Plato to prepare the basis for giving an account of motion, it is the introduction of mathematical notions into natural philosophy that allows Plato to show in detail the intelligibility of motion (to the degree that he considers it to be intelligible). It is to these mathematical notions that we will turn in the next chapter.

126 By contrast, for the atomists the phenomena themselves, taken up as they appear to us, would not meet the criterion of rational admissibility, though the explanation of them in terms of atoms does.

6

The Possibility of Natural Philosophy According to Plato II: Mathematical Advances and Ultimate Problems

6.1 Introduction

We saw in the last chapter that Plato's *Sophist* prepares a new account of separation and connection as well as a new understanding of the law of non-contradiction, and that the *Timaeus* further develops rational admissibility and the principle of sufficient reason. These changes introduced by Plato are necessary conditions for understanding natural philosophy as a proper scientific endeavour. For Parmenides, criteria for inquiry and the logical basis restrict rigorous philosophy to metaphysics. The atomists extend both – and thus also the ontology of the Eleatics – *implicitly* in a way that also allows for integrating their investigation of nature. But it is only with Plato that we can see how the logical basis and the criteria for philosophy are *explicitly* developed in a way that can begin to answer Parmenides's challenge, maintaining both the new rigour introduced by Parmenides's criteria for philosophy and the realm of motion and change as an object of philosophical investigation.

Plato's understanding of the logical operators allows for making complexity and relations intelligible: if x is not F, all this means, according to Plato's new understanding of negation, is that x is different from F; so x can still *be* and even *be F* in some way. As every motion involves a relation between different times (like the times at which movement begins and ends), as well as different places or conditions, such an understanding of negation is a precondition for making processes intelligible: it allows for conceptualising motion and change as 'x is here first, but not here later' or 'x is F at t_1 but not F at t_2' without getting into inconsistencies. Later in this chapter, we will see how Plato develops an account of time, space, motion, and change in the *Timaeus* that rests on these logical developments. Given that the *Sophist* showed that Being and non-Being can be combined when we talk about the Forms, which are the main objects of thought, it seems clear that connecting Being and non-Being, which is necessary for understanding motion, does not necessarily lead to inconsistencies, but can in principle be intelligible.

Developing the logical basis, however, allows for conceiving all kinds of complex objects consistently and for bringing Being and non-Being together and is thus not specific enough to show that *motion* can indeed be conceptualised so as to satisfy the criteria for rigorous philosophy. We have no problem with spatial and temporal differences qua differences, as we saw with Parmenides. Yet accounting for differences and complexity is not specific to motion. It is also required if, for example, we want to give an account of something that is partly green and partly blue: the part that is blue is not green but nevertheless is and vice versa.

What is specific to motion and change, however, is that we can understand and measure them in terms of time and a second magnitude, that is, in terms of time and spatial magnitude in the case of locomotion and growth, or in terms of time and some quality in the case of change. Measuring them means connecting them to numbers, and it is not until mathematical notions are incorporated into the description of natural phenomena in addition to the logical developments discussed previously that we come close to having a real foundation for natural science adequate to explain the phenomena of time, space, and motion. This connection of mathematics and logic in accounting for natural philosophy happens in Plato's *Timaeus*.[1] We will see in this chapter how something we take for granted in modern science and as crucial for its success, namely that it employs mathematical structures, starts to be systematically developed with Plato.[2]

6.2 Introducing Mathematical Structures

Taking mathematical structures as crucial for an account of the physical realm is an approach that is not completely new with Plato. Such a 'mathematical approach' to nature is already adopted by the Pythagoreans, and Plato builds on some of their insights. While we can say that Plato takes up a Pythagorean approach very broadly speaking,[3] there are also important differences between the Pythagoreans and Plato: perhaps most importantly, the Pythagoreans do

[1] It seems to me that scholars seriously underestimate the function of mathematics when they assert that for Plato the world is intelligible *only* because it is made after an intelligible model.

[2] The combination of mathematics and physics also made the *Timaeus* attractive to the sciences in early modern times. Cf. Martens 2003. Johannes Kepler uses the *Timaeus* to justify his own merging of mathematics and physics in his astronomy, which is a central point in his *Astronomia nova*.

[3] The figure of Timaeus, who presents the cosmology in Plato's work, has often been seen as a Pythagorean; see, for example, Taylor, p. ix. Locri, the place Timaeus allegedly comes from, was known in ancient times for its Pythagorean community, and when the pseudo-Pythagorean treatise "On the Soul of the World and Nature" was plagiarised in the first century CE and claimed to be the paradigm of Plato's dialogue, it was never disputed that Plato's *Timaeus* seemed influenced by the Pythagoreans.

not connect their mathematical approach to logical developments of the sort we see Plato engaged with. And the mathematical structures crucial for Plato are not just arithmetical, as with the Pythagoreans, but also geometrical. Furthermore, with the Pythagoreans, mathematics usually seems to provide the guiding principles for their account of the natural world. For example, it is due to the mathematical prominence they ascribe to the number ten that the Pythagoreans assume a counter-earth – a heavenly body that we cannot perceive because our earth and the counter-earth are supposed to be on opposite sides of the central fire. The assumption of the counter-earth allows the Pythagoreans to posit ten heavenly bodies,[4] but there is nothing in the physical world that supports this postulation. By contrast, Plato's assumption of seven heavenly bodies fits the number of heavenly bodies that are not fixed stars and can be observed with the naked eye.[5] Mathematics leads Plato not to introduce new entities, but only to show that what we perceive as chaotic or irregular, as for example, the motions of the planets, can be understood as the combination of regular processes, which allows us to save the phenomena to some degree.[6] Nevertheless, we will see that mathematical principles also sometimes determine assumptions in the *Timaeus* without support from the physical realm.

Finally, the Pythagoreans, at least on Aristotle's' interpretation,[7] make basic mathematical elements and structures, like proportions and numbers, the very essence of sensible objects,[8] while for Plato mathematical structures express

[4] The five inner planets known in ancient times, plus sun, earth, moon, the central fire, and the counter-earth.

[5] While in many respects Plato is a rationalist in natural philosophy, too, the *Timaeus* nevertheless takes up important observable data. Cf. Vlastos 1975c, p. 50.

[6] Plato's basic triangles constituting the four elements may seem a counterexample to this claim, since they are also entities that we cannot perceive introduced into the physical world for theoretical reasons. However, we cannot perceive these triangles because they are too small for our sense organs, and thus this introduction is more reminiscent of atomistic theories than of the Pythagorean counter-earth, which cannot be perceived because of where it is situated. The Pythagorean case is, nevertheless, not singular, if we think of the nineteenth-century arguments for an undiscovered planet.

[7] Burkert 1972 distinguishes two traditions of the reception of Pythagoreanism. The first begins in Plato's Academy and treats Pythagoreanism as largely identical with later Platonic thought. The second, represented by Aristotle, claims that fifth-century Pythagoreanism had some influence on Plato but was radically different from Plato by not distinguishing between the intelligible and sensible realms.

[8] For a weaker understanding of the Pythagorean claim, see Huffman, p. 204 ff., who proposes that at least Philolaos asserts that things "give signs of" or "point to" numbers rather than that they are identical with numbers. However, Huffman agrees with Aristotle that for the Pythagoreans mathematical structures are essential for what is, although for Philolaos it is limiter and limited that are the most basic mathematical structures, rather than numbers. Numbers play a metaphysical role in defining the essence of things, but they are not the material out of which something is made. Cf. Schibli 1996 for a defence of Aristotle's assertion as historically accurate.

the intelligible way in which the cosmos is set up without being the essence of natural things. Part of the reason for this change seems to have been that the Pythagorean assumption is problematic for setting up a physics: as Aristotle points out,[9] starting with mathematica as the very essence of the physical world leaves us badly suited for explaining the natural world, and seems especially inappropriate to explain locomotion. Avoiding the stronger assumption that mathematica are the very essence of physical things, Plato tries to take up solely the basic insight of Philolaos and the Pythagoreans that if something possesses number[10] or, more generally speaking, mathematical structure, we can know it, for if we can express something in mathematical terms, it must have some intelligible structure and hence can be known. In this way Plato presents a set-up of the universe where numbers and geometrical figures *indicate* the rational set-up of the universe without thus creating a problem for natural philosophy: motion is understood not to be number in essence, but rather to proceed according to number (i.e., number proves motion's intelligibility). Numbers do not constitute physical things. Mathematical structures do not make things what they are,[11] but rather guarantee that these things are indeed possible objects of knowledge.[12]

How, then, does Plato use mathematical structures to demonstrate the intelligibility of the world, and which mathematical structures does he use? The most important mathematical notions that Plato imports into his natural philosophy are the number series, proportions, and geometrical forms. These mathematical notions allow him to conceptualise features of the natural world that may otherwise have seemed unintelligible.

Proportions secure the unity of the World Body and determine the order according to which the orbits of the heavens are established; geometrical forms determine the basic building blocks of the appearances (such as fire, earth, etc.) and thus also the spatial order of objects; and the number series is what allows us to give an account of the motions of sensible objects (once the demiurge has bestowed order). These motions can be assigned to numbers and hence be measured with the help of time. Let us briefly run through the first two features

9 So in his *Metaphysics* A, 989b29 ff.

10 See Philolaos fr. 4, which reads: "All things that are known have number. For it is not possible that anything whatsoever be understood or known without this". There is a strong reading of this fragment according to which number is really constitutive of all things, which is what Plato tries to avoid. Huffman, however, reads this fragment merely as claiming that all things are wholes of parts and thus ordered pluralities.

11 As they seem to do according to Aristotle's account of the Pythagoreans: *Metaphysics* A, 5; M, 4 and 8; N, 3.

12 While the elements seem to be made up by geometrical bodies, the geometrical bodies on their own do not yet constitute them. For example, it is not enough to have a pyramid in order to have fire; rather we also need a cluster of these pyramids, pyramids with a tendency to move, and a connection to the Form of fire in order to derive fire. See Sattler 2012.

before we delve into the third with an investigation of the measure of motion in the *Timaeus*.

We will begin by looking briefly at the proportions according to which the World Body and the World Soul, and thus the orbits of the heavens, are set up.[13] Proportions guarantee the unity of the stuff of which the World Body is made, that is, the thorough connection of the elements that constitute the World Body. In addition, these proportions are one factor of the explanation of why there are exactly four elements. Fire and earth must be among the constituents of the World Body, we are told in *Timaeus* 31b, because it has come into being and thus is visible and touchable, the former requiring fire, the latter earth. But in order to unite two things, a third is needed as a bond, and the best connector is a proportion in the ratio a:b=b:c (the first should be in the same relation to the middle as the middle is to the last, according to 32a), thus a geometrical proportion. While this would suffice for a plane, given that the world is three-dimensional, we in fact need two bonds, we are told, and so we get air and water, which allow us to form a geometrical proportion between fire and earth in three dimensions: as fire is to air, so air is to water, and so water is to earth.[14]

We see that Plato here starts with what at that time seems to be taken as a physical datum, namely that there are four basic elements. He introduces geometrical proportions in order to account for the unity of the material plurality and in order to explain why there are exactly four elements. Thus mathematics is originally introduced in order to *ensure* the *intelligibility* and unity of the physical world. Later on, however, with Plato's introduction of the basic triangles, mathematics also *shapes basic assumptions* about the physical world.

Let us now turn to the World Soul, which is set up according to mathematical proportions; since the revolutions of the heavenly bodies are the visible manifestation of the motions of the World Soul, the heavenly revolutions are guaranteed to be mathematically structured. The World Soul is composed of rather peculiar ingredients: Being, Difference, and Sameness, each of which has indivisible and divisible components (35a). What is important for us is that this whole mixture is arranged according to the ratio of the powers of 2 and 3 (35b ff.) – 2 and 3 seem to be a natural starting point since they are often seen as the most basic numbers there are.[15] The arithmetical proportions we get

[13] Proportions seem to be used in two different ways in the two cases – to unify dissimilar things in case of the World Body and to divide the uniform soul stuff in the case of the World Soul. See Glenn 2011.

[14] Plato does not say in which respect this proportion holds, but presumably it is a proportion of the quantity of their respective volumes. Cf. Cornford's commentary, pp. 51–2.

[15] In the mathematics of Plato's time, 1 is for the most part seen not as a number but as a basic unit (see Euclid *Elements*, book VII, def. 1 and 2) and as containing evenness as well

through the progression of the two series, one following the powers of 2, the other the powers of 3, not only give us basic musical intervals[16] but also ensure that the two bands that are set up according to these proportions are intimately tied to a mathematical structure. These two bands are then used to describe the sphere that is our cosmos[17] and are seen to form the basis for the orbits of the heavenly bodies. We see that the set-up of the planetary orbits is thus based on a mathematical structure.

More specifically, one of the two circles, called the circle of the Same, which commentators normally understand to be situated along the Equator, and so parallel to the Tropics,[18] is undivided, while the circle of the Different, which is situated along the Ecliptic, is split up into seven circles. The fixed stars move along the circle of the Same only (40a), whereas the sun, the moon, and the five planets are set into the seven circles of the Different in such a way that each circle of the Different accommodates one of these seven heavenly bodies (38c–d). Sun, moon, and the planets move according to both circles, the circle of the Different specific to them and the circle of the Same, which affects them all, as it influences the circle of the Different as a whole (38c–39b).

So far the introduction of mathematics primarily secures the intelligibility of the physical world, without interfering with any assumptions derived from observing the sensible world. When we now look at the second crucial introduction from the realm of mathematics, the geometrical bodies underlying the four elements, we will see that the introduction of mathematics leads Plato to assumptions that come solely from mathematical considerations, not from physical considerations. But relying solely on mathematics may mislead us in our assumptions about the nature of the physical world – after all, assumptions deriving solely from mathematics can be used for, but may not be isomorphic to, the physical realm. This is a danger that we will see also with Aristotle, in the last chapter of the book.

Plato takes up the traditional 'four elements', fire, air, water, and earth, as constituting the sensible things. However, Timaeus claims that they are not the most fundamental constituents (and hence 'elements' in this sense). Rather they themselves are made up of something yet more basic: an arrangement of triangles (53d–55c), the 'atoms' of the *Timaeus*.

as oddness in the Pythagoreans (see Philolaos fr. 5). Accordingly, 2 and 3 are the first proper even and odd number, respectively; cf. also Cornford, pp. 60–1. The numbers 2 and 3 and their powers occur repeatedly in the *Timaeus* as the numbers whose relation grants the unity the demiurge establishes in his creation.

16 E.g., 4:3 which is a fourth in music. Cf. Cornford, p. 71.

17 This can be imagined as similar to the two bands of an armillary sphere. See Cornford, p. 74.

18 See, e.g., Proclus *Commentary on Plato's* Timaeus book III, 237–8; Cornford, pp. 74–86; Taylor, pp. 147–52.

In his account of the elements, Plato combines atomistic and Pythagorean features: following the Pythagoreans he understands physical bodies as based on mathematical structures, namely, geometrical bodies;[19] and like the atomists he thinks that what we perceive is not the basic building blocks – only a cluster of the basic building blocks is perceptible and what we perceive is thus quite different from the geometrical forms. In this way Plato combines a mathematical basis with the non-mathematical phenomena we perceive. Fire is derived if four basic triangles form a tetrahedron and a bunch of such tetrahedra gather together, air if the same happens with octahedra, water with isocahedra, and earth with cubes.[20] But the triangles of two tetrahedra can also join to form an octahedron, or the triangles of an icosahedron can split up into two and a half octahedra.

This geometrical basis of the four traditional elements of the natural world allows Plato to guarantee the intelligibility of the physical world on at least three levels: (1) it allows him to give an account of fire, air, water, and earth themselves in terms of a secure science, geometry – the elemental bodies are fully graspable by reason because they inherit the rationality of the triangles on which they are based; (2) it makes intelligible the transformation of the elements into each other as a transformation of one kind of geometrical body into another; and (3) it allows Plato to explain further physical features in geometrical terms: for example, the hotness of fire is explained in terms of the acuteness of the angles of the geometrical bodies underlying fire (61d ff.).

Plato connects geometrical with physical structure by looking for abstract similarities in both realms: pointedness, for example, can be seen as a feature of acute angles in mathematics and also as a feature of a piercing experience in the physical world. Referring to such similarities allows Plato to explain physical features in mathematical terms and to answer the pressing question raised by the atomistic account of Leucippus and Democritus – how can perceptible qualities arise out of an arrangement of atoms none of which possesses these qualities? For Plato, perceptible qualities that the geometrical shapes in themselves do not possess can arise out of their arrangement in the phenomena by translating mathematical features of the geometrical shapes into physical features.[21]

The way in which Plato introduces these geometrical shapes into his account of the cosmos not only ensures the intelligibility of the basic building blocks of

[19] In contrast to the Pythagoreans, it is geometrical rather than arithmetical structures that Plato takes to be basic here.

[20] For the additional requirements needed in order to get from the basic triangles to fire, earth, water, and air, see below and footnote 12 above.

[21] We may find a predecessor of this geometrical basis of the physical features in the Greek atomist explanation of different qualia in terms of the shapes and sizes of the atoms; see for example, DK68 A129. But the atomists do not seem to look for similarities between the physical and the mathematical in the way we find in the *Timaeus*.

the sensible world, however, but also leads to assumptions about the basic structure of the physical world that derive solely from geometry, not from physics: given that no triangle can serve as a basis for all four geometrical solids, Plato assumes two fundamental kinds of triangle that are not reducible to each other: a scalene right-angled triangle forms the basis for the tetrahedron, octahedron, and isocahedron, while an isosceles right-angled triangle is the building block for the cube. As a consequence, only three of the four physical elements can be transformed into each other, for only three of the basic geometrical bodies share the same basic triangle. Earth is excluded from the cycle of transformation (54b–d), not because of any physical reason or evidence but because of the underlying geometrical structure.[22] In the initial account of the physical phenomena, Plato talks about the transformation of all four elements into each other;[23] it is only once we are given a mathematical explanation of the four elements that we are told that earth has a special status – thus it seems clear that this status is due to the underlying mathematics, not to physics.

We find a similar problem of assumptions deriving solely from the mathematical realm with Plato's account of the basic geometrical solids. Plato seems to take up new geometrical research of his time here, presumably by Theaetetus,[24] to develop his physics further: this research showed that there can only be five polyhedra where all faces are congruent regular polygons and the same number of faces meet at every vertex;[25] we now call these polyhedra 'Platonic bodies' because of their employment in the *Timaeus*. Plato takes up these polyhedra as the "most perfect bodies" (53d–e) underlying the four elements. But there are *four* elements, and *five* basic geometrical bodies. Accordingly, a function must be found in the physical world for the fifth geometrical body, for the dodecahedron. It is finally assumed to be the form of the world as a whole (55c).[26] But this assumption also derives from Plato's usage of geometrical structures in the physical world; it does not arise from the physical world itself.[27] In addition, employing these basic polyhedra, because

[22] See Aristotle's criticism of this assumption in *De caelo* 306a. Cf. Zeyl, p. lxix and Cornford, p. 216.

[23] "We *believe we see* [*ôs dokoumen horômen*, 49b–c] all four elements turning into each other" – this expression of caution is meant to make the passage compatible with the later mathematical account and is not based on the observation of the physical phenomena.

[24] See Waterhouse 1972–3.

[25] See the proof in Euclid's *Elements* at the end of book XIII.

[26] Presumably because a dodecahedron is close enough in form to a sphere, as we are told in the *Phaedo*. A sphere is the form Timaeus ascribes to the world in 33b for reasons of perfection, as it "comprehends in itself all the [presumably: regular geometrical] figures there are", and is "equidistant every way from centre to extremity". Cf. Cornford, p. 54.

[27] However, in contrast to the Pythagorean assumption of a counter-earth, mathematics does not lead Plato to introduce a new physical entity. Rather, it leads Plato to separate two sets of physical phenomena (the transformation of water, air, and fire into each other,

their number is exhaustive, also raises the question for Plato whether there may be more than one world (55c–d), one corresponding presumably to each geometrical form. If the dodecahedron is the form of our world, are the other geometrical bodies each the form of some other world? Should we assume that there are five worlds rather than just one?

While physics determines the phenomena that are ultimately meant to be explained, we see that it is geometry that drives the account of the basic structure of the elements such that the phenomena must be appropriated to it to a certain degree, as when earth is excluded from the circle of transformation even though in the original account it seems to be part of the intertransformation of elements. We will encounter further problems with the introduction of mathematics into the natural realm now in Plato's account of a measure of locomotion.

6.3 Locomotion and Mathematical Structures

So far we have seen the extent to which Plato gives an account of the basic elements of the sensible world in terms of geometrical figures, and of the basic set-up of the orbits of the heavens and the unity of the World Body in terms of proportions. For his account of locomotion in the sensible world – once the demiurge has set it up rationally – Plato once again employs mathematical structures, by introducing time as a measure of (regular) motion. We saw in the section on measurement in the first chapter that a measure allows movements to be quantified and compared. For Plato, motion is quantifiable (solely) in terms of time and thus consistent and rationally admissible statements about movement are possible. Accordingly, we will start this section by looking at his definition of time. But we must first consider briefly the relationship between time and eternity, before we can turn all our attention to time as measuring motion.

6.3.1 Time and Eternity

> Ὡς δὲ κινηθὲν αὐτὸ καὶ ζῶν ἐνόησεν τῶν ἀιδίων θεῶν γεγονὸς ἄγαλμα ὁ γεννήσας πατήρ, ἠγάσθη τε καὶ εὐφρανθεὶς ἔτι δὴ μᾶλλον ὅμοιον πρὸς τὸ παράδειγμα ἐπενόησεν ἀπεργάσασθαι. καθάπερ οὖν αὐτὸ τυγχάνει ζῷον ἀίδιον ὄν, καὶ τόδε τὸ πᾶν οὕτως εἰς δύναμιν ἐπεχείρησε τοιοῦτον ἀποτελεῖν. ἡ μὲν οὖν τοῦ ζῴου φύσις ἐτύγχανεν οὖσα αἰώνιος, καὶ τοῦτο μὲν δὴ τῷ γεννητῷ παντελῶς προσάπτειν οὐκ ἦν δυνατόν· εἰκὼ δ' ἐπενόει κινητόν τινα αἰῶνος ποιῆσαι, καὶ διακοσμῶν ἅμα οὐρανὸν ποιεῖ μένοντος

on the one hand, and the transformations of earth on the other), that mere observation would not have separated. It also leads him to assume that the fifth perfect solid must also be the shape of something created, in this case the world as a whole.

> αἰῶνος ἐν ἑνὶ κατ᾽ ἀριθμὸν ἰοῦσαν αἰώνιον εἰκόνα, τοῦτον ὃν δὴ χρόνον ὠνομάκαμεν. ἡμέρας γὰρ καὶ νύκτας καὶ μῆνας καὶ ἐνιαυτούς, οὐκ ὄντας πρὶν οὐρανὸν γενέσθαι, τότε ἅμα ἐκείνῳ συνισταμένῳ τὴν γένεσιν αὐτῶν μηχανᾶται· ταῦτα δὲ πάντα μέρη χρόνου, καὶ τό τ᾽ ἦν τό τ᾽ ἔσται χρόνου γεγονότα εἴδη.
>
> Now when the father who had begotten the universe observed it set in motion and alive, a thing that has come to be as a shrine for the everlasting gods, he was well pleased, and in his delight he thought of making it more like its model still. So, as the model was itself an everlasting Living Thing, he set himself to bringing this universe to completion in such a way that it, too, would have the character to the extent that was possible. Now it was the Living Thing's nature to be eternal, but it isn't possible to bestow eternity fully upon anything that is begotten. And so he began to think of making a moving image of eternity: by bringing order to the universe, he makes an everlasting image of eternity remaining in unity, (an image) moving according to number; this[28] is, of course, what we have named 'time'. For before the heavens came to be, there were no days or nights, no months or years. But now, together with framing the heavens, he devised their coming to be. These all are parts of time, and *was* and *will be* are forms of time that have come to be. (37c6–e4; Zeyl's translation with modifications, emphasis original)

In the passage cited and in the following paragraphs, the demiurge aims to make the created world even more similar to the model (μᾶλλον ὅμοιον πρὸς τὸ παράδειγμα, 37c8). Hence, as the model is an *eternal living being* (ζῷον ἀίδιον), the world needs to become as like an eternal living being as possible. The possibility of transferring the *eternity* of the model to the world is the concern of the passage we are dealing with. We can infer this also from 39d7–e2, where it is stressed that the stars and hence the *organa*, "instruments" (for example, 41e), of time were created in order to make the world as similar as possible to the model in imitating its *eternal* nature (διαιωνίας φύσεως).

The imitation of the eternity of the model is problematic, however, since the universe has come into being and "it isn't possible to bestow eternity fully upon anything that is begotten" (37d4–5). How can the cosmos, as something that has not always existed, be made more eternity-like? How it is at all possible to transfer eternity to the world, and is something lost or changed in this transformation?

All that can be accomplished for the sensible world, we are told, is an *image* of eternity. And this image is time (*chronos*). We learn that eternity is "remaining in unity" (37d7),[29] and later that only "is appropriately said of it", not "was"

[28] I follow the tradition in understanding "this" (*touton*) as referring back to the whole preceding clause. Brague 1982 has suggested that it be understood as referring back only to *arithmon*; I argue in detail against this reading and for translating *aiôn* as "eternity" in chapter 5 of my manuscript *Ancient Notions of Time from Homer to Plato*.

[29] Cf. Brague 1982, pp. 57–8.

or "will be" (37e6–7). So Plato is concerned with two aspects of eternity: (1) the unity in which it remains, and (2) the fact that it is not part of a temporal series, but only *is*. These two aspects can be understood as two sides of one and the same attribute. Eternity is thought to be without past and future, it is, so to say, all 'at once', not stretched out, since it always "is" and never "was" or "will be" (37e6–38a1).[30] Eternity can thus be understood as a special form of 'temporal' unity, namely being all at once – in this sense, eternity is remaining in unity. This kind of unity cannot be conferred on the created world, since we must conceive of its existence as a plurality of succeeding states – at least of a state 'before' and a state 'after' creation – and thus of temporal extension.

But how can time help bestow the monolithic eternity, at least in some sense, upon the created pluralistic world? Given that the intelligible unity of eternity is all at once, there is no temporal end; and endlessness is bestowed upon the sensible world by having the cosmos go on for all time – the demiurge could end it, but promises he will not. Like the eternal model, the created cosmos will have no temporal end because it is everlasting.[31] The two forms of endlessness do not provide us with exactly the same kind of unity, however, since with the eternal model everything *is* all at once, while the temporal cosmos is stretched out into 'was' and 'will be'. The forms of time accomplish the task of translating the permanent 'is' into the stretched-out order of 'was' and 'will be'. This order can also be read as the assumption that time has a certain direction, what we have come to call the 'arrow of time'. We usually assume that within the temporal world there is only one direction to the succession of events, but Plato's remark in 38a2 about the model growing neither older nor younger seems to indicate that in principle both directions are possible. So what is established seems to be an extended time without end, rather than a certain direction.

But it is not only eternity's lack of a temporal end that can be imitated by the sensible world; it also adopts the unity of eternity in some form. The simple unity of eternity is transferred to the pluralistic world by transforming the unity that is all "at once" into a *regular succession*.[32] As we will see below, it is because of the parts of time (days, nights, etc.) that this succession occurs

[30] Plato seems to express the idea that the model is not just throughout all time with an 'is' that can be understood in a logical sense, echoing Parmenides' timeless now in fr. 8, line 5. See Owen 1966, especially pp. 329–30; Taylor, p. 189; Cornford 1948, p. 102. We saw in Chapter 2 that Parmenides refuses a temporal succession for his one Being in order not to introduce any difference into it. Similarly, Plato considers such temporal differences unsuitable for the model. Plato's characterisation of the model as being *akinêtôs* in 38a2 seems to refer to Parmenides' description of the one single Being as *akinêton* in fr. 8, line 26 (the adverbial use 'being *akinêtôs* always-in-the-same-condition' does not bar the verbal echo of Parmenides). However, in contrast to Parmenides, Plato's true Being can be imitated by time. Cf. Taylor, p. 679.

[31] The cosmos can thus be called sempiternal.

[32] That the problem of the unity of the created world is one of the major topics of this dialogue is signalled by its beginning and end: it starts with the word "One" (Εἷς) spoken

"according to number", as the parts permit measurement of various processes.[33] Thus eternity's "remaining in unity" is taken up by the image "moving according to number". The image is moving since it is an image in the realm of becoming; *remaining* or abiding in the model corresponds to an *ordered movement* in the world of becoming. The order of this motion is established by having it run according to *numbers* and thus according to one *unified* principle, as can be made evident with the help of the parts of time.

Thus, time is to be understood as an order in the sensible world that works according to numbers. It is the numerically ordered motions of the heavens. With the help of these motions it can be shown that nature and its changes are compatible with certain intelligible structures, namely numerical structures. The intelligibility of these changes is thus established.[34] More specifically, time shows the order of the processes in the created cosmos in two respects: first, since the heavenly movements used as instruments of time move "according to number" and hence are connected to the sequence of natural numbers, it is by means of time that the plurality of events[35] can be integrated into the order of a countable succession – for example, we can say that event *x* occurs two days (using the parts of time) later (using the forms of time) than event *y*.[36] Secondly, it is by means of time that single movements (at least regular ones) can be measured.

Let us now look more closely at time's function as a measure of motion, since it is with the help of measurement that the processes in the sensible world are shown to be intelligible in the *Timaeus*.

6.3.2 *Time as the Measure of Motion*

Timaeus claims that the world becomes more similar to its model, which also means more intelligible, thanks to time imitating the eternity of the intelligible

by the counting Socrates, and ends with "εἷς οὐρανὸς ὅδε μονογενὴς ὤν" ("this one heaven being the only one of its kind").

33 The orbits of the planets are formed according to numerical proportions even 'before' the introduction of time. Originally, the circular motions along these orbits are just the psychic motions of the World Soul. But once the heavenly bodies are created and set into these orbits, we have *physical movements* along them that manifest the psychic movements and are connected with number – connecting *physical movements* with numbers is what we get with the introduction of time.

34 Unity and intelligibility can thus be understood as closely connected: the imitation of the unity of the model also proves the intelligibility of the sensible world. In order for something to be intelligible, it must be something, that is, some one thing, and hence it must possess some unity.

35 I think it is obvious that the order and unity established with the help of time in the passage under discussion (37c6–e5) are the order and unity of events and changes (to establish order and unity of individual unmoved physical things would not require the introduction of time). Thus, time shows intelligibility at the heart of change, while change in earlier dialogues seems to be to blame for the failure of physical things to be intelligible.

36 For a more detailed account, see Sattler 2010.

realm. An essential aspect of this 'imitation' lies in the fact that time secures the measurability of the processes in the sensible world – it is thus proof of their intelligibility. For Plato, however, time is not connected to all motion, but solely to measurable motion, which excludes the disorderly motion of the elemental traces 'before' the creation of the universe. This may strike us as odd, since we usually assume that all processes are in time; however, this assumption is not to be found in early Greek thinking.[37] In fact, Plato's *Timaeus* is one of the central texts leading to such a unified assumption, and only allows for one exemption of a process in the physical world from being in time, namely the pre-cosmic chaotic motion that is 'outside' of time.[38] Let us now start the investigation of time as a measure by showing that measurability is indeed an appropriate notion for interpreting the *Timaeus*.

6.3.2.1 The Centrality of a Measure in the *Timaeus*

We have already seen that what is all one in an indivisible unity in the model is divided into different aspects in the created world (37d7, 57e–58a). So the one eternal Living Being is imitated by the features of livingness, temporality, uniqueness, and ordered plurality within the created world. We saw in the first chapter that a necessary condition for measurability is that what is to be measured possesses a certain aspect with respect to which it is investigated – what I called dimensionality. In the created cosmos there is a plurality of aspects that can in principle be measured. With respect to locomotion, time and space would be the most obvious aspects to measure. We can gather from the function of time that in the *Timaeus* measurability is not merely possible but an important feature in the dialogue: it is in terms of time that motions are connected to numbers in a way that allows for their quantification. And with the help of the units of time, these motions are quantified. So if we recall the features of a measure explained in the first chapter, we will see how time can serve as the measure of movement in the *Timaeus*:

(1) Time is the aspect of motion that is measured in the *Timaeus*, that is, what we called the dimension of the measurand.
(2) Time allows for assigning motions to numbers as it is in terms of time that movements can be connected to numbers (i.e., we can measure the (temporal) extension of a motion).
(3) The units used to measure movement are units of time: days, nights, etc.

We see that time is a measure in the sense described in the first chapter. While the measurement of time is not something new with Plato, the *Timaeus* is the first text we have where all (but one) processes, changes, and motions in the created world can be brought under one measure in the sense of one common respect in

37 See Sattler 2017b.
38 Aristotle will get rid of this exception too.

which they can all be measured[39] – time. Thus, time can be understood as a valid standard for all change.[40] Employing such a common measure may seem trivial, but if we take into consideration the disjointed understandings of time in the ancient Greek world before Plato,[41] it is by no means a given.

Although Plato does not discuss the different aspects required for measuring explicitly, he frequently employs cognates of the term "measure" in the dialogue. In the stretch of text most relevant to our project (37d–39d), the word "measure" (*metron*) and its derivatives are used three times. Let us briefly look at these occurrences and see why measurement is crucial for Plato's understanding of movement and time.

(1) The diurnal movement of the sun is said to be a conspicuous "measure" (μέτρον ἐναργές) of the relative speed of the planets as it is a simple motion along the circle of the Same (39c1–2)[42] and lit. In this way, human beings[43] can "share in number", learning numbers from the revolution of the Same and the Uniform (39b2–8).
(2) Apart from the movements of the sun and the moon, the revolutions of the other heavenly bodies, of the planets, are not noticed by most people and so are neither named nor measured (συμμετροῦνται) against each other by numbers. Hence, people do not realise that these other movements also signify time (39c6–d1).
(3) The perfect number of time fulfils the perfect (or great) year, "when the relative speeds of all eight revolutions have accomplished their courses together", as measured (ἀναμετρηθέντα) by the circle of the Same (39d2–7).

The first passage shows the (diurnal) movement of the sun to be a paradigmatic "measure" (in the sense of a measurement unit) for the movements of the planets. The second makes clear that the movements of the planets, not just of the sun, also serve as markers of time. The third passage explains that all these heavenly movements[44] can be measured with one common temporal unit, the perfect year. A brief analysis of these three passages will support the claim that Plato's notion of measure corresponds to the kind of measure discussed in the first chapter, and that the measure for created physical movements[45] in Plato is time.

39 This is one way in which the unity of all processes, changes, and motions in the created world is secured.
40 At least once the world has been set up by the demiurge.
41 See Sattler 2017b.
42 Cf. Cornford, p. 115.
43 As the creation of mankind has not yet been introduced, the text speaks not of human beings but of "those living things appropriately endowed".
44 What we would call solar, lunar, and planetary movements.
45 Created physical movements are in contrast to the uncreated (disorderly) motions of the elements 'before' the creation, which cannot be measured, and the created but unphysical

(1) Given that the sun is the only heavenly body that is conspicuously lit, its movement is recognisable by everybody throughout the universe. It thus paradigmatically fulfils two functions:

a. First, its movement is understood as a measurement unit for the speed of the planets, the swiftness and slowness of their motions in relation to each other. In order to bring the different but regular movements of the planets into a ratio with each other, the movement of the sun is presumed as a basic unit,[46] used to measure all the other movements. And this unit is thought of as a *temporal* unit, a unit to measure the time needed by the planetary movements: the unit of one day and night. This is the smallest natural unit for the purposes of Plato's cosmology and thus the one most appropriate for determining other, longer periods.[47]

b. Secondly, this shining measure is prepared so that human beings can participate in number with the help of their eyes. Since in its movement creating day and night the sun always returns to its starting point (wherever we may fix this point on the continuous circle), it creates regular sections. A complete revolution can be understood as 'one', the next as 'two', and so forth. In this way, the units of time make visible the succession of numbers for human beings (see 47a).

motions of the World Soul, which need not be measured. In contrast to the merely intelligible movement of the World Soul, the intelligibility of physical movements is not immediately obvious; rather they need to be connected to numbers first to show that they can be accessed by reason, and this connection requires a measure.

[46] The motion of the sun is basic in two ways. It is the unit that is most useful for our measurements (while in principle the motion of any of the heavenly bodies could be used to measure motion), and the motion of the sun creating day and night follows solely the orbit of the Same (while, the lunar motion producing the temporal unit of a month also follows the circle of the Different); see below.

[47] Plato makes clear in 39c1–3 that the temporal unit provided by the movement of the sun and used to determine all other heavenly motions is "one day and night": "Thus and for these reasons day and night came into being, the period of the single and most intelligent revolution." The movement creating day and night is the "single and most intelligible one" in so far as the sun follows only the orbit of the Same: the god lit the sun "in order that he might fill the whole heaven with his shining and that all living things for whom it was meet might possess number, learning it from the *revolution of the Same and Uniform*" (39b6–c1). Only the yearly movement of the sun also follows the one orbit of the Different specific to the sun: "a year [has passed] when the sun has completed its *own* circle" (39c5). Hence, night and day make obvious a movement that all the planets share and which thus can be called a "single" movement, namely the movement along the circle of the Same, while the planets differ in their movements along the orbit of the Different; see 38c8–9 and also Cornford 1948, p. 115. While with day and night we get a unit which in principle we could gain from all planetary movements along the circle of the Same, the different movements of the planets along their particular circle of the Different produce additional units of time which are bigger than day and night.

The passage introducing the sun as a common measure also makes clear that Plato ensures the rational admissibility of motion in two respects: the sun and thus a singular element in its revolution is an obvious measure (39b), for it can be seen by everybody. Its use should give the same result for everybody who employs their reason, as it enables us to tell the duration of a movement in an objective way. Furthermore, all (circular) physical movements can be compared to the movement of the sun. Thus, we have a criterion which enables us to prove the sameness or difference of two movements in respect to their duration in an objective way.

(2) This passage tells us three things for our inquiry:
  a. The function of measuring duration, fulfilled by the movement of the sun, could in principle also be fulfilled by the movement of any of the planets; each of them could be used as a timer.[48] This seems to be the reason why in 41e5 Plato uses the plural "times" when talking about the planets as the "instruments of time" (ὄργανα *χρόνων*): we could say that each planet gives us the unit for measuring a time and hence we could talk about "times" in the plural.
  b. The movements of the different planets can be measured against each other so that each provides the unit for one unified measure. Thus, the planets are also the instruments of one, singular time (ὄργανα *χρόνου*, 42d5).
  c. However, most of the planets look "bewildering in number and surprisingly variegated" (39d1–2) to human beings. They cannot see that these planets perform a regular movement[49] as they do not "measure them by numbers" (i.e., they do not understand that they can indeed be assigned to numbers in a regular way so that they can be used as timers).[50]

(3) The third, somewhat complicated passage refers to the fact that after a certain period of time, all the planets come back to the same relative position on the circle of the Same. This description shows that in Plato, too, to measure means to measure something *with respect to* some dimension. The occurrence of the perfect year is realised when all the different revolutions of

[48] Cf. Taylor, pp. 215–16 and Brague 1982, p. 62.

[49] The overall motion of the planets is regular, according to the *Timaeus*, whether or not this means that the motion of the Different, which is one aspect of the overall motion, is uniform and continuous. For Eudoxus' account of celestial motion, see Chapter 9.

[50] The Greek word for planets, *planêtes*, originally means the one who wanders and thus indicates that the planets had been seen as moving in an irregular fashion. In 40b Plato points out that they are only wanderers in the sense that they have (regular) turnings, due to their following two different paths, the circle of the Same and the circle of the Different. In *Laws* 822a, finally, Plato simply claims that the heavenly bodies do not wander but rather always follow one and the same track.

the planets are *measured by the circle of the Same* (39d6–7). If they were measured by something else, for example, by the circles of the Different, which are different for the different planets (36c5–d7), we would not get the perfect year Plato is talking about.[51]

These three passages show that Plato does in fact work with a notion of measurement that very much resembles the one I gave above. In accordance with our third criterion for measurement, that measurement units are needed, he discusses days, nights, months, and years as units of time to measure movements and how these units can be gained (passages 1 and 2 just discussed). Our second measurement criterion, assigning a physical structure to a numerical one, is hinted at in passage 2. Furthermore, the whole set-up of time in the *Timaeus* accords with this criterion, as is shown by the fact that we can learn numbers from the heavenly motions (39b, see passage 1), which presupposes that the heavenly motions are systematically connected to numbers, for example, one diurnal motion of the sun gives us one unit. And if other motions are compared to the motion of the sun, the latter can be used to assign these other movements to the number series.[52] Under the same conditions the numbers will always be connected to the motions in the same way, as is implied in 38c5–7, where we are told that the motions of the heavenly bodies preserve the numbers of time (see below). Finally, our first criterion for measurement, that a dimension must be specified in respect to which something is measured, corresponds to Plato's statement in the third passage, that the different movements must be measured with respect to something. Although prima facie the orbit of the Same seems to be the measure of the movements and speed here, what is actually taken as a measure is the time they take to traverse this orbit of the Same. That the circle of the Same is not a measure different from time is made clear by the reference to the unit used: the unit of measurement here is

51 Both the circle of the Different and the circle of the Same could be taken as a measure of the movements of the planets. The planets are moving according to the movement of the Different and the Same as they follow their track and can thus be measured according to both (although only the movement of the Same allows easy comparison between the motions of the different planets). In Plato two different orbits are necessary for each planet to explain their seemingly variegated (39d1–2) movements as regular movements; this seems to be closely related to the tradition of Eudoxos, which explains the movement of sun and moon with the help of three spheres and the movement of the five outer planets by the revolutions of four different spheres. See Aristotle *Metaphysics* Λ, 1073b17–32; Cornford 1948, p. 116; and Chapter 9. While Eudoxus' system is obviously more sophisticated than the two-sphere model here in the *Timaeus*, it seems clear that the basic principle – to explain seemingly non-uniform and non-circular motions with the help of a combination of different uniform and circular motion – is the same in both accounts, pace Knorr 1990. Cf. also Karfik 2004, p. 178, n. 40.

52 In contrast to modern measurement theory, Mohr 1986, p. 41 understands the assignment of the empirical to the numerical realm as an assignment of elements to each other (although allowing for one-to-many assignments).

the perfect *year*, hence a temporal unit, which can only measure something temporal – all other aspects of the revolutions of the planets, such as change in brightness or distance travelled, are not relevant. Time is thus the aspect of the planetary movements measured, which is also implied in passages 1 and 2.

Hence, time as introduced by Plato can be understood as a measure of movement in the way we defined measure in the first chapter. In addition, as a background for Aristotle's conception of measurement, which we will discuss in Chapter 8, it will be useful to mention a few specific features of Plato's measure:

(1) In the context of natural philosophy, measure, according to Plato, is independent of the observer, since planetary movements that are not observed by anyone also signify time. The importance of the observer looking at the movement of the planets (i.e., of the timers) lies in the fact that in this way the observer learns numbers and can adjust her own movements to the regular motions of the heavens – rather than, as in Aristotle, in the fact that an observer is needed for counting and measuring.
(2) Time qua measure has a normative aspect in Plato, since the ordered movements of the planets, being the markers of time, function as a paradigm to which other motions are meant to be assimilated. Most importantly, human beings *should* follow the example of the regular movements of the universe in the movements of their thoughts.[53] Aristotle, by contrast, strictly separates the kind of measure that may be of interest for normative concerns from the measure needed in the context of natural philosophy.[54]
(3) Plato employs a simple measure: in the *Timaeus* time is the only dimension to measure movement. We will also find this feature in Aristotle's notion of measurement in the *Metaphysics* and it will concern us in chapters 8 and 9.
(4) Time as a measure passes on number to human beings – there will be no equivalent to this in Aristotle.

We have determined time as a measure of movement in so far as it is the dimension in respect to which motions are measured. We now need to investigate how the process of measuring is understood in Plato. For that process, discrete units are needed; the way such units are acquired will be the subject of the next section.

[53] See 47b6–c4, 90c–d, and 42c5–6, where we are told that human beings should follow the revolution of the Same and Uniform within them. In 81a6–b2 we read that the ingredients of our blood necessarily imitate the movement of the universe. See also 38c5–7 and 41e6.

[54] A question to ask here, but one that I cannot pursue, is whether Aristotelian teleology in nature does not also bring in the idea of normativity.

6.3.2.2 Time in the Process of Measuring: Units of Time

To establish how Plato conceptualises the process of measurement, it will be helpful to consider briefly the conditions that a measurement unit must meet in modern conceptions, which will allow us to see whether Plato's account of measurement units can be understood along the same lines. During this investigation we should keep in mind that while Plato is concerned with *temporal* units, the time measured is meant as an answer to the question 'how long does motion *x* or change *y* last?'[55] Plato's interest is in measuring *motion* and *change*.

When measuring time, we are confronted with the well-known phenomenon that movement and time seem to be a measure for each other: on the one hand, we need time in order to measure movements, but on the other we must employ certain movements to get units of time and to measure temporal duration. The mutual dependence of time and movement is usually addressed by assuming that time is dependent on motion in one respect, and motion on time in another: there is one movement, a regular one, which is used for getting units of time, and there is another movement which is measured by time with the help of these units. In the *Timaeus* we find that with the help of the regular circular movements of the heavenly bodies the continuous time is divided into units of time (such as days and nights), while these temporal units are in turn used to 'divide' and measure other movements.[56]

6.3.2.1.1 How to Acquire Units of Time As we saw in the first chapter, motion, time, and space do not come individually prepacked. But we need discrete units for measuring the time a certain motion takes.[57] In order to get from a continuous process to something which can be quantitatively determined in a measuring process, we need a unit that meets four conditions:

(1) The main task of a unit for measuring continua is to help us to set limits within such a stream, that is, to divide it into parts that can then be counted. Acquiring clearly marked-off parts and thus achieving

[55] Cornford, p. 102 and Zeyl, p. xliii take it that the circular movements of the heavenly bodies measure time. Although this is correct, it is only half the story, for we usually measure a certain time with the help of these units in order to characterise a particular process.

[56] In Plato, time, which is meant to guarantee the countability and measurability of movement, gets its own units, and thus its countability, from the movement of the planets (see 38c 5–7 and above). Accordingly, Plato can say that time is "produced" by the movement of the stars (38c3–7 and 38e3–5), for they provide what we understand as temporal units: "days, nights, month, and years". With the help of these first discrete units of time, other movements can be made countable and measurable.

[57] We will gain units of time from one motion in order to measure another. But the temporal units we have gained can also be used to measure the time *between* two motions.

countability is not just a matter of course in the case of uninterrupted movements and changes. For example, my becoming tanned or the steady advance of a tortoise do not usually in themselves provide us with any natural limits (in some processes, but by no means all, beginning and end are naturally given, but not necessarily any natural parts). For Plato, it is in terms of temporal units that we can mark off parts of processes and thus count them.[58]

(2) The units must be *appropriate to the dimension* measured, as we already saw in Chapter 1; for example, we cannot use centimetres to measure the time a motion takes, but need hours or days.[59]

(3) The unit must be of finite quantity (thus making feature (1) possible).

(4) This finite quantity must be always the same (i.e., it must be constant so that a reasonable comparison is possible).[60]

All four of these features can be shown to be employed by Plato for temporal units. (1) and (4) we find at 38c5–7: The sun, the moon and the five other stars "came to be in order to set limits to [*diorismon*] and to stand guard over [*phylaken*, i.e., they preserve the regularity] the numbers of time". They set limits (i.e., they delimit parts in a continuum) and they stand guard over the number of time (i.e., they are responsible for ensuring that the number, that is, the unit used to determine the number, is always the same).[61] They are able to set limits because the units established by their circular movements – day, night, month, year, and even the great year – are of finite quantity, and hence (3) is met. Finally, we find (2), for example, at 41e6, where it is explicitly said that the planets whose motions operate as temporal units are "instruments of *time*". But in 38c5–7 (cited above) too it is obvious that when the planets are there to keep the numbers of time, time must be that in respect to which our measurement is carried out. Hence, it is clear that the dimension assumed to determine the processes in the universe is time and that Plato introduces units of measurement appropriate to this dimension, namely temporal units.

58 See Zeyl, p. xlii, who assumes that the "'movement' of time 'according' to number indicates both the everlasting flow of time and its divisibility into parts that are numerically discrete". Cf. Vlastos 1995, p. 272.

59 Units can be understood as an expression of a certain dimension, i.e., they are the terms in which a certain dimension is employed in the measurement procedure. See Ellis 1968, pp. 128 and 142, who understands dimension names as generalised unit names.

60 See Kretzmann 1976, p. 96, who thinks that for a part to be able to measure out a whole (and this we can understand as serving as a unit) it "must (a) have some finite quantity and (b) be quantitatively stable".

61 This does not hold for the day as a unit, since the length of the day changes throughout the year, as Plato complains in *Republic* VII. In the *Timaeus* Plato avoids this problem by working with the day-night period as one unit (what later on is called the νυχθήμερον), which is always the same. See 39b–d and Cornford, p. 115.

Two other features of a unit which we take for granted in modern science can also be found in Plato (they are named here for the sake of completeness, but separately since they will not be important for this chapter):

(5) The units chosen are in principle arbitrary but suitable.[62] Accordingly, Plato shows in 39c6–d1 that any movement of the planets, which in their circular movements are the only suitable units at hand, can be employed.
(6) The units chosen are compatible with each other.[63] In Plato this compatibility is expressed by the fact that the daily movement of the sun can serve as a basic unit for all the other planetary movements operating as units, and by the fact that they all come together in the great year.

So far it is clear what conditions a unit must fulfil. But how Plato thinks we might acquire such a unit it is not yet evident. The kind of unit we are looking for depends on the dimension we want to measure (condition 2). As here the dimension in question is time, we need temporal units. With time we face the problem that it is not enough to determine a unit just once, unlike, for instance, setting up a kilogram (or a talent) cylinder to measure mass. Rather, the unit must constantly be produced anew[64] – in an everlasting universe this production must go on forever. Thus while the unit needed must be finite in extension (condition 3), its production process must be infinite in the sense of everlasting. The constancy of the unit (condition 4) can be guaranteed, in spite of its continuously new production, by having it produced with the help of an everlasting *regular* process. Finally, the temporal unit must enable us to set limits within the processes measured (condition 1).

For Plato, all these conditions are met by the revolution of the planets – the "instruments of time" (41e6). The position, the regularity, and the circularity of their motions together provide us with everything we need for temporal measurement. The *circularity* of the heavenly motions allows us to break down continuous processes into discrete units: the return of the planets to the starting point of their revolution (no matter what we take as their starting point) allows us to mark off a unit – a day, a month, a year, what Plato calls "parts of time". While producing finite units, the motions of the heavenly bodies nevertheless go on everlastingly with their revolutions, and hence can

[62] See Ellis 1968, pp. 152–9.
[63] Usually there are different units for a given dimension, but they must be compatible with each other, i.e., it must be possible to translate the result of a measuring procedure given in units of one scale, e.g., centimetres, into the corresponding result using the units of another scale, e.g., inches. If this is not possible, we are no longer dealing with one and the same dimension.
[64] According to Brague 1982, pp. 30 and 49, this is the reason why Plato uses the present tense *poiei* in 37d6, claiming that the demiurge *makes* the heavens, although it means that he must change the sequence of tenses quite brusquely.

be used for an infinite production of units. In this way a circular motion (i.e., a motion returning in itself) allows us to combine infinity with finiteness.

The *regularity* of these motions[65] ensures that we can always derive the same unit and thus can compare different motions that take place at different times. Finally, the *position* of the planets and the stars means they are accessible from everywhere in Plato's finite universe – one of the orbits is even furnished with light, which enhances the accessibility of the regular motions. The accessibility of the planetary motions allows us to compare all other regular processes with them.

So the circularity, which allows us to gain units, the regularity, which allows us always to gain the same unit, and the position of the heavenly bodies and their revolutions, which makes them accessible everywhere, together allow for measuring the time of any motion and for comparing measurements universally. The first discrete units of time we gain from the planetary motions allow us to measure other motions in turn: with the help of the regular units of time, each process in the physical world can be divided into temporal parts so that the discrete intervals thus gained can be "connected" with the series of natural numbers. Furthermore, we can determine not only the length of a process, and compare it to other processes – whether it is as long as, shorter, or longer than another one – but also whether it happened before, after, or simultaneously with some other event.[66] Hence, a movement can be counted and compared to other movements, and statements about its quantity and timing are possible.

What is more, as we have already seen above, one of the "instruments of time", the sun in its motion, allows us human beings to learn how to count in the first place (39b). Thus, time not only demonstrates the rationality of the regular processes in the world, but also enables us to develop our rational ability by endowing us with a "share in number".

6.3.3 Space as Excluded from the Measurement Process

As the main kind of *kinêsis* we are dealing with here is locomotion, it seems we must also look at Plato's account of distance and space.[67] After all, if we want to fully measure motion, we must determine not only the time taken but also how much distance is covered. A thing moves faster if it covers more space in the same time, and slower if it covers less distance in the same time.

[65] For Plato, circular movement is the only truly regular movement and thus the movement most akin to reason.

[66] Time in fact allows for the order of an everlasting sequence of before and after, which we can further determine with the help of the parts of time. See Sattler 2010.

[67] In my manuscript in progress *Conceptions of Space in Ancient Greek Thought*, I discuss the different notions of space, place, distance, extension, and so forth that are important for an account of motion.

Plato does indeed provide an account of something that can be seen at least as a basis for space:[68] an account of what he introduces as "the receptacle", also calls "the wet nurse of becoming", and eventually identifies with *chôra*, space. This receptacle is introduced as a novelty, a third metaphysical principle, over and above Being and Becoming (52a), well-known from earlier Platonic texts. The receptacle fulfils at least four tasks within Plato's cosmology:

(1) According to the *Timaeus*, the sensible things are copies or images of intelligible Forms. As images they need something in which they can appear, and this is the receptacle, which does not possess any of the features these copies possess and therefore can receive them all equally. The Forms guarantee that a sensible thing is the thing it is, for example, fire is fire because it is an image of the Form of fire. But the receptacle is responsible for the fact that the thing that comes into being is a sensible thing, since it is once the image of the Form appears in the receptacle that the image is the sensible thing we perceive.

(2) The receptacle is that in which the triangles and the bodies made up of them move. Like the atomists, Plato assumes that the basic elements can only move if there is something over and above these elements for otherwise two elements would coincide in the same place – only for Plato this something is not void, as with the atomists, but a metaphysical novelty.

(3) The receptacle helps explain the initiation of the original motion in the world:[69] even before the divine demiurge started his 'creation process', the receptacle "was filled with powers [of the traces of the four elements] that were neither alike nor evenly balanced, there was no equipoise in any region". Plato assumes a like-to-like principle at work in the cosmos. And since the traces are not evenly balanced and because the traces next to each other are unlike each other, they move the receptacle and are moved by it in turn; thus the receptacle is itself moved and causes motion (52e–53a).

While the initial motion of the traces and the receptacle is chaotic, it leads nevertheless to some sorting, to a first spatial order. This is why Plato compares it to the motion of a winnowing basket – shaking the traces, we get the heavy and dense parts on one side and the light and rare ones on another.

(4) Finally, while the elements constantly change into each other, that in which they appear, the receptacle, stays stable. While the receptacle is

[68] In Sattler 2012 I show that the receptacle cannot itself be space since important features of what we would usually understand by 'space' derive in fact from the form of the World Body and the content of the receptacle (features such as dimensionality or distance). The receptacle is nevertheless a basis for space since it also possesses a set of potentialities for basic spatial features, which are developed as an actual geometrical and physical space with the help of the traces, the elements, and the form of the World Body.

[69] A point that the atomists failed to explain, according to Aristotle, as we saw in Chapter 4.

> itself moved, it does not change its character as a receptacle, that is, it does not change into something else, in contrast to the elements. In this way the receptacle ensures some stability. It is a stable "this", as Plato calls it, by being that in which something moves and in which the elements change into each other (49e7–50a2), and by ensuring the continuous possibility for these processes to take place.

While the receptacle grants stability, is that in which all the copies of the Forms can move and appear as sensible things, and contributes to the initiation of the first motion, it is not employed in any way when Plato deals with the measurement of motion, as we will see in more detail below; nor are the elemental traces, which may be seen as a 'pre-creational' basis for determining spatial distances. Measurement is connected solely to time, which is 'created' by the divine craftsman in order to make the world even more similar to its model.[70] The receptacle, by contrast, is something already given to the demiurge, something uncreated. Plato calls it a 'necessary cause', in contrast to the rational causes, which are the demiurge, the Forms, and the model.

Time and space are thus situated on two completely different ontological levels: the receptacle is uncreated and essentially connected to the chaotic motions before creation, while time is created by the demiurge so as to bestow rational order upon the world.[71] It remains unclear in the *Timaeus* how the two could possibly be connected if we want to give an account of speed.

In accordance with this ontological division, we also find two different kinds of 'natural motion' in the *Timaeus*: the regular motions of the heavens connected with time and the completely irregular motions of the traces of the elements in the receptacle. The first can be an object of science to some degree, while the latter cannot. The idea that the receptacle and the traces in it are given to the demiurge 'before creation' also leads to the problem that the motions of the receptacle and the traces seem to occur 'before' the creation of time, that is, 'outside' of time.[72] I will not go into this problem here, but it already foreshadows one problem with Plato's account of motion that we will see more clearly below, namely that Plato's account of motion is only of limited generality.

[70] We should not forget that measurement does not amount to the same thing as ratios; numerical ratios appear in the initial construction of the World Body and of its soul – and indeed in geometrical particle theory, the sizes of triangles, and the particles themselves. See 54b ff., 56c.

[71] The motions of the receptacle lead to the separation of the elemental traces in such a way that fire traces gather together here, water traces there. But this order is an inadvertent side effect of the motion of the receptacle, connected neither to reason nor to any intention.

[72] See, for example, Vlastos 1995.

6.4 Problems with a Simple Measure

The units Plato uses for measuring time are gained in such a way that the regular circular movements of the planets over the distance of their orbits are equated with units of time. Although today scientists do not typically define a unit of time in terms of a thing's covering a certain distance,[73] so far this production of units is unproblematic. What is problematic in the *Timaeus*, however, is that while the measure for movement is time, these temporal units are used to measure not solely the time a motion takes but also its speed. That Plato is in fact really speaking of units of time, on the one hand, and speed measured, on the other, can be seen in his remarks on the relationship of the different speeds of the planets. Because the length of an orbit is merely translated into time and not taken into account as a magnitude of its own, the different orbits, the larger or smaller circles, can be immediately related to different speeds:

> τὸ μὲν μείζονα αὐτῶν, τὸ δ' ἐλάττω κύκλον ἰόν, θᾶττον μὲν τὰ τὸν ἐλάττω, τὰ δὲ τὸν μείζω βραδύτερον περιῄειν
>
> [Some planets] would move in a larger circle, others in a smaller one, the latter moving more quickly and the former more slowly. (39a2–4)

Prima facie this statement seems to leave open two options for the relation between the size of the orbits and the speed of the planets:

(1) Some bodies move in smaller circles and are in addition faster; other bodies move in bigger circles and are in addition slower. Here speed and the length of the path are only externally and accidentally connected. If this interpretation is correct, the speed of each planet must be determined independently of the size of its orbit.

(2) There is an essential connection between the speed of a heavenly body and the length of the path it covers, as the speed is a 'result' of the length of the path: some planets are faster *because* they follow smaller circles and are thus back at their starting point earlier than those following a larger circle. On this reading, the length of the path is immediately translated into the length of time – the longer the path, the more time it takes.[74] And the time

[73] E.g., with atomic clocks we define a unit of time by the transition between certain energy levels of a caesium 133 atom, see the IS-standard.

[74] You may object to this interpretation by pointing out that Plato claims that the sun, Venus, and Mercury move at the same speed (38d), but as these three heavenly bodies must be situated in different circles, the length of these circles must differ, and thus Plato is really talking about speed in the sense of distance covered over the time taken. However, while the sun, Venus, and Mercury are situated on different circles, Plato nowhere states that these three circles are of different length (the circle of the moon is identified as that closest to the earth and thus the smallest, and those of the outer planets are presumably further away and thus bigger, but nothing suggests here that Plato is not

it takes the planets to get back to where they started is the measure of their speed. This reading would require us to show that in Plato's text the speed of the planets is thought to be dependent merely on the size of the orbit (for the longer the path of a planet's circle, the more time it will take to complete it, and thus, according to Plato, the slower the planet).

In deciding between these two alternatives, we should take a look at the passage immediately following the one just quoted, since there Plato clarifies how the speed of the planetary movements is measured:

> τῇ δὴ ταὐτοῦ φορᾷ τὰ τάχιστα περιιόντα ὑπὸ τῶν βραδύτερον ἰόντων ἐφαίνετο καταλαμβάνοντα καταλαμβάνεσθαι πάντας γὰρ τοὺς κύκλους αὐτῶν στρέφουσα ἕλικα διὰ τὸ διχῇ κατὰ τὰ ἐναντία ἅμα προϊέναι τὸ βραδύτατα ἀπιὸν ἀφ' αὑτῆς οὔσης ταχίστης ἐγγύτατα ἀπέφαινεν. ἵνα δ' εἴη μέτρον ἐναργές τι πρὸς ἄλληλα βραδυτῆτι καὶ τάχει καὶ τὰ περὶ τὰς ὀκτὼ φορὰς πορεύοιτο, φῶς ὁ θεὸς ἀνῆψεν ἐν τῇ πρὸς γῆν δευτέρᾳ τῶν περιόδων, ὃ δὴ νῦν κεκλήκαμεν ἥλιον
>
> Indeed, because of the movement of the Same, the ones that go around most quickly appeared to be overtaken by those going more slowly, even though in fact they were overtaking them. For as it revolves, this movement gives to all these circles a spiral twist, because they are moving forward in two contrary directions at once. As a result, it makes that body which departs most slowly from it – and the movement of the Same is the fastest of the movements – appear closest to it.[75] And so that there might be a conspicuous measure of their relative slowness and quickness with which they move along in their eight revolutions, the god kindled a light in the orbit second from the earth, the light that we now call the sun. (39a4–b6)

According to the last sentence of this passage, the motion of the sun is sufficient to measure the relative speed of the planetary movements. This shows that the temporal units prepared by the motion of the sun are the

assuming these three circles to be of the same size). For Plato the revolutions of the sun, Venus, and Mercury share the same periodic time (see Cornford, p. 105 ff. and Calcidius *ad locum*), which seems to result from their covering circles of the same distance (and the circles are first and foremost geometrical circles, so need not be staggered). While this special case of the relationship between the sun, Venus, and Mercury thus does not undermine the interpretation given, it may be easiest initially to set it aside and concentrate only on the cases where different heavenly bodies explicitly cover orbits of different size, as when we compare the motions of the moon and the sun, for example.

75 Cornford's translation and commentary make it obvious that this closeness means "appearing to keep pace with the movement of the Same most closely", i.e., appearing closest to it in speed (see pp. 112–13). For our purposes we can leave out the interpretation of the spiral twist as it is only meant to explain further phenomena that we can observe in the sky, such as retrogradation, and does not add anything to the basic idea of how speed is measured.

only thing needed to determine the speed of a movement in Plato's conception.[76] This temporal unit will only measure temporal duration, however, and the durations thus compared can only be the time the planets need to complete their orbits, as the text does not mention anything else that could be measured. Speed is thus reduced to the time taken, and the time taken depends only on the size of the orbit. Thus, time and space are actually not taken into account as two independent magnitudes in order to measure speed, a problem we have already seen in a different version in Zeno's moving rows paradox. In Plato, too, speed is not understood as a certain distance covered in a certain time. Rather, the bigger a circle, the longer a planet will need to traverse it.

Hence, interpretation (1), which prima facie appears to be a possible and favourable interpretation, cannot be what Plato is actually talking about, as there is no hint in the text that the speed of the planets can be accounted for independently of the circles of their orbits. There are no grounds independent of the length of the distance traversed that determine the speed of the planets. Therefore, interpretation (2) seems to remain as the valid interpretation of the connection between the size of the orbits and the speed of the planets. Speed then is determined only by the length of the path a planet travels, and this spatial distance is translated, for Plato, into the amount of time a body needs to complete its circle (i.e., to get back to the starting point in its orbit). A body which gets back to its starting point within only a few units of time is fast; if it needs many units, it is slow. Thus, the body with the spatially smaller orbit is *for that reason* faster than the one with a larger orbit, because it has a smaller task to complete.[77]

This interpretation seems to be challenged by Cornford's understanding of the speed in question as angular velocity. For angular velocity, the speed to be determined is not connected to the amount of distance travelled, but is related to the angle covered.[78] It seems that Plato can neglect the second dimension needed for the quantification of speed, the angle, since he always compares the completion of one circle, an angle of 360 degrees.

However, we saw above that Plato does indeed mention the sizes of the orbits – some are larger, others smaller (39a2–4) – and intimately connects them to the speed of the heavenly bodies, as the following passage also shows:

> After the god had made the body of each of them [the sun, moon, and the five planets] he put the seven into seven orbits along which the revolution

[76] The movement of the sun can easily be taken as a unit of time, but for a unit of speed according to our modern understanding of speed, we would need the distance traversed as well.

[77] Plato is in fact right that the inner planets, travelling on smaller orbits, are faster than the outer ones. Not for the reason he gives, however, but rather because of the decreasing strength of the gravitational force of the celestial centre. See also Taylor, p. 204.

[78] See Cornford, p. 106.

> of the Different was moving. The moon he placed into the first circle around the earth, the sun in the second above the earth. The Dawnbearer [Venus] and the star said to be sacred to Hermes [Mercury] he set to run in circles that equal the sun's in speed. (38c7–d3)

The moon is placed in the first (thus in the smallest) circle around the earth, while the sun is placed the second smallest and Venus and Mercury in circles that "equal the sun's in speed". It is the three *circles*, and not just the motions in them, that are of the same *speed*. This connection between speed and circle can only be explained by the size of the circles: circles of the same size have the same speed not because the planets in them happen to move with the same speed, but because if something goes with constant speed, it will cover the three orbits of the same size in the same time.[79] Accordingly, for Plato orbits of different sizes have different speeds, because if something moves with constant speed, it will cover different orbits in different times.[80] If Plato were focusing only on angular velocity, without taking distance into account, then planets in orbits of different sizes, like Jupiter and the moon, could in principle cover the same angle in the same time and so it would not make sense to talk of 'circles of the same speed'. But if we start with the assumption that two bodies move at the same *linear speed* in orbits of different sizes, then they will also show different angular speed (since linear speed equals angular speed times the radius).[81]

Modern commentators simply take over this connection of speed and the size of the orbit without further ado, as we can see, for example, in Cornford's commentary: "The 'swiftest' of them is the one which completes his journey in the shortest time, namely the moon. The 'slowest' is the outermost, Saturn, 'the body which departs most slowly from the swiftest of all movements'. *Thus the smaller the orbit, the quicker the body*."[82] Only Taylor seems to notice that the distance is problematic in Plato's account of speed: "Saturn, from his point of view, is the slowest planet simply because Saturn takes so long to get round his orbit; he does not allow for the much greater distance which Saturn has to go."[83]

[79] Plato assumes that Venus, Mercury, and the sun complete their orbits in a solar year; see Cornford, p. 106.

[80] This thought is already introduced with the creation of the orbits in 36d: "He set the circles [of the Different] to go in contrary directions: three to go at the same speed, and the other four to go at speeds different from both each other's and that of the other three."

[81] Rather than working with the concept of angular velocity, Plato seems to be dealing with a more basic notion expressing something like 'time in which a thing accomplishes what it is supposed to'.

[82] Cornford, p. 113; emphasis added. See also Gregory 2000, p. 125 for a discussion of the corresponding passage in *Republic* 617a–b.

[83] Taylor, p. 204. On the same page, however, Taylor remarks with respect to the passage we are considering, 39a2–3, that the "words make it clear that Timaeus regards the moon as the swiftest, Saturn as the slowest, planet, and thus show beyond dispute what has been assumed in earlier notes, that by the τάχη of the circles he means simply the 'periodic times' of the planets, not their 'velocities in their orbits'". But if Plato was indeed

We may think Plato is just using the word 'speed' in the way we do today when we say things like, 'The journey from A to B is much quicker if you take route *x* rather than route *y*' to mean simply that *x* is shorter and so it should take less time if you travel at the same speed on both. But such an everyday sense of speed does not give us an account of how to measure speed that would be useful for natural philosophy and for a more scientific understanding of speed.

We have seen that Plato's measure of speed is a simple measure: the difference in the size of the orbits (i.e., in distance) is immediately translated into the difference in time it takes to cover them; consequently, speed is determined by only one dimension, namely time. What is measured is only the period of time the heavenly bodies take to complete their orbit, without taking into account the size of the orbit as its own magnitude.[84]

We may object that not only is the time required taken into account, but also what it is required for – namely a complete turn on a celestial orbit. 'The completion of an orbit' only tells us when we can stop measuring time, however; it does not tell us the size of the task or whether the task of completing one circle is bigger, smaller, or the same as that of completing another circle.

There is one passage, 39d5–7, where a second magnitude seems to be used to determine speed. This second magnitude is not space but 'the Same'. However, the magnitude of the circle of the Same is not taken into account in the way we would expect were the Same indeed treated as an independent second magnitude. Rather, there are two circles along which the sun, moon, and the five planets revolve, the circle of the Same and the circle of the Different.[85] This passage tells us that we measure the time needed from one constellation of the planets to the recurrence of the very same constellation with the temporal unit provided by the motion along the circle of the Same; temporal duration is also the only thing measured here.

But is measuring speed solely with the help of time in the way Plato does strange only for us, who are no longer used to it? We will see that it is in fact also a problem for Plato's account of motion. More precisely, there are two major problems with his account of speed: it is not universally applicable as not all kinds of motion are measurable and hence comparable in accordance with it. Furthermore, it will turn out to be problematic as soon as we try to compare movements covering different distances. We will spell out these two problems in the following sections.

interested only in periodic times all along (i.e., only in determining the time the planets take), the size of the orbit needed not concern him.

84 The additional complication in 39a–b, quoted above, between apparent and actual quickness, refers to a difference between the regular speed assumed by Plato for each heavenly body and the phenomenal speed we seem to experience in the sky; it does not concern the point discussed in the main text.

85 The motion of these heavenly bodies is thus a combination of a motion of the circle of the Same (which is the same for every one of them) and a motion of their specific circle of the Different (different for each of these heavenly bodies).

6.4.1 Restricted Comparability

While we take every movement to be connected to time, for Plato time seems to be connected only with regular and circular physical motion, or movement that can be understood in terms of regular, circular motion. Time is not employed when Plato talks about the disorderly motion 'before' creation or about the merely intelligible motion of the World Soul.

Circular motion is the paradigmatic kind of motion as it is "most akin to reason" (34a). And within the created world the circular celestial motion is the one on which we should model ourselves (90c–d). The speed of rectilinear motions can be measured with the help of the movements of the heavens within the Platonic framework, but only if the motion has a given end point, since speed for Plato is the time necessary to reach this point.[86] Plato's conception of speed cannot be applied to a motion that does not have such an end point, for example to a locomotion going on indefinitely in a straight line. For speed in Plato is nothing but the amount of time a movement needs to reach its point of accomplishment or final point.

Given that his universe is finite and spherical, Plato may think that all motion either has an end point (if it is linear) or a point of accomplishment (if it is circular). But for us, Plato's conception of measuring speed is limited to what we would think of as special cases. It is not a universal conception for measuring the speed of physical movement, which would be needed for a general philosophy of nature. As mentioned in Chapter 1, I do not assume that an astronomical account of speed would be applicable without modification to non-celestial motions, but the *Timaeus* provides us with the most elaborate discussion of speed we find in the Platonic corpus and makes some assumptions, as we saw above, about linear speed – so if we were to find a general account of speed in Plato, it would be here. The account we find, however, not only lacks universal applicability but also does not provide us with a consistent conceptualisation of speed, as we shall see in the following.

6.4.2 Lacking Consistency: The Tortoise Wins the Race

Plato's conception of the measurement of speed reduces speed to the duration needed to accomplish a task, without taking the size of the task into account. If Athens had to choose whether to send Achilles, the fastest runner in the world, or a tortoise to the Olympic Games, and the tortoise had always practised in a

[86] Another way to frame this thought is as follows: take Taylor's point that what Plato means by 'speed' is what we mean by 'periodic time' and generalise the idea of periodic time, which gives us the idea that what is important for accomplishing a task is simply the time it takes; then if your task is to get from St Andrews to Edinburgh while mine is to get from St Andrews to Vienna, you have moved faster than me if (given that we start at the same moment) you get to your destination before I get to mine.

tiny little stadium and Achilles in a huge one, according to Plato's conception of speed, Athens might well choose the tortoise. For within the Platonic framework, the tortoise could be faster than Achilles under these circumstances. Only if Achilles and the tortoise are asked to practise in the same stadium, and so perform the same task, will Achilles be the faster of the two.[87] Should Achilles and the tortoise leave Athens at the same time to travel to Larissa, Plato's account of who is faster would be the same as ours – the one who reaches Larissa first. But in this case the space is fixed – the distance between Athens and Larissa – and we are measuring solely the time it takes both to get to Larissa. Plato's concept of speed does not allow for a comparison of movements covering different distances and hence a comparison of different tasks.

If we compare different tasks, Plato's account turns out to be problematic, for while talking about different speeds, he merely compares different durations, the amount of time needed, which is not enough to measure different speeds. In order to measure speed, two different magnitudes are needed, time and – for locomotion – distance travelled. Hence, today we would not talk about things being faster or slower within the framework of the *Timaeus*, but rather about them requiring a longer or shorter time – for in our conception of speed, the bodies moving may all share the same speed.

Plato does not consider how much distance is covered in how much time for at least two reasons: first, the paths of the orbits travelled are needed to 'produce' time (38c4). Hence, the distance covered is used only as an aid to obtain temporal units – the distance the sun covers from its rising to its setting is translated into the temporal unit of a day. Today, too, we usually consider the movements of the sun and the moon solely with respect to the time 'produced' by them, a day or a month.[88] But even though the movement of each planet is primarily used to measure time, if we are interested in the speed of this planet, we must also determine the distance traversed. We cannot measure speed solely by the movement of the sun (see 39b2–8) as this movement is taken only as a timer.

Secondly, space is on a fundamentally different ontological level from time in Plato's metaphysics, as we saw above.[89] Time and space are of essentially

[87] Plato does not think that all the planets travel on a circle of equal size, as is obvious from 39a, cited above, and 38d.

[88] There are also certain astronomical contexts where an immediate translation of a spatial magnitude into a time, in the way we find it in the *Timaeus*, is appropriate. See Kuhn's 1985, pp. 8–9 account of the Babylonian and Egyptian usage of the *gnômôn*.

[89] See also Cornford 1948, pp. 102–3: "We are apt to speak of Becoming as going on 'in time and space', as if these two conditions were on the same footing. Plato does not so regard them . . . This Receptacle, finally identified with Space (52A), is treated as a given frame, independent of the demiurge and a necessary condition antecedent to all his operations. Time is not a given frame; it is 'produced' by the celestial revolutions (38E), which are

different character, Plato does not put them on the same footing, and it is not obvious that they can be combined in such a way as to determine speed.

We may think that this ontological difference need not concern us, since we need only distance, not space, for an account of motion and speed. But we cannot conceive of distance in the *Timaeus* without the traces (and later elements) in the receptacle.[90] We might also think that Plato's introduction of mathematical structures should allow him to tackle this problem, given that he uses mathematical elements for spatial as well as temporal extensions. We have seen, however, that Plato allocates different mathematical structures to time and space – time is connected to the number series and thus to an arithmetical notion, whereas space is connected to geometry. Thus while mathematical structures seem to help bridge the gap between time and space prima facie, they may in fact deepen the divide.[91]

Time and space are understood as having fundamentally different structures, which does not allow for measuring them in the same way, and so they cannot be combined in an account of motion. Speed as we understand it, in terms of the distance covered over the time taken, cannot be conceptualised in this framework. Plato's account of motion is given only in temporal terms; Zeno's paradoxes of motion still cannot be dealt with. Aristotle will attempt to solve this problem of how we can bring together time and space in order to measure motion by understanding both time and space as continua, and thus as possessing a similar internal structure. It is to Aristotle and this attempt that we will now turn.

themselves the work of the demiurge . . . Space is a condition without which Reason could not produce the visible order. Time is a feature of that order".

90 See Sattler 2012.

91 See Chapter 9 and Aristotle *Posterior Analytics* I, 7, 75a–b, who claims that arithmetic and geometry are different in genus.

7

Aristotle's Notion of Continuity: The Structure Underlying Motion

7.1 Introduction

Aristotle's *Physics* is the final point in the development of natural philosophy for our project since it presents the first account of motion and speed as a relation of time and space. The foundation for quantifying the relation between time and space is a specific notion of continuity, more precisely, a mathematical understanding of continuous magnitudes as incorporated into and developed in Aristotle's account of both spatial magnitude and time. The current chapter will concentrate on this notion of continuity as developed in Aristotle's *Physics*. However, by demonstrating that time and space[1] are essentially such continua and share the same basic internal structure, Aristotle prepares the ground for connecting time and space and for showing how we can in principle use them to measure motion; this second step will be fully developed in the next chapter. Thus, while the current chapter deals with Aristotle's reply to what I called the 'continuum problem' in Zeno's paradoxes of motion, the following chapter will examine Aristotle's response to the motion problem. The discussion here will also show ways in which Aristotle implicitly replies to crucial features of Zeno's paradoxes of plurality.

Perhaps somewhat surprisingly, given the status of natural philosophy before him, Aristotle simply starts the book we call *Physics*, without further ado, by claiming it to be *physeôs epistêmê*, a science of nature (184a15),[2] when it seemed only an object of *doxa* for Plato and Parmenides. Aristotle has several reasons of his own for the enhanced epistemic status of physics: for example, that it deals with the first principles and causes of the natural realm.[3] But with respect to other possible preparations for a science, namely logical operators, criteria of inquiry, and the employment of mathematics, he can also take over and further develop important points from Plato.

We have seen that Plato clearly distinguished between operators and operands in the *Sophist*, even if he did not always stick to this distinction. Aristotle develops this further with his distinction between primary substances

[1] 'Space' in this chapter is shorthand for spatial magnitude or distance.
[2] Compare the first sentence of *De caelo*, 268a1 ff. and *Metaphysics* E, 1.
[3] See *Physics* I, 1, 184a 10–18, and book II.

(operands) and the other categories (operators).[4] Plato also prepared the ground for Aristotle with respect to the criteria of inquiry – consistency, rational admissibility, and the principle of sufficient reason – in the *Sophist* and *Timaeus*. For Plato, too, studying the natural world must meet these criteria, even if only in a restricted sense, as is appropriate for an *eikôs mythos*.

Aristotle famously devotes almost a whole book of the *Metaphysics* to the notion of consistency, to the different formulations of the principle of non-contradiction. These important developments need not detain us here.[5] What matters for the current project is that Aristotle does not restrict this criterion to the realm of metaphysics proper but, following Plato's expansion, explicitly applies it to all philosophically or scientifically rigorous investigations[6] (though different qualifications will be appropriate when applied to different realms).[7]

Furthermore, even if Aristotle may not subscribe to universal acceptance of the principle of sufficient reason, he develops the notion of a reason in play here further by distinguishing four kinds of causes and reasons in his *Physics*, what we have come to call efficient, formal, final, and material cause. I cannot discuss this fourfold distinction and its partly Platonic roots here, but we will get an idea of the role that final causation plays in his account of the unity of motion.

Aristotle takes up the criterion of rational admissibility from the atomists and Plato whereby the basic ontological constituents must be not only testable by our own reason but also explain the phenomena.[8] We will see in the next chapter how Aristotle takes up the idea that a measure allows motions in the sensible realm to be precisely quantified and hence statements about these motions to be testable by every rational being using her own reason. The current chapter will prepare the way by showing how mathematical notions helped to make the natural realm intelligible and thus a topic for rationally admissible discourse.

This is an idea Aristotle could also take over from Plato's *Timaeus* which, as we saw in Chapter 6, applies mathematical notions to natural philosophy in order to demonstrate the intelligibility not only of the celestial motions and the set-up of the World Soul, but also of the most basic elements of all bodies. And

4 See, for example, his *Categories* and *Metaphysics* Γ, 2, 1003b5 ff. Cf. Palmer 2009, pp. 129–33.

5 They are briefly discussed in Chapter 5. See *Metaphysics* Γ, 3–6 and 8; cf. Wedin 2004; Rapp 1993.

6 See, for example, 1005b15–16.

7 For example, in sublunary physics we must usually qualify that some physical thing cannot be F and not-F 'at the same time' or 'both actually', while when we talk about heavenly bodies, such differences of respect are not necessary (see, e.g., 1010a25 ff.).

8 In *De generatione et corruptione* I, 5, 321a17–29 Aristotle explicitly claims that a satisfying account of growth and diminution has to "save" (*sôzein*) what holds true of things growing or diminishing. See Bodnár 2012, p. 7, who takes this usage of *sôzein* as strikingly similar to the one in the motto *sôzein ta phainomena*.

this is the point that I want to concentrate on in this chapter: how mathematics contributes to establishing an *epistêmê* of nature for Aristotle.

In Aristotle's *Physics*, mathematics is used to show that natural philosophy meets the requirements of a science in at least two respects: by taking over central notions and *concepts* from the mathematicians (for example, the crucial notion of continuity) and by employing or imitating mathematical *methods*. Aristotle understands the mathematical sciences[9] as cardinal sciences, as is clear from passages like *De caelo* 306a23 ff. There he claims that those who assume indivisible triangles out of which bodies are constructed (i.e., Plato's *Timaeus*) must assert that not all bodies are divisible and thus come into conflict with "our most accurate sciences, namely the mathematical".[10] It is the methods of these "most accurate sciences" that we can find in part in the *Physics*, especially in book VI, which is the focus of attention in this chapter. The deductive structure of *Physics* VI also shows similarities with Euclidean geometry:[11] it can be read as a series of theorems stated and then proved, from which further proofs can be deduced.[12] Giving at least a part of the *Physics* a quasi-mathematical form[13] also contributes to understanding Aristotle's *Physics* as a science (a science of *kinêsis*) and *kinêsis* as a proper object of philosophy.[14]

Before I give an outline of this chapter, let me deal briefly with a few possible objections that my investigation in this and the following chapter might face. The main text for these chapters is Aristotle's *Physics* books IV–VI and VIII. I will treat these books as an attempt to provide one consistent account of motion. This presupposition might be questioned, given that the *Physics* is usually thought to have consisted originally of two different treatises, one on the principles of nature (*peri archôn, physika legomena*, or *peri physeôs*, which is usually taken to refer to books I–IV), and one on motion (*peri kinêseôs*, which is usually seen as encompassing books V, VI, and VIII with book VII probably inserted later).[15] I may therefore appear to be using passages from

9 For Aristotle there are several sciences that can be called 'mathematical sciences', some more physical, such as optics, mechanics, and harmonics, others pure, like geometry and arithmetic. Mendell 1998 has shown that geometry is a cardinal science according to the criteria for scientific investigations laid down in the *Analytics*.

10 We see that taking over the basic idea of employing mathematical notions in order to prove the intelligibility of the natural world does not prevent Aristotle from criticising the particular version in which Plato employs this idea.

11 See Jope 1972.

12 We might characterise some parts not as theorems and demonstrations but as equivalents to scholia, however.

13 Whether we think that it is indeed structured like a mathematical treatise or only echoes some features of a mathematical treatise, and whether or not we think that book VI is different from the others in dealing with the fundamentals of physics rather than physics.

14 Proclus in his *Elements of Physics* recasts *Physics* VI and VIII in axiomatic form.

15 See Ross' commentary, p. 1 ff. for these divisions. Other scholars have seen VII as more closely connected to the remaining books of the *Physics*. Understanding the book *peri*

what were originally two different treatises. However, Aristotle himself sometimes refers to the whole of the *Physics* as one treatise,[16] and we also find that the commentators propose different divisions of the books into two treatises: Andronicus and Simplicius, for instance, took books I–V as constituting the treatise on the principles of nature and books VI, VII, and VIII as constituting the treatise on motion. This speaks in favour of not making a strict division between books I–IV, on the one hand, and books V–VIII, on the other, and we will see that with respect to what is important for the current project, these two treatises work with the same basic assumptions.

A further objection might be that book VI will be the centre of attention in the current chapter while books V and VIII will be important supplements to it, and yet the *Physics*, or at least the *peri kinêseôs* treatise, is sometimes seen as culminating in book VIII, in the proof that there is only one type of motion and it is the cause of continuous generation.[17] However, considering the emphasis of the current chapter, book VIII can be read as developing the thoughts of book VI one step further (a step that is not essential for the investigation here): while book VI demonstrates that motion, time, and spatial magnitude are continuous, book VIII shows that there is a first continuous motion that is infinite. Book VIII also explicitly introduces the notion of potentiality and actuality into the account of continuity given in book VI, as we will see below.

Finally, it might be seen as surprising that I do not concentrate on books I and III of the *Physics*, since the first book provides Aristotle's solution to the problem of how motion can be accounted for despite its connection to non-Being, which the Eleatics pointed out, and the third book gives Aristotle's definition of motion. These two books are indeed crucial for Aristotle's understanding of motion as they provide what we can call his metaphysical account of *kinêsis*, that is, they show how *kinêsis* fits into his ontology. However, for our project, which concentrates on Aristotle's physical account of *kinêsis*, all we need are the points of books I and III that follow now.[18]

In *Physics* I, 8 Aristotle takes up the basic problem of generation that Parmenides raised in fr. 8, as we saw in Chapter 2: "What comes to be must do so either from Being or from non-Being, and both are impossible. For Being cannot come to be, since it already is, and nothing can come to be from

kinêseôs as referring to V, VI, and VIII goes back to Porphyry. See also Odzuck 2014, pp. 14–41.

16 For instance, at the beginning of the *Meteorologica*, and probably in *Metaphysics* Λ, 1073a32 and *Physics* 257a34 (but Aristotle also refers to the books on motion and on principles/nature separately; see Ross, p. 2).

17 One type with different instances. See, for example, *De caelo* II, 3.

18 It is not possible to do justice to Aristotle's metaphysical basis of motion in this book, nor can the causal aspects of motion in book III (in his discussion of mover and moved), which we do not find in VI.

non-Being, since something must be underlying" (*Physics* 191a28–31). While Parmenides understands Being and non-Being as Being and non-Being simpliciter, in the discussion of the *Sophist* we have seen Plato preparing the path to a solution: Being also is not and non-Being also is (i.e., each is or is not something specific). However, it does not become clear in the *Sophist* how this understanding helps us to grasp processes in particular. We can understand Aristotle to be preparing this application here.

His solution in the *Physics* is that changes[19] do not have to be understood as something changing from non-Being to Being or vice versa, but rather as something changing from not being some particular F to being this particular F such that something stays the same – the Being that undergoes the changes – and something becomes different – its not-being-F turns into its being-F. For Aristotle, a change involves one thing that underlies the change, which stays the same, and two opposites, F and not-F. There is no Being or non-Being simpliciter involved in change. So while the Eleatics were right that nothing can come into Being either from Being or from non-Being simpliciter, something can come to be some specific F from not being it before.

This analysis has an important predecessor in Plato's *Phaedo*, especially in 102d–103c, which presents a three-principle scheme: something underlying that takes up opposing features and remains during the change, like Socrates and the two opposites of tallness and shortness.[20] But only Aristotle casts this scheme in the traditional (but now corrected) idiom of being and non-being, as we can see in his example for the structure of change – the man who is not educated becomes a man who is educated (189b34 ff.)[21] – while at the same time transforming it into a more technical language. Furthermore, Aristotle always sticks to this scheme, while in other works, like the *Philebus*, Plato seems to use a two-principle scheme.

Aristotle does not apply this solution to locomotion in particular in book I but, given the Eleatic background, one possible application reads: if something moves, it can be described as changing from not being at A to being at A – a description of motion that can rest on Aristotle's solution for change, even if it does not yet give us what is specific to locomotion.[22]

In book I, Aristotle makes it clear that change, and therefore also motion, does not involve Being and non-Being simpliciter, but only a certain being (being F) and a certain non-being (not being F) belonging

[19] While Parmenides' problem is meant to exclude all kinds of processes – generation, change, as well as motion – Aristotle is not dealing with generation as such in the *Physics* (this topic he reserves for *De generatione et corruptione*); all his examples are examples of (mere) change.

[20] See Broadie 2019 for a detailed analysis of the *Phaedo* passage in connection with Aristotle's *Physics*.

[21] See also *Physics* I, 8.

[22] Cf. also *Physics* VIII, 3, 254a11–14.

to something.[23] But it is not obvious yet how *kinêsis*, which involves both a certain being and non-being, can be understood more specifically. Aristotle answers this question in book III with the help of the distinction between *dynamis* and *energeia* which he only hints at in book I. Before that, in book II, he gives an account of things being by nature (*physei*) as that which "has the *archê* of *kinêsis* and rest in itself" (192b13–14). Here for the first time in Western philosophy we get an explicit account of nature in terms of what can itself start or stop processes without any external support. Hence, nature refers to a specific realm that corresponds roughly, though not completely, to the realm in which later natural philosophy is interested. This understanding of *physis* also requires an account of *kinêsis* (200b12–15), which we find in the first chapters of book III.

Famously, Aristotle defines *kinêsis* in book III as the actuality of what is potential in so far as it is potential.[24] What is important in the case of motion is that we need actuality and potentiality simultaneously: to be in motion something must be actualising its potentiality, for example, actualising the potentiality to be in the marketplace by going to the marketplace.[25] But it cannot already fully have actualised this potentiality of being in the marketplace. If the actualisation is complete – being in the marketplace – there is no motion any longer, and if the potentiality is not yet actualised at all, there is no motion yet.

Accordingly, Aristotle calls *kinêsis* an *atelês energeia*, an incomplete actuality, in *Physics* 201b 32. This understanding also accords with Aristotle's

[23] In the case of actual generation of a substance there is also Being simpliciter, but it does not derive from non-Being *haplôs*.

[24] In 201a10–11, 201a27–9, and 201b4–5. It is a notorious question how to translate the Greek term *entelecheia* (*energeia*) here: translating it as 'actualisation' raises the problem that actualisation itself expresses a process (the process of becoming actual) so that we would define motion and change in terms of a change to actuality, and thus work with a circular definition. If, however, we translate it as 'actuality', we face the problem that an actuality could also refer to a product, state, or a skill, and thus would not help us to capture motion and change. Thus, we need to make sure that the translation points to a process when some of the potential is still potential, and that means unrealised potential. For simplicity's sake I will talk about 'actuality' in the following, but we should keep in mind that in the context here actuality is not meant to refer to some product, state, or skill. A similar translation problem arises with the last clause of the sentence "in so far as it is potential": if we understand this to mean potentially *being F* then again we seem to be dealing with the potentiality for being a product (like a house or a statue); if, on the other hand, we understand it as potentially *becoming F*, then again we are presupposing a process in order to define a process. See Gill 2003. In contrast to an actuality in the narrower sense, a process essentially involves having an unrealised potential. See Coope 2012, p. 279.

[25] Strictly speaking, *kinêsis* involves a mover and a moved, the former with an active potentiality, the latter with a passive potentiality, while the change is the joint actuality of both. See Gill 2003, pp. 8–11.

distinction of complete and incomplete activities and his account of *kinêsis* and *energeia/entelecheia* in his *Metaphysics* book Θ, 6: there motion is used as an example for a wider notion of actuality, but it is contrasted with a narrower notion of actuality by pointing out that *kinêsis* is never complete and aims at an actuality other than itself, while *energeia/entelecheia* in the narrower sense is complete in every moment and is (or includes) its end.[26] While the latter passage is somewhat disputed,[27] almost all scholars agree that some distinction between complete and incomplete activity can be found in Aristotle – and *kinêsis*, at least in the strict sense, is an imperfect activity.[28]

This distinction between incomplete and complete activity also implies a distinction with respect to homogeneity, which is of interest for our discussion later on. If I am engaged in a narrow actuality, like perception, the actuality in every temporal stretch is as complete in itself as in any other stretch, and in this sense all temporal parts of an *energeia* seem to be completely homogeneous. By contrast, parts of a motion (and of other continua) are not fully homogeneous in this respect, since, for example, with a motion from Athens to Thebes, a part closer to Thebes has realised more of the actuality than a part closer to Athens.

There are also motions for Aristotle that are not goal-oriented and thus simply continuous, however; for example, the heavenly bodies' circular motion, breathing, or walking for the sake of health and not in order to arrive anywhere. In these cases it is better to think of the movement as actualised potential for moving. What is, nevertheless, essential for *kinêsis* also in these cases is the fact that there is always some unrealised potential (even if it is not a goal-oriented motion) in contrast to a mere actuality without unfulfilled potential. The fact that motion always involves both actuality and potentiality, never only one of them, implies that the actuality in question is restricted in some respects, which will be spelt out in detail below.

In order to describe the internal structure of motion, time, and spatial magnitude, Aristotle uses a familiar term – *suneches*, being continuous. But his particular understanding of *suneches* is new to the philosophical discussion. The most important earlier usage of the term *suneches* is found in Parmenides' poem. *Suneches* as expounded by Aristotle in the *Physics* is one of the central notions he employs from the realm of mathematics and it is developed further

[26] So a *kinêsis*, as, for instance, the process of building a house, as long as it is going on, always points to its end (the fully built house) but is necessarily incomplete – the house is not yet fully built. As soon as completion is reached, the *process* of building is over. By contrast, actuality in the narrow sense, like seeing red, does not aim at something beyond itself, it has its end in itself. And usually an actuality in the narrow sense is complete at every moment, while *kinêsis* takes time. See Makin, pp. 141–50.

[27] See Burnyeat 2008 for the claim that *Metaphysics* Θ, 6 is not part of the original book.

[28] See, for example, *Physics* 201b, 257b or *NE* X, 4 1174a14–b10. There is, however, also a wider sense of *kinêsis* in which it seems to include perfect activities.

for a physical context. Accordingly, the current chapter will start by looking briefly at two accounts of *suneches* that preceded Aristotle, one philosophical and one mathematical, and by providing a reminder of the most important account of magnitudes that does not understand them as continua, namely, the atomistic account. Aristotle's notion of continuity responds to all three accounts, but in different ways. That he is reacting to different accounts of magnitude will also explain in part why we find two different characterisations of continuity in the *Physics* whose relationship with each other Aristotle does not make explicit. We will see that while Aristotle takes up the term *suneches* from Parmenides, his understanding of it is strongly influenced by the practice of the mathematicians.

Subsequently we will look at Aristotle's own notion of continuity as developed in books V, VI, and VIII of the *Physics*. We will first consider his two different characterisations of continuity before the bulk of the chapter spells out the most important features of continuity as understood by Aristotle: a new part-whole relation, a new understanding of limits, and a new notion of infinity. Finally, I will show how this Aristotelian understanding of continuity attempts to resolve the difficulties stemming from what I have called the 'continuum problem' of Zeno's paradoxes.

7.2 Notions of Magnitude Influencing Aristotle's Concept of a Continuum

Most of this section concentrates on two notions of continuity that preceded Aristotle – the philosophical understanding we find in Parmenides and a mathematical understanding that is implicitly at work in the mathematical practice of the time and that Aristotle takes up as a basis for his own philosophical work. Continuity per se is not the focus of our investigation, however, but rather continuity in so far as it is a feature of different magnitudes. Accordingly, a brief consideration of atomistic conceptions, the most important way of conceiving magnitudes that does not assume them to be continua, is also called for.[29]

The current section will start by considering the notion of *suneches* in Parmenides, which implies indivisibility and absolute homogeneity, an understanding taken up by Zeno. I will then provide a brief summary of atomistic accounts of magnitude, which keep indivisibility as an essential feature, but abandon (global) homogeneity. The reverse is true of the last notion we will consider, the mathematical notion of *suneches*, which rests on the idea of unrestricted divisibility but also preserves some idea of homogeneity. This mathematical notion serves as a basis for Aristotle's account of continuity in

[29] We should bear in mind that the distinction between continuous and discrete magnitudes we are used to today is only gradually developed in early Greek thinking. See below.

the *Physics*, while the Parmenidean and the atomistic notions of magnitudes are the principal points of attack against which Aristotle's account is directed.

7.2.1 Parmenides' Suneches

We turn first to the most important philosophical understanding of *suneches* before Aristotle, that found in Parmenides. The fact that Parmenides' Being is called *suneches* might suggest a close connection to Aristotle's continuous magnitudes, but there is in fact a crucial difference between Parmenides' *suneches* and Aristotle's: continuity implies *divisibility* for Aristotle, but *indivisibility* for Parmenides. Behind Parmenides' understanding seems to lie the assumption that a division is possible only where there is a difference – if something is divisible then it must be divisible by virtue of a difference within itself. But since for Parmenides what is *suneches* is homogeneous in every respect, it is necessarily indivisible.[30]

Both thinkers start from an understanding of that which is continuous as homogeneous, as what is internally uniform. However, they draw opposite inferences from this assumption: Parmenides claims that homogeneity implies absolute indivisibility, whereas for Aristotle it entails indefinite divisibility. This difference can partly be explained by their different starting points. Aristotle, coming, as we will see, from a notion of continua relevant to the realms of mathematics and physics, starts with something spatially extended; accordingly, differences can be drawn simply from the fact that one part of what is spatially extended is here, while another is there (231b4–6). Thus, such continua seem to be divisible, unless something speaks against it. By contrast, Parmenides starts from something that shall be shown to be consistently conceivable on his terms, which implies that it has no difference and is not physical – accordingly, there does not seem to be a reason for dividing this consistently thinkable homogeneous thing.[31] In addition, as we have seen, Zeno's plurality paradoxes claim negative consequences for divisibility: assuming divisibility undermines a strong notion of unity (fr. 1) and the parts of such a division cannot be conceived in a consistent way (fr. 2). Aristotle will

[30] See Makin 1993 on Zeno on this topic. As we saw in the second chapter, Parmenides thinks he has an argument for excluding the possibility of any differences by showing that any difference implies non-Being and that the notion of non-Being is inconsistent.

[31] Part of the difference may also be based on a different understanding of what counts as a sufficient reason for assuming divisibility. If we take the principle of sufficient reason as requiring a positive reason to assume something, then we may claim with Parmenides that there is no reason for dividing what is homogeneous (since there is no difference anywhere), and thus it is indivisible. But if the principle is understood as demanding only that there is no reason against the assumption, then we may follow Aristotle and claim that since no reason speaks against any division, what is homogeneous is divisible in whatever way we like.

question these negative consequences and show that there is no reason to deny the divisibility of what is homogeneous; what is homogeneous is divisible.[32]

Let us first look into Parmenides' notion in a bit more detail by examining three passages from fr. 8 that show being *suneches* to imply absolute homogeneity and indivisibility.[33] In the first passage, Parmenides calls his Being *suneches* for the first time:

> οὐδέ ποτ' ἦν οὐδ' ἔσται, ἐπεὶ νῦν ἔστιν ὁμοῦ πᾶν,
> ἕν, συνεχές·
> never was it, nor will it be, since it is now all
> together, one, continuous. (fr. 8, lines 5–6a)

Being "now all together, one, continuous" is given as the reason why "was" and "will be" cannot be truly said of Parmenides' Being. What was and will be seem to be the things belonging to what mortals assume on their way of *doxa*, as well as what we deal with in our everyday world. These things are spread out temporally, they are extended in time: they were there in (some part of) the past and will be there in (some part of) the future. By contrast, what truly is not subject to these temporal differences, since it is "now all together". As we have already seen in the chapter on Parmenides, "now" in this passage is best interpreted as indicating atemporality.

Eon is *all together* – "being all together" combines two basic features: it is all (πᾶν), which seems to be taken up by calling it "one", and it is together (ὁμοῦ), which seems to be taken up by calling it "continuous". And since "being all together" is named as the reason why *eon* cannot be subject to temporal difference, it in itself seems to deny any temporal extension, any "was" and "will be".

For Parmenides, being *suneches* excludes not only temporal differences but also other kinds of difference, as the second passage demonstrates:

> οὐδὲ διαιρετόν ἐστιν, ἐπεὶ πᾶν ἐστιν ὁμοῖον·
> οὐδέ τι τῆι μᾶλλον, τό κεν εἴργοι μιν συνέχεσθαι,
> οὐδέ τι χειρότερον, πᾶν δ' ἔμπλεόν ἐστιν ἐόντος.
> τῶι ξυνεχὲς πᾶν ἐστιν· ἐὸν γὰρ ἐόντι πελάζει.

(1) And it is not divisible since it is all homogeneous.
(2) Nor is it more anywhere, which would prevent it from being one continuous, nor less, but it is all full of being.
(3) Therefore it is all continuous, for Being is in contact with[34] Being. (fr. 8, lines 22–5)

[32] Ultimately, this also seems to lead to a difference in the understanding of homogeneity. While for the Eleatics what is homogeneous must be completely uniform, for Aristotle this uniformity need not be assumed in all respects, as we will see below.
[33] For a more extended discussion see Sattler 2019a.
[34] *Pelazei* can be understood in a figurative sense here which avoids restricting it to a physical context. See McKirahan 2008.

The first step in this argument claims that Parmenides' Being is all homogeneous (ὁμοῖον), which implies that it is not divisible. The second step rules out a condition that would prevent it from being continuous, namely, being more or less; instead it is as a whole full of being, which seems to mean equally full, neither more nor less. The third step, "Being is in contact with Being", points out that all of Being is connected, and so, presumably, there is nothing in between anywhere that is not Being, which would undermine the homogeneity of *eon*. We will see that this formulation "Being *is in contact with* Being" is echoed by Aristotle's first definition of the continuum as "those things whose limit touch and are one" (227a10–12).[35]

"Therefore it is all continuous" suggests a conclusion – what precedes should explain why *eon* is "all continuous". The features in steps (1) and (2) that seem to guarantee *eon* to be *suneches* are that it is not divisible, it is all homogeneous, it is not more anywhere nor less, and it is all full of being. Accordingly, being homogeneous seems to be a weaker notion than being *suneches*; for something to be *suneches* means being *homoion* plus fulfilling some further criteria. The Greek word *homoion* basically means 'being like something' or 'being of the same kind'[36] – Being is not divisible into different kinds. If understood in this way, claiming *eon* to be *homoion* would still leave the possibility of other differences, like quantitative or qualitative differences. At least some of these possibilities are then excluded with the following lines: there is no more or less that would prevent Being from being continuous. 'More or less' does not refer to indivisibility according to kind, but rather seems to refer to some other features that would allow for difference and thus for divisibility. Thus being *suneches* not only implies indivisibility in kind, but also excludes some kind of qualitative or quantitative difference (be it temporal, spatial, ontological, or logical).

The third passage takes up the discussion of conditions that would prevent Being from being *suneches*, although it does not use the word; it not only denies any more and less, but also introduces a denial of *eon* being larger or smaller:

> αὐτὰρ ἐπεὶ πεῖρας πύματον, τετελεσμένον ἐστί
> πάντοθεν, εὐκύκλου σφαίρης ἐναλίγκιον ὄγκωι,
> μεσσόθεν ἰσοπαλὲς πάντηι· τὸ γὰρ οὔτε τι μεῖζον
> οὔτε τι βαιότερον πελέναι χρεόν ἐστι τῆι ἢ τῆι.
> οὔτε γὰρ οὐκ ἐὸν ἔστι, τό κεν παύοι μιν ἱκνεῖσθαι
> εἰς ὁμόν, οὔτ' ἐὸν ἔστιν ὅπως εἴη κεν ἐόντος
> τῆι μᾶλλον τῆι δ' ἧσσον, ἐπεὶ πᾶν ἐστιν ἄσυλον·
> οἷ γὰρ πάντοθεν ἶσον, ὁμῶς ἐν πείρασι κύρει.

[35] See below. Parmenides talks of *pelazei*; Aristotle will use the term *hapthesthai*.
[36] See Pape 1880.

> Since there is a final limit it is everywhere complete, like the mass of a well-rounded sphere, equally balanced everywhere from the centre. For it is necessary that it is not any larger or smaller here or there, since there is no non-Being that would prevent it from being homogeneous; and it is not being in a way that there would be here more there less of Being, since it is as a whole unscathed. For being everywhere equal to itself, it is present equally [ὁμῶς] within its limits. (fr. 8, lines 42–9)

The main aim of the argumentation here is not the continuity of *eon*, but its completeness, which is established with the help of three steps. Steps two and three can nevertheless be understood as giving an account of what it means to be continuous, which seems to be a necessary condition for completeness, while the first step is specific for demonstrating completeness. This first step is given right at the beginning, while the other two follow the conclusion that *eon* is everywhere complete:

(1) There is a final limit.
(2) There is no non-Being, hence Being cannot be larger or smaller.
(3) Being is as a whole unscathed, hence it is not more or less.

Conclusion: Being is everywhere complete.

The main claims of steps (1)–(3) have been established earlier in Parmenides' poem and therefore can be used here as premises with which the reader is already familiar.[37] They are used to explicate that the conditions under which the One would not be what we could call 'continuous' are absent. What would prevent the One from being continuous and thus everywhere complete could be either non-Being or an unequally distributed Being. The former would lead to the One being larger or smaller here or there,[38] the latter to more or less being.[39] But since there is no non-Being (as shown in frr. 2, 6, and 7) and not more of Being here and less there (as claimed in lines 22–5), the continuity of Being as a whole is granted.

While 'larger' and 'smaller' suggest quantitative difference, 'more' and 'less' could also cover qualitative differences (for example, more or less hotness, blueness, etc.). Given that Parmenides seems to employ these terms for distinct kinds of difference, it seems plausible that he wants to

[37] Premise 1 had been introduced in lines 26 and 29 ff. Premise 3 seems to take up the argument from passage 2. The main claim from Premise 2, that there is no non-Being, is already familiar from fr. 2, even if so far it has not been connected to not being larger or smaller or being continuous.

[38] This argument, that being smaller or larger is eliminated if there is no non-Being, seems to rest on the thought that non-Being could increase and reduce Being by being included within Being.

[39] This may also refer to being denser or rarer as we find it as modifications of Anaximenes' air.

rule out something like what we would call quantitative as well as qualitative differences here.[40]

The argument reconstructed so far ensures the continuity of Being; but it is not enough to support the *completeness* of Being. This requires, in addition, a final limit (premise 1). Accordingly, the summary that rounds off this argument takes up all the features necessary to grant completeness: everywhere equal to itself, *eon* is present equally (an apparent reference to continuity) within its limits. Thus our passage can also be seen as echoing and in part further developing lines 29 ff.: there the completeness of Being relies on a limit as well as on Being not lacking anything. The notion of such a final limit will also be of great importance for Aristotle, who takes it up as what we will call an 'outer limit' below. For Parmenides this notion of a final limit does not necessarily entail that something is limited by something else – there is no other thing apart from the One Being that could limit it, so the limit does not seem to be a constraint from outside, but is rather self-imposed by Being.[41] By contrast, Aristotle's notion of an outer limit usually implies, as we will see, that there is something else outside (with the exception of the case of the world as a whole).

7.2.2 Atomistic Notions of Magnitude

In Chapter 4 we looked at the oldest explicit atomistic position of which we have clear evidence, that held by Leucippus and Democritus. Theirs is one crucial atomistic position against which Aristotle argues, but not the only one. Aristotle also seems to have in mind atomistic tendencies flourishing in the Academy, which are clear, for example, in Plato's *Timaeus*.[42] And in book VI Aristotle offers arguments directed against point atomism, so against the assumption of atoms that are of zero-size, like points.

But let us concentrate here on positions employing extended atoms. They assume ultimate particles that are without internal or qualitative differences,[43] but of different size and shape.[44] These extended atoms cannot be further divided, and they function as parts for all the phenomena we perceive. Like

40 This does not require that Parmenides would have distinguished between quality and quantity in the way familiar to us since Aristotle.

41 Within the Parmenidean context, the meaning of such a self-imposed limit seems to be that Being is fully determined by itself (it does not need something outside to determine it), even though we may wonder whether the notion of being limited or unlimited fits the Parmenidean framework at all. See also Owen 1960, p. 65, who understands Parmenides' usage of *peras* here as "the mark of invariance", of constancy.

42 With respect to the early Academy's belief in indivisible magnitudes, see also *On Indivisible Lines*.

43 See Joachim, p. 166.

44 See Aristotle's *Physics* 188a 22 ff., 203a33 ff.; *De caelo* 303a4 ff.; *De generatione et corruptione* 314a21 ff., 315b6 ff.; *Metaphysics* A, 4, 985b4 ff.; Simplicius *In Phys.* 1882, 36,1 ff., Simplicius *In de Caelo* 1894, 242,15 ff., 294,33 ff.; Cicero *De natura deorum*, I, §66.

Parmenides, the atomists claim indivisibility for their basic entities but, unlike Parmenides, they do not require absolute homogeneity of everything there is;[45] they hold that there is only local, not global, homogeneity. This position allows the atomists to reply to the problem of infinite divisibility raised in Zeno's paradoxes, or at least this is what we gather from the beginning of Aristotle's *Physics*:

> ἔνιοι δ᾽ ἐνέδοσαν τοῖς λόγοις ἀμφοτέροις, τῷ μὲν ὅτι πάντα ἕν, εἰ τὸ ὂν ἓν σημαίνει, ὅτι ἔστι τὸ μὴ ὄν, τῷ δὲ ἐκ τῆς διχοτομίας, ἄτομα ποιήσαντες μεγέθη.
>
> Some gave in to both of these [sc. Eleatic] arguments – to the argument that all is one if Being means one, by saying that non-Being is, and to the argument from dichotomy, by positing atomic magnitudes. (187a1–3; translation by Furley, slightly modified)

The positing of atomic magnitudes is understood to be a reaction to the dichotomy argument. The people who posited atomic magnitudes have been identified either as Leucippus and Democritus or as members of Plato's Academy.[46] And the dichotomy argument has been interpreted as referring either to Zeno's first paradox of motion or to his plurality paradoxes.[47] For our current purposes we have no need to settle this question. All that is important is that the argument for the existence of atoms, whether Leucippian or Academic, was used to avoid some of Zeno's paradoxes.

Zeno's paradoxes, those of motion as well as of plurality, showed that the divisibility of magnitudes seems to lead into contradictions, as we saw in Chapter 3. Atomists try to avoid these problems by assuming that the divisibility of magnitudes will not go on infinitely, in which case we do not need to deal with the problem of infinitely many parts making up a finite whole. Rather, they argue, the division of magnitudes stops after a finite number of divisions, namely, when we reach the ultimate constituents of a magnitude, its

45 In Democritus and Leucippus there is usually non-Being, void in between two atoms.

46 Alexander and Porphyry understood this to be a reference to Plato and Xenocrates. See also Furley 1967, pp. 88, 104–10 and Sedley 2007. Makin 1993, p. 51 and others have understood it to refer to Leucippus and Democritus. While the example of the biped animal immediately before the passage quoted seems to speak in favour of Academic atomism, Aristotle's explicit reference to Leucippus in 325a23 ff. resembles crucial features of our passage and the very same people are claimed to assume "that non-Being is", "non-Being" being a common name for the atomistic vacuum. See, e.g., *Metaphysics* A, 985b4–10; *De generatione et corruptione* 325a. See also Ross, ad loc., p. 479 ff.; Burnet 1930, §173 for this interpretation.

47 For the understanding of this as Zeno's first paradox of motion, see 239b22, where Aristotle himself calls this paradox "dichotomy". See also Zekl's commentary, ad loc., and Furley 1967, p. 82. By contrast, in his 1936 commentary, p. 479, Ross follows the ancient commentators in thinking that it refers to Zeno's arguments against plurality (DK29 B2 and B3, Lee frr. 9 and 11).

atoms. Thus, the existence of atoms not only prevents the seemingly absurd implication that a finite magnitude 'contains' infinitely many extended parts, but also blocks the alternative and equally dubious road that an extended magnitude consists of parts that are not extended.

When Aristotle goes back to the assumption of infinite divisibility, rejecting indivisible atoms, he must show that his account of magnitude can equally well avoid such seeming contradictions. We will see that he does even more: he shows not only that his account of continuous magnitudes can block Zeno's paradoxes, but also that the atomistic solution, if used against the motion paradox called 'dichotomy', leads to an uncomfortable side effect: since time and space are not infinitely divisible in an atomistic picture, different speeds can only be compared up to a certain point.[48] Aristotle's own solution, by contrast, allows for unrestricted comparability.[49]

7.2.3 *A Mathematical Notion of* Suneches

As we will see below, Aristotle understands magnitudes as continua and continua as being divisible without end. This understanding of continua is one of the crucial notions Aristotle employs from a mathematical context in order to establish a science of locomotion. Surprisingly, however, we do not find a discussion or explicit definition of *suneches* as such in the mathematical texts handed down to us from the time before or just after Aristotle. In Euclid, our best mathematical source close to the time of Aristotle, the term *suneches* is not defined and seldom found: there are only thirty-four occurrences in the *Elements*, most of them in book VIII, and most of the time the term refers to a continued proportion – to a continuous ratio as in book VIII, prop. 8.[50]

[48] If we compare a faster and a slower motion (as Aristotle does in *Physics* VI), we can say that what moves faster will cover a certain distance in less time than what moves slower – up to the point where we reach atomistic sizes. If the slower moves a certain distance in one time atom, the faster should cover this distance in less than an atom of time, which is not possible, however, if we assume time atoms. And the same holds true for space atomism: at a certain point we cannot say that what moves slower will cover less distance in a certain time than what moves faster, because there is no distance smaller than a space atom.

[49] See *Physics* VI, 2 and the next chapter.

[50] Heath 1921, p. 11 ff. assumes that mathematics itself from the Pythagoreans onwards distinguished arithmetic (and music) as the study of *posa* (multitude or discrete quantity) and geometry (and astronomy) as the study of *pêlikon* (magnitude or *continuous* quantity). According to Aristotle *Metaphysics* B, 11, 1001b15–16, however, Zeno has not yet distinguished sufficiently between discrete and continuous quantities; but in Gorgias DK82 B3 (Sextus VII, 73) we find the idea of *poson* as a divisible *suneches* that is cuttable (*tmêthêsetai*).

Nevertheless, we find several clear indications that the mathematicians understood lines, surfaces, and solids as continua in the sense Aristotle makes explicit in the *Physics*: first, in a couple of passages of the *Elements*, Euclid uses the term *suneches* to mean 'successive' in the way continuous lines are, a two-place understanding of the term (AB is continuous with BC).[51] Furthermore, geometers must understand their geometrical objects as being *suneches* in the sense of being always further divisible for their mathematical activities. This becomes clear, for example, from the discussion in *On Indivisible Lines*, where the reply to the postulate of indivisible lines commonly relies on the assumption that mathematicians treat their geometrical objects as being always further divisible (969b20 ff.); a one-place understanding of *suneches*. While not explicitly discussed, this notion of *suneches* seems to have been taken for granted by mathematicians in geometrical constructions: it is clear that when they assume crossing lines and similar constructions, there is no reflection of atomistic worries, such as the concern that a line crossing the first line would need to go between two atoms; rather, infinite divisibility just seems to be assumed.

Moreover, while Euclid does not explicitly discuss the difference between numbers and magnitudes (a difference that could be taken to reflect the difference between discrete and continuous things), we find enough passages that clearly presuppose it, for example, *Elements* X, proposition 6: "If two magnitudes have to one another the ratio which a number has to a number, then the magnitudes are commensurable." This proposition obviously presupposes a distinction between *megethos* and number, since it singles out those magnitudes that resemble numbers, in that they also have a ratio to each other as numbers have, from those magnitudes that do not have this kind of relation (and that we will see in the next proposition to be incommensurable).

Finally, Aristotle himself claims that mathematicians understand magnitudes in this sense when he remarks in his discussion about the infinite in *Physics* book III that the mathematicians use it when they talk about their magnitudes as being infinitely divisible (203b17–18) – the very same notion we will see Aristotle using as the basis for his own definition of *suneches*. And in 200b18–20 Aristotle refers to existing definitions of being *suneches* as being infinitely divisible that are likely to be mathematical accounts. Accordingly, Aristotle's understanding of *suneches* seems to come out of a reflection on Greek mathematics – he can take up this understanding of magnitudes from Greek geometry (as a notion of magnitude the mathematicians constantly

[51] For example, in book XI, prop. 1, line 7 we read: "For, if possible, let a part *AB* of the straight line *ABC* be in the plane of reference, and a part *BC* in a plane more elevated. There will then be in the plane of reference some straight line continuous with *AB* in a straight line." And in book I, postulate 2: "to produce a finite straight line continuously in a straight line" (*kata to suneches* – so one thing picks up where another leaves off). See also book IV, prop. 16.

assume in their constructions), even if it is not prominently captured by the term *suneches* there.

The terminology Aristotle seems to pick up from the Eleatic thinkers, for whom, as we have seen, being *suneches* played a crucial role. Understanding Aristotle as combining Eleatic terminology with a mathematical understanding is also supported by Aristotle's *Categories* chapter 6. In 4b20 Aristotle divides quantities into those that are discrete and those that are continuous and then argues for understanding numbers as discrete and lines (surfaces, bodies) as continuous. The fact that he does not simply take it for granted that numbers are discrete, but rather feels he must argue that there is no contact between the parts of numbers, shows that he is not just taking over ready-made terminology. On the other hand, Aristotle's primary example in each case is taken from the realm of mathematics – numbers and lines.[52] So Aristotle seems to take up a distinction that is implicit in mathematical thinking and, by employing and expanding[53] Eleatic terminology, he introduces the distinction between continuity and discreteness into the philosophical realm.

While Aristotle seems to take up the *terminology* of being *suneches* from Parmenides, the mathematical *understanding* of magnitudes on which Aristotle bases his notion of *suneches*, is crucially different from the Eleatic notion. For mathematicians, magnitudes (and thus what is *suneches*) are divisible without any restriction; we can in principle go on dividing *ad infinitum*. While the mathematicians share with the Eleatics the assumption that being *suneches* implies being homogeneous – for the mathematicians, in the sense that each possible part of a continuous magnitude is treated alike – the infinite divisibility of the mathematical continuum is the very opposite of the indivisibility that Parmenides assumed.[54] The mathematicians must constantly assume continuity in the sense of infinite divisibility – it is presupposed by any geometrical operation that involves the mathematical bisection of a line, surface, or body, as is evident, for instance, in Aristotle's claim in *De caelo* 303a2 that the assumption of indivisibles is impossible, since then mathematics would not be possible.[55]

To claim that Aristotle employs a mathematical understanding of continuity in order to set up a science of locomotion is not to say, however, that Aristotle

[52] Once he has made the basic distinction clear with the help of the mathematical examples, Aristotle also gives examples from the empirical realm – spoken syllables on the one hand, and time and place on the other.

[53] The term *diôrismenon*, discreteness, employed in the *Categories*, does not seem to go back to the Eleatics.

[54] Waschkies 1977 thinks that Aristotle tried to bring together two unrelated theories about extended magnitudes, a Parmenidean one and one going back to conversations in the Academy between Plato and his students. For Waschkies, it was ideas from Eudoxus and the geometers of his time that allowed Aristotle to do so. Accordingly, the geometry of Aristotle's time is one essential strand in his notion of continuity.

[55] Cf. *On Indivisible Lines* 969b29–70a5.

simply takes up the mathematical notion and leaves it at that. On the contrary, he explicitly develops this understanding of *suneches* in such a way as to be able to deal with problems specific to motion and to account for the specific form of unity required for physical, but not mathematical, continua.[56]

Aristotle must make adjustments, especially at one crucial point, as he introduces the mathematical notion of continuity into his natural philosophy: in the physical realm there is a difference between being continuous and being contiguous, as he points out in V, 3, while this difference is not to be found in mathematics. For mathematicians, two lines in the same plane that touch are one[57] (as are two surfaces or bodies); they are continuous.[58] In physical contexts, however, two things that are continuous in themselves and next to each other do not become one thing simply by touching. Accordingly, in V, 3 Aristotle gives us two criteria for being continuous. Continuous are those things whose limits touch – this is what makes for continuous things in mathematics. Additionally, however, in the physical realm, these limits must be one, for otherwise we would be dealing only with neighbouring things. In the physical context we can distinguish limits that are merely touching from those that have become one, since where the limits are one we have an object that moves as a whole. Aristotle's examples in V, 3 show different ways in which the limits can be one.

We saw in Chapter 3 that Zeno heavily attacks the assumption of infinite divisibility in his paradoxes. We do not have the textual evidence to say whether the mathematicians did indeed respond to Zeno or simply ignored him, or perhaps they did not feel attacked because Zeno showed that infinite divisibility leads to paradoxes for physical things (like motion and a plurality of physical things), not explicitly for mathematical ones. But whether or not the mathematicians responded in any way, it is clear that when Aristotle takes over infinite divisibility as a crucial feature of continuous magnitudes in the *physical* realm, he must react to Zeno's attacks and show that they ultimately do have no force. Accordingly, Zeno's paradoxes are crucial as background to Aristotle's consideration of continuity in book VI, even if the basic understanding of *suneches* is imported from the mathematicians.

[56] For instance, with motion the potentiality of certain parts of the space through which the object moves must necessarily be actualised for the motion to take place, which is not true in the case of continuous spatial extensions the mathematicians deal with. In contrast to mathematicians, Aristotle must also deal with the question of how to conceive the transition from the continuum of motion to the continuum of rest.

[57] As long as they do not simply intersect or form an angle.

[58] And two points cannot touch but must coincide and thus be one (see, for example, *Physics* 231a29 ff. or *Metaphysics* K, 1069a12 ff.). The fact that Aristotle sometimes uses mathematical examples in his discussion of continuity in the *Physics* does not speak in favour of seeing a distinction between continuity and contiguity also at play in the mathematical realm, since he never uses mathematical examples to show that contiguous bodies are not necessarily continuous.

Since natural philosophers in the period investigated take over mathematical concepts in order to solve problems of conceptualising nature, as we see with Plato and Aristotle, it seems that to this extent at least mathematics had an influence on philosophy. In many cases, however, the influence is difficult to trace back with any precision, since our mathematical sources before Euclid are very sparse – parts of a history of geometry by Eudemus are preserved in Proclus' commentary on Euclid, there is a bit of Antiphon and Hippasus, we have passages in Plato and Aristotle referring to mathematical problems, examples, and discussions of their time, and there are fragments of Philolaus and Archytas; but most of the earlier mathematical texts seem not to have been handed down after Euclid.[59]

While our standard modern notion of the continuum[60] also stems from the realm of mathematics, as does Aristotle's notion originally, Aristotle develops this notion, as we will see, with the aim of applying it to the physical realm.[61] Thus for our investigation of Aristotle's notion of continuity, it will be useful to put aside our modern understanding of continuity for the time being. Not only have there been further conceptual developments (such as the assumption of actual infinity since Cantor) that are not possible within Aristotle's framework, but also Aristotle's and our modern accounts can be seen as dealing with two different answers to two different questions.[62]

With this note of caution let us now move on to Aristotle's account of continua.

7.3 Aristotle's Two Accounts of the Continuum

Aristotle provides us with two different characterisations of continuity in the *Physics*, the first in V, 3, the second in VI, 2:

(1) Continuous are those things whose limit, at which they touch, is one.
(2) Continuous is that which is divisible into what is always further divisible.[63]

We see that the first account of continuity is two-place, 'A is continuous with B', while the second is one-place, 'A is continuous' – we have already seen the

[59] The inverse direction, whether philosophy had any influence on the mathematicians, is heavily disputed. I will not take a stance on this question.

[60] Which for the most part is now seen as a property of functions of real numbers (or points).

[61] See, for instance, 204a34–b4, where Aristotle restricts to the realm of *aistheta* his investigation of the notion of infinity that is crucial for the concept of the continuum.

[62] Cf. Wieland 1962, p. 287, n. 7.

[63] Cf. Herold 1976. There is some disagreement in the secondary literature about the exact passages that give the definition of Aristotle's continuum. However, the basic features of the continuum seem to be more or less agreed upon. The second characterisation is sometimes also captured as 'what is infinitely divisible'.

distinction between one-place and two-place usages of continuity prefigured with the mathematicians.

But why does Aristotle give us two different characterisations and how are they related to each other? He does not explicitly answer these questions, but he starts book VI with a reminder of the first characterisation of continuity, from which he moves on, without any further ado, to the second characterisation in chapters 1 and 2; so it seems clear that he takes the two characterisations to be closely related.[64]

On my reading, we can understand the relation between the two accounts as two different approaches to the very same notion of a continuum that we may call synthetic and analytic, respectively. The first, the synthetic account, tells us what a continuum is like if we begin our consideration with individual things and want to know how a continuum can be 'made up' from this starting point – metaphorically speaking, since continua, at least strictly defined, cannot be conceived of as the result of adding individual single things. By contrast, the second account is an analysis of the continuum that takes it apart conceptually.[65] The first does not yet presuppose the existence of a continuum[66] and thus could be understood, according to Aristotle's *Analytics*, as a nominal definition, while the second is preceded by an existence proof – we are shown that motion, time, and spatial magnitude must be thought of as continua – and thus can be understood as a real definition.[67]

Furthermore, each account can be read as taking up one of the positions discussed above and then turning against it – the first responds to an atomistic notion of magnitudes; the second to Parmenides' conception. Accordingly, Aristotle gives us two definitions of continuous things, one nominal and one real, not only in order to fulfil the requirements of a scientific theory, but also to show that the main alternatives on offer for understanding magnitudes, the Eleatic and the atomistic, cannot hold. This should become clear when we turn now to look at both definitions in somewhat more detail.

7.3.1 Things Whose Limits Touch and Are One

In *Physics* V, 3 Aristotle analyses different types of physical unity.[68] For his purposes, the most important of these is being *suneches*:

64 Scholars agree that the two accounts are closely related, but there is some discussion of whether Aristotle does indeed provide us with two definitions in his *Physics* or whether one passage is the actual definition, while the other is either preparatory or a further development.

65 For this understanding of analytic and synthetic accounts, see Kant, *Prolegomena*, note on §5.

66 The immediate context of this account, 227a17 ff., indicates that being continuous is later in the order of becoming and a narrower term than being in contact and successive.

67 I owe the connection to the *Analytics* to Henry Mendell.

68 See especially 228a19–b15.

> τὸ δὲ συνεχὲς ἔστι μὲν ὅπερ ἐχόμενόν τι, λέγω δ' εἶναι συνεχὲς ὅταν ταὐτὸ γένηται καὶ ἓν τὸ ἑκατέρου πέρας οἷς ἅπτονται, καὶ ὥσπερ σημαίνει τοὔνομα, συνέχηται. τοῦτο δ' οὐχ οἷόν τε δυοῖν ὄντοιν εἶναι τοῖν ἐσχάτοιν. τούτου δὲ διωρισμένου φανερὸν ὅτι ἐν τούτοις ἐστὶ τὸ συνεχές, ἐξ ὧν ἕν τι πέφυκε γίγνεσθαι κατὰ τὴν σύναψιν. καὶ ὥς ποτε γίγνεται τὸ συνέχον ἕν, οὕτω καὶ τὸ ὅλον ἔσται ἕν, οἷον ἢ γόμφῳ ἢ κόλλῃ ἢ ἁφῇ ἢ προσφύσει.
>
> The 'continuous' is a subdivision of the contiguous: things are called continuous when the limit at which both touch has become one and the same[69] and is, as the word implies, held together: continuity is impossible if the extremities are two. This definition makes it plain that continuity belongs to things that naturally in virtue of their mutual contact form a unity. And in whatever way that which holds together is one, so too will the whole be one, e.g., by a rivet or glue or contact or organic union. (227a10–17)[70]

Aristotle begins his account of continuity by supposing two different things that get so close as to become one – a starting point that is a consistent continuation of his analysis so far in book V. His analysis opens by investigating the unity of physical things that are spatially apart, and then brings them closer together with each new concept of unity, as can be seen from the notions "in succession" and "contiguous" that precede the concept of continuity.[71] The run of examples – rivet, glue, organic union – also marks a gradual intensification of closeness.[72] This very order in the presentation of different unities allows Aristotle to choose a starting point that does not presuppose the existence of continua as what is infinitely divisible and that is therefore a starting point atomists could also share.[73] As the physical things come closer and closer together, Aristotle can move little by little from a notion acceptable to the atomists to his own conception, which adds the 'fusion' of the limits.

However, we thus get an account that seems to give a rather weak notion of continuity, as is revealed by closer scrutiny of the kind of *unity* that Aristotle is talking about in the passage cited. The examples in which things are connected by glue or a rivet seem to be especially constructed to illustrate the way two things that touch can *become* one – for example, if we try to

69 See *Metaphysics* B, 1002b2–3 and White 1992, pp. 12–13.

70 Translations of the *Physics* are by Hardie and Gaye, unless otherwise noted, often with some modifications.

71 The notions 'together', 'apart', and 'in between' seem to constitute a series of its own. For the difficult relation between 'in contact' and 'contiguous', see Ross, p. 626.

72 Cf. *Metaphysics* Δ, 1016a, where the continuous by nature is conceived of as being more one than the continuous by art.

73 Atomists cannot accept Aristotle's idea of a two-place notion of continuity for atoms, since two atoms cannot become one thing, as Democritus explicitly claims. They can accept it, however, for things in the phenomenal world, while they would reject infinite divisibility also for the phenomenal world.

displace one thing, the other one will now move with it. But these examples have not necessarily been selected to demonstrate all the aspects of a continuum.[74] The strong clinging together of the whole is well illustrated by the glue image, but these pieces glued together are not a uniform whole in every respect in the way Parmenides' *suneches* seemed to be (the glued whole could still be more here and less there). How then can Aristotle take being glued or being riveted as examples of a continuum? If we glue together two pieces of wood to form one plank, then insofar as we regard this glued result as a plank, it is continuous. Viewed in another way, however – for instance, in light of the grain of the wood or its colour – the result may not be considered continuous. Accordingly, in order to understand something as continuous, it need not necessarily be homogeneous in all respects; all we need is one respect in which it can be considered continuous.[75] This account of continuity is clearly a weaker notion than the one we saw Parmenides employing – it tells of a unity in at least one respect but not necessarily in all.[76] It also introduces a feature that will be crucial for Aristotle's understanding of continua:

Considering something as continuous in just one respect enables us also to recognise differences that may serve as starting points for a division – and it would allow Aristotle to show an open-minded Eleatic a transition from an Eleatic understanding of *suneches* to an Aristotelian understanding. For example, if we consider the length of a multicoloured stick, it is easy to imagine that the stick – while in fact continuous in its extension – could be cut at any of the points where the colour changes. Differences in colour could serve as a reason for understanding the length of the stick as divisible, even though it is continuous without any differences. Importantly, Aristotle goes one step farther – if the stick in its continuous length is divisible at a point where we find a difference in colour, then it should be divisible at any other point, too, for with respect to the continuous length no possible partition is in any way more natural than any other. Thus the stick is as divisible as one likes. This further point is exactly what we find in Aristotle's second definition.

But before we move on to this second definition, I should point out briefly three possible problems we face with Aristotle's first characteristic of continuity:

[74] Cf. *Metaphysics* Δ, 1016a.

[75] The 'physical' proof for the continuity of A and B employed by Aristotle in the *Metaphysics* employs force and motion: if A and B are joined in such a way that the resulting whole moves when force is applied just to A or just to B, they are continuous and not merely together. It is hard to see how this criterion can be applied in the case of motion itself in order to figure out whether a motion is continuous.

[76] Cf. Aristotle's characterisation of a continuous quantity in the *Categories* as that which "has a common boundary at which its parts join together" (5a1 f.).

(1) Aristotle sometimes talks about "the limit" in the singular being one – for example in the passage quoted at the beginning of this section (227a12), where he claims the *peras* to be one – and sometimes about "the limits" in the plural, for example, in 231a22, where he takes up the account of being *suneches* from book V as that whose *eschata* are one.[77]
(2) It seems unclear whether he actually claims that two things become one or that their limits become one (in the passage quoted, he seems to start out with the limit being one, but in the following he seems to assume that the things are thus one).
(3) In our passage Aristotle talks about the limit "being one", but sometimes there does not seem to be a limit left between the things that have become one, and in book VIII, he seems to think only of potential limits within a continuum.

I think all three problems can be solved if we take into account once more Aristotle's starting point here: if we start with two distinct things, such as two drops of mercury, which get ever closer together, then originally they have different limits (plural), at which the drops touch, so that first the two limits form one limit (singular), in such a way that we can still see that originally there were two different drops, and finally they form one drop so that the two drops themselves have become one.[78] Thus, two physical things become one continuous thing through their limits first becoming one. And this limit, while still perceptible at some point in the process, eventually disappears, so that the limits left within the continuous whole are only potential limits, limits that we can set as we please. (To assume that there is still an actualised limit between the two original wholes would be a problem for Aristotle's account of *suneches* as something that is divisible as one likes, since then the new whole is already prejudiced in favour of these two parts – if I then wanted to divide the new continuous drop into three parts, it would still, in addition, reflect the two original wholes as parts.)

7.3.2 Things Being Divisible without Limits

The first account started from premises the atomist could share and resulted in a notion of a whole that would still allow for absolutely simple unities à la

[77] We may be worried about Aristotle's use of the plural, since the limits of lines are points and he explicitly claims that points cannot touch (and if limits touch, would they not have to have limits themselves?).

[78] The question that seems to remain here is how two limits can touch – wouldn't they themselves need limits for that? I think this is only a problem for conceptual or mathematical limits, like points, not for physical limits, like the limit of the mercury drops, where we are never really dealing with points. Alternatively, we may think that Aristotle is talking loosely when he talks about 'limits' in the plural (as he also seems to in his talk about points touching in 227a29), and in fact thinks about a limit *at* which things touch.

Parmenides.[79] The second account, by contrast, starts from an undifferentiated whole and shows us a position neither the atomists nor Parmenides could share: it makes a point against Parmenides by requiring a continuum to be divisible, and against the atomists by claiming that the continuum is divisible *ad infinitum*. Against Zeno's paradoxes, Aristotle tries to show how the divisibility of physical magnitudes can consistently be thought even in its last consequences (i.e., as infinite divisibility).[80] In this section we will first look briefly at Aristotle's second account, before considering how he moves from the first to the second. A comparison of the different presuppositions employed in Aristotle's and Parmenides' notions of continuity constitutes the final part of this section. Let us start with Aristotle's second account: "I call continuous what is divisible into what is always further divisible" (λέγω δὲ συνεχὲς τὸ διαιρετὸν εἰς αἰεὶ διαιρετά, 232b24–5).

Continuous things are things that can always be divided further. The parts that result from division are of the same kind as the whole. For example, if we divide a length, each part will also be a length. And each part will also be a continuum, a "divisible". Continuous things are, borrowing a term from fractal geometry, 'self-similar' wholes.[81] This definition of continuity as what is always further divisible into divisibles is equivalent to infinite indivisibility, as can be seen not only from αἰεὶ (always) in the definition, but also from the prelude to this definition:

> διὸ καὶ τοῖς ὁριζομένοις τὸ συνεχὲς συμβαίνει προσχρήσασθαι πολλάκις τῷ λόγῳ τῷ τοῦ ἀπείρου, ὡς τὸ εἰς ἄπειρον διαιρετὸν συνεχὲς ὄν.
>
> Hence in acts of defining the continuous, the *logos* [definition] of the infinite is often used, since what is infinitely divisible is continuous. (200b18–20)

We see here that Aristotle is referring to existing accounts about the close relationship between being *suneches* and infinite divisibility, and thus not inventing this conceptual link himself. Though he does not name any names, it seems likely that he is referring here to some mathematical (more specifically, geometrical) definitions; they may have been by mathematicians

[79] However, it also allows for other kinds of wholes – divisible ones.

[80] While the atomists seem to claim that infinite divisibility *does not* hold, Zeno attempts to show that infinite divisibility *cannot hold* – and Aristotle seems to react to both.

[81] This self-similarity is expressed in the quotation above: "in whatever way that which holds together is one, so too will the whole be one" (227a15–16). In his arrow paradox Zeno also implicitly assumes self-similarity between whole and parts since he infers from the fact that the arrow does not move in the indivisible now that it does not move in the extended period. But while Zeno mistakenly assumes self-similarity between an extended period and an extensionless now, Aristotle deals with a true self-similarity for the extended parts behave like their extended whole.

associated with the Academy, like Eudoxus, or just refer to some general mathematical practice.

Aristotle's second account of continuity claims that continua can be divided wherever we please, and we will always get proper parts that are themselves continua and hence further divisible. Because there are no fixed given boundaries within the continuum, it is "always further divisible" without restrictions, a conclusion that can also be drawn from the very beginning of book VI, where we see how Aristotle moves from the first to the second account:

> Εἰ δ' ἐστὶ συνεχὲς καὶ ἁπτόμενον καὶ ἐφεξῆς, ὡς διώρισται πρότερον, συνεχῆ μὲν ὧν τὰ ἔσχατα ἕν, . . . ἀδύνατον ἐξ ἀδιαιρέτων εἶναί τι συνεχές, οἷον γραμμὴν ἐκ στιγμῶν, εἴπερ ἡ γραμμὴ μὲν συνεχές, ἡ στιγμὴ δὲ ἀδιαίρετον. οὔτε γὰρ ἕν τὰ ἔσχατα τῶν στιγμῶν (οὐ γάρ ἐστι τὸ μὲν ἔσχατον τὸ δ' ἄλλο τι μόριον τοῦ ἀδιαιρέτου), οὔθ' ἅμα τὰ ἔσχατα (οὐ γάρ ἐστιν ἔσχατον τοῦ ἀμεροῦς οὐδέν· ἕτερον γὰρ τὸ ἔσχατον καὶ οὗ ἔσχατον).
>
> Now if the terms 'continuous', 'in contact', and 'in succession' are understood as defined above, things being 'continuous' if their extremities are one . . . nothing that is continuous can be composed 'of indivisibles': e.g., a line cannot be composed of points, the line being continuous and the point indivisible. For the extremities of two points can neither be one (since of an indivisible there can be no extremity as distinct from some other part) nor together (since that which has no parts can have no extremity, the extremity and the thing of which it is the extremity being distinct). (231a21–9)

And a few lines further on:

> φανερὸν δὲ καὶ ὅτι πᾶν συνεχὲς διαιρετὸν εἰς αἰεὶ διαιρετά· εἰ γὰρ εἰς ἀδιαίρετα, ἔσται ἀδιαίρετον ἀδιαιρέτου ἁπτόμενον· ἓν γὰρ τὸ ἔσχατον καὶ ἅπτεται τῶν συνεχῶν.
>
> Moreover, it is plain that everything continuous is divisible into divisibles that are always further divisible: for if it were divisible into indivisibles, we should have an indivisible in contact with an indivisible, since the extremity of things that are continuous with one another are one and are in contact. (231b15–18)

The first passage demonstrates that nothing indivisible can be a part of a *suneches*, because the limits of indivisibles can never become one. For a limit must be different from what it limits, and so for the limits of something indivisible to become one would require the indivisible thing to be divisible to a certain degree, namely, into the limit and that which is limited. Thus, indivisibles cannot fulfil the necessary condition for being a *suneches* identified in the first definition. And this position prepares the ground for the second account. For if a continuum cannot be *composed* of indivisible parts, then neither can it be *divided* into indivisibles, as the second passage shows.

Accordingly, possible limits can be set within a continuum as one pleases, without any restriction.[82] And if something is continuous in the sense of being always further divisible and thus continuous in a one-place sense, then its potential parts are continuous in the two-place sense.

The first chapter of book VI shows that Aristotle is arguing not only against extended atoms, which we find in Leucippus and Democritus as well as in the Academy, but also against point atomism.[83] The argument in the first passage just quoted, 231a21–9, is explicitly directed against points. While this argument would in principle also work against extended atoms, a few lines later, 231b6 ff., Aristotle gives an argument that works only against point atomism, as it turns on the fact that between two points there is always a line (and thus potentially further points) and between two moments there is always time (and thus potentially further moments); hence point-like indivisibles cannot even be successive, let alone continuous.[84] In this way Aristotle ensures that he has covered the main variations of atomism (either assuming extended or non-extended atoms), whether or not they were adhered to by his contemporaries.

My second chapter showed that Parmenides, like the point atomists, takes something indivisible that is not extended as fundamental. Parmenides' assumption seemed to rest on the idea that divisions are possible only where there are differences in reality. Since Parmenides' *suneches* does not allow for ontological differences in any respect, no logical or epistemic division is possible either, for example, as merely conceptual divisions in a measurement process. By contrast, Aristotle allows for epistemic divisions even if there are no ontological differences: I can mark off a part of a continuum, even if there are no differences within the whole. Such a division seems to carry the stigma of arbitrariness, but it can provide additional information about the thing epistemically divided, allowing us, for example, to quantify it – a point whose crucial importance we will see in the next chapter. Furthermore, in order to consider something as a continuum, that thing is reduced to a certain respect, what I called a 'dimension',[85] and only needs to be homogeneous in this specific respect. It is not necessary that the thing be homogeneous in every respect, as with Parmenides, in order to qualify as

[82] More precisely, the first chapter of book VI initially shows that Aristotle's definition of the term 'continuous' implies that things to which it applies cannot consist of indivisibles; and he goes on to show that not consisting of indivisibles means being always divisible into further divisibles. The second part of the chapter is then dedicated to a demonstration of the existence of the continuous structures he is especially interested in, that is, of time, space, and motion.

[83] Thus he also argues against a continuum being composed of parts with nil extension, our version (α) in the Zeno chapter.

[84] This argument is not valid against extended atoms, since with extended atoms there need not be anything homogeneous in between the indivisible atoms.

[85] When a rod is considered only with respect to its length, we look at one dimension only. Speed, however, does not come into view unless we look at the relation of two dimensions, time and space.

continuous. Our multicoloured stick would still count as continuous for Aristotle, since it is enough if it is homogeneous in length, even if it is not homogeneous with respect to colour. It is the one dimension in question, length, that is always further divisible into similar parts.[86]

However, if a complex thing is reduced to one dimension (in the example above, to a length) is not this dimension as simple as Parmenides' One? How can we make any divisions if there are no differences with respect to the dimension in question? Aristotle can allow for possible divisions by comparing the continuous thing in question with another thing of the same dimension. This process is prominent in measurement processes, when, for example, a standard metre, as a unit, is compared with a beam that is to be measured.[87] Every comparison requires the assumption of a plurality – we need at least two things that we can compare, for example, the beam and the standard metre. Hence, such possible divisions are gained with a postulation the Eleatics did not share: the assumption of plurality.

Against Parmenides, Aristotle shows that the divisibility of a thing can be thought without contradictions. However, it is not yet clear how *infinite* divisibility can consistently be thought, as Zeno's paradoxes require. This needs a closer look at the implications of Aristotle's concept of continuity.

7.4 Implications of Aristotle's Concept of a Continuum

Aristotle's notion of continuity requires a new understanding of unity:[88] continua can have parts (see Aristotle's second account of continua), but

86 For example, dividing the length of the stick into parts will always give me parts of length, and nothing else; what happens to the colour – whether some green is now neighbouring other green or blue – is of no relevance. It seems that 226b27–31 is an objection to that since there "being continuously moved" is determined as leaving "no gap or only the smallest possible gap in the material – not in the time (for a gap in the time does not prevent things having a 'between'), while, on the other hand, there is nothing to prevent the highest note sounding immediately after the lowest)". Here the temporal interruptions are taken to be irrelevant even though these gaps mean that the continuous motion cannot always be further divisible into similar parts. However, in this passage, "being continuous" is not used in its terminological sense, which is only introduced later (see the commentaries of Zekl and Ross, and their proposal for the arrangement of the text ad loc.); it is only used in order to demonstrate the meaning of 'in between', and not to explain the unity of a continuous motion. In the passage under discussion Aristotle starts from a discrete row (he is dealing with the row of the discrete intervals of tones rather than with the continuous change of a pitch shift) that necessarily has gaps in between the different tones. Hence, he can also allow for gaps in time since time has nothing to do with the aspect or dimension in question, namely the tonal intervals.

87 It also requires what Euclid will call a ratio (*logos*) between the two things. See Euclid *Elements* V, definitions 3 and 4.

88 Compare as a contrast the relation of parts and whole as sketched in Plato's *Theaetetus* 204e (whether or not Plato endorses it). See also Burnyeat's edition, p. 205 ff.

they cannot be defined as the sum of such parts in the sense that the parts would be prior to the whole.[89] Furthermore, continua must be divisible *ad infinitum*, which, as we will see in the next chapter, grants measurability and comparability. But it is not yet clear how the unity of continua, such as motions, can be thought consistently. In order to see how Aristotle's conception of continua addresses this crucial question, we will split the problem into three sub-inquiries, each of which deals with a feature crucial to Aristotle's notion of continua: the part-whole relation, Aristotle's account of limits, and his discussion of infinity.

We should keep in mind that the paradigm continua for Aristotle are physical bodies, time, distance, and locomotion (liquid continua, like water, and things like moulding dough may have certain different features – for example, the distinction between contiguity and continuity that Aristotle introduces in the *Physics* works with solid bodies, but does not seem to work with water).[90] Aristotle's account of continuity is meant to fit the physical realm in so far as it contains bodies and their motions, what is involved in these motions, like continua over which a locomotion will take place, such as a path on the ground, and the time in which this locomotion is performed.

Since the potentiality of the parts that a continuum possesses is only introduced by Aristotle in book VIII, and not explicitly to be found in books V–VI, it has been questioned whether Aristotle's accounts of continua in V–VI on the one hand and in VIII on the other do indeed provide the same understanding of continua, and whether they are even compatible.[91] I take them as different contributions to the same idea of what continua are, even if the context and aim of argumentation differs in the treatises and thus Aristotle brings in different points at different times. But I do not think that the two main points of worry discussed in the literature show that these two accounts are in tension with each other:

(1) Homogeneity versus heterogeneity of motion: it has been claimed that while VIII, 8 demands that motion be homogeneous, books V–VI assume motion to be heterogeneous. As a premise for its proof that only circular motion can be eternal, VIII, 8 requires that motion is homogeneous in the sense that each part of the change is one in kind with each other part. By contrast, books V–VI assume heterogeneity of motion in the sense that the different parts of a motion are not of one kind. One part of a change is of the same kind as another if it has the same end point. But let us assume that the motion AC has as parts a motion from A to B and a motion from B

[89] While a mathematician nowadays may understand a continuum, like a line of the length 1, as the sum of ½ and ¼ and ⅛, etc., this only works for mathematical continua, not for physical ones.

[90] The universe as a whole seems to be not continuous, but only contiguous.

[91] Most recently so by Rosen 2015. See ibid. for the following two points.

to C. Then the two parts do not have the same end point and are thus not of the same kind, hence the motion is heterogeneous.

This understanding of motion as heterogeneous in books V–VI seems to be mistaken, however, as the two parts do have the same end point – in so far as AB is a *part of AC*, it also has C as its ultimate end point. We can find such a treatment of AB as part of AC in *Physics* 232b20–233a31, quoted towards the end of Chapter 8. Only if B is physically actualised (for example, by the body stopping at B) do AB and BC differ in their end point. But then we do indeed have two different motions (which may also show in that the body moving from A to B starts to slow down as it approaches B, in a way it does not when AB is just part of the motion AC).

(2) Potentiality doctrine: the fact that Aristotle does not mention potentiality in books V–VI, but only introduces it in book VIII has been seen as a sign of a conflict. However, it seems to me that Aristotle only introduces those parts of his overall account of the metaphysics of motion that he needs in any given context; he does not need the claim about potential parts yet when introducing continuity in books V–VI, only once he goes into more detail later on.[92]

7.4.1 A New Understanding of the Part-Whole Relation

One of the first problems we encountered with Zeno was a difficulty posed by the first paradox of motion, that covering a *finite* distance, a runner must traverse an *infinite* number of spatial pieces – or, expressed differently, that a finite whole seems to contain infinitely many parts. We called this the 'continuum problem", since it is not specific to motion but is the same for every continuum. Before we investigate whether Aristotle's notion of a continuum can help us to solve this problem, we must first have a clear understanding of the notion of parts and whole that Aristotle employs for continua.[93]

We saw in Chapter 3 that Zeno used two incompatible part-whole relationships, one giving priority to parts that constitute the whole, and one giving priority to the whole from which parts may be derived. In order to avoid this paradox, atomists vote for the first kind of relationship, assuming indivisible atoms as the ultimate things given, which are arranged as parts to form new wholes. In contrast, Aristotle's notion of a continuum is a part-whole

[92] While book VI introduces the notion that continua are always divisible into divisibles, book VIII then elaborates that the parts thus constructed are only potential parts (see, for example, 262a22–5).

[93] Aristotle of course knows many different part-whole relations; see, for example, *Metaphysics* Δ, chs. 25 and 26, or *De caelo* 274a31. However, for our purposes we will restrict ourselves to the whole-part relation implied in the notion of a continuum.

relationship along the lines of the second option, where a continuous whole is given and parts are dependent on it.[94]

The part-whole relationship employed by the atomists Leucippus and Democritus raises the question of how it can account for the unity of a whole constituted by atoms as parts. By contrast, the part-whole relationship which Aristotle employs leads us to consider how we can talk about parts at all in such a conception and to ask what makes for a proper part. We saw in the chapter on Zeno that parts of a continuum can be defined neither in a qualitative sense nor by their extension, which can be freely chosen. This observation is reinforced by Aristotle's two definitions of continuity:

(1) According to Aristotle's first account, something is continuous if the limit where two things touch has become one and, ultimately, the two things have become one. Hence, as long as we are dealing with parts of a continuum and not independent wholes, any possible limits marked between these parts must be such that the parts can become one, no longer distinguished as two different parts. Therefore, there cannot be any intrinsic distinction between such parts with respect to the continuous extension in question: parts of the multicoloured stick can have different colours and thus different qualities with respect to colour, but the parts do not differ in quality with respect to the continuous length of the stick.[95]

(2) Since Aristotle's second account states that something continuous must always be further divisible, the parts of such a continuum must be divisible themselves without restriction. Otherwise, the whole continuum would not be divisible into *divisibles*. But unrestricted divisibility implies that the parts of such a whole are not fixed, that there are no naturally given parts, in contrast to the atomists' conception of the atoms as ultimate parts. Hence, any extension can be further divided and there are no ultimate extensions with the help of which we could define a proper part once and for all.

Not much can be said about such parts: we get neither a quantitative nor any relevant qualitative characterisation that would distinguish them from each other decisively. I may for the time being put a mark in the middle of some

[94] For Aristotle's idea that the whole is prior to the part, see also 1041b12–33 (where it is stated that the whole which is not a compound, like a heap, is "not only its elements") and *Politics* 1253a18–25.

[95] The only *relevant* qualitative characteristic of the parts of a continuum is that with respect to the dimension in question, the parts have the same qualitative characteristic as the whole, e.g., each part of something spatially extended will be a spatial part. But this is not a feature that would distinguish parts from each other; it merely distinguishes them from parts of wholes that belong to other dimensions as, e.g., spatial parts from temporal parts. And we may claim that each spatial part has a different position, but this only works once we have marked out parts (that would otherwise not be there) and changes once we go on dividing these parts. Thus, we cannot characterise parts by their location definitively (though we may do so for a certain time or purpose).

distance so as to conceptually divide it into two and then distinguish the part to the right-hand side of the mark from the part to the left-hand side, but again these parts could be further divided and the whole continuum could be divided in such a way that our mark is no longer of any importance, as when I now divide the whole continuum into three equal parts. The parts of a continuum are alterable (i.e., further divisible into smaller and smaller parts). Accordingly, a continuum cannot be thought of as the sum of its parts in the sense that the parts are prior to and constitute the whole, which is the natural way to think of, for example, the whole of a heap of grains, a part-whole relation along the lines of the alternative chosen by the atomists. In the heap of grains we have given parts, the grains, the collection of which makes up the whole heap in the sense that the heap can be defined as nothing but the sum of these parts. The only thing that we can say of the parts of a continuum, by contrast, is that they are parts of a specific whole; only this *relation* will always be true.[96] This may seem to be of no significance, since the discrete parts of the heap of grains are also thought of in relation to the whole, for otherwise it would not be a *heap* of grains. However, these discrete parts, the grains, are to a certain degree also independent of the whole, since they make up the whole, and thus are prior to it, while the parts of a continuum are completely dependent on their whole, which is in every sense prior to them.

We will see that it still makes good sense to talk about parts with respect to a continuum, since we can mark off a stretch of a continuum – arbitrarily – as a part and measure out the whole with the help of this part. The possibility of partition in Aristotle's second account of the continuum prepares the grounds for such possible parts.

As the parts of a continuum are not given, the whole only potentially possesses parts. Note that the potentiality involved here is an attribute of the whole, not of the parts, since otherwise the existence of parts would be presupposed, as potential, but the only thing that can be presupposed here is the existence of the whole that has the potentiality to be divided into parts.[97]

Parts of whatever number and size can be actualised, but the possibility of being divided into parts can simply remain potential, it need not be actualised. An Aristotelian continuum has the potential to be divided anywhere, and so it seems that it has the potential to possess infinitely many parts. In comparison to other potentialities, however, the potentiality of actualising infinitely many parts, and indeed of actualising any parts of the continuum at all, is restricted in important ways, as we can see from *Metaphysics* Θ, 1048b14–16:

> τὸ δ' ἄπειρον οὐχ οὕτω δυνάμει ἔστιν ὡς ἐνεργείᾳ ἐσόμενον χωριστόν, ἀλλὰ γνώσει. τὸ γὰρ μὴ ὑπολείπειν τὴν διαίρεσιν ἀποδίδωσι τὸ εἶναι δυνάμει ταύτην τὴν ἐνέργειαν, τὸ δὲ χωρίζεσθαι οὔ.

96 Cf. also Harte 2002, p. 273.
97 Cf. also Charlton 1991, pp. 133–4.

> But the infinite is not potentially in such a way that it will ever be separated in actuality; but it will be separated [only] conceptually.[98] For the fact that the process of dividing never comes to an end ensures that this activity exists potentially, but not that the infinite exists separately.[99]

Taking into account this as well as other passages, we will see that for Aristotle the potentiality for there being parts of a continuum is restricted in three ways: (a) if such parts are physically actualised, we usually lose the original whole; (b) even if we do not care about preserving the original whole, not all parts that are possible theoretically can physically be actualised, but only some of them;[100] and (c) also theoretically, the whole cannot be thought of as being divided all at once into all possible parts. Let me explain these three restrictions in somewhat more detail.

(a) If parts of a continuum are actualised, this could be done either conceptually or physically. We actualise a part conceptually, for example, when we measure out a plank. This conceptual division in turn can be carried out either by simply conceiving or imagining a mark on the continuous plank or by physically marking off a part, say, with a pencil, without changing the continuum – a mark that can be removed again later on. By contrast, a part is physically actualised when the initial plank is cut into two pieces, in which case the two potential parts are transformed into two new continuous wholes. The parts therefore only stay parts as long as they are solely conceptually, not physically, separate entities. There are some continua where a physical actualisation of parts can be reversed. For example, if we divide the water in a basin with the help of a sheet of glass, we can remove the sheet and get back our original whole. But in the case of most solid continua, any attempt to regain the original whole after a physical separation, for example, with the help of glue or nails, will only arrive at a weakened unity of the whole.

(b) The actualisation of the possibility of being divided into parts is also limited by the fact that only conceptually can a division go on and on. In the

98 It has been suggested to emend *gnôsei* to *genesei* (Burnyeat 2008), so that instead of "conceptually" we would read "in a process of coming to be" (against it, however, see Weidemann 2017, p. 221). For my purposes, we do not have to decide between these two alternatives – we will see below that certain divisions of a continuum are only *conceptually* as is the infinity aimed for in such a *process* of division.

99 For a discussion of the different possibilities for translating this ambiguous passage, see Weidemann 2017. On p. 220, he captures the potentiality discussed here as follows: "It is the potentiality of the line to be divided, given any natural number n, $n + 1$ times."

100 Aristotle does not discuss this second restriction explicitly, but there are traces that he clearly restricts some divisibility to theory and thus distinguishes it from physical divisibility, see Miller 1982, pp. 89–90. Furthermore, *Physics* VII, 5 shows that while a force moving a certain thing may in theory be always further divisible, this does not correspond to the actual physical division possible. For in the empirical realm there is a lower threshold up to which a force could still move a thing, while we have no reason to assume such a lower limit in mere theory.

sensible realm, on reaching a certain smallness, a physical division can no longer be actualised – at a certain point it will not be possible to divide the plank any further with our means. Thus, not all parts that can be actualised in theory can become physically actual.

(c) The actualisation of possible divisions, and thus of having parts, is also restricted in theory. Although continua are infinitely divisible, as we have seen, this does not mean, even in theory, that they can have been divided infinitely in such a way that we now have infinitely many parts. This restriction can be seen from a passage in *Physics* book III:

> τὸ δὲ μέγεθος ὅτι μὲν κατ᾽ ἐνέργειαν οὐκ ἔστιν ἄπειρον, εἴρηται, διαιρέσει δ᾽ ἐστίν· οὐ γὰρ χαλεπὸν ἀνελεῖν τὰς ἀτόμους γραμμάς· λείπεται οὖν δυνάμει εἶναι τὸ ἄπειρον. οὐ δεῖ δὲ τὸ δυνάμει ὂν λαμβάνειν, ὥσπερ εἰ δυνατὸν τοῦτ᾽ ἀνδριάντα εἶναι, ὡς καὶ ἔσται τοῦτ᾽ ἀνδριάς, οὕτω καὶ ἄπειρον ὃ ἔσται ἐνεργείᾳ· ἀλλ᾽ ἐπεὶ πολλαχῶς τὸ εἶναι, ὥσπερ ἡ ἡμέρα ἔστι καὶ ὁ ἀγὼν τῷ ἀεὶ ἄλλο καὶ ἄλλο γίγνεσθαι, οὕτω καὶ τὸ ἄπειρον (καὶ γὰρ ἐπὶ τούτων ἔστι καὶ δυνάμει καὶ ἐνεργείᾳ· Ὀλύμπια γὰρ ἔστι καὶ τῷ δύνασθαι τὸν ἀγῶνα γίγνεσθαι καὶ τῷ γίγνεσθαι).
>
> As we have seen, magnitude is not actually infinite. But by division it is infinite. For it is not hard to refute the 'Indivisible Lines'. The alternative then remains that the infinite is in potentiality. But 'being in potentiality' should not be taken in the following way: if we say that it is possible that this is a statue, then this will be a statue, so that the infinite will be in actuality. Rather, 'being' is said in many ways, so that in the way the day and the games are, in that it is always one thing after another coming about, so too is the infinite (for of these things too we say that they are in potentiality and in actuality; for 'there are the Olympian Games' either means they may occur or that they are actually occurring). (206a16–25)

In this passage and its context, Aristotle discusses the sense in which a magnitude can be said to be infinite. After dismissing the idea of a magnitude of infinite size, he discusses infinite divisibility. The potentiality of infinite division is clearly set apart from the potentiality involved in the process of a statue coming into being. They differ in their potential being. But how they differ depends on whether we focus on *potential* or on *being* in Aristotle's discussion of the *potential being* of the infinite in question (which in the literature is often phrased in terms of *potential existence*). Hintikka and Bostock think that the important distinction here is in the notion of existence, which is different for processes and substances. The infinite is potential as well as actual in the same way as a process, in that one part occurs after another, rather than all parts being there at once.[101] Lear, by

[101] See Hintikka 1966; Bostock 1972–3, p. 38; Coope 2012, pp. 274–5. Bostock 1991 claims that it is only book III that understands infinity as merely potential, while book VI accepts that a line or stretch of time does actually contain infinitely many points – a claim for which I cannot find any basis in the Aristotelian text.

contrast, emphasises the *potentiality* of existence by pointing out that an actualisation of an infinite division cannot occur, so any dividing process will be finite.[102] Coope combines both points of view in arguing that infinite division is essentially only potential in that it has no complete occurrence – such processes are occurring, but cannot occur, which would mean that the whole of it occurs (i.e. the process of dividing is actualized completely, all divisions are made) – but it is also actual in that it is fulfilling its potential; however, it is doing so incompletely. To do justice to the complex account of the potential existence of infinite division, an interpretation along the lines of Coope seems to be needed.[103]

The passage quoted starts out by claiming that the infinite divisibility of a continuous magnitude is only potential. But 'being potential' can mean different things, and so Aristotle goes on to clarify in which sense infinite divisibility is potentially. We are left with a twofold restriction: first, the potential in question is such that not all of it can be actualised at the very same time, and second, not all of the potential can be actualised over a given period of time. A quantity of bronze can fully actualise its potential to become a statue *at one time* – when a sculptor creating a statue finishes her work, all of the potential of the bronze to be a statue is actualised. By contrast, a magnitude that is potentially divisible *ad infinitum* cannot have all of its potential actualised at the very same time. Rather, the actualisation, as well as the actuality, of such a potential requires time, analogous to the way that all the parts of a temporal event, like a day or the Olympic Games, cannot be actualised at the same time – only when one part is gone can another part be actualised.[104] The possibly infinitely many parts of a continuum cannot all be actualised simultaneously.[105] Furthermore, in contrast to temporal events like a day or the Olympic Games, infinite divisibility cannot be completely actualised over a period of time, as a day can be, once it is over.[106]

If we attempt to actualise the potential of a magnitude to be infinitely divisible, there will always be some potentiality left. For not *every* part that

[102] Lear 1979–80. While Lear's emphasis seems right – the existence Aristotle is discussing is potential existence, δυνάμει ὄν – the reasons he gives are the finiteness of us human beings and the size of physical cuts, and thus reasons that do not fully capture the ultimate structural reason why infinite division is essentially potential. See also Coope 2012, p. 277.

[103] Given Aristotle's claim in *De caelo* 274b13–14 and *Physics* 241b3–11, that something cannot be changing to that to which it cannot have changed, however, we should add to Coope's account that while dividing processes are occurring, they can only be understood as dividing processes *ad infinitum* potentially.

[104] This understanding of actualisation is analogous to the one we use for time, which should not be a surprise, since time is itself a continuum.

[105] This is true for time qua continuum in the most restrictive sense. With spatial continua, several parts can be actualised at the same time though never all at once as Aristotle states. See *De generatione et corruptione* 316b19–27 and below.

[106] And this is the point Hintikka and Bostock seem to miss in their account. See Coope 2012 for the idea that in a day there is complete actualisation.

can potentially be conceived of can actually exist – neither at a particular time nor over some extended stretch of time – but *any* part can be actualised (i.e., 'constructed' by a division). In this way infinitely many parts are possible, even though not infinitely many parts can ever be actual, not even in theory. One important consequence of this understanding of the potentiality in question is that the whole continuum can never be thought of as the sum of its parts, if by 'sum' we understand that all parts are given prior to the sum. And in this way Aristotle presents a new model of a part-whole relationship, markedly different from the one the atomists had worked with.

We will see that this new understanding of the part-whole relation also helps to counter Zeno's paradoxes. The threefold restriction of the potentiality of parts of continua allows Aristotle to counter one of the assumptions we saw at work in Zeno's paradoxes: that something's being divided is directly inferred from something's being divisible. Zeno infers that if a one were divisible, it would be divisible everywhere, and if it were divisible everywhere, it could be divided everywhere, which would lead to paradoxical assumptions about the parts thus produced.[107] Zeno deduces the state of being divided simply from that of being divisible – an assumption that seems to be supported by the usage of Greek verbal adjectives ending in *-tos*, which can denote possibility as well as a passive resulting state. By contrast, Aristotle shows that while any potential division can be actualised, this does not imply that all divisions can be actualised at the same time.[108]

Clarifying the possibility of division is necessary to avoid the problem that something extended dissolves into extensionless points,[109] which for Aristotle would be to confuse the notion of a part with the notion of a limit. His conception of the limits of continua allows us to avoid this problem.

7.4.2 *A New Twofold Concept of a Limit*

From an atomistic point of view, what delimits a whole we perceive from another whole is a first and a last part (and, ultimately, a first and a last atom).[110] But if we understand such wholes as being continuous, there are no parts that *constitute* the whole and therefore no first and last part that could

[107] Lee fr. 2; Simplicius *In Phys.* 139, 27 ff.

[108] See also his discussion in *De generatione et corruptione* I, 2, 316b19 ff.

[109] This was possibility (α) of the three problematic possibilities the dichotomy paradox led to – see Chapter 3. Aristotle shows that although division is possible and a point exists everywhere potentially, it does not follow that magnitudes reduce to points, since, as Miller 1982, p. 98 expresses Aristotle's view, "the existence of every actually existing point is conditional upon the existence of two segments with magnitude into which the subsection is divided". See also *De generatione et corruptione* I, 2, 317a2 ff.

[110] Whatever we can perceive is already too big to be an individual atom; it is an arrangement of atoms.

serve as what delimits this whole from another whole. Instead, the function of delimiting the whole is fulfilled by what Aristotle understands as *limits* of the continuum, which work in a way crucially different from parts of continua.[111] In *Metaphysics* Δ, 17 Aristotle gives an overview of possible understandings of the notion of a limit:

> 'Limit' means (1) the last (point) of each thing, i.e. the first point beyond which it is not possible to find any part, and the first point within which every part is; (2) the form, whatever is the form, of a magnitude or of a thing that has magnitude; (3) the end of each thing (and of this nature is that towards which the movement and the action are, not that from which they are – though sometimes it is both, that from which and that to which the movement is, i.e. the final cause); (4) the substance of each thing, and the essence of each; for this is the limit of knowledge; and if of knowledge, of the object also. Evidently, therefore, 'limit' has as many senses as 'beginning', and yet more; for the beginning is a limit, but not every limit is a beginning. (1022a4–13; translation by Ross)

Meaning (1) captures the notion of a limit that allows us to determine where a thing begins and ends, and thus marks it off from its surroundings. It is the limit that allows us to separate one whole from another. Meaning (2) seems to apply especially to spatially extended continua,[112] since with such continua – a rod, for example – the form of the whole thing will separate it from its surroundings (while with a motion, for instance, we take only beginning and end as limits). By contrast, meaning (3) is important for motions and actions. In an Aristotelian context, a motion is essentially understood with respect to what it aims at (i.e., its final end point; for example, my walk to Athens); thus meaning (3) appears to be a subcase of (1). And it seems that both (1) and (3) can capture physical or conceptual limits – for example, if I plan to walk to Athens but never make it there, my 'Athens' for this very walk is only a conceptual limit, but if I am successful, it is a physical one. Meaning (4) is not relevant for our project.

For Aristotle in the *Physics*, limits are dependent on what they limit.[113] And in contrast to parts, limits are indivisible (in the dimension in question) and always

[111] In *Metaphysics* Δ, 25, Aristotle lists several understandings of parts, but we will use only his understanding of parts as employed in the distinction between the parts and the limits of a continuum: a now as an unextended limit of time is not a part, since it cannot measure out a stretch of time, it is no fraction of time, and no bunch of nows can make up the whole (see, for example, 218b6–8 and 220a18–19; Hussey incorrectly translates *to de nun ton chronon orizei* (219b11–13) as "it is the now that *measures* time", when what Aristotle describes there is that the now marks off time). For the notion of a limit, see Hasper 2006, pp. 72–3.

[112] Accordingly, Ross translates *megethos* here as "spatial magnitude". It may also apply to time if, for example, we understand the "form" of the day as 'from sunrise to sunset'. But it seems to work most naturally for magnitudes extended in space.

[113] By contrast, for a mathematician, limits may be prior to what they limit, since we construct a surface out of lines, for example. For Euclid, a sphere is generated out of a (rotating)

possess one dimension fewer than what they limit – the limits of three-dimensional bodies are two-dimensional surfaces, the limits of two-dimensional surfaces are one-dimensional lines, and the limits of one-dimensional lines are extensionless points.[114]

So what *kinds* of limits are relevant for individuating continua? The passage from the *Metaphysics* counts only the beginning and the end as what defines the unity of continua. In his account of motion, time, and space, however, Aristotle also uses a notion of what we can call a limit that is crucial for continua but is not one of the four mentioned above – a limit between parts. This additional notion of a limit – the second kind of limit that is important for our project – is not examined for continua in general, but we will see Aristotle discussing it in depth with regard to specific continua.[115] The function of this second kind of limit will become clear by comparing it to wholes that are made up of discrete parts, like a heap of grains.

The limit of one of the grains that make up a heap delimits one grain from other grains *within* the whole heap. But the limit of that grain can (a least in part) also be a section of the *outer* limit of the whole heap. What delimits a possible part of a continuum from another possible part cannot, however, function as an outer limit of the whole continuum, as is clear from the restricted actualisability we saw above: as long as we are dealing with proper parts, the limits of these parts cannot be physically actualised and can be actualised conceptually only to a limited degree. If a part is indeed physically actualised, then its limit can function as an outer limit. But the original part has thus turned into a new whole. Its limit stops being a limit *within* the original whole and becomes an outer limit. With wholes made up of discrete parts, like the heap of grains, we find only one kind of limit, because the limits of a part and the limits of a whole function in exactly the same way. By contrast, with continua we are dealing with two different kinds of limit: there is a clear difference between what can be called an inner limit (between parts) and an outer limit (that delimits one thing from another).[116] I should add that while

circle, and Speusippus seems to have claimed that geometrical figures in general are generated out of points. Aristotle himself claims in the *Topics* 141b5 f. that for definition a point is prior to a line, and so forth (see also Aristotle's *Protrepticus* VI, 38, line 10 ff.). In the context of Aristotle's natural philosophy, however, the limits depend on what is limited, which seems unproblematic in cases (1) and (2) above. It is less clear with (3), since we may think that the limit as that towards which the process aims (for example, Athens) is independent of the process (my walking to Athens). However, qua the final limit of the actualised process of my walking to Athens, this 'Athens' is dependent on the process, since only if the process comes to an end in Athens can the arrival in Athens be understood as the final limit of the process (and not just as what was aimed at).

114 In the following I will often focus on the case of lines limited by points as the simplest case and thus deal with limits that are dimensionless and indivisible.

115 The most intensive discussion of a limit we get for time.

116 This difference may become clearer if we illustrate it with the help of a mathematical function, where points within a given interval work like inner limits, while points

Aristotle uses the terms *perata* and *eschata* for outer limits, he does not explicitly use any terminology like 'inner limits'. Rather, Aristotle usually talks about *sêmeia* (marks or points) or *stigmata* (points) in the passages I am discussing. I will nevertheless talk, anachronistically, about these division points as 'inner limits', not only because of ease of expression, but also because it allows us to see the connection with the end points of continua more easily. This choice is also supported by the fact that Aristotle himself uses the same word, *akra* (high point or end), for what we would call outer limits (for example, in 227a25) as well as what we would call inner limits (for example, in 262a23).[117]

The two kinds of limit that we find with Aristotle's notion of continuity can be characterised as follows:

(1) 'Outer limits' are boundaries that mark off a continuum from its surroundings, one whole thing from another. Within the Aristotelian framework, these limits are normally either both given (for example, when we measure a certain rod, the beginning and end of that rod are given), or the beginning is given and the end is clear, like an aim or goal (for instance, the final condition at which a certain change is aimed),[118] or the end will eventually become clear, as in a stroll.

(2) Division points or marks – what I want to call 'inner limits' – are limits that mark off possible parts *within* a continuum. These division points are usually not given but constructed for certain purposes: for example, when measuring a long table with a small ruler we mark off parts of the table with the help of such marks. Like outer limits, inner limits possess one dimension fewer than what they limit for Aristotle. For example, the inner limits of a line are without extension, they are points. And inner limits not only allow for divisions within a continuum but also, as the flip side, guarantee internal continuity. Thus while outer limits differentiate one whole from another and thus guarantee the unity of a continuum *vis-à-vis* another continuum, inner limits guarantee the internal unity (i.e., its continuity and gaplessness).

delimiting the interval can be understood as outer limits: given a function, e.g., $f(x)=2x$ within the interval [0,4], the slope of the graph of this function at any point 'within' this interval will always be 2 (since the derivative of the function $f'(x)=2$). At the end points 0 and 4, however, no slope is defined. Thus we see that points within the defined interval have crucially different characteristics than points delimiting the interval. The first predecessor of such a distinction between two kinds of limit may be taken to be Anaxagoras, for whom the initially homogeneous mass is characterised by two features: (1) there is no smallest part, no 'internal limit' (which implies infinite divisibility); and (2) there is no biggest part, no external limit. See frr. B1 and 3, and below.

117 And while he uses *semeion* as an inner limit in 262a29, in 262b7 it can also refer to an outer limit.

118 The *hou heneka* is taken as the limit of a motion, for instance, in *De motu* 700b15–16; see also meaning (3) in the passage from the *Metaphysics* quoted above.

These two kinds of limit are clearly distinguished: marks or inner limits can be set as one pleases within the continuum and may always stay potential; outer limits cannot be set as one likes and they, or at least the limit marking the beginning, are necessarily actualised. If, however, inner limits are actualised, two different kinds of actualisation are possible. We can actualise an inner limit in a solely conceptual way, as when we 'create' a mark during a measurement process by applying a measurement unit. This actualisation leads to parts that are still only potential parts and it preserves the special character of the inner limits, even when it has a physical expression, as for instance when we mark our conceptual part with a pencil. If, by contrast, we actualise such parts by actualising the inner limits physically, we turn these inner limits into outer limits, as when the initial rod is cut into two pieces.

Let us now examine the outer and inner limits specific to the three continua in which we are especially interested – time, space, and motion. We will first consider to what extent time, space, and motion each have an outer limit and how we shall conceive of these limits, and then look at their respective inner limits. As for spatial continua, individual magnitudes that are extended in space have a beginning and an end,[119] and thus outer limits. Furthermore, for Aristotle, space itself (i.e., as the whole of our universe) has an outer limit.[120] By contrast, time does not, since it has neither beginning nor end:

> Since time cannot exist and is unthinkable apart from the now, and the now is a kind of middle point, uniting as it does both a beginning and an end in itself, a beginning of future time and an end of past time, it follows that there must always be time: for the extremity of the last period of time that we take must be found in some now, since there is nothing to grasp in time except the now. Therefore, since the now is both a beginning and an end, there must always be time on both sides of it. But if this is true of time, it is evident that it must also be true of motion, time being a kind of affection of motion. (251b14–28)

Time per se does not have an outer limit. But a *certain* time, such as an hour, does have a beginning and an end. The time a motion takes will have outer limits, the original inner limits of infinite time that are treated like outer limits when time is perceived with respect to a certain motion. While sublunar motions also have a beginning and an end, the circular motion of the heavens, to which Aristotle refers in the passage just quoted, is infinite.

The infinite extension of time and the heavenly motion show that outer limits are not necessary features of all continua. But if there are outer limits, they must be clearly distinguished from possible inner limits. Even if inner and outer limits are called by the same name – 'point' in the case of a line extended

[119] With the exception of spheres.
[120] See, for example, *Physics* IV, 5, 212b20–2.

in space[121] and 'now' in the case of time – their functions are clearly distinguished.

In the case of motion, outer limits raise an additional problem. While a certain time and spatial magnitude, appropriately perceived, will have outer limits of their own dimension – spatial limits for space and temporal limits for time – the same does not seem to be true of motion, for the beginning and end of a motion are determined not by 'motional' limits but by rest. Motion is usually delimited by two phases of rest, or more precisely by the point where a phase of motion turns into a phase of rest. However, motion and rest are both spatio-temporal, so that the spatio-temporal entity 'motion' is delimited by another spatio-temporal entity, 'rest', just as one spatial entity is delimited by another spatial entity. And it is the *same thing*, for example Coriscus, that either rests or moves (i.e., the 'behaviour' of the thing moved determines the outer limits of the motion): if the thing comes to a standstill, the motion stops. Aristotle does not call motion and rest something spatio-temporal. But he talks about the realm of the moveable, which embraces motion and rest, for in principle we can only say that something is at rest if it can be moved, but is not in motion at the moment.[122] And what is moveable is either covering some spatial distance during a certain time, and is therefore in motion, or is not covering any distance during that time and is therefore at rest.

The outer limits of a motion can also be determined merely spatially or merely temporally. But it is only by finding the two sections of rest that delimit a motion that we can be sure that the motion in question is indeed one and unintermittent,[123] as we saw in the discussion of Zeno's first motion paradox: while we may determine the outer limits of a run merely by spatial points A and B, the runner could rest in between and thus perform two motions between A and B, rather than one. Thus, referring to the spatial points A and B does not necessarily guarantee a continuous motion. Alternatively, it could be the case that the runner goes on after point B all the way to point C, and hence the motion from A to B is not one motion, but rather part of one motion. To understand motion solely in temporal *or* solely in spatial terms is therefore insufficient; rather motion must be understood in terms of time *and* space – a point we will discuss in some detail in the next chapter.

But defining the outer limits of a motion raises yet another problem: while atomists can easily talk about a first and a last part of motion, Aristotelian continua are always further divisible and accordingly, as he points out, do not

121 For the point as an outer limit of lines, see Euclid *Elements* I, def. 3: γραμμῆς δὲ πέρατα σημεῖα ("the extremities of a line are points"); Euclid also defines the limits of a solid as surfaces, see book XI, def. 2.

122 See below.

123 As required according to 227b21–32. See also 228b4–6, which claims that if a movement is divided by periods of rest, it is no longer one and continuous.

possess a first or last point; thus, there is no last moment when something is still in motion. If we assume, for example, that Coriscus rests for a while, then starts a little race with the tortoise, and finally rests again, how can the transition from resting to running and from running to resting be conceived? Looking at the transition from motion to rest, we find that whichever interval close to the finishing point we may choose, there will always be a smaller one closer to it; and from the finishing point onwards Coriscus is already resting again. Similarly for the transition from rest to motion:[124]

> φανερὸν τοίνυν ὅτι οὐκ ἔστιν ἐν ᾧ πρώτῳ μεταβέβληκεν· ἄπειροι γὰρ αἱ διαιρέσεις. οὐδὲ δὴ τοῦ μεταβεβληκότος ἔστιν τι πρῶτον ὃ μεταβέβληκεν. . . . ὅτι μὲν οὖν οὔτε τοῦ μεταβάλλοντος οὔτ᾽ ἐν ᾧ μεταβάλλει χρόνῳ πρῶτον οὐθέν ἐστιν, φανερὸν ἐκ τῶν εἰρημένων·. . . οἷον ἐν τοῖς μεγέθεσιν. ἔστω γὰρ τὸ ἐφ᾽ ᾧ ΑΒ μέγεθος, κεκινήσθω δ᾽ ἐκ τοῦ Β εἰς τὸ Γ πρῶτον. οὐκοῦν εἰ μὲν ἀδιαίρετον ἔσται τὸ ΒΓ, ἀμερὲς ἀμεροῦς ἔσται ἐχόμενον· εἰ δὲ διαιρετόν, ἔσται τι τοῦ Γ πρότερον, εἰς ὃ μεταβέβληκεν, κἀκείνου πάλιν ἄλλο, καὶ ἀεὶ οὕτως διὰ τὸ μηδέποτε ὑπολείπειν τὴν διαίρεσιν. ὥστ᾽ οὐκ ἔσται πρῶτον εἰς ὃ μεταβέβληκεν.
>
> It is evident, then, that there isn't anything in which first something has changed,[125] for the divisions are infinite. So, too, of that which has changed there is no primary part that has changed . . . It is evident, then, from what has been said, that neither of that which changes nor of the time in which it changes is there any primary part . . . Take the case of magnitudes: let AB be a magnitude, and suppose that a motion has taken place from B to a primary 'where' C. Then if BC is taken to be indivisible, two things without parts will have to be contiguous [which is impossible]: if on the other hand it is taken to be divisible, there will be something prior to C to which the thing moved has changed, and something else again prior to that, and so on to infinity, because the process of division may be continued without end. Thus there can be no primary 'where' to which a thing has changed. (236a26–b16)

This passage points out that there is no *primary* part of movement or time at the beginning of a change. We find the same difficulty that there is no first part before the end point of a change, for instance the coming to a standstill of a locomotion and the beginning of rest, as 238b36–239a22 makes clear. Since

[124] For a detailed discussion of the problems of transition, see *Physics* VI, 5 ff. See also Wieland 1962, pp. 310–13; Kretzmann 1976 and 1982; Lear 1981, p. 103, n. 10; Sorabji 1983, ch. 26; Strobach 1998, p. 55 ff.

[125] We are not talking about the impossibility of a first point of accomplishment at the end of a process here, but rather about the impossibility of a first part of movement in which something has already changed while still being in the process of movement. Hardie and Gaye probably added "with reference to the beginning of change" in the first sentence of their translation (which I have removed here since it does not refer to anything in the Greek) in order to indicate this focus.

motion, rest, time, space, and the thing moved are all continuous and hence infinitely divisible, there cannot be a first or last point of any of them.[126] If we look at a motion from A to B, we cannot determine a first instant of motion taking place after A or a last instant taking place before B.

A quick glance at Plato, who seems to have a similar problem in his *Parmenides*, might give us an idea of how such a shift of motion to rest can be thought. For Plato, nothing can change immediately from rest to motion, since motion and rest are contradictories,[127] and changes seem to be thought of as continuous processes, at least to some degree.[128] Accordingly, with the occurrence of motion, rest must have already ceased to be, since contradictories can in no way admit their opposite (i.e., they must be strictly separate).[129] The transition from movement to rest, from one contradictory opposite to the other, must therefore be conceived of in a way that preserves this separation. Plato's solution in the *Parmenides* is to have this transition happen at an instant (*exaiphnês*) that is itself an instant of neither motion nor rest, but rather is 'between' both. An instant is not a part of time since then either motion or rest would take place in it; it is outside of time and not in time at all (*Parmenides* 156d).[130] In this way, motion and rest are clearly separated, with the help of something that is in between movement and rest and is neither the one nor the other.

Aristotle does not fully embrace this Platonic solution: he takes on board neither an instant beyond the sphere of the movable and of time at which the

[126] This seems to make outer limits indeterminate. However, while it is not possible to say at which point a motion comes to a standstill, Aristotle thinks there is a point when we can say that no motion or change is going on any longer. See below.

[127] For Aristotle some things are neither in motion nor at rest, as for example the unmoved mover – since he does not have the potential for motion he cannot be at rest either. By contrast, for Plato everything seems to be either in motion or at rest. At least the *Phaedo* (78b–79b) and the *Sophist* suggest that everything, even the Forms, are unchanging and at rest and *Sophist* 250a determines motion and rest as *enantiôtata*; thus we seem to deal with exhaustive and exclusive opposites, and hence contradictory opposites in Plato. Plato's distinction between being at rest and being unmoved in the first deduction of the *Parmenides* 139b seems to suggest that motion and rest are, after all, only contraries, since there is a third state, 'being unmoved' (see Sattler 2019c). However, while this distinction was probably influential for Aristotle, in Plato this distinction is made in a paradoxical context, where the fact that the One seems to be unmoved but yet cannot be at rest is meant to show the difficulty into which the assumption of the One has led us. It is not positively taken up, and the *Sophist* still assumes the contradictory nature of motion and rest.

[128] See Chapter 5; *Parmenides* 156c; *Sophist* 252d.

[129] See, e.g., Plato's conception of contradictories in the *Phaedo* 105d–e.

[130] For Plato, an instant is not just not a *part* of time, as we also find in Aristotle's account of *nun*, but it is also outside of time, μηδ ἐν ἑνὶ χρόνῳ (156c2) and ἐν οὐδενὶ χρόνῳ (156e6). By contrast, Aristotle could not say this, as, for example, 219b33 ff. makes clear, where we read that if there were no *chronos* there would be no *nun* and vice versa. See Sattler 2019c for a more detailed discussion of Plato's notion of *exaiphnês*.

change from rest to movement happens as a discontinuous jump, nor that motion and rest are contradictory. Of contradictories, one must always hold, while 221b7–222a9 makes it clear that there can be things that are neither in motion nor at rest. For Aristotle, motion and rest are both part of the one domain of what can in principle be moved; motion passes over to rest, and vice versa.[131] However, Aristotle seems to integrate Plato's suggestion by employing an indivisible and extensionless instant as a limit that is *simultaneously the outer limit of rest and of motion* in the very domain of the movable:[132]

> ὅτι μὲν τοίνυν τὸ μεταβεβληκός, ὅτε μεταβέβληκε πρῶτον, ἐν ἐκείνῳ ἐστίν, δῆλον· ἐν ᾧ δὲ πρώτῳ μεταβέβληκεν τὸ μεταβεβληκός, ἀνάγκη ἄτομον εἶναι. λέγω δὲ πρῶτον ὃ μὴ τῷ ἕτερόν τι αὐτοῦ εἶναι τοιοῦτόν ἐστιν.
>
> It is clear, then, that that which has changed, at the moment when it has first changed, is in that to which it has changed. The 'primary when' in which that which has changed effected the completion of its change must be indivisible, where by 'primary' I mean possessing the characteristics in question of itself and not in virtue of the possession of them by something else belonging to it. (235b30–236a2)

"The 'primary when' in which that which has changed effected the completion of its change" is the limit between motion and rest.[133] As a limit it is indivisible and without extension. As the limit between motion and change, it

[131] This fits with Aristotle's definition of rest as "not being changed of that which admits of change" (202a3–5). *Kinêsis* is thought as the actuality of something which is there in rest potentially. As we saw above, with *kinêsis* the actuality cannot take place all of a sudden at once but rather must take place gradually. Thus it must take time. And the way Aristotle talks about the start and end point of motions, which we just saw in the main text, suggests that the transition from rest to motion (which is when motion starts) and from motion to rest (when motion ends) must also be thought of as gradual. Hence motion and rest do not exclude each other in the way contradictory opposites do. Rather, both belong to the realm of the moveable, to which not everything belongs, and thus can only be contrary opposites. This does not mean, however, that something can be in motion and at rest at the same time, but only that there can be something that is neither at rest nor in motion, like a number can only be odd or even, not both, while there can be things that are neither odd nor even (see also Chapter 1). Contradictory opposites only allow for abrupt jumps. It is true that Aristotle does allow for some alterations to be all-at-once changes, like the freezing of a pond (253b25; see also Waterlow 1982, ch. 5). But there, too, we presumably have a gradual building up of the conditions allowing for the freezing (a gradual lowering of temperature, for example), and the freezing is then the outer limit of this process. In any case, Aristotle does not allow for such all-at-once changes in the case of locomotion, which is the main kind of change we are interested in.

[132] While the limit is ontologically dependent on the moveable (and thus belongs to the domain of the moveable), it is itself neither in motion nor at rest. In the following quotation Aristotle does not talk about rest since he is talking in more general terms of the contrast between a change and the state into which something has changed.

[133] For a fuller discussion of the 'primary when', see Morison 2013.

is the only point within a motion or rest that does not separate motion from motion or rest from rest.[134] Although this limit is 'in between' motion and rest, Aristotle sometimes ascribes it to rest.[135] In these cases, where rest seems to be understood as the outer limit of motion, rest (qua such a limit) must be conceived of as an extensionless point, even though what follows will have to be an extended phase of rest (otherwise we could not properly talk of rest). So while there is no first moment of change after a state of rest and thus no first now in which a thing *has first accomplished some change*, as *Physics* VI, 6 makes clear,[136] there is a first now in which a thing *has changed in the sense of being done with its change* (VI, 5) – the extensionless outer limit.[137]

[134] This can be seen in analogy to the way we think of continuous open intervals nowadays: it is the point of an interval that in contrast to all other possible points does not have an ε-surrounding.

[135] It seems that in principle this could also be done the other way round – that this limit is ascribed to motion. And so Bostock 1991, p. 193 claims that all that is needed is a "*decision* as to what to say about these special instants of change". According to Strobach 1998, p. 57, however, Aristotle does not just state that any problematic instant should be assigned to the later state (here: the rest after something has moved); rather, there are good reasons for this. If we take, for example, something which is darkening all over at once, there is a continuum of shades and thus no last shade before black. But if there is no last instant at which something is not the case, there is a first one at which it is the case. Although something cannot rest or remain at an instant, it can be something, e.g., white or at a point at an instant. A change can have been completed and a new state of affairs can have come into being at an instant. Sorabji 1983, ch. 26 argues similarly: the state of which there is no last (or first) instant is one which involves changing, while the state admitting of a first instant does not (p. 413).

[136] Let us say that up to point t there was a state of rest, and from t onwards a thing is changing. Then, for every $t_n > t$ you might come up with as a first now in which our thing was changing, there will be an earlier now $t_{(n-1)} > t$ at which our thing was already changing. For problems with applying this Aristotelian understanding of change to changes between contradictories, see Bostock 1991, pp. 196–200.

[137] In *Metaphysics* Θ, 1048b18–35 Aristotle states that the perfect tense claims the reaching of a *telos*, while the present tense refers to an *energeia*, so both usages, present and perfect, cannot be true together in the case of motion (while they can in the case of a complete *energeia*; I do not follow Ross' emendation in 1048b22–3 but accept the manuscript tradition apart from reading the dative ἐκείνῃ instead of ἐκείνη). See also *Physics* VI, 1, 231b30–232a1, where Aristotle claims that "if somebody walks to Thebes, he cannot be walking to Thebes and at the same time have walked to Thebes". However, in book VI of the *Physics*, Aristotle claims that because there is no first point when a motion starts, whenever I say something is moving, it must also be true that something has moved already. Accordingly, Aristotle seems to work with two different senses of 'having moved' with respect to motion, one that indicates that the *telos* has been reached, which is not true together with the present tense form of the verb; the other is compatible with the present tense, since there is no first point of motion, so whenever it is true that something is moving, it is also true that it has moved. Coope 2012, p. 279 understands the perfect tense in this passage of the *Metaphysics* as a marker of aspect, rather than of tense, which seems to allow for a similar distinction. The atomist Diodorus Cronos claimed that only the perfective sense of motion is true – accordingly, we can never say

While we may think that this idea that there is a first point where something has changed ('having become F' qua 'being F') may hold true in the cases of alteration, this does not show sufficiently that it works in the case of locomotion. However, if we understand locomotion, as Aristotle usually does, as moving from E to F and the motion coming to an end at F, we see that the same structure holds also for locomotion.[138] The thing moving may not stop its motion after having arrived at F, however. Thus we have to distinguish between defining a motion as being a motion from E to F from defining it as what occurs until the mover rests. For example, if Coriscus takes an evening stroll from his house to the next village, I could understand this motion either as a motion from his house to the next village – in which case it ends in the next village whether or not he rests there. Or I could determine it as what is limited by rest, in which case this motion need not end in the next village but could go on as the same motion if he does not rest there but comes back in one go. Hence, there are two different ways of marking the outer limit of a motion. 'Being at rest at the next village' is logically different from 'being at the next village' where this is considered as the end point which specifies the direction of the prior *kinêsis*. We may think that 'being at the next village' is part of what makes it the *kinêsis* to the next village,[139] while 'being at rest at the next village' is, so to speak, definitionally posterior. On the other hand, Aristotle's definition of motion in *Physics* III may rather seem to speak in favour of a motion being essentially determined by not being at rest (at which point the potentiality in question would be fully actualised). We will see that in the passages we are discussing Aristotle is mainly interested in cases where both coincide, being and resting at the next village,[140] and where resting can be seen as the proper indication that a motion has come to an end.

Let us now move on to the 'inner limits' of continua. We will look first at the function of marks or inner limits in general, before considering the specific inner limits for time, space, and motion; again, with motion we shall encounter additional problems.

One function of the inner limits of a continuum is to serve as a potential division point that helps us to measure continua.[141] Let us show this with the help of a simple example, the measuring of a spatial extension, such as the

that something is moving, only that it has moved. This restriction is another disadvantage the atomists face compared to Aristotle.

138 See also 219b33 ff. and the main text below; 262a21 ff.; Graham, ad loc.

139 Aristotle often specifies a motion by its end point, for example in 262a6–8, where we are told that the motion from A to B is contrary to that from B to A. In 264a14 ff. Aristotle uses the contrariety of these two motions determined by their end point in order to show that there must be rest in between these two motions.

140 See, for example, 265a30 ff.

141 This very function (together with Aristotle's notion of infinity) will explain how Aristotle can conceive of infinite divisibility consistently.

length of a beam. In order to measure the length of the beam, we employ another smaller continuum, such as the length of a ruler, which is applied to the beam. In order to allow for precise repetition of the application necessary for measuring – to measure the beam we must establish how many times the ruler fits into it – we set internal marks within the measurand (i.e., 'internal limits' like a point). This marking is not a physical division but merely a conceptual division, and we are therefore not dealing with outer limits but with 'inner limits'.[142]

Aristotle uses such 'inner limits' when he compares the motion of two things of different speeds: the faster one will cover the same distance in less time, thus dividing the time, while the slower one will cover less distance in the same time, thus dividing the distance, see especially 233a4–12:

> If the magnitude is divided, the time will also be divided. And we can carry on this process forever, taking the slower after the quicker and the quicker after the slower alternately, and using what has been demonstrated at each stage as a new point of departure: for the quicker will divide the time and the slower will divide the length. If, then, this alternation always holds good, and at every turn involves a division, it is evident that all time must be continuous. And at the same time it is clear that all magnitude is also continuous; for the divisions of which time and magnitude respectively are susceptible are the same and equal. (233a4–12)[143]

All the divisions of time and space used for comparing the quicker and the slower thing are 'inner limits': they do not separate one whole from another, but rather mark off a possible part within the spatial continuum or the temporal continuum, respectively. These marks are not given as physical realities in the way the beginning and the end of a motion are given, but they can be 'constructed' by us to compare the two motions performed at different speeds. If we construct such marks, we actualise a possible part of the continuum in question, not physically, but conceptually. In his treatise on time, Aristotle points out another basic function of such marks:

> Clearly, too, if there were no time, there would be no 'now', and vice versa. Just as the moving body and its locomotion involve each other mutually, so, too, do the number of the moving body and the number of its locomotion. For the number of the locomotion is time, while the 'now' corresponds to the moving body, and is like the unit of number. Time, then, is also both continuous through the 'now' and divided at it. For here, too, there is a correspondence with the locomotion and the moving body. For the motion or locomotion is one by virtue of the moving thing,

[142] In principle, it would not matter for Aristotle's notion of continuity whether there are empirically any atoms, since the unrestricted divisibility he needs is only conceptual, not physical.

[143] I will discuss this passage in greater detail in Chapter 8.

> because that is one – not because it is one *ho pote on*, for then it might leave a gap, but because it is one in *logos*[144] – for this determines the movement as 'before' and 'after'. Here, too, there is a correspondence with the point; for the point also both connects and terminates the length – it is the beginning of one and the end of another. But when you take it in this way, using the one point as two, a pause is necessary if the same point is to be the beginning and the end. (219b33–220a13)

While referring to the separation function already mentioned, this passage also introduces a holding-together function: continua are continuous and one, thanks to their inner limits. Given that Aristotle does not think that first there are parts that then can be made one, the holding-together function is not to be understood as connecting separately given parts. Rather, it demonstrates that wherever we cut, there is a part, and all possible parts hold together.

Thanks to this second function, inner limits also play an important role for the unity of a continuum. Thus inner limits seem to have two different tasks:

(1) by serving as potential division points, they divide and thus help to create 'sections' for measurement purposes, and
(2) they are proof for the holding together of the whole continuum and thus for its continuity.

We saw that the outer limit determines the section of reality that we take as one thing; it is what marks off one thing from its surroundings. The internal unity of this section (i.e., its continuity and gaplessness) is granted by the 'inner limit'. Far from destroying continuity, as Parmenides thought, such limits set within a continuum stand surety for the continuity of continuous magnitudes. The first passage quoted for inner limits explicates that they guarantee continuity: "If, then, this alternation always holds good, and at every turn involves a division, it is evident that all time must be continuous" (233a8–10). The fact that within a continuum we can set division points wherever we please and there will always be a length or a time that we thus divide and not an undefined gap, guarantees that we are dealing with something continuous (see also the second of Aristotle's two characterisations of a continuum). That there are inner limits guaranteeing continuity is even more obvious from their connecting function (see the first characterisation of the continuum): it is the inner limits that show the connection and thus hold together any two possible parts of the continuum, as the transition from one to the other. Given that inner limits need not necessarily be actualised, it seems that inner limits *show* the connection, rather than *doing* any connecting themselves. There are no parts already there waiting to be divided that would then need to be connected.

An inner limit is one possible instant within the whole continuum. As such, it can be used for uniting as well as for dividing: if we look to the right and left

[144] For a discussion of the sense in which the thing moving must be one, see Chapter 8.

of a point, then the point connects the right- and left-hand parts, while if the point and what is to its right is contrasted with what is to the left of it, then we are dividing the continuum. The very same point or now, spatial and temporal inner limits respectively, can be perceived as dividing two parts of a continuous thing[145] or combining them. However, if this one point is used as two, such that it is the end point of one part and the starting point of a another, the one inner limit has become two outer limits, as the last line of the passage just quoted makes clear for the case of motion (220a12–13).

The now and the point fulfil these functions of connecting and dividing in the very same way. The thing moved, paralleled in the passage just cited with the now and the point, seems to work analogously to the other two, but in a somewhat looser sense, as it needs to be captured in terms of nows and points. We can use the thing moved in order to set marks within a continuum, by marking off a part of the motion it has just completed, for example, where it is at a certain time or when it is at a certain point.[146] However, even when we use the thing moved, and not a certain time or distance, to mark off a part of motion, we have to use nows and points.

Nevertheless, the thing moved is one important factor in establishing the continuity of the motion it performs. It allows us to distinguish, for instance, Coriscus' motion to the marketplace from Socrates' motion to the marketplace. Time and space each on their own could determine many different motions. For example, if Coriscus and Socrates walked to the marketplace from the Lyceum at the same time, the temporal and spatial limits of these two walks would not help us to single out Coriscus' motion; but taking into account the temporal and spatial limits as well as the thing moving, Coriscus, allows us to single out the very motion we want to investigate.[147]

What are the consequences of Aristotle's account of limits for Zeno's paradoxes? We will see that Aristotle's notion of outer limits helps to solve the problem raised by the plurality paradox, that in order to distinguish two things we seem to need a third thing in between the two things *ad infinitum*

[145] Thus, it can be perceived as the end of one part and the beginning of a second. The line or time itself is, of course, not affected by this potential division, continuity is not interrupted.

[146] The thing in motion can be used as a '*potential* inner limit' since it exists and thus itself has the potentiality to set a mark. Now and point, on the other hand, are '*potentially* inner limits' since they do not exist independent of us actualising them, so they themselves do not have a certain potentiality. Rather, they are only functions of division and combination (for example, a moving object could stop at some point, thus actualising its being at that position, whereas this cannot be done with the now).

[147] If we only looked at Coriscus and did not take the temporal and spatial limits into account, we would face the problem that he could perform different motions; thus it would not be enough to refer to a motion as 'Coriscus' motion' to single out the one motion of Coriscus we are interested in.

(DK B3). And his notion of what we called inner limits helps to deal with the arrow paradox.

Let us start with the first point. In the third chapter we saw Zeno's claim against what we called 'external pluralists' that a collection of a finite number of things must also contain an infinite number of things,[148] on the basis that between any two elements of a finite plurality there must be another element in order to ensure that the two are indeed separate and thus two different things; and we need another thing between this thing in between and each of the first two elements, *ad infinitum*. This argument can be proven wrong by showing that what separates one thing from another is in fact not a third thing, but the limit of a thing. A limit is not a thing in its own right, but rather something that is ontologically dependent on an object, a feature of it. The limit is extensionless and indivisible and thus does not lead to an infinite plurality of things.

In the paradox of the flying arrow, what is true for the whole time is concluded from what is true for a 'now', namely that the arrow is at rest. With continua, parts share their basic features with their whole, so it is possible to infer what holds of the whole from what holds of a possible part. However, we saw in Chapter 3 that in Zeno's paradox the nows are treated as indivisible and not extended, while the period of time is divisible and extended; here whole and parts do not share the same basic features.

While Aristotle takes up the now from Zeno's paradox as unextended and indivisible, he shows, against Zeno, that such a now is not a part of time:

> ὅτι οὐδὲν μόριον τὸ νῦν τοῦ χρόνου, οὐδ' ἡ διαίρεσις τῆς κινήσεως, ὥσπερ οὐδ' ἡ στιγμὴ τῆς γραμμῆς· αἱ δὲ γραμμαὶ αἱ δύο τῆς μιᾶς μόρια.

> the 'now' is no part of time nor the point of division any part of the movement, any more than the points are parts of the line – for it is two lines that are parts of one line. (220a19–21)

Rather than a part, a now is a limit[149] and thus functions quite differently from a part: while parts of continua are themselves continua (i.e., extended and always further divisible), limits are indivisible and not extended. Given these characteristics, limits can be used to mark off parts and are thus features of parts, rather than parts themselves. Accordingly, we cannot infer what happens during a period of time from what happens in the now. The paradox of the arrow demonstrates the problems we get into if such unextended and indivisible limits are conceived of as parts.

Not only the possibility of motion but also the possibility of rest is excluded 'within' such a limit:

[148] DK29 B3; Lee fr. 11.

[149] See also 218a6–7. The thought that something is an 'element' of a whole and at the same time not extended is shown below to be the intuition of a limit as a possible mark in a continuum.

ὅτι δ᾽ οὐθὲν ἐν τῷ νῦν κινεῖται, ἐκ τῶνδε φανερόν ἐστιν. εἰ γάρ, ἐνδέχεται καὶ θᾶττον κινεῖσθαι καὶ βραδύτερον. ἔστω δὴ τὸ νῦν ἐφ᾽ ᾧ N, κεκινήσθω δ᾽ ἐν αὐτῷ τὸ θᾶττον τὴν AB. οὐκοῦν τὸ βραδύτερον ἐν τῷ αὐτῷ ἐλάττω τῆς AB κινηθήσεται, οἷον τὴν AΓ. ἐπεὶ δὲ τὸ βραδύτερον ἐν ὅλῳ τῷ νῦν κεκίνηται τὴν AΓ, τὸ θᾶττον ἐν ἐλάττονι τούτου κινηθήσεται, ὥστε διαιρεθήσεται τὸ νῦν. ἀλλ᾽ ἦν ἀδιαίρετον. οὐκ ἄρα ἔστιν κινεῖσθαι ἐν τῷ νῦν. ἀλλὰ μὴν οὐδ᾽ ἠρεμεῖν· ἠρεμεῖν γὰρ λέγομεν τὸ πεφυκὸς κινεῖσθαι μὴ κινούμενον ὅτε πέφυκεν καὶ οὗ καὶ ὥς, ὥστ᾽ ἐπεὶ ἐν τῷ νῦν οὐθὲν πέφυκε κινεῖσθαι, δῆλον ὡς οὐδ᾽ ἠρεμεῖν . . . ἔτι δ᾽ ἠρεμεῖν μὲν λέγομεν τὸ ὁμοίως ἔχον καὶ αὐτὸ καὶ τὰ μέρη νῦν καὶ πρότερον· ἐν δὲ τῷ νῦν οὐκ ἔστι τὸ πρότερον, ὥστ᾽ οὐδ᾽ ἠρεμεῖν.

We will now show that nothing can be in motion in a now. For if this is possible, there can be both quicker and slower motion in the now. Suppose then that in the now N the quicker has traversed the distance AB. That being so, the slower will in the same now traverse a distance less than AB, say AC. But since the slower will have occupied the whole now in traversing AC, the quicker will occupy less than this in traversing it. Thus we shall have a division of the now, whereas we found it to be indivisible. It is impossible, therefore, for anything to be in motion in a now. Nor can anything be at rest in a now: for, as we were saying, only that can be at rest which is naturally designed to be in motion but is not in motion when, where, or as it would naturally be so: since, therefore, nothing is naturally designed to be in motion in a now, it is clear that nothing can be at rest in a now either . . .[150] Again, when we say that a thing is at rest, we imply that its condition in whole and in part is at the time of speaking uniform with what it was previously: but the now contains no 'previously': consequently, there can be no rest in it. (234a24–b7)[151]

We cannot think of motion, or rest, at an instant (i.e., at a now), since motion and rest must be conceived of as taking place *between* two points, i.e., in order to detect whether motion or rest takes place, comparison between a previous state and a later state is required. Looking simply at an indivisible instant will not tell us whether something is moving or resting; everything is unmoved in such an instant.[152] If something were to move during an instant, this instant would have to be divisible.[153] Aristotle reinforces this argument by employing a comparison of different speeds, which is conceivable only if the space and the time of this motion are taken to be divisible.[154] Although

[150] What is left out of the quotation here is the proof that denying that movement or rest to take place at an instant also avoids the contradiction that the instant of transition between movement and rest is both motion and rest simultaneously.

[151] See also 239a23 ff.

[152] See Moore 1990, pp. 42–3.

[153] See Russell 1970, p. 51.

[154] Owen 1957–8 thinks that Aristotle's denial of movement at an instant is only due to what he calls Aristotle's "clumsy dynamic". For Owen, Aristotle's focus on motion over a

Aristotle sometimes uses the 'now' as a colloquial way of expressing the present *span* of time and thus a part of time, in a systematic context the 'now' is always unextended and indivisible for him.[155]

In order to block Zeno's paradox of the flying arrow, we could either conceive of the nows as proper parts (i.e., as extended and divisible), in which case the arrow would be moving in such a now; or we could take the nows in the way Zeno did, as unextended and indivisible, in which case they are not parts of time and we cannot infer what is true over an extended period of time from what is true at an unextended now. In both cases we avoid Zeno's paradox.

Aristotle chooses the latter possibility (i.e., he understands the nows as inner limits of time). He gives various arguments for proving that these limits have to be indivisible and extensionless. The most general argument claims that were a limit not extensionless and indivisible, the two parts that are divided by it would have no boundary in common. Rather, there would be an extended interval between the two parts: the one part would have the left bit of this interval as its limit, the other part the right bit of the interval. The parts divided would no longer be continuous (234a3–9).

While this argument is valid for points and nows, there is a further argument for the indivisibility and extensionlessness only of the now (233b33–234a24). Carrying within it the remains of the thought of the present, the now is used not only to mark off one time from another but also to mark off the future from the past. If the now were understood as a divisible extension, it would have to be made up of past times as well as future times (234a11–14), which we usually prefer to separate clearly from each other;[156] and inner limits as conceived by

period prevents him from thinking about the very useful concept of an instant velocity. However, today we, too, cannot simply conceive of motion or rest at an instant. Rather, we must start out with a period of time since we can only conceive of motion as something happening *between two points of time* (very much in the way Aristotle conceives of it in 238b36 ff.). Only starting out from a stretch of time and space can we reduce the distance between the two points further and further so that they converge to a limit of the initial period. Our notion of an instantaneous velocity thus depends also on extended periods and distances (even if the limit is distinct from any member of the series). See Lear 1981.

155 Hence Waterlow 1984, pp. 137–42 points out that past and present are not treated in the same way by Aristotle. The first is the stretch of time that has already elapsed while the second is the limit that delimits this past stretch from the future one to come. That Aristotle conceptualises the present as an instant, according to Waterlow, is due to his definition of movement in *Physics* III, 1, where the moment of completion of a process is taken as the primary use of the present tense; and the completion cannot have a beginning or an end and thus must be instantaneous. Accordingly, it is the teleological character of the final condition (see VI, 5) that allows for connecting Aristotle's discussion of the continuum in book VI to his teleological definition in book III.

156 The arguments for indivisibility and extensionlessness are not valid for the thing moved.

Aristotle allow us to do so. Since these inner limits can be set *ad infinitum*, we should finally look briefly at the notion of infinity employed here.

7.4.3 A New Conception of Infinity

In the previous section we specified the kind of limit (*perata*) that continua and their unity require within the Aristotelian framework. However, continua are also related to what is without limit (the *apeiron*), to what is 'infinite'.[157] How can these two features go together? Something that has outer limits, such as a spatial magnitude or a finite process, seems to be a whole and thus complete – how can it then have any connection to what is *apeiron* in the sense characterised in the *Physics*?

> συμβαίνει δὲ τοὐναντίον εἶναι ἄπειρον ἢ ὡς λέγουσιν. οὐ γὰρ οὗ μηδὲν ἔξω, ἀλλ' οὗ ἀεί τι ἔξω ἐστί, τοῦτο ἄπειρόν ἐστιν . . . ἄπειρον μὲν οὖν ἐστιν οὗ κατὰ τὸ ποσὸν λαμβάνουσιν αἰεί τι λαμβάνειν ἔστιν ἔξω. οὗ δὲ μηδὲν ἔξω, τοῦτ' ἔστι τέλειον καὶ ὅλον· οὕτω γὰρ ὁριζόμεθα τὸ ὅλον, οὗ μηδὲν ἄπεστιν, οἷον ἄνθρωπον ὅλον ἢ κιβώτιον. ὥσπερ δὲ τὸ καθ' ἕκαστον, οὕτω καὶ τὸ κυρίως, οἷον τὸ ὅλον οὗ μηδέν ἐστιν ἔξω· οὗ δ' ἔστιν ἀπουσία ἔξω, οὐ πᾶν, ὅ τι ἂν ἀπῇ. ὅλον δὲ καὶ τέλειον ἢ τὸ αὐτὸ πάμπαν ἢ σύνεγγυς τὴν φύσιν. τέλειον δ' οὐδὲν μὴ ἔχον τέλος· τὸ δὲ τέλος πέρας.
>
> The infinite turns out to be the contrary of what it is said to be. It is not what has nothing outside it that is infinite, but what always has something outside it . . . Our account then is as follows: a quantity is infinite if it is such that we can always take a part outside what has been already taken. On the other hand, what has nothing outside it is complete and whole. For thus we define the whole – that from which nothing is wanting, as a whole man or a whole box. What is true of each particular is true of the whole as such – the whole is that of which nothing is outside. On the other hand, that from which something is absent and outside, however small it may be, is not 'all'. 'Whole' and 'complete' are either quite identical or closely akin. Nothing is complete [*teleion*] which has no end [*telos*]; and the end is a limit. (206b33–207a15)

How can continua be thought to be finite and complete and thus wholes while at the same time *apeiron*? This is exactly the question posed by Zeno's dichotomy paradox: in order for a run over the finite distance from A to B to be completed, an infinite number of spatial pieces must be covered in a finite time.[158] We will see in the next chapter how Aristotle responds to what we called the motion problem of this paradox – how can *infinitely many spatial*

[157] Something which has no limits cannot have any ends, since they are (or are at least determined by) a certain kind of limit; hence it is infinite. Cf. also Plato's *Philebus* 26b, where *peras* and *apeiron* are introduced as two basic principles.

[158] Cf. *Physics* VI, 7.

pieces be covered in a *finite time*? In what remains of the present chapter we will concentrate on what we have termed the continuum problem: how can a *finite* continuous *whole*, be it spatial or temporal, contain *infinitely many parts*. That the parts in question can only potentially be 'constructed' became clear in the section on parts and wholes. What remains to be shown is how the infinity involved here can be thought.

Let us first look briefly at the two discussions of infinity that seem to be in the background of Aristotle's understanding of it, that of the mathematicians and that of the natural philosophers. The most important accounts of infinity by the Presocratic philosophers can be found with Anaximander and Anaxagoras. Anaximander calls the ultimate thing, which grounds all the rest, the *apeiron* (fr. B1), which in Greek means unlimited as well as inexhaustible. Accordingly, Anaximander's *apeiron* is understood by scholars as what is unlimited in extent (i.e., physically unlimited),[159] but also as what is undetermined.[160] Anaximander's *apeiron* can be seen as a predecessor to Aristotle's understanding of *apeiron* in two ways: in the idea that time is extended without limits and in the idea that *apeiron* can be understood not only as physical limitlessness but also as indeterminacy.

Another understanding of limitlessness appears to be perhaps even more important for Aristotle: the differentiation between limitless in smallness and limitless in largeness that we find in Anaxagoras. Fr. B1 indicates that in the beginning everything was together and limitless in multitude/magnitude as well as in smallness. Limitlessness in smallness is further determined in fr. B3: with the small there is never a smallest but always something that is yet smaller. Thus, it seems clear that infinity need not be connected to infinite extent (limitlessness in multitude/magnitude).

For the mathematicians, the notion of infinity seems to be of crucial importance in arithmetic as well as in geometry. They seem to require not only an infinity of numbers but also infinitely many numbers or infinitely extended or many lines or geometrical figures. Aristotle points out in his *Physics*, however, that the mathematicians do not require that there be infinitely many numbers, only that there be no limit to the size of the number they can choose, and that they in fact do not require infinite lines (or infinitely many lines), but only as large (or as many) as they need for any construction.[161]

[159] This grants that generation never ceases. The extension can be temporal, spatial, and/or material, since Anaximander's *apeiron* seems to be described in terms of all three. Ch. 1 of my manuscript in progress, *Conceptions of Space in Ancient Greek Thought*, discusses this.

[160] See Kirk, Raven, and Schofield 1983, p. 110 and Kahn 1994, p. 233. Thus the *apeiron* can turn into all the elements. The problems with the whole-part relationship we saw raised by Zeno's paradoxes also show how the lack of physical limits can lead to conceptual indeterminedness.

[161] The method of exhaustion is no objection to this claim. There we approximate the area of a circle by similar polygons in such a way that we 'square' the circle by going on with

The influence of both discussions, of the Presocratics and the mathematicians, can be seen in Aristotle's first enumeration of the various senses of 'infinite' in book III:

> μάλιστα δὲ φυσικοῦ ἐστιν σκέψασθαι εἰ ἔστι μέγεθος αἰσθητὸν ἄπειρον. πρῶτον οὖν διοριστέον ποσαχῶς λέγεται τὸ ἄπειρον. ἕνα μὲν δὴ τρόπον τὸ ἀδύνατον διελθεῖν τῷ μὴ πεφυκέναι διιέναι, ὥσπερ ἡ φωνὴ ἀόρατος· ἄλλως δὲ τὸ διέξοδον ἔχον ἀτελεύτητον, ἢ ὃ μόγις, ἢ ὃ πεφυκὸς ἔχειν μὴ ἔχει διέξοδον ἢ πέρας. ἔτι ἄπειρον ἅπαν ἢ κατὰ πρόσθεσιν ἢ κατὰ διαίρεσιν ἢ ἀμφοτέρως.
>
> The problem, however, which specially belongs to the physicist, is to investigate whether there is a sensible magnitude which is infinite. We must begin by distinguishing the various senses in which the term 'infinite' is used. (1) What is incapable of being gone through, because it is not in its nature to be gone through (the sense in which the voice is 'invisible'). (2) What admits of being gone through, the process however having no termination, or what scarcely admits of being gone through; or what naturally admits of being gone through, but is not actually gone through or does not actually reach an end. (3) Further, everything that is infinite may be so in respect of (a) addition or (b) division or (ab) both. (204a1–7)

Aristotle can dismiss the first sense of infinity (1) as irrelevant for an investigation within the realm of natural philosophy. In his first paradox Zeno assumes infinity in the second of the senses given (2), while Aristotle will work mainly with the options given with the third sense (3).

As the first sentence of the passage quoted above makes clear, Aristotle's principal concern in the third book of the *Physics* is proving that no *infinitely extended sensible magnitude* can exist. He first proves the conceptual impossibility (204b4–10, an infinite magnitude would violate the definition of a magnitude) and physical impossibility (204b10 ff.) of such an assumption, before determining the special kind of infinity that can indeed be attributed to a magnitude: infinity of division.

With infinity of addition and division Aristotle seems to take up, in a modified form, Anaxagoras' distinction between two kinds of infinity, in smallness and bigness. He makes clear, however, that in contrast to his predecessors, he does not conceive of infinity or the infinite as a principle.[162] The

inscribing a sequence of regular polygons inside it, starting with a square, seemingly going on *ad infinitum*. For if we look at where this method is used, as in *Elements* XII, 2, we see that Euclid stops after a finite number of steps, just enough to show that the difference between the original figure and the inscribed figure decreases by at least half at each step of the sequence.

[162] See, e.g., 203a3–4 and Moore 2001, pp. 34–5. Infinity in smallness and bigness is also what Aristotle supposes Plato to have incorporated into his notion of the Great and the Small. Aristotle calls them the "two infinites" since they allow for division as well as augmentation to go on infinitely. See *Physics* 203a15–16 and 206b27–9.

way he fills out infinity in smallness takes up the mathematicians' notion of infinity of division, as he makes clear in 203b18:[163]

> τοῦ δ' εἶναί τι ἄπειρον ἡ πίστις ἐκ πέντε μάλιστ' ἂν συμβαίνοι σκοποῦσιν, ἔκ τε τοῦ χρόνου (οὗτος γὰρ ἄπειρος) καὶ ἐκ τῆς ἐν τοῖς μεγέθεσι διαιρέσεως (χρῶνται γὰρ καὶ οἱ μαθηματικοὶ τῷ ἀπείρῳ).
>
> Belief in the existence of the infinite comes mainly from five considerations: (1) From the nature of time – for it is infinite. (2) From the division of magnitudes – for the mathematicians also use the notion of the infinite . . . (203b15–18)

According to Aristotle, the infinity we are dealing with in Zeno's dichotomy and Achilles paradox is the infinity of division, named above as possibility (3b). These two paradoxes claim that running over a finite distance requires covering an infinite number of spatial pieces. Thus they seem to confront us with three equally untenable positions, as we saw in Chapter 3: either (α) we get infinitely many parts of nil extension, so that something extended is made up of unextended parts, or (β) the parts are extended, in which case an infinity of these extended parts should lead to an infinitely extended whole, or (γ) we assume that the parts of the continuous finite distance are simply always further divisible and thus not determined. Aristotle's analysis of infinity shows that the second possibility, (β), rests on a lack of distinguishing strictly between infinity of addition and division, for this position assumes that an infinite division leads to infinitely many non-converging extended parts which, added back together, would lead to an infinite extended whole.[164] Infinite divisibility thus seems to lead to infinite extension – a confusion Aristotle diagnoses as one of the problems underlying Zeno's paradoxes in 233a21 ff. (see Chapter 8).

But even if infinity of addition and infinity of division are clearly distinguished, the latter must be further specified in order to avoid all paradoxical results, as we learn from a passage part of which was already quoted above:

> Things are said to be either potentially or actually; and a thing is infinite either by addition or by division. Now, as we have seen, magnitude is not actually infinite. But by division it is infinite. (There is no difficulty in refuting the theory of indivisible lines.) The alternative then remains that the infinite has a potential existence. (206a15–18)

[163] See Moore 2001, p. 44: "Properly understood, the mathematical infinite and the potential infinite [i.e., the infinite Aristotle finally encounters] were, for Aristotle, one and the same. Far from abhorring the mathematical infinite, he was the first philosopher to seriously champion it."

[164] We have already seen in the chapter on the atomists that if we divide something into non-converging extended parts, we will not in fact get infinitely many of them. Aristotle also seems to be aware of converging series, as *Physics* 206b7–9 suggests.

The infinity of division is not *kat'energeian*, actual, but only potential. Immediately after this passage, Aristotle explains the restrictions on the potentiality dealt with here,[165] which we have already discussed.

According to Aristotle, the infinity rightly attributed to continua is an infinity of division, not of extension; it is only potential and can never be fully actualised. Thus, continua, being infinite, are indeed not complete; but this does not mean that they are incomplete with respect to their outer limits. Rather, they are incomplete with respect to division, since if we try to divide them, we will never reach an end.[166] This kind of incompleteness of continua is easily combinable with the fact that continua are almost always finite things. Nonetheless, continua seem to be indeterminate, since their parts are not determined.[167] This would only be a problem, however, if we thought of continua as constituted by their parts, which, as we have seen, they are not. The potential infinity of division is crucial for the refutation of Zeno's first argument, since the dichotomy paradox would indeed be problematic if the infinity in question could not be brought into a relation with finite continuous things.[168]

Infinity of division holds for all three continua in question: time, motion, and space. However, within the Aristotelian framework, time and the motion of the heavens also admit of another kind of infinity, the infinity of having no beginning or end. Is Aristotle therefore facing the problem of having to assume the rejected infinity of extended magnitudes after all?

For Aristotle, the infinity of time and the heavenly motions belongs to what he calls "infinity of addition", which he splits up into addition of magnitudes and addition of "time and human beings" (206a25–b3). While all parts persist in the case of addition of magnitudes, addition of time and human beings means that a new 'part' of the thing comes to be after another part has ceased to be (207b13–15); "human beings" here is obviously taken to mean the species which, according to Aristotle, never ceases to be – although individual human beings pass away, new individual human beings constantly come into being. Aristotle claims infinity of addition of magnitudes to be impossible, since it

165 See Hussey's commentary, ad loc. Moore 2001, p. 40 describes the difference between actual and potential infinite as follows: "The actual infinite is that whose infinitude exists, or is given, at some point *in* time. The potential infinite is that whose infinitude exists, or is given, *over* time; it is never wholly present." While this is a useful way of capturing a notion of actual and potential infinity, it cannot be the one Aristotle is working with, since this account would not allow for time itself to be actually infinite, which Aristotle claims.

166 Or, as Coope 2012, p. 277, expresses it, "in the case of an infinite process, there is no such thing as the whole of it".

167 This is the problem we saw connected to choosing alternative (γ) for the part-whole relationship in the dichotomy paradox.

168 On the potentiality of this infinity, see Thomson 1954, p. 3: "To speak of an infinity of possibilities is not to speak of the possibility of infinity."

would lead to the abhorred assumption of a body that is infinitely extended (205a7–8). Infinity of addition "with time and human being" is, however, possible within the Aristotelian framework, since not all parts of the thing in question exist simultaneously. This is the case for time as a whole and the never-ending motion of the heavens.[169] Aristotle conceives of the series of numbers in the same way: "its infinity [i.e., the infinity of number] is not remaining [actually] but consists only in a process of coming to be, like time and the number of time" (207b14–15) – a process which can be understood as the process of counting.

This special potential infinity of addition that Aristotle assumes for time, natural species, and the number series is not relevant for his solution to Zeno's paradox, however. The continuum problem questions the possibility of a finite *spatial distance* having infinitely many parts – and thus focuses on what Aristotle understands as infinity of division.

Many features of Aristotle's notion of continuity call for further discussion – for instance, how exactly can we conceive of the passage from motion to rest and what is the size of a part of time that we take to be actualised – for which there is no space here. But before moving on to Aristotle's account of the measure of motion, I will give a brief summary of how Aristotle's notion of the continuum responds to Zeno's motion paradoxes.[170]

For his account of continua, Aristotle employs one of the two incompatible part-whole relationships used in Zeno's dichotomy paradoxes – the relationship that gives clear priority to the whole over the parts. The parts are 'produced' from the whole through a conceptual division and thus do not constitute the whole. Zeno was right in his dichotomy paradox that this division can go on *ad infinitum*. But, according to the part-whole relationship that is appropriate for continua, he is wrong to ask how these parts can make up the original whole, for these parts always stay potential – if they were fully actualised, they would constitute a new whole. And not *every* part that can potentially be conceived of can actually be there, but *any* part can be actualised, and in this way infinitely many parts are possible. Accordingly, we get neither parts with nil extension nor actually infinitely many extended parts. That the parts are to a certain degree left indeterminate is not a problem, as they do not constitute the whole.

The characterisation of a whole in terms of its parts is replaced by a characterisation of the whole in terms of the outer and inner limits of the continuum in question. The conceptual inner limits, which we can set as

[169] See, for example, 251b19–252a5. Moore 2001, p. 44 thinks that this gets Aristotle into trouble with regard to the past since "it is hard not to see already in this – the past now *being* past – a presentation of the actual infinite".

[170] With the exception of the moving rows paradox, which raises exclusively a motion problem, and will thus be dealt with in the next chapter.

we please, must not be confused with the outer limits, which are given and determine the whole continuum *vis-à-vis* other wholes. For, as Aristotle claims in 263b3–6, the reply to "the question of whether it is possible to pass through an infinite number of units" depends on whether the units are actual – "if the units are actual, it is not possible; if they are potential, it is possible".

For Aristotle, the infinity involved in motion is not a feature of the parts that would make up a whole but only a possibility for conceptualising a continuous whole. The parts 'constructed' in a concrete analysis will always be finite; only the possibility of partition will be infinite. We are thus dealing with (1) an infinity that is potential rather than actual, and (2) an infinity of operation[171] rather than of extension or size. And such an infinity can belong to finite things, which is necessary for solving the dichotomy and the Achilles paradox.

Mistaking inner limits for possible parts of a continuum led to Zeno's arrow paradox, where an indivisible inner limit is treated like a part of a motion. From the impossibility of conceiving motion in such an indivisible limit, in a now, it was concluded that motion is not possible over a period. According to Aristotle, such extensionless limits cannot be understood as possible parts of an extended whole. Neither is it possible to think of a continuous whole as consisting of indivisible limits, nor can we infer what is possible in an extended whole from what is possible in or at these indivisible limits.

Only once the part-whole relation, the conception of limits, and of infinity are newly determined is it possible to conceive of physical continua consistently – a necessary condition for solving Zeno's paradoxes. However, these conceptual changes do not yet allow for a sufficient account of motion. Accounting for just *one* continuum is not enough to determine motion and speed, which are essentially complex. Rather, the *relation* of two continua, time and space, is needed for determining motion. This is exactly what Aristotle employs in his comparison of different speeds – as we will see in the next chapter.

[171] Hussey translates *energeia* as "operation" or "actual operation", which seems unhelpful to me in the context of Aristotle's discussion of infinity. In Hussey's sense of "operation", which refers to the actual existence of something, there cannot be infinity of operation for Aristotle, while there is infinity of operation if we understand it to mean that a process, like division, has no natural end.

8

Time and Space: The Implicit Measure of Motion in Aristotle's *Physics*

In the previous chapter, we saw how Aristotle's understanding of time, space, and motion as continua allows him to respond to the continuum problem raised by Zeno's paradoxes – how a finite whole can contain infinitely many parts. But how does Aristotle respond to what we called the motion problem – how an infinite number of *spatial* pieces can be covered in a finite *time*, or, in more general terms, how time and space can be combined in a motion? Aristotle's notion of continua, as magnitudes that are always further divisible, prepares the ground for solving this problem too. As the current chapter will demonstrate, Aristotle's understanding of continuity is the basis for relating time and space in a way that can account for motion.

The previous chapter showed that continua can not only have outer limits that separate them from one another, but also possess possible inner division points (what we called 'inner limits') that can mark off one possible part from another. This chapter will show that in addition to these different kinds of limit, motion also allows for limits of different dimensions. A motion can be determined by temporal or spatial limits: it can be seen as the temporal unity of an uninterrupted motion going on from t_1 to t_2, or as the spatial unity of a motion from s_1 to s_2.[1] However, in the strict sense, the unity of a motion is determined by both limits (i.e., in spatio-temporal terms); we will see below how Aristotle gradually builds up to such an account.

First, however, we need to look at Aristotle's notion of a measure of motion. We saw in Chapter 6 that a measure allows for quantifying and comparing motions and thus for giving consistent and rationally admissible statements. Chapter 6 also showed that Plato's measure of motion faces two problems: (1) it only allows for restricted measurability, since it can only be used for motions that possess a point of accomplishment or a final point; and (2) although it is a measure for one motion being faster than another, only the time of a motion is taken into account so that we are dealing with a simple measure. Both problems created serious difficulties

[1] Qua spatial unity it may allow for a possible temporal interruption; see Chapter 3.

for any attempt to establish a philosophy or science of motion. The current chapter will reconstruct Aristotle's reaction to these problems in the *Physics*.

We will see that unrestricted divisibility permits unrestricted measurability[2] and thus quantification of time, distance, and motion that does not require a point of accomplishment. Moreover, Aristotle's *Physics* is the first text handed down to us where we clearly find a complex measure for motion: a measure that takes into account time as well as distance covered.[3]

We will start this chapter with Aristotle's account of measure in the *Metaphysics* that lays down his theory of measurement explicitly. It is reminiscent of Plato, as it understands measure not only as homogeneous with the measurand, but also as one-dimensional. For a full account of motion and speed and a response to Zeno's challenge, however, we need a complex measure, not a one-dimensional one, taking into account the time taken as well as the distance covered. We will see that this is exactly what Aristotle implicitly develops in the *Physics*, when comparing different motions and responding to Zeno's challenge.

In his explicit account, however, Aristotle calls only time a measure of motion in the *Physics* and therefore seems to abide by the one-dimensionality required in the *Metaphysics*, neglecting the homogeneity requirement. For the complex measure of speed, the two requirements of the *Metaphysics* – one-dimensionality and homogeneity – cannot be fulfilled together. If we use the complex measure of time taken over distance covered for determining speed, it is not one-dimensional, and if we use a one-dimensional measure, say, only the time taken, we do not have a measure homogenous with the measurand, since we are no longer measuring speed but only duration. Thus Aristotle is forced to employ a notion of measurement for speed that is not in line with his general account in the *Metaphysics*.[4] His implicit account of a complex measure prepares our modern measure of motion; but we will see that he cannot

[2] If magnitudes were not divisible as we please but atomistic, we could only use scales whose units correspond to the indivisible parts of the magnitude in question (the units would have to be either of the same size or an integer multiple of these indivisible parts, but could not be of a size that includes a fraction of these parts).

[3] I understand 'being simple' as meaning 'involving only one dimension or aspect to be measured' and 'being complex' as meaning 'involving the relation of more than one dimension or aspect to be measured'; see Chapter 1.

[4] While Aristotle's general explanation of sublunary motion in the *Physics* is the first attempt to develop a complex measure, he employs solely time as a measure of movement for cosmology. See, for example, *De caelo* 287a25–8. This is possible, since for astronomical purposes the most important kind of speed is angular velocity, where we are interested not in the time necessary to cover a certain distance, but in the time necessary to cover a certain angle, which is usually taken to be 360 degrees, so that we can concentrate on the time.

explicitly endorse it. The reasons for this lack of endorsement will be spelt out in Chapter 9.

8.1 The General Concept of Measure in Aristotle's *Metaphysics*

Before we examine Aristotle's definition of measure in the *Metaphysics*, let us remind ourselves of the basic features necessary for all kinds of measurement. As we saw in the first chapter, a measure normally allows us to quantify sensible things.[5] It therefore enables us to understand the physical world as intelligible by concentrating on one aspect of the thing investigated, on a certain feature or quality it possesses.[6] The amount of this quality can be measured by assigning it to numbers with the help of measurement units. Therefore, in order to measure something, three things must be taken into account:

(1) We must determine the respect in which it is to be measured – what we called the *dimension* measured (we are going to measure the temperature or weight of our tortoise). The thing to be measured with respect to a certain dimension we called the measurand.
(2) The dimension must be quantified by *assigning* the measurand to the *number series*.
(3) *Units* must be defined to carry out the quantification. A certain amount of the dimension is taken as a unit – for example, a kilogram to measure weight – so that the measurand can be determined as a multiple of that amount.

Aristotle's conception of measurement is a special case of this general understanding of measurement, as we will see when we look at his explicit account of measure in the *Metaphysics*. But before we do so, I should make it clear that measure will be discussed not so much with respect to concrete units (like the yards I gain from my yard stick), but rather with respect to general measurement units belonging to a certain dimension (for example, spatial units that allow us to measure distance). Furthermore, while with simple measures there are usually measuring instruments specific to what we want to measure (such as scales for measuring weight), for measuring speed we use a combination of different measuring instruments (one for measuring time, another for distance, both of which are conveniently combined in our modern speedometers).

[5] It also enables us to quantify mathematical things, but sensible things are the focus of attention in natural philosophy.

[6] Note that the notion of quality we will come across later in Aristotle's account of measurement units is a different notion.

8.1.1 A Simple Measure: Being One-Dimensional and of the Same Kind as What Is Measured

The most detailed account of Aristotle's understanding of measurement in general is found in *Metaphysics* I, 1 (1052b18–1053a8).[7] In our analysis of his account we will see that in Aristotle the Greek word *metron* can mean 'measure' in the sense of both dimension and unit of measurement;[8] *metron* will therefore be translated according to context.

As we will see, measure in Aristotle is characterised by three basic features:

(1) Measure is that by which the quantity of something is known.
(2) Measure is always of the same kind as the measurand.
(3) Measure is simple either in quantity or quality.

Let us briefly investigate these three features in turn:[9]

(1) According to the passage in I, 1, the essential task of a measure is to allow us to know the *quantity* of something:

> μέτρον γάρ ἐστιν ᾧ τὸ ποσὸν γιγνώσκεται.
>
> For measure is that by which the quantity is known. (1052b20)

We always want to know how much (or many) a quantity is,[10] which a measure enables us to determine. To fulfil this function, a measure must possess certain characteristics:

(2) In Aristotle, a measure (by which he understands a measurement unit here) must always be *sungenes*, homogeneous, with the thing measured:

> ἀεὶ δὲ συγγενὲς τὸ μέτρον· μεγεθῶν μὲν γὰρ μέγεθος, καὶ καθ' ἕκαστον μήκους μῆκος, πλάτους πλάτος, φωνῆς φωνή, βάρους βάρος, μονάδων μονάς.
>
> But the measure is always homogeneous [with the measurand],[11] it is of magnitudes a magnitude, and in particular of length a length, of breadth

[7] *Metaphysics* Δ, 6 (1016b17–25) and N, 1 (1087b33–1088a14) are variations of it. See also 1056b and 1072a.
[8] We will see that 'measure' does not refer to any physical instrument in the context that interests us here.
[9] There is also an accuracy requirement in Aristotle's account of measurement – a measure is most precise if nothing can be added or taken away without it being noticed – but I will have to leave this aside here. For a more detailed discussion of Aristotle's account of measurement in the *Metaphysics*, see Sattler 2017a.
[10] The Greek word *poson*, which I translate as "quantity", literally means "how much" or, when in the plural, "how many".
[11] The Greek text literally only says that the measure must always be *sungenes*, not what it is *sungenes* with. But it seems clear that the only thing with which the measure could be homogeneous is the thing to be measured, the measurand (the measure must be of the

> a breadth, of sounds a sound, of weights a weight, of units a unit. (1053a24–7)

The measurement unit must be of the same kind as what is measured – if we want to measure a length, we need another (smaller) length to measure it, while for the measurement of weight we need another weight (that the dimensionality has to be the same for the measurand and the measurement unit is also required by modern accounts of measurement). From the passage above, however, we can infer not only (a) that Aristotle takes what we called dimension to be an important feature of measurement, but also, if we look a bit closer, (b) that he understands the dimension in question as always being one-dimensional.

(a) The measure used must be *sungenes* with what is to be measured. By this, Aristotle cannot mean that the measurement unit is homogeneous with the measurand in all respects, for then only feet could be measured with the unit 'foot', not tables or rooms.[12] Rather, it seems clear that the unit and the measurand must be *sungenes* in the aspect that is measured, for example, to measure the length of a table we will need a measurement unit that also possesses length, like a centimetre on a ruler. Hence, we see that Aristotle takes into account that the respect to be measured – what we called dimension – must be clear.

Aristotle's awareness of dimensionality can also be seen in his explanation that we attribute heaviness and speed both to what has any weight or speed whatsoever and to what has excessive weight or speed:

> τὸ μέτρον ἑκάστου ἕν, ἐν μήκει, ἐν πλάτει, ἐν βάθει, ἐν βάρει, ἐν τάχει (τὸ γὰρ βάρος καὶ τάχος κοινὸν ἐν τοῖς ἐναντίοις: διττὸν γὰρ ἑκάτερον αὐτῶν, οἷον βάρος τό τε ὁποσηνοῦν ἔχον ῥοπὴν καὶ τὸ ἔχον ὑπεροχὴν ῥοπῆς, καὶ τάχος τό τε ὁποσηνοῦν κίνησιν ἔχον καὶ τὸ ὑπεροχὴν κινήσεως: ἔστι γάρ τι τάχος καὶ τοῦ βραδέος καὶ βάρος τοῦ κουφοτέρου).
>
> The measure of each is a one – in length, in breadth, in depth, in weight, in speed (for 'weight' and 'speed' are common to both contraries; for each of them has two meanings – 'weight' means both that which has any amount of heaviness and that which has an excess of heaviness, and 'speed' both that which has any amount of movement and that which has an excess of movement; for even the slow has a certain speed and the comparatively light a certain weight).(1052b25–31; Ross' translation with modification)

This passage shows that Aristotle expressly differentiates between two meanings of terms like "weight" and "speed": between the dimensionality

same *genos* as the thing to be measured, otherwise no measurement is possible – how could we measure, for example, weight with a length?). Accordingly, translations normally add something like this; see, e.g., the translations by Bonitz, Ross, and Tricot.

[12] In Aristotle's account, a foot can measure all kinds of length.

of certain magnitudes (weight and speed as *dimensions* are also attributed to light and slow things, and thus to "both contraries", the light and the heavy, the slow and the fast) and the different degrees that tell us the number of times a unit of these magnitudes is contained in a measurand[13] (what is called heavy and speedy is what possesses a considerable degree of weight and speed, i.e., what contains the basic unit multiple times).[14]

(b) The dimension to be measured is conceived as simple; we always deal with a one-dimensional measure. This is evident from the examples given for the *sungenes* requirement: length is measured with one measure and breadth with another, yet in principle the measure of breadth could have been traced back to the measure of length plus a certain sense of direction in space, as is the case today. The fact that Aristotle treats breadth as a dimension to be measured separately instead shows that he prefers to treat each dimension as conceptually simple and not further divisible; that is, it cannot be traced back to other dimensions. This passage is not the only place in the *Metaphysics* where we find this conceptual simplicity, for nowhere in this work does Aristotle describe a measure as the relation of two different qualities.[15]

We may assume that Aristotle has to allow for complex dimensions when dealing with surfaces or bodies. However, the way he measures an area seems to be with the help of a smaller area, and a cube is measured by a smaller cube.[16] Surfaces and bodies are thus treated as simple dimensions by Aristotle that are not traced back to the combination of two or more simpler dimensions.[17]

This strict one-dimensionality can also be seen in Aristotle's account of the measure of motion in 1053a8–12:

> καὶ δὴ καὶ κίνησιν [εἰδῶσι] τῇ ἁπλῇ κινήσει καὶ τῇ ταχίστῃ (ὀλίγιστον γὰρ αὕτη ἔχει χρόνον)· διὸ ἐν τῇ ἀστρολογίᾳ τὸ τοιοῦτον ἓν ἀρχὴ καὶ μέτρον (τὴν κίνησιν γὰρ ὁμαλὴν ὑποτίθενται καὶ ταχίστην τὴν τοῦ οὐρανοῦ, πρὸς ἣν κρίνουσι τὰς ἄλλας)
>
> And indeed [they know] movement too by the simple movement and the fastest (for this takes least time). Thus also in astronomy, such a

[13] The passage itself does not actually say anything about the number of times a unit is contained in a measurand, but rather talks about weight and speed in the sense in which something is said to possess it just because it has more of it than light and slow things.

[14] βάρος and τάχος are genuinely ambiguous in Greek in a way that is difficult to mirror exactly in English: τάχος can mean swiftness as well as speed, βάρος heaviness as well as weight.

[15] On the simplicity of the measure, see *Metaphysics* 992a10 ff., 1016b19 ff., 1085a7 ff., and below.

[16] See below and Chapter 9.

[17] While Aristotle seems to allow a comparison of length to surfaces in *Topics* 158b30 ff., what he actually compares there are the *ratios* of the areas to the *ratios* of the sides – just what we should expect, given the mathematics of his time (see Chapter 9).

> 'one' is the starting point and measure (for they assume the movement of the heavens to be uniform and the fastest, and judge the others by reference to it).

In order to measure a movement we take another movement, the fastest one.[18] This fastest movement for Aristotle is the rotation of the fixed stars, which gives us a day as the smallest unit by which other units can be measured. It is the smallest because it's the one that occurs most frequently and returns to its starting point most quickly.

We should note that for Aristotle the fastest unit for measuring motion is the one that needs least time[19] – that different motions cover different distances is not taken into account here. Only time is considered in determining the fastest movement for measuring other motions. The dimension to be measured cannot be further analysed as the relation of two simpler dimensions (like the relation of time and distance),[20] but is simply time. Thus, Aristotle seems to understand motion and speed here in roughly the same way as Plato in the *Timaeus* – the fastest is what "takes least time", no matter what distance is traversed.

It may be objected that what we are measuring with is another motion, and so something that is in itself complex (covering some distance in a certain time). However, all that is taken into account of this motion with which we measure is the time it takes, as Aristotle states explicitly; and while we could also use a motion to measure a distance,[21] we usually cannot use the very same motion to measure the time and distance of another motion simultaneously. For example, the *distance* the sun covers during a day (speaking from an ancient geocentric perspective), plays no role when I use the sun's motion to measure the journey from Athens to Sparta as needing, say, three days.[22] We may understand the passage as tracing back one dimension (motion) to another one (time); however, we are tracing back motion to a dimension that is simple in itself, solely time. We will see this prominently taken up in Aristotle's *Physics*, where time (and only time) is claimed to be "the measure of

[18] The fastest motion must also be simple, we are told in our passage, and by this Aristotle presumably means that it just goes in one direction on a circle (as the fixed stars move in a circle without changing directions or deviating to right or left). See also *Physics* VIII, 9 for Aristotle's explanation of why only circular motion can be simple and complete.

[19] Here Aristotle probably has in mind that part of the heavenly motion that produces the smallest temporal unit, namely a day (which does not take least time in the sense of coming to a halt then, but in the sense that it requires the least duration to get back to its starting point).

[20] Or the relation of time and angle, if we are dealing with angular velocity.

[21] As Aristotle does in his *Physics*; see below.

[22] Otherwise we would get into problems like how to measure a motion if the motion we measure with proceeds in another direction from the one that is measured, or if the motion used for measurement purposes is a different kind of *kinêsis*, etc.

motion".[23] Regarding the *Metaphysics*, we can conclude that Aristotle's model of measurement for movement is a one-dimensional measure employing one-dimensional units.[24]

(3) Finally, for Aristotle a measure must be one and indivisible (i.e., it must be simple either in quantity or quality):

> πανταχοῦ γὰρ τὸ μέτρον ἕν τι ζητοῦσι καὶ ἀδιαίρετον· τοῦτο δὲ τὸ ἁπλοῦν ἢ τῷ ποιῷ ἢ τῷ ποσῷ.
>
> For everywhere they seek the measure to be one and indivisible; and this is what is simple either in quality or quantity. (1052b33–5)

The talk of indivisibility here should not make us think of any atomistic ideas – Aristotle clearly argues against atomism in his natural philosophy. He assumes that for measurement purposes we *treat* the measure as indivisible, but this does not mean that the magnitude used as a measure is in fact atomistic, as we will see in a moment. Nevertheless, what seems surprising in light of our interpretation so far is that Aristotle talks about simplicity in quality *or* quantity. As we saw above, each measure necessarily requires dealing with quantity *as well as* quality, since it is always a quality (what we called 'dimension') that is to be quantified by measurement. When we consider the context, however, we see that the quality or quantity to be investigated is only raised once the appropriate *scale* for the measurand is discussed (1052b25–35).[25] Accordingly, indivisibility in quantity or quality seems to be a further specification of the kind of things measured. Aristotle apparently wants to examine two different kinds of magnitude, which need different kinds of scales to measure them: some magnitudes need scales using elements that are simple in quality as a basis; other magnitudes need scales working with elements that are quantitatively simple. In both cases the basic elements, that is, the units used for measurement, must be treated as indivisible. For they are meant to serve as a basis in such a way that the measurand can be expressed as the multiple of this basis (for example, a foot is taken as being indivisible for measuring a length so that the length of the beam we want to measure can be expressed as three times this one foot, that is, as three feet).

Aristotle further characterises being indivisible in quantity as being indivisible with respect to perception, while indivisibility in quality is captured as being indivisible in *eidos*.[26] Still to be explained, however, is what indivisibility

[23] Note that Aristotle claims that we measure motion by time and time by motion (for example, in *Physics* 220b31–2) as well as that time is the measure of motion (*Physics* 220b32–221a1; see below).

[24] The units must be one-dimensional as well for they must be chosen in accordance with the dimension.

[25] For a more detailed demonstration that Aristotle is indeed introducing differences in scale here, see Sattler 2018b.

[26] See 1016b23–4, 1020a33–b1, 1053a19–20, and 1087b33–1088a3.

in *eidos* and *perception* means and to what scales they are relevant. Aristotle does not explain this distinction any further explicitly, but he gives a couple of examples for each. As an example for indivisibility in quantity or to perception he gives us a foot:

> τὸ δ' εἰς ἀδιαίρετα πρὸς τὴν αἴσθησιν θετέον, ὥσπερ εἴρηται ἤδη· ἴσως γὰρ πᾶν συνεχὲς διαιρετόν.
>
> The one [the foot] must be placed among things which are undivided with respect to perception, as has been said already – for every continuum is equally divisible. (1053a23–4)

Of course, a foot is divisible in principle, since it is a continuum. However, for measurement purposes we can treat it as if it were indivisible, since for our perception the foot is given as a whole in nature and hence as something we perceive as one thing. It is something that cannot be further divided in so far as it is a given whole (1052a22–3). We use such units in order to quantify continuously extended magnitudes. These units seem to be 'quantitatively indivisible' because in order to serve as a basis for measuring, these units must be treated as if they were indivisible in their quantity; for example, the foot is treated as indivisible in its extension. (They are, however, not indivisible in quantity in the way the atoms of Leucippus and Democritus are indivisible in their extension.)

Being indivisible in *eidos* is explained as that which is indivisible in or for knowledge and science in 1052a32–3 – what we may term 'conceptually indivisible'. The basic idea seems to be that if one tries to divide such a unit further, it will no longer fit the definition of its dimension. For example, dividing a semitone in the diatonic scale or a quarter-tone in the enharmonic scale would lead to something that can no longer be used as a basis[27] for the respective musical scales (while in principle we could use a half foot as our basic unit). Unlike the units indivisible to perception, the units indivisible in quality are not continuously extended; rather, they are discrete.[28]

The important difference between indivisibility in quantity and in quality pointed out by Aristotle is that with a scale employing units indivisible in quantity we are quantifying something continuous, while with a scale using units qualitatively (i.e., conceptually indivisible) we are quantifying something that is not continuous. In modern measurement procedures we are not really concerned with what Aristotle calls the "indivisible in quality", for we take what is "indivisible in quality" as the basis for counting, rather than for

27 We can of course divide the difference between two wavelengths further or the string on a monochord, let's say, between what we in modern parlance would call E and F, but we will no longer get a tonal unit of our musical system. Rather, we will interpret such a tone as a badly played E or F. Accordingly, the semitones are indivisible as far as hearing goes.

28 For a more in-depth discussion of the distinction between indivisibility in quality and in quantity, see Sattler 2017a.

measuring (see below). We will see that Aristotle does not always count what is indivisible in quality as part of the field of measurable quantities either.

8.1.2 Comparison with a Modern Conception and the Relation between Counting and Measuring

Aristotle's understanding of measurement can be shown to be a borderline case of the modern conception of measurement presented in the first chapter, for (1) it possesses all the basic features named above, but (2) these features are determined in Aristotle's *Metaphysics* in a very specific way, not in the general way discussed above.

(1) The basic features we postulated as necessary for all kinds of measurement are dimensionality, the assignment of the measurand to the number series, and units to carry out this quantification. All three features of measurement can be found in the passages of Aristotle's *Metaphysics* referred to above: Aristotle's awareness of the need to determine the *dimension* to be measured is evident in his claim that the measure must be of the same kind as the thing measured. Furthermore, Aristotle presents a couple of different *measurement units*, for example, a foot or a semitone, and thus shows that he takes into account that different basic units of measurement must be found for measurands of different dimensions – the semitone quantifies the dimension of pitch, the foot quantifies the dimension of length.[29] Finally, Aristotle's remark that number means a plurality measured by a one – a measured plurality[30] indicates that we can understand the *measurand* to be *assigned to numbers* with the help of units. This assignment makes it possible to answer the question, How much (many) of the measurand in question are we dealing with?

(2) However, Aristotle understands these features of measurement in a very specific way. He assumes the dimension of the measurand to be essentially simple (i.e., one-dimensional), while for us the measurand can be *n*-dimensional. Furthermore, whatever our metaphysics of numbers, we normally *treat* the numbers to which the measurand is assigned as abstract and separate, a least conceptually, from the thing to be measured. For

[29] The ancient understanding of measurement units is slightly different from our modern one: for Aristotle a unit must be understood as indivisible for measurement purposes so that the measurand can be seen as a multiple of this basic one. By contrast, there is no expectation of indivisibility in modern theories of measurement. Furthermore, the ancient Greeks had two words for units: μονάς and μέτρον. *Metron* can be a unit of all kinds of things (a centimetre, a kilogram, etc.). By contrast, *monas* is primarily tied to numbers – it is a unit of numbers in the sense that, for example, the number seven consists of seven *monades*. Counting obviously involves this sense of units; see Euclid VII, def. 2: "A number is a multitude composed of units [*monades*]."

[30] 1088a5, discussed on the next page.

Aristotle, by contrast, numbers are necessarily always dimensional. Aristotle's account of numbers is a controversial subject in the secondary literature, but judging from books M and N of the *Metaphysics*,[31] it seems that he assumes numbers always to be connected to a single sensible domain or respect, what we called 'dimension' above. For Aristotle, empirical things and numbers are strictly joined,[32] which, as we will see shortly, is one reason why counting and measuring are not necessarily always distinguished.

Let us look at Aristotle's idea of the dimensionality of numbers in so far as it is relevant for our inquiry of measurement in a bit more detail. For him, the one is understood as being a unit tied to a particular respect or dimension and the necessary starting point for quantifying something: "The essence of what is one is to be some kind of beginning of number; for the first measure is the beginning" (1016b17–19). This understanding is spelled out in more detail in 1087b33–1088a14, especially in 1088a4–8:

> (1) 'the one' means the measure of some plurality, and 'number' means a measured plurality and (2) a plurality of measures (thus it is natural that one is not a number;[33] for the measure is not measures, but both the measure and the one are starting points).

The two principal parts of the first sentence of this passage, which I have labelled (1) and (2), can be analysed as follows:

(1) I understand "'The one' means the measure of some plurality" as claiming that the one is always a measure in the sense of a measurement unit with

[31] Aristotle is much more explicit in his account of geometry than of arithmetic, but given his stance on geometry, which makes mathematical objects dependent on substances, I take the following to be a plausible reconstruction of his arithmetic.

[32] Cf. Annas 1987, pp. 142–3, where she interprets *Metaphysics* M, 1–3 as showing that mathematical objects are not *ousiai*, but merely *onta*, for *ousiai* exist separately on their own, in contrast to mathematical objects. On the idea that mathematical objects and hence numbers must have some sort of matter in Aristotle's thought, see Hussey, p. 183; Cleary 1995, p. 375.

[33] Not understanding one as a number was common in ancient Greece; see Euclid *Elements* VII, defs. 1–2, and Heath 1921. See, however, Annas 1975, p. 100 and her 1976 translation, p. 39 for passages where Plato and Aristotle do use one as a number; and Cleary 1995, p. 383 (Pritchard 1995, ch. 5, however, claims that the one as unit only shares certain predicates with *arithmoi*, without thus being understood as an *arithmos*). For Aristotle, the one as well as numbers are thought to be tied to a particular respect, e.g., if we count our cups, the one as well as any number we come up with in our counting are a one and a number of cups. Hence, the one can also be treated as a number by Aristotle, since one and numbers are both dimensional in the sense that one or two are one or two *of something*, not one or two simpliciter. In Plato, the same double treatment of the one as a number or as the starting point for numbers is possible because he understands the one as well as numbers to be strictly non-dimensional and intelligible.

which we quantify a plurality.[34] Number as a "*measured* plurality", then, is the number of the thing quantified; for example, the three we determine (by measuring) is a three of inches, three inches. Therefore, number cannot be thought of as independent of the unit. Rather, number is the numerical value of the magnitude specified by the unit.

(2) Number is also determined as "a plurality of measures", which serves to explain why "one is not a number". Accordingly, this expression proposes that a number must be understood as the number of times a basic one (an inch, for example) must be used in order to measure out something (3 inches are a plurality of the basic measure 'inch'). The one, by contrast, gives us this basic unit and is not itself a plurality.

Thus, for Aristotle, the one tells us the respect or dimension measured as well as the unit with which this dimension can be measured: if our one is a kilogram, for example, we know we are measuring weight and we have a unit for measuring other, heavier things, namely 1 kg.[35] Each number is a multiple of such a basic unit, and thus necessarily tied to the dimension of this basic unit. It is the result of the measurement process and, hence, the numerical value of the magnitude specified by the unit.[36] This understanding of number seems to be the main reason why Aristotle does not differentiate between counting and measuring in the passages we have investigated so far, but presumes them both to be processes of measurement.[37]

Aristotle does not seem to be interested in the difference between measuring and counting we encountered in Chapter 1, namely, that when we measure, the dimension must be determined first and the units are then derived from a continuum, while when we count, discrete units are given (like a horse, for example), as is the dimensionality. So, for instance, at 1088a8–11 Aristotle describes counting the number of horses as 'measuring' them, which shows that for him having a group of discrete elements is not necessarily what distinguishes counting from measuring;[38] he seems to work with a broader notion of measuring here that includes counting.[39]

[34] For this understanding of number, see also *Physics* 224a2–4.

[35] Given that Aristotle thinks of length and breadth as different dimensions (since amounts of them cannot, qua length and breadth, simply be added together), it seems that he is not thinking of the units in questions as the units the physical instrument gives us – the yard ruler – since this could be used for both length and breadth. Rather, he seems to be thinking of what we could call abstract units – the yard-of-length versus the yard-of-breadth.

[36] Euclid's account of 'unit' (*monas*) and 'number' (*arithmos*) in his *Elements* can be understood along similar lines; see the beginning of book VII, where we read in definition 1 that *monas* is that in virtue of which each of the things that exists (*ton onton*) is called one (*hen*), and in definition 2 that number is a multitude composed of units.

[37] Cf. Annas 1975, pp. 99–100.

[38] Cf. Annas 1975, pp. 98–9.

[39] What we would probably capture as 'quantifying', which can include measuring as well as counting.

However, in *Metaphysics* Δ, 13, Aristotle distinguishes between numerable and measurable entities, between counting and measuring in very much the way we do:

> πλῆθος μὲν οὖν ποσόν τι ἐὰν ἀριθμητὸν ᾖ, μέγεθος δὲ ἂν μετρητὸν ᾖ. λέγεται δὲ πλῆθος μὲν τὸ διαιρετὸν δυνάμει εἰς μὴ συνεχῆ, μέγεθος δὲ τὸ εἰς συνεχῆ.
>
> A quantum is a plurality if it is countable, a magnitude if it is a measurable. ‘Plurality’ means that which is divisible potentially into non-continuous parts, ‘magnitude’ that which is divisible into continuous parts. (1020a8–11; Ross’ translation with modifications)

In light of the passages investigated so far, it is prima facie surprising to find this differentiation between *arithmêton* and *metrêton* in Aristotle’s *Metaphysics*.[40] But judging from the previous subsection, this or a similar differentiation is what we should expect, since there all dimensions were divided into two groups according to the scales needed to quantify them. On the one hand, a measurable magnitude can be quantified with the help of units ‘indivisible in quantity’ (or to perception), that is, with units that are continua, like a foot. They are divisible in principle, but for the purpose of measuring, they are treated as indivisible to perception. On the other hand, a numerable magnitude is quantified with the help of units that are ‘indivisible in quality’ (i. e., conceptually), like the horse with which we quantify the number of horses in a group. Aristotle’s distinction between measurable and numerable magnitudes can be seen as corresponding to our distinction between measuring and counting.

Accordingly, our investigation so far suggests distinguishing two senses of measuring and counting in Aristotle. In a wider sense, counting and measuring mean the quantification of something with a certain dimension and there is no essential difference between them. This wider sense is employed in passages such as 1053a1, where Aristotle discusses the measure (qua unit) of number, as being more exact than a furlong or talent. We also saw this understanding of measuring in 1088a8–11 in Aristotle’s talk about “measuring horses”; and it can also be found in Euclid.[41] It is helped by the fact that the Greek word μετρέω can mean counting as well as measuring.[42]

However, since different dimensions need different scales and are thus quantified differently, with the help of units indivisible either in quantity or quality, we can also distinguish between counting and measuring in a narrower

40 Though we find a similar distinction already in the *Categories*, ch. 6.

41 For example, in *Elements* VII, defs. 13 and 14: “A *composite number* is that which is measured by some number . . . Numbers *relatively composite* are those which are measured by some number as a common measure”.

42 Cf. LSJ. For example, Theocritus *Idylls* 16, 6 seems to talk about *counting* waves (κύματα μετρεῖν) and Herodotus 2, 6 about *measuring* land (μεμετρήκασι τὴν χώρην).

sense. Measuring in this narrower sense means the quantification of a continuous magnitude using units indivisible in quantity. This sense corresponds to our understanding of measuring. Counting in a narrower sense is understood as the quantification of a non-continuous plurality using units indivisible in quality, which corresponds to our understanding of counting as determining the cardinality of a plurality.[43]

Aristotle's understanding of numbers explains not only why counting and measuring can be equated when taken in a wider sense, but also why we do not need a specific operation to assign the measurand to numbers. Within Aristotle's framework, there is no ontological gap between numbers and measurands, for a number just is a multiple of a basic unit.[44] If we nevertheless want to think of the relation of number and measurand in terms of assignment, with Aristotle we have to think of an isomorphism rather than a homomorphism, since there is a one-to-one correspondence between elements and numbers.[45] This isomorphism and Aristotle's conception of numbers indicate that the measurand is unlikely to be regarded as having more than one dimension, for in order to measure something *n*-dimensional, the measurand must be assigned to *n* different numbers that must be related to each other.[46]

Let me briefly anticipate the problems this understanding of numbers raises for measuring motion and speed, where the measurand has to be assigned to two different number series: the duration must be assigned to a number of time with the help of temporal units, while the distance covered is assigned to a number of space with the help of spatial units. We then must find a way how these two different numbers can be related to each other. Modern mathematical conceptions of number allow us, with the help of different units, to assign different aspects of a measurand to the same number series

43 We saw in the previous chapter on Aristotle's notion of continuity that continua have different kinds of limits from unities of discrete elements, like a group of horses. This difference is also reflected in the way these two kinds of magnitude can be quantified: since the elements of a group of horses are clearly and fixedly delimited from each other, the quantity of the group can be determined by *counting* these parts. With a continuum, on the other hand, there are no internal limits given and hence no parts that could be counted. Accordingly, internal marks within the continuum must be constructed first with the help of a measurement unit in order for us to *measure* (in the narrow sense) a continuum.

44 See also Wieland 1962, p. 317: "Eine Zahl ist immer eine Zahl von Dingen". This seems to solve all assignment problems with Aristotle, but it can be shown that he exchanges them for other kinds of problem.

45 It can be understood as a map that is both, one-to-one and onto; and concatenation just *is* addition.

46 Unless the *n*-dimensional measurand is understood as simple, in the way Aristotle seems to understand surfaces. In the case of speed, however, this leads to the kind of problems that the assumption of speed units bring with it; see below and Chapter 9.

that is itself independent of any specific dimension.[47] As the different numbers of the different aspects are still numbers of the one number series, they can be related to each other without any problem and we can express this relationship, for instance, as 'x kilometres per y hours', where 'per' indicates the relationship.

According to Aristotle's conception of numbers, however, each aspect of the measurand to be measured opens up a new dimensional series of numbers, since the number 'assigned' to the dimension measured was shown to be a multiple of the basic one. In this case, it is not clear whether the different numbers of these different series can be related to each other at all (in the way we would need to do for complex measures which work with numbers related to more than one dimension). This problem with the measurement of n-dimensional magnitudes that results from Aristotle's theory of numbers will preoccupy us in Chapter 9.

We may think that Aristotle could treat the measurand as n-dimensional simply by assuming that our basic unit is itself a relation – for example, by using the equivalent of 'one metre per one second' as a basic unit for speed. Speed would then be understood as the multiple of such a 'speed unit', as the multiple of one metre per one second. This treatment would mean, however, that the two aspects of motion, time and space, are fixedly assigned to each other,[48] with one unit of time, a second, always connected to one unit of space, a metre.[49] Such a fixed relation will lead into problems if the thing measured changes speed, for example, if it starts to cover two metres in one second, or one metre in two seconds. It is unclear how the basic unit of one metre per one second would allow for measuring these speeds, since with such a 'speed unit' the relationship between the different aspects cannot vary.[50] Accordingly, such a speed unit would lead directly into Zeno's paradox of the moving rows in which the amount of time required for a motion is understood to be fixedly connected to the space covered.[51] If something were moving 7 metres in 9 seconds, and something else 10 metres in 13 seconds,

47 This is possible because we treat numbers in themselves as having no dimension; see Chapter 9.

48 We may think the Greeks could avoid this problem with the help of axiomatisation, where we get the ratios of times to the ratios of distance covered. For a brief discussion of this possibility and why it would not help capture speed, see Chapter 9.

49 If we did not treat them as fixedly assigned to each other, but rather as being in a flexible relation, then time and space would be treated as two independent magnitudes which require numbers of time on the one hand and numbers of space on the other – the very problem that the idea of a 'speed unit' is meant to avoid.

50 We cannot simply say that if a thing doubles its speed, it is now moving at the speed unit 'times two', since it is not even clear what 'times two' of our speed unit amounts to – 1 metre in 2 seconds, or 2 metres per 1 second, or 2 metres per 2 seconds?

51 See Chapter 3 and below.

our speed unit 'one metre per one second' will not even enable us to determine which motion is faster.

Plato's conception of measurement in the *Timaeus* seems to be somewhat closer to ours, as for Plato numbers are not strictly connected to the sensible realm;[52] a result not of Plato's thinking about measurement but of his basic philosophical assumptions. Aristotle's critical response to these assumptions includes presupposing numbers to be dimensional, as this prevents them from being substantial, independent entities.[53] Nevertheless, it is Aristotle's concept of measurement that prepared the ground for our modern conception of the measure of motion, as we will see in the discussion of Aristotle's conception of measurement in his *Physics* that follows.

8.2 The Measure of Movement in Aristotle's *Physics*

How does Aristotle's understanding of measurement in the *Metaphysics* compare to his *Physics*, where the measure he is interested in first and foremost is that of motion (*kinêsis*)? The attempt to conceptualise the measure of motion as simple will lead into the problems we saw in Plato's *Timaeus*. But attempting to avoid these problems by employing a multidimensional measure seems to create insurmountable problems, as we have just seen in our discussion of measurement in Aristotle's *Metaphysics*. Aristotle seems caught on the horns of a dilemma. If he conceptualises the measure of movement and speed as one-dimensional, in accordance with his conception of measurement in the *Metaphysics*, his account will generate problems similar to those we encountered in Plato (of two motions covering different distances, the one which takes less time will be faster, even if it covers less distance than the other motion, etc.). Alternatively, if Aristotle conceptualises the measure of motion as complex (i.e., more-dimensional), his conception of measurement will generate the problem of having to combine different measurement numbers.

We will see that in the *Physics*, Aristotle begins by conceptualising movement and its measure as one-dimensional – time is shown to be the simple measure of motion. But he then turns, at least implicitly, to a complex measure for measuring motion. This implicit approach to measuring motion is in tension with his explicit philosophical commitments, however, and the final chapter will present a brief discussion of why this tension cannot be resolved. Let us start now with Aristotle's account of time as the measure of motion and the problems it raises.

[52] Cf. Annas 1987, p. 146.

[53] See Aristotle's criticism of Plato's and Platonic conceptions of numbers in books M and N of his *Metaphysics*; his *Physics* 204a17–20; Hussey, pp. 78 and 89; and Annas' understanding of what we called 'the dimensionality of numbers' as an anti-Platonic reaction (1975, pp. 99–100).

8.2.1 Time as a One-Dimensional Measure and Number of Motion

Prima facie the measure of movement in the *Physics* seems to be simple, as is required according to the *Metaphysics*. In 223b16–18 Aristotle explains that in the same way that we quantify the *poson* of horses with the unit 'horse', so we measure motion through time and thus can determine how much of it we have – no mention of distance covered. In general, Aristotle names only time as a measure of movement, as we can also see, for example, in 220b32–221a1: "Time is measure of motion and of being moved" (ἐπεὶ δ' ἐστὶν ὁ χρόνος μέτρον κινήσεως καὶ τοῦ κινεῖσθαι). Aristotle calls time "measure of motion". He does not use an article in front of "measure", as it is part of the predicate here. Aristotle calls nothing else "measure of motion", and time is a simple measure that cannot be reduced to any other physical magnitude.[54] Hence, in Aristotle's *Physics* time clearly is *the* (one-dimensional) measure of movement.[55]

While the passage just quoted explicitly understands time as a measure of motion, in other passages Aristotle calls time "*number* of motion", not its measure. For example, in 219b1–2 time is determined as "the number of motion in respect of before and after".[56] Most scholars do not think that Aristotle makes a distinction between time as number and time as measure of motion in the *Physics*.[57] But this is usually taken as a given, while the discussion above provides good *reason* for understanding number and measure as expressing the same account – since number for Aristotle is always the multiple of a basic unit, in a wider sense he does not distinguish between counting and measuring.

Some scholars have taken these two characterisations of time, as number and as measure, to be significantly different, however, and Coope even argues that time is defined only as a number, not as a measure.[58] So is Aristotle working with two different notions of time? Or is time not understood as a measure of motion at all in Aristotle's *Physics*? Let us look briefly at this scholarly debate to see whether we have overlooked something in Aristotle's account of measurement.

For those scholars who think there is a significant difference between time as number of motion and time as measure of motion, the former is usually regarded as prior. Thus Conen thinks that only time qua number is related

[54] While Aristotle claims that there would be no time without any motion or change, this does not mean that time is nothing but motion (for example, it also requires some counting 'psyche'; see below).

[55] While Aristotle talks about motion and distance measuring each other mutually, he never calls distance or space a measure or number of motion so that we could say that time quantifies the before and after of motion and distance something else.

[56] Cf. 223a30–b1.

[57] See, for instance, Zeller 1879, p. 299; Annas 1975, pp. 97–113; Hussey, p. xxxviii.

[58] See Coope 2005.

to all motion and is therefore fundamental, while time as measure is connected in a special way to the motion of the heavens. Similarly, Sorabji claims time as number to be prior, arguing that we start with counting, and only if the counting is done in a regular way can the resulting stretches of time constitute units of measurement.[59]

However, the differences between counting and measuring that we usually employ and find in modern measurement theory are not used in Aristotle's *Physics* in order to separate time as a number and time as a measure. Usually, measuring and counting are distinguished by (a) the fact that we count given discrete units while we measure continua, which must first be divided into units; and (b) the fact that we count with numbers that are themselves dimensionless, while we measure with the help of units of a certain dimension.

We have just shown that these features do not establish a difference between counting and measuring understood in a wider sense in the *Metaphysics*. As the relevant passages of the *Physics* are concerned with continuous magnitudes throughout, the narrower sense of counting and measuring we saw in Aristotle will not be at play in his account of time. Difference (a), given discrete units versus continua that must be divided, is not employed by Aristotle in the two accounts of time, as at least two points make clear. Firstly, Aristotle explains the "mutual measurement" of time and movement in 220b14–24 with the help of the analogy of counting horses.[60] With counting horses, the unit 'horse' is already given, while with time and movement the unit must be constructed – but evidently Aristotle does not take this difference to be relevant here. Secondly, units of measurement are required for time characterised as number as well as for time as measure,[61] which shows that the distinction between given discrete units and continua that must first be divided is not significant in our context.

Nor can difference (b), between units having and lacking a dimension, be employed for the definition of time in the *Physics*. This is apparent from Aristotle's theory of numbers, which is confirmed in his *Physics* by passages such as 224a2–4, where he claims that when we count ten dogs and ten sheep in a certain sense the number is the same, but the tens are not, since one is a ten of sheep, the other a ten of dogs. As we saw above, for Aristotle number is always the multiple of a basic one that has a certain dimension[62] – so we have a dimensional unit that allows us to quantify either a continuum or a group of things (depending on whether we are measuring or counting in the narrower sense). Hence, number cannot be prior to measure within the

59 Conen 1964, p. 140; Sorabji 1983, p. 86.

60 We will hear more about the problem of mutual measurement later on; cf. Coope 2005, p. 107, n. 13.

61 As we can see when we compare passages like 220b32 ff. and 220b14 ff., where time is called a measure, with passages like 223b12–14, where time is called a number of motion.

62 Unless we are mathematicians and abstract from what is to be counted.

Aristotelian framework. If there were a priority, measure would be prior to number as we always quantify with the help of dimensional units.[63] Furthermore, Aristotle claims time to be a specific number, namely the number that is counted (219b6–8), which also displays the importance of dimension: "each thing is *counted* by something homogeneous, units by a unit, horses by a horse, and time by some [bit of] time"[64] (223b13–14). That which is counted is counted by something homogeneous, thus time as a number has to be understood as having a dimension.

Accordingly, I do not think it possible to understand time qua number as more fundamental than time qua measure, as Sorabji and Conen seek to do. Sorabji's claim that we only deal with measuring once our counting is done in a regular way goes against any understanding of counting we find in Aristotle. For it presupposes that the counting of time can be done in a disorderly way – that we could, for example, 'count' nows in a random fashion. But what kind of counting would this be? It could certainly not be counting *time* for Aristotle. And given that even in Sorabji's random fashion, the nows are quantifying something continuous, what he calls "counting time" should rather be seen as measuring time (even if the units used are not equal).

Similarly, Conen's claim that time as the number of motion is prior to time as a measure of motion fits neither the Aristotelian framework nor a modern understanding: since movement is a continuum, we have to divide it up into parts before we can start counting. If this division is not to be completely arbitrary, it is only possible with the help of a measure supplying us with a discrete basic unit; the result of our quantification will thus be a multiple of this basic unit. Hence, we can only count continua by measuring them. Furthermore, Conen confuses the use of the movement of the heavens as units for measurement with their being measured, for 223b12–20, the passage Conen takes to prove that time qua measure is originally merely the measure of the movement of the heavens,[65] discusses the movements of the heavens only as the *most suitable units* of measurement.[66] The heavenly movements are

[63] That time qua measure is not restricted to certain motions is clear from passages like 221b25–30; 221b21–2 explains that "being in time" means "being measured by time" – so whatever is in time can be measured by time.

[64] "Units by a unit", according to Aristotle, is an abstraction in accordance with the mathematicians; see *Metaphysics* K, 4.

[65] See Conen 1964, pp. 123 and 138.

[66] This passage is one big conclusion (and Conen also translates it like this):

(1) Since there is locomotion and among this circular motion,
(2) and as everything is counted with something homogeneous
(3) and if that which is prior is the measure of all homogeneous things,
then
(4) the regular circular motion is above all the measure,
(5) for its number is best known.

units of measurement, not what is originally measured by time. And they are the most suitable, and hence the 'original' measure of all other movements since, with the help of the heavenly movements, movement in general can be reliably measured within the Aristotelian framework.

Finally, according to Coope, Aristotle defines time only as number, not as measure of motion, since time is defined as an order, not a quantity.[67] For "Counting the before and after in time is a way of ordering changes; it is not in itself a way of finding out how much change has passed."[68] However, we saw in the first chapter that counting does not mean ordering. To count is simply to determine the cardinality of a plurality of things by coordinating two functions: the function that allows us to consider each single element of the plurality – no matter in which order – is coordinated with the operation that takes us through the series of the natural numbers. And, as Coope herself points out, Aristotle's understanding of time as something countable cannot mean that it is countable in the way a discrete plurality is. Rather, time is countable in the *Physics* in the way something continuous is countable once we use units of measurement that allow us to mark off parts of time. But understanding time as countable in this sense also means, contrary to Coope's claim, that we determine its quantity. That time is indeed understood as determined in its quantity and as a measure of motion in Aristotle's *Physics* is supported by the following points.

First, Coope claims that Aristotle is not defining time as measure, even in passages where he seems to do exactly that (such as 220b32–221a1, quoted above), since he is not talking about units in this context. However, determining concrete units is only the practical side of measuring, which as a first step requires what Aristotle does first – establish time as the dimension in terms of which motion is measured. And Aristotle eventually deals with the practical side of measuring when he talks about the motions of the heavens as the *most suitable units* of measurement.[69]

(5) is an explanation of why condition (3), the priority condition, is met with regular circular motion. (2) is further explained by examples – horses are measured by a horse (223b14) – so it is clear that here Aristotle is talking about units used to determine the quantity of something. Thus, if we know that everything is quantified with a unit homogeneous to the thing quantified (2) and in some (epistemological, cf. (5)) sense prior (3), then we can conclude that the regular circular motion is the measurement unit to measure other motions. For as a motion it is homogeneous with other motions (2), it is prior (3) since its number is best known (5), and it exists (1). The passage investigated is meant to answer the question of what the appropriate measurement unit is in order to measure motion. And Aristotle's answer is: the regular circular motion, which we can understand as the movement of the heavens, i.e., the diurnal rotation of the fixed stars.

[67] See Coope 2005, pp. 99 and 104. Coope thus seems to invert part of Sorabji's central claim that we only deal with measuring once our counting is done in a regular way.

[68] Coope 2005, pp. 99–100.

[69] For example in 223b12–20.

Secondly, in order to defend her claim that time is number but not measure of motion, Coope has to assume that Aristotle is talking about number "in an extended sense", since time is continuous, while numbers deal with discrete pluralities.[70] But this extended sense of number captures exactly what is understood by measuring – quantifying continuous magnitudes. Several times Coope has to point out that Aristotle talks about measuring when, according to her interpretation, he should talk about counting.[71]

Aristotle does not use any of the differences available to him (which we saw in the narrow definitions of counting and measuring in the *Metaphysics*) to distinguish time as a number and time as a measure from each other. Hence, in the *Physics* Aristotle seems to use the characterisation of time qua number and time qua measure to indicate the same idea. That the difference between counting and measuring is not significant for Aristotle's *Physics* is also supported by the fact that he draws conclusions about measuring from premises about counting.[72]

Let us explore now which feature of the things that are in time are measured by time, which is specified by passages such as 227b23–6:

> τρία γάρ ἐστι τὸν ἀριθμὸν περὶ ἃ λέγομεν τὴν κίνησιν, ὃ καὶ ἐν ᾧ καὶ ὅτε. λέγω δ' ὅτι ἀνάγκη εἶναί τι τὸ κινούμενον, οἷον ἄνθρωπον ἢ χρυσόν, καὶ ἔν τινι τοῦτο κινεῖσθαι, οἷον ἐν τόπῳ ἢ ἐν πάθει, καὶ ποτέ· ἐν χρόνῳ γὰρ πᾶν κινεῖται.
>
> There are three things with respect to which we speak of motion, the 'that which', the 'that in which', and the 'when' [that during which]. I mean that there must be something that is in motion, for example, a human being or gold, and it must be in motion in something, for example, in a place or in an affection,[73] and when, for all motion takes place during a time.

This passage makes clear that time determines what we would call 'duration' (i.e., the temporal extension of something). In the Greek text, however, we do not find a word with the exact meaning of 'duration'. Here the words *hote*, *pote*, and *en chronô* are often translated as 'during'.[74] Similarly, in 221a4–5 *einai* is sometimes translated as 'duration'.[75] What is measured by time, according to 221a4–5, seems to be the 'being of movement' (so Hussey) or its 'essence' (so Hardie and Gaye). But the 'being of movement measured by time' can only be

70 Coope 2005, p. 95.

71 See, for instance, Coope 2005, p. 108, n. 14.

72 For example, in 223b12–20 discussed above, Aristotle talks about counting (*arithmeitai*) in premise (2) but he draws a conclusion from this about measuring (*metreitai*) in (4).

73 While 'that in which something changes' is understood as place or affection here, it is understood as time in 236b2–4.

74 See, for example, Hardie's and Gaye's translation. For the relationship between *chronos* and duration, see my manuscript *Ancient Notions of Time*, ch. 3.

75 As, for example, in Zekl's translation.

the temporal being of movement,[76] the time 'in' which it takes place (*en chronô*), or what in modern terms we would call 'duration'.

For Aristotle, only the duration of a movement is measured and we are dealing with a simple measure of motion, time – as we saw it in Plato's *Timaeus*. But, in contrast to Plato, for Aristotle time measures all kinds of movements, with no exception.

8.2.2 *The Search for a Measure of the Same Kind as Motion*

We saw above that Aristotle faces a dilemma with the measure of motion in his *Physics*: either he understands it as one-dimensional, as he had in the *Metaphysics*, in which case there will be problems with accounting for speed (i.e., the relation of time and space), or he conceptualises the measure of motion as complex, as the time taken as well as the distance covered – then he will need to combine different measurement numbers, which goes against the one-dimensionality established in the *Metaphysics* and does not fit his ontology of numbers. So far, we have seen that Aristotle employs a one-dimensional measure for measuring movement: it is simply time. Thus, Aristotle seems to have gone for the one-dimensionality horn of the dilemma, and we should expect him to get into problems similar to those encountered by Plato in the *Timaeus*. And matters get worse for Aristotle, as two crucial features of a measure according to the *Metaphysics* – that it is of the same kind as the measurand and that it is one-dimensional – exclude each other with respect to measuring speed.

The measurement conception of the *Metaphysics* requires the measure to be of the same kind as the measurand, for example, a length for measuring length. In the *Physics*, however, *time* is used as the measure of movement, and therefore movement does not seem to be measured with a unit of the same dimension, as the *Metaphysics* demands. If a small amount of one dimension is always used to measure larger amounts of that same dimension, movement should be measured with a movement. Why then is a movement not used to measure the amount of another movement in the *Physics*?[77]

Handling a motion is not as easy as handling, say, a length. Unlike a measuring tape (or a *kanôn* in ancient Greece), which could be handily stored and transported to be pulled from a bag whenever and wherever needed for

[76] It cannot be its spatial being.

[77] We may object that when measuring a rod, we are only using a measure homogeneous with one aspect of the rod, e.g., either with its length *or* with its weight. Similarly, we can either measure the duration of a motion or the distance it has covered. However, Aristotle himself acknowledges speed as what is specific for motion (in contrast to time), so one would expect him also to introduce a measure for it. And in his *Metaphysics* he does indeed claim that a motion is measured by a motion. For the objection that motion is indeed measured by a motion, since temporal units are gained from motions, see below.

measuring purposes, it was not possible for the ancients to transport motions, as we do, for example, with the mechanism in a watch. And even if we do not take into account this technical problem of the availability of units for motions, how that measurement would be conceived is also unclear. What is a *small* motion used for measuring others – a very fast movement, a very short movement, or perhaps a movement that covers only a small distance?

Furthermore, what if the movement used as a measure goes in one direction and the movement to be measured goes in another? It is not clear, for example, how I could use my walk north to measure someone else's run to the east.[78] We could avoid this difficulty by measuring a movement with the help of another movement that moves linearly in the same direction or by having both movements proceed in concentric circles. Alternatively, we could use only one aspect of motion to measure another motion, namely, its duration. The time a motion takes can be measured independently of the direction and kind of movement employed for measurement.

8.2.2.1 Measuring Movement with the Help of Time: Technical and Theoretical Challenges

8.2.2.1.1 Technical Demands Whereas in the *Metaphysics* the units appropriate for measuring movement are treated as if given, the *Physics* makes it clear that units of time are not given in the sense that a unit of weight is given – once weights are set as standard for a scale, they can be used over and over again.[79] The units we need in order to measure a motion cannot be stored; rather they must be constantly produced anew. Although they can be produced with the help of a movement, their production is a rather complicated task. Employing simply any old movement would enable us to decide whether the movement measured took more, less, or as much time as the movement used to measure. But usually such a rough comparative measurement is not enough; we also require information about how long a motion lasts, or how it compares to other motions in the past or future. We saw days, nights, and so forth used for this purpose in Plato's *Timaeus*. With Aristotle, in order to measure the duration of motion A, we need to employ another motion, B, that displays the following features:

(1) Motion B must go on *continuously* without any interruptions. For if motion B, employed in order to measure, were interrupted at any point while motion A, the motion to be measured, carried on, this interval would be indeterminate: without B we would not know for how many units A had gone on. Furthermore, if B is to provide us with units not only

[78] Furthermore, we saw that for Aristotle length and breadth are their own dimensions; see also 205b31–4 and Chapter 9. To attempt to measure a motion in one direction with another motion in a different direction thus seems to result in dimensional confusion.

[79] This was true even in the IS system until the most recent changes in 2019.

for the measurement of one particular motion, but also for measuring and comparing all the different movements and changes taking place at different times, B should go on uninterruptedly and without any end. We will then be able to measure not only *a* time, the time a certain motion takes, but also *the* time.[80]

(2) In order to be able to measure movements taking place in different places, we need a *ubiquitous* measure. Motion B must be accessible from everywhere (at least within a certain geographical location).

(3) The continuous movement B should enable us to *set* reasonable *marks* in order that a unit can be marked off from the continuous process.

(4) In order that these sections of movement always correspond to the same section of time (i.e., that they will lead to regular units allowing for sensible comparisons), B must be *regular*. For Aristotle, regularity implies uniformity (i.e., moving with constant speed).[81]

These four features – continuity, ubiquity, mark setting, and regularity – are demanded of the motion used to measure, in order to gain reliable and repeatable temporal units.[82] For Aristotle, the only movement that meets all these criteria is the motion of the heavenly bodies,[83] which thus play a crucial role for temporal measurement not only in Plato but also in Aristotle. Hence Plato and Aristotle require the same characteristics of the motion providing units to measure motion.[84]

[80] The time, that is, that everything in a certain geographical location shares. Alternatively, requirement (1) could be met by different motions, granted that there will never be an interruption between one movement stopping and the next one starting.

[81] See 223b12–20 and 1053a10–12 for the regularity and uniformity of the motions used for measurement purposes. While we would also consider some non-uniform motions as regular – like regular pendulum swings – for Aristotle, only uniform motions are regular.

[82] Most of these features are also required from the thing used for spatial measurement, as for instance a foot: it must be continuous in the sense of being without interruption, ubiquitous in the sense that it must be useable everywhere and any time, and regular in the sense that it is always the same unit used to measure. Given that a foot is given as a whole, it need not provide us with the possibility of setting marks within itself (as the continuous motion must in order to gain temporal units in the first place); but it must allow us to mark off one foot after another within a continuous length to be measured.

[83] Today, we use the changes in the energy level of the caesium 133 atom as basic temporal units. Hence, while we are sticking to the requirement of regularity, we are in fact not using a continuous motion but rather a succession of discrete states for measuring time. The regular marks needed to gain temporal units are thus set naturally, and we understand the unity of these alternating transitions as time. We are no longer fighting with the severe theoretical problems Aristotle faced in conceptualising time and determining its relation to movement; thus, the continuity we are employing for the concept of time need not be given 'in nature'.

[84] We investigated the features of a unit to measure motion in the *Timaeus*, while here we are looking at the character of a motion that should allow us to gain such units. But the

The regular circular motions of the heavenly bodies easily solve the most urgent problem facing the acquisition of units of measurement, namely, how to get discrete parts that can be used as measurement units from a continuous motion. We can choose any point we please in the revolution of the heavenly bodies and the heavenly bodies will return regularly to this starting point,[85] which enables us to mark off a part of movement naturally.[86] The units gained from the circular motions of the heavenly bodies are ubiquitous – they can be perceived everywhere in the universe. These movements are available everywhere for everybody, solving the problem of availability.[87]

Furthermore, only circular motion can be *without end*, since an infinite linear motion would presuppose an infinite universe, contrary to Aristotle's assumption of a finite universe.[88] Additionally, only circular motion can continue without any change of speed and, thus, produce *regular* units, that is, units that are always the same.[89] For the Aristotelian framework presupposes that the world is set up in such a way that the four basic elements – fire, air, water, and earth – are concentrically arranged around the centre of the universe, each element forming a separate circle. The fifth element, out of which the heavenly bodies are made, forms a circle beyond the sublunary sphere. Only when the heavenly bodies move in a circle do they not change position relative to their natural place and thus can move always at the same speed; but they would change their natural position if they moved rectilinearly.[90]

Hence, the revolutions of the heavens allow us to determine reliable and repeatable units of time accessible to everybody. But in order to gain temporal

similarities should nevertheless be clear. Some such set of requirements may have been common in their time, even if Aristotle is the first Greek thinker who explicitly spells them out. We learned that Plato cared about ubiquity from his special interest in the units gained from the motions of the sun as units perceptible everywhere.

85 Depending, however, on the celestial motion we choose – it holds without any restrictions for the motion of the fixed stars, but the sun's motion from sunrise to sunrise changes over the course of the year.

86 A unit thus gained is not just taken up like a foot given in nature; rather we must use the possibility of drawing a mark ourselves.

87 Measuring with the help of a circular motion, however, is also the reason why people thought time to be the movement of the sphere, and human affairs to form a circle, mistakenly transferring the circularity of the movements used to derive temporal units to what is measured; cf. *Physics* 223b21–33.

88 As is evident in his *De caelo* and in *Physics* 241b18–20; see also *Physics* VIII, 8–9, which argue that motion in a straight line would have to double back and thus would include moments of rest.

89 Strictly speaking, the regularity is required by the time used to measure, since units of time cannot be stored like units of weight, but must be constantly produced anew. Hence, the movement used for their generation must be uninterrupted so that there is always a temporal unit available; and it must be without any change of speed so that different measurands measured with units produced at different 'times' are still comparable.

90 See *De caelo* 288a14–b10 and *Physics* 227a27–9.

units, Aristotle has to abstract from any spatial characteristic and to concentrate solely on the temporal aspect of the heavenly motions used to acquire units. This temporal aspect is straightforwardly comparable to every other motion, independent of any issues of direction, and thus is exactly what we seem to need for the purposes of measuring.[91]

8.2.2.1.2 Theoretical Problems We have seen the advantages of measuring only the duration of a motion and how Aristotle's framework allows temporal units to be derived. However, a significant theoretical problem lingers if *only time* can be called the measure of motion, as the *Physics* appears to suggest. That position seems similar to claiming that the measure of a table can only be length, and not also, for example, weight. If only time can be the measure of motion (and is not just the only measure of the *duration* of motion), is motion not simply the same as time? In this case either (I) time is reducible to motion or (II) motion is nothing but time.

Let us look at the first possibility, that time is reducible to motion (I). The beginning of Aristotle's treatise on time makes it clear that time depends on motion or change:[92] "it is obvious that time is not without movement or change" (φανερὸν ὅτι οὐκ ἔστιν ἄνευ κινήσεως καὶ μεταβολῆς χρόνος, 218b33–219a1).[93] As Aristotle then spells out, this dependence means that

[91] Aristotle does indeed also talk about the heavens when dealing with *topos* in *Physics* IV, but in the context of measurement they are only used to provide temporal units.

[92] He has already sketched out that time cannot be the same as movement or change though we necessarily always perceive them together, and seems to conclude from this that there is no time without movement or change and that time is something *of* movement (τί τῆς κινήσεώς ἐστιν), that is, it is some feature, characteristic, or attribute of motion. This conclusion, rather than that movement is something of time, seems to be due to the fact that time does not have a 'substratum' of its own, while motion has as its 'substratum' the thing moved. However, it is not clear whether this conclusion in 218b29–219a10 is valid, for movement could also be just one *conditio sine qua non* of time, or movement and time could mutually be a necessary condition for each other without one belonging to the other.

[93] How does Aristotle derive the claim that there is no time without motion from the observation that if there is no motion, no time seems to have elapsed, and whenever we perceive motion, we are aware that some time has elapsed? The secondary literature offers three explanations: (1) the so-called verificationists, like Sorabji and Hussey, assume some implicit premise to be at work to the effect that all intervals of time can in principle be perceived. If all temporal intervals can in principle be perceived, and we cannot perceive time without motion, then there is no time without motion (see Sorabji 1983, p. 75 and Hussey, p. 142). (2) Coope 2005 assumes Aristotle to be correct in his conclusion due to the endoxic method he uses: the assumption that there is no time without motion turns out to underlie the judgements we ordinarily make about time and thus to be an *endoxon* on which further investigations rest. (3) Finally, for Roark 2011, Aristotle's conclusion is supported by time being an "evident proper feature" of motion, which Roark understands as follows: "For any pair of type-perceptibles $< \Phi, \Psi >$, Φ is an evident proper feature of Ψ iff every token Ψ_i (and nothing else, except by virtue of its relation to some such token)

we require motions or changes in order to gain temporal units. While time is a measure of movement that quantifies the duration of a motion, time is dependent on movement for the production of temporal units. We appear to be employing motions in order to measure a motion after all,[94] and time seems to have fallen out of the picture. But while motions thus simply seem to measure other motions, time makes clear which aspect we are interested in for our measurement – we are focusing on the duration of the motion. The direction of a motion or the distance covered are not taken into account. Time is thus not reducible to motion, rather time as a measure specifies the particular aspect of motion we are quantifying.

However, if time is the only aspect measured, the second possibility comes into play, that in the measurement process motion appears as nothing but time (II). What we measure of a movement here is identical to what we measure of a time. Therefore, the result of the measurement process (for example, that it lasted five seconds) seems to be the same, whether we measure a time or a movement. Until now, however, it seemed to be the case in Aristotle's discussion that the measurement of every dimension had to have results specific to that dimension, and thus its own "measurement number" (for example, five feet when measuring length, but five pounds when measuring weight). If time is all that we can measure of motion and our measurement number is merely temporal, then motion seems to be reducible to time. Aristotle is clear, however, that time and motion cannot simply be equated:

> ἔτι δὲ μεταβολὴ μέν ἐστι θάττων καὶ βραδυτέρα, χρόνος δ' οὐκ ἔστιν· τὸ γὰρ βραδὺ καὶ ταχὺ χρόνῳ ὥρισται, ταχὺ μὲν τὸ ἐν ὀλίγῳ πολὺ κινούμενον, βραδὺ δὲ τὸ ἐν πολλῷ ὀλίγον· ὁ δὲ χρόνος οὐχ ὥρισται χρόνῳ, οὔτε τῷ ποσός τις εἶναι οὔτε τῷ ποιός.
>
> Again, change is always faster or slower, whereas time is not: for 'fast' and 'slow' are defined by time – 'fast' is what moves much in a short time, 'slow' what moves little in a long time; but time is not defined by time, by being either a certain amount or a certain kind of it. (218b13–18; translation by Hardie and Gaye)

This passage shows that time and motion cannot be the same thing, since motion is always fast or slow, while time is not. But, then, can what seems to be specific to motion and what Aristotle finally deals with as the quantity of movement – the speed of a motion – be measured at all with the help of time? Can time be of the same kind (συγγενές) as speed in the same way as a measure was required to be of the same kind as what is measured? It seems that

has some token Φ_i as one of its features, and for any percipient *z*, *z* cannot perceive any Ψ_i as a Ψ without also perceiving its Φ_i as a Φ." We cannot perceive time without motion, because time is an evident proper feature of motion, and that means time is something of motion.

[94] Cf. Wieland 1962, p. 327.

a measure of the same kind as speed has to be more complex than time alone. The passage cited introduces the idea of a second magnitude besides time which has to be taken into account when measuring speed: if much of this second magnitude is covered, then the change is fast, if little, the change is slow. For locomotion, this second magnitude is the aspect from which we abstracted earlier on, namely the distance covered by a motion. But how can this aspect be reintroduced without leading to the dimensional confusions we have talked about? In the next section, we will see Aristotle's first steps towards solving this problem.

8.2.2.2 Aristotle's First Steps Towards a Complex Measure of Motion

In order to conceive of a complex measure of motion, it must first be clear that time and motion are not simply identical. Aristotle secures a difference between time and motion in a way that we will analyse in three steps. First, time is defined as a feature of motion, more precisely, a measure or number *of* motion with respect to before and after – thus, the motion measured involves more than the time it takes (1). The difference between time and motion is fleshed out with reference to the thing moved (2) and with reference to the account (*logos*) of the thing moved (3).

8.2.2.2.1 The Difference between Time and Motion (1) Motion measured involves more than the time it takes:

> τοῦτο γάρ ἐστιν ὁ χρόνος, ἀριθμὸς κινήσεως κατὰ τὸ πρότερον καὶ ὕστερον. οὐκ ἄρα κίνησις ὁ χρόνος ἀλλ' ᾗ ἀριθμὸν ἔχει ἡ κίνησις. σημεῖον δέ· τὸ μὲν γὰρ πλεῖον καὶ ἔλαττον κρίνομεν ἀριθμῷ, κίνησιν δὲ πλείω καὶ ἐλάττω χρόνῳ· ἀριθμὸς ἄρα τις ὁ χρόνος. ἐπεὶ δ' ἀριθμός ἐστι διχῶς (καὶ γὰρ τὸ ἀριθμούμενον καὶ τὸ ἀριθμητὸν ἀριθμὸν λέγομεν, καὶ ᾧ ἀριθμοῦμεν), ὁ δὴ χρόνος ἐστὶν τὸ ἀριθμούμενον καὶ οὐχ ᾧ ἀριθμοῦμεν. ἔστι δ' ἕτερον ᾧ ἀριθμοῦμεν καὶ τὸ ἀριθμούμενον.
>
> For this is time – number of motion in respect of 'before' and 'after'. Hence time is not motion, but only motion in so far as it has number. A proof of this: we judge the more or the less by number, but more or less motion by time. Time then is a kind of number. But number, we must note, is used in two senses – both of what is counted or the countable and also of that with which we count. Time obviously is what is counted, not that with which we count. That with which we count is different from that which is counted. (219b1–9; translation by Hardie and Gaye with modifications)[95]

While time is named in this passage as the only aspect of motion measured, Aristotle explicitly claims that time is not motion. Rather, time is "number of

[95] See also 220b8–9.

motion in respect of 'before' and 'after'", and more precisely 'what is counted *of* motion' – it is solely an aspect of motion that we can quantify.[96] In order to better understand the idea of time as what is counted of motion, let us first clarify the difference between number as what is counted and number as that with which we count, which Aristotle introduces here.[97]

This distinction is commonly understood as the distinction between a number that is the same in all counting processes – the number with which we count – and a number that refers to the particular group counted – the number as what is counted.[98] I want to suggest a somewhat different account of this distinction that allows for making good sense of some peculiarities of Aristotle's treatment of time, as we will see below.[99] The distinction that I suggest Aristotle has in mind here may be easiest to grasp if we consider how this difference is manifested in the measurement of something simple, such as the length of a rod. *What is 'counted'* (actually) or 'countable' (potentially) is the length of this rod, which can be expressed by a number. Accordingly, number in the sense of what is counted is the length of the rod as expressed by a certain number, for instance, 3 feet.[100] *That with which we count* is a

[96] It may be objected that even if time is not motion, but some aspect of motion, it may still be the only aspect that can be quantified, and thus can be treated as equal to motion within a measurement context. However, we saw in the speed passage above that for Aristotle what is fast is what "moves *much* in a short time" – so the second magnitude is obviously quantifiable too (it is either much or little that is covered in time). For reasons of space, I cannot discuss here Aristotle's qualification that time is number of motion *in respect of before and after*.

[97] Note that Aristotle here uses 'counting' in the wider sense, since he talks about time as what is counted, even though time is something continuous.

[98] See Hussey, p. 151. Coope 2005, p. 89 understands it as the "distinction between those numbers that are only countable and those that are both countable and also of the kind we count with". One of her examples is a group of three dogs, where the three dogs are what is counted and the numerals 1, 2, and 3 are the numbers with which we count (p. 90). Coope claims the distinction between numbers with which we count and numbers that are only countable to be "a distinction between the kind of number that is a discrete plurality and another kind of number that is continuous". However, this clearly does not fit her own dog example (if we count dogs, what we count is clearly not continuous but given as a group of discrete elements).

[99] The remainder of my interpretation of Aristotle in this chapter does not, however, depend on whether the reader accepts this reading of the distinction.

[100] What is counted is not the number 3, but the 3 feet that make up the length of the rod. This number will be bigger or smaller depending on the unit we chose: if we measure with a foot, our rod may be assigned the number 3, while if we measure with a yard, the number of the same rod will be 1. The length measured as such, however, is an independent magnitude; it is in no way affected by the number that is assigned to it. We may be surprised to find Aristotle calling a continuous magnitude like time 'number', but this is a problem all interpretations of this passage face (see also Coope 2005, p. 88 and p. 90, n. 13). I have tried to explain Aristotle's switches between discrete and continuous quantities above.

measurement unit, for instance a foot, with the help of which we can measure out the whole length of the rod. Number as that with which we count is this basic unit, our foot, and the multiple thereof. In the case of time measurement, what is counted is the time taken by a certain motion, A, and that with which we count is a certain motion, B, used as a temporal unit. The time of the second motion, B, can be called a number because it determines the 'one' (for example, 1 day) of which all other numbers (for instance, 5 days) will be a multiple. The time of the first motion, A, can be called a number because it can be expressed with the help of the second.[101]

Understanding time as what is counted *of* motion implies that motion is not only its time, in the same way the rod is more than the length that is measured: it has density, weight, colour, etc., features of the rod that are not taken into account when measuring its length. Even if the rod is measured with a shorter rod, only its length is being measured, which is something that is *of* the rod, not the rod itself.

In this last example, the length of a rod appears on both sides of the measurement process: the length of the shorter rod is what we measure with, while the length of the rod in question is what is measured. Time can also be used in both senses: as the aspect of motion that is measured[102] and as that with which we measure, for we use temporal units for our measurement of the temporal aspect of motion. Although we refer to a certain *motion* as a unit – for example, one revolution of the sun – all aspects of this motion except its duration are disregarded, and what is left is used as the basic unit for our time measurement. In the case of the motion measured as well as that of the motion used to measure, every feature of the motion apart from its time has been abstracted.[103]

In accordance with this result, we find Aristotle not only identifying time as a number in the sense of that which is counted of a motion, but also indicating that he understands time as a number with which we count. Time is what is

[101] It may be objected that this identification of the number with which we count with the unit primarily, and with the multiple only derivatively, overlooks the essential plurality of the Greek term *arithmos*. However, while Aristotle in general follows the practice of Greek mathematicians to distinguish between one (*hen*) as the unit and *arithmoi* as a plurality, there are other passages where he understands one as *arithmos*; for example, in *Categories* 5a31, in *Physics* 220a27–32, and in *Metaphysics* 1056b25, 1080a24, 1082b35, 1085b10; see also Annas 1975. Furthermore, if the number with which we count is tied closely to the specific unit, e.g., a foot, then it seems there is a danger that Aristotle has no means of expressing the fact that the number of feet along my garden path is the same as the number of dogs that are running around in my garden. However, Aristotle can still allow for expressing this fact with the help of an extra step of abstraction from the concrete units – a step of abstraction, however, that will be of more interest to mathematicians than to philosophers of nature.

[102] See, for example, 219a3.

[103] Hence, time is the only respect that the two motions must have in common.

counted of a motion but, if the motion in question is one we use in order to measure other motions, then time is also the number with which we count. In 221b10–11 Aristotle makes clear that time as the number of motion does not require something to move, for it can allow for something being at rest 'in this number'. That is, time is not only *what is counted* of a particular motion. In the case of something resting, time is something *with which we count* the period of rest.[104]

In 220b3–5 Aristotle claims that time as something continuous is long or short, while as a number time is many or few.[105] Prima facie this passage seems to make a difference between time as measure and time as number, which runs counter to our conclusions above. However, Aristotle is not suggesting here that there is a time that is a measure and a time that is a number, but rather that time can be seen as something continuous – which we can understand as the continuous duration that we quantify – and as a number: once we have gained a temporal measurement unit, time can be treated as something discrete with which we can quantify a duration.[106]

For Annas, Aristotle's use of time as what is counted as well as that with which we count is the expression of a conflict that stems from the "confusion" of number with numbered groups.[107] By contrast, the interpretation developed here suggests that Aristotle's use of time in both forms is just what we should expect.[108] While time as that aspect of a motion that is counted is a time *tied to the motion to be quantified*, time as that with which we count is time *used as a reference for all motions*. We find this distinction in Aristotle, without any confusion of these two kinds of number.

Since Aristotle has shown that time is only one aspect of motion, he must show, first, what else there may be to motion that allows for a distinction between time and motion, and, second, how it is related to the temporal aspect of motion. In order to find an answer to the first question, we will explore Aristotle's reference to the thing moved (2), as well as to the account (*logos*) of

[104] See also 220a15–21, 221b10 f.; Sorabji 1983, p. 86; Annas 1975, p. 111.

[105] Even if one were tempted to translate *polus* and *oligos* here as 'much' and 'little', we would have to understand 'much' in the sense of many units of time and 'little' as few units of time in order to account for the contrast Aristotle is pointing out here.

[106] Coope 2005, p. 89 ff. claims that time is only a number that is countable, not a number with which we count, because time is not itself a discrete plurality, which is what we need in order to count. Coope is right that time, for Aristotle, is not a discrete plurality, but rather something continuous. However, a temporal unit gained from a continuous motion, such as, for example, a second, can be used to count (in the wider sense) the times it fits into a certain duration; and in this sense, time can also be that with which we count. The passages in which Aristotle takes time as a number with which we count, quoted above, support this interpretation.

[107] Annas 1975, p. 111.

[108] Time as that with which we count is usually not discussed as time but rather as number (which is the multiple of a temporal basic unit).

the thing as being in motion (3), as candidates that establish a difference between time and motion.

(2) In trying to refute the opinion that time and motion are identical, Aristotle first employs a reference to the thing moved:

> ἐπεὶ δὲ δοκεῖ μάλιστα κίνησις εἶναι καὶ μεταβολή τις ὁ χρόνος, τοῦτ' ἂν εἴη σκεπτέον. ἡ μὲν οὖν ἑκάστου μεταβολὴ καὶ κίνησις ἐν αὐτῷ τῷ μεταβάλλοντι μόνον ἐστίν, ἢ οὗ ἂν τύχῃ ὂν αὐτὸ τὸ κινούμενον καὶ μεταβάλλον· ὁ δὲ χρόνος ὁμοίως καὶ πανταχοῦ καὶ παρὰ πᾶσιν.
>
> Since time is above all thought to be motion and a kind of change, this is what must be examined. Now the change or movement of each thing is only in the thing which changes or where the thing itself which moves or changes may chance to be. But time is present equally everywhere and with all things. (218b9–13; translation by Hardie and Gaye with modifications)

Finishing off a collection of puzzles concerning time, Aristotle investigates the hypothesis here that time is a kind of motion or change. He is never seriously tempted to adopt this position. But it seems to have been a position that had some attraction and was held by some people. So in light of the close connection Aristotle assumes between time and change, others may believe this position to be in line with his thinking, which makes it necessary for him to spell out why this collapse of motion and time does not hold. He starts this inquiry by naming the most important objection to this hypothesis: while time is ubiquitous,[109] motion is not. Whereas movement can only be in the thing undergoing a particular motion, time is everywhere alike with all things[110] and not bound to a single thing.[111]

Prima facie it seems that motion is tied to the thing moved while time is not, so that a reference to the thing moved should be enough to distinguish between time and motion. However, if the thing that moves now and whose existence is a condition of that very motion, is at rest later on, it will still be linked to time but no longer be linked to motion. Reference to a thing that moves and rests can, nevertheless, be used to strengthen the claim that there is a difference between time and motion, for whatever is moving is in time, while not everything that is in time is moving.[112] The kind of difference the moving thing does in fact establish becomes clearer in the following passage:

> ἐπεὶ δ' ἐστὶν ὁ χρόνος μέτρον κινήσεως, ἔσται καὶ ἠρεμίας μέτρον [κατὰ συμβεβηκός]·[113] πᾶσα γὰρ ἠρεμία ἐν χρόνῳ. οὐ γὰρ ὥσπερ τὸ ἐν κινήσει

[109] It is independent of any particular motion.

[110] God is not in time, so, strictly speaking, time is not with all things, but at least with all movable things.

[111] See also 223a17–18 and 223b1–12.

[112] While we saw above that motion is more than the time it takes, we see here that time is more than the time a certain motion takes.

[113] Ross brackets κατὰ συμβεβηκός since it is omitted by Themistius and not mentioned in Alexander's commentary.

ὂν ἀνάγκη κινεῖσθαι, οὕτω καὶ τὸ ἐν χρόνῳ· οὐ γὰρ κίνησις ὁ χρόνος, ἀλλ' ἀριθμὸς κινήσεως, ἐν ἀριθμῷ δὲ κινήσεως ἐνδέχεται εἶναι καὶ τὸ ἠρεμοῦν. οὐ γὰρ πᾶν τὸ ἀκίνητον ἠρεμεῖ, ἀλλὰ τὸ ἐστερημένον κινήσεως πεφυκὸς δὲ κινεῖσθαι, καθάπερ εἴρηται ἐν τοῖς πρότερον.

Since time is the measure of motion, it will be the measure of rest too, since all rest is in time. For it is not the case that there is any necessity for what is in time to be moved as there is for what is in motion to be moved. For time is not motion, but number of motion: and what is at rest can also be in the *number of motion*. Not everything that is not in motion can be said to be 'at rest', but only that which is of such a nature as to be moved, though it is [currently] deprived of motion, as was said above. (221b7–14)[114]

The difference between what can be moved and that which is not movable is equivalent to the difference between what is in time and what is not. Thus, we see that the reference to the individual thing as the potential bearer of a motion does not allow us to discriminate between time and motion as such, but rather helps us to define the realm within which motion is possible. The realm of the movable thus established corresponds to the realm of what is in time.

(3) Aristotle manages to use the thing moved for identifying a particular motion, nevertheless, by taking the thing moved into account with respect to its *logos*, not as a particular individual thing:

καὶ γὰρ ἡ κίνησις καὶ ἡ φορὰ μία τῷ φερομένῳ, ὅτι ἕν (καὶ οὐχ ὅ ποτε ὄν – καὶ γὰρ ἂν διαλίποι – ἀλλὰ τῷ λόγῳ)·

And the motion and locomotion are one by virtue of the moving thing, because that is one (not because it is one ὅ ποτε ὄν, for then it might leave a gap, but because it is one τῷ λόγῳ). (220a6–8)

The moving thing grants the unity of motion not in so far as it is one *ho pote on*, but in so far as it is one *tô logô*. The principal translations understand *ho pote on* as referring to the unity of the individual thing qua individual single thing.[115]

114 For the realm of what is in time, see also 221b25–222a9. That time as the number of motion can also be used to measure rest (221b27–8) shows that Aristotle does not understand motion and rest to be contradictory. Rather, both belong to the realm of the movable and there are things that do not belong to this realm – both features are characteristic of contraries.

115 Ross translates the *ho pote on* phrase as "not by its individual unity", Hussey as "not [one] X, whatever X it may be that makes it what it is", Hardie and Gaye as "not because it is one in its own nature", and Pellegrin as "non pas un par ce qui fait qu'il est ce qu'il est". Zekl takes *pote* in a temporal sense and thus translates "nicht was es jeweils in irgendeinem Zeitpunkt ist"; similarly, Bostock 1980 and Stevens, in the Vrin edition, translates "non pas ce qu'étant à un certain moment il est transporté". Cf. *Physics* 219a19–21, Ross, ad loc., p. 598, and Coope's appendix on this expression. According to Coope 2005, p. 177, *ho pote on X esti* refers to "a ground or basis for the being of X", but what she takes as the ground of a motion is exactly the

Following this suggestion, the first part of our passage claims that it is not the thing moved qua being a particular individual thing, like being Coriscus, that founds the unity of the motion. The individual thing could stop moving (i.e., leave a gap in what might prima facie be understood to be one motion). For example, the motion of Coriscus from the Lyceum to the marketplace might be interrupted by Coriscus stopping to rest in a wine shop before going on to the marketplace. Coriscus will still be the same Coriscus, but, at least from a certain perspective, he will have performed two different motions, one before stopping and one after stopping.[116] Given that an individual thing performs many different movements (in kind and/or number), the individual thing on its own does not secure the unity of a certain motion, and it cannot explain the difference between time and motion.[117] (When Aristotle claims that the *pheromenon* grants unity not qua being one *ho pote on*, but qua being one *tô logô*, this does not mean that the moving thing could 'leave a gap' in taking a break from being the thing it is, say, from being Coriscus. Rather, as we will see more clearly below, it means that Coriscus remaining Coriscus does not in itself guarantee the unity of his motion, for he could perform two different motions.)

Understood thus, this position contains no surprise, for if the individual thing moved cannot on its own settle the difference between motion and time, then nor can it guarantee the unity of a motion, since it is still the very same individual thing whether it is in motion or at rest. According to the second part of the passage quoted, the unity of a motion is guaranteed by the *logos* of the thing moved. Let us investigate whether the *logos* of the thing moved thus may also establish the difference between time and motion.

Being one in *logos* is commonly translated into English as 'being one in definition'.[118] This phrase cannot refer to definition in the sense of determining the genus or kind of the thing moved, for the whole kind qua kind may obviously remain unmoved while the single individual is moving;[119] definition

individual thing, as the stone is for the stone-in-motion. For an extensive discussion of the phrase *ho pote on* in the Aristotelian corpus, see Ledermann 2014.

116 *Physics* V, 4 spells out different senses of the unity of a motion, and we see there that in some qualified sense, even if Coriscus stops in a wine shop this may still count as one motion. However, in the passage under discussion, Aristotle is interested in what in V, 4 he calls motion being one in a strict or unqualified sense (ἁπλῶς, 227b21), what we can understand as individualising a single motion.

117 However, in a cosmological context, the thing moved is exactly what grants the continuity of *kinêsis*; see *De generatione et corruptione* 337a25–33. We will see that in the *Physics*, the thing moved is one of three principles that together grant the unity; see 227b23–6, discussed as part of 227b21–32, below.

118 See the editions of Hussey, Ross, and Hardie and Gaye, ad loc.; similarly, Stevens and Pellegrin translate 'being one in *logos*' as being one "par la definition" and "selon sa definition", respectively.

119 For example, humankind as a kind will not be moved because Coriscus sets out to go to the marketplace.

in this sense is of no importance for the unity of a motion. Following suggestions from Ross and Hussey, we can understand the thing moved being one in *logos* as the unity of this thing with respect to a certain account that can be given of it or with respect to a certain state it is in.[120] This interpretation is confirmed by 219b18–21, which is the inverse of the passage we are investigating, where the sameness of the *ho pote on* and a difference in *logos* are used to explain the *identity of a thing*:

> [τὸ φερόμενον] τοῦτο δὲ ὅ μέν ποτε ὂν τὸ αὐτό (ἢ στιγμὴ γὰρ ἢ λίθος ἤ τι ἄλλο τοιοῦτόν ἐστι), τῷ λόγῳ δὲ ἄλλο, ὥσπερ οἱ σοφισταὶ λαμβάνουσιν ἕτερον τὸ Κορίσκον ἐν Λυκείῳ εἶναι καὶ τὸ Κορίσκον ἐν ἀγορᾷ.
>
> This [the moving thing] is, in respect of what makes it what it is [*ho pote on*], the same (as the point is, so is a stone or something else of that sort); but in definition [*logos*] it is different, in the way in which the sophists assume[121] that being Coriscus-in-the-Lyceum is different from being Coriscus-in-the-marketplace. (Hussey's translation with modifications)

Judging from the example here, *logos* has to be understood as the account of something in a certain, non-essential respect. Whereas 220a6–8 explains that the unity of a motion is guaranteed by the moving thing's *logos*, this passage shows that the identity of a thing is ensured by whatever it is (*ho pote on*) while its *logos* may differ. The basic notions of *ho pote on* and *logos* seem to stay the same, however, and thus 219b19–21 can help us to understand 220a6–8.

To guarantee the unity of a movement, *logos* must be understood as the thing moved, not in so far as it is this individual single thing but in so far as it can be further determined in a certain respect (quality, place, quantity) that stops being true when that movement comes to an end. For example, the thing moved can be determined as *x* being A changing into B[122] (as something white changing into black) or as *x* moving from A towards B.[123] The individual thing itself is considered only as the bearer of a movement and of features necessary for this movement. The unity of a motion that Coriscus performs is not guaranteed by the person Coriscus alone, for Coriscus performs many motions during his lifetime, but by 'Coriscus moving in the marketplace' (if this is the motion whose unity we are interested in). Coriscus may whistle a march as he moves in the marketplace, but this is a different *kinêsis* with a different *logos*. Once Coriscus sits down and rests, the *logos* of the thing moved in which we

120 See Ross, ad loc., p. 602. Hussey claims that "Coriscus must be the same 'in definition' in the sense of 219b19–21, i.e., in the same state" (p.158).

121 The sophists presumably assume it in the sense of taking it as a premise with which they make their interlocutor agree.

122 This is how the Eleatic notion of motion as not being A and being A is transformed in Aristotle.

123 For the unity of a motion, it is important to give an account of the motion in terms of its end point, as is done in *Physics* III.

are interested, namely "Coriscus moving in the marketplace", ceases to be true – but Coriscus may still be whistling his march. The motion under focus has come to an end, while the whistling motion continues.[124]

Logos thus considered guarantees the unity of a motion in three respects:

(1) It determines the starting and end point and thus the outer boundaries of the motion: Coriscus moving *from A to B*.[125] These conceptual limits do not necessarily coincide with the limits of the motion that in fact takes place; for example, Coriscus may collapse before he reaches the intended goal B, or, having reached B, he might suddenly decide to go further. However, the starting and end points gives us a criterion to decide whether something belongs to the motion taking place within these limits and thus contributes to identifying a particular motion.[126]
(2) Since the *logos* is specified as '*Coriscus* moving from A to B', it allows us to single out Coriscus' motion from all other motions taking place at the same time or in the same space (the motion of Aristotle, of a tortoise, etc.).
(3) Finally, the *logos* also guarantees the continuity of the motion in the sense that we can employ 'Coriscus *moving* from A to B' as a criterion so that whenever this statement is true for a certain interval, we are dealing with the very same continuous motion. If Coriscus were suddenly to rest during his journey from A to B, there would be a moment when 'Coriscus *moving* from A to B' was not true. Hence, the occurrence of rest actualises, and thus settles, the outer limits of a complete and non-intermittent motion.

We can learn in more detail how Aristotle understands 'Coriscus *moving* from A to B' from a passage that prima facie is not concerned with this problem at all, but deals with the conditions for the existence of a now: "for it is by reference to the moving thing that we recognise the before and after in a motion, and the now exists in so far as the before and after are countable" (219b23–5).

Aristotle has already shown that there is no time without motion (218b21), and as we cannot define a now without a reference to time, a now depends on

[124] Let us assume, for example, that Coriscus is moving in the marketplace to do his food shopping. Once this is done, he wants to do some sports and leaves the marketplace for the Lyceum – then we have a new motion: Coriscus moving from the marketplace to the Lyceum. Both motions could in principle also be one continuous motion, the motion of walking around in the marketplace and going off to the Lyceum. But if this is indeed a continuous motion, then the *logos* of this motion would also be a different *logos*: it is Coriscus' motion around the marketplace and to the Lyceum.

[125] I focus on the case of locomotion, but it would also hold for changes, like C changing from being A to being B.

[126] But what about motions whose account does not include a clear end point, e.g., my stroll across the marketplace? In this case, beginning and end point seem to be only given with me actually starting and stopping my stroll, not with any spatial or conceptual boundaries.

motion as well. Thus, the now can only 'exist' if there is some motion. But why does the now exist, as this passage claims, only with reference to the countability of the before and after of a motion?

Let us assume that we pick out a certain now of a certain motion. Only if we can determine that there is a before and after this now that belongs to the same motion will it be guaranteed that there is a measurable extended surrounding – the distance, no matter how small, between the now and whatever before and after we choose[127] – within which the *logos* 'Coriscus is moving' must be true. Only that which has such a neighbourhood can be understood as a now of this motion.[128] A now in this sense does not exist at the starting and end points of a motion, for in the case of the starting point there is no measurable before belonging to the same *logos* and in the case of the end point there is no measurable after. Of course, some now will still 'exist' there, but only with reference to a different motion that allows us to define these nows.[129]

The *logos* 'Coriscus is moving from A to B' is thus our criterion for the *continuity* of a motion. However, the *logos* cannot *make* the movement continuous. Aristotle must simply presuppose the continuity of the motion 'in reality'. Nevertheless, the *logos* of a motion can be determined in such a way that it allows us to check for every stage, whether that stage still belongs to the motion in question. Although the *ho pote on* of what is moved cannot guarantee the unity of a motion from beginning to end, it is nevertheless a kind of reference point needed when we ask whether something belongs to a motion determined by a certain *logos*. For example, we need to refer to *Coriscus* when we ask whether a certain state belongs to *Coriscus'* motion from A to B. Accordingly, we need all three elements of the *logos* in order to secure the unity of a motion: (a) the reference to 'Coriscus' differentiates his motion from all other motions taking place at the same time; (b) 'is moving', if it holds true at every part I choose within A and B, shows that we are dealing with a continuous motion; and finally (c) 'from A to B' determines the interval in question, and thus singles out this motion from all the other motions that Coriscus performs during his lifetime.

127 We cannot choose the interval to be arbitrarily big as we are restricted by the starting and end points A and B of the motion in question.

128 The thing moved, '*Coriscus* moving from A to B', thus connects preceding and succeeding states as belonging to one motion, in the same way as a point connects preceding and succeeding points of a certain distance and the now connects preceding and succeeding nows of a certain time; see Chapter 7. In this way we can understand Aristotle's claim in 220a4–11 that the motion is continuous on account of the moving thing, just as time is continuous on account of the now, and length is continuous on account of the point.

129 If we have gained a now with the help of a certain motion, we can use it for all simultaneous motions. In this sense, a now is ubiquitous while still being dependent (epistemically) on the specific motion it was gained from. Because of this possible ubiquitousness, starting and end points of a motion can also be determined temporally.

This understanding of how the *logos* of a motion guarantees its unity fits Aristotle's investigation of the different senses in which *kinêsis* can be said to be one:

> Motion is one in an unqualified sense when it is one in essence and number: and what this motion is will be made clear by the following distinctions. There are three things with respect to which we speak of motion, the 'that which', the 'that in which', and the 'when' [that during which]. I mean that there must be something that is in motion, for example, a human being or gold, and it must be in motion in something, for example, in a place or in an affection, and when, for all motion takes place during a time. Of these, it is the *thing in which* the motion takes place which makes it one in genus or kind; it is the *thing moved* that makes it one in subject; and it is the *time* that makes it consecutive. But *it is the three together that make it one without qualification*: to effect this, the 'that in which' the motion occurs must be one and indivisible, for example, the species; also the 'when' in which the motion takes place, so the time must be one and non-intermittent, and that which is in motion must be one – not in an accidental sense. (227b21–32; Hardie and Gaye's translation, strongly modified with emphasis added)

The unity of a continuous *kinêsis* is granted by the persistence of the thing moved, the time taken, and the respect in which *kinêsis* takes place (which Gill calls "the track" of the change).[130] These three taken together are exactly what the *logos* of a motion provides:[131] the 'that which' (i.e., *Coriscus*), the 'that in which' (i.e., Coriscus moving *from A to B*), and the 'when' (that during which, i.e., 'Coriscus is now moving from A to B').

We see that if we want to give an account of locomotion in terms of the *logos* of the thing moved, we need to take into account 'that in which' the motion takes place. In the case of locomotion, this means referring to space or

[130] See Gill 1984, pp. 15–16. According to Gill, two changes are the same in genus if their respective track "consists of contraries that fall within the same genus". And they are one in species if, in addition, their track is an indivisible species, i.e., a species that cannot be divided further into subspecies. The indivisibility in species of that in which the motion takes place is a necessary but not a sufficient condition for motion being "one without qualification", i.e., in essence and number.

[131] More precisely, it is the unity of these three factors involved taken in a sufficiently abstract way; for instance, the thing moved that must remain one is not a specific human being with all his characteristics, but the human being in so far as he is the moving thing in question that grants the unity of motion. For example, it is still the same motion from A to B even if the person moved is first cold but then becomes hot; if, let us say, Coriscus is shivering when he starts out his motion from the Lyceum to the marketplace, but starts sweating after a little bit. However, this does not mean that the thing moved could be one accidentally (for example, the white thing blackening and Coriscus walking are one thing accidentally) or one in respect to something common (for example, two men being restored to health from the same inflammation of the eye), as Aristotle points out in 227b31–228a3.

distance:[132] the beginning and end of a locomotion, and hence important aspects of its unity, are determined not solely by time, but also spatially.

8.2.2.2.2 Space Returning One crucial characteristic of space employed by Aristotle for his account of motion is its continuity – according to *Physics* IV, the continuity of space *founds* the continuity of locomotion, which in turn establishes the continuity of time:

> Now since what is moved is moved from something to something, and every magnitude is continuous, the movement follows the magnitude: it is because the magnitude is continuous that the movement is too. And it is because the movement is that the time is. (For the time always seems to have been of the same amount as the movement.) Now the 'before' and 'after' holds primarily, then, in place [*en topôi*]; and there in virtue of relative position. But since 'before' and 'after' hold in magnitude, it is necessary that the before and after also hold in movement, these corresponding to those. But also in time the 'before' and 'after' must hold, because the one [the temporal before and after] always follows the other [the before and after of a motion] of them. (219a10–19)

Movement "follows" (*akolouthei*) spatial magnitude and 'is followed' by time so that movement and time 'inherit' an important feature from the spatial magnitude. This is explained by the fact that as something moves over a continuous spatial distance, its movement will be as continuous as the distance over which it moves – and as time belongs to movement, it too will be continuous.[133] Time thus seems to be dependent on space and inherits its continuity and the possibility of marking off a before and after from spatial distance via movement. On the other hand, time cannot be reduced to space, for *time* is the only entity essential for *all* kinds of *changes* – all of them are in time, but not all cover a certain distance;[134] by contrast, space is necessary solely for the definition of locomotion. In order to give an account of locomotion, time and space must be irreducible magnitudes, which is to say that they are (also) independent of each other.

[132] Most of the time Aristotle talks simply about *megethos*, since he wants to give an account of all kinds of change. That *megethos* must nevertheless be understood as spatial magnitude in his fundamental investigation of 'motion' can be inferred from passages like 219a10–15, where the continuous *megethos* is connected to before and after according to *topos*; or from 219b15–18, where Aristotle shifts from *megethos* to the point (*stigme*) understood in a spatial sense.

[133] See also 207b23–7. Sometimes, however, Aristotle states it the other way round – if time is continuous, so is distance traversed (see 233a13–21); but here the order seems to be based on our way of talking, and not intended to show that the continuity of time is grounding the continuity of the distance.

[134] However, locomotion is primary *vis-à-vis* all other kinds of change; see *Physics* VIII, 7, where this is explained by the fact that all other kinds of motion involve some form of locomotion. See also Odzuck 2014.

Space and motion can also be used to measure each other. We encountered mutual measurability with time and motion above. For space and motion, it can be seen, for example, in 220b28–31:

> And we measure both the distance by the movement and the movement by the distance; for we say that the road is long, if the journey is long, and that this is long, if the road is long.[135]

Not only will a distance help us to measure a motion, but also a motion can be used to measure a distance.[136] As we will see in the next section, however, we need to refer to space *and* time in order to measure the full locomotion, its speed.[137] Speaking more precisely, the measure of motion is complex: it is a certain *relation* of time and space. We will see that only such a measure of motion fulfils the homogeneity requirement that Aristotle put forth in his *Metaphysics* – only the relation of time and space is a homogeneous measure for measuring the speed of motion.[138]

8.2.3 The Relation of Time and Space

We have seen that in Aristotle's *Physics*, time is *a* measure of motion in the sense that no motion can be thought, let alone measured, without time. Only time is explicitly called a measure of motion and in fact it seems to be *the* measure of motion for Aristotle. However, if we do not restrict ourselves to measuring the duration of a motion and look at Aristotle's *implicit account*, we see that time is, after all, not treated as *the* measure of motion. Time is not homogenous with motion in the sense required by the measurement conception of the *Metaphysics*.[139] For in order to measure how fast a locomotion is, we need to consider time as well as the distance covered, also according to Aristotle's implicit account. And we need the ratio of these two magnitudes, since it is the specific relation of these two dimensions that characterises a

[135] Hence, a motion could also provide us with spatial units (not only temporal ones).

[136] Especially in an ancient context, it was natural to measure a distance by human activity, for example, by travel days (how much a person or an army could cover in a day by walking).

[137] The close connection of time and space with respect to motion is also apparent in the proof of *Physics* VI, 7 that it cannot be the case that of time or distance, one is infinite while the other is finite.

[138] The homogeneity requirement does not mean that the physical tool with which we measure must be homogenous with the measurand, but only the conceptual measure. For example, if we measure the speed of my run, we may use a measuring tape to measure the distance I travel, and a watch to measure the time I take. Neither the measuring tape nor the watch is homogenous with my speed, but the conceptual measure I use for measuring my speed is, namely the ratio of distance covered to time taken.

[139] It would be homogeneous only with the duration of a motion. If we measure the speed of a motion, however, time alone is not homogeneous with what is to be measured.

motion. For instance, a motion covering 3 metres in 1 second is different from a motion covering 2 metres in 1 second – the one is faster than the other.[140]

Taking the relation of time and space as the measure of motion makes for a complicated story: on the one hand, time and space are independent magnitudes and must both be taken into account. On the other hand, we are interested not simply in time and space each on its own, but rather in the specific relation between time and space that a particular motion displays. Understanding the measure of motion as a relation between time and space allows one to avoid both the moving rows problem, in which the relationship between time and space is fixed, and the dichotomy and the Achilles paradox, which take into account solely distance but not time.

Let us spell out the specific relationship of time and space needed in order to measure the speed of a motion. It is not enough simply to consider first the time a certain motion takes and then the distance the motion covers, for we would not then be able to measure and hence compare the speed of different motions, as a brief example shows: let us assume that Achilles travels 27 metres in 9 seconds and Patroclus requires 16 seconds to traverse 32 metres. If we compare first their times and then the distances travelled, all we can say is that Achilles finished sooner and Patroclus covered a longer distance. But we cannot say whether Patroclus or Achilles was faster unless we compare the *relationship* of time and space for both. Only then can we say that Achilles, covering 27 m/9 sec and thus 3 m/1 sec, is faster than Patroclus, who traverses 32 m/16 sec and thus 2 m/1 sec. We must take into consideration the *relation* of time and space, two initially independent magnitudes, and not simply each magnitude on its own. The measure of motion thus cannot be the mere addition of two simple measures – one simple measure for time, another simple measure for space – but must be inherently complex: *the relation of time and space*.

In order to determine the speed of a motion, Aristotle does in fact take into account not only spatial and temporal magnitudes but also their relationship, since he considers how much *time* is needed *for* traversing a certain *spatial distance*, as we see in passages such as 222b30–223a4:

> it is evident that every change and everything that moves is in time; for the distinction of faster and slower exists in reference to all change, since it is found in every instance. I say that moves faster which changes before another into the condition in question, when it moves over the same interval and with a regular movement; e.g., in the case of locomotion, if both things move along the circumference of a circle, or both along a

140 Today, we talk of the ratio of time and space, rather than of their relation, but a ratio is nothing but a specific form of relation.

> straight line;[141] and similarly in all other cases. (translation by Hardie and Gaye with modifications)

For Aristotle, that which changes in less time over the same extension (or into a certain condition) is faster. The distance is taken into account, but treated as set, in order to compare the different times needed to cover it. We also see the reverse in Aristotle, where the time is taken as set and the different distances covered within this time are compared:

> Let A be faster than B. Since that is faster which changes first [before something else does], in the time ZH in which A has changed from G to D, in this time B will not yet have arrived at D but will be short of it: so that in the same time the faster will pass over more magnitude. (232a27–31)

The fact that in Aristotle one of the two magnitudes, the temporal or spatial, is understood as set does not in itself mean that they are not both taken into account as independent magnitudes or that their relation is taken as fixed. *Either* the time *or* the distance is taken as set here simply to enable comparison, a method that corresponds to our practice: in order to compare the speed of Achilles and Patroclus in the example given above, we transform the fractions representing the relation of temporal and spatial magnitude in both cases, 27 m/9 sec and 32 m/16 sec, so as to produce a common denominator. Then we have Achilles moving 3 metres in 1 second and Patroclus running 2 metres in 1 second, and can tell that Achilles is faster than Patroclus. For this comparison, we have set one magnitude to be the same in both instances – here the time, at 1 second – just as Aristotle also fixes one magnitude. It need not always be time that is taken as fixed; we could also say that to cover 1 metre (thus fixing the distance travelled) Achilles needs one third of a second and Patroclus needs half a second.

Thus, in contrast to scenario (2) in Chapter 1, where we looked at two ships covering the same distance at different times, we can compare different distances here, but for comparison purposes we look at the time needed to cover the same distance, 1 metre. While Aristotle does not connect his comparison to numbers, the fact that he changes between taking a certain time as set and taking a certain distance as set, and both just as examples, shows that he is in fact taking both time taken and distance covered into account.

We see that to take one magnitude as set does not mean that duration and distance are not both taken into account as independent magnitudes, since the magnitude that is fixed can be combined with different amounts of the other magnitude at different speeds. However, time and space are no longer taken into account as two independent magnitudes if we take not only either space or time as fixed, but also their relation. We saw such a situation in Zeno's paradox

[141] With 'motions along a circle' Aristotle may here have motions of heavenly bodies in mind, while 'motions along a straight line' seems to capture sublunar motions.

of the moving rows: covering one unit of space (one letter element) was understood as *always* requiring one unit of time, which meant that the relation of time and space was fixed. Accordingly, when the moving row B passed two resting As at the same time as four Cs moving towards B, this enterprise seemed to require two units of time on the one hand (to pass two As) and four units of time on the other (to pass four Cs).

The problem in Zeno's paradox arose because he was looking at a motion that proceeded simultaneously past resting As and moving Cs and assumed the same relation of time and space to hold in both cases. But Aristotle does not work with this situation. He only considers cases where the speed of the thing moved and the relevant circumstances stay the same – in this case, the relation of time and space will indeed also stay the same. And Aristotle shows that the measure of motion is all-encompassing – unlike Plato's measure, it is not restricted to motions that possess a point of accomplishment.

Furthermore, Aristotle also clarifies that with respect to uniform motion, whenever one of the two magnitudes, time and space, is divided, the other magnitude is divided in the same ratio. This second point will be central for solving the motion problem of the dichotomy and the Achilles paradox. In order to see this point, we will need to quote a rather long passage from *Physics* VI. This passage is a high point of Aristotle's *Physics* in that it clearly understands motion in terms of the relation of time and space. Today understanding motion in terms of this relation is a matter of course, but as we have seen throughout this book, to reach this point required much preparation by Aristotle and his predecessors.

I have divided the text into six parts for ease of reference:

(1) ἐπεὶ δὲ πᾶσα μὲν κίνησις ἐν χρόνῳ καὶ ἐν ἅπαντι χρόνῳ δυνατὸν κινηθῆναι, πᾶν δὲ τὸ κινούμενον ἐνδέχεται καὶ θᾶττον κινεῖσθαι καὶ βραδύτερον, ἐν ἅπαντι χρόνῳ ἔσται τὸ θᾶττον κινεῖσθαι καὶ βραδύτερον. τούτων δ' ὄντων ἀνάγκη καὶ τὸν χρόνον συνεχῆ εἶναι . . .

(2) ἐπεὶ γὰρ δέδεικται ὅτι τὸ θᾶττον ἐν ἐλάττονι χρόνῳ δίεισιν τὸ ἴσον, ἔστω τὸ μὲν ἐφ' ᾧ Α θᾶττον, τὸ δ' ἐφ' ᾧ Β βραδύτερον, καὶ κεκινήσθω τὸ βραδύτερον τὸ ἐφ' ᾧ ΓΔ μέγεθος ἐν τῷ ΖΗ χρόνῳ. δῆλον τοίνυν ὅτι τὸ θᾶττον ἐν ἐλάττονι τούτου κινήσεται τὸ αὐτὸ μέγεθος· καὶ κεκινήσθω ἐν τῷ ΖΘ. πάλιν δ' ἐπεὶ τὸ θᾶττον ἐν τῷ ΖΘ διελήλυθεν τὴν ὅλην τὴν ΓΔ, τὸ βραδύτερον ἐν τῷ αὐτῷ χρόνῳ τὴν ἐλάττω δίεισιν· ἔστω οὖν ἐφ' ἧς ΓΚ. ἐπεὶ δὲ τὸ βραδύτερον τὸ Β ἐν τῷ ΖΘ χρόνῳ τὴν ΓΚ διελήλυθεν, τὸ θᾶττον ἐν ἐλάττονι δίεισιν, ὥστε πάλιν διαιρεθήσεται ὁ ΖΘ χρόνος. τούτου δὲ διαιρουμένου καὶ τὸ ΓΚ μέγεθος διαιρεθήσεται *κατὰ τὸν αὐτὸν λόγον*. εἰ δὲ τὸ μέγεθος, καὶ ὁ χρόνος.

(3) καὶ ἀεὶ τοῦτ' ἔσται μεταλαμβάνουσιν ἀπὸ τοῦ θάττονος τὸ βραδύτερον καὶ ἀπὸ τοῦ βραδυτέρου τὸ θᾶττον, καὶ τῷ

ἀποδεδειγμένῳ χρωμένοις· διαιρήσει γὰρ τὸ μὲν θᾶττον τὸν χρόνον, τὸ δὲ βραδύτερον τὸ μῆκος.

(4) εἰ οὖν αἰεὶ μὲν ἀντιστρέφειν ἀληθές, ἀντιστρεφομένου δὲ αἰεὶ γίγνεται διαίρεσις, φανερὸν ὅτι πᾶς χρόνος ἔσται συνεχής. ἅμα δὲ δῆλον καὶ ὅτι μέγεθος ἅπαν ἐστὶ συνεχές· τὰς αὐτὰς γὰρ καὶ τὰς ἴσας διαιρέσεις ὁ χρόνος διαιρεῖται καὶ τὸ μέγεθος. ἔτι δὲ καὶ ἐκ τῶν εἰωθότων λόγων λέγεσθαι φανερὸν ὡς εἴπερ ὁ χρόνος ἐστὶ συνεχής, ὅτι καὶ τὸ μέγεθος, εἴπερ ἐν τῷ ἡμίσει χρόνῳ ἥμισυ διέρχεται καὶ ἁπλῶς ἐν τῷ ἐλάττονι ἔλαττον·. . .

(5) καὶ εἰ ὁποτερονοῦν ἄπειρον, καὶ θάτερον . . .

(6) διὸ καὶ ὁ Ζήνωνος λόγος ψεῦδος λαμβάνει τὸ μὴ ἐνδέχεσθαι τὰ ἄπειρα διελθεῖν ἢ ἅψασθαι τῶν ἀπείρων καθ' ἕκαστον ἐν πεπερασμένῳ χρόνῳ. διχῶς γὰρ λέγεται καὶ τὸ μῆκος καὶ ὁ χρόνος ἄπειρον, καὶ ὅλως πᾶν τὸ συνεχές, ἤτοι κατὰ διαίρεσιν ἢ τοῖς ἐσχάτοις. τῶν μὲν οὖν κατὰ τὸ ποσὸν ἀπείρων οὐκ ἐνδέχεται ἅψασθαι ἐν πεπερασμένῳ χρόνῳ, τῶν δὲ κατὰ διαίρεσιν ἐνδέχεται·καὶ γὰρ αὐτὸς ὁ χρόνος οὕτως ἄπειρος. ὥστε ἐν τῷ ἀπείρῳ καὶ οὐκ ἐν τῷ πεπερασμένῳ συμβαίνει διιέναι τὸ ἄπειρον, καὶ ἅπτεσθαι τῶν ἀπείρων τοῖς ἀπείροις, οὐ τοῖς πεπερασμένοις.

(1) And since every motion is in time and in any time there can be a motion, and the motion of everything that is in motion may be either quicker or slower, in any time there can be a quicker and slower motion. And this being so, it necessarily follows that time also is continuous . . .

(2) For since it has been shown that the quicker will pass over an equal magnitude in less time than the slower, suppose that X is quicker and Y slower, and that the slower has traversed the magnitude AB in the time FG. Now it is clear that the quicker will traverse the same magnitude in less time than this: let us say in the time FH. Again, since the quicker has passed over the whole AB in the time FH, the slower will in the same time pass over AC, say, which is less than AB. And since Y, the slower, has passed over AC in the time FG, the quicker will pass over it in less time: so that the time FH will again be divided. And if this is divided the magnitude AC will also be divided *according to the same rule* [*logos*] and again, if the magnitude is divided, the time will also be divided.

(3) And we can carry on this process forever, taking the slower after the quicker and the quicker after the slower alternately, and using what has been demonstrated at each stage as a new point of departure: for the quicker will divide the time and the slower will divide the length.

(4) If, then, this alternation always holds good, and at every turn involves a division, it is evident that all time must be continuous. And at the same time it is clear that all magnitude is also continuous; for the divisions of which time and magnitude respectively are susceptible are the same and equal. Moreover, the customary arguments make it plain that, if time is continuous, magnitude is continuous also,

inasmuch as a thing passes over half a given magnitude in half the time taken to cover the whole . . .

(5) And if either is infinite, so is the other, and the one is so in the same way as the other . . .

(6) Hence Zeno's argument makes a false assumption in asserting that it is impossible for a thing to pass over or severally to come in contact with infinite things in a finite time. For there are two senses in which length and time and generally anything continuous are called 'infinite': they are called so either in respect of divisibility or in respect of their extremities. So while a thing in a finite time cannot come in contact with things quantitatively infinite, it can come in contact with things infinite in respect of divisibility: for in this sense the time itself is also infinite: and so we find that the time occupied by the passage over the infinite is not a finite but an infinite time, and the contact with the infinites is made by means of moments not finite but infinite in number. (232b20–233a31; translation largely follows Hardie and Gaye, emphasis added)

The passage quoted forms the core of the second chapter of book VI. In the first chapter Aristotle argues that time, spatial distance, and motion must be of the same internal structure (either atomic or continuous) and against point atomism he attempts to establish that a continuum cannot be thought of as being made up of extensionless and thus indivisible points. In chapter 2, by contrast, Aristotle argues against continua consisting of extended atoms,[142] as he seeks to prove that the parts of continua must be thought of not only as extended, but also as always further divisible. In order to show the latter, and, in particular, that the continua time and distance[143] are always further divisible, Aristotle uses a comparison of two motions of different speeds. According to Aristotle, this very proof can be used to show one crucial assumption of Zeno's dichotomy paradox to be wrong, namely that infinitely many spatial pieces will have to be covered in a finite time. The comparison of two motions also suggests that the motion to be measured does not need to have a point of accomplishment, as we can single out any stretch of time or space of this motion for comparison and quantification.[144]

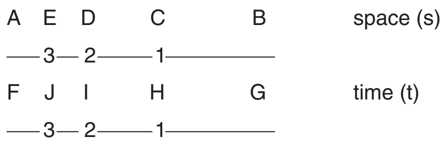
[142] That is atoms of the kind postulated by Leucippus and Democritus and in Plato's Academy.

[143] In the passage quoted, Aristotle talks more neutrally about "magnitude", presumably because he wants to capture the principle structure of different kinds of change, not only of locomotion. But his application of the results to Zeno's dichotomy paradox makes it clear that locomotion is among the changes that have such a continuous structure, and for our purposes we will restrict ourselves to locomotion and spatial distance.

[144] This implies that in principle we could also measure the speed of an infinite uniform motion, like the motion of the heavenly bodies, even if this would require a perspective on motion that may no longer square with Aristotle's account of motion in *Physics* III, 1.

When two motions of different speed are compared, as in the passage quoted,[145] the slower motion will always divide the distance while the faster divides time (see section (3) of the quotation): during the same time, the slower motion will cover less distance than the faster motion and thus divide the distance, while the faster motion will cover the same distance in less time and thus divide the time.[146]

So let us assume that the slower has traversed the magnitude AB in the time FG; the faster – let us say that it is twice as quick – will then traverse the same magnitude AB in half the time, in FH. In the time FH, the slower will only cover half the original magnitude, AC. AC will be covered by the faster motion in less time, in FI, etc.:[147]



This comparison shows that time and space can be divided as needed by bodies moving at different speeds; they are divisible as we please, for any speed we choose will produce a division of space or time. In this comparison, time and space can vary in becoming as small as one wishes, since we can always divide them further – this is what Aristotle wants to prove, that there are no indivisible parts. In addition, he thereby shows that time and space are divided in such a way that the division of one is performed in the same way as the division of the other, *kata ton auton logon*, "according to the same ratio" or "according to the same rule" (Section 2).[148] If the

145 Given that Aristotle's analysis in this passage will be a crucial step to answering the first two paradoxes of motion, it is especially interesting that the passage has sometimes been seen as an inverse Achilles, that is, as a sequence of divisions *ad infinitum*, where each step does not add more distance covered (as in the Achilles), but subtracts a bit of distance.

146 It is actually not important whether the two motions compared do indeed cover the very same track, as they do in Zeno's Achilles paradox, or only a track of the same length; we just need a generic length of, say 100 metres, not the 100 metres of a particular track.

147 '1', '2', etc. indicate the first, the second, etc. division of distance and time in the figure.

148 For *logos* qua 'rule', cf. LSJ *logos* 2.d, which we find as early as Pindar, and also in Plato and Aristotle. The phrase *kata ton auton logon* in 233a4 could also be translated as 'according to the same argument' (as suggested by Henry Mendell in personal communication), so we would not have to assume that Aristotle is comparing two motions with constant speed. Rather, the speed could change. Mendell 2007, p. 12, n. 16 points out that throughout *Physics* Z, Aristotle does not need constant speed; rather the weaker notion of uniform periodicity is sufficient (i.e., if 'a movement over a distance is divided into equal distances (periods), every equal distance travelled is travelled in an equal time'). I think this is right for the passages Mendell explicitly names as examples (233b4–5 and 233b26–7). However, if we were to understand *logos* as 'argument' here, it would not be clear which argument Aristotle is referring to. More importantly, in order to solve Zeno's motion paradox, where we deal with infinite divisibility, mere uniform periodicity would

slower body is not given time FG, but only the shorter time FH that the faster body needs in order to cover the distance AB, then the slower will not manage to cover the whole of AB, but only some smaller distance AC.[149] Thus, every division of the time taken leads to a *corresponding* division of the space covered, and vice versa. The only thing that stays constant is the specific relation of time and space of each motion. For example, if the slower body covers one unit of space in two units of time, while the faster body covers one unit of space in one unit of time, the relationship of one spatial unit per two temporal ones for the slower motion and the relationship of one spatial unit per one temporal unit for the faster motion will stay the same, no matter how far we divide time and space. As long as the speed stays the same, the rule of division that captures this relation stays the same. This very relation is the measure of motion,[150] and it answers what we called the motion problem of Zeno's dichotomy, as Aristotle makes clear in section (6).

As we read in the passage just quoted and saw in Chapter 3, Zeno's dichotomy paradox assumed that "it is impossible for a thing to pass over . . . infinite things in a finite time". That is, it seems to be impossible to go through infinitely many spatial parts in a finite time – this is what we called the 'motion paradox' challenging Zeno's runner: the time available is too short to cover something infinite. Accordingly, the finite time cannot be correlated with the infinitely many spatial parts. It seems we would need an infinitely extended time to traverse the infinitely many parts. But even if the runner of the dichotomy paradox did indeed have an infinite amount of time available (if, for example, we asked immortal Apollo to do the run for us), we would still be caught in a paradox, since the infinitely many spatial parts are the parts of a *finitely* extended distance, which could not be paired with an *infinitely* extended time.[151]

However, the idea that motion requires infinitely many spatial parts to be passed in a finite time rests on an implicit assumption – an assumption that section (2) of the long passage quoted above has already proven wrong – namely that each division of the distance covered does not entail an equal division of time. Zeno does not divide the time of a motion whenever he

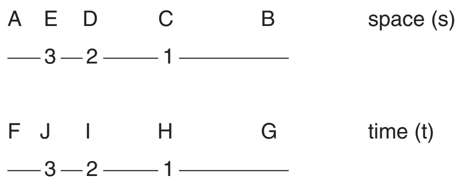
not guarantee that the rule of division would always stay the same, no matter where we cut. The usage of *aiei* (always) in our passage suggests that Aristotle uses uniform motion here and Aristotle explicitly talks about uniform motion (ὁμαλὴν κίνησιν) when comparing two motions of different speed in IV, 222b30–223a4. Heath 1949, p. 129 also assumes constant speed.

149 If we assume that A is twice as fast as B, the rule of division would be 1 over 2 to the power of n.

150 It is the measure of motion in the sense of determining the dimension that we need to measure if we measure speed – the relation of time and space, which is the first and a crucial step for measuring; Aristotle is not here specifying measurement units nor ascribing time and distance covered to numbers, which are the next steps.

151 We thus see how the two problems of the dichotomy – a finite distance is meant to possess infinitely many parts that in turn shall be covered in a finite time – are combined.

divides its distance. Otherwise, Zeno would have to put the division somewhat like this: first the runner has to cover half the racecourse AC in half the time FH. But before he can cover this first half, he first has to cover half of this half, AD, in half of the half time, FI, and so on *ad infinitum*.



In fact, this is exactly the relation we just saw when comparing the two motions of different speed – each division of the distance of a motion leads to an equal division of time. Dividing time and space always by the same rule resolves the problem that the infinitely many spatial parts cannot be related to a finite time. And this is Aristotle's answer to the motion problem: when we look at a particular motion, we are not facing infinitely many spatial pieces that need to be covered in a finite time, but rather a division *ad infinitum* for time as well as space.[152]

Given that for Zeno time is not divided when the distance is divided, it seems that in his paradox the infinity of division and infinity of extension are not sufficiently distinguished: the distance gets divided *ad infinitum* so that each part gets smaller and smaller, while the time is not divided and thus stays finite; accordingly, it seems that the infinitely many parts could at best be covered in an infinitely extended time. Such an inference can only be drawn, however, if we allow ourselves to infer infinity of extension from infinity of division.[153] Showing that the same kind of infinity (5) has to be ascribed to both time and space can solve this confusion – as Aristotle demonstrates in our

[152] Aristotle assumes time per se to be infinite; see especially *Physics* VIII. Moreover, for him the first thing moved by the unmoved mover must be forever in this motion (see 259b32 ff.). The main point he needs in order to deal with Zeno's first two paradoxes of motion, however, is the similarity of the internal structure of time and spatial distance – both are divisible *ad infinitum*. This holds true whether we talk about a finite motion or the infinite motion of the heavens.

[153] Similarly, in his plurality paradox DK29 B2, Zeno seems to infer things to be so big as to be infinite from infinite divisibility. In the motion paradox above, infinite divisibility seems to be employed for the distance covered, while infinity of extension seems to be required with respect to time. Mere temporal atomism would also lead to this problem. For if time is taken to contain extended indivisible parts, an ongoing division of space would lead to increasingly small parts of space, while the parts of time, after a certain division, would stay the same. Thus, as the parts of space getting infinitely smaller would be assigned to parts of time that always maintain their extension, the time needed to traverse a finite distance would in fact be infinitely extended. Aristotle addresses this version of the problem in *Physics* VI, 2 by showing that time is a continuum.

passage (6): "while a thing in a finite time cannot come in contact with things quantitatively infinite, it can come in contact with things infinite in respect of divisibility: for in this sense the time itself is also infinite" (233a26–8). When the distance travelled is infinite in the sense of being infinitely divisible, the time taken to cover that distance is necessarily also *infinite in the very same sense*, since in the case of motion every division of the distance covered requires an equal division of the time taken. By showing that time is continuous in the very same way that space is continuous (4), Aristotle can demonstrate that in the dichotomy paradox infinity of division is required for space as well as time. A finite distance that can be divided *ad infinitum* is covered in a finite time that can also be divided *ad infinitum*.

That time and space are always divisible by the same rule is based on their being divisible *ad infinitum*, which is a feature of their continuous structure, as we saw in the previous chapter. According to Aristotle, this very structure allows us to bring time and space into a consistent relationship with each other, and thus to answer the question raised by our investigation of Plato: how can time and space, as seemingly completely different entities, be consistently combined in such a way as to determine speed? Aristotle answers that whatever else we can say about time and spatial distances, their fundamental structure as magnitudes is the same – they are continua. Accordingly, we can combine them. And Aristotle goes even further, as we saw above, in his comparison of different speeds: not only *can* time and space be divided in the very same way, since they are both infinitely divisible, they also *have to* be divided according to the same rule. Every division of the distance covered leads necessarily to a division of the time taken, and vice versa. This may seem to bring us back to the moving rows paradox, where the time needed was understood to be completely dependent on the space covered. However, time and space are correlated differently with different motions (faster ones have a different correlation than slower ones). And time and space are correlated in the same way with a particular motion only as long as the speed and the relevant circumstances stay the same, as is the case in the Aristotelian passage cited above, but was not the case in Zeno's fourth paradox of motion. Zeno introduced a complex scenario where the distance to be covered is, on the one hand, a row at rest and, on the other, a row moving towards the mover. The relation between time and space in one part of the scenario seemed to be the same as the relation between time and space in the other part of the scenario. However, as the distance to be passed moves in one part of the scenario (the Cs move towards the Bs), but does not move in the other (the As are resting), the relevant circumstances of these two parts of the scenario are different and lead to a difference in the specific relationship of time and space.

While Aristotle explicitly spells out his solution to the motion problem with respect to the dichotomy (233a17–21), he does not spell out his solution to the paradox of the moving rows. Indeed, his treatment of the latter is rather wanting, because ultimately he himself has problems dealing with the relationship of time and space in his account of motion.

There are several passages, however, indicating that Aristotle deals with the relation of time and space in a way that provides the basis for a complex measure of speed; most importantly, the long *Physics* passage just discussed, where Aristotle implicitly uses the correlation of the two independent magnitudes time and space in his account of speed when countering Zeno. Furthermore, just before this long passage, in 232a25–7, Aristotle understands 'faster' in a way that takes into account time and a second magnitude (like space).[154] Similarly, in 218b13–18 quoted above, where time and space are not only taken into account, but can also vary in extent: 'fast' is what moves much in a short time, 'slow' what moves little in a long time. Hence, motion does not seem to be reduced to time.

However, this last passage does not serve as a starting point for measuring speed, but rather provides a rough explanation of being fast and slow (in order to show that time and motion cannot be identified).[155] And Aristotle introduces speed by saying that fast and slow are defined (ὥρισται) by time. So, again, Aristotle is not including the second magnitude in the *definition*. Moreover, this last passage can account for speed also only in a comparative way. While 218b13–18 seems non-comparatively prima facie, as it does not talk about some A being faster than some B, the introduction of the passage makes it clear that also here motion is always understood as being faster or slower (θάττων καὶ βραδυτέρα). This, of course, means being faster or slower in comparison to something else.

Aristotle's *explicit account* of the measure of motion names only time; space[156] is quietly dropped.[157] From our modern perspective, Aristotle appears to stop just short of what we would recognise as our modern understanding of speed, unable to give an *explicit* account of motion in terms of time and space.[158] Let us finish our project by sketching why Aristotle has difficulty dealing with this relation in the next and final chapter and by looking at whether his contemporaries may have been more successful with a complex measure of motion.

[154] See Chapter 9.
[155] See Chapter 9 for further discussion.
[156] Or another, second magnitude in the case of changes other than the change of place.
[157] Similarly, the passages where Aristotle talks about the proportionality between weight and time taken to cover a certain distance (for example, *Physics* 216a13 ff., 249b30 ff.; *De caelo* 273b30 ff.) suggest that he is dealing with the foundation of a complex measure. But, again, there is no attempt on Aristotle's part to capture these relations as exact rules, or to formulate precisely the exceptions he sees at play with lower thresholds. See also Lloyd 1987, pp. 218–22.
[158] It is worth noting that accounting for speed in terms of time and space is open to an at/at theory of motion, but can be captured by other theories as well.

9

Time as the Simple Measure of Motion

In Chapter 8 we saw that Aristotle does not incorporate the complex measure he uses into his explicit account of measuring motion. Before we consider what may have prevented him from doing so, we should first look briefly at whether contemporaries of Aristotle, notably Eudoxus, may have conceptualised a complex measure of motion. This broader viewpoint will allow us to see whether conceptualising the measure of speed as the relationship of distance covered and time taken was a difficulty for Aristotle's framework specifically or for Aristotle's time more generally.

9.1 Other Accounts of Speed

Aristotle is not the only one of his time to investigate speed, as he himself makes clear in *Physics* VI, 2 when he gives an account of 'being faster' that he claims to be "in conformity with the definition of faster given by some [*tines*]". Thus Aristotle's discussion of measuring motion seems to have been part of a larger contemporary debate about conceptualising speed. It has been suggested that Aristotle is referring here to Eudoxus' influential but lost treatise *Peri tachôn*.[1] Let us take a brief look at Aristotle's passage before we turn to its possible background:

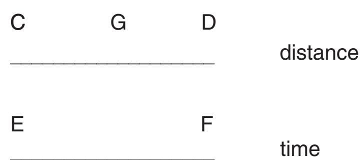
> ἀνάγκη τὸ θᾶττον ἐν τῷ ἴσῳ χρόνῳ μεῖζον καὶ ἐν τῷ ἐλάττονι ἴσον καὶ ἐν τῷ ἐλάττονι πλεῖον κινεῖσθαι, καθάπερ ὁρίζονταί τινες τὸ θᾶττον.
>
> the faster of two things traverses (a) more in an equal time, (b) an equal amount in less time, and (c) even more in less time, in conformity with the definition of faster given by some. (232a25–7)[2]

[1] So Mendell 2007, p. 21.

[2] See Mendell 2007, pp. 3–37 for a discussion of these three claims, which he characterises as "three necessary conditions for showing that A is faster than B under different conditions" (p. 22).

Spelt out explicitly for locomotion, this passage claims that what is faster moves (a) in the same time[3] over more distance,[4] and (b) in less time over the same or (c) even more distance. Case (a) makes clear that speed is not reduced to time, as we can hold the time constant in our comparison of two different movers and compare only the distances they have covered in this time. As in the long passage from the previous chapter (232b20–233a31), so also here in (a) and (b), either the distance is held constant and the difference in time taken indicates the difference in speed, or the time is held constant and the different distances covered indicate which mover is faster.[5] In addition, however, we also have case (c), which seems to confute my claim in Chapter 1 that the ancients always held constant one of the two magnitudes involved in measuring speed, either time or space. Here we seem to be dealing with different times and different distances – the faster covers more distance in less time than the slower.

However, closer examination reveals that this third case does not in fact give a separate account of speed in which time and magnitude covered both vary; rather it is an extension of the previous cases.[6] Obviously, it is not always true that something faster, A, will cover more distance in less time than something slower, B. Such a case only works under certain circumstances: let us assume, for example, both A and B are travelling along a distance CD, and while A covers CD in time EF, during the same time B covers only a part of CD, say CG. Then for any point between G and D, A will reach this point (and thus cover more distance) in less time than B:



[3] Aristotle seems to assume that the slower and the faster move in the same time, although strictly speaking it would be enough for the time to be equal (they need not start at the very same time).

[4] Since Aristotle wants to give a general account of one *kinêsis* being faster than another, he literally only talks about the faster moving "in the same time over more". Hardie and Gaye add that the faster is moving over more "magnitude"; for our purposes we can focus on locomotion and understand the more that is covered as more 'distance'.

[5] In 232b20–233a31, we found either the faster mover traversing the same distance in less time or the slower covering less distance during the same time – in the case of the faster, the distance was held constant; in the case of the slower, the time was held constant. However, in principle both cases could have been expressed either by keeping the time constant (during the same time, the faster will move a greater distance while the slower will travel less distance) or by keeping the distance constant (in order to cover the same distance, the faster will need less time, the slower more time).

[6] Mendell 2007 thinks that (c) is dependent on (a), while I see it as derived from (b). In any case, it seems clear that it is dependent on one of the previous cases; it is comparative and cannot give the speed of one thing as such.

However, A will not cover more distance in less time for any point from D onwards, as it will need more time than EF to cover the distance beyond D.[7] So if one thing is faster than another, (a) and (b) always hold true, while (c) does not always hold, but spells out the consequences of the previous cases for a particular stretch of the motion in question. It does not give an account of speed in which time as well as distance can vary. In 222b33–223a4 Aristotle gives a summary of claims such as the one just looked at by understanding faster as "what changes before another into the condition in question", a formulation that emphasises the temporal aspect of motion.[8]

So far we have seen a variation of Aristotle's account of speed in the passage investigated, but we have not seen anything completely different. But is it possible that Eudoxus in his famous treatise *Peri tachôn* – whether or not Aristotle's passage just discussed refers to it – may have understood the measure of motion explicitly as the relationship or ratio of distance covered and time taken?[9] That might then suggest that the reluctance to conceive of the measure of motion in this way in his *Physics* may be specific to Aristotle.

As Eudoxus' work is lost, our answer to this question can only be somewhat speculative. However, we do have a couple of testimonies of *Peri tachôn*.[10] With their help we can reconstruct something of its nature: it was a work on the motions of the heavenly bodies and provided an account of the different spheres we must assume for sun and moon (three each) and the planets (four each) in order to explain their seemingly irregular motions.[11] This is a more sophisticated version of what we found in Plato's *Timaeus* – seemingly irregular motions of the heavenly bodies are traced back to a combination of regular circular

7 See Mendell 2007, p. 17.

8 Mendell 2007, p. 11 thinks that Aristotle goes on in Z, 2 to prove the three cases from a more fundamental notion: " What changes earlier is faster" (232a28–9), which also emphasises the temporal aspect.

9 Eudoxus probably built up theorems about ratios.

10 See Lasserre, frr. F121–6, which collect testimonies from Eudemus, Alexander, and Simplicius; and Überweg 2004, p. 58. We also find some reports of Eudoxus' work in Aristotle *Metaphysics* Lambda 8 and *De caelo* II, 12–14; see below. The most extensive fragment is the one by Simplicius on Aristotle's *De caelo* 492 ff., F 124; the other fragments in Lasserre do not add much.

11 While Eudoxus is credited with the geometrisation of astronomy (Überweg 2004, p. 66), so that we should expect his *Peri tachôn* to give us the mathematics of motion, in the testimonies we possesses of this work, his proportion theory does not seem to be used explicitly (so Lasserre pp. 182–3, who sees the fact that Aristotle in *Metaphysics* 1073b39–1074a14 contrasts his own system with that of Eudoxus and Callippus without any mathematical description as a sign that Eudoxus only described the system of the spheres but did not set it up mathematically; for a different view, see Knorr 1990). Proportion theory is used, however, for capturing the distances of earth, sun, and moon in fragments D7–13, as Lasserre points out.

motions.[12] Aristotle explicitly reacts to this model in *Metaphysics* Lambda,[13] first briefly summarising Eudoxus' basic idea (1073b16 ff.), which would lead to the assumption of twenty-six spheres,[14] then characterising Callippus' suggestions for additional spheres (1073b33 ff.) as claiming that sun and moon as well as the five planets would each need five spheres, which would require thirty-five spheres in total, and finally presenting his own account, which assumes that in order to fully capture the *phainomena* we need yet more spheres that set back the first sphere for each following planet (1074a), leading to either fifty-five or forty-seven spheres,[15] depending on whether the setback also needs to be done for sun and moon.

The speed Eudoxus is dealing with is angular speed or, more precisely, rotational speed, since he is concerned with the rotations of spheres which are involved in the motion of the planets as we perceive them.[16] Eudoxus compares the rotations of different spheres and calls the circular motion of one sphere fast and that of another slow, but the distance travelled is of no concern and not taken into account anywhere. Thus the speed we are interested in for sublunary motion – the relationship between distance covered and time taken – does not seem to be relevant in Eudoxus' work, for the actual distances play no role.[17]

But we also find traces of Eudoxus' thinking on speed in works other than the fragments of *Peri tachôn*, for example in Lasserre's fr. D10, Aristotle's *De caelo* 291a29 ff. (if this is indeed, as Lasserre suggests, referring to Eudoxus). Here we are told that the speed of each heavenly body is proportional to its distance from the sphere of the fixed star, and that it is this distance that is responsible for their speeds: the rotation of the fixed stars is assumed to be simple and the fastest; all the other bodies must be slower, since they are affected by the motion of the fixed stars, while the sphere specific to them moves in the direction opposite to the heavens. As the planets closest to the fixed stars are most affected by the motion of the fixed stars, they are claimed to be the slowest. Again, the speed is determined not by how much distance a planet covers in how much time, but rather by the distance of a planet's position from the sphere of the fixed stars – the closer the planet is, the slower

[12] Regularity seems to be constituted in Eudoxus by having a fixed orientation towards another sphere – usually it is assumed that the second sphere is oriented towards the first, the third towards the second, and the fourth towards the third, but Yavetz 2003 assumes that the fourth sphere is also oriented towards the second.

[13] For a discussion of Aristotle's account, defending its reliability, see Mendell 2000, p. 78 ff.

[14] Cf. also Simplicius, Lasserre, fr. F124. Cornford, p. 116 claims twenty-seven spheres, but without any explanation.

[15] Though forty-nine seems more plausible than the transmitted forty-seven, as Mendell has pointed out in an unpublished review of Bowen.

[16] The spheres seem to be thought of as solid objects.

[17] The distances between the different spheres (and thus the widths of the spheres) of a planet may in fact be insignificant.

it will be. We are not given a measure that could be used as a complex measure of sublunary speed.

So far it seems that the astronomical works of Eudoxus do not help us find a complex measure of time and space for motion – the distance covered is not taken into account as its own magnitude. With angular velocity – the kind of velocity with which astronomy is mainly concerned – we can easily disregard the second magnitude, the angle, by assuming it to be 360 degrees, which allows us to concentrate solely on time.

But perhaps other astronomical works give an account of speed that could have been used also for sublunary motions. Do we find some hints of such an account of speed even in Aristotle's cosmology itself? In the previous chapter we saw Aristotle's brief reference to the motion of the heavens as the quickest in *Metaphysics* Iota and we find somewhat more detail in his *De caelo* 287a23 ff.: in the context of proving the sphericity of the world, Aristotle shows that the heavens must also move in a circle and be spherical. His argument is that the continuous motion of the heavens is the measure of all motion, and since the measure must be a minimum of the dimension measured, and a minimum of motion is the swiftest motion, the heavenly motion must be the swiftest; the swiftest motion, however, is the motion that follows the shortest line;[18] and the shortest line for something that returns upon itself is a circle, so the heavens must move in a circle. Again, we see that what is fastest is understood simply as what covers the least distance – it is the thing that follows the shortest line (thus it will cover the same angle in less time). Again, from this astronomical way of dealing with motion, we do not gain a complex measure of motion allowing us to measure how much distance is covered in how much time. Rather, the fastest body is taken to be simply the one covering the shortest distance, whether or not it is the fastest in our sense. As in the passage from *Metaphysics* Iota, 1, so also here in *De caelo* the cosmological account of the speed of motion will not help us if our goal is to gain a complex measure of sublunary motion where both distance and time are taken into account as two distinct magnitudes.

Finally, let us look briefly at the work of Autolycus,[19] a mathematician and astronomer probably some twenty-five years younger than Aristotle, and more

[18] The swiftest motion covers the shortest line in the same way in which, if I must cross a square, the shortest line will require me to move along the diagonal, rather than along two of the sides.

[19] As my study ends with Aristotle (and his time), I am unable to examine Apollonius as reported in Ptolemy's *Almagest* XII, roughly 150 years after Aristotle, who does use ratios of speeds. Apollonius claimed that if the ratio of a certain line to another line is the same as the ratio of the speed of the epicycle to the speed of the deferent, then a certain point will be the "stationary point" of the planet, the limit between its forward motion and its retrogradation (Heiberg, pp. 450–1; Toomer, pp. 555–62). However, this account, too, seems not to capture speed as being measured by two magnitudes that are in principle

precisely at his *On the Moving Sphere*, a textbook on mathematical astronomy which has been seen as the oldest Greek mathematical text that is preserved in its entirety, very likely influenced by Eudoxus.[20] At the beginning of this book, we find the following two definitions or general principles (*horoi*):[21]

1. Ὁμαλῶς λέγεται φέρεσθαι σημεῖα ὅταν ἐν ἴσῳ χρόνῳ ἴσα τε ἢ καὶ ὅμοια μεγέθη διεξέρχηται

It is said that points are moved uniformly if in equal time they traverse equal magnitude.

2. ἐὰν δὲ ἐπί τινος γραμμῆς φερόμενόν τι σημεῖον ὁμαλῶς δύο γραμμὰς διεξέλθῃ, τὸν αὐτὸν ἕξει λόγον ὅ τε χρόνος πρὸς τὸν χρόνον ἐν ᾧ τὸ σημεῖον ἑκατέραν τῶν γραμμῶν διεξῆλθεν καὶ ἡ γραμμὴ πρὸς τὴν γραμμήν.

But if some point moved along some line goes through two segments of the line uniformly, the time will have the same ratio to the time in which the point has gone through each of the segments as [the size of] the segment to [the size of] the segment.

Here, in the case of a uniform motion, we have a ratio of a time to a time that is the same as the ratio of a distance to a distance. Several elements in this passage are of interest for our purposes. First, this passage explicitly talks about lines and thus distances that are not related to angles, which we might expect in astronomical thinking. Furthermore, the second definition refers to the speed of only one moving point, rather than comparing the speeds of two different things. It therefore seems to give an account of speed of one thing non-comparatively, while in our texts so far, speed was always expressed as a comparison of two different motions. And, finally, it captures the uniformity of the speed of a motion with the help of ratios: the ratio of t_1 to t_2 is the same as the ratio of d_1 to d_2.

Thus it seems that speed is conceptualised by ratios in a way that captures exactly what we do when we think of the speed of a body as the distance it covers over the time it takes. But is it? In fact, this conceptualisation does not give us the relationship of time and distance covered, but rather the relationship of two times, which is the same as the relationship of the distances covered; thus it expresses the uniformity of a motion during t_1 and t_2, not its speed. Now it is true that $t_1{:}t_2 = d_1{:}d_2$ is equivalent to d_1 over $t_1 = d_2$ over t_2.

independent, time and space, and are seen as forming a specific ratio. In general, ancient mathematicians do not assume ratios between inhomogeneous magnitudes, as the ancient theory of ratios does not allow for it.

20 It is, however, an elementary work, and Eudoxus had probably done more sophisticated work previously.

21 Hultsch calls them “definitiones” in his edition.

However, such a transformation is not made here, not only because it is the uniformity of motion that is at issue, not its speed, but also because for ancient Greeks, ratios only held between magnitudes of the same kind.[22] We can have t_1:t_2 being the same as d_1:d_2, but no mixed ratio.[23] There is nothing here about the measure of motion consisting of a ratio of distance covered and time taken. Thus our modern understanding of speed, as a ratio of distance covered to time taken, would not fit the understanding of ratios in the time under investigation.

Furthermore, we are dealing with only one uniform motion here. If we take it as equivalent to comparing two motions (point one moving d_1 in t_1, point two moving d_2 in t_2),[24] we see that this means of capturing speed, with the ratio of times to times being the same as that of distances to distances, only works if we are dealing with two bodies moving at the same speed (for example, one covering 2 metres in 1 second and the other 4 metres in 2 seconds). It does not work if we compare things that move at different speeds (for example, one covering 5 metres in 2 seconds, the other covering 9 metres in 7 seconds). If we try to compare these motions of different speeds, we find ourselves back at the problem of Achilles and Patroclus from the previous chapter: all we can say is that one covers more distance and the other needs less time.

Aristotle's comparison of the motions of a faster body and a slower body in *Physics* VI, 2 is in fact more complex than Autolycus' passage.[25] But Aristotle never introduces the relationship of distance covered and time taken as measure of motion.[26] And speed is always dealt with comparatively,

[22] The ancient Greeks could express things like 'if the ratio of d_1 to d_2 is the same as the ratio of t_1 to t_2', but alternando only works with ratios of the same kind; thus they cannot deal with ratios of distance to time as when we deal with d_1 over t_1.

[23] We also see this with Archimedes, who cannot multiply weight by a distance, but must claim something like 'there is balance if the left weight is to the right weight as the right distance is to the left distance'. We may think that the ancient Greeks could have avoided using ratios between different magnitudes, by using speed units instead. For the problem such speed units would raise, however, see below.

[24] I translate the different segments of the one motion into a comparison of two motions, in order to see whether this position differs from the other positions we have looked at so far. Given that we are dealing with equal ratios, point one and point two do not actually need to start at the same time; it would be enough if the time taken is equal.

[25] We find an even more complex proportion used in *Physics* VII, 5, where Aristotle takes into account not only a body moving another one in a certain time over a certain distance, but also the force (*dunamis*) needed.

[26] We saw in Aristotle's comparisons that the two bodies either cover the same distance (then we measure how much time they take to do so) or take the same time (then we measure how much distance they cover during this same time). But what we do not get in Aristotle's comparison is a case where A covers one distance in one time, and B covers some different distance in some different time. Following Mendell 2007, we may also think that Aristotle's account of the speeds of the bodies could be expressed as a kind of proportion: the relationship of d_1 over t_1 is > or < than the relationship of d_2 over t_2.

comparing a faster with a slower motion; we never get an account of the speed of one individual motion. By contrast, time is used as a non-comparative measure.

Why then does Aristotle not turn the relationship of time and space into an explicit measure of motion? And why do we not get an account of the speed of one individual motion? Aristotle uses the distance-covered-over-time-taken of different motions as grounds for holding that this one is faster than that one, but he does not seem to see the distance-covered-over-time-taken in each case as a measure of something called 'speed' considered non-comparatively.

Measuring speed poses at least two problems that we do not find when measuring length, time, or weight: first, speeds do not add up in the way lengths or weights do – if I have two bags and each weighs 2 kg, then together they weigh 4 kg; two lengths of 20 cm put together are 40 cm in length; but two bodies moving at a speed of 20 km/hour do not combine to move at 40 km/hour. Secondly, to measure length A, I can use a smaller length, B, but nothing comparatively simple works for measuring speed – to measure the speed of motion A, I cannot simply use the speed of motion B, a simple speed unit treated as its own magnitude. Zeno's paradox of the moving rows showed the problems such a speed unit gets us into: a speed unit like 1 m/1 sec seems fine as long as we measure a uniform motion, but how might we use it if we want to determine the speed of one body moving 7 metres in 9 seconds and another one moving 10 metres in 13 seconds? Our speed unit will not help us to determine the speed of these two motions or which one is faster.

We may think these cases where two motions move neither for an equal amount of time nor over the same distance could be dealt with by compounding ratios[27] – that is, by using ratios of speeds compounded out of ratios of (linear or angular) distances and ratios of times. While it seems that ancient ratio theory could capture this, we never find such an account in any of the texts handed down to us.[28] And the discussion in the following section will

However, this would again require that Aristotle and his contemporaries dealt with proportions of different magnitudes.

27 What Euclid calls ratios *di' isou* (*ex aequali*) in *Elements* V, def. 17.

28 Nor is it found where we may expect it to appear later, as in *Almagest* XII, where Ptolemy cites Apollonius. Similarly, we may think that while a speed unit cannot be added to itself or multiplied in the way units of length can, nevertheless there may be a way to deal with two motions that share neither the amount of time nor the amount of distance covered, as in the example above, by using a third motion that shares features with both. So if A moves 7 metres in 9 seconds and B moves 10 metres in 13 seconds, we now assume a third thing, C, which moves 7 metres in 13 seconds (so it shares the distance covered with A and the time taken with B). But all we can say of the motion of C is that it is slower than A and B (for it takes more time than A to cover the same distance, and it covers less distance in the same time as B), so it will not help us to decide whether A or B is faster. If we assume a fourth thing, D, which moves 10 metres in 9 seconds (sharing the distance covered with

show that conceptual problems in fact hinder thinking of the ratio of time and distance in a way that would allow us to understand it as a measure of speed and thus ultimately to deal with Zeno's paradox of the moving rows.

This paradox is largely neglected in the secondary literature and yet it is the only paradox that Aristotle cannot really answer, for it requires understanding time and space as two independent magnitudes that nevertheless stand in a certain relation to each other. We saw in the previous chapter that Aristotle can in principle take time as well as distance covered into account when determining speed. However, he only considers cases where the speed of the thing moved and the relevant circumstances[29] stay the same, but we do not find him providing any means for dealing with a situation like that in Zeno's paradox, where a motion past some resting As is simultaneously a motion past some moving Cs.

Let us now look at possible reasons why this paradox is still a problem for Aristotle and why we do not find a complex measure of motion as a relationship of time and space in Aristotle's explicit account of the measure of motion.

9.2 Reasons Why Aristotle did not Explicitly Use a Complex Measure

In the first chapter we discussed the conceptual stages to understanding speed. The first involves two objects starting at the same time and covering the same distance – the faster object is then simply the one that arrives first at the end point. Things get somewhat more complicated if the two objects cover the same distance but do not start at the same time – then we need to measure the time to determine which one is faster. However, in both of these cases measuring speed can be reduced to capturing time. Only when two objects cover different distances in different times do we need to measure not only the time taken in each case but also the distance covered in each; and we also need to relate time and space in such a way that we can measure the speed.

The long passage quoted in the previous chapter (232b20–233a31) seems to show that no problem stands in the way of Aristotle's comparing motions covering different distances in different times. He seems to have everything needed in order to conceptualise a complex measure and to understand motion in terms of time *and* space. But while Aristotle uses what we can understand as all the ingredients for a complex measure when challenging Zeno, he does not introduce such a complex measure into his account of

B and the time taken with A), all we can say about it is that it is faster than both A and B, without being able to decide whether A or B is faster.

29 See chapters 3 and 8.

measuring motion. Also with Aristotle we get only a comparative account of speed and only time as the explicit measure of motion.

From a modern perspective, Aristotle may seem to stumble at the finishing line. As the failure to account for speed and motion in terms of time and space leads to severe problems, which Aristotle has the conceptual tools to avoid, there must be significant reasons why he sticks to time alone in his explicit one-dimensional measure of motion. Prima facie three reasons appear to explain why Aristotle identifies only time as the measure of motion.

First, the second magnitude can appear unimportant after all. When we speak of the motion of the sun, for example, no second dimension is usually mentioned in order to determine speed. However, the second magnitude relevant for this motion is the angle traversed, as we saw in Chapter 6 and just above. In taking one full revolution into account as a unit, for reasons of simplicity we set this second magnitude, 360 degrees, as constant and may forget that it is nevertheless needed (we may get very different results for the question of which body is faster if we compare the time one body requires for covering 360 degrees with the time another requires for covering 180 degrees). And in 232b20–233a31 Aristotle clearly deals with the second magnitude involved in each change. His employment of a one-dimensional measure cannot be explained by the seeming insignificance of the second magnitude that a cosmological context may suggest.

Secondly, Aristotle may fail to claim distance as one aspect of the measure of motion because he is giving an account of any kind of change in his *Physics*, not just of locomotion. The relation between time and distance is relevant to the measure of locomotion, but only time is needed in a measure for all kinds of change. However, a full account of change, no matter which kind, always requires time in relation to a second magnitude. For example, when you and I measure which of us is getting a tan more quickly, we need both time and degree of tan. In order to give an account of change, we need a complex measure, the relation between time and a second magnitude, where that second magnitude could be spatial distance, but it could also be colour or some other quality.

Finally, the relationship between time and space can also be found with what is at rest; thus this relationship may not be specific to motion after all, and hence not adequate for measuring motion. However, Aristotle understands rest as a kind of privation or absence of motion.[30] Only those things that have a *dunamis* for motion can be said to be at rest. The relation between time and space is therefore the right measure for rest and motion. The space covered amounts to zero then, but this does not mean that space can be left out. For it is an important piece of information that this thing that can move in principle is so 'slow' at this very moment as not to cover any space at all.

[30] See the discussion about the moveable as what can be in time in *Physics* V, 6 and Chapter 7.

While the three prima facie reasons why Aristotle identifies only time as the measure of motion are not robust, there are two serious problems which, I think, prevent him from employing the *relation* between time and space explicitly for measuring motion, one *metaphysical* and one stemming from the realm of *mathematics*. The mathematical background also prevents Aristotle's contemporaries from understanding speed as a ratio of time and space, and the notion of a relation would remain problematic for philosophers for a long time to come.

We can see that this notion is problematic for Aristotle from the *Categories*,[31] where he defines the category of relation a second time, after the first definition appears to be problematic, extending into the category of substance (8a28 ff.).[32] And in his *Metaphysics*, Aristotle shows relations to be the least essential of all categories since qua relative they are based on something simpler and more fundamental:

> τὸ δὲ πρός τι πάντων ἥκιστα φύσις τις ἢ οὐσία τῶν κατηγοριῶν ἐστι, καὶ ὑστέρα τοῦ ποιοῦ καὶ ποσοῦ . . . οὐθὲν γάρ ἐστιν οὔτε μέγα οὔτε μικρόν, οὔτε πολὺ οὔτε ὀλίγον, οὔτε ὅλως πρός τι, ὃ οὐχ ἕτερόν τι ὂν πολὺ ἢ ὀλίγον ἢ μέγα ἢ μικρὸν ἢ πρός τί ἐστιν.
>
> What is relative is least of all categories a kind of nature or substance, and is posterior to quality and quantity . . . For there is nothing either great or small, many or few, or in general relative to something which is not [also] something else in virtue of which it then is great or small, many or few, or generally relative to something. (1088a22–8)

For Aristotle, whatever is relative to something is first and foremost something else; it has its own nature and, in addition, it is also relative to something else.[33] Thus a relation is always derived from the relata.[34] The relata must be given in the first place in order for a relation to obtain. However, in the previous chapter, in discussing 232b20–233a31, we saw that when establishing continuity and disputing Zeno, Aristotle deals with the relation between time and space as something primary, in the sense of being the only thing that stays the same: the sections of time and space in question change with every division (the space is divided by the slower motion, the time by the faster motion), while for both motions the specific relation between time and space stays constant and thus provides us with the rule of potential divisions (the relation of one spatial unit per two temporal ones for the slower motion, and the relation of

[31] A ratio is also a relation.

[32] The literature contains different ways of dealing with these two definitions, but in any case at 8a13–28 Aristotle worries that the first definition he gave may include too many things; cf. Duncombe 2015.

[33] Bonitz's translation of the last sentence also makes this very clear: "überhaupt hat nichts eine Relationsbestimmung an sich, ohne dass es etwas anderes wäre".

[34] While in other places Aristotle also understands the other non-substantial categories as dependent on substances, in the passage quoted, *pros ti* is *least of all* a substance.

one spatial unit per one temporal unit for the faster motion in our example in the previous chapter). It might seem that as time is determined, so to speak, by the faster thing and distance by the slower thing, the relation between time and space is dependent on the things that divide them. However, the faster and slower things are only needed to motivate the continuous division, which could also be gained with one single motion (we can also say of a single uniform motion that half of the distance is covered in half the time, and so forth). In the passage quoted, Aristotle therefore uses the relation of time and space as the basic measure of movement. But this treatment of speed as a complex magnitude is not part of Aristotle's official doctrine. Aristotle's metaphysics cannot accommodate a relation as a measure of motion.

In addition, the ratio of time and space would not fit any of the kinds of *pros ti* that Aristotle lists in *Metaphysics* Delta 15 – it is neither a relative according to number (like half and double) nor a relative of doing and having done (like cutting and being cut), nor a relative in the sense of measure in relation to what is measured so that one of them, say time, would be the measure of the other, say space. Furthermore, for the ancients a measure, in the strictest sense, has to be a quantity, as Aristotle states in *Metaphysics* 1053b16–20. A relation is thus by no means a natural candidate for a measure.

The second main reason for Aristotle to stick to his notion of measurement from the *Metaphysics* in his explicit account is mathematical. He takes over important limitations from the mathematics of his time. They are reinforced by his own notion of numbers, which is driven by anti-Platonist concerns. Let us start with the more general mathematical background that influenced Aristotle and that precluded the idea of a uniform measure for the relation of different dimensions, as is needed for determining speed.[35]

The crucial point is the reliance of Greek mathematics on a principle of homogeneity which requires the magnitudes of a mathematical operation to be of the same dimension. That principle seems to have been derived from an emphasis in Greek mathematics on geometrical rather than arithmetical proofs. This approach seems to have been normal practice in Greek mathematics,[36] and may have been reinforced by the discovery of the incommensurable some time in the fifth century BCE.[37]

[35] The following account provides a brief sketch that would need to be filled out in another project.

[36] See, for example, Unguru 1975; and Fowler 1999, p. 10, who claims that Greek mathematics was completely non-arithmetised and that geometry was "the main ingredient of Greek mathematics". Aristotle hints at pebble demonstrations, and thus arithmetic demonstrations, in the Pythagoreans. But geometrical demonstrations seem to have been the norm later on (as we can see, for example, from the way Euclid deals with arithmetic in *Elements* VII–IX).

[37] Plato's *Theaetetus* is in fact the earliest document we possess that mentions this problem, and in the *Laws* it is referred to as something still not widely known among non-experts.

Discovering the incommensurable means discovering two lengths that cannot be understood as the multiple of a common measure. No matter which basic unit we choose, we will not be able to use it as a measure for the side as well as for the diagonal of a square. If we chose a foot as our basic unit and assume that the length of the side of our square is 1 foot, we will not be able to say how many feet the diagonal is, since what we would understand as the square root of 2 cannot be expressed as a whole multiple of the basic unit '1 foot'. A number that is not determinable as a multiple of the basic unit is not rationally determinable in Greek arithmetic. In the realm of arithmetic, the relation between these two lengths thus remains indeterminate to some degree,[38] whereas it is determinable in the realm of geometry (for example, as the diagonal of a square whose side is one basic unit). Geometry thus appeared to have a wider scope of mathematical explanation than arithmetic; it can deal with cases that arithmetic cannot, and there seems to have been a tendency to leave the burden of mathematical proofs to geometry in order to obtain a uniform method. This does not mean that there are no arithmetical proofs. It does mean, however, that geometrical proofs are used in many instances where to us arithmetical ones are far simpler,[39] and that geometrical proofs seem to have been the standard in ancient times.

The mathematical consequences of focusing on geometrical proofs are quite remarkable. Proving numerical hypotheses geometrically requires that the operands of a mathematical operation are of the same dimension.[40] If I try, for instance, to add a solid and a surface, the result will not be defined.[41] In modern parlance, if by a^3 we understand a solid and by b^2 a surface, for the Greeks a statement such as '$a^3 + b^2 = c$' would be meaningless.[42] And if we have

Some scholars have proposed that this discovery led to a first fundamental crisis of mathematics, mostly clearly perhaps Hasse and Schulz 1928; cf. Caveing 1998. For the following argument, whether there was indeed such a crisis is not relevant.

38 Even though in mathematical textbooks such numbers would be expressed either via the approximation of fractions/proportions (where this was possible) or, later on, via an upper and a lower limit (as we find, for example, with Archimedes).

39 While this preference for geometrical proofs is not explicitly stated, it seems to be confirmed by ancient practice. And not only arithmetic but also mechanics and astronomy seem to have been influenced by this move.

40 See, for example, Boyer 1968, p. 85, who describes what he sees as a shift from the arithmetical algebra of the Pythagoreans to the geometrical algebra of classical times: "A 'geometrical algebra' had to take the place of the older 'arithmetical algebra,' and in this new algebra there could be no adding of lines to areas or of areas to volumes. From now on there had to be a strict homogeneity of terms in equations."

41 This holds true for the mathematics of Aristotle's and Euclid's time, while later mathematicians, most notably Hero of Alexandria, move more freely between different dimensions within one operation.

42 One may think that adding a surface and a line ($a^2 + b = c$) may be less of a problem, if they use pebble squares and rectangles for numbers, for then b and c will be areas too. It is harder to see how we would then deal with a^2, however. Be this as it may, the examples given here in the text are clearly problematic for Greek mathematics of Aristotle's time.

a rectangle with sides x and c and we know that with respect to another rectangle ab, a:x = b:c, x cannot simply be derived by dividing ac by b, in the manner to which we are accustomed, for we would then be dividing a surface, ac, by a line, b.[43] For the Greeks it was necessary to find a way of solving such an equation so that both sides could always be understood as describing a surface.[44] The geometrical method of proof thus requires all elements within such a mathematical operation to be homogeneous, to be of the same kind. This can also be seen from Euclid's definition of ratios in book V, definition 3 as a "relation in respect of size between two *homogeneous* magnitudes".[45]

Such a "principle of homogeneity" is characteristic of the mathematics of Aristotle's time.[46] It makes it impossible to use numbers of different dimensions in a single numerical operation, for the geometrical basis of mathematical statements does not allow for operations involving different geometrical magnitudes. The quantity of a line can be compared only to that of another line, and not to that of a surface; ratios can only hold between magnitudes of the same kind.[47]

Furthermore, different dimensions cannot be assigned to one and the same number (as we do when we talk of 30 km/h, for example), and one dimension cannot be understood as the relation between two simpler dimensions. So the principle of homogeneity derived from the realm of Greek mathematics prevents the combination of different dimensions in a single uniform measure not only in mathematics but also in a physics that works with mathematical notions and tools in the way Aristotle's physics does. A complex measure of the kind needed for determining speed cannot be developed on the basis of a

43 Greek mathematics of the time considered here neither worked with variables nor did division work in our modern way; these examples are intended simply to make the problem as clear as possible.

44 Such a solution can be found, for example, in Euclid I, prop. 44, which shows how a parallelogram equal to a given triangle can be applied to a given straight line (in a given rectilinear angle).

45 Mueller 1981, p. 136 points out Euclid's concern for the homogeneity conditions of propositions in book V (even though on p. 133 Mueller claims that Euclid is in fact concerned with minimising homogeneity assumptions and thus takes V, 23 as an "inexplicable exception" – though for Euclid to attempt to minimise homogeneity assumptions, they must first have been dominant).

46 See Alten et al. 2003, p. 64.

47 See the scholion to Euclid V, 15, which makes clear that magnitudes which are not homogeneous cannot have a ratio to each other: not a line to a surface nor a surface to a body. But a line has a ratio to a line and a surface to a surface. See also Waschkies 1977, p. 264 and Boyer 1968, p. 99. What is possible, however, is multiplying a magnitude by a number (as it only means repeated addition of the same element) and comparing the ratio between two homogeneous things to the ratio of two other things, as we saw above and can also see in passages like *Elements* VI, 1. But in these cases, too, we do not face an operation between inhomogeneous magnitudes. Rather, we are shown that, say, between two distances the numerical relation of 1:2 may hold in the same way as between two triangles.

mathematics so understood. And Aristotle was too grounded in the mathematics of his time to overcome this restriction.[48]

The mathematical principle of homogeneity seems also to have reinforced Aristotle's own notion of measurement and numbers.[49] Judging from his account in the *Metaphysics*, Aristotle developed his conception of numbers against Scylla and Charybdis, so to say: countering the Scylla of the Pythagoreans, who thought numbers were the nature of sensible things,[50] and the Charybdis of Plato and the Academy, for whom numbers per se were independent of and separate from the sensible world.[51] Against the latter, Aristotle understands numbers as being essentially connected to a certain sensible dimension: each number is the number of something specific, of a sensible item. Thus, the number two is not two as such, but two cups, two inches, etc.[52] It is the multiple of a certain unit, which is one either in perception (for example, an inch) or conceptually (for example, a human being).

[48] Aristotle's mastery of mathematics has seen opposing interpretations – some claim he had only a superficial knowledge of mathematics, others that he had a deep understanding of the subject. But neither interpretation provides grounds to assume that he would conceive of a mathematics not influenced by this principle (which is not to say that there are not also differences between the conception of mathematics in Aristotle and in his contemporary mathematicians). This is supported by Aristotle's reference to common mathematical problems (like incommensurability in *Prior Analytics* 41a26), mention of particular mathematicians (for example, in *Metaphysics* 1073b and *Sophistical Refutations* 171b), and use of mathematics as a paradigm of scientific knowledge.

[49] In *Posterior Analytics* I, 5 Aristotle suggests, however, that we can prove proportions universally for numbers, lines, times, and solids.

[50] Huffman argues against understanding Philolaos as assuming that numbers constitute the essence of things. The Pythagoreans I am taking up here, however, are those whom Aristotle portrays in his *Metaphysics*, books M and N, whether or not this account is indeed historically accurate.

[51] See, for example, Aristotle's *Metaphysics* A, 987b27 ff. and N, 1090a30 ff. (for the Pythagoreans, see also Rowett 2013, pp. 19–20). Speusippus' and Xenocrates' positions cannot be dealt with here separately. Cleary 1995, p. 366 also thinks that Aristotle wants to position himself between two groups with his theory of numbers, but he proposes that these two groups are Plato and the Pythagoreans, on the one hand, and the natural philosophers on the other. I think that Cleary underestimates the important differences between the number theories of Plato and the Pythagoreans, which Aristotle himself points out in his *Metaphysics*; and Cleary himself makes clear that Aristotle ignores the natural philosophers when discussing number theories. We may read the *aporia* in *Metaphysics* B, 5 as pointing to certain natural philosophers as a third problematic group, namely those thinkers who reduce mathematics to physics (cf. also Menn, unpublished manuscript, ch. Ib3). However, this third group is less relevant to the current debate, and I will concentrate on the other two.

[52] Aristotle's understanding of number may seem less strange if we take into account that in the acrophonic number system, which was the dominant system at his time, different signs were used for writing a number of drachmas, talents, or *khoes*, and so for money, weight, and volume; see Mendell 2009, p. 137.

An immediate consequence of this conception of numbers is that numbers are not strictly separate from sensible things. In Chapter 6 we saw how Plato deals with the question of how to combine the realm of numbers with the sensible world by understanding the world to be set up according to numerical ratios. Furthermore, Plato assumed different 'mediators' between numbers and the sensible world, such as time. By contrast, given his understanding of numbers, Aristotle has no problem combining sensible things and numbers when measuring, since he assumes there is no ontological gap between them. While today we assign the empirical measurand to an abstract series of numbers, for Aristotle number is nothing but the quantification of a plurality of physical objects; there is therefore no need to assign numbers to the things to be quantified in a separate step.

This tight bond between numbers and the sensible world helps Aristotle shift away from the Charybdis of Plato, but brings him close to the Scylla of the Pythagoreans. For the Pythagoreans, as understood by Aristotle in *Metaphysics* M and N, number is the *essence* of the sensible world. Aristotle avoids this Pythagorean consequence by binding numbers not to the form and *ousia* of something – to what makes a thing the thing it is – but to matter, that is, to intelligible or noetic matter.[53] Being the multiple of a material basic unit, numbers are assumed to be dependent on matter. Thus Aristotle manages to steer between both the Charybdis of Plato and the Scylla of the Pythagoreans. But the price he pays is the reinforcement of the problems introduced by the principle of homogeneity. Aristotle's understanding of numbers means that even if the principle of homogeneity had not held universally in his time – in fact, it was held universally until the time of Descartes and Fermat and only overturned by Leibniz and Newton – we still could not bring temporal and spatial numbers together in a single uniform measure.

Aristotle's notion of numbers implies that each is understood as an integer; more precisely, as an integer multiple of the basic unit. Fractions and other divisions are not possible: if your basic unit is a cup, then eight ninths does not seem to be a proper number.[54] The ancient Greeks may have dealt with this problem by using an adequate subunit as the new unit,[55] but an adequate subunit may not always have been available. And if fractions are not possible, then the comparison of different lengths, say, with the help of a single basic

[53] See *Metaphysics* Z, 1036a9–12, which divides matter into perceptible (*aisthêtê*) and intelligible (*noêtê*) matter, claiming that the latter belongs to the object of mathematics; see also *Metaphysics* Z, 1036b32–1037a5. See Mueller 1970 on intelligible matter in the case of geometrical objects and Mendell 2004 on the question of whether Aristotle has different notions of intelligible matter. *Metaphysics* N, 1092b17–18, while dealing with a rather specific contrast, suggests that number is bound not to *ousia* but to *hyle*.

[54] See Wieland 1962, p. 319. An understanding of numbers as ordered pluralities was widespread; see Huffman, p. 173 ff.

[55] See Mendell 2009.

scale, may also be impossible, as this method may lead to fractions. Rather we will need different scales, different 'ones' to measure such different lengths.

The problem gets worse once different dimensions come into play, as happens in the case of motion. In order to be able to take different dimensions into account in a single measurement procedure, we need numbers that are independent of both dimensions, so that both dimensions can be assigned to the numbers. But in Aristotle numbers are so tightly bound to a dimension that no common basis can be found to combine numbers of different dimensions in the way necessary for a complex measure. Aristotle's conception of numbers does not allow for the conception of a dimensionless number series, as this would lead to a separation of sensible reality and numbers.[56] Aristotle can allow for dimensionless numbers as an abstraction for mathematicians, but not for natural philosophers.

Finally, dimensions themselves do not seem to be sufficiently abstract in the Aristotelian framework to allow for applying mathematical operations to them. Yet such a procedure, too, would be necessary for a complex measure. What we regard as a complex dimension (s/t), that is, a dimension which is a relation between two or more simpler dimensions, the ancient Greeks would instead have understood as a simple dimension of its own. For example, Aristotle shows in his *Metaphysics* that a solid cannot be traced back to a relation between simpler dimensions (in the way we would trace it back to the relation of length, breadth, and height): in 992a10 ff. and 1085a7 ff., Aristotle distinguishes solids, surfaces, and lines as three different genera, and 1092b30 ff. seems to exclude the possibility that different genera can be measured by the same measure. Thus, in the *Metaphysics* the measure of the volume of a cube cannot be thought of as the relation between the length of its sides in the way that we do when we calculate length times breadth times height, $a \text{ x } a \text{ x } a = a^3$. Rather, a cube is measured by a smaller cube and thus by something Aristotle regards as a simple dimension.[57]

[56] Whether or not we are (in the modern sense) Platonists with respect to numbers, we usually assume that we can work with dimensionless number series for purposes of measurement. For a contrast, see Aristotle's account of the difference between counting ten horses and counting ten dogs at the end of *Physics* IV, 224a3 ff.

[57] The passages quoted suggest that Aristotle would reject calculating the volume of a cube as length times breadth times height as theoretically inadequate, even though he may allow it for practical purposes to keep calculations simple. See also 1016b17–31 and his criticism in *De caelo* 298b33 ff. of the dimensional confusion he thinks Plato gets into with his assumption of basic triangles forming the elemental bodies. Unfortunately, Aristotle is not very explicit, and he seems to allow different things for the purposes of calculation and construction (see, for example, *De anima* 413a11–20 and *Metaphysics* 996b18–22 for squaring a rectangle, that is, finding an equilateral rectangle equal to an oblong rectangle with the help of a mean proportional between two straight lines). But for Aristotle there does not seem to be an operation like length times length, and he does not work with complex measurement units. By

Although today we also work with numbers bound to a certain empirical dimension in our physics, we are nevertheless able – independent of the specific metaphysics of number we in fact hold – to assign numbers of different dimensions to a single series of numbers and hence to unite them. We are also able to understand a combination of different dimensions (for example, km/h) as forming a new but internally complex dimension. Aristotle, however, has bound numbers to sensible things to such a high degree that he cannot combine numbers of different dimensions. Consequently, he cannot develop a truly complex measure conceptually.

When dealing with Zeno's paradoxes, however, we saw Aristotle using what we can understand as a complex measure, the relation of time and space – a measure that truly meets the criterion of being homogeneous with the measurand, motion and speed. Aristotle can use such a measure in this case because to do so does not require an explicit account of the relation of time and space as the measure of motion and because the sections of time and space are not assigned to numbers.[58]

The powerful tool that enables Aristotle to connect time and space is his notion of a continuum. That notion would in principle also allow for a theoretical foundation of a relation of time and space. But because of Aristotle's conceptions of relations and numbers and the mathematical treatment of magnitudes in his time, the notion of a complex or relational measure cannot enter his conception of a measure explicitly. A measure that is itself a relation would mean giving relations their own role that cannot be simply derived from their relata and thus separating them, and indeed also numbers, from the perceptible relata in some sense – a position that for Aristotle would have been too close to the whirlpool of Platonic doctrine. While Aristotle's analyses in the *Physics* show that he can work with a complex measure of motion, his position in the *Metaphysics* keeps him from explicitly conceptualising this complex relation. Science will fully capture this complex relation only some two thousand years later, after important changes in the understanding of metaphysics and mathematics, which include, for example, changes in what is understood by mathematical operations and numbers in early modern times.[59]

contrast, in the *Theaetetus* 147e ff. Plato seems to suggest that an area is determined as a product, one unit times one unit. And in *Laws* 820a ff., in the context of introducing the idea of the incommensurable, the Athenian stranger claims that it is usually assumed that we can measure a length and a breadth against each other.

58 In his comparison of different speeds, time and space are used like variables; they can take on whatever numerical value we like.

59 See Pritchard 1995, ch. 4.

9.3 Constructive Developments: A Résumé

While we do not find an explicit account of speed in terms of a ratio of time and space in the period looked at in this book, there are great achievements in preparing the ground for such an account and for an understanding of time and space on which later philosophers and scientists could build. We saw how Parmenides introduced crucial criteria for philosophical/scientific investigations that have stayed with us ever since. Given Parmenides' logical operators, these criteria were originally understood in a far more restrictive sense than later on. These criteria were reinforced by Zeno and were used to show from within the problems that such standards seem to raise for any attempt to do natural philosophy – not only for any account of speed, but also for an account simply of time and spatial distance. The atomists Leucippus and Democritus were among the first to implicitly expand Parmenides' logical understanding by putting non-Being on a par with Being, and their understanding of non-Being qua void established a predecessor of a notion of an infinite, homogeneous, and isotropic space .

In Plato we found further explicit advances of the criteria for philosophy – especially of the principle of non-contradiction and the principle of sufficient reason – as well as important developments of the logical operators. We saw that the *Timaeus* also provides a rich and elaborate account of time that is connected to an explicit account of measuring motion. It is not, however, related to Plato's account of space, such as is found in his natural philosophy, and we saw the consequent problems for an account of sublunar speed.

This is exactly where Aristotle's *Physics* comes in: by demonstrating that time and space are continua and thus share the same basic internal structure, Aristotle prepares the ground for connecting time and space. Letting them share their basic internal structure is the foundation on which later philosophers could build in order to measure motion in terms of time and space – but an explicit conceptualisation of this last step had to wait until long after Aristotle.

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